Nearly Perfect Fluidity: From Cold Atoms to Hot Quarks

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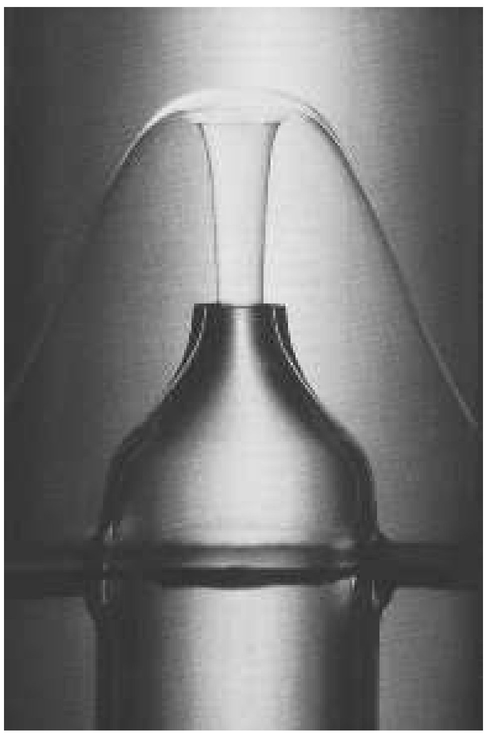


RHIC serves the perfect fluid

Heavy Ion Collisions are very complicated (time-dependent, strongly correlated quantum many body physics), but at RHIC & LHC a very simple theory appears to work.

$\pi\alpha\nu\tau\alpha\ \rho\varepsilon\iota$ (*everything flows*)

Heraclitus of Ephesus, 535 - 475 BC

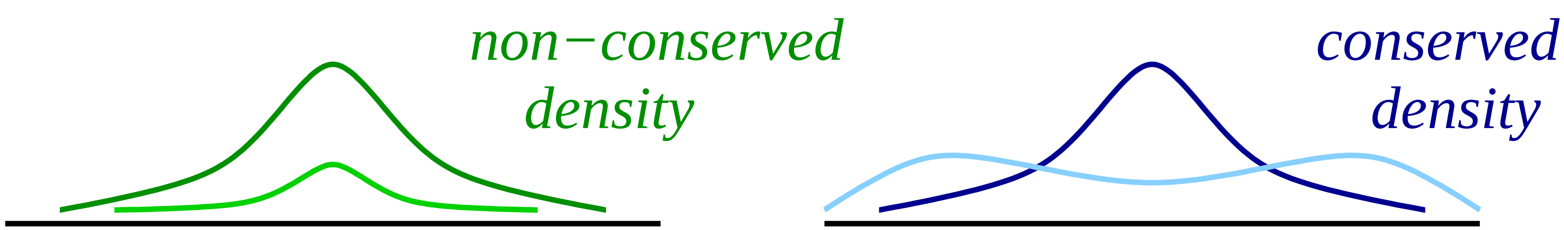
In this talk I will try to address the questions "Why?" and "How general is this phenomenon?".

Hydrodynamics

Hydrodynamics (undergraduate version): Newton's law for continuous, deformable media.

Fluids: Gases, liquids, plasmas, . . .

Hydrodynamics (postmodern): Effective theory of non-equilibrium long-wavelength, low-frequency dynamics of any many-body system.



$\tau \sim \tau_{micro}$ $\tau \sim \lambda$

$\tau \gg \tau_{micro}$: Dynamics of conserved charges.

Water: $(\rho, \epsilon, \vec{\pi})$

Simple non-relativistic fluid

Simple fluid: Conservation laws for mass, energy, momentum

$$\frac{\partial \rho}{\partial t} = -\vec{\nabla}(\rho \vec{v}) \qquad \frac{\partial \epsilon}{\partial t} = -\vec{\nabla}\vec{j}^{\epsilon}$$

$$\frac{\partial}{\partial t}(\rho v_i) = -\nabla_j \Pi_{ij}$$

mass $\times$ acceleration $=$ force

Constitutive relations: Stress tensor

$$\Pi_{ij} = P\delta_{ij} + \rho v_i v_j + \eta \left(\nabla_i v_j + \nabla_j v_i - \frac{2}{3}\delta_{ij}\nabla_k v_k \right) + O(\nabla^2)$$

reactive dissipative 2nd order

Expansion $\Pi^0_{ij} \gg \delta\Pi^1_{ij} \gg \delta\Pi^2_{ij}$

Regime of applicability

Expansion parameter $Re^{-1} = \dfrac{\eta(\partial v)}{\rho v^2} = \dfrac{\eta}{\rho L v} \ll 1$

$$\frac{1}{Re} = \underbrace{\frac{\eta}{\hbar n}}_{\text{fluid property}} \times \underbrace{\frac{\hbar}{m v L}}_{\text{flow property}}$$

-1

$$\text{Bath tub}: \quad mvL \gg \hbar \quad \text{hydro reliable}$$

$$\text{Heavy ions}: \quad mvL \sim \hbar \quad \text{need} \;\; \eta < \hbar n$$

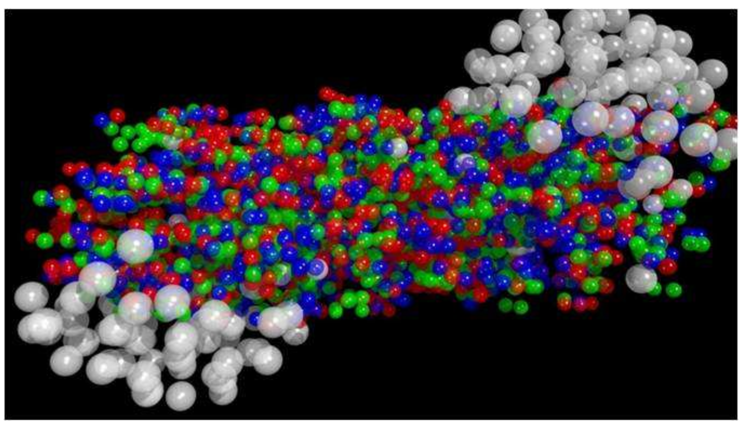
Note: Bacteria swim in the regime $Re^{-1} \gg 1$ but $Ma^2 \cdot Re^{-1} \ll 1$.

Breakdown of fluid dynamics

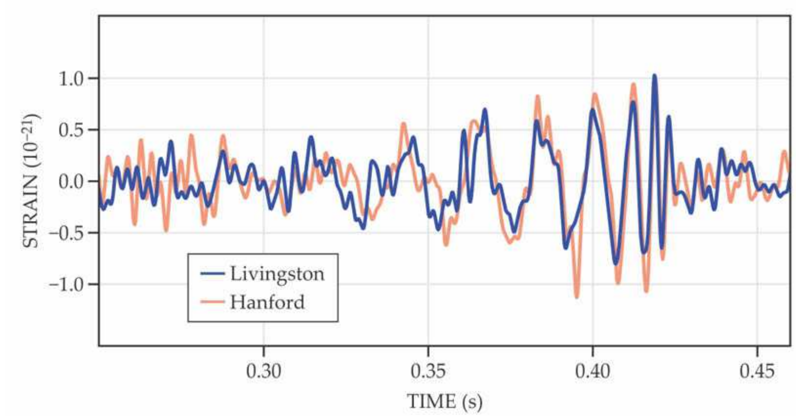
Fluid dynamics is a universal theory but the breakdown of hydro, the emergence of non-hydrodynamic modes, is not.

Two extreme cases: Non-interacting particles or strongly collective, but non-hydrodynamic $(\omega_m(q \to 0) \neq 0)$ modes.

Ballistic motion



Quasi-normal modes



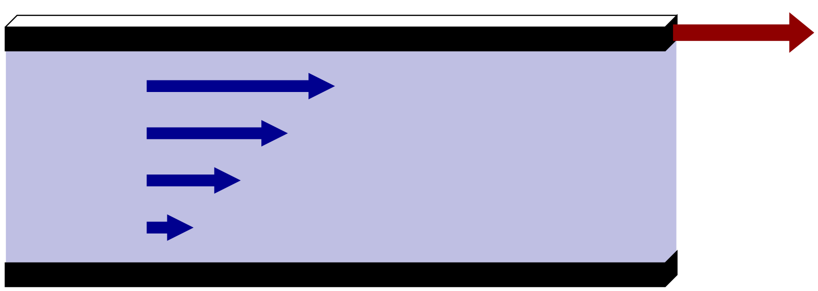
Shear viscosity and friction

Momentum conservation at $O(\nabla v)$

$$\rho \left(\frac{\partial}{\partial t}\vec{v} + (\vec{v}\cdot\vec{\nabla})\vec{v} \right) = -\vec{\nabla}P + \eta\nabla^2\vec{v}$$

Navier-Stokes equation

Viscosity determines shear stress ("friction") in fluid flow



$$F = A\,\eta\,\frac{\partial v_x}{\partial y}$$

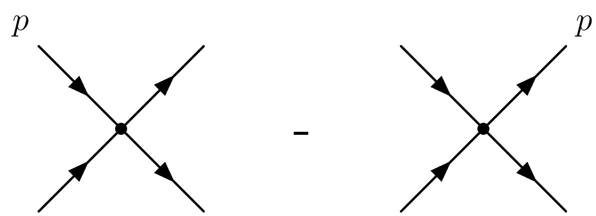
Kinetic theory

Kinetic theory: conserved quantities carried by quasi-particles.

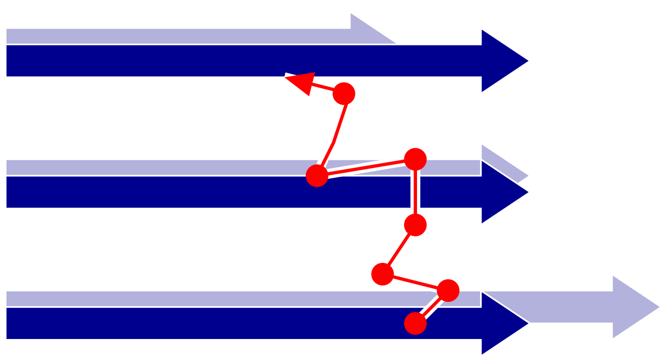
Quasi-particles described by distribution functions $f(x,p,t)$.

$$\frac{\partial f_p}{\partial t} + \vec{v}\cdot\vec{\nabla}_x f_p + \vec{F}\cdot\vec{\nabla}_p f_p = -C[f_p]$$

$$C[f_p] \;=$$



Shear viscosity corresponds to momentum diffusion



$$\eta \sim \frac{1}{3} n\,\bar{p}\,l_{mfp}$$

Shear viscosity: Low density limit

Weakly interacting gas, $l_{mfp} \sim \dfrac{1}{n\sigma}$

$$\eta \sim \frac{1}{3}\frac{\bar{p}}{\sigma}$$

shear viscosity independent of density

Maxwell (1860): "Such a consequence of the mathematical theory is very startling and the only experiment I have met with on the subject does not seem to confirm it."

Shear viscosity: Additional properties

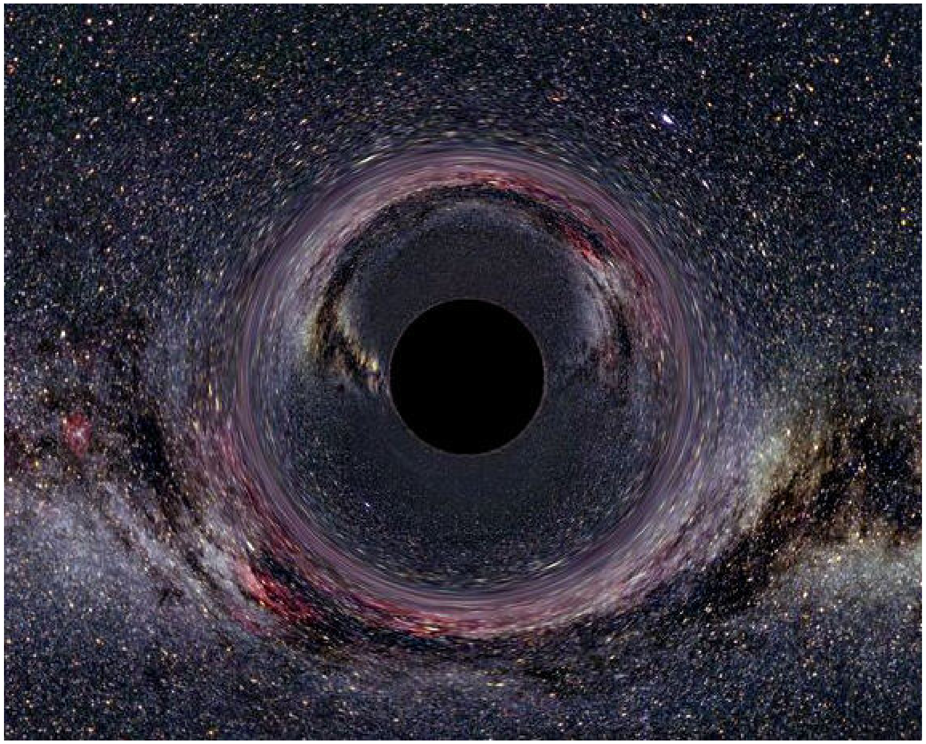
Non-interacting gas ($\sigma \to 0$): $\eta \to \infty$

non-interacting and hydro limit ($T \to \infty$) limit do not commute

Strongly interacting gas: $\frac{\eta}{n} \sim \bar{p} l_{mfp} \geq \hbar$

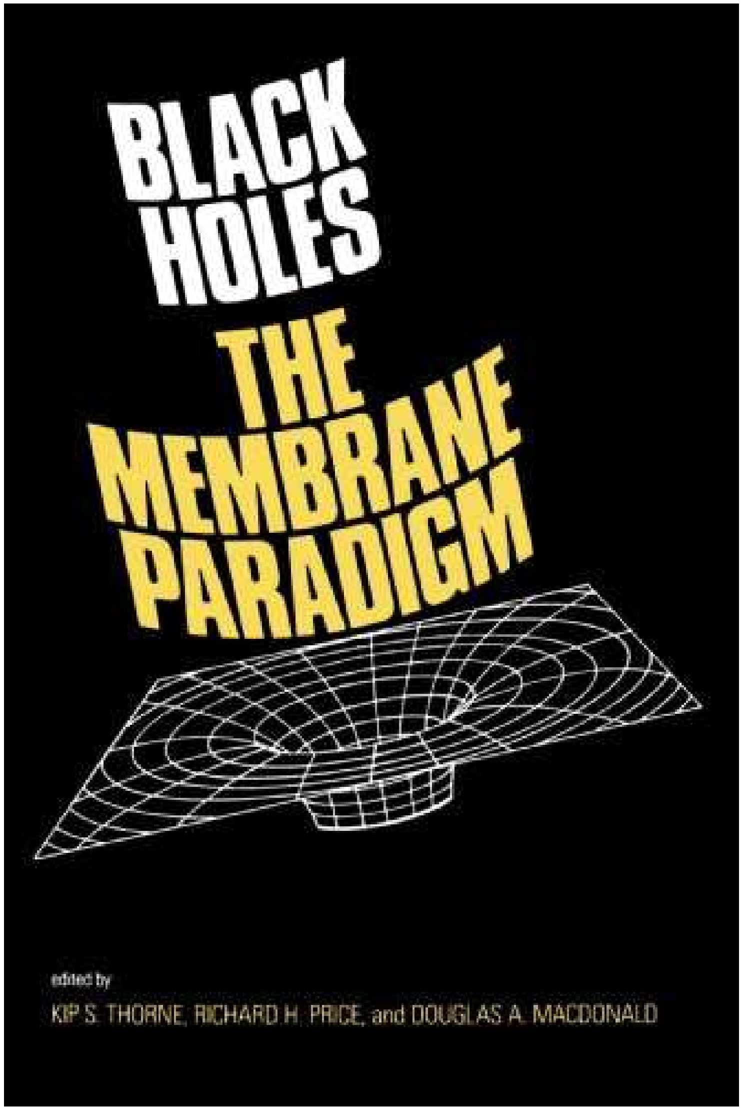
Quantum bound. But: Kinetic theory mat not be reliable!

And now for something completely different . . .



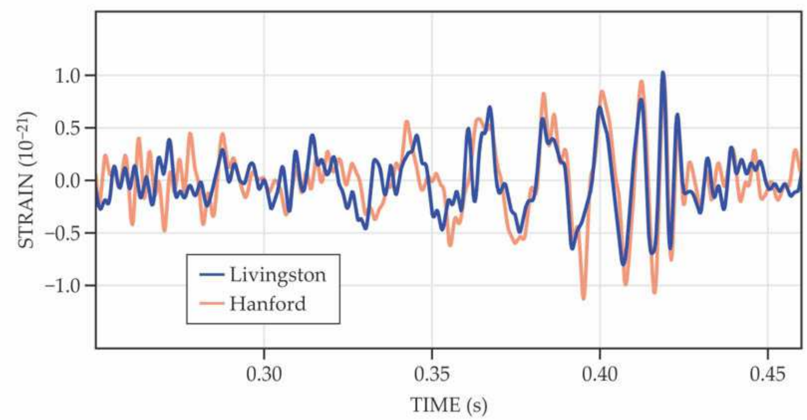
This is an irreversible process, $\Delta S > 0$.

And now for something completely different . . .



Ringdown can be described in terms of stretched horizon that behaves as a sheared fluid

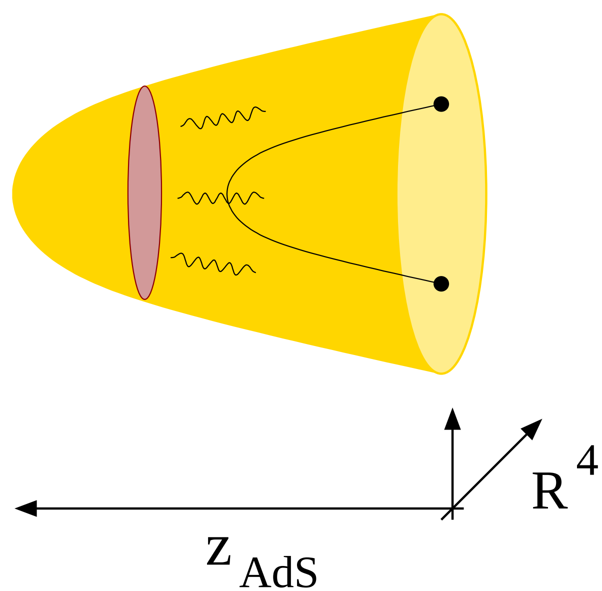
$$\eta = \frac{s}{4\pi}$$



Note: Unusual thermodynamics, e.g. $\zeta, C < 0$.

Idea can be made precise using the "AdS/CFT correspondence"

Strongly coupled thermal field theory on R^4	$\Leftrightarrow$	Weakly coupled string theory on AdS_5 black hole
CFT temperature	$\Leftrightarrow$	Hawking temperature of black hole
CFT entropy	$\Leftrightarrow$	Hawking-Bekenstein entropy $\sim$ area of event horizon



Holographic duals: Transport properties

Thermal (conformal) field theory $\equiv AdS_5$ black hole

CFT entropy $\Leftrightarrow$ Hawking-Bekenstein entropy

$\sim$ area of event horizon

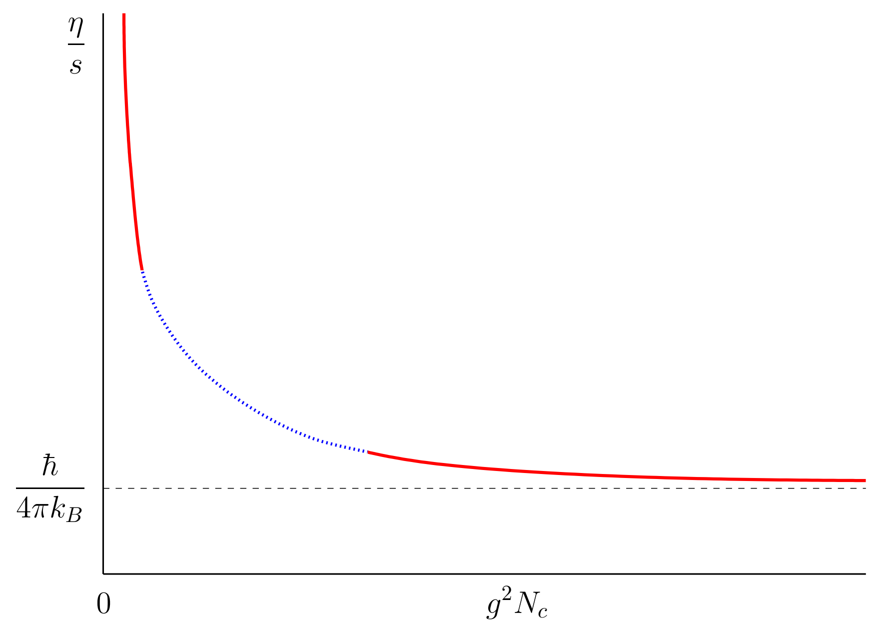
shear viscosity $\Leftrightarrow$ Graviton absorption cross section

$\sim$ area of event horizon

Strong coupling limit

$$\frac{\eta}{s} = \frac{\hbar}{4\pi k_B}$$

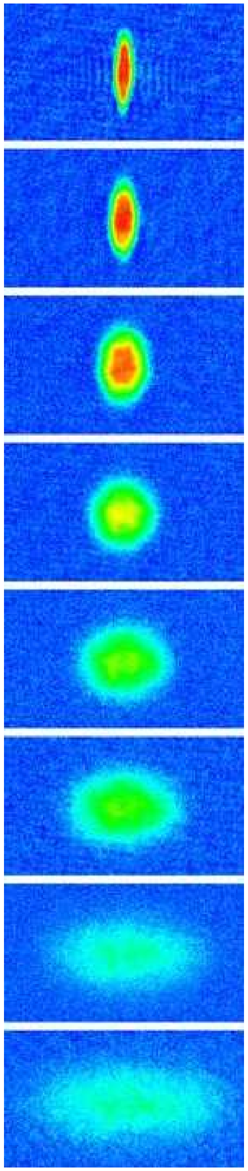
Son and Starinets (2001)

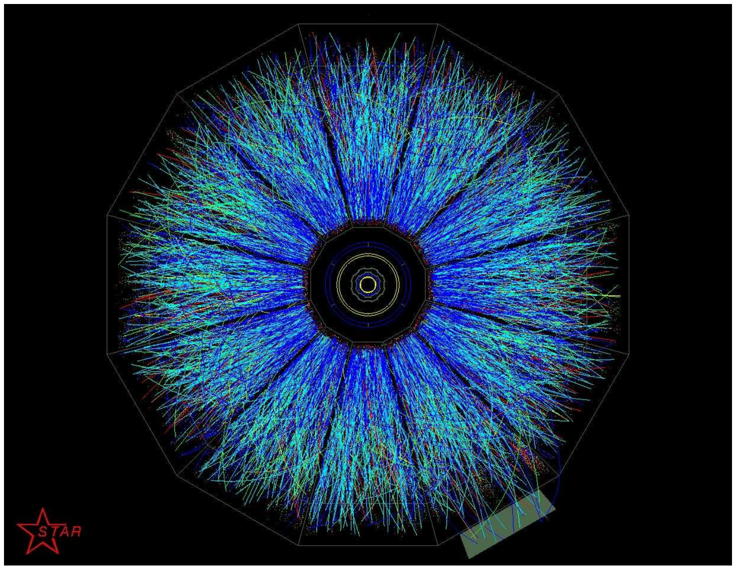


Strong coupling limit universal? Provides lower bound for all theories?

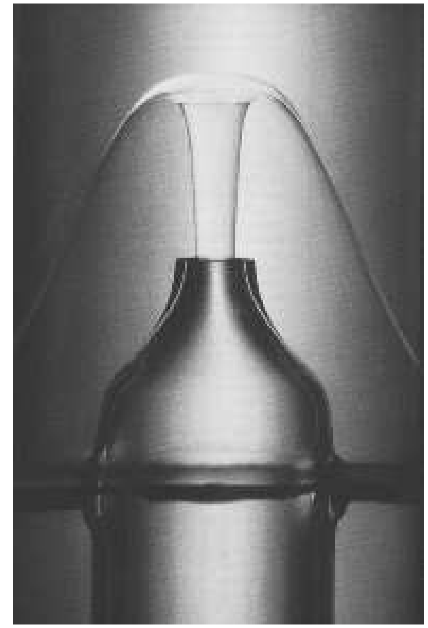
Answer appears to be no; e.g. theories with higher derivative gravity duals.

Perfect Fluids: The contenders





QGP (T=180 MeV)



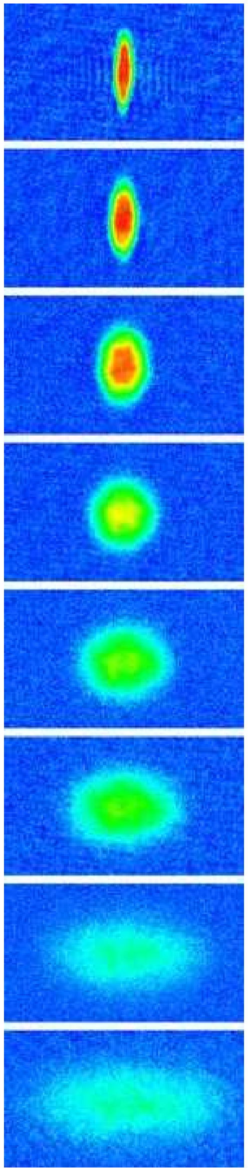
Liquid Helium

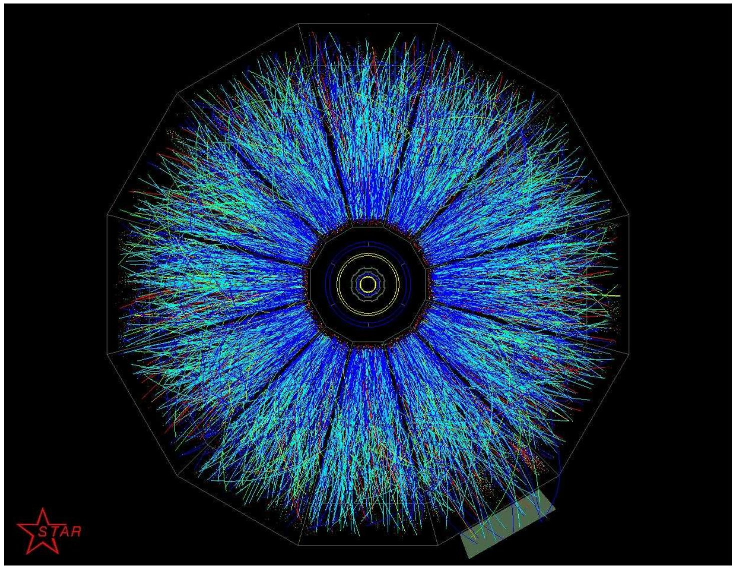
(T=0.1 meV)

Trapped Atoms

(T=0.1 neV)

Perfect Fluids: The contenders

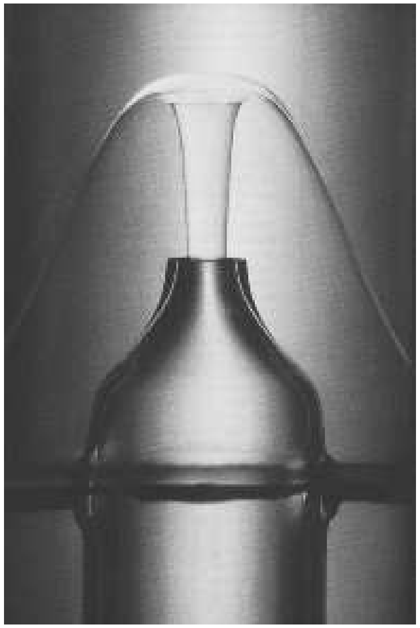




QGP $\eta = 5 \cdot 10^{11} Pa \cdot s$

Trapped Atoms

$\eta = 1.7 \cdot 10^{-15} Pa \cdot s$



Liquid Helium

$\eta = 1.7 \cdot 10^{-6} Pa \cdot s$

Consider ratios

η / s

Perfect Fluids: Not a contender

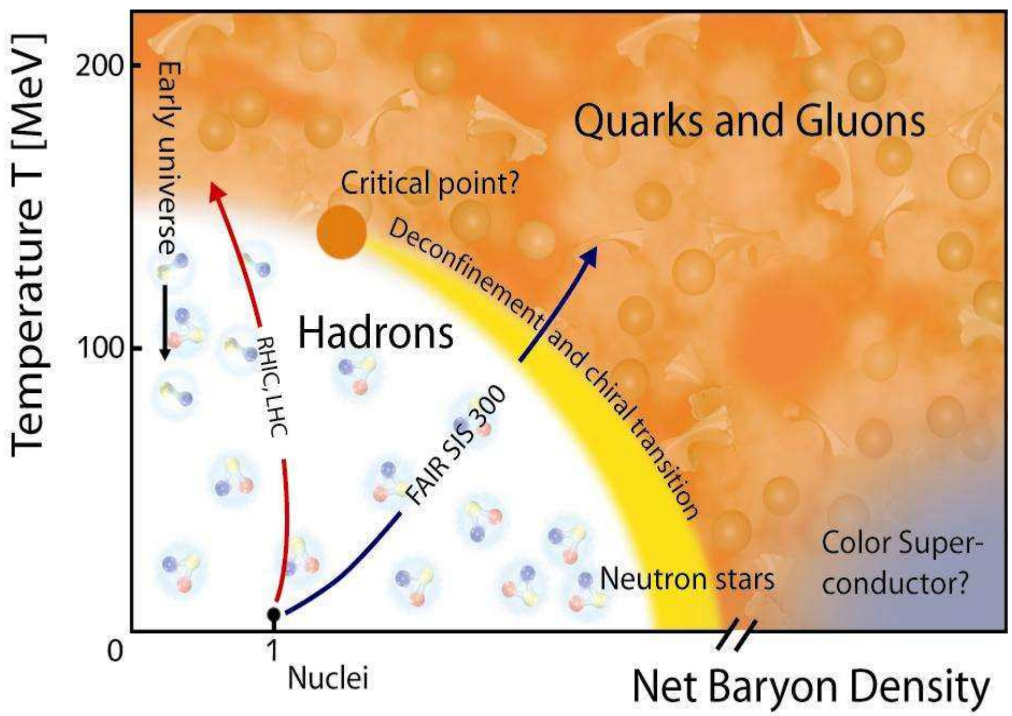
Queensland pitch-drop experiment

1927-2011 (8 drops)

$$\eta = (2.3 \pm 0.5) \cdot 10^8 \, Pa \, s$$

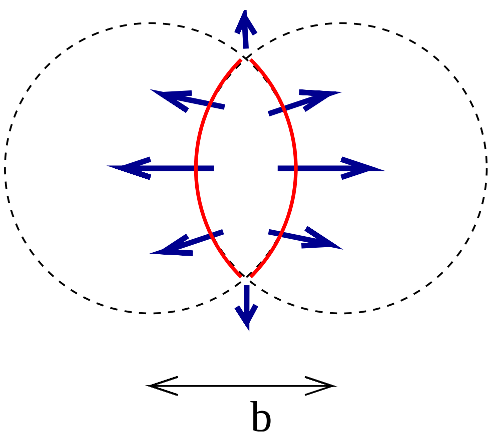
I. QCD and the Quark Gluon Plasma

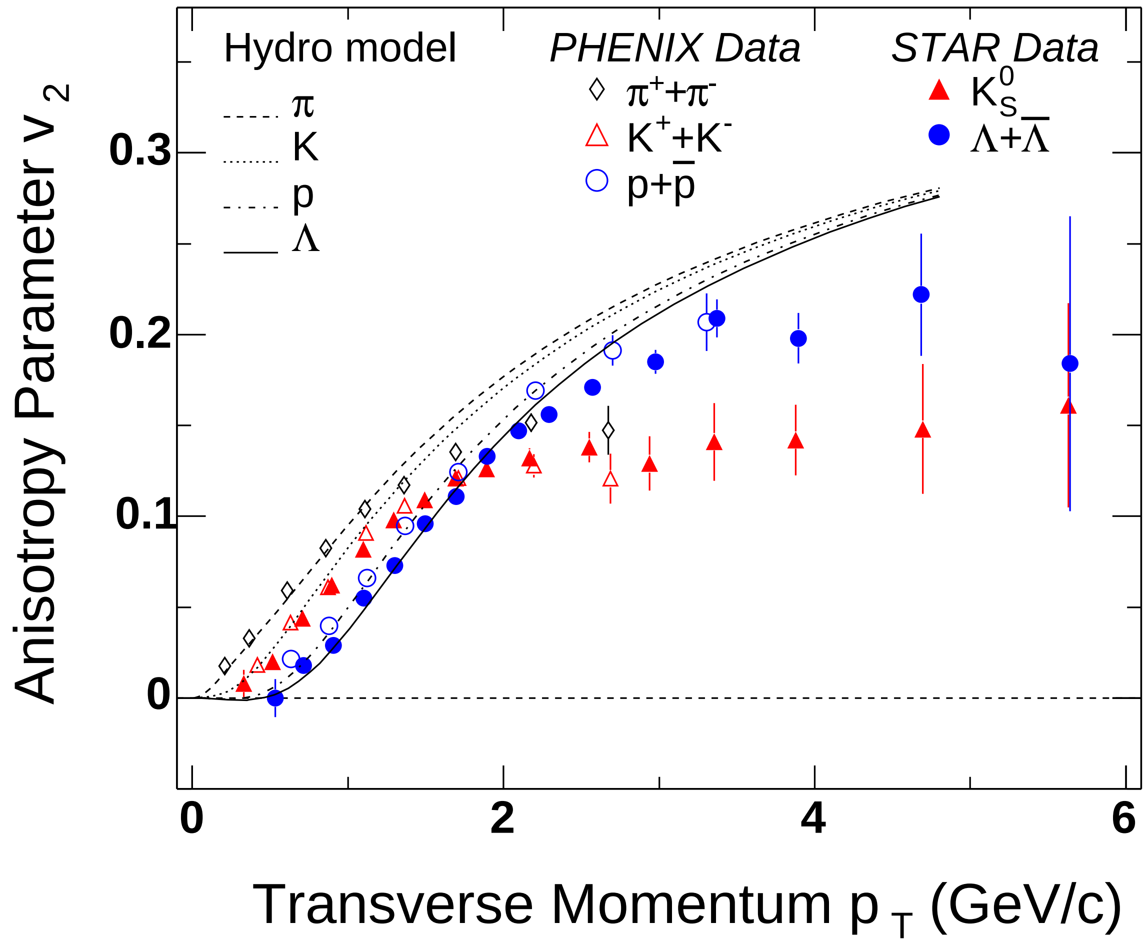
$$\mathcal{L} = \bar{q}_f (i\not{D} - m_f) q_f - \frac{1}{4g^2} G^a_{\mu\nu} G^a_{\mu\nu}$$



Elliptic Flow (QGP)

Hydrodynamic expansion converts coordinate space anisotropy to momentum space anisotropy





source: U. Heinz (2005)

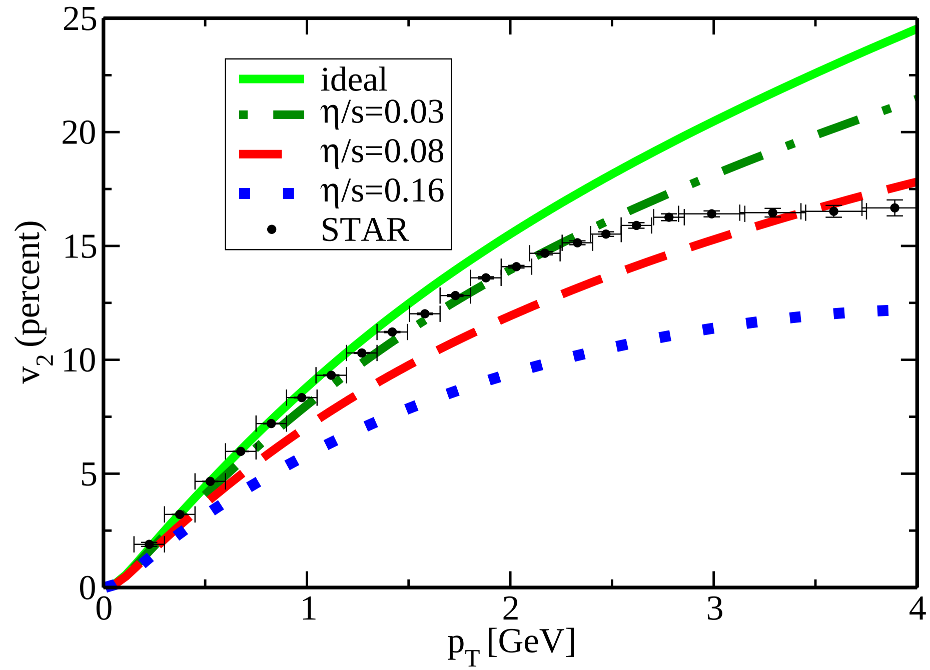
$$p_0 \, \frac{dN}{d^3p}\bigg|_{p_z=0} = v_0(p_\perp)\,(1 + 2v_2(p_\perp)\cos(2\phi) + \ldots)$$

Viscosity and Elliptic Flow

Viscous correction to v_2 (blast wave model)

$$\frac{\delta v_2}{v_2} = -\frac{1}{3}\frac{1}{\tau_f T_f}\left(\frac{\eta}{s}\right)\left(\frac{p_\perp}{T_f}\right)^2$$

Grows with $p_\perp$, decreases with system size



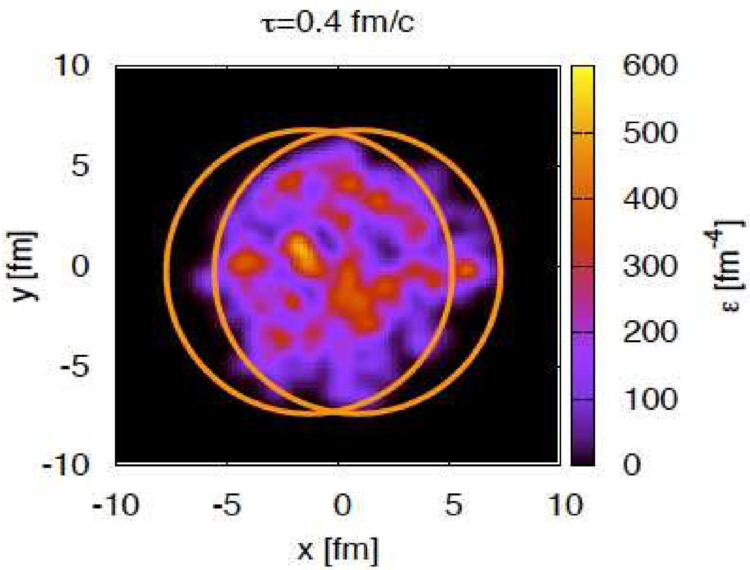
Romatschke (2007), Teaney (2003)

Many details: Dependence on initial conditions, freeze out, etc.

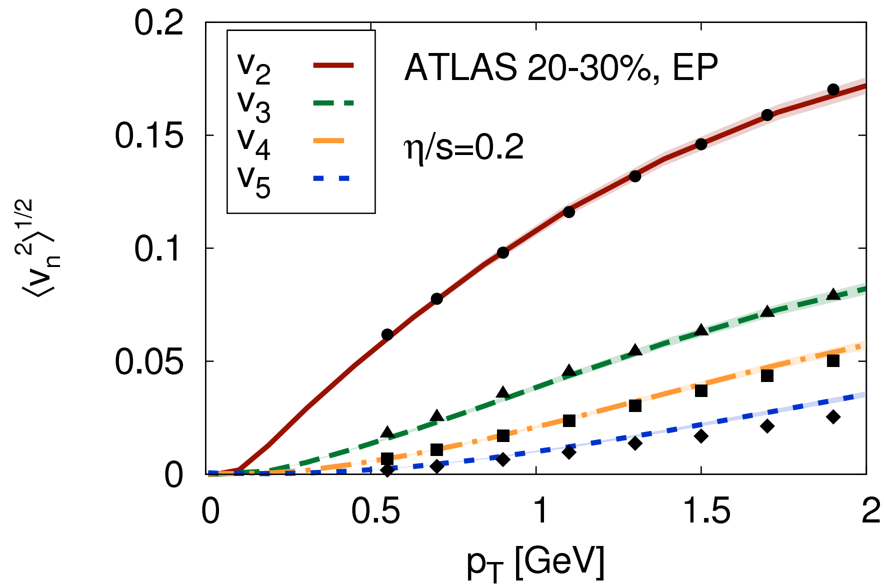
conservative bound $\quad \frac{\eta}{s} < 0.25$

Higher moments of flow

Hydro converts moments of initial deformation to moments of flow



B. Schenke



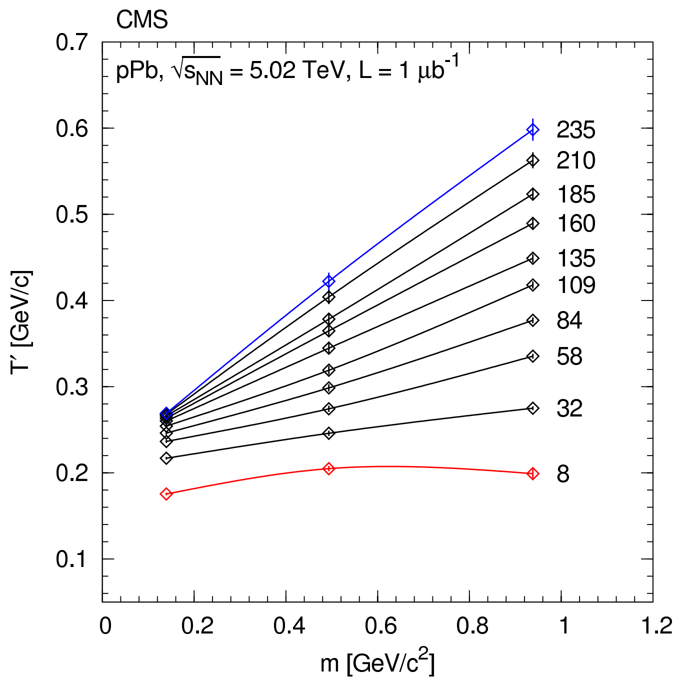
C. Gale et al.

Glauber predicts flat initial spectrum ($n \geq 3$). Observed flow spectrum consistent with sound attenuation

$$\delta T^{\mu\nu}(t) = \exp\left(-\frac{2}{3}\frac{\eta}{s}\frac{k^2 t}{T}\right)\delta T^{\mu\nu}(0)$$

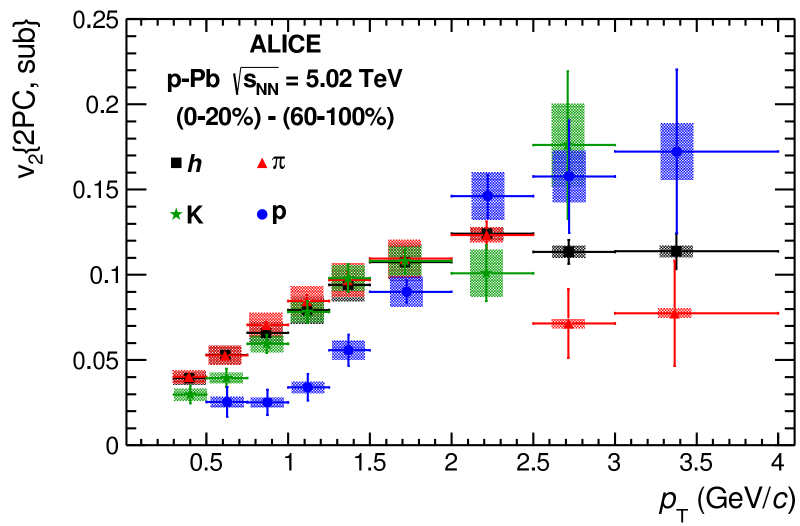
Everything flows (including p+Pb, and maybe even p+p)

Signatures of collective expansion (radial and elliptic flow) in high multiplicity p+Pb collisions.



Mass ordering of mean p_T

CMS (2013)



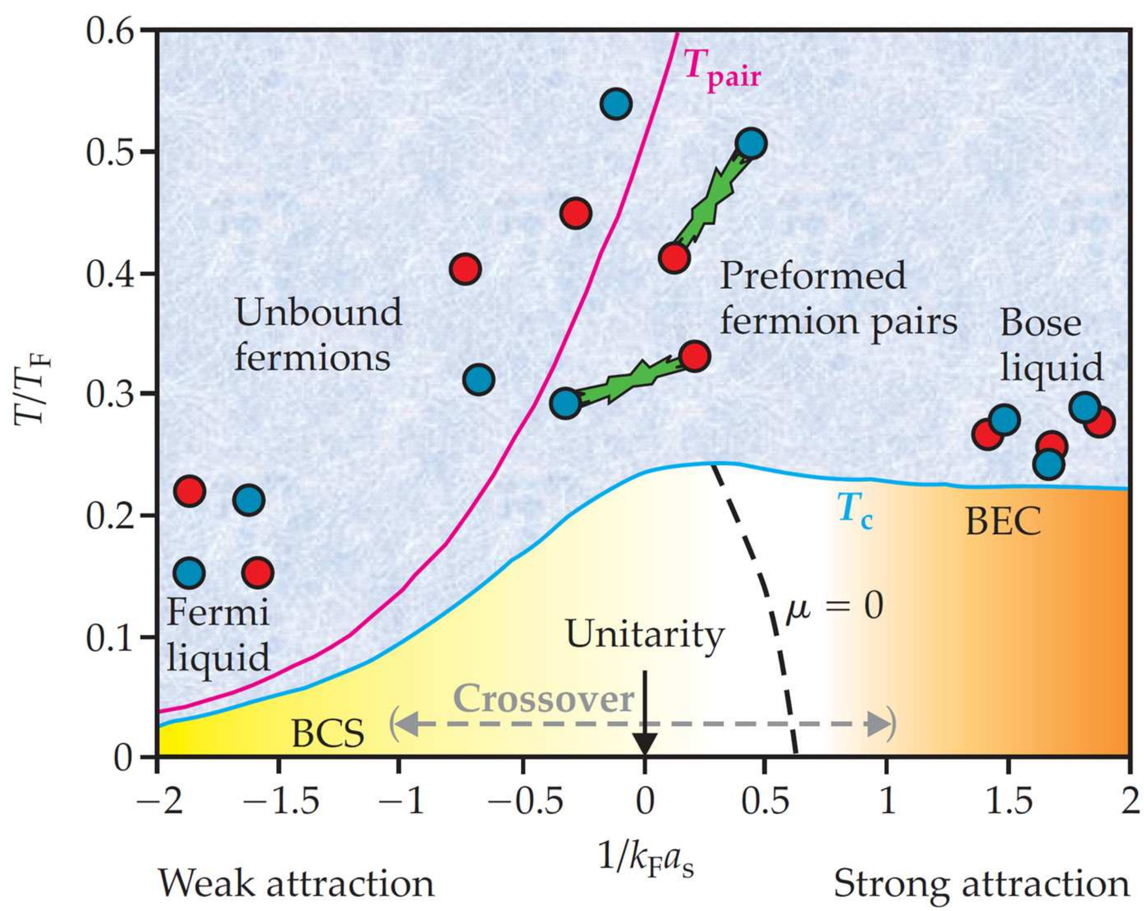
Mass ordering of $v_2(p_T)$

Alice (2013)

Further evidence for short mean free path? Or suppression of non-hydrodynamic modes?

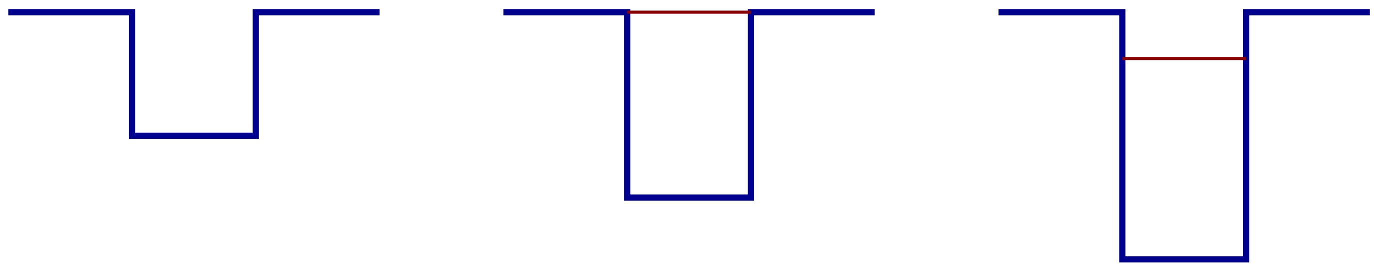
II. Dilute Fermi gas: BCS-BEC crossover

$$\mathcal{L}_{\text{eff}} = \psi^\dagger \left(i\partial_0 + \frac{\nabla^2}{2M} \right) \psi - \frac{C_0}{2} (\psi^\dagger \psi)^2$$



Unitarity limit

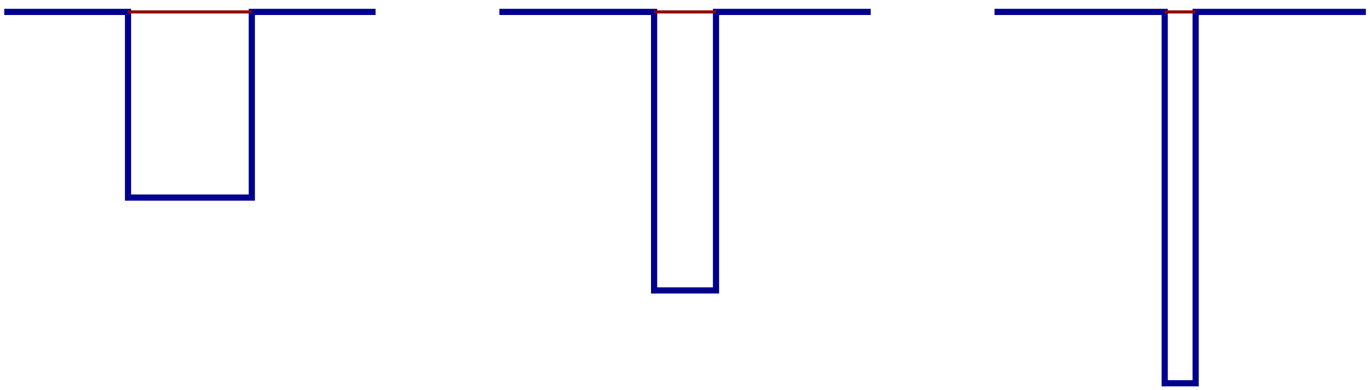
Consider simple square well potential



$a < 0$ $\qquad$ $a = \infty,\ \epsilon_B = 0$ $\qquad$ $a > 0,\ \epsilon_B > 0$

Unitarity limit

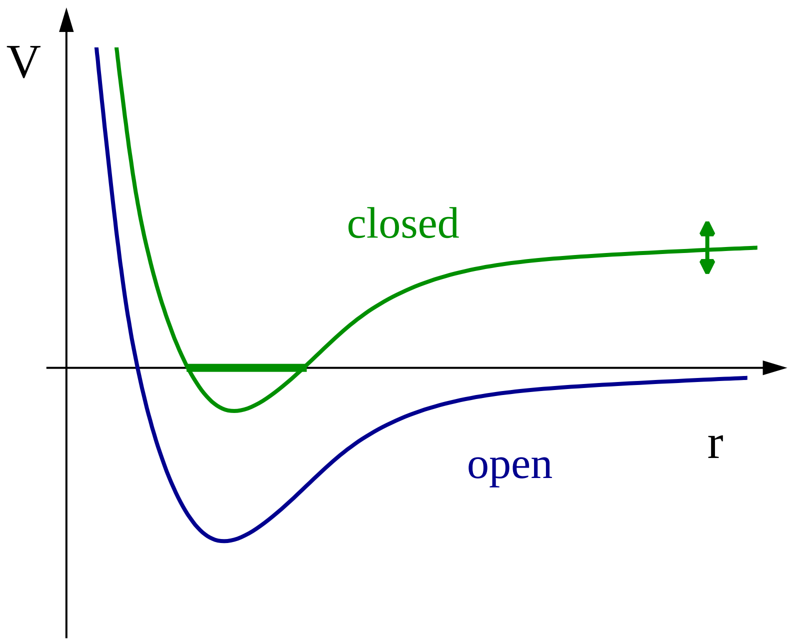
Now take the range to zero, keeping $\epsilon_B \simeq 0$

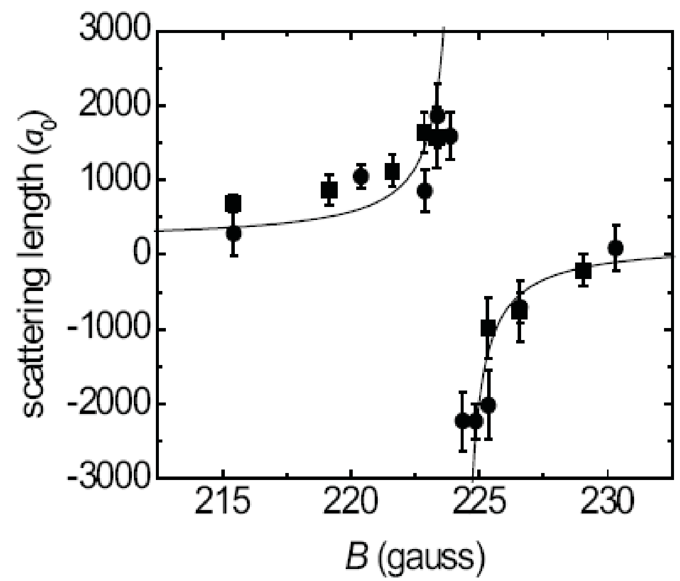


Universal scattering amplitude $\mathcal{T} = \dfrac{1}{ik}$

Feshbach resonances

Atomic gas with two spin states: "↑" and "↓"





Feshbach resonance $$a(B) = a_0\left(1 + \frac{\Delta}{B - B_0}\right)$$

Fermi gas at unitarity: Field Theory

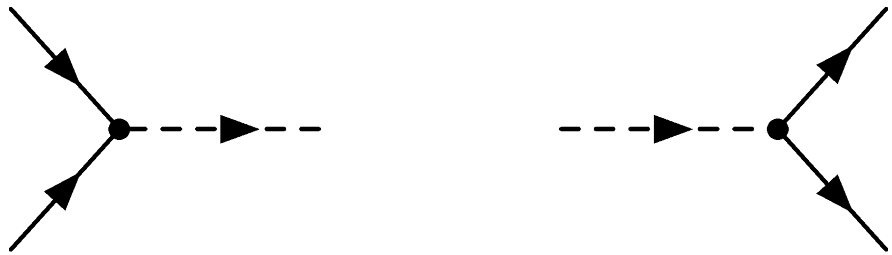
Non-relativistic fermions at low momentum

$$\mathcal{L}_{\rm eff} = \psi^\dagger\left(i\partial_0 + \frac{\nabla^2}{2M}\right)\psi - \frac{C_0}{2}(\psi^\dagger\psi)^2$$

Unitary limit: $a \to \infty$ (DR: $C_0 \to \infty$)

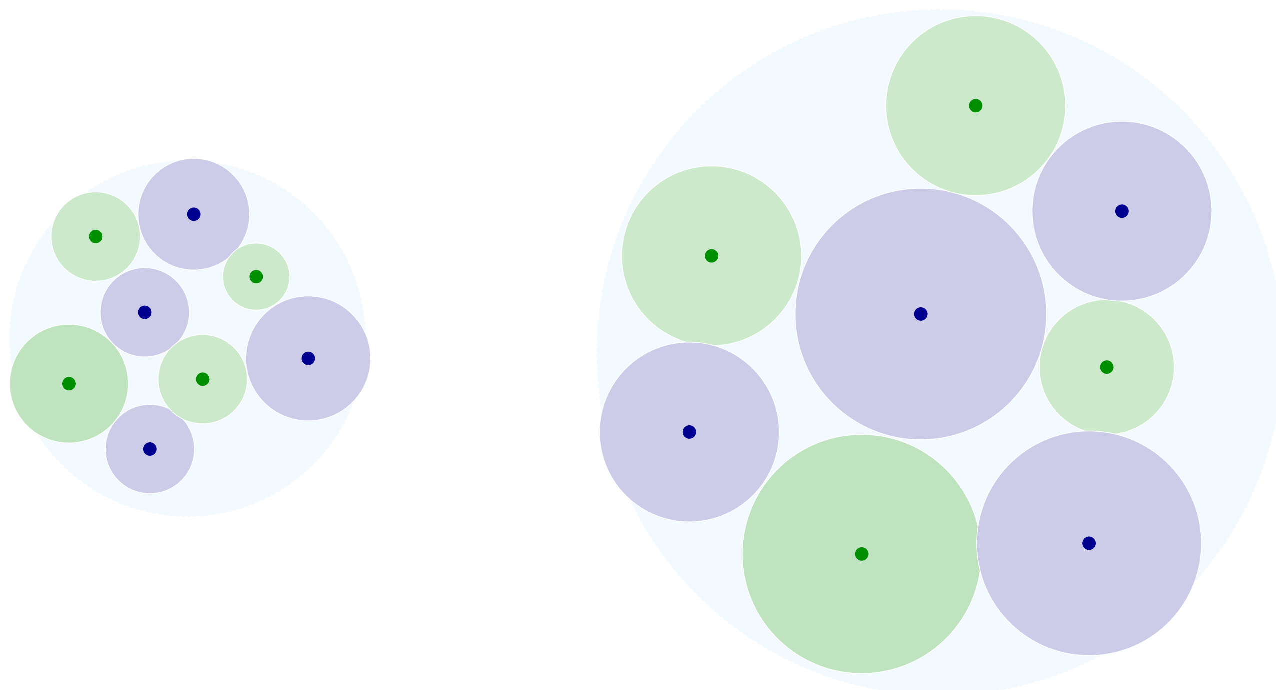
This limit is smooth (HS-trafo, $\Psi = (\psi_\uparrow, \psi_\downarrow^\dagger)$)

$$\mathcal{L} = \Psi^\dagger\left[i\partial_0 + \sigma_3\frac{\vec{\nabla}^2}{2m}\right]\Psi + \left(\Psi^\dagger\sigma_+\Psi\phi + h.c.\right) - \frac{1}{C_0}\phi^*\phi\ ,$$



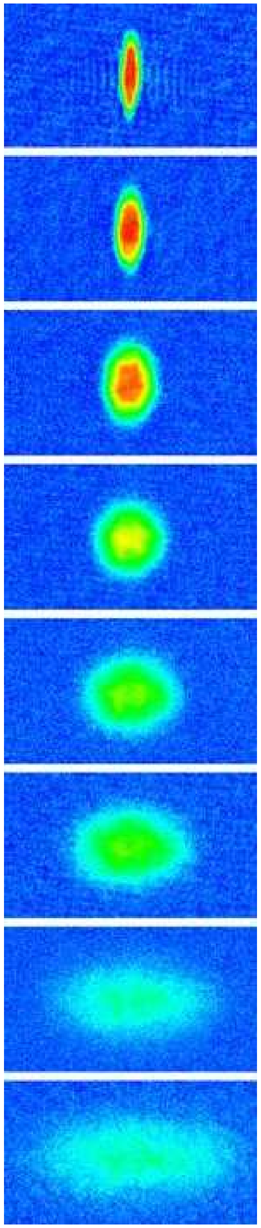
Universal fluid dynamics

Many body system: Effective cross section $\sigma_{tr} \sim n^{-2/3}$ (or $\sigma_{tr} \sim \lambda^2$)

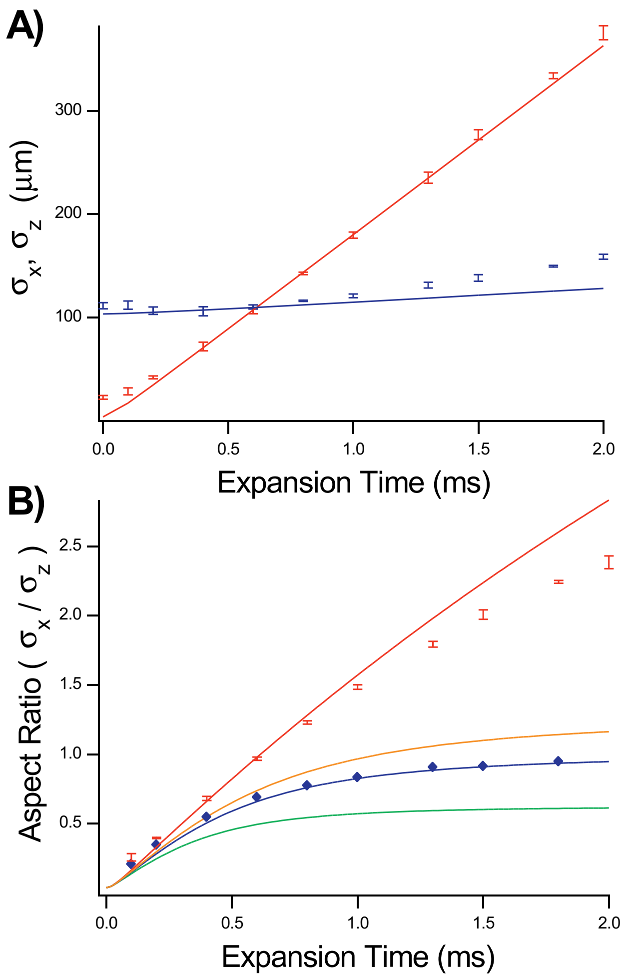


Systems remains hydrodynamic despite expansion

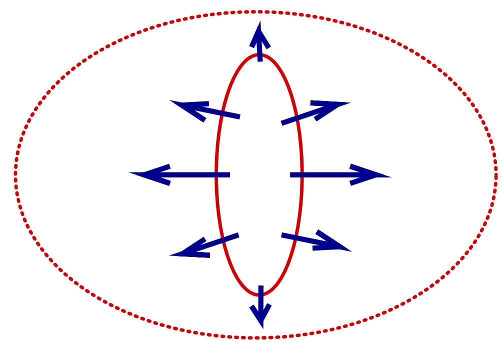
Almost ideal fluid dynamics



O'Hara et al. (2002)

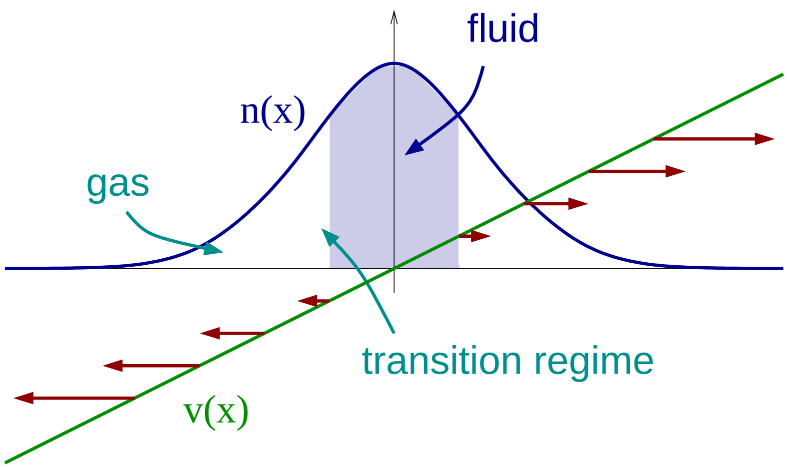


Hydrodynamic expansion converts coordinate space anisotropy to momentum space anisotropy



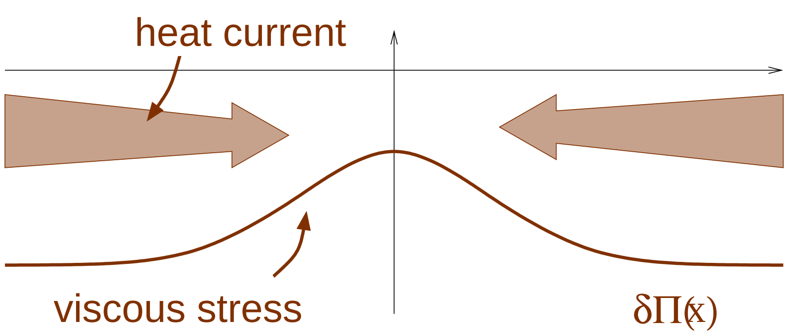
Determination of $\eta(n, T)$

Measurement of $A_R(t, E_0)$ determines $\eta(n, T)$. But:



The corona is not a fluid.

Can we ignore this issue?



No. Hubble flow & low density viscosity $\eta \sim T^{3/2}$ lead to paradoxical fluid dynamics.

$$\dot{Q} = \int \sigma \cdot \delta\Pi = \infty$$

Possible Solutions

Combine hydrodynamics & Boltzmann equation. Not straightforward.

Hydrodynamics + non-hydro degrees of freedom ($\mathcal{E}_a$; $a = x, y, z$)

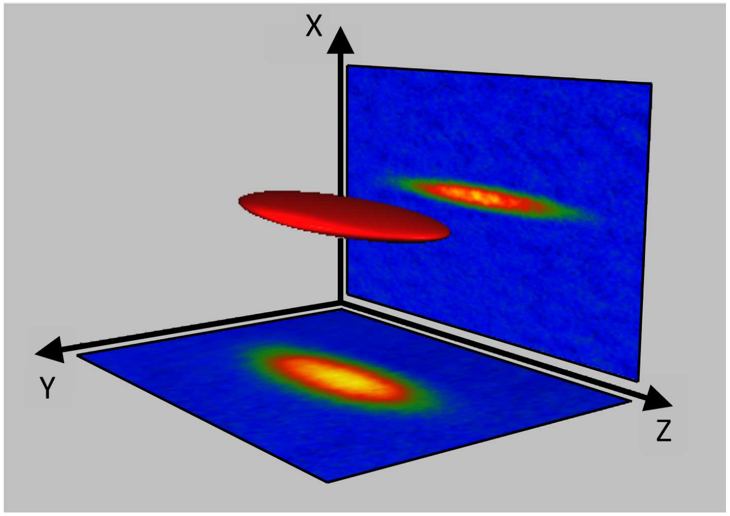
$$\frac{\partial \mathcal{E}_a}{\partial t} + \vec{\nabla} \cdot \vec{j}_a^{\epsilon} = -\frac{\Delta P_a}{2\tau} \qquad \Delta P_a = P_a - P$$

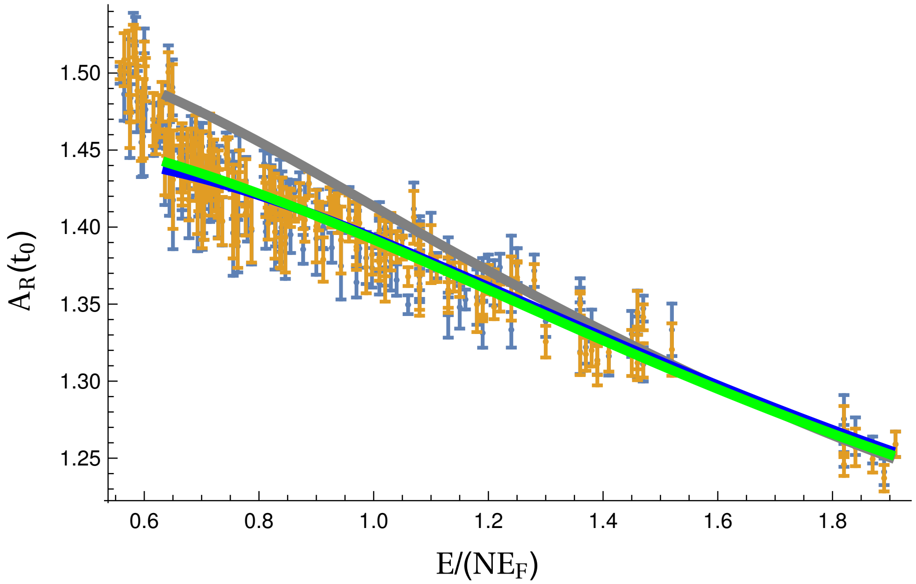
$$\frac{\partial \mathcal{E}}{\partial t} + \vec{\nabla} \cdot \vec{j}^{\epsilon} = 0 \qquad \mathcal{E} = \sum_a \mathcal{E}_a$$

τ small: Fast relaxation to Navier-Stokes with $\tau = \eta/P$

τ large: Additional conservation laws. Ballistic expansion.

Anisotropic fluid dynamics analysis



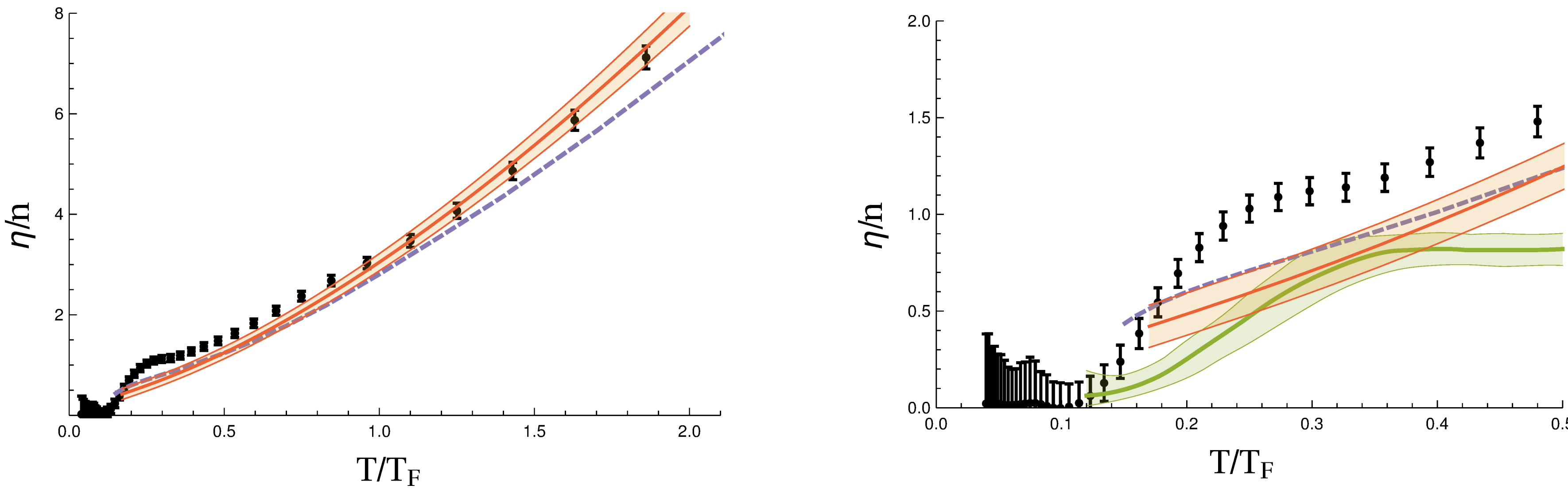


$A_R = \sigma_x/\sigma_y$ as function of total energy. Data: Joseph et al (2016). $E/(NE_F) \sim 0.6$ is the superfluid transition.

Grey, Blue, Green: LO, NLO, NNLO fit.

$$\eta = \eta_0(mT)^{3/2}\left\{1 + \eta_2 n\lambda^3 + \eta_3(n\lambda^3)^2 + \ldots\right\}$$

Reconstructed η/n



Red band: This work. Right figure zooms in on $T_c/s \sim 0.17T_F$.

Black points: Same data, simplified theory. Dashed line: T-matrix theory (Enss et al.). Green band: QMC (Bulgac et al.)

$$\eta(T \gg T_c) = (0.28 \pm 0.02)(mT)^{3/2} \qquad \eta_0(th) = \frac{15}{32\sqrt{\pi}} = 0.269$$

$$\eta/n|_{T_c} = 0.41 \pm 0.15 \qquad \eta/s|_{T_c} = 0.56 \pm 0.20$$

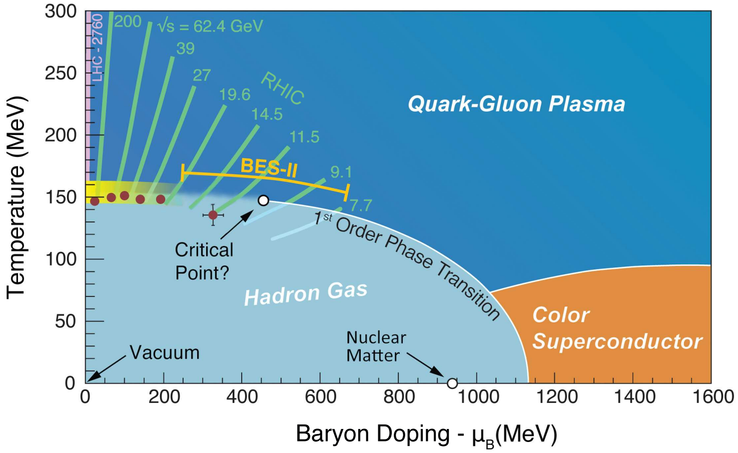
The bottom-line

Remarkably, the best fluids that have been observed are the *coldest* and the *hottest* fluid ever created in the laboratory, cold atomic gases (10^{-6}K) and the quark gluon plasma (10^{12}K) at RHIC.

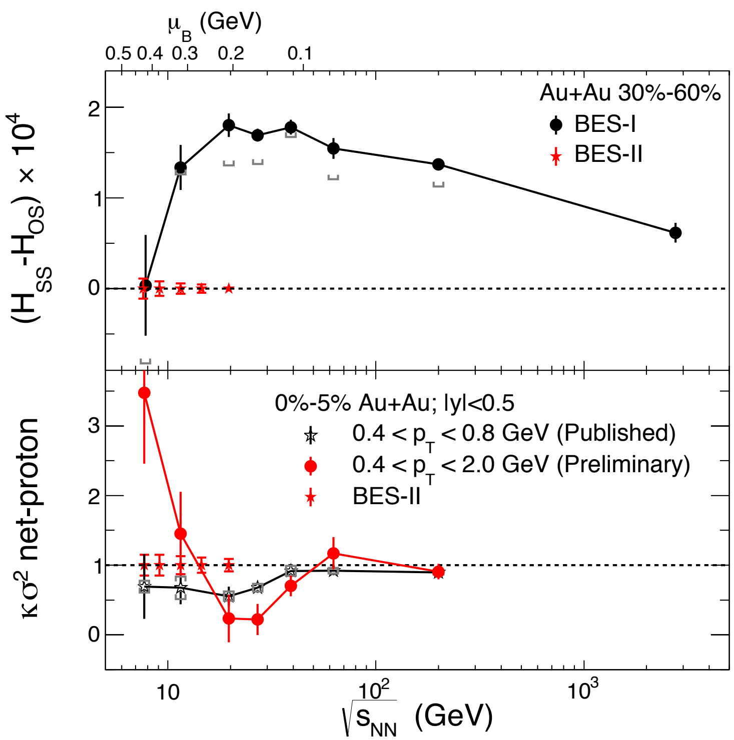
Both of these fluids come close to a bound on the shear viscosity that was first proposed based on calculations in string theory, involving non-equilibrium evolution of back holes in 5 (and more) dimensions.

Quantum limited viscosity and relaxation time explain applicability of fluid dynamics in very small, very short lived systems. Nature of non-hydrodynamic modes remains to be explored.

Outlook: Critical Fluctuations



BEST COLLABORATION



Historical digressions

Historical digression: Mott's minimal conductivity

(Sir) Nevill Mott predicted that the metal-insulator transition cannot be continuous; there is a minimal conductivity.

Conduction in Non-crystalline Systems

IX. The Minimum Metallic Conductivity

By N. F. Mott
Cavendish Laboratory, Cambridge

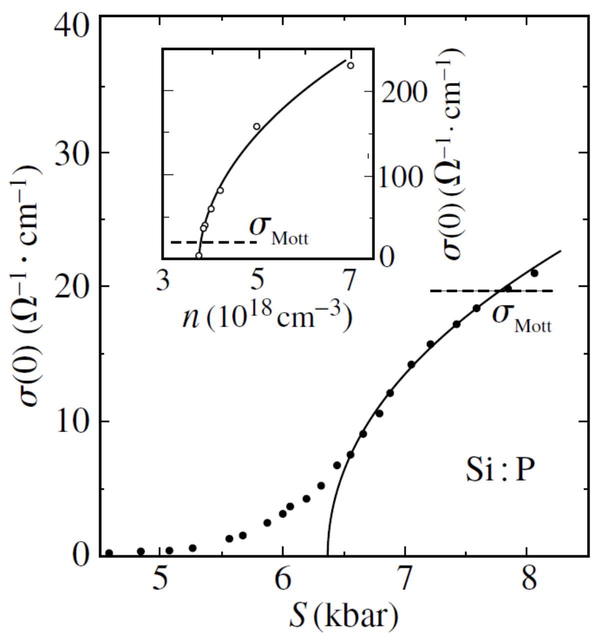
[Received 27 July 1972]

$$\frac{\sigma}{n^{1/3}} \geq \frac{1}{(3\pi^2)^{2/3}} \frac{e^2}{\hbar}$$

This idea is not correct,

the metal-insulator transition can

be continuous.



Historical digression: Minimal shear viscosity

Danielewicz & Gyulassy argue that the shear viscosity cannot be zero.

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Dissipative phenomena in quark-gluon plasmas

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than $\langle p \rangle^{-1}$. Requiring $\lambda_i \gtrsim \langle p \rangle_i^{-1}$ leads to the lower bound

$$\eta \gtrsim \frac{1}{3} n \ , \qquad (3.3)$$

where $n = \sum n_i$ is the total density of quanta. What seems amazing about (3.3) is that it is independent of dynamical details. There is a finite viscosity regardless of

Is this idea correct?