Spin Alignment in $^{12}$C+$^{12}$C Inelastic Scattering

Stephen Joseph Willett
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The excitation function of $^{12}$C+$^{12}$C scattering has been characterized by numerous resonances (total width $\Gamma=50-500$ keV) of unique spin in the energy range $E_{CM}=5-20$ MeV and broader structures ($\Gamma=2-4$ MeV) at $E>10$ MeV. The latter have been proposed to be a continuation of the former: molecular states of narrow width near and above the Coulomb barrier. However, other researchers have proposed alternate, non-resonant mechanisms that might also give rise to the broad maxima.

To help establish the physics underlying the gross structures, we have undertaken a measurement of the spin alignment in the inelastic scattering. These measurements were performed over a wide range of particle scattering angles ($53^\circ<\theta_{CM}<101^\circ$) and at 21 energies ($15<E_{CM}<33$ MeV) chosen to map over both the broad and narrow
structures observed in the cross section data \( \sigma(E) \). Our method employed particle-gamma radiation correlations in the scattering plane, so that both alignment \( A(E, \theta) \) and phase angle \( \beta(E, \theta) \) information could be extracted. Separate experiments established an absolute calibration for the alignments.

Our results demonstrate that the alignment is only weakly a function of scattering angle. Furthermore, the angle-averaged alignment \( \langle A \rangle \) is enhanced at energies where either broad or narrow maxima occur in \( \sigma(E) \) for \( E < 26 \text{ MeV} \), but above that energy \( A(E) \) becomes relatively constant with respect to both \( E \) and \( \theta \). The phase angle \( \beta \) is approximately linear in \( \theta \), and exhibits a trend of increasing slope of \( \beta(\theta) \) with increasing interaction energy. Deviations from this general behavior at certain energies may be interpreted as signs of true quasimolecular resonances.

We also compared theoretical predictions to our results. Resonant theories, such as the single particle or the band crossing models, do not reproduce the alignment data. Non-resonant theories, such as the DWBA or the diffraction model, can simultaneously reproduce gross features of both \( A(E) \) and \( \sigma(E) \), and also satisfactorily predict \( \beta(\theta) \). These predictions fail at higher energies, indicating that their simplifying assumptions are no longer valid in that regime. The behavior correlated with the narrow structure still awaits detailed explanation, most probably incorporating a resonance origin.
SPIN ALIGNMENT IN $^{12}$C+$^{12}$C INELASTIC SCATTERING

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I also thank the other members of my thesis committee--Peter Parker, Steve Sanders, Denise Caldwell, and Tom Appelquist--for their attentive reading of this manuscript and suggestions for its improvement.

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Although at times I felt the contrary to be true, I did not in fact spend all my waking hours in New Haven on physics. On the whole, graduate school has been an enjoyable experience, due chiefly to activities and friendships with the people mentioned above, fellow physics students, and their counterparts in other departments at Yale. Their companionship is deeply treasured.

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1. INTRODUCTION

1.1 Preface

Characteristic of most fields in physical and biological sciences is the close interplay between theory and experiment. The initial motivation may come from either source, but its rapid and continual development depends upon a synergistic effort from both areas. Obstacles are inevitable in either theory or experiment—too many or too few choices, too little or too much information— but input from one area can help overcome difficulties in the other. Alternatively, inactivity in one area may lead to disinterest in the other, and the entire field will lie fallow, perhaps for years.

Both modes of interaction are seen in the history of the study of nuclear molecules, as can be discerned in the review in this chapter.
In fact, the present study was stimulated by the multitude of theories proposed to explain carbon-carbon inelastic scattering within (or without) the quasimolecular picture. We hope that the presentation of new data in this field will serve to continue the iterative process between theory and experiment, and lead to greater understanding of this small segment of natural phenomena.

1.2 Nuclear molecular phenomena

a) Early evidence and models

With the advent of high resolution detectors and accelerators for heavy ions two decades ago came the first evidence for nuclear molecular phenomena. Bromley, Kuehner, and Almqvist observed structure in the $^{12}$C+$^{12}$C 90° elastic scattering excitation function (Br60) and correlated structures near the Coulomb barrier ($E_{CM}$=6 MeV) in the proton, neutron, alpha, and gamma exit channels of the same reaction (Al60). This behavior is contrasted with the structureless results obtained for $^{16}$O+$^{16}$O. The correlated behavior made a statistical explanation (Er63,Mo67) unlikely, and suggested that special states in $^{24}$Mg near $E_x$=20 MeV were being populated. The sharpness of the states indicated that they did not readily mix with the numerous (hundreds per MeV) nearby compound nuclear levels, and the characteristic width $\Gamma$<100 keV indicated the states had a lifetime $\tau$=h/$\Gamma$~10$^{-20}$sec. This compares with a transit time of $\sim$10$^{-22}$ sec for the passing ions. Semiclassical arguments would indicate a peripheral interaction: while each carbon
nucleus has a radius of approximately 3 fm, the distance of closest approach under Coulomb repulsion is more than 8 fm at 6 MeV. The Chalk River group proposed that two carbon nuclei entered a "molecular" state distinct from a fused $^{24}\text{Mg}$, but the binding mechanism was still to be explained.

One early theory by Davis (Da60) proposed that the resonances were rotation-vibration states in the potential of the relative motion of the two ions: i.e., the optical potential. In a sense this begged the question: an optical potential can hide a lot of physics. Moreover, no physically reasonable choice of optical model parameters was able to reproduce the experimentally-observed spectrum. Vogt and McManus (Vo60) saw the ground state deformation and easy deformability of $^{12}\text{C}$ as essential ingredients; the rigid closed-shell $^{16}\text{O}$ nucleus was seen to be incapable of molecular binding. Decay to compound nuclear states was thought to be inhibited by the centrifugal barrier for high spin states, and by a high nucleonic rearrangement energy reflecting the Pauli principle (Wi61).

b) Later experiments and theories

Additional data upon which to judge the theories were called for. For each resonance one could in principle measure the cross section $\sigma$, the intrinsic spin $J$, the total width $\Gamma$, and the partial decay widths $\Gamma_i$ to various exit channels $i$. In particular, large widths to the elastic and specific reaction channels could indicate that the excitation function fluctuations were not merely statistical artifacts from compound nuclear levels. Further experimental work at Chalk River
(Br61,Al63,Ku63) estimated the carbon partial width and established the spins of the resonances to be fairly small (2+, 4+).

These new data did not give a ringing endorsement to any extant theory. The centrifugal barrier was not high enough to prevent spreading to nearby compound states, and the effect of the exclusion principle was not easily tested. During this period new models were needed to explain the data, and the field lay relatively inactive. The top panel of fig. 101 illustrates the difficulties in the models. Depicted is the real part of the potential (Coulomb plus nuclear plus centrifugal contributions) for the one-body description of the relative motion of the two carbon nuclei. Structures seen in the data were not broad enough (a few MeV) to be virtual shape resonances (E2) above the potential well. The narrow width (i.e., long lifetime) might dictate quasibound states within the well, but direct population at E3 seemed to require tunneling through a substantial barrier, implying a small cross section.

Acting upon a suggestion by Nogami, Imanishi proposed (Im68,Im69) that the underlying binding mechanism was a virtual excitation of one of the carbon ions to the 21+ state. This level at E_x=4.439 MeV is known to be strongly collective and a member of the ground state band of quadrupole shape deformations. The two carbon ions, approaching each other at E3 in the lower panel of fig. 101, are dropped by the Q-value of the virtual inelastic scattering into quasi-bound states of the interaction potential. This mechanism reproduces the observed narrow widths without requiring tunneling, and with a suitable set of parameters reproduced the energies and spins of the three resonances.
101) SCHEMATIC POTENTIAL FOR QUASIMOLECULAR INTERACTIONS

The real part $V(r)$ of the optical potential is plotted versus the relative distance $r$ between the two nuclei. Interaction energy $E_1$ corresponds to a quasibound state in the potential well. A virtual, or shape, resonance occurs at $E_2$. $E_3$ is at an energy $E_X$ greater than a quasibound state, where $E_X$ is the excitation energy of one or both of the individual nuclei. $E_4$ corresponds to a shape resonance which is $E_X$ above a quasibound state. The relation of these four cases to quasimolecular models is discussed in sects. 1.2b and 1.3b.
QUASIMOLECULAR INTERACTIONS

ENERGY

SEPARATION DISTANCE

E_1

E_2

E_3

E_4

Fig. 101
single and mutual $2^+$ and single $3^-\ (E_x=9.641\ MeV)$ excitations—denoted respectively as $(0^+,0^+), (2^+,0^+), (2^+,2^+),$ and $(3^-,0^+)$. This calculation takes the Imanishi model to its logical conclusion by including all possible excitation modes of the carbon nuclei thought to be important in this energy region.

In yet another extension of Nogami's original idea, Tanimura (Ta78, Ta80) has coupled those same four channels and calculated $^{12}$C+$^{12}$C scattering using a modified adiabatic potential (MAP), in contrast to the sudden potential of the double-resonance model. In sect. 1.4 the predictions of BCM and MAP calculations for inelastic scattering are compared.

c) Spectrum generation

Other approaches have been followed in generating a spectrum of resonances that might reproduce the experimental sequence. For example, Cosman, et al., (Co81) have recently proposed that the $^{12}$C+$^{12}$C resonances be grouped into five collective band multiplets, and suggested their basis to be shell model shape isomers of $^{24}$Mg.

From a different theoretical basis, Iachello (Ta81) has argued that quasimolecular resonances, instead of being governed by the quadrupole degrees of freedom that dominate low-lying excited states of the nuclei, should be governed by dipole operators characteristic of diatomic molecules. Approaching the subject in an algebraic manner, he points out that the appropriate group symmetry of the problem should be $U(4),$ and its decomposition into the chain $U(4)\supset O(4)\supset O(3)\supset O(2)$ would
generate a spectrum of resonances with energies

\[ E(J,v) = C_0 + C_1(v+\frac{1}{2}) + C_2(v+\frac{1}{2})^2 + C_3 J(J+1), \]

in which \( v \) and \( J \) are the vibrational and spin quantum numbers, respectively.

Erb and Bromley (Er81) have applied this formula, which corresponds to approximate solutions of the Morse potential for molecules (Sc68, sect. 49), to the 28 correlated \(^{12}\text{C}+^{12}\text{C}\) resonances below 13 MeV whose spins are established. They discovered that the energies can be reproduced with four free parameters \( C_1 \) (one of which, \( C_3 \), is inversely proportional to the moment of inertia) and two quantum numbers, \( J \) and \( v \). Erb and Bromley also suggested that the \( 0_{2}^{+} \) state at 7.654 MeV may prove to be more important in the molecular binding than previously suspected. Previously, Tohsaki-Suzuki (To78) had proposed that coupling to this "3α" state would explain why the \(^{8}\text{Be}+^{16}\text{O}\) exit channel is so prominent in the \(^{12}\text{C}+^{12}\text{C}\) interaction.

d) Microscopic models

In contrast to the preceding macroscopic approaches, other researchers have tried to understand the experimental data on microscopic bases. Two basic approaches have been attempted: shell model and self-consistent calculations.

Within the former, Leander and Larsson (Le75) have calculated potential energy surfaces using a modified Nilsson one-center shell model. For \(^{24}\text{Mg}\) they obtain secondary minima at large deformations, and suggest that molecular states may be associated with these, in analogy
with fission isomers.

More recently, Chandra and Mosel (Ch78) have obtained, from two-center shell model calculations, an interaction potential with a shoulder near 4.5 fm. They argue that this is associated with an avoided level crossing between the \(f_{7/2}\) and \(p_{1/2}\) levels in the compound nucleus, which rearranges the nucleonic configuration in \(^{24}\text{Mg}\) to minimize energy in the separated system. The authors draw parallels to the preceding work, but argue that a full two-center calculation is needed to identify which potential minimum of Leander and Larsson corresponds to the molecular states.

Microscopic calculations starting from the nucleon-nucleon interaction have used self-consistent methods such as time-dependent Hartree-Fock (TDHF). Early studies by Maruhn and Cusson (Ma76) could consider only head-on collisions (\(L=0\)), but density profiles indicated that after the collision the system spent a significant time in a molecule-like configuration. Later three-dimensional constrained TDHF calculations by the Orsay-Livermore group (Cu79,F180) demonstrated that the orientation of the oblate carbon ground states has a large effect on the scattering. In particular, the axial configuration (face-on-face scattering) gives results similar to the usual optical potential in this mass region. In contrast, triaxial scattering (edge-on-edge) is strikingly different, giving a shallow potential shape with a minimum at a larger radius that is needed to support the collective molecular states.\(^2\) Density contours in this work suggested a long-lived molecular

\(^2\) Chandra and Mosel had reached similar conclusions.
configuration. These authors also demonstrated, by calculating a collective mass parameter, that the nuclear structure of the individual carbon ions changes rapidly in the collision. This implies that models based on static properties of the $^{12}\text{C}$ nucleus can be only approximate at best, and a full dynamical understanding of the reaction relies on future microscopic calculations of this kind.

1.4 Inelastic scattering

As the Imanishi model and its extensions recognized, the wavefunction for the $2^+$ state has a large overlap with the ground state of $^{12}\text{C}$. If it is energetically possible, we would expect a large cross section for inelastic scattering to this state. Moreover, because of its presumed role in the binding mechanism of the quasimolecules, it may be expected that this exit channel will demonstrate resonant phenomena. But at higher energies and spins it becomes increasingly difficult to establish a non-statistical origin to structures in an excitation function. In particular, not all exit channels may have an appreciable branching from each resonance, and interference from a resonance background amplitude may shift a peak some fraction of its width. In particular, Shapira, et al., (Sh74) demonstrated that data known at that time (1974) in elastic, inelastic, and alpha particle channels above 13.5 MeV were consistent with-- although they did not require-- a Hauser-Feshbach statistical explanation.

With this motivation, Cormier, et al., (Co77, Co78) measured the total cross section for single and mutual inelastic scattering to the $2^+$ state
as a function of beam energy; their results are shown in fig. 104. The persistence of the narrow ($\Gamma \sim$ a few hundred keV) structures up to about 20 MeV bears notice. In addition, striking and regularly-spaced broad structures ($\Gamma \sim$ a few MeV), that continue to the highest energies measured, also appear. The energies of the centroids of these structures are roughly correlated among the two inelastic channels and the fusion data of Sperr, et al., (Sp76).

Based on energy systematics of the broad structures, Cormier interpreted them as a continuation of the dinuclear resonance states seen at the lower energies. If one were to plot the centroids of these structures versus the postulated spins indicated in fig. 104, the points would fall on the same locus in fig. 103 that encompassed the lower energy resonances. A partial width decomposition (Co78) of the gross features indicated $\Gamma_{\text{elas}}/\Gamma_{\text{tot}} = .18-.33$ for the presumed resonances, while reduced widths for the $(2^+,0^+)$ and $(2^+,2^+)$ channels were found to be comparable$^3$ to those of the $(0^+,0^+)$ channel. These facts are evidence of the molecular nature of these structures. Cormier interpreted the broad structures as doorway states, slightly more complicated than simple shape resonances, with coupling leading to more complex configurations and fragmentation below 20 MeV. A further discussion of this interpretation of the data can be found in ref. Co80.

Kondo, et al., (Ko79a) and Tanimura (Ta80) have reproduced the energies of the broad structures via coupled-channel treatment of the

$^3$ In fact, the inelastic cross sections are huge; the sum of the single and mutual $2^+$ excitation are 10-25% of the total reaction cross section in this energy region, so any successful theory must account for the key role the inelastic channels play in the scattering.
104) INELASTIC SCATTERING EXCITATION FUNCTIONS

The upper panel indicates the integrated cross section for $^{12}\text{C}+^{12}\text{C}$ scattering which leaves one of the nuclei in the 2$^+$ state; the lower panel shows $\sigma$ for the case when both exiting ions are in that state. The data for $E > 15$ MeV are from Co77 and Co78. The data for $E < 15$ MeV are from Em73, with the relative normalization as noted in Fu80. An earlier version of this figure (Co78) had the Em73 cross sections too large by nearly a factor of 2.

105) COMPARISON OF RESONANCE MODELS WITH INELASTIC SCATTERING

The solid line represents the cross section for the $^{12}\text{C}(^{12}\text{C},^{12}\text{C})^{12}\text{C}(2^+)$ reaction, as presented in fig. 104. The other lines are coupled-channel predictions for this excitation function: the dotted line using the band crossing model (Ko79a) and the dashed line using the modified adiabatic potential (Ta80).
Fig. 104
More recently, Chandra and Mosel (Ch78) have obtained, from two-center shell model calculations, an interaction potential with a shoulder near 4.5 fm. They argue that this is associated with an avoided level crossing between the $f_{7/2}$ and $p_{1/2}$ levels in the compound nucleus, which rearranges the nucleonic configuration in $^{24}$Mg to minimize energy in the separated system. The authors draw parallels to the preceding work, but argue that a full two-center calculation is needed to identify which potential minimum of Leander and Larsson corresponds to the molecular states.

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Kondo, et al., (Ko79a) and Tanimura (Ta80) have reproduced the energies of the broad structures via coupled-channel treatment of the

\[ \text{-----------------------------} \]

\(^3\) In fact, the inelastic cross sections are huge; the sum of the single and mutual $2^+$ excitation are 10-25% of the total reaction cross section in this energy region, so any successful theory must account for the key role the inelastic channels play in the scattering.
104) INELASTIC SCATTERING EXCITATION FUNCTIONS

The upper panel indicates the integrated cross section for $^{12}\text{C} + ^{12}\text{C}$ scattering which leaves one of the nuclei in the $2^+$ state; the lower panel shows $\sigma$ for the case when both exiting ions are in that state. The data for $E > 15$ MeV are from Co77 and Co78. The data for $E < 15$ MeV are from Em73, with the relative normalization as noted in Fu80. An earlier version of this figure (Co78) had the Em73 cross sections too large by nearly a factor of 2.

105) COMPARISON OF RESONANCE MODELS WITH INELASTIC SCATTERING

The solid line represents the cross section for the $^{12}\text{C}(^{12}\text{C},^{12}\text{C})^{12}\text{C}(2^+)$ reaction, as presented in fig. 104. The other lines are coupled-channel predictions for this excitation function: the dotted line using the band crossing model (Ko79a) and the dashed line using the modified adiabatic potential (Ta80).
Fig. 104
Fig. 105
(0+,0+),(2+,0+),(2+,2+), and (3-,0+) exit channels. Their calculations for the (2+,0+) final state are shown in fig. 105, together with the data of Cormier, et al. Of course these models require the resonances to be states of good angular momentum.

In an attempt to confirm the resonance interpretation of these data, Cannell, et al., (Ca79) used a particle-gamma correlation technique to measure the spins of the compound system in the inelastic channel. They discovered that the broad structures could not be characterized by a single spin value, which called the resonance interpretation into question. In fact, considering that the characteristic width \( \Gamma \sim 3 \text{ MeV} \) implied lifetimes \( \tau \sim 10^{-22} \text{sec} \), roughly the collision time, the possibility was raised that the broad structures were not doorway states to molecular resonances, but simply kinematical artifacts of direct inelastic scattering.

In summary, while the molecular character of the resonances in the barrier region is firmly established, the nature of those at higher energies has remained an open question. In the following section we explore the various models based on non-resonant direct reaction mechanisms.
1.5 Direct reaction models

a) DWBA

At present, the standard way to treat any nuclear reaction is via distorted wave Born approximation (DWBA). The optical potential obtained from a fit to elastic scattering data is used to calculate inelastic scattering. By this method Cannell and coworkers reproduced the gross structures observed in the inelastic excitation function. (More details are given in chap. 6.) The authors also found that a DWBA treatment gave adequate explanation of the particle-gamma correlation for the $2^+$ state measured out of the reaction plane at 30 MeV, while the corresponding resonance interpretation predicted an out-of-plane gamma radiation yield six times larger than observed. But the in-plane/out-of-plane ratio for the other broad maxima at $E=14.5$, 18.75, and 24.25 MeV showed the agreement between the calculations and the data to be worse than that for 30 MeV.

b) Diffraction

Another direct interaction approach is that via the diffraction model of Phillips, et al (Ph79). This approximation replaces the DWBA inelastic scattering amplitudes by those given by the Austern-Blair relation (Au65). Using a suitable parametrization these authors were able to reproduce not only the total inelastic cross sections but also the inelastic angular distributions. Within this model, maxima in the excitation function arise from angular momentum and Q-value matching,
and the appearance of striking structure reflects the presence of only even partial waves in the identical-boson system. The essential aspects of the optical potential are strong absorption for small impact parameters, surface transparency, and a high centrifugal barrier.

c) Barrier top

Friedman, et al., (Fr79) have interpreted the broad, higher energy structures in $^{12}\text{C}+^{12}\text{C}$ inelastic scattering within the framework of a barrier top model. The authors classify these as orbiting resonances, but the model incorporates many of the same physical features that underlie the diffraction model. Strong absorption within the potential barrier and the higher centrifugal barrier for large $L$ lead to an angular momentum window for the reaction around the grazing value $L_{gr}$.

We must face the problem of defining what actually constitutes a physical resonance. In the present context the narrow, long-lived quasibound states in the potential seem to better fit the concept of true molecular resonances, since the virtual states of shape or orbiting resonances are short-lived. Keeping this distinction in mind, the barrier top model is discussed in chap. 6 together with others that are claimed to be non-resonant. In fact, at least some authors (Le81) make no distinction between the diffraction and the barrier top models.

d) Wave interference

The diffraction and the barrier top models deal only with the part of the incoming wave reflected at the outer barrier. The strong absorption
in the nuclear interior is presumed to damp to insignificance any wave reflected from the interior. Lee, et al., (Le81) have argued that the parameters adopted by Phillips, et al., and Friedman, et al., are unrealistic. Instead, they have employed a commonly-made suggestion and have reproduced the gross structures via interference between the surface and the interior reflected waves.

From fig. 106 we observe that the predictions of the four models discussed in this section for the inelastic scattering cross section are qualitatively similar.

1.6 Alignment

We are thus confronted with conflicting interpretations of the data, and the most reasonable basis on which to accept some and reject others is, as always, that of obtaining additional experimental information which will distinguish among the theories and which any successful theory must reproduce. The spin alignment of the inelastically-scattered carbon is one such quantity that might distinguish among the models. Determination of this alignment is time-consuming and not often performed in heavy ion studies, but it is of particular interest here because specific predictions of various models can be tested with it. For example, the band crossing model bases the origin of resonances in $^{12}\text{C}+^{12}\text{C}$ upon the dominance of the "aligned configuration" $J=I'+2$, whose band mixes with the ground state band in this energy region. The diffraction model explains the gross maxima in the cross section as the dominance of $L'=L-2$ orbital angular momenta; again, this implies
The predicted excitation function for $^{12}\text{C}(^{12}\text{C},^{12}\text{C})^{12}\text{C}(2^+)$ is plotted as the solid line for the DWBA (Ca79), the dashed line for the barrier top model (Fr79), the dash-dot line for the diffraction model (Ph79), and the dotted line for the wave interference model (Le81).
Fig. 106
enhanced alignment. The purpose of the present experimental study is the measurement of the alignment of the inelastic scattering system—i.e., the degree to which the spin I of the excited state is aligned with the orbital angular momenta—and the use of these data to judge among the various models that have been proposed. The measurement of the alignment is completely model independent, so it provides physical information that any model must reproduce.

In general, the alignment $A$ of an interacting nuclear system varies with both the interaction energy $E$ and the scattering angle $\theta$. While the structure of $A(E)$ is clearly interesting from the above considerations, the behavior of $A(\theta)$ is also important in this study. In general, interference can affect physical observables at different angles, and in drawing conclusions from the data we must guard against being misled by such angular phenomena. Additionally, the diffraction model has been successful in reproducing the inelastic angular distribution only for $\theta_{CM} < 70^\circ$. It was considered of interest to examine whether this (or any other) model might suffice to explain the scattering in some angular ranges but not in others.

While our investigation was underway, data were published by Trombik, et al., (Tr80) for the $^{12}\text{C}+^{12}\text{C}$ system. They reported alignment measurements at only 12 combinations of energy and angle—in our view insufficient to permit the drawing of firm conclusions about the correspondence of the alignment with either the cross section data or the various model predictions. We have measured alignments and phase angles for 333 energy-angle pairs in the ranges $\theta_{CM} = 53-103^\circ$ and $E_{CM} = 15.6-32.4$ MeV, tracing over both broad and narrow features
obtained yield a substantially better basis upon which to judge among the proposed interaction models.

1.7 Results

We have found that the alignment $A(E, \theta)$ is only weakly a function of scattering angle, and that the angle-averaged alignment $<A>$ is enhanced at energies $E < 26 \text{ MeV}$ where either broad or narrow maxima occur in $\sigma(E)$. For $E > 26 \text{ MeV}$, however, $A$ becomes relatively constant with respect to both $E$ and $\theta$. Resonant theories cannot reproduce the data, but non-resonant models can explain the gross features in $A(E)$ and $\sigma(E)$ for $E < 26 \text{ MeV}$. At higher energies the latter predictions also fail, which is understood by examination of the partial wave contributions to the scattering. We have also measured the relative substate phase angle $\beta(E, \theta)$ and compared theoretical predictions for it to the data. Preliminary reports of the alignment results were presented to the American Physical Society (Wi80) and at the 1980 International Conference on Nuclear Physics (Wi80a).

Finally, we outline the organization of the present thesis. There are a number of methods available for the measurement of spin alignments in nuclear reactions; we chose a particle-gamma correlation technique in which the $\gamma$-angle was varied for a fixed particle angle. The formalism pertinent to this technique is outlined in chapter 2, as well as the effects identical particle symmetry have on our analyses and results. The experimental technique and data analysis for the
alignments constitute chapter 3. Because of the particular technique used, we must perform separate measurements to obtain an absolute normalization for our data; these are summarized in chapter 4. Our results and a comparison with previous cross section and alignment data are presented in chapter 5. In chapter 6 we compare predictions made by various models with our experimental data. Finally, chapter 7 summarizes the present work and indicates new directions to which it points in heavy ion physics.
2. FORMALISM

2.1 Conservation principles and definition of alignment

A discussion of alignment in a nuclear reaction starts with the most general observations on conservation of angular momentum and parity. Any two body reaction must conserve total angular momentum \( J \):

\[
\vec{J} = \vec{L} + \vec{S} = \vec{L}' + \vec{S}', \tag{1}
\]

Here \( L \) is the relative orbital angular momentum, \( S \) the channel spin, and the unprimed and primed quantities refer to entrance and exit channels, respectively. If states of definite parity \( P \) are populated, Bohr (Bo59) has demonstrated that reflection symmetry relative to the reaction plane must imply

\[
P e^{i\pi m} = P' e^{i\pi m'}, \tag{2}
\]
where $m$ is the projection of $S$ on a quantization axis perpendicular to the plane (i.e., $\hat{z} = \hat{k} \times \hat{k}'$ in fig. 201).

For our reaction $^{12}\text{C}(^{12}\text{C},^{12}\text{C})^{12}\text{C}^+$ we note $S=0$, $S'=2$, and $P=P'=+1$. Hence, the Bohr theorem implies that only even magnetic substates $m$ of the excited nucleus may be populated: $m=0, +2, \text{ and } -2$.

The amplitude $a_m$ for population of a given substate is a complex number written as $a_m e^{i\beta_m}$, with $\alpha$ and $\beta$ real and $\alpha$ non-negative. These six substate parameters for our reaction constitute only four independent quantities, since unitarity requires $\Sigma_m (\alpha_m)^2=1$, and only the relative phases between the substates are physically meaningful. It bears emphasis that $a_m$ and $\beta_m$ are each implicit functions of the energy $E$ and of the scattering angle $\theta$ of the reaction. Since $+2$ and $-2$ are the only non-zero $m$ values allowed, we hereafter denote them by subscripts $+$ and $-$. 

The probability for population of substate $m$ is $(\alpha_m)^2$. We define alignment of a nuclear reaction as the probability that, given that the reaction has occurred, the largest (in absolute value) possible magnetic substate is populated. Hence, in our case the alignment would be defined as:

$$A = \alpha_+^2 + \alpha_-^2.$$  

An alternate definition of alignment is used by Trombik, et al., (Tr80):

$$P_{zz} = \frac{1}{3} \Sigma_m (m \alpha_m)^2 - 1 = \alpha_+^2 + \alpha_-^2 - \alpha_0^2.$$  

The correspondence between the notations is clear, from the case of no alignment ($A=0$ or $P_{zz}=-1$) to the situation of complete alignment.
201) SCHEMATIC DIAGRAM OF $^{12}\text{C}+^{12}\text{C}$ INELASTIC SCATTERING

Both nuclei before the collision and one of them afterward have intrinsic spin $S=0$. As demonstrated in Bo59, the remaining nucleus must have even spin projection on a quantization axis $z=k\times k'$.

202) E2 RADIATION PATTERNS

For our case the x-y plane is the reaction plane. In this plane the $m=\pm2$ radiation (solid line) is maximum, while $m=0$ radiation (dashed line) vanishes. $m=\pm1$ radiation (dotted line), the only substate classically allowed to have any amplitude in the $\hat{z}$ direction, is forbidden in the specific nuclear reaction we consider.
Bohr Theorem \[ S_z = \text{even} \]

\[ S = 2 \Rightarrow S_z = 0, \pm 2 \]
E2 RADIATION PATTERNS

Fig. 202
\[ A = \frac{2}{3} \quad \text{or} \quad P = \frac{1}{3}. \]

The foregoing discussion applies to any reaction in which the only non-zero spin is possessed by the inelastically-scattered \( 2^+ \) particle, as was the case for Hayward's (Ha70) \( ^{12}\text{C}(a,a')^{12}\text{C}_2^+ \) experiment, or Steadman's (St74) \( ^{56}\text{Fe}(^{16}\text{O},^{16}\text{O})^{56}\text{Fe}_2^+ \) study. In our study the identical particles in the entrance channel imply that \( J = L \) must be even; parity conservation leads to even \( L' \) as well. Thus, (1) allows only \( L' = L, L-2, \) or \( L+2 \), and conservation of \( m_j = m_L + m_j \) means that for \( L' = L-2 \) (the "aligned configuration" of the band crossing model), \( A \) must be unity.

\[ A = P_{zz} = 1. \]

Equal populations of the three substates is represented by \( A = \frac{2}{3} \) or \( P_{zz} = \frac{1}{3} \).

2.2 Particle-gamma correlation function

Because the 4.439 MeV state in \( ^{12}\text{C} \) has a lifetime of 61 fsec (Aj80), it decays (via pure E2 radiation) long before it can be detected experimentally. The substate it was in while excited gives rise to a characteristic radiation pattern illustrated in fig. 202; thus, particle-gamma coincidence data can be used to deduce the magnetic substate population parameters in \( ^{12}\text{C}+^{12}\text{C} \) inelastic scattering.

Numerous authors have obtained angular correlation formulae, although comparisons of the expressions have been difficult because of differing phase conventions. Recently Rose and Brink (Ro67) have derived phase consistent expressions for gamma angular distributions, and Rybicki, Tamura, and Satchler (Ry70) have extended the work to include particle-gamma correlations following nuclear reactions: \( A(a,b)B(Y)C \). Our
carbon-carbon scattering is a particular case of the latter, so we use their density matrix approach to obtain the correlation function $W$ for our experiment. Equations from these two papers are referenced as RB or RTS.

In the reaction $^{12}\text{C}(^{12}\text{C},^{12}\text{C})^{12}\text{C} + (\gamma)^{12}\text{C}$, $J_{A}^a = s_{a} = s_{b} = J_{C} = 0$, $J_{B} = 2$, and the excited state de-excites to the ground state via $\gamma$-emission 100% of the time. We define the correlation function as the ratio of the double to the single differential cross section:

$$W(\psi, \phi) = \frac{\frac{d^2 \sigma}{d\Omega_b} d\Omega}{\frac{d\sigma}{d\Omega_b}} ,$$

where $\psi$ and $\phi$ are the polar and azimuthal angles of $\gamma$-emission with respect to a $z$-axis perpendicular to the reaction plane. (We reserve the symbol $\omega$ for the scattering angle of particle $b$ with respect to the beam axis.) (5) is normalized such that

$$\int_{4\pi} W(\psi, \phi) d\Omega = 1 ;$$

thus, our correlation function is $1/4\pi$ times the expression in (RTS9).

The density matrix for the substates is (Br68, p.108)

$$\rho_{m_m} = a_m a_m^* = \alpha_m \alpha_m e^{i(\beta_m - \beta_{m'})} .$$

Since $\Sigma_m (\alpha_m)^2 = 1$, the trace of $\rho$ is simply the differential cross section $d\sigma/d\Omega_b$. From the density matrix are formed the statistical tensors (RTS7)

$$T_{KQ} = \Sigma_m^', \rho_{m_m} (-)^{2-m'} < 2 2 m - m' | KQ >$$
appropriate for the case of an unpolarized beam and target, and when the
polarization of neither b nor γ is observed. RTS and RB contain the
detailed derivation of the single and double differential cross sections
from the amplitudes for inelastic scattering and γ-decay, which in
turn depend on \( T_{KQ} \) and spherical harmonics \( Y_K^Q(\psi, \phi) \). The
result is the correlation function (RTS10-12)

\[
W(\psi, \phi) = \frac{1}{4\pi} \sum_{KQ} \frac{T_{KQ}}{T_{00}} R_K \sqrt{\frac{4\pi}{2K+1}} Y_K^Q(\psi, \phi).
\]

\[
= \frac{1}{20\pi} \sum_{KQ} \frac{1}{2K+1} T_{KQ} R_K Y_K^Q(\psi, \phi).
\]  

(9)

\( R_K \) are angular distribution coefficients given by RB. In general they
allow for mixed gamma multipolarities \( L, L' \), but in the present case
\( L = L' = 2 \) only and (RB3.36) reduces to

\[
R_K(2 2 2 0) = -\sqrt{5} <2 2 1 -1 | K 0 > W(2 2 2 2; K 0).
\]

(10)

Multipolarity selection rules (RTS17)

\[
| L - L' | \leq K \leq | L + L' | \quad \text{and} \quad 0 \leq K \leq 2 J_B
\]

and non-observance of the γ-polarization (implying \( K = \text{even only} \)) limit
\( K \) to 0,2,4. Also, since \( Q = m - m' \) and \( -K \leq Q \leq K \) from the Clebsch-Gordon
coefficient in (8), our choice of quantization axis and the Bohr theorem
dictate \( Q \) to be even as well. Evaluation of Clebsch-Gordon and Racah
coefficients in (10) yields

\[
R_0 = 1 , \quad R_2 = -\sqrt{5/14} , \quad R_4 = -\sqrt{8/7} .
\]

(11)

Working out the algebra of (9) yields:

\[
W(\psi, \phi) = \left[ 1 - \alpha_0^2 + (\alpha_+^2 + \alpha_-^2)/4 + \frac{15}{2} \alpha_0^2 \cos^2 \psi \right.
\]

\[
- \left[ \frac{5}{4} (\alpha_+^2 + \alpha_-^2) + \frac{15}{2} \alpha_0^2 \right] \cos^4 \psi
\]
Using $\sum_m (\alpha_m)^2 = 1$ to simplify the algebra, (12) reduces to

$$W(\psi, \varphi) = \frac{5}{16\pi} \sin^2 \psi \left[ \alpha_+^2 + \alpha_-^2 + (1 + 5\alpha_0^2) \cos^2 \psi \right. \\
- 2\sqrt{5} \cos^2 \psi \left[ \alpha_+ \alpha_0 \cos(2\varphi + \beta_+ - \beta) + \alpha_0 \alpha_- \cos(2\varphi + \beta_0 - \beta) \right] \\
- 2\alpha_+ \alpha_- \sin^2 \psi \cos(4\varphi + \beta_+ - \beta_0) \right]. \quad (13)$$

If $\psi = 90^\circ$, then $\cos^2 \psi$ vanishes and the expression simplifies to

$$W(90^\circ, \varphi) = \frac{5}{16\pi} \left[ \alpha_+^2 + \alpha_-^2 - 2\alpha_+ \alpha_- \cos(4\varphi + \beta_+ - \beta_0) \right]. \quad (14)$$

This is independent of $\alpha_0$ and $\beta_0$, as we would expect because there is no $m=0$ radiation at $\psi = 90^\circ$. Also, $W(90^\circ, \varphi)$ is completely symmetric with respect to interchange of $m=+2$ and $-2$.

### 2.3 Identical particle symmetry

The two exiting carbon ions—one which had been excited and one which had not—are in principle distinguishable, but that is difficult to accomplish experimentally. The former would recoil after emission of a photon, so the peak in its energy spectrum would be Doppler broadened. In fact, recent experiments on $^{12}\text{C} + ^{16}\text{O} \to ^{12}\text{C}_2 + ^{16}\text{O}$ (Be80,Bo78) have exploited this fact to measure the energy spectrum of the carbon with a high resolution magnetic spectrometer and deduce substate populations from line shape analysis. But in our case the unexcited carbon arrives at practically the same energy as the excited, and resolution of the two would be extremely difficult. Therefore, our experiment measures both
\[ ^{12}C(^{12}C, ^{12}C_2^+)^{12}C \] and \[ ^{12}C(^{12}C, ^{12}C)^{12}C_2^+ \] intensities.

Fortuitously, this ambiguity in detection does not affect the formulae to be applied. We consider the scattering \(2(1,3)4^*\) in the center-of-mass system, depicted in fig. 203A. While keeping the detectors fixed in the laboratory, we rotate the reactants by 180° about the z-axis (panel B). Labels for identical particles 1 and 2 can be exchanged (panel C), yielding the reaction \(2(1,4^*)3\). A and C correspond to our two experimental cases, and the transformation from one to another is simply \(\phi \rightarrow \phi+180°\). (The change in angle from photon recoil is neglected here. At maximum it would be 1.0° CM, which is smaller than our particle angular acceptance.) Examination of (13) indicates that this transformation leaves \(W(\psi,\phi)\) unchanged. Thus, it is irrelevant to the analysis which particle, 3 or 4\(^*\), is detected from the inelastic scattering.

The position of the \(\gamma\)-detector in fig. 203 now enters into consideration. Inelastic scattering is detected in both A and C, while particle-gamma coincidences are seen only in A. In order to have our data correspond to the defined correlation function (5), the singles yield must be divided by 2. This is nothing more than the familiar factor needed for proper normalization of the differential cross section for identical particle scattering.

Although (14) is symmetric with respect to \(m=\pm 2\), the polarization of the excited state can be determined in general by a measurement of the circular polarization of the photon (Ha70). Such a determination is not possible in the present case because the z-axes of fig. 203A and C are antiparallel. By accepting both cases we accept both directions as
203) EFFECT OF NON-DISTINCTION OF FINAL PRODUCTS

The inelastic scattering is portrayed in the center-of-mass system, with nucleus 4 left in an excited state, and the detectors denoted by p (particle) and γ (γ-radiation). In all panels the quantization axis is $z = k_1 \times k_2$, and $\theta$ and $\phi$ are measured from the incident beam direction. After rotation of the system by 180° and interchange of the labels for identical particles 1 and 2, panels A and C differ in three important respects: i) the excited particle is detected in C; ii) the quantization axis is reversed; iii) the γ-radiation occurs at angle $\phi + \pi$ and is not observed in C. Our setup cannot distinguish between particles 3 and 4, so our data analysis must take ii) and iii) into account. Since we are interested in alignment but not polarization, ii) is not relevant. The transformation $\phi \rightarrow \phi + \pi$ implicit from iii) leaves the correlation function, eqn. (13), unchanged. However, we must scale the coincidence yield by a factor of 2 because of the non-detection of γ-radiation in panel C.
Fig. 203

\[ \hat{Z} = \hat{k}_1 \times \hat{k}_3 \]

2(1,3)4*

(A)

rotate by \( \pi \) about \( \hat{Z} \)

1(2,4*)3

(B)

interchange \( \downarrow \) labels 1, 2

2(1,4*)3

(C)
204) 90° INELASTIC SCATTERING OF IDENTICAL PARTICLES

Since the interaction regions denoted by the boxes are physically inaccessible, panels A and B correspond to quantum-mechanically indistinguishable reactions. Yet the quantization axes of the two cases have the opposite sense. Thus, as argued in the text, identical particle symmetry implies $a_+ = a_-$ and $b_+ = b_-$ for this scattering angle.
\[ \hat{Z} = \hat{k}_i \times \hat{k}_f \]
positive \hat{z}, and the distinction between \(m=+2\) and \(-2\) is blurred. Yet we can salvage some information from the difference between \(\alpha_+\) and \(\alpha_-\).

For the case of \(\theta=90^\circ\), fig. 204 illustrates two possible scatterings of identical particles that are quantum mechanically indistinguishable and, by symmetry, equally probable. As illustrated inside the boxes (corresponding to the physically inaccessible region), panels A and B correspond to anti-parallel \(z\)-axes. Hence, by symmetry we must have \(\alpha_+ = \alpha_-\) and \(\beta_+ = \beta_-\). Observation of this behavior at \(\theta=90^\circ\) is used to provide confirmation that our data are accurate.

2.4 Experimental measurement of the correlation function

For a given beam energy and scattering angle, (12) may be written as:

\[
W(\phi, \varphi) = c_0 + c_1 \cos^2 \phi + c_2 \cos^4 \phi + c_3 \sin^2 \phi \cos^2 \phi \cos 2\varphi
\]

\[
+ c_4 \sin^2 \phi \cos^2 \phi \sin 2\varphi + c_5 \sin^4 \phi \cos 4\varphi + c_6 \sin^4 \phi \sin 4\varphi. \tag{15}
\]

Hence, to obtain information from a "complete" angular correlation experiment, we need to fit the data to a six term function of \(\varphi\)-angle. As indicated in (14), if we choose to measure only the in-plane correlation, all terms with \(m=0\) drop out and (15) simplifies to:

\[
W(90^\circ, \varphi) = c_0 + c_5 \cos 4\varphi + c_6 \sin 4\varphi. \tag{16}
\]

Good fits to (16) are obtained with only a few well-spaced settings of \(\phi\) and with good counting statistics.
If the γ-detector subtends an appreciable solid angle—and it must if a reasonable coincidence rate is to be achieved—the general forms of $W$ must be adjusted to reflect that fact. The particle detector typically has a much smaller solid angle, and in consequence, averaging over its aperture is neglected.

The standard technique for correction of solid angle effects in angular correlation data is outlined in Marion and Young (Ma68). Correction factors to be applied to the various orders of Legendre polynomials in the fit are calculated and graphed in Ma68, and they apply either to the photopeak only or to the entire gamma spectrum. However, in our analysis we chose to include the photo and escape peaks of the photon spectrum. In addition, a fit to Legendre polynomials instead of to eqn. (16) will lead to undesired complications in obtaining the physical quantities of interest. Thus, we have defined the measured correlation function $W'$ to be

$$W'(\psi, \omega) = \frac{\int W(\psi, \omega) \, d\Omega}{\int d\Omega} \frac{\int_{\psi-\delta\psi}^{\psi+\delta\psi} \int_{\phi-\delta\phi}^{\phi+\delta\phi} W(\psi, \phi) \sin \psi \, d\psi}{\int_{\psi-\delta\psi}^{\psi+\delta\psi} \sin \psi \, d\psi}$$

where $\delta\psi$ and $\delta\phi$ are effective half angles subtended by the γ detector. The $\delta\psi$ and $\delta\phi$ are estimated from the geometry by replacing the circular cross section of the NaI crystal halfway through its depth by a square of equal area. It is assumed that the detector response function is flat over that solid angle. These definitions mock up the detailed interaction of the photons with matter. Our choice of
gamma spectrum acceptance lies between the two cases of Marion and Young. We performed fits to the data along the lines of the latter, and obtained results in close agreement to those corresponding to eqn. (16).

Letting $x = \cos \psi$, $W'$ is written as:

$$W'(\psi, \varphi) = c_0 + c_1 Q_2 x^2 + c_2 Q_4 x^4$$
$$+ K_2 (Q_2 x^2 - Q_4 x^4) (c_3 \cos 2\varphi + c_4 \sin 2\varphi)$$
$$+ K_4 (1 - 2Q_2 x^2 + Q_4 x^4) (c_5 \cos 4\varphi + c_6 \sin 4\varphi),$$

where

$$Q_2 = \cos^2 \delta\psi + \frac{1}{3} \tan^2 \psi \sin^2 \delta\psi$$
$$Q_4 = \cos^4 \delta\psi + 2 \tan^2 \psi \cos^2 \delta\psi \sin^2 \delta\psi + \frac{1}{5} \tan^4 \psi \sin^4 \delta\psi$$

$$K_2 = \frac{\sin 2\delta\varphi}{2\delta\varphi}, \quad K_4 = \frac{\sin 4\delta\varphi}{4\delta\varphi}.$$  \hfill (18)

In the reaction plane the correction factors included in (18) for $c_0$, $c_5$, and $c_6$ apply. Fig. 205 illustrates a typical measured in-plane correlation, a least squares fit to terms 0, 5, and 6 of (18) for $\delta\psi = 12^\circ$, and a curve generated with the same coefficients but without solid angle corrections.

Finally, we establish the relation between the fitted coefficients and the substate parameters. Comparison of (15) with (13) yields

$$c_0 = \frac{1}{4\pi} \left[ 1 - \alpha_0^2 + \frac{1}{4} (\alpha_+^2 + \alpha_-^2) \right] = \frac{5}{16\pi} (\alpha_+^2 + \alpha_-^2)$$

with proper normalization

$$c_1 = \frac{15}{8\pi} \alpha_0^2, \quad c_2 = -\frac{5}{8\pi} \left[ \frac{1}{2} (\alpha_+^2 + \alpha_-^2) + 3 \alpha_0^2 \right]$$

$$c_3 = -\frac{5\sqrt{6}}{8\pi} \left[ \alpha_+ \alpha_0 \cos (\beta_+ - \beta_0) + \alpha_0 \alpha_- \cos (\beta_0 - \beta_-) \right]$$

$$c_4 = \frac{5\sqrt{6}}{8\pi} \left[ \alpha_+ \alpha_0 \sin (\beta_+ - \beta_0) + \alpha_0 \alpha_- \sin (\beta_0 - \beta_-) \right]$$
205) TYPICAL IN-PLANE CORRELATION FUNCTION

These data were taken at $E=17.31$ MeV and $\theta=78.86^\circ$. The solid curve is a least squares fit to the data incorporating the correction factors $K$ and $Q$ of eqn. (18). To illustrate the effect of finite $\gamma$-angular acceptance, the dashed line is generated from the coefficients obtained from the least squares fit, but with the solid angle correction factors neglected.
Fig. 205
\[ c_5 = \frac{-5}{8 \pi} \alpha_+ \alpha_- \cos (\beta_+ - \beta_-), \quad c_6 = \frac{5}{8 \pi} \alpha_+ \alpha_- \sin (\beta_+ - \beta_-) \] (19)

Inverting these relations gives us our parameters of interest:

\[ \beta = \beta_+ - \beta_- = \tan^{-1} \left( \frac{-c_6}{c_5} \right) \]

\[ A = \alpha_+^2 + \alpha_-^2 = \frac{16 \pi}{5} c_0 \]

\[ P = \frac{|\alpha_+^2 - \alpha_-^2|}{\alpha_+^2 + \alpha_-^2} = \frac{\sqrt{c_0^2 - c_5^2 - c_6^2}}{c_0} \] (20)

2.5 Absolute normalization

From definition (5) it is clear that in order to measure a correlation function \( W \), we need only obtain simultaneous particle (p) and particle-gamma (p\( \gamma \)) yields—call them SING and COIN—and multiply by the appropriate factors to transform to differential cross sections. Because \( d/d\Omega \) is common to both numerator and denominator, factors appropriate to it need not be introduced because they cancel in the ratio. In fact, the only factor we need is a division of the \( p\gamma \) yield by the efficiency \( \varepsilon \) of the \( \gamma \) detector times its solid angle \( \Omega \). Let \( N=1/\varepsilon \Omega \), then \( W=N \times \text{COIN} / (\text{SING}/2) \).

Such a normalization is crucial in our experiment, since all the formulae assume \( \alpha_m \) are known in absolute terms. If we measure an in-plane correlation, we are sensitive only to \( \alpha_+ \) and \( \alpha_- \) directly, and the scale to be assigned to them is unknown.

In general we could perform Monte Carlo calculations for \( \varepsilon \Omega \) for a
given geometry, but the effects of attenuation and Compton scattering from shielding and other objects near the crystals might invalidate the results. It was most desirable to determine N experimentally for our particular experimental apparatus.

Because of its significance to our final results, three different determinations of N were performed. First, following the method used by Hayward and Schmidt (Ha70), low energy inelastic proton scattering on $^{12}$C was performed, and the $p'$ and $\gamma$ angular distributions were separately measured and integrated to give N. Second, we examined alpha particle scattering on $^{12}$C and compared our measured alignment to the results in ref. Ha69. Finally, we performed a "complete" angular correlation experiment, measuring $W$ both in and out of the reaction plane. By fitting those data to the general form (15) and extracting values for $a_0^2$ and $a_+^2 + a_-^2$, imposition of unitarity yielded the value of N. The details of these techniques, their results and errors, and the final value adopted for N are discussed in chapter 4.
3. EXPERIMENTAL TECHNIQUE AND DATA REDUCTION

3.1 Beam and targets

The experiments were performed using beams accelerated at Yale's MP-1 Tandem Van de Graaff facility. The $^{12}$C beams of energy 31.5 to 65 MeV used for the measurements and for one normalization procedure were extracted from a hollow carbon cone in a UNIS sputter-ion source, the latter manufactured by Extrion. The other two normalization runs used 10.3 MeV protons and 22.75 MeV alpha beams extracted from the duoplasmatron source manufactured by High Voltage Engineering Corporation. All beams emerged as $q$=-1 ions from the source, and were stripped to $q$=+1 to +5 charge states by carbon foils or a nitrogen gas stripper in the accelerator terminal. The accelerated ions were momentum analyzed to ±0.1% resolution and transported to the laboratory
gamma cave, where all the experiments were performed. In this cave the background $\gamma$-radiation is low because of extensive shielding and walls constructed from concrete low in radioactive Th and K. In addition, a beam dump was constructed 5 m downstream of the scattering chamber and shielded by paraffin, borax, and lead (in that order) to absorb neutrons and photons. This feature was most important for normalization runs in which $\gamma$-singles were measured, as in the (p,p') experiment described in sect. 4.1. The beam dump was electrically isolated and used as a Faraday cup to collect beam currents, which were typically 40-200 na for experiments using $^{12}$C beams.

The targets were natural carbon foils of nominal areal density 100 $\mu$g/cm$^2$. Background scattering from the 1% $^{13}$C and other contaminants present in the target was found to be of negligible importance, reflecting the relatively high cross section for the $^{12}$C+$^{12}$C$^*$ interaction. A $^{12}$C beam of 31.5-65 MeV loses 200-120 keV as it passes through these targets (No70). All measurements are quoted for center-of-mass energies E corresponding to a point midway through the target. The resultant reaction energy resolution, 0.3-0.1%, is about a factor of 3-1 larger than the intrinsic resolution of the beam from the analyzing magnet.

3.2 Detection of particles and gamma radiation

Fig. 301 is a schematic diagram of the reaction. Particles are scattered at an angle $\theta$ and gamma rays are emitted at polar angle $\psi$ and azimuthal angle $\phi$ with respect to the magnetic quantization axis.
301) DEFINITION OF ANGLES AND DETECTORS

18 slits define particle scattering angles $\theta$ into a position sensitive detector PSD1. PSD2 detects recoil particles in kinematic coincidence in the "p-p singles" mode. $\gamma$-radiation is detected at angles $(\psi, \phi)$ with respect to the coordinate axes as shown.

302) IN-PLANE CORRELATION EXPERIMENTS

This figure gives the same information as fig. 301, but includes the details of beam collimation and shielding. A tin sheet is placed between the lead and the NaI crystal to absorb x-rays. Detectors NaI B and PSD2 were added only for the later experimental runs; for specifics see table I.
Fig. 301

$^{12}\text{C}(^{12}\text{C},^{12}\text{C})^{12}\text{C}^*$

Beam

$\phi$

$\theta$

PSD 1

PSD 2

5x5" NaI
### I) PARAMETERS OF IN-PLANE RUNS

<table>
<thead>
<tr>
<th>Code</th>
<th>K</th>
<th>L</th>
<th>M</th>
<th>N</th>
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<td>20.1</td>
<td>20.1</td>
<td>25.6</td>
<td>25.6</td>
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<tr>
<td>$\delta \psi = \delta \phi$</td>
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<td>$12^\circ$</td>
<td>$10^\circ$</td>
<td>$10^\circ$</td>
</tr>
<tr>
<td>PSD2 (p-p singles)</td>
<td>no</td>
<td>no</td>
<td>yes</td>
<td>no</td>
</tr>
<tr>
<td>NaI B</td>
<td>no</td>
<td>no</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Energies (MeV)</td>
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<td>17.56</td>
<td>17.91</td>
<td>16.81</td>
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<td></td>
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perpendicular to the reaction plane. Fig. 302 shows the experimental instrumentation. A Si surface barrier position sensitive detector (active area 8 x 47 mm, manufactured by Ortec), denoted by PSD1, is mounted inside the scattering chamber. A thin brass mask having 18 vertical slits placed in front of the detector defines the scattering angle $\theta$. For runs with $E \geq 21.5$ MeV a second PSD was positioned to capture all recoils from $^{12}\text{C}+^{12}\text{C} \rightarrow ^{12}\text{C}+^{12}\text{C}_2^+$ reactions in which a reaction product was detected in PSD1; its purpose is discussed later in this section.

Gamma rays were detected in a 12.7x12.7 cm cylindrical NaI(Tl) scintillator whose front face was 20-25 cm from the target. For some runs it was mounted on a goniometer, such that it could be positioned at any desired angle in the upper hemisphere ($0 \leq \psi \leq 90^\circ$). For runs M and N (see table I for a summary of relevant data and nomenclature on all runs of this experiment) two identical 12.7 x 12.7 cm crystals mounted on a rotatable table, as shown in fig. 302, were used to expedite the in-plane data collection.

Attenuation of 4.44 MeV photons in the reaction plane by the 1/4 in. thick Al scattering chamber can be estimated (using $\mu/\rho = 0.030$ cm$^2$/gm from Ev55, p. 715) to be 5%. A check of the $\gamma$-singles yields indicates the attenuation is uniform with respect to $\psi$ to within 1%, which means the effects of the target frame and the recoil detector on the attenuation can be ignored. Even if these latter factors were significant, they would only play a small effect in the determination of $A$, $P$, and $\beta$ from the coefficients of the least-squares fit to the data $W(90^\circ, \phi)$. The effects of the $\phi$-independent attenuation are included.
in the determination of the absolute normalization $N$. Attenuation out of the reaction plane is discussed in sect. 4.3.

The slits in front of the defining detector each subtended 0.7° in the laboratory system, and were positioned by optical telescopic alignment. The beam was defined by a 2 mm diameter Ta collimator 50 cm upstream of the target, with a maximum possible shift of 1.0° of the measured lab scattering angle because of beam wandering. Thus, the data are an average over 1.4° CM. The quoted angles at the centers of the slits are accurate to within ±2.0° CM.

The cleanest way to perform the experiment is through use of a particle identification system. One possibility is a ΔE-E telescope. The standard thin Si ΔE detectors (10 μm) would be too thick for the carbon ions of interest to us, but a gas ΔE cell could be constructed to surround the surface barrier PSD. The most readily available particle identification system, however, was a kinematic coincidence between the ejectile and recoiling nucleus of the two body reaction. The following discussion explains the technique and demonstrates its adequacy for the task at hand.

Fig. 303 presents the two body kinematics for $^{12}\text{C}+^{12}\text{C}$ reactions at $E=21.92$ MeV, with panel A displaying energy vs. angle in the defining detector for five possible reactions. In the angular range accepted by PSD1 (25-45° in the laboratory frame), it follows that alpha particles from $^{12}\text{C}(^{12}\text{C},\alpha)^{20}\text{Ne}^*(E_X=7-15$ MeV) or two alpha particles from $^{12}\text{C}(^{12}\text{C},^8\text{Be})^{16}\text{O}^*(E_X=5-9$ MeV) might be mistaken for a carbon from $^{12}\text{C}+^{12}\text{C}^*$. Here values of $E_X=11$ and 7 MeV respectively are taken for display purposes. If these events are gated by 4.439 MeV photons, then
303) TWO BODY KINEMATICS

The five most significant exit channels of the $^{12}\text{C}+^{12}\text{C}$ reaction are considered, with the energy and angle of particles detected in PSD1 and PSD2 graphed pairwise. Sect. 3.2 discusses how this technique separates other events from the carbon-carbon exit channels of interest.
Fig. 303
such background is unlikely to occur in the coincidence yield. But the singles yield is not so guarded, and such spectral contamination would tend to lower $W$ and hence decrease the measured alignment. Thus, a second detector was installed to detect recoiling particles from the reaction. The range subtended by $\theta_2$ was chosen to insure complete kinematic coincidence with inelastic events. As panel B indicates, there is still the possibility of detecting alpha particles from the $^8\text{Be}$ reaction. But as panel C illustrates, if we analyze the data as $E_2$ vs. $\theta_1$, then $^{12}\text{C}$ events are grouped in a band of energies whereas $^8\text{Be}$ ions are much less energetic.

Our recoil detector was position sensitive, but since the two body kinematics indicated that only the energy signal was needed to eliminate the background, we chose to use only that signal; i.e., PSD2 operated only as a large area particle energy detector. Because of its large solid angle—required for complete kinematic coincidence—PSD2 could only be exposed to about one-fifth the beam desired for standard particle-gamma coincidence (p$\gamma$) runs. Thus, brief kinematic coincidence (pp) runs were taken both before and after a sequence of p$\gamma$ runs in order to obtain the singles angular distribution. These yields were then scaled to the longer p$\gamma$ runs via integrated beam charge. During p$\gamma$ runs PSD2 was protected from excessive particle flux by a thick metal shield positioned between it and the target.
3.3 Electronics

Signals from the particle and gamma detectors were pre-amplified in the gamma cave, then routed to a cubicle where coincidence timing was performed and signals prepared for input into the IBM 4341 computer-based data acquisition system for event-by-event logging and on-line analysis.

Fig. 304A illustrates the preamps used and the source of the fast (rise time ~50 ns) signals for the defining particle detector and the gamma detector(s). Fig. 304B shows how these 8 signals were then amplified, put in proper time sequence, and used to generate logic and TAC (time to amplitude conversion) signals. Much of this instrumentation is standard nuclear physics technique, using commercially available NIM modules, but two points concerning fig. 304B bear mentioning.

First, in the fast timing system, constant fraction discrimination is used to obtain timing resolution of ~10 ns full width at half maximum in the TAC peak. This process is somewhat trickier than setting fast timing with just scintillators or small Si detectors, because the timing quality of the PSD varies with position along the detector's length. The slowest rise times are from signals originating in the center, and adjustments must be made to optimize the resolution across the entire length of the PSD.

The second feature of note in fig. 304B is the prescaling operation (PS) inserted after the generation of γ and particle singles events. We recall from sect. 2.5 that we need to measure the ratio COIN/SING,
Panel A summarizes the detectors and the preamplifiers at the scattering chamber. The output signals are then sent to NIM electronics in the control area for amplification, timing, and generation of logic signals, as indicated in panel B. Final logic (S or C) or linear signals denoted by the circles are input to the computer interface, which treats them in a manner summarized in table II. All abbreviations and model numbers in this figure refer to solid state components manufactured by Ortec, with the exceptions of the Canberra 2003 preamplifier and the NaI preamplifiers built by the WNSL electronics staff.
Front and Back Gamma Detectors

Fig. 304 A
II) FRONT END CONFIGURATION

The rows correspond to the linear output signals in fig. 304B. The columns correspond to the logic signals of each event.

<table>
<thead>
<tr>
<th>Event</th>
<th>CF</th>
<th>CB</th>
<th>SF</th>
<th>SB</th>
<th>SP</th>
</tr>
</thead>
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<tr>
<td>TF</td>
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</tr>
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<tr>
<td>TB</td>
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</tr>
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<td></td>
<td></td>
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<td>X</td>
</tr>
<tr>
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</tr>
<tr>
<td>E1</td>
<td>X</td>
<td></td>
<td>X</td>
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</tr>
</tbody>
</table>
and in order to obtain results with small statistical errors, a given total number of events analyzed by the computer should be approximately half COIN and half SING. The rate-determining step in this experiment is the time for data acquisition and analysis, so we are free to arrange our modular electronics to optimize the mix of signals going into the computer interface, known locally as the "front end." From published εΩ values for NaI crystals we expect <1% efficiency for our system, so that a "hardwired" prescale of 100 in the singles means that we can accept a greater beam flux and improve our counting statistics without increasing the dead time of the computer. The prescale factors were set at 1000 for γ singles, 100 for normal particle singles (when the mask is in place in front of PSD2 and the AND simply passes E₁ events), and 1 when pp events were being accumulated.

The result of the operations shown in fig. 304B is 5 distinct events: singles in the particle detector(s), singles in the front (F) or back (B) γ-detectors, and coincidences between PSD1 and either F or B. Table II indicates which of the 7 output linear signals correspond to each of these events.

Signals denoted by circles in fig. 304B were routed to the front end, where the linear signals were fed into 12-bit analog-to-digital converters (ADC) whose contents were read by the computer's CPU in response to the appropriate logic signal. The electronics displayed in fig. 304B do not obviously preclude an event being simultaneously a singles and a coincidence, say S_p and C_B; in fact, we would expect 1% of C_B to also be S_p. The event exclusion feature on the front end, plus the relative timing of C_B or C_F 1 μs before S_p,
insured that such events were analyzed as coincidences, thus retaining the most information available. Scalers read by the front end provided a measure of how much the $S_p$ yield should be increased to compensate for this exclusion.

3.4 Event analysis

The event routine in use was capable of analyzing data on line allowing us to monitor the progress and obtain at least preliminary real-time results during our experiments. All events were also written onto tape, enabling the data to be "replayed" for final analysis so that the gates could be optimally set.

To obtain the correlation function $W=N \times \text{COIN/(SING/2)}$, two types of events were analyzed in somewhat different fashion because of the different information available from them.

Fig. 305 outlines the logic of the pp event analysis. As discussed above in relation to fig. 303, parameters $E_2$ and $\theta_1$ were first tested in order to separate carbon ions from other possible background. The software gate depicted in fig. 305A was drawn, and all events which lie within it were processed further. Panel B illustrates the $E_1$ vs. $\theta_1$ analyzer, exhibiting the parallel lines we would expect for elastic, inelastic, and mutual inelastic carbon scattering. Fig. 306 is a photograph of the display for the analyzer from a typical pp run. The intensity of the dots indicates on a logarithmic scale the number of counts for particular bins in $E_1$ and $\theta_1$. Finally, as fig. 305C
305) ANALYSIS OF P-P EVENTS

These three panels summarize the on-line computer analysis described in sect. 3.4, leading to a yield for \( ^{12}\text{C}(^{12}\text{C},^{12}\text{C})^{12}\text{C}(2^+) \) events for 18 distinct angles.

306) TYPICAL \( E_1 \) VS. \( \theta_1 \) SPECTRUM

This photograph from the computer-controlled display is of the analyzer depicted in fig. 305B. The intensity of the dots is proportional, on a logarithmic scale, to the counts in each \((E_1,X_1)\) bin. The relationship between \( X \), the position along the length of the PSD, and \( \theta \), the particle scattering angle, is practically linear.
ANALYSIS OF p-p EVENTS

(A) (B) (C)

- carbon
- elast
- inel
- mut
illustrates, each of the gates in \( E_1 \) vs. \( \theta_1 \) admits only one reaction. Events that pass through give a histogram whose peak sums are points on the angular distribution. As seen in fig. 307, the slits are distinct, yielding unambiguous sums for each peak. These are the SING yields, with errors assigned statistically (i.e., the square root of the number of counts).

As noted in table I, some runs did not utilize PSD2 or NaI B, so the portions of figs. 301-304 relevant to those detectors were excluded in the analysis. In particular, the analysis outlined in fig. 305 would start in panel B.

The analysis of particle-gamma coincidence (p\( \gamma \)) events is summarized in fig. 308. The four linear signals per event were studied pairwise to obtain the final yield. First the TAC signal was analyzed vs. \( \theta_1 \), and a software gate drawn as shown in fig. 309 for further processing of true coincidences. Random coincidences, indicated by the scattered, less intense dots in fig. 309, were subtracted from events falling within the gate by generating a second gate in T vs. \( \theta_1 \) space with a simple T translation of the first gate. The limits of the gate vary with \( \theta_1 \), as expected from the poorer timing qualities of signals from the center of PSD1. If all events within the gate of fig. 309 were projected onto the T axis, the full width at half maximum (FWHM) of the TAC peak would be 20-30 ns. If only one slit near the end of PSD1 is considered, the time resolution improves to \( \sim 10 \) ns.

Analyzers depicted in panels B and C of fig. 308 were similar to those previously described in the pp analysis. A notable difference is that elastic scattering is missing here-- an expected result because it
307) TYPICAL ANGULAR DISTRIBUTION SPECTRUM

This display photograph shows, as a linear histogram, the raw data for elastic scattering to 17 angles defined by the slit mask in front of PSD1. (Signals from the first slit were lost in the electronic noise.) The peaks are cleanly separated, making the assignment of practically every event to a particular scattering angle unambiguous.

308) ANALYSIS OF P-γ EVENTS

These four panels summarize the on-line computer analysis, described in sect. 3.4, leading to the yield for particle-gamma radiation coincidences in the reaction $^{12}$C($^{12}$C,$^{12}$C)$^{12}$C*(γ)$^{12}$C.

309) TYPICAL T₁ VS. θ₁ SPECTRUM

This display photograph illustrates the analyzer of fig. 308A on a logarithmic scale. The area with a large number of counts, representing true p-γ coincidences, are surrounded by the limits of the software gate for further event analysis.
ANALYSIS OF $p$-$\gamma$ EVENTS

Fig. 308
is not accompanied by emission of a photon. Gates in B and bins in C are identical for both pp and pγ analysis, so that the ratio COIN/SING is obtained consistently.

Finally we consider the photon energy, depicted in panel D, in order to further eliminate background. Our coincidence yield is taken from only the photo and escape peaks for the 4.439 MeV radiation. Some 6.13 MeV photons, originating from the reaction $^{12}\text{C}(^{12}\text{C},2\alpha)^{16}\text{O}^*$, were detected in the experiment, and Compton scattering within the NaI crystal of these photons leads to a background in the 4.439 MeV event window that varies with the probability of that reaction—hence, with energy and angle. But by previously measuring the spectrum shape of this detector for 6.13 MeV photons only, a known fraction (35\%, with uncertainty ±3\%) of the counts in the dotted region of fig. 308 are subtracted as the Compton background in the shaded region. The result is the coincidence yield COIN, with an error assigned statistically to both the dotted and shaded sums and propagated in quadrature for the difference of the two numbers.

3.5 Experimental correlation function

Our data are now used to generate the correlation function

\[ W = N \times \text{COIN}/(\text{SING}/2) \]

the overall normalization constant \( N \) is discussed in sect. 2.5 and chap. 4. In addition, there are a number of other corrections to be made in extracting the true value of \( W \) from our data.

In order to confirm the reliability of our data, it is important to
compared our inelastic angular distribution (SING multiplied by an experimentally-measured slit-to-slit relative solid angle factor) with differential cross sections previously measured at Yale (Wi73,Er79b) and at Heidelberg (Em73). The pp runs produced excellent agreement with these other $d\sigma/d\Omega$ data, while the p runs— at least for certain energies and angles where the $^{12}\text{C}+^{12}\text{C}^*$ cross section is low— resulted in a significant background in our data. We scaled SING for specific energies and angles by a factor determined by comparison with the Wi73, Er79b, and Em73 data. We estimate the systematic error from this procedure to be <5%. This correction is only made on SING because COIN utilized the $T$ and $E_\gamma$ signals to reduce such contamination to a negligible level.

We also applied other corrections necessitated by the particular configuration of the instrumentation. The event exclusion mentioned previously— in which one event was interpreted as both $p\gamma$ and $p$ by the electronics but analyzed only as $p\gamma$ by the computer— increased SING by approximately 1%. When the pp method was utilized, a slightly different computer dead time correction for the pp and $p\gamma$ runs (0.1%) was needed to properly divide COIN and SING. With separate NaI detectors used to measure forward ($\phi=40,60,80^\circ$) and backward ($100,120,140^\circ$) $\gamma$-angles, a match-on factor of 0.98 was introduced because of the slightly different performance of the two crystals.

The most significant additional correction is the change in $p\gamma$ solid angle because of relativistic effects. For typical angles and energies we measured, carbon ions emerge from the reaction with
velocities approximately 7% of the speed of light, c. Thus, a first order relativistic correction is sufficient:

\[ W(\psi, \varphi) = W(\psi', \varphi') \frac{d\Omega'}{d\Omega} = W(\psi, \varphi')(1 + 2\frac{v}{c}\cos \Theta + \ldots) . \]  

(21)

The primed angles refer to the rest frame of the source, the unprimed angles refer to the laboratory frame, and \( \Theta \) is the angle between photon emission and particle velocity \( \vec{v} \). Neglect of this factor skews the data and leads to erroneous fits. If \( ^{12}\text{C} \) and \( ^{12}\text{C}^* \) were distinguished by our experiment, the correction could vary as much as 14%. Since we do not make that distinction, the relativistic correction factor is averaged over this ambiguity. The factor can be as great as 8%, which is quite significant within the accuracy of our experiment.

A number of correction factors normally needed for differential cross section measurements are unnecessary in this correlation work. For example, target thickness buildup, integrated beam charge, electronic dead time, and equilibrium charge state corrections all affect COIN and SING equally, and thus cancel out when calculating \( W \).

Some angular correlation experiments must include corrections for the hyperfine interaction between the nuclear spin and the atomic electrons. The magnetic field \( H_0 \) at the nucleus because of the unpaired orbiting electrons can change the magnetic substate of the nuclear spin \( I \), so the angular correlation formulae should include attenuation coefficients \( Q_K \) to account for this effect. Berant, et al., (Be71) give these as

\[ Q_K = 1 - \frac{K(K+1)}{(2I+1)^2} \frac{(\omega \tau)^2}{1 + (\omega \tau)^2} , \text{ where } \omega = \frac{\mu N g H_0}{\hbar} (2I+1). \]  

(22)
For our experiment we estimate $H_0 = 36$ MG and $\omega = 4 \times 10^{11}$ sec$^{-1}$, assuming $g = 0.5$ for the $2_1^+$ state of $^{12}$C. But the lifetime of that state is $\tau = 61$ fsec, so evaluation of (22) implies that $Q_K$ differs from 1 by less than one part per thousand for $K = 2$ or 4, and thus it is negligible in this experiment.

3.6 Least squares fit to the correlation function

The final experimental data for $W$ are tabulated in appendix A.1. That table reflects the value of $N$ adopted in sect. 4.4, but the errors listed arise only from statistical and experimental factors listed in this chapter. The systematic error assigned to the final value of $N$ is not properly attached to the data until after the fitting, so it is not included in A.1.

A standard least squares fit (see Be69, program REGRES) is made to the data of A.1 by eqn. (16), and the errors calculated for the coefficients by the fitting procedure are propagated in quadrature to obtain errors for $A$, $P$, and $\beta$. These results are tabulated in A.2. The errors for $A$ should be considered only relative: angle-to-angle and

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1 There has been great recent interest in the measurement of moments of short-lived nuclear states via the method of fast ions traversing ferromagnetic media (Be80a). Yet this advanced method is still not adequate for $^{12}$C($2^+$) -- in fact, because of its brief lifetime, that state has been used to probe the method itself (Go76). A reasonable estimate of the g-factor is given by the collective model (Al64): $g = Z/A = 0.5$ for $^{12}$C. Measured values (Gr68) for $2^+$ rotational states in other even-even nuclei ($A = 150-192$) are in the range $g = 0.25-0.40$, which is slightly less than the rigid rotor prediction $Z/A$, due to the pairing interactions of the nucleons.
energy-to-energy. In order to interpret these in terms of absolute alignment, the error in N must be folded in, as discussed in sect. 4.4.
4. ABSOLUTE NORMALIZATION

4.1 Inelastic proton scattering on $^{12}\text{C}$

We need the factor $\varepsilon\Omega$ for the $\gamma$-detector in order to properly normalize the angular correlation data. In principle, any reaction we can measure that produces photons of the desired energy will suffice. In practice, we wish to choose a simple reaction at as low an energy as possible, to keep background at a minimum. Thus, we elected to examine the reaction $^{12}\text{C}(p,p')^{12}\text{C}_2^+$. For each proton exciting the first $2^+$ state in $^{12}\text{C}$ there is exactly one 4.439 MeV photon emitted. Hence, if $Y$ is the singles yield, then

$$N = \frac{1}{\varepsilon\Omega} \frac{\int Y(p) \, d\Omega_p}{\int Y(\gamma) \, d\Omega_\gamma} = \frac{2\pi}{2\pi} \frac{\int Y(p) \sin\theta \, d\theta}{\int Y(\gamma) \sin\theta \, d\theta} \quad (23)$$
where we have exploited the cylindrical symmetry about the beam axis to eliminate integration over $\phi$, and assumed unit efficiency for the proton detector.

Fig. 401 shows the experimental arrangement used. Inelastically scattered protons were detected by a Si surface barrier detector. Its solid angle was defined by a rectangular collimator, behind which was placed a 2.3 mg/cm$^2$ Ni foil that absorbed recoiling carbon ions but allowed protons to pass. This detector was in a rotatable mount in the same scattering chamber as was used for the alignment measurement, so the value of $e\Omega$ obtained is applicable to the normalization of the latter data.

The beam impinged on a natural carbon foil of nominal areal density 100 $\mu$g/cm$^2$, as in the other experiments. The target was placed at angle $\tau=45^\circ$ with respect to the beam axis for measurement of protons forward of 90$^\circ$, and at $\tau=135^\circ$ for backward protons, in order that the target frame did not block the ejectiles. A monitor detector, fixed below the reaction plane (as defined by the particle detector) at 158$^\circ$ with respect to the beam, detected elastic scattering events for each run. These normalization data were used to facilitate the match of $\tau=45^\circ$ and 135$^\circ$ data near $\theta=90^\circ$, and also to monitor possible target thickness buildup during the runs. (None was detected.)

Gamma radiation was detected by the same 12.7x12.7 cm NaI scintillator used in the other runs. The shielding, the geometry, and the scattering chamber are also identical to those discussed in chap. 3. In addition to measuring the gamma angular distribution, the detector was varied in distance D from the target at a fixed angle. Knowledge of
401) INELASTIC PROTON SCATTERING NORMALIZATION EXPERIMENT

The experimental arrangement is similar to fig. 302, except that the particles are detected by a small surface barrier Si detector, and a Ni foil absorbs recoiling carbon ions.

402) GAMMA RADIATION SPECTRUM

The largest yield arises from de-excitation of the 4.439 MeV state in $^{12}$C, and the summing window (shaded region) for those events includes the photo and escape peaks. The background from $^{16}$O(3$^-$) and neutrons is an order of magnitude smaller, and is subtracted as shown. Other discrete peaks arising (see inset) from (p,p') and (p,α) reactions on $^{27}$Al (En78) do not interfere with our sum.
\( \gamma \)-radiation spectrum

Counts

Channel Number

10^5

10^4

10^3

10^2

10

0

200

400

600

800

1000

\( \beta^- \)\n
24\text{No}

2.98

2.73

2.21

1.37

24\text{Mg}

27\text{Al}

0.840

1.01

6.13 D.E.

6.13 S.E.

6.13

4.44 S.E.

4.44 D.E.

2.21 S.E.

2.98 S.E.

1.37 S.E.

0.840
the D-dependence of εΩ is needed to match alignment runs that took place at different distances.

The beam energy chosen for the protons was 10.3 MeV. Although this is somewhat above the threshold of 6.52 MeV for the $^{16}$O(p,p')$^{16}$O(6.13 MeV) reaction, a negligible number of 6.13 MeV photons were detected, indicating that $^{16}$O contamination of the target was not significant. The higher energy beam was necessary for stable operation of the accelerator. As mentioned in sect. 3.1, the beam dump was 5 m away from the θ-detector and heavily shielded with paraffin, borax, and lead; this kept the neutron background small, as shown in fig. 402. Other gamma radiation, identified in the figure as originating from (p,p') and (p,α) reactions on $^{27}$Al, appeared in the spectrum below 3 MeV, and thus did not affect our region of interest, the photo and escape peaks of 4.439 MeV.

The proton and gamma angular distributions are illustrated in figs. 403 and 404. The data were divided by a pulser yield that was triggered by the integrated beam charge. This normalized the proton and gamma yields not only to total beam flux, but also to the electronic dead time for each run.

In addition, the particle counts were multiplied by the solid angle factor sin θ. Thus, to get an angle-integrated yield, we need only integrate the area under the curve from 0 to 180°. Using Simpson's rule, the trapezoidal rule, and a parabolic interpolation for 26°<θ<146° give results consistent to within 1%:

$$\Delta \Omega \int_{26^°}^{146^°} Y(p) \sin \theta \, d\theta = 1.082 \pm 0.024 \, /nC.$$ (24)
403) PROTON ANGULAR DISTRIBUTION

The inelastically-scattered proton yields are multiplied by the solid angle factor $\sin \theta$ and normalized to the total beam flux (corrected for electronic dead time). The curve passing through the experimental points is extrapolated to zero at $0^\circ$ and $180^\circ$, and the area under it is proportional to the integrated cross section.

404) GAMMA RADIATION ANGULAR DISTRIBUTION

The 4.439 MeV $\gamma$-radiation yields, obtained from the spectra as indicated in fig. 402, are normalized to the total beam flux and plotted versus angle. Statistical errors are smaller than the size of the dots. The solid line is a least squares fit of the data to eqn. (28); those parameters are then used to calculate the area under the curve.
Fig. 4.3

\[ \text{CTS} \cdot \sin \theta / \text{PULSER} \]

\[ \theta_{\text{lab}} \]
We extrapolate to zero at $\theta=0$ and $180^\circ$ by a straight line, with the error in calculating the areas of those triangles estimated at 5\%.

Angular regions 0-26$^\circ$ and 146-180$^\circ$ account for 13\% of the total area under the curve, which is $1.249\pm.032$. The solid angle of the collimator in front of the detector was $0.765\pm.090$ mrs, so using axial symmetry we obtain

$$2\pi \int_0^\pi Y(p) \sin \theta \, d\theta = 10260 \pm 1230 / \text{nC}. \quad (25)$$

The photons in fig. 404 emerged from the inelastic scattering with a characteristic angular distribution $W$:

$$W(\theta) = c_1 + c_2 \cos 2\theta + c_3 \cos 4\theta. \quad (26)$$

Experimentally we measured an angular distribution $W'$ averaged over the solid angle of the detector:

$$W' = \frac{\int W \, d\Omega}{\int d\Omega} = \frac{\int W(\theta) \sin \theta \, d\theta}{\int \sin \theta \, d\theta}. \quad (27)$$

To obtain the coefficients in (26) we must fit the experimental data to an expression that takes the finite solid angle of the NaI into account. To first order, that is

$$W' = c_1 + c_2 K_2 \cos 2\theta + c_3 K_4 \cos 4\theta. \quad (28)$$

where $K_n = \sin(n\delta\theta)/n\delta\theta$ and $\delta\theta =$ half angle of the NaI, as in sect. 2.4.

The solid line in fig. 404 is a fit to (28). The coefficients are inserted into (26) to obtain $W(\theta)$, which replaces $Y(\gamma)$ for our integration. The total number of photons that were detected in $4\pi$ sr

...
\[ 2 \pi \int_0^{\pi} W(\theta) \sin \theta \, d\theta = 618.0 \pm 18.5 \, \text{nC}. \tag{29} \]

Finally, substitution of (25) and (29) into (23) yields

\[ N = \frac{10260 \pm 1230}{618.0 \pm 18.5} = 16.60 \pm 2.05 \tag{30} \]

for \( D = 26.5 \, \text{cm} \), with an uncertainty of 12% reflecting, almost entirely, the uncertainty in the determination of the solid angle of the particle detector.

Fig. 405 gives the distance variation of \( Y(\gamma) \) for constant \( \theta_\gamma \).

The data are not only from the present experiment, but also from a later check using a PuBe source of 4.439 MeV photons replacing the target. Angle-averaging effects must be included in the former data, because of the characteristic distribution (25) with respect to the beam direction, but such effects do not enter for an isotropic source. Assuming an exponential law \( Y(\gamma) = cD^n \), we find a least squares fit to

\[ \log Y = \log c + n \log D \tag{31} \]

yields \( n = -1.61 \) for the \((p,p')\) data and \( n = -1.74 \) for the PuBe data. The configuration leading to the former result is more closely similar to the correlation measurements than the latter. Two reasons are the finite extent of the PuBe source and the necessary absence of the scattering chamber lid during the measurements. Thus, we adopt \( n = -1.61 \) as the correct exponent, but in consideration of the fit to the PuBe data, we fold in an error of \( \sqrt{1 - (D/26.5 \, \text{cm})^{-13}} \) in quadrature with the 12% above.
405) VARIATION OF $\gamma$-YIELD WITH DISTANCE

The indicated yields were normalized to the total beam flux (for the $^{12}\text{C}(p,p'\pi)$ data) or to time (for the PuBe source of 4.439 MeV photons), with suitable corrections for electronic dead time. For all points the statistical errors are smaller than the circles or triangles. The solid lines are least squares fits to eqn. (31).

406) COMPARISON OF $^{12}\text{C}(\alpha',\alpha)^{12}\text{C}(2^+)$ ALIGNMENT MEASUREMENTS

Open circles indicate the measurements of Hayward (Ha69), and the solid dots are our results, multiplied by a single factor for normalization to the former data. Except where indicated, the errors are smaller than the size of the circles.
Fig. 405

$\gamma$ yield (relative)

$D (\text{cm})$

$^{12}\text{C} (p, p')$ experiment

PuBe Source
From this \((p,p')\) experiment the absolute normalization factors for the alignment runs are determined to be:

\[ N = 15.70 \pm 1.94 \text{ for } D = 25.6 \text{ cm} \]

\[ N = 10.64 \pm 1.33 \text{ for } D = 20.1 \text{ cm}. \]

4.2 Inelastic alpha particle scattering on \(^{12}\text{C}\)

As mentioned in sect. 4.1, the method of \((p,p')\) scattering was used by Hayward (Ha69) to normalize his \(^{12}\text{C}(\alpha,\alpha')^{12}\text{C}^*\) substate population data. Since our experiment uses a similar technique, another way for us to obtain \(N\) is to remeasure part of Hayward's experiment performed with a 22.75 MeV alpha particle beam, and to check the agreement. This is rather easy with our experimental arrangement, since the position sensitive detector measures many particle angles simultaneously. Our slits were positioned to subtend \(1.2^\circ\) \text{lab} versus \(2^\circ\) for Hayward; our target is at \(\tau=45^\circ\) for an effective thickness of 141 \(\mu\text{g/cm}^2\) versus 200 \(\mu\text{g/cm}^2\) for Hayward. We position PSD1 at \(70^\circ<\theta_{\text{lab}}<113^\circ\) in order that no \(^{12}\text{C}\) recoils are detected. The spectrum is very clean, with only the elastic and inelastic alpha particle lines being of any significance.

The data analysis proceeded along the same lines as for \(^{12}\text{C}+^{12}\text{C}^*\) in sect. 3.4, except that it was not necessary to use a second particle detector for the recoils. Because alpha particles and \(^{12}\text{C}\) ions are clearly separable, we have \(W=N \times \text{COIN/SING}\). This experiment was performed at \(D=25.6 \text{ cm}\), so we took \(N=15.70\) from the \((p,p')\) determination. Agreement between our results and Hayward's was
excellent for $\beta(\theta)$ and the relative values of $A(\theta)$. However, we found that $N$ had to be scaled down by .855 in order for the absolute values of the alignment to agree with the Seattle measurement (see fig. 406). Hayward's points have stated uncertainty of 2-16%, and by computing the average error over this angular range, we estimate that our determination of $N = 13.42$ by this measurement to have uncertainty ±9%.

4.3 Complete $^{12}\text{C}^+^{12}\text{C}^*$ angular correlation

Finally, $N$ is determined by measurement of a "complete" angular correlation of $^{12}\text{C}(^{12}\text{C},^{12}\text{C})^{12}\text{C}^*$, that is, a measurement of $W(\psi,\phi)$ both in and out of the reaction plane and a fitting of these data to eqn. (18) of chap. 2. From those coefficients $\alpha_0^2$ and $\alpha_+^2+\alpha_-^2$ are extracted separately, and by imposing the constraint $\alpha_0^2+\alpha_+^2+\alpha_-^2=1$, $N$ is established.

We attempted to measure a complete angular correlation at $E=17.91$ MeV using PSD1 without the pp singles, but it was unsuccessful. We obtained $N$ varying with particle angle $\theta$, a result which is clearly not correct, because $N$ depends only on the characteristics of the $\gamma$-detector. This measurement is sensitive to particle background, so we decided to use solid state particle telescopes for positive particle identification. The drawback to this system, apart from the long time required for data accumulation, is that it can be performed only at energies high enough such that the carbon ions to be detected do not lose most or all their energy in the $\Delta E$ detector. Since the in-plane
alignment data are satisfactorily normalized relative to each other, a telescope run was necessary only at one previously measured energy. We chose 29.72 MeV.

Fig. 407 shows the experimental arrangement with two sets of ΔE-E telescopes fixed 13.7° apart in the laboratory frame. A brass plate with slits of approximately the same dimensions as those in chap. 3 defines the angular acceptance of the particle detectors. These components are fastened onto the same rotatable mount used for PSD1, in order that two angles are measured simultaneously for a large number of γ-angles, then another particle position can be set.

The instrumentation for this experiment is quite similar to fig. 305, except here the γ-detector is in fast coincidence with one of two E detectors, and the ΔE and E signals are electronically summed. The ΔE vs. E spectrum (fig. 408B) is typical, with the most prominent particles detected being carbon and helium ions. Fig. 408 is a logic diagram of the pγ event analysis. Particle singles analysis is a subset of that figure, namely panels B and C, with the elastic peak appearing. The gates and peak limits are set the same for both coincidence and singles analysis. Again, as indicated in panel D, only the 4.439 MeV photo and escape peaks are accepted. But because particle identification has been performed, there are no Compton tails of higher energy photons to worry about.

One additional correction that enters here but not in the other experiments is the attenuation of gamma radiation. All other data were obtained in the reaction plane only, where the scattering chamber (and hence the absorption) was found to be uniform to within 1% with respect
407) COMPLETE ANGULAR CORRELATION EXPERIMENT

This arrangement is similar to fig. 401, except particle identification is performed by two solid state $\Delta E-E$ telescopes. Angular correlations are measured with the NaI crystal positioned both in and out of the reaction plane.

408) ANALYSIS OF $\Delta E-E-\gamma$ EVENTS

The treatments for coincidence and singles events are analogous to those shown in figs. 305 and 308. The TAC spectrum (panel A) for coincidence events is one dimensional in this case. Panel B is a familiar $\Delta E-E$ spectrum for particle identification. The Q-value of the reaction is obtained in panel C; the peak corresponding to elastic scattering appears only in the singles mode. Finally, panel D shows the $\gamma$-spectrum window for coincidence events as the shaded region.
NaI at angle $(90^\circ - \psi)$ out of reaction plane
ANALYSIS OF $\Delta E - E - \gamma$ EVENTS

(A)

true

random

T

(C)

mut inel elas

E

(B)

carbon

(D)

$\Delta E$

E

$E_\gamma$
to $\phi$. But the fairly thick aluminum lid and lip on the scattering chamber meant that attenuation out of the plane may be appreciably greater than in plane, so before fitting the yields to (18), we must correct the raw data for this non-uniform absorption. We measured the attenuation experimentally by looking at $\gamma$-singles. The NaI was set at $\phi=90^\circ$ so that while the goniometer brought it out of the plane ($\phi$ from $90^\circ$ to 0), the angle with respect to the beam axis remained constant. This is necessary because any singles measurement has azimuthal symmetry about the beam axis. Defining $B$ as the attenuation of 4.439 MeV photons in this geometry, then the correction factor $B(\phi)/B(90^\circ)$ was found to be as much as 1.30.

The relativistic correction discussed in sect. 3.5 and eqn. (21) must also be applied, but the calculation of the angle $\Theta$ between the photon and its source is more involved. Defining $\chi$ as the angle between the velocity of the excited carbon and the projection in the reaction plane of the photon emission, then

$$\cos \Theta = \cos \chi \sin \phi . \tag{32}$$

Two settings of the two-telescope array were used, in order to measure the complete angular correlation of four particle angles. Data were taken for 6 $\phi$ angles with $\phi=90^\circ$ (the regular in-plane angular correlation), 7 $\phi$ angles with $\phi=60^\circ$, and 6 or 7 (depending on the setting of the telescope array) $\psi$ angles, with $\phi$ set at the minimum of $W(90^\circ, \phi)$ for one of the telescopes. This minimum was chosen in order that the $m=0$ polar dependence illustrated in fig. 202 could be observed clearly as a check on the data. These 17 or 18 data points for each of 4 particle angles $\Theta$ are fitted to a seven term function of
\( \gamma \)-angle (18). Working out the algebra of the coefficients implies that \( c_1 + \frac{c_1}{3} + \frac{c_2}{5} \) scales as \( N \) and should equal \( \frac{1}{4\pi} \) if \( N \) is correct. The data and the results are summarized in app. A.3, and imply that \( N=13.86 \pm 1.39 \), which is 10\% lower than the normalization obtained from the \((p,p')\) experiment.

4.4 Determination of the normalization constant

The preceding sections give values for \( N \) of 15.70 \( \pm \) 1.94, 13.42 \( \pm \) 1.25, and 13.86 \( \pm \) 1.39 for \( D=25.6 \) cm. The last two numbers, each obtained from a particle-gamma correlation experiment, are in very close agreement, while the first value, obtained via a singles technique, is approximately one standard deviation higher. Because of possible systematic effects that may have entered into the singles experiment, we discard the first value of \( N \) listed above, and adopt the mean of the other two-- 13.64 \( \pm \) 1.91 (14\%)-- as our value for the absolute normalization constant.
5. RESULTS AND COMPARISON WITH OTHER DATA

5.1 Alignment

a) Relative normalization

As mentioned in chap. 3, our measured correlation function is insensitive to such variables as target thickness, integrated beam flux, and the solid angle of the particle detector (or the slits in front of it); thus the relative normalization of the data, angle-to-angle and energy-to-energy, should be automatic. Yet because the data were accumulated in four separate runs,\(^1\) with more crucial parameters such as

\[\text{--------------------------}\]

\(^1\) Details were given in table I.
γ-detector solid angle and electronic thresholds open to variation, we have performed overlap runs to match these different sets of data.

The most important difference among the different data sets is that run M (for E>21.5 MeV) employed the kinematic coincidence technique to measure the singles yield, while the other runs which took place earlier did not. The results are easily corrected for this difference. Comparison of the data at energies where both techniques had been used indicates that after minor corrections (sect. 3.5), the two techniques give the same relative results, and one overall scaling constant matches the absolute normalizations. Thus, the kinematic coincidence technique was not essential to obtain reliable data; with proper corrections the earlier measurements are also useable. In-plane angular distributions were measured using both techniques at E=17.91, 21.92, and 24.43 MeV. The agreement of $A(\theta)$ for each energy was excellent, with the application of a single angle-independent factor. This factor was identical for 17.91 and 21.92 MeV, and 1.36 times smaller for 24.43 MeV, reflecting the fact that at higher energies the background possibilities were kinematically more limited.

For runs K, L, and N, in which the kinematic particle coincidence was not employed, an overlap measurement at 17.91 MeV provided the basis for the relative normalization of the data. Given the internal consistency of the data set for 21 energies, the absolute normalization was then set by a match at 29.69 MeV to the renormalized results of sect. 4.3 (see fig. 501).
501) ABSOLUTE NORMALIZATION OF THE 29.69 MEV ALIGNMENT DATA

The solid dots are normalized alignments from the complete angular correlation experiment (see sects. 4.3 and 4.4). A least squares match for $\theta=60^\circ, 70^\circ$, and $90^\circ$ to the data from run M establishes the absolute normalization for the PSD data.

502) ALIGNMENT VS. ANGLE FOR EACH ENERGY

The error bars incorporate statistical and other relative errors, but not the absolute normalization uncertainty $\pm 14\%$. The lines are drawn merely to guide the eye.

503) ALIGNMENT VS. ENERGY FOR EACH ANGLE

The data of fig. 502 are replotted for the indicated average angles; the angular bins are 2-3° wide. The same comments concerning error bars and solid lines in fig. 502 apply here.
\( E_{CM} = 29.69 \text{ MeV} \)
Fig. 502 A
Fig. 502 B
Fig. 503 A
The alignments thus obtained are plotted versus $\theta$ in figs. 502B (runs with the kinematic coincidence) and 502A (without). The error bars reflect statistical effects only, in order that the angle-to-angle relative behavior can be accurately gauged.

b) Energy dependence

The alignments depicted in fig. 502 are replotted in fig. 503 as a function of energy for particular angular ranges. Since only 10 energies had useable data in the first or second slits, data from them are omitted from the new figures. The angles which label the curves are approximate; the data actually fall within a range of $\pm (1-1.5)^\circ$ relative to the values listed. The important feature to note in these figures is that, despite significant variations in portions of $A(E)$ for certain particle angles, for the most part the energy dependence of the alignment is insensitive to the angle.

In order to get a clearer overall picture of the information contained in these two figures, we have averaged the data with respect to angle and present it in fig. 504A. In this figure the center of the shaded region is determined by the mean alignment $\langle A \rangle$ for angles between $59^\circ$ and $90^\circ$ CM. A check of fig. 502 reveals that the data are reasonably symmetric with respect to $90^\circ$, as required by the identical particles in the entrance channel. Also, because the final carbon ions were not distinguished in the particle detector, data beyond $90^\circ$ were excluded from the averaging in order to avoid double counting. The shading indicates the spread in $A$ for the angles measured. The width corresponds to $\pm 1$ standard deviation; thus, for each energy, roughly
504) $\langle A \rangle$ AND $\sigma$ VS. ENERGY

The data of fig. 503 are averaged and plotted in panel A, as described in the text. Panel B displays the cross section integrated over a similar angular range. The data are from Em73 (triangles), Wi73 (open circles), and Er79b (solid circles), with the curve drawn for visual guidance.
Fig. 504

(A) $\theta_{\text{CM}} = 59^\circ - 90^\circ$

(B) $\theta_{\text{CM}} = 42^\circ - 90^\circ$
two-thirds of the angles measured have \( \Delta \) falling within the shaded
region. The smallest angles measured for \( E=19.5-21.2 \text{ MeV} \) were \( 69-75^\circ \).
<\( \Delta \)> in fig. 504 is over a more restricted angular range for those
energies, but this fact does not significantly affect the utility of the
figure.

Panel B of fig. 504 is the angle-integrated cross section for
inelastic scattering in a comparable angular range. The similarity of
the two excitation functions is apparent: for \( E<26 \text{ MeV} \), both broad and
narrow maxima in \( \sigma \) are accompanied by enhanced alignments. But for
energies above 26 MeV, the alignment is not as large as might have been
expected from a simple extrapolation of the lower energy correlation
between \( \Delta \) and \( \sigma \). The narrow width of the shaded band implies the
variation of \( \Delta \) with \( \sigma \) is much smaller at these higher energies. These
features of the alignment data must be reproduced by any successful
model of the inelastic scattering. A detailed comparison of theory with
experiment is presented in the next chapter.

While the origin of broad structures observed in \( \sigma(E) \) may be in
doubt, the narrow peaks are widely believed to reflect genuine
quasimolecular behavior. Evidence for this conclusion comes from
correlated maxima in other exit channels. While similar structure has
not been reported for \( 16.8 \text{ MeV} \), it has been observed near \( 17.9 \text{ MeV} \):
James and Fletcher (Ja78) report a \( 12^+ \) resonance at \( 17.78 \text{ MeV} \) from their
study of the \( ^8\text{Be} \) exit channel. A maximum near \( 19.3 \text{ MeV} \) has been seen in
a number of other channels: \( p \) (Va74), \( n \) (Sp74a), \( d \) (Ke74), \( ^8\text{Be} \) (\( 12^+ \) at
\( 19.46 \), Ja78), and elastic (\( 12^+ \) at \( 19.4 \), Co81). It is significant that
at energies \( 16.8, 17.9, \) and \( 19.3 \text{ MeV} \), where structure of width
$T=450-750$ keV was observed in the cross section, the angle-averaged alignment is enhanced relative to nearby energies.

The behavior of the correlation function with respect to particle scattering angle undergoes a striking change as the interaction energy is varied through the region of the narrow resonances. Fig. 505 shows $W(\theta)$ for $\psi=90^\circ$, $\phi=40^\circ$, and the lowest 12 energies measured. The periodicity of the correlation function changes abruptly in passing from 17.31 to 17.56 MeV, and again from 18.91 to 19.31 to 19.52 to 20.02 MeV. These features may be characteristic of the genuine resonances in this energy region and their interference with a non-resonant background. This method might also be useful for assignment of characteristic spins, but the angular range of particle detection would probably have to be wider.

5.2 "Polarization"

The fits to the in-plane correlation function yield two values of substate amplitudes $a_{\text{min}}$ and $a_{\text{max}}$ corresponding (but not necessarily respectively) to $a_+$ and $a_-$ in the formalism. The sum of the squares gives the alignment, and one may naively expect the difference of the squares to give a polarization. But as discussed in chap. 2, the identical particle symmetry and non-distinction of the exiting carbon ions in our technique make the polarization experimentally inaccessible. Nevertheless, behavior of a polarization-like quantity at 90° can be checked to confirm the validity of our results.
505) VARIATION OF THE CORRELATION FUNCTION WITH ENERGY AND ANGLE

For a fixed \( \gamma \)-angle \( \phi=90^\circ \) and \( \phi=40^\circ \), the correlation function \( W \) is plotted versus particle scattering angle for the lowest 12 energies measured. Where no error bars are shown, the statistical errors are smaller than the dots. The lines are drawn only to guide the eye.

506) PHASE ANGLE VS. PARTICLE ANGLE

The angle \( \beta \) from the least-squares fit to the correlation function data is plotted versus scattering angle. The different lines (solid, dashed, dotted, and dash-dot) indicate the energies (the same as those listed in fig. 502) which increase monotonically from left to right in the figure. We observe \( \beta(90^\circ)=0 \), as expected. Further discussion of the data can be found in sect. 6.4.
Fig. 505
We define

\[ P = \frac{(\alpha_{\text{max}})^2 - (\alpha_{\text{min}})^2}{(\alpha_{\text{max}})^2 + (\alpha_{\text{min}})^2} \]  

(33)

in analogy with the usual definition of polarization. Arguments in sect. 2.3d imply \( P = 0 \) at \( \theta = 90^\circ \). In fact, a check of the tabular data (app. A.2) indicates that for \( E < 24 \text{ MeV} \), 11 out of 13 measured energies exhibit \( P \) consistent with zero for at least one angle near \( 90^\circ \). For \( E > 24 \text{ MeV} \), \( P \) near \( 90^\circ \) is well-bounded away from zero for the remaining 8 energies, although \( P \) does drop significantly near \( 90^\circ \) compared to neighboring angles at the same energy. Thus, the expected trend of decreasing \( P \) is observed, bolstering our confidence in the data. Some undetermined effect at higher energies (perhaps greater sensitivity to the acceptance range of the particle angle) may prevent our measured \( P \) from completely vanishing near \( 90^\circ \).

It is interesting that in most cases away from \( 90^\circ \), structure in \( A(E) \) or \( A(\theta) \) is mimicked by structure in \( P(E) \) or \( P(\theta) \). Yet because we don't distinguish between the excited and the unexcited carbon ions, the physical meaning of \( P \) is obscure away from \( 90^\circ \). Thus, the empirical observation concerning the data may not be too revealing of the underlying physics of the scattering process.

5.3 Phase angle

Finally, our 3 parameter fit to the in-plane correlation function yields the phase angle \( \beta = \beta_+ - \beta_- \). As shown in fig. 506, generally \( \beta \) varies smoothly with both angle and energy. In addition, \( \beta = 0 \) at
\( \theta = 90^\circ \), independent of energy, as required by the symmetry discussed in chap. 2 in connection with fig. 204. These facts again support the internal consistency of our data. There are other interesting angle- and energy-related effects, which are discussed in conjunction with the models in sect. 6.4.

5.4 Comparison with previous data

As mentioned in sect. 1.6, Trombik, et al., (Tr80) have recently published alignment data on this system; their data are compared with ours in fig. 507. The two experiments give consistent results for all 4 energies at \( \theta = 74^\circ \), and for 3 of 4 energies near \( 84^\circ \). It is reassuring to see that the two different methods, with independent techniques used to attach an absolute normalization to the alignment, agree so well.

The Munich experiment, while more difficult than ours, yielded much less information. They determined \( \Lambda \) by detecting \( \gamma \)-radiation perpendicular to the reaction plane. The finite solid angle of gamma radiation acceptance plays a central role here, because as demonstrated in chap. 2, no radiation occurs at exactly \( \psi = 0^\circ \). Trombik and collaborators relied on a Monte Carlo calculation of the NaI efficiency to extract the population of the \( m = \pm 2 \) and 0 substates, which may have led to the somewhat larger uncertainties attached to their data than to ours. Moreover, since all in-plane angles \( \phi \) were measured at once, their technique made it impossible to determine \( P \) or \( \beta \), which may be important for a complete understanding of the reaction process.
COMPARISON WITH THE MUNICH DATA

Our data are the solid dots, joined by the full line. All the alignments measured by the Munich group (Tr80) are indicated by X's. The two panels are labelled by the approximate particle scattering angles. Specific angles for each of the solid points can be found in table A.2; all are within 1° of the indicated value. The Munich group measured alignments at 75°, 84°, and 107°, but symmetry about 90° implies that alignments for the first and the last be practically identical. As seen in the upper panel, those values are consistent for three energies measured, but at E=24.1 MeV the alignments at 75° and 107° of Tr80 are strikingly different. The authors do not discuss this discrepancy in their paper.
Fig. 507

$\theta_{CM} \approx 74^\circ$

$\theta_{CM} \approx 84^\circ$
The most serious drawback to the Munich work, however, is the paucity of data. They were able to obtain data at only 8 independent energy-angle pairs, and they acknowledged that a larger set of data are needed before it is possible to draw firm conclusions. In particular, contrary to the Munich observation, we have found that the cross section and the alignment exhibit a remarkable correlation, at least for energies below 26 MeV. Especially in view of the rapid variation of A(E) observed in our data, measurements at only a few energies cannot give a true picture of the alignment behavior in this region.
6. THEORETICAL CALCULATIONS

6.1 Rotation of the quantization axis and the statistical prediction

Calculations of differential cross sections within any model involve an incoherent sum over magnetic substates:

\[
\frac{d\sigma(\theta)}{d\Omega} = \sum_M |T_L^M|^2 ,
\]

where \(T_L^M\) are the transition matrix elements to the final state of interest. The probability of population of state \(M\) is proportional to \(|T_L^M|^2\). The summation in eqn. (34) obscures many of the details of the reaction mechanism. Thus alignment or polarization experiments are important to more sensitively evaluate models by differentiating among the magnetic substates.
In most computer codes, reaction calculations are based on coordinate axes in which \( \hat{z} = \hat{k} \) -- i.e., the beam direction -- and azimuthal symmetry simplifies the calculation. But in our case we want \( \hat{z} = \hat{k} \times \hat{k}' \), perpendicular to the reaction plane, in order to exploit the Bohr symmetry to eliminate odd integral substates. The transformation from one frame to the other is accomplished by rotation through Euler angle \( \beta = 90^\circ \). Since \( T_L^M \) transforms under rotation (Sa64) like \( Y_L^M \), it follows (Ja79a) that

\[
T_{LM'} = (-)^M \sum_M T_L^M d_{LM}^L \left( \frac{\pi}{2} \right),
\]

where \( T_L^M \) is the amplitude in the frame where \( z \) is parallel to the beam, and \( T_{LM'} \) refers to the case where \( z \) is normal to the scattering plane. Also, \( T_L^{-M} = (-)^M T_L^M \) implies (Ha67) that \( T_{LM'} = 0 \) for \( L + M' \) odd, consistent with the Bohr theorem.

For our case \( L = 2 \), and using the formulae for rotation matrices (Br68, p.24) we obtain from (35):

\[
\begin{align*}
T_{\pm 2} &= \frac{3}{8} T^0 - \frac{1}{2} T^2 \pm i T^1, \\
T_0 &= -\frac{1}{2} T^0 - \frac{3}{2} T^2,
\end{align*}
\]

where the label \( L \) has been dropped for conciseness. The alignment \( A \) is given simply by:

\[
A = \frac{|T_0|^2 + |T_{-2}|^2}{|T_2|^2 + |T_0|^2 + |T_{-2}|^2}.
\]

Given an angular momentum \( L = 2 \), a statistical distribution would imply equal population of all five allowed substates \( M = -2, -1, 0, 1, 2 \). To calculate the alignment this would predict, we take \( T_M = \nu_M e^{i\sigma_m} \)
such that $v_M$ is a constant independent of $M$, and the phases $\sigma_M$ are assumed to be random. (Setting all the phases equal to zero yields the same result.) Eqn. (37) gives $A = 0.405$ for the statistical expectation of the alignment. A glance at fig. 502 indicates that for most angles and energies, the measured alignment exceeds the random statistical prediction.

A simple statistical prediction does not take into account any specific reaction mechanism. As outlined in chap. 1, a large number of models, both resonant and non-resonant, have been proposed to explain the broad structure observed in the inelastic cross sections. The purpose of this chapter is to confront those models with experimentally-measured (and model-independent) alignments. Calculations based on the double resonance model, which require considerable computer resources, are in progress. Some of the other models discussed in chap. 1 are not sufficiently developed to predict an alignment or cannot give one in principle.

For those models which do yield predictions for alignment, we discuss them in the next two sections as resonant and non-resonant. Generally, the calculations were performed with $z$ parallel to the beam direction, and the rotation discussed above is necessary to obtain alignments perpendicular to the reaction plane. All calculations are based on parameters adopted by the proponents of the various theories. We have made no attempt to vary the parameters in order to obtain optimum agreement with both the cross section and the alignment data. Rather, we compare the calculations with our data and with each other, in hope of establishing which are the most promising in terms of reproduction of
the data. We then indicate qualitatively how the parameters may need to be changed to better reproduce the experimental data.

6.2 Resonant models

a) Single-particle shape resonances

The broad structures observed in the inelastic scattering excitation function were first interpreted by Cormier, et al., (Co78) to be predominantly single particle shape resonances in the potential of the relative motion of the two nuclei. They tentatively assigned spins based on energy systematics, and estimated total and partial widths for the (10+), (12+), and (14+) peaks from the (2+,0+), (2+,2+), and fusion cross sections. But their results do not explicitly indicate the alignments that would be expected within the model.

We have incorporated their widths into a calculation that includes isolated Breit-Wigner resonances. The resonance amplitude must be added to the background \(<S>\) to give the total scattering matrix (Wi71,p.118):

\[
S_L(E) = \langle S_L(E) \rangle + e^{i\phi} \frac{1}{\sqrt{\Gamma_{el}}} \frac{\Gamma_{inel}}{E_0 - E - i\Gamma_{tot}/2}
\]

The resonance energies \(E_0\) are taken at the centroids of the gross structures (13.9, 18.4, and 24.1 MeV), and the mixing phase \(\phi\) is taken to be zero. The biggest obstacle in this analysis is establishing \(<S>\); the background from standard optical models yields gross structures (see
fig. 605), while we seek a background that yields a smooth and slowly-rising excitation function. We have used the S-matrix parametrization of Ph79 with a large width ($\Delta = 2.5$ MeV) to generate a smooth background. (Fig. 601 illustrates the effect a variation of $\Delta$ has on $\sigma(E)$.) We then add the resonance term to the background S-matrix and observe how the integrated cross section and the alignments are affected.

The results are displayed in fig. 602. The background is smooth and of a magnitude consistent with a line drawn under the broad peaks in the Co78 data. Introduction of the Breit-Wigner term leads to interference, and the resulting cross section does not exhibit structures as prominent as those observed experimentally. Slight maxima do persist in $\sigma$, but they do not have a consistent effect on the predicted alignment averaged over the angular range studied. These inconclusive results are not unusual; in fact, identification of resonance parameters has long been plagued by the extreme difficulty in separating a non-resonant background from the structures of interest.

We can take a different tack and argue that if the broad maxima Cormier and coworkers observed have a common resonance origin, then other physical observables associated with these structures, such as alignment, should also exhibit common behavior. Yet as indicated in fig. 504, while there is broad enhancement in $\Lambda$ near 19 and 24 MeV, there is not a similar enhancement near 30 MeV. This observation suggests that other explanations for the structure should be pursued.
601) DIFFRACTION MODEL: $\sigma$ PREDICTIONS VS. $\Delta$

Increasing the width parameter $\Delta$ from 1.0 MeV (solid line) to 1.5 (dashed), 2.0 (dotted), and 2.5 MeV (dash-dot) leads to progressively less structure in $\sigma(E)$ within the diffraction model of Ph79.

602) BREIT-WIGNER RESONANCES: $A(E)$ AND $\sigma(E)$

The dashed lines are calculations using the background amplitude only, taken from the diffraction model parametrization with $\Delta=2.5$ MeV (see fig. 601). The solid lines are calculations which added three Breit-Wigner resonance terms, using the parameters obtained in Co78.
Fig. 601

Diffraction Model
$^{12}\text{C} + ^{12}\text{C}^* (2^+)$

$\Delta = 1.0 \text{ MeV}$

$\sigma_{\text{mb}}$

$E_{\text{CM}} (\text{MeV})$
Fig. 602

- \( \theta_{CM} = 59^\circ - 90^\circ \)
- \( \theta_{CM} = 0^\circ - 90^\circ \)

\( \sigma_{mb} \) vs. \( E_{CM} \) (MeV)
b) Band crossing model

The band crossing model (BCM), one of the versions of the extended Imanishi model, has been successful in reproducing energies and spins of Coulomb barrier resonances (Ko75) as well as the broader structures in the inelastic and fusion cross sections at higher energies (Ko79a). We have used these authors' coupled-channel program and the parameters listed in Ko79a to calculate the alignment predicted in the higher energy region. The detailed predictions of \( A(E, \theta) \) are in fig. 606A. The angle-averaged results are compared to the experimental data in fig. 603.

Unlike fig. 3 in Ko79a, fig. 603B addresses the cross section integrated over only in a limited angular region. While the detailed agreement is not as good as for \( \sigma_{\text{tot}} \), the general features of three broad maxima are still present.

On the other hand, fig. 603A presents a puzzling prediction for the alignment. Over the entire energy range, the system is strongly aligned: a result we expect because the model relies on the crossing of the "aligned inelastic band" with the \( n=0 \) molecular band of refs. Ar72 and Fi72 to produce resonances. But the relatively small oscillations in the predicted \( A(E) \) do not bear any apparent relationship to peaks in \( \sigma(E) \). This apparent anomaly is cleared up in fig. 604, which illustrates the partial cross sections which comprise \( \sigma_{\text{tot}} \) and their dominant configurations (aligned or non-aligned). The maxima in the oscillations in \( A(E) \) arise from peaks or shoulders in the aligned configurations for the individual partial waves. As it turns out, the alignment is a more sensitive measure of the fragmentation of the
603) BAND CROSSING MODEL: $A(E)$ AND $\sigma(E)$

The data are from fig. 504, and the dashed lines are BCM calculations (Ko79a) for $A(E)$ and $\sigma(E)$, each in the angular ranges indicated.

604) BAND CROSSING MODEL: PARTIAL WAVE CONTRIBUTIONS

The numerous oscillations in the predicted $A(E)$ are compared to the partial wave cross sections, denoted by thin solid, dashed, dotted, and dash-dot lines. At energies where the total cross section is dominated by a single component with $L' = J - 2$ (denoted by an asterisk), the alignment reaches a maximum. Energies where $L' = J$ are dominant (crosses) do not have enhanced alignments.
Fig. 603
Fig. 604

\( \sigma \) (mb)

\( E_{CM} \) (MeV)

- \( \ast : L^* = J-2 \) dominant: aligned configuration
- \( \dagger : L^* = J \) dominant: non-aligned configuration

\[ \sigma_{TOT} = \sum \sigma_L(E) \]
resonances within this model than is $\sigma(E)$.

It is clear from fig. 603A that the BCM prediction of alignment does not correspond to the experimental data. The BCM calls for a consistently large alignment, and reflects neither the dramatic enhancements in $A$ near 18 and 24 MeV, nor the lack of that enhancement near 30 MeV. Therefore, the band crossing model does not give an adequate account of the experimental alignment data in this energy region of $^{12}\text{C}+^{12}\text{C}$ scattering.

6.3 Non-resonant models

a) Plane waves

In the limit as the scattering angle $\theta \to 0^\circ$— Litherland and Ferguson's (Li61) method II-- an alignment can be calculated in plane wave approximation. Using the beam direction as the quantization axis, $M=0$ only, so setting $T^1$ and $T^2$ in eqn. (36) equal to zero yields $A=0.75$. Our alignments are measured for $\theta>50^\circ$, far away from $0^\circ$, so that this limit does not prove to be very useful.

b) Distorted waves

More useful in heavy ion scattering is the distorted waves Born approximation (DWBA) formalism, described in detail by Satchler (Sa64). By use of a suitable optical potential, generally derived from a fit to
elastic scattering data, the inelastic scattering can be calculated within a one-step direct interaction picture. The form factor appropriate for $^{12}\text{C} + ^{12}\text{C}(2^+)$ is from the collective model. Computer codes such as DWUCK (Ku69), using a zero-range approximation, or finite range PTOLEMY (Gl78) have been developed for these standard calculations.

Reilly and collaborators (Re73) established the optical model parameters in this energy range. Cannell, et al. (Ca79) demonstrated that symmetrized DWUCK calculations using these potential well parameters, but with the radii decreased by 4.75% (a change claimed in ref. Ca79 not to do violence to the fits to the elastic scattering data), can reproduce the broad features observed in the $^{12}\text{C} + ^{12}\text{C}$ inelastic scattering between 15 and 32 MeV.

We have modified DWUCK to properly account for identical particle symmetry. A comparison of our results to PTOLEMY output (in which symmetrization is built into the original code) confirms that our alterations were correct. Using the same parameters as the Pennsylvania group, we reproduced their DWBA calculation for the cross section, then proceeded to extract the inelastic amplitudes from that calculation, rotate them to the new quantization axis, and derive predictions for alignment. The results are graphed in fig. 605A.

The dotted line in fig. 605B is from a similar calculation using the Reilly potential. The small alteration in the potential radius proposed by Cannell essentially changes the phase of the excitation function structures by $180^\circ$, bringing them into agreement with the data. For Cannell's parameter set we have calculated the alignment (fig. 606B),
Data are indicated by solid lines; in panel A they are from fig. 504, and in B from Co78. The dotted line is the prediction of the DWBA using the optical model parameters of Re73. Adjustment of the well radii by Ca79 yields the dashed-line predictions.
Fig. 605
then averaged it over the same angular range as our data, and plotted these in fig. 605A. While the DWBA predicts broad oscillations in A as well as in $\sigma$, the maxima in A occur approximately 1 MeV higher in energy. The oscillations in A are far too gentle to reproduce the trend of the experimental data. The small amplitude of the predicted alignment oscillations does not arise primarily from the averaging of A over angle, as seen in fig. 606, but instead originates from the optical model parameters used in the calculation. While this comparison with the data does not definitively prove that the DWBA approach is wrong, it does at least indicate that different parameters are needed to simultaneously reproduce the elastic, inelastic, and alignment data. It may well be that no reasonable parameter set exists.

c) Diffraction model

A particular approximation to the DWBA treatment was made by Austern and Blair (Au65). They considered that heavy ion collisions were characterized by strong absorption of interior waves, but only weak absorption of surface grazing waves. Coupled with the high centrifugal barrier for higher partial waves, these facts give rise to a diffraction-like scattering pattern. The inelastic amplitudes can be expressed (Au65, Ha67) as derivatives of the elastic S-matrix elements with respect to the entrance and exit channel orbital angular momenta $L$ and $L'$. Thus, as Phillips, et al., (Ph79) demonstrated, large cross sections arise at energies where there is a large overlap of those derivatives. For $^{12}\text{C}+^{12}\text{C}(2^+)$, centrifugal and Q-value effects imply that the $L'=L-2$ configuration dominates the scattering, and the
constraint that \( L, L' = \text{even} \) from identical particle symmetry leads to structure in the cross section. In fact, the broad maxima observed near 14, 18, 24, and 30 MeV have been reproduced in this model, which was claimed to be constructed free of resonances.\(^1\) The model also implies enhanced alignments at those energies.

In addition, dominance of a single value of \( L \) and \( L' = L - 2 \) at certain energies implies that the alignment is independent of particle angle. This fact is demonstrated analytically in app. B. Fig. 606C illustrates the angle-independence of the predicted alignments within this model, in contrast to variations with respect to angle seen in the BCM and DWBA predictions (panels A and B).

The diffraction model predictions and the data for \( \sigma \) and \( A \) are compared in fig. 607. The gross enhancements in both quantities near 18 and 24 MeV are well reproduced within the model, while the finer structures found below 20 MeV are acknowledged to be beyond the scope of any single-step non-resonant model. Two features of the alignment data are unexplained: the drop in \( A \) at 24.4 MeV compared to 23.4 and 25.2 MeV, and the total failure of the diffraction model to reproduce the relative constancy of \( A(E) \) above 26 MeV. The former might be understood from the detailed angular behavior, but the latter requires a more fundamental explanation.

Maxima in the cross section arise within the diffraction model from the dominance of a single pair \((L, L')\) of well-matched surface grazing partial waves. In particular, \( L' = L - 2 \) leads to enhanced alignment. But

\[ \text{-----------------------------} \]

\(^1\) For a further discussion see the following subsection.
Alignments are calculated for 14 equally-spaced angles from 54° to 90° over the energy range studied in the present experiment. Within the diffraction model the energies where A(E) is maximum do not change with particle angle, nor does the magnitude of A vary at those energies. In contrast, the behavior of A(E) changes with particle angle in both the BCM and DWBA calculations.

The data (solid lines) are the same as in fig. 605. The dashed lines indicate diffraction model calculations with A=1.0 MeV, as used in Ph79. Panel B is an integration over 0-90°, while A is an average over 59°-90°.
Figure 6.07

Graph showing 

\[ \sigma_{\text{TOTAL}} \text{ (mb)} \]

vs.

Ecm (MeV)

Alignment A

A

B

\[ \theta_{\text{cm}} = 59^\circ - 90^\circ \]
if the partial wave cut-off were not so sharp, or equivalently if the optical potential were more diffuse, then several values of $L$ or $L'$ (or both) could contribute at every energy. This situation washes out not only the structure in the predicted cross section, but more especially in the alignment, which depends on the more sensitive requirement that $L' = L - 2$. Analysis of $^{12}\text{C}+^{12}\text{C}$ elastic and inelastic scattering data taken by Stokstad, et al., (St79) at $E=35-63$ MeV indicates that the optical potential appropriate for these higher energies is, indeed, more diffuse. This fact allows several pairs of partial waves to be significant, and means that the parametrization of ref. Ph79 is less appropriate to higher energy $^{12}\text{C}+^{12}\text{C}$ scattering. An examination of the inelastic cross section data indicates that the gross structures (of characteristic width a few MeV) that dominate the results of Co78 become broader, more widely spaced, and less prominent above 32 MeV in St79. Moreover, the present data indicate that the alignment changes character near 26 MeV, becoming relatively constant with respect to angle and energy, thus indicating the onset of a transition region between two distinct behaviors.

d) Barrier top model

Structure arises within the diffraction model from energy-angular momentum windows for inelastic scattering; these windows move up in $L$-space as the beam energy is increased. Friedman, et al., (Fr79) point out that such a condition is consistent with the occurrence of orbiting-type "barrier top resonances" (Fr77), which exist when strong internal absorption makes classical potential "pocket" resonances untenable.
These authors have described the inelastic amplitude in terms of two Regge poles moving in the complex L-plane, similar to a previous treatment of anomalous heavy ion transfer reactions by Fuller and Dragun (Fu74). The Wisconsin group demonstrated this resonance formalism reproduces the $^{16}\text{O}+^{16}\text{O}(3^-)$ and $^{12}\text{C}+^{12}\text{C}(2^+)$ scattering as adequately as does the diffraction model. For the particular case of $^{12}\text{C}+^{12}\text{C}$, they note that the pocket in their folded potential fills in at high energies (near 30 MeV), leading to a great increase in width and consequent damping of the resonances, as had been experimentally observed (St79).

There has been great difficulty in sorting out the differences between the diffraction and the barrier top models. Each interpretation arises from the same physical situation—strong internal absorption and surface transparency in the short-ranged optical potential—and each relies on energy-angular momentum windows to generate gross structures in the cross section. Yet one model is labeled resonant, while the other is claimed to be manifestly non-resonant.

A major difference may lie in the character of the width parameter of each model, which we have analyzed with an Argand diagram technique. The barrier top model uses $\Gamma_L$ which, as the beam energy is increased, gives a constant width of the resonances in L-space and a rapidly increasing energy width. On the other hand the diffraction model employs $\Delta$, a constant energy width which leads to decreasing L-width for higher incident energies. The diffraction model has been criticized for using an unrealistically narrow width (Fr80a), and as figs. 601 and 602 show, an increase in $\Delta$ leads to less pronounced structure in $\sigma$ and unpredictable behavior in the angle-averaged
alignment.

Nonetheless, all non-resonant models do reasonably well in reproducing the inelastic cross section, so it has been our hope that a measurement of alignment would provide a basis to choose one or more models over the others. As noted previously, the diffraction model as implemented by Phillips is more successful than the DWBA as used by Cannell. Unfortunately, the barrier top model cannot predict either differential cross sections or alignments because the phases of the radial integrals are not specified (Fr80).

We have, however, performed some hybrid calculations to plumb the relation between the diffraction and the barrier top models. First, we have tried to parametrize the S-matrix not in E-space as was done in Ph79, but in L-space (Fr77):

\[ \eta_L = \left\{ 1 - \exp \left[ - \frac{2 \pi \left( L - L_{\text{orb}}(E) \right)}{\Gamma_L} \right] \right\}^{-1/2}. \]  

(39)

Again we used the Austern-Blair-Hahne relation to obtain the radial integrals, and we obtained cross sections quite similar to Ph79 and alignments very close to those in fig. 607. Apparently the diffraction model is not sensitive to the particular method of parametrizing the S-matrix.

Second, we have replaced the radial integral in the diffraction calculation by the two-pole prescription of Friedman (Fr79). The resulting angle-integrated cross sections were similar to those published in Fr79, while the predicted angle-averaged alignments decreased smoothly with energy, showing no gross structure. But the lack of phase specification within the barrier top model is probably
crucial, and an additional ansatz concerning the phase may be necessary to reproduce the alignment data.

6.4 Phase angle

In addition to alignment, the quantities "polarization" P and phase angle \( \beta \) are extracted from the fits to the correlation functions. Except for the expected behavior, \( P \rightarrow 0 \) at \( \theta = 90^\circ \), information useful for the evaluation of models is not available in \( P \). In addition to its property \( \beta = 0 \) at \( \theta = 90^\circ \), the behavior of \( \beta(E, \theta) \) may hold additional physical information. For example, Steadman, et al., (St74) used the relative phase between the \( m=+2 \) and \( -2 \) substates to distinguish between coupled-channel and DWBA predictions for Coulomb-nuclear interference in the \( ^{56}\text{Fe}^{(16}\text{O},^{16}\text{O})^{56}\text{Fe}^+(2^+) \) reaction. Perhaps \( \beta \) might also differentiate among the models we have considered for the \( ^{12}\text{C}^{(12}\text{C},^{12}\text{C})^{12}\text{C}^+(2^+) \) reaction.

As demonstrated in app. B, the application of the high angular momentum limit simplifies analytical expressions for the correlation and produces easily-tested relations. In particular, the further assumption of \( L' = L - 2 \) leads to a predicted linear behavior of \( \beta(\theta) \), with a slope equal to \(-2L'+1\).

With the same amplitudes used to obtain alignment predictions, we have calculated the phase angles expected within the band crossing, diffraction, and DWBA models. For all energies considered, the band crossing and diffraction models each predict \( \beta \) to be linear in \( \theta \), as
expected from the treatment in app. B. The slopes $S$ of those loci are tabulated in table III and plotted in fig. 608. Near energies where maxima occur in $\sigma(E)$ (18, 24, and 30 MeV), the slope calculated within each model is very close to $-(2L'+1)$. This agreement confirms the dominance of $L'=L-2$, the aligned configuration, at those energies. The energy dependence of $S$ for energies between these plateaus (near 15, 21, and 27 MeV) is slightly different for the two calculations. Yet it is noteworthy that these two models— one involving coupled channels and claiming the existence of resonances, the other a one-step process with no resonances— give predictions for $\beta$ that are practically identical.

A DWBA treatment, as applied by Cannell and collaborators, yields a somewhat different behavior of the phase angle, as shown in fig. 609 together with the data. For $E=18-21$, 23-26, and 28-33 MeV, the DWBA predicts $\beta(\theta)$ to be very similar to the BCM or diffraction model results, with only gentle oscillations about a straight-line trend. But for the other energies calculated, the DWBA phase angle deviates from monotonic behavior at the more forward angles, and for some energies (17-18 and 22 MeV) yields a fragmented prediction for $\beta(\theta)$. At one energy, 17.91 MeV, this fragmented behavior is actually observed in the data.

The correspondence of the theories to each other and to experiment is summarized in table III. Near 18, 24, and 30 MeV, all theories successfully reproduce the data, which exhibit a linear behavior. Approximately 1 MeV above those energies, the data follow the theoretically-expected behavior for $\theta>70^\circ$, but are 180$^\circ$ out of phase forward of that angle. Then near 21 and 26 MeV the predictions become
608) SLOPES OF $\beta(\theta)$

The data listed in table III are plotted versus energy. Solid dots are the experimental values, the dashed line is the band crossing model prediction, and the dotted line is the diffraction model prediction. Horizontal solid lines indicate the slope $S$ expected from even integral exit angular momenta $L'$, assuming $S=-(2L'+1)$.

609) PHASE ANGLES VS. PARTICLE SCATTERING ANGLES

The solid dots and error bars are the experimental data, as listed in app. A.2. The solid lines are the DWBA predictions using the parameters of ref. Ca79. At energies where the diffraction and band crossing model predictions (which are practically identical) differ substantially from the DWBA, the BCM values are shown as dashed lines. Some of the error bars seem skewed because the formalism actually derives $\tan \beta$, and not $\beta$, from the least squares parameters (see chap. 2, eqn. (20)).
Fig. 608
Fig. 609 A
Fig. 609 B
Fig. 609 C
### III) PHASE ANGLE: EXPERIMENT AND THEORY

Column 1 contains $E_{CM}$ in MeV. Cols. 3-5 give the slope of $\beta(\theta)$ for the experimental data (in the angular range indicated in col. 2), the diffraction model prediction, and the band crossing model prediction. For energies less than 18.5 MeV, no meaningful experimental slope could be assigned. Col. 6 indicates the nature of the DWBA prediction as drawn in fig. 609, and col. 7 gives the comparison of the theoretical predictions to the experimental $\beta(\theta)$. The code for the last two columns follows the table.

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**KEY**
- **F**- Fragmented
- **L**- Linear
- **K**- Linear, except for a "kink" at forward angles
- **A**- All 3 theories reproduce the data
- **S**- Experimental slope is anomalously small
- **D**- DWBA fits data better than BCM or Diff
- **SB**- S for 65-85°; good fit by BCM or Diff for 85-101°
- **O**- Theory is 180° out of phase with experiment
- **P**- O for forward angles (<70°); A for backward angles
completely out of phase with the data at all angles. These observations may be related to the comment made by Phillips, et al., (Ph79) that the diffraction model was unsuccessful in reproducing d\sigma/d\Omega for 70°<θ<110° at energies where \sigma was maximum. However, the physics underlying these phenomena is not yet established. We hope that further theoretical work will be stimulated and the information made available by the present experiment can be fully exploited to understand the details of the \(^{12}\text{C}+^{12}\text{C}\) reaction mechanism(s).

In addition, as indicated in figs. 608 and 609 and table III, the apparent slopes of \(\beta(\theta)\) over all or part of the angular range measured are much smaller than predicted by the models for \(E=16-17.8\) and 19.5-20 MeV. This behavior might be related to the narrow resonances observed in other exit channels of the \(^{12}\text{C}+^{12}\text{C}\) reaction near this energy. Again, further theoretical work needs to be done to establish whether \(\beta(\theta,E)\) may contain a signature for genuine resonance phenomena.

Distinct from a plane- or distorted-waves approach, an alternate method to obtain the particle-gamma correlation function is via the adiabatic approximation. Blair and Wilets (B161) have demonstrated that in this treatment, too, \(W(90°,\phi)\) is proportional to \(\sin^22(\phi-\phi_0)\). The symmetry angle is \(\phi_0=(\pi-\theta_{\text{CM}})/2\), where \(\theta\) is, as before, the particle scattering angle. Adapting this expression to our formalism implies an adiabatic prediction of \(\beta=2(\theta-\pi)\). This result is in clear disagreement with our data for all energies measured, as displayed in fig. 506. Similar results were obtained by Hayward and Schmidt (Ha70) in their study of the \(^{12}\text{C}(\alpha,\alpha')^{12}\text{C}(2^+)\) reaction.
7. CONCLUSION

7.1 Summary of present work

Faced with a number of theories put forth to explain structures observed in the cross section for $^{12}\text{C} + ^{12}\text{C}(2^+)$ scattering at higher energies, we have undertaken to measure additional physical observables of the reaction. Using a particle-gamma correlation technique, we obtained the alignments and the relative substate phases for a wide range of particle angles, and for energies that map over both the broad and narrow structures in $\sigma(E)$. Calculations for various theories are compared with the alignment data in fig. 701.

Except in the range $E=19.5-21.2$ MeV, the alignment averaged over the particle angles is substantially greater than the value expected from random population of the magnetic substates. Moreover, $A(E)$ is strongly
701) A(\(E\)): DATA AND PREDICTIONS

The data of fig. 504 are presented (solid lines) along with the calculations from the band crossing model (dotted line, from fig. 603), the DWBA (dash-dot line, from fig. 605), and the diffraction model (dashed line, from fig. 607), as well as the energy-independent predictions of the plane waves (PW) and the random statistical (STAT) models.
Fig. 201

Alignment A

$E_{CM}$ (MeV)

$\theta_{CM} = 59^\circ - 90^\circ$
(and positively) correlated with both the broad and narrow structures observed in \( \sigma(E) \) for \( E<26 \text{ MeV} \). Above that energy, the alignment is fairly insensitive to variations in either \( \theta \) or \( E \), although it is still substantially greater than the random prediction.

The resonant band crossing model was found inadequate to explain \( A(E) \), while non-resonant direct reaction mechanisms, particularly those incorporated in the diffraction model, were successful in reproducing the broad structure observed in \( \sigma \) and \( A \). The failure of these models at higher energies is attributed to greater diffuseness of the nuclear potential well, perhaps indicating a significant change in the nature of the \( ^{12}\text{C}+^{12}\text{C} \) interaction. The transition to this energy region, in which several pairs of partial waves contribute to the scattering, is reflected in the behavior of \( \beta(\theta) \), and leads to a change in the characteristic behavior of \( A(E) \) and \( \sigma(E) \).

Again, this experimental work does not prove that the diffraction model is the correct one for this reaction. Although the distorted waves and the Austern-Blair approaches to the inelastic scattering problem use different approximations, to lowest order (which is what we consider here) they should yield similar predictions. The disparity in the results for \( A(E) \), therefore, arises from differences in the parametrization of the physical system between Phillips, et al., and Cannell, et al. A reasonable parametrization within any model which fits all the data now available is called for. Current theories, both resonant and non-resonant, need to be further developed (and perhaps new models devised) in order to reproduce the features of the \( ^{12}\text{C}+^{12}\text{C} \) interaction. Our point is that the gross structures observed at \( E>15 \)
MeV do not seem to be inconsistent with a direct, non-resonant interpretation.

The narrow structures below 20 MeV, whose width and correlations with other exit channels indicate a genuine quasimolecular origin, are characterized in our work by enhanced alignment, unusually small slopes in $\beta(0)$, and a striking change in the pattern of the correlation function $W(\theta)$. These findings should be employed to gain greater understanding of the mechanisms that underlie this resonance process.

7.2 Extensions to other work

Because of their difficult and time-consuming nature, experiments to measure parameters of magnetic substate populations are rarely performed in heavy ion physics. They become worthwhile only when the information they contain is needed to establish the physics involved in a particular nuclear reaction or set of reactions. The techniques we used can be extended both to other energies and angles in the same system ($^{12}$C+$^{12}$C*) and to studies of other systems.

As discussed in the preceding chapter, the phase angles and alignments measured for $E=26-32$ MeV augur a different character in the $^{12}$C+$^{12}$C scattering that has been apparent in cross section data at higher energies. Further studies in this energy region may better delineate this transition.

There is a more obvious transition near the lower energies we have studied. Narrow quasimolecular structures have been established only
for $E<20$ MeV, while the broad structures appear for $E>12$ MeV. In our work we have examined the character of narrow structures at 16.8, 17.9, and 19.3 MeV that appear to be superposed on the broad structure centered at 18 MeV. Enhanced alignments occur at those energies, and the correlation function $W(\theta)$ undergoes a striking change of character as the energy increases and passes over the resonances. A detailed study down to $E=12$ MeV might bring out more details of the interplay between the resonance effects and the non-resonant background. In addition to its time-consuming nature, such an experiment would have to cope with both the Doppler recoil of the particles\textsuperscript{1} and with a lower counting rate because of a smaller inelastic cross section and thinner targets needed for better energy resolution.

The behavior of $^{12}\text{C}+^{12}\text{C}$ scattering at more forward angles $\theta<50^\circ$ may also prove interesting, although a study of angular distributions (Ph79) indicates that region is well reproduced by non-resonant direct reaction mechanisms. Near $0^\circ$ the plane wave limit should be observed experimentally.

The particle-gamma technique might profitably be applied to other nuclear systems. In particular, the $^{16}\text{O}+^{16}\text{O}(3^-)$ excitation function has shown broad structures similar to $^{12}\text{C}+^{12}\text{C}(2^+)$, and has been addressed by the same models (Ko80, Ph79, Fr79, Le81). If the effects giving rise to the narrow structures are masking the physics underlying the broad maxima, then $^{16}\text{O}+^{16}\text{O}$ will be a good system in which to study only the gross features in $\sigma(E)$. In addition, contamination of the spectrum by

\textsuperscript{1} This effect was not a problem for us, as discussed in sect. 2.3, but it would be more significant at lower velocities.
higher energy photons would not be a concern. Since fragmentation of
the broad structures occurs in $^{12}\text{C}+^{12}\text{C}$ only for $E<20$ MeV, we would
expect our results above 20 MeV to be similar to the outcome of a
$^{16}\text{O}+^{16}\text{O}$ study. This expectation might also apply to broad, regularly-
spaced structures observed in inelastic scattering of other identical
particle systems, such as $^{14}\text{C}+^{14}\text{C}$ (Ko80a) and $^{28}\text{Si}+^{28}\text{Si}$ (Be81).

The question arises whether the gross structures observed in $^{12}\text{C}+^{12}\text{C}^*$
and $^{16}\text{O}+^{16}\text{O}^*$ merely reflect angular momentum and Q-value matching, as
claimed in refs. Ph79 and Fr79. Freeman, et al., (Fr80a) have studied
the $^{16}\text{O}+^{16}\text{O}(0_2^+)$ and $^{12}\text{C}+^{16}\text{O}(0_2^+)$ excitation functions and concluded
that structures persist even with a severe angular momentum mismatch.
Perhaps these reactions, which involve only a small fraction of the flux
associated with the grazing angular momentum, arise from a different,
and possibly resonant, mechanism. Analogous results for $^{12}\text{C}+^{12}\text{C}^*$
scattering to the $3_1^-$, $4_1^+$, and $0_2^+$ states have been reported by Fulton,
et al (Fu80).2 While all these $^{12}\text{C}$ states decay via alpha particle
emission, a different technique to measure alignments (for non-zero spin
states, of course) might be devised, and the resulting data prove useful
for understanding these recent studies.

Related experiments have been performed in inelastic scattering
involving non-identical particles. Jachcinski, et al., (Ja79) have used
particle-gamma techniques measuring $W(\theta)$ for fixed $\psi$ and $\phi$ to
derive the spins of resonances in $^{12}\text{C}^*+^{16}\text{O}$ and $^{12}\text{C}+^{16}\text{O}^*$ scattering.
These were the same techniques used in $^{12}\text{C}+^{12}\text{C}^*$ by Cannell, et al.,

2 Observation of structure in the $0^+$ inelastic scattering excitation
function is also damaging to the band crossing interpretation.
The $^{12}\text{C}^{*}+^{16}\text{O}$ magnetic substates have also been measured using a line-shape analysis of high resolution particle spectra from magnetic spectrographs (Bo78, Be80). This technique promises to be a convenient and powerful tool to use inelastic scattering in order to analyze direct reaction mechanisms, by studying their predicted partial cross sections to different magnetic substates.

Finally, other researchers have recently performed alignment and polarization experiments in heavy ion transfer reactions. Pougheon, et al., (Po79) have observed strong $^{20}\text{Ne}^*$ polarizations in $^{16}\text{O}(^{16}\text{O},^{12}\text{C})^{20}\text{Ne}$. Takahashi, et al., (Ta78a, Ta80a) have measured the $^{12}\text{B}$ polarization in $(^{14}\text{N},^{12}\text{B})$ reactions on a wide range of heavy ion targets. And Trautmann, et al., (Tr81) have found that polarizations of ejectiles from the $^{16}\text{O}+^{58}\text{Ni}$ reaction are consistent with Q-value matching effects of a direct nuclear reaction. This is a young field, and only recently have models been examined for alignment (Va79) and polarization (Bo80) in heavy ion transfer reactions. It remains to be seen whether the DWBA or related models are adequate for this purpose.

As experimental techniques become more sophisticated, heavy ion reactions are studied in ever-increasing detail in terms of cross section, substate populations, polarization, alignment, and phase angle. This large body of data challenges theorists to devise more precise models that are able to reproduce the information recently made available.
APPENDIX A. TABLES OF DATA

A.1 IN-PLANE ANGULAR CORRELATIONS

The energy $E_{\text{CM}}$ (in MeV) is listed above each block of data. The particle scattering angle $\theta_{\text{CM}}$ (in degrees) is in col. 1. The remaining columns are, in order, the in-plane $\gamma$-angle $\phi$ (in degrees), the correlation function $W(90^\circ, \phi)$, and the standard deviation $\sigma_W$ on $W$. $\sigma_W$ includes statistical and other errors propagated through the analysis, but does not include the 14% uncertainty in the determination of the absolute normalization constant $N$. 
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A.2 ALIGNMENTS, "POLARIZATIONS", AND PHASE ANGLES

The first two columns are $E_{CM}$ (in MeV) and $\theta_{CM}$ (in degrees). The next five columns are, in order, $A$, $\sigma_A$, $P$, $\sigma_P$, and $\beta$. The last two numbers, separated by a hyphen, give the range of $\beta$ values corresponding to one standard deviation in both $c_5$ and $c_6$ (see chap. 2, eqn. (20)). The standard deviations $\sigma$ are obtained from the least-squares fitting procedure. Only $A$ depends on the absolute normalization of the correlation function, and in these tables $\sigma_A$ does not include the 14% uncertainty in the determination of $N$. 
A.3 COMPLETE ANGULAR CORRELATIONS

The following table lists the correlation function $W$ corrected for $\gamma$-attenuation, and the coefficients from a fit to eqn. (18) pertinent to the determination of $N$. The data are normalized with $N = \frac{15.70}{0.479} = 32.77$ (the value from sect. 4.1 times the fraction $f'$ of the $\gamma$-spectrum in the photo and escape peaks). For the four particle angles, the mean value of the quantity $c_3 + c_4/3 + c_5/5$ is $M = .1882$. A comparison of $M$ to $\frac{1}{4\pi}$ implies the value of $N$ should be 13.86. We estimate an uncertainty of 10% in this result based on the errors in $M$, $f$, and other systematic effects.
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$c_0$  
0.1586 ± 0.0027  
0.1858 ± 0.0043

$c_1$  
0.3239 ± 0.0238  
0.2508 ± 0.0401

$c_2$  
-0.4365 ± 0.0256  
-0.4270 ± 0.0416

$c_0 + c_1/3 + c_2/5$  
0.1793 ± 0.0098  
0.1840 ± 0.0163
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\[
c_0 = .1834 ± .0027 \quad 2385 ± .0058
\]
\[
c_1 = .2923 .0231 \quad -.0359 .0505
\]
\[
c_2 = -.4420 .0241 \quad -1.477 .0516
\]
\[
c_6+c_1/3+c_2/5 = .1924 ± .0095 \quad .1970 ± .0206
\]
APPENDIX B. ANGULAR DEPENDENCE OF ALIGNMENT AND PHASE ANGLE

Using the limit of high orbital angular momentum and the simplifying assumption of \( L' = L - 2 \), the angular behavior of \( A \) and \( B \) can be derived analytically (We79).

We start with an expression for the angular dependence of the wavefunction \( \Psi(\theta, \phi) \):

\[
\sqrt{\sin \theta} \Psi = \sum_{\mu} \left< L' \mid -\mu \right. \mid L \mid 0 \left. \right> \sqrt{\sin \theta} Y_{L'}^{\mu}(\theta, \phi) \chi_2^{\mu}. \tag{40}
\]

Here we have dropped constants not essential to our derivation, and multiplied by the factor \( \sqrt{\sin \theta} \) that would normally be introduced into an integration over solid angle,\(^1\) so that all angle-dependent terms are included. We sum over the magnetic substates \( \mu \) of the \( I=2 \) state.

\(^1\) For example, the integrated cross section is given by

\[
\sigma = \int | \Psi |^2 d\Omega = \int | \Psi |^2 \sin \theta \, d\theta \, d\phi = \int | \sqrt{\sin \theta} \, \Psi |^2 \, d\theta \, d\phi \tag{41}
\]
excited in the scattering; $\chi_{2}^{\mu}$ is the spin wavefunction for that
substate. $L$ and $L'$ are the incoming and outgoing orbital angular
momenta, respectively. (40) is the standard expression obtained by
defining the beam axis $k$ as the quantization axis $z$. A rotation of the
coordinate axes introduces different spin wavefunctions:

$$\chi_{2}^{\mu} = \sum_{m} \mathcal{D}_{\mu m}^{2} \chi_{2}^{m} . \quad (42)$$

Inserting (42) into (40) yields:

$$\sqrt{\sin \theta} \psi \propto \sum_{m} \sum_{\mu} \left[ \sum_{L} L^{-\mu} 2 \mu | L 0 > \sqrt{\sin \theta} Y_{L}^{\mu}(\theta, \phi) \mathcal{D}_{\mu m}^{2}(R) \right] \chi_{2}^{m} . \quad (43)$$

The quantity in brackets is $a_{m} = a_{m} e^{i \phi_{m}}$, the complex amplitude
to populate substate $m$ with any chosen quantization axis. We want
$z=k \times k'$ perpendicular to the reaction plane, and we make the arbitrary
choice $y=k'$, which later leads to $a_{m}$ independent of $\phi$. The Euler
angles effecting this transformation are $\alpha = -\phi + \pi/2$, $\beta = -\pi/2$,
and $\gamma = 0$, and the rotation matrix is (Br68, eqn. 2.17):

$$\mathcal{D}_{\mu m}^{2}(R) = e^{i \mu \phi} e^{-i \mu \pi/2} d_{\mu m}^{2}(-\pi/2) . \quad (44)$$

The spherical harmonics are written in terms of associated Legendre
polynomials (Sc68, eqn. 14.16), and we take the asymptotic value of the
latter in the limit of large angular momentum $L$ (Ab65, eqn. 8.10.7) to
obtain:

$$Y_{L}^{m}(\theta, \phi) \approx (-)^{m} \frac{2L+1}{8 \sin \theta} \frac{(L-m)!}{(L+m)!} \frac{\Gamma(L+m+1)}{\Gamma(L+3/2)} \cdot$$

$$\cdot \cos \left[ (L+\frac{1}{2}) \theta - \frac{\pi}{4} + \frac{m \pi}{2} \right] e^{i m \phi} . \quad (45)$$

For our case $L \geq 10$, which is sufficiently large to make this limit
valid for θ not close to 0 or 180°. Also, \( g \gg m \), and we drop factors in (45) that are constant in this approximation. Insertion of (44) and (45) into (43) then gives

\[
a_m = \sum <L' -\mu 2 \mu | L0> \cos (\omega - \frac{3}{2} \mu \pi) e^{-i\mu \pi/2} d^2 \mu m (-\frac{\pi}{2}). \tag{46}
\]

Here we have defined \( \omega \) as \((L'+1/2)\theta-\pi/4\). Making the further assumption \( L'=L-2 \) leads to evaluation of the Clebsch-Gordon coefficients (Ab65, table 27.9.4):

\[
<L' -\mu 2 \mu | L0> = \sqrt{\frac{L'(L' - 1)}{16 (L' + \frac{3}{2})(L' + 3/2)}} \approx \frac{1}{4} \text{ for } \mu = \pm 2
\]

\[
\sqrt{\frac{L'(L' + 2)}{4 (L' + \frac{3}{2})(L' + 3/2)}} \approx \frac{1}{2} \text{ for } \mu = \pm 1
\]

\[
\sqrt{\frac{3 (L' + 1)(L' + 2)}{8 (L' + \frac{3}{2})(L' + 3/2)}} \approx \frac{3}{8} \text{ for } \mu = 0. \tag{47}
\]

Values for \( d^2 \mu m (-\pi/2) \) are calculated from Br68 table 1, p. 24, and with (47) they are used to evaluate the sum in (46). The results are:

\[
a_m = \frac{1}{2} (\cos \omega \mp i \sin \omega) \text{ for } m = \pm 2
\]

\[
= 0 \text{ for } m = 0, \pm 1. \tag{48}
\]

Hence, \( a_+^2 = a_-^2 \) independent of \( \omega \), while \( a_0^2 = 0 \). With the approximations \( L'=L-2 \) and \( L' \) large, alignment is complete \( (A=1) \) and independent of particle scattering angle. A variation of alignment with angle would indicate a significant admixture of the \( L=L' \) component of the scattering.

The phase angle from (48) is

\[
\beta = \beta_+ - \beta_- = -2 \omega = \frac{\pi}{2} - (2L'+1)\theta. \tag{49}
\]

Linear behavior of \( \beta(\theta) \) with slope \(-(2L'+1)\) would most probably
reflect dominance of $L'=L-2$ in the scattering, although a combination of $(L,L')$ values at a given energy might conspire to give a linear character to $\beta(\theta)$ with an apparently anomalous slope.
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