APPLICATION OF CORIOLIS-COUPLING MODEL TO

ODD-ODD NUCLEI IN THE 2s-1d AND 1f-2p SHELLS

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ABSTRACT

The strong-coupling collective rotational model of Bohr and Mottelson is used to calculate the energy level spectra and electromagnetic properties of odd-odd nuclei in the 2s-1d and 1f-2p shells. The model places the last odd neutron and odd proton in Nilsson orbitals generated by the axially-symmetric deformed core. The model Hamiltonian consists of the collective rotational energy, including the Coriolis term, plus the Nilsson Hamiltonians of the two odd particles plus a residual interaction between them. The wave functions are obtained by diagonalizing this Hamiltonian on a basis consisting of the rotational bands built on all the allowed single-particle states within the major shell. Magnetic dipole moments, electric quadrupole moments, and M1 and E2 transition rates are calculated. Calculations have been performed for F^{18}, Na^{22}, Al^{26}, P^{30}, Sc^{46}, and Mn^{56}. Agreement with the experimental spectrum is poor for F^{18}. For Na^{22}, agreement with the experimental spectrum and electromagnetic properties is good for states below 2 MeV, but too many states are predicted between 2 and 4 MeV. For Al^{26}, there is good agreement with the experimental energy spectrum, but there are discrepancies in the transition rates between low-lying levels. For P^{30}, the experimental spectrum is well reproduced below 4 MeV, but transition rates from states between 2 and 4 MeV do not agree with experiment. For Sc^{46} and Mn^{56}, the theoretical results are in fair accord with existing experimental data, but there is not enough data available to provide an adequate test of the model.
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Chapter I

Introduction

In this thesis we present the results of calculations of some properties of several light nuclei using the unified model of Bohr, Mottelson, and Nilsson. We precede our discussion of the formalism of this model and the implications of the results with a more general discussion of the role of models in nuclear physics and of the motivation for this work.

In principle, one might hope to develop a theoretical description of the atomic nucleus by treating it as a finite many-body system. One would then write a nuclear Hamiltonian in terms of the interactions between individual nucleons as determined from experiment or from some more fundamental theory. The Schrödinger equation for this Hamiltonian could then be solved and the resulting wave function used to predict experimentally observed nuclear properties. In practice, such a frontal assault on the problem of nuclear structure is far beyond the capabilities of the theoretical physicist, and a less fundamental avenue of approach must be sought.

This simpler approach is generally provided by the concept of a nuclear model. Instead of beginning with the properties of the constituents of the nucleus, the individual nucleons, we begin with the properties of the nucleus itself and attempt to construct a physically reasonable model which will reproduce these latter properties. In the worst case, this can amount to nothing more than a parameterization of the experimental data. But in more favorable cases nuclear models can play an important role in furthering the development of nuclear physics. A model which successfully interprets some experimental data may be useful in suggesting...
additional experiments which provide a more stringent test of the model. Also, the success or failure of a phenomenological model may provide a guide to a more fundamental theoretical approach to the same problem.

One of the most interesting features of nuclear physics has been the wide applicability of two models which are apparently contradictory in their initial assumptions. The nuclear shell model assumes that the individual nucleons occupy independent single-particle orbitals and that the properties of the nucleus are largely determined by the last few "active" nucleons with the others serving as an inert core. The collective model, on the other hand, assumes that the motion of the individual nucleons is highly correlated and that the nuclear degrees of freedom can be well represented by a few collective variables that involve averages over the individual nucleon coordinates. The common feature of these two models is the reduction of the number of variables needed to describe the nucleus to a manageable level. Each of these models has enjoyed considerable success in interpreting nuclear properties. In particular, the collective model gives a good account of the rotational properties of nuclei in the deformed regions of the periodic table, of enhanced electromagnetic transition moments, and of a wide variety of quasi rotational phenomena in nuclei in which the deformation is not yet fully stabilized; the shell model accurately predicts properties of nuclei with only a few particles or holes outside a closed shell, such as nuclei in the vicinity of Pb$^{208}$.

These two aspects of nuclear structure are combined in the unified model, which is used in the calculations presented in this thesis. This
model, like the shell model, places the last few active nucleons in single-particle orbitals, but the core is allowed collective degrees of freedom, and the single-particle orbitals depend on the collective coordinates of the core. The unified model has been applied successfully to both light and heavy nuclei in the deformed regions of the periodic table.

The nuclear s-d shell is a particularly interesting region of the periodic table from the point of view of nuclear models because both single-particle shell models and collective models have enjoyed some success in interpreting the properties of nuclei in this region. Early shell model calculations with configuration mixing by Elliott and Flowers (El 55) gave a good account of properties of nuclei with only a few particles in the s-d shell. At about the same time, Litherland et al. (Li 56) suggested that some properties of nuclei around mass 25 indicated a rotational character for these nuclei and could best be explained by the unified model. The success of these two approaches in the same region of the periodic table suggested that the collective model and the shell model may in fact be equivalent. This equivalence was established by Elliott in his developments of the $SU_3$ classification of nuclear states (El 58), which demonstrated the microscopic origin of nuclear rotational properties. More recently, Hartree-Fock calculations by Levinson and Kelson (Ke 64) have provided further insight into the connection between microscopic and collective aspects of nuclear structure by providing a self-consistent derivation of deformed single-particle orbitals similar to the orbitals used in the unified model.

Considerable progress has been made in understanding the properties of s-d shell nuclei in terms of these models. However, up to the present
time, no applications of the unified model to odd-odd nuclei in the lower half of the s-d shell have been reported that utilize the full complexity of the model, including mixing of rotational bands due to the Coriolis term in the rotational Hamiltonian. One purpose of the present investigation was to determine to what extent the self-conjugate odd-odd nuclei in the lower half of the s-d shell can be understood in terms of a unified model including a complete treatment of band mixing and a realistic residual interaction between the odd neutron and the odd proton. The calculations presented here will also provide some insight into features of the unified model with band mixing - which we will call the Coriolis model - that are not found in simple unified model calculations without band mixing.

Nuclei in the $^{7/2}$ subshell generally have been considered to be good candidates for shell model calculations because the $^{7/2}$ shell is well separated in energy from neighboring shells. However, calculations in which the active nucleons are restricted to the $^{7/2}$ shell have failed to account for the properties of some of these nuclei (Mc 64). A recent study by Malik and Scholz for odd-even $^{7/2}$ shell nuclei has suggested that a unified model can explain many of these discrepancies, when the Coriolis coupling is properly included. In this thesis, we extend the work of Malik and Scholz to odd-odd nuclei in the lf-2p shell in order to determine whether the Coriolis model also provides an appropriate framework for the description of odd-odd nuclei in this region.
In this chapter, we present a brief sketch of the "unified" model as it was developed by Aage Bohr and his collaborators. A detailed description of the theory in a form suitable for application to light odd-odd nuclei will be given in the following chapter. The emphasis here will be on the foundations of the model rather than on the mathematical details necessary for applications. Some calculations based on the unified model will be mentioned in order to indicate the range of nuclei to which this model has been applied and the success it has had. Finally, some microscopic theories bearing on the unified model will be discussed.

**Collective degrees of freedom**

The nuclear surface with respect to the origin is given by 
\[ R(\theta, \phi). \]

We expand \( R \) in spherical harmonics,
\[ R(\theta, \phi) = R_0 \left[ 1 + \sum_{\lambda\mu} \alpha_{\lambda\mu} Y_{\lambda\mu}(\theta, \phi) \right], \quad (1) \]

where \( R \) is the radius of the nucleus in the limit of a spherical shape. The coefficients \( \alpha_{\lambda\mu} \) in this expansion are taken as generalized coordinates describing the degrees of freedom of the nuclear surface. For \( \lambda = 0 \), we get an increment in the nuclear radius. An excitation of this type would involve the compression of nuclear matter and would be expected to occur at much higher energy than the oscillations described by higher values of \( \lambda \), which conserve the nuclear volume. The \( \lambda = 1 \) terms describe motions of the nuclear center of mass and so are not internal degrees of freedom. Hence the first terms of physical interest for the description of low-lying states have \( \lambda = 2 \). Terms with \( \lambda > 2 \) are expected to be less important.
Therefore, only \( \lambda = 2 \) will be discussed in the present work and the subscript \( \lambda \) will be dropped.

For small values of the coefficients \( \alpha \), the potential energy can be written

\[
V = \frac{1}{2} \sum_{\mu} C|\alpha_{\mu}|^2 ,
\]

and the kinetic energy is

\[
T = \frac{1}{2} \sum_{\mu} B|\dot{\alpha}_{\mu}|^2 .
\]

Then the conjugate momentum to \( \alpha_{\mu} \) is

\[
\pi_{\mu} = \frac{\partial T}{\partial \dot{\alpha}_{\mu}} = B\dot{\alpha}_{\mu} * .
\]

The quantities \( B \) and \( C \) can be estimated using the classical hydrodynamic model, but we will now be interested only in the general form of the equations and will reserve a discussion of the parameters until later.

The general deformation of order 2 represents an ellipsoid which may be specified by three angular parameters giving its orientation in space and two internal parameters characterizing its shape. The orientation can be specified by the Euler angles of the principle axes of the ellipsoid with respect to a space-fixed coordinate system. The shape of the ellipsoid in a body-fixed coordinate system whose axes coincide with the principal axes of the ellipsoid is then given by the coefficients \( a_{\nu} \), with

\[
a_{\nu} = \sum_{\mu=-2}^{2} a_{\mu} \mathbf{S}^2_{\mu\nu}(0),
\]

where \( \mathbf{S}^2_{\mu\nu}(0) \) are the transformation functions for the spherical harmonics of order 2 and \( 0 \) represents the three Euler angles giving the orientation of the ellipsoid. Since the \( a_{\nu} \) are defined in the principal axis system, we must have \( a_2 = a_{-2} \) and \( a_1 = a_{-1} = 0 \). Equation (5) thus defines a
transformation from the five \( a_v \) to new coordinates \( a_0, a_2, \) and \( \theta_i \).

However, the transformation from the \( a_v \) to the \( a_0, \theta_i \) coordinates is not unique, since the principal axes of the ellipsoid can be chosen in several different ways. The lack of uniqueness of the \( a_v, \theta_i \) coordinates will impose symmetry requirements on the wave functions defined in terms of these coordinates that will be discussed later.

We now define the deformation parameters \( \beta \) and \( \gamma \) by

\[
\begin{align*}
a_0 &= \beta \cos \gamma \\
a_2 &= \frac{1}{\sqrt{2}} \beta \sin \gamma .
\end{align*}
\]

The coordinate \( \beta \) is a measure of the total deformation of the nucleus, and \( \gamma \) describes the deviation from axial symmetry. The set of coordinates \( \beta, \gamma, \) and \( \theta_i \) will be referred to as \( \beta_\mu \).

The potential energy can be obtained in terms of the \( \beta_\mu \) by using Equations (2), (5), (6), and the unitary character of the \( \mathcal{D}_{\mu v} \). The result is simply

\[
V = \frac{1}{2} C \beta^2 .
\]

The kinetic energy given by Equation (3) can be written in terms of the new coordinates \( \beta_\mu \). The resulting expression splits into two terms, the first of which represents vibrations that change the shape of the nucleus without changing the orientation, and the second of which represents rotation of the nucleus without changing the shape. We therefore write

\[
T = T_{\text{vib}} + T_{\text{rot}} .
\]

The vibrational term is given by

\[
T_{\text{vib}} = \frac{1}{2} B (\dot{\beta}^2 + \beta^2 \dot{\gamma}^2) .
\]
angular momentum of the nucleus $\hat{\mathbf{L}}$ as

$$\hat{\mathbf{L}} = \int \rho \mathbf{r} \times \mathbf{v} \, dr$$  \hspace{1cm} (10)

where the integral is taken over the volume of the nucleus, $\rho$ is the nuclear mass density, and $\mathbf{v}$ is the velocity of the nuclear fluid. Under the assumption of irrotational flow, one then finds (Bo 52):

$$T_{\text{rot}} = \frac{1}{2} \sum_{\kappa} \frac{\mathbf{L}_{\kappa}^2}{\mathcal{J}_{\kappa}}$$  \hspace{1cm} (11)

with

$$\mathcal{J}_{\kappa} = 4B_\kappa^2 \sin^2 \left( \gamma - \frac{2\pi}{3} \right) .$$  \hspace{1cm} (12)

we note that for the axially symmetric case $\mathcal{J}_3 = 0$, and $\mathcal{J}_1 = \mathcal{J}_2 = \mathcal{J}$.

This brief outline shows that the classical theory of second order nuclear surface oscillations provides the rotational and vibrational degrees of freedom that we need to describe the collective aspects of the nucleus.

The transition to quantum mechanics is made as in Bo 52. In general, this results in a complicated system of coupled differential equations involving the $\beta_\mu$ coordinates, since the moment of inertia tensor depends on the vibrational coordinates. But for many nuclei in the region of deformed equilibrium shapes, empirical evidence suggests that this vibrational-rotational coupling can be neglected. In the lower half of the $s$-$d$ shell in particular, the experimentally observed constancy of the quadrupole moment for rotationally excited states suggests that these nuclei are not subject to vibrational excitations. This assumption will be made in all the calculations described below, so we now discuss only the rotational aspect of the problem.
The rotational energy can be written in terms of the angular momentum components along the principal axes of the ellipsoid as

\[ T_{\text{rot}} = \frac{3}{2} \hbar^2 \sum_{\kappa=1}^{3} R_\kappa^2 \]

with

\[ R_\kappa = \frac{\gamma \kappa}{\hbar} \]

(We will use the convention that indices 1, 2, and 3 refer to the body-fixed axes and indices x, y, and z refer to the space-fixed axes.)

The operators \( R_\kappa \) can be calculated as differential operators in the \( \ell_\mu \).

They satisfy the commutation relation

\[ [R_1, R_2] = -iR_3; \]

that is, the commutation relation has the opposite sign from the commutation relation for the components of an angular momentum operator in a space-fixed coordinate system. An appropriate set of basis functions for the rotational wave function is formed by the functions \( \mathcal{D}^I_{MK}(\theta_i) \), which form a \((2I + 1)\) - dimensional representation of the rotation group. These functions are eigenfunctions of \( R_3 \), \( R_z \), and \( R_\theta \) with eigenvalues \( K \), \( M \), and \( I(I + 1) \) respectively. An eigenfunction for the total collective Hamiltonian in the axially symmetric case could then be written as

\[ \psi = f(\beta)g(\gamma) \mathcal{D}^I_{MK}(\theta_i), \]

where \( f(\beta) \) and \( g(\gamma) \) are eigenfunctions of the vibrational parts of the Hamiltonian. When we take into account the symmetry requirements imposed by the ambiguity of the coordinates, we find that the appropriate nuclear wave function is (Pr 62)

\[ \psi = f(\beta)g(\gamma) \left[ \mathcal{D}^I_{MK}(\theta_i) + (-)^I \mathcal{D}^I_{M-K}(\theta_i) \right]. \]
Moreover, only even values of K are allowed, and for K = 0 there are only even values of I.

We expect this wave function to describe an even-even nucleus where both vibrational and single-particle excitations are much higher than the rotational excitations. For such a nucleus the model predicts rotational bands based on the various vibrational states with the energy levels proportional to \(I(I + 1)\). And in fact, there are many nuclei whose energy spectra are in remarkably close agreement with this prediction. In the "deformed" regions of the periodic table with \(A = 155\) to \(190\) and \(A > 220\), there are several cases in which rotational band spectra follow the \(I(I + 1)\) rule to within one or two percent for spins up to \(10\) or \(12\). This close agreement between a simple theory and experimental data is virtually unknown in other areas of nuclear structure physics. It serves as a very strong support for the validity of the collective rotational picture.

**Coupling of Collective and Particle Motion**

This simple picture must be modified for the case of odd \(A\) and odd \(N\)-odd \(Z\) nuclei. For these nuclei, we need a model in which the total angular momentum is shared between the core and the last one or two nucleons. If the interaction between the odd particles and the deformed core is strong, then we can expect the odd particles to move in the average field generated by the core and to be affected only adiabatically by the rotation of the core. This "strong-coupling" approximation will be used in all the calculations described below.

The total angular momentum of the nucleus \(\hat{I}\) is then taken as the sum of \(\hat{R}\), the nuclear surface oscillation angular momentum, and \(\hat{j}\).
the single-particle angular momentum. The rotational energy, given by (11), can now be written in terms of $I$ and $J$:

$$T_{\text{rot}} = \sum_k \frac{\hbar^2}{2\mu_k} (\mathbf{I}_k - \mathbf{J}_k)^2.$$  \hspace{1cm} (19)

We express the wave functions in terms of a representation in which $I_3$ and $J_3$ are diagonal. The basis functions are of the form

$$\psi = f(\theta)g(\gamma)x_n(r)\Theta_{MK}(\theta,\gamma)$$  \hspace{1cm} (20)

for the case of axial symmetry. The function $x_n(r)$ is the wave function for the odd particles in the field generated by the nuclear core. Since this field does not have spherical symmetry, the particle angular momentum $J$ is not a good quantum number, but the projection of $J$ on the axis of symmetry, $\Omega$, is a good quantum number. Axial symmetry also implies that the rotation of the core can have no component along the axis of symmetry, so we have $K = \Omega$.

The basis function (18) is not diagonal in the energy operator (19) because of the presence of the cross terms $\mathbf{I} \cdot \mathbf{J}$ that couple the particle angular momentum with the total angular momentum in (19). These terms are similar to the potential giving rise to the "Coriolis force" in the classical mechanics of rotating coordinate systems, and hence they are frequently referred to as "Coriolis-coupling" terms. They will connect members of two rotational bands of the form (18) with projections $K$ and $K'$ for which $|K-K'| = 1$. When the wave function is properly symmetrized, there is also a diagonal contribution from the Coriolis term in the case of a $K = \frac{1}{2}$ band. The energies of such a "decoupled" band are generally satisfactorily reproduced in heavy nuclei by first order perturbation theory. When two bands with $\Delta K = 1$ lie close together in energy, as in the case of $^{183}$W, the energy levels and wave functions can be calculated.
by diagonalizing the Coriolis term on a basis consisting of the un-
perturbed members of the two bands (Ke 56), ignoring possible effects of
higher lying bands.

The resulting picture for the odd-even and odd-odd nuclei is
similar to that for the even-even nuclei. The model predicts rotational
bands based on different single-particle or vibrational states and inter-
acting via the Coriolis coupling. Agreement between model predictions and
experimentally observed spectra in the deformed regions of the periodic
table is generally about as good as for the even-even nuclei.

**Single-particle Wave Functions**

As stated above, the interaction between the odd nucleons and the
core is taken as a central potential generated by the core in which the
odd nucleons move while adiabatically following the rotation of the core.
As in most calculations with the unified model, we follow the suggestion
of Nilsson (Ni 55) in using a deformed harmonic oscillator potential
with a spin-orbit term and an $\omega^2$ term for this potential. The single-
particle Hamiltonian is then:

$$
\hat{H} = -\frac{\hbar^2}{2M} \nabla^2 + \frac{N}{2}(\omega_p^2 x^2 + \omega_p^2 y^2 + \omega_z^2 z^2) + C\vec{\sigma} \cdot \vec{s} + D \vec{\ell} \cdot \vec{L} .
$$

The ratio $\omega_z/\omega_p$ is determined by requiring the surface of the nuclear
core to be an equipotential. The quantity $\omega_0 = (\omega_p^2 \omega_z^2)^{1/3}$ is determined
by requiring the root mean square radius of a particle in the oscillator
potential to match the observed nuclear radius. Thus shape and depth
of the potential well are related to the collective aspects of the nucleus.
The parameters $C$ and $D$ are generally chosen empirically for each harmonic
oscillator shell.
The single-particle Nilsson wave function $\chi_\Omega$ is obtained by diagonalizing the Nilsson Hamiltonian (19) on a basis consisting of the spherical harmonic oscillator wave functions $\psi_j^\Omega$ with the same total number $N$ of oscillator quanta. Because of the lack of spherical symmetry, the particle angular momentum $j$ is not a good quantum number, and the eigenfunctions involve sums over $j$:

$$\chi_\Omega = \sum_j c_{j\Omega} \psi_j^\Omega.$$  \hspace{1cm} (22)

The coefficients $c_{j\Omega}$ are usually referred to as Nilsson coefficients.

The Hamiltonian (19) also connects states with $\Delta N = 2$. However, such states are separated by about $2\hbar\omega$, and so in the Nilsson approximation this coupling is neglected.

The Nilsson model predicts for a given deformation parameter the relative positions of the intrinsic states on which rotational bands are constructed. Hence the experimentally observed positions of the rotational bands in odd $A$ nuclei give an indication of the nuclear deformation. Moreover, if all the nucleons in the nucleus occupy the lowest allowed Nilsson orbitals, then the total energy of the nucleus can be calculated approximately by adding up the Nilsson energies. The minimum of this energy as a function of deformation also serves to predict the static deformation of the nucleus. The deformations indicated by these two methods are found to be consistent with each other and with the deformation calculated from the observed quadrupole moment for many nuclei in the deformed regions (Mo 59). The magnetic moments and transition rates calculated from the Nilsson wave functions are also generally consistent with experimental data for these same nuclei. These facts lend strong
support to the validity of the unified model for deformed nuclei.

Application of Collective Model to Light Nuclei

As already mentioned above, the most successful applications of the unified rotational model have been in the rare earth region (155 ≤ A ≤ 190) and in the actinide region (A > 226). There are two reasons why we would expect this model to be less appropriate for light nuclei. First, as a general rule, the fewer the number of particles in a system, the less appropriate is a collective description of the system. Second, for lighter nuclei the moment of inertia becomes smaller. This increases the spacing between members of a rotational band and therefore casts doubt on the validity of the adiabatic treatment of the particle motion with respect to the rotating core. Nonetheless, there is strong evidence to suggest that another region of statically deformed nuclei occurs in the lower half of the 2s-1d shell, for A between 20 and 30. The first application of the unified model to an s-d nucleus was made by Litherland et al. (Li 56), who showed that the low-lying levels of Al^{25} could be interpreted naturally as a group of overlapping rotational bands. This interpretation also provided an explanation for some inhibited M1 transitions and enhanced E2 transitions in this nucleus. Further experimental studies and theoretical calculations have led to other successes for the rotational model in this region, so that it is now an accepted tool in the study of s-d shell nuclei. Recent calculations (Sc 66, Ma 66, Sc 67) indicate that many properties of odd nuclei in the 1f_{7/2} shell (N or Z = 21-28) can also be understood in terms of the unified model with suitable treatment of the Coriolis-coupling term.
The application of the unified model to light nuclei is in principle the same as to heavy nuclei. But in practice the different characteristics of light and heavy nuclei lead to somewhat different approaches. In heavy deformed nuclei, the rotational bands can usually be readily identified in the experimental spectrum. Coriolis mixing among these bands can be included by diagonalizing a relatively small matrix corresponding to the few bands that are observed. Mixing with possible high-lying bands that have not been observed experimentally is usually not explicitly taken into account, because there is no simple way of deciding which bands will be important in view of the sensitivity of the band head energies to small changes in the parameters of the calculation. The effect of high-lying bands has sometimes been simulated by introducing "higher order" Coriolis terms that connect states with $\Delta K > 1$. In general, it is hoped that the parameters of the calculation allow enough flexibility to absorb such effects, and this is borne out by the good agreement with experiment provided by the unified model for heavy deformed nuclei.

For light nuclei, on the other hand, the effects of Coriolis coupling are both more important and easier to include in a systematic manner. The moment of inertia is expected to be roughly proportional to $A^{2/3}$, so the Coriolis-coupling term, which is inversely proportional to the moment of inertia, will be much larger for light nuclei than for heavy nuclei. Strong band mixing may disguise the rotational character of the spectrum, making it difficult to evaluate either the band head energies or the moment of inertia from the experimental spectrum. This strong mixing makes it desirable to include as many bands as feasible in the calculation, rather
than limiting the basis set to those bands for which there is direct experimental evidence. In light nuclei, states which differ in their principal harmonic oscillator quantum number $N$ but have the same parity are fairly widely separated, and the number of states with the same $N$ is manageably small. It is therefore natural to include all possible bands based on states with the same value of $N$, and this procedure has been adopted in the present calculations. Band head energies are calculated directly from the Nilsson model. The moment of inertia is taken as a parameter of the calculation, but the same value is used for all the bands and for off-diagonal matrix elements between bands.

**Microscopic Aspects**

Our discussion of the unified model has concentrated on the application of this model to the prediction of nuclear properties. We now turn to a discussion of the theoretical basis for the use of this model.

The basic assumption of the unified model is that macroscopic coordinates defining the nuclear surface can adequately represent the degrees of freedom involved in nuclear collective motion. This *ad hoc* assumption is justified mainly by the fact that it gives rise to a theory of nuclear rotations which is in excellent agreement with experiment for many nuclei. From a more fundamental point of view, we would expect to describe the nucleus as a finite many-body system of strongly interacting fermions. Ideally, the assumptions of the unified model would be derived from this more fundamental viewpoint. This program has not been carried out, but there are nonetheless several interesting connections between the fundamental microscopic theories and the phenomenological macroscopic approach of the unified model.
One such connection is provided by Hartree-Fock calculations for deformed light nuclei (Ri 68). In the Hartree-Fock approximation, each nucleon of the nucleus moves independently in an average one-body potential generated by the two-body interaction with all the other nucleons. The nucleons occupy the lowest orbitals determined by solving the Schrödinger equation for the one-body potential. Since the potential in turn depends on these orbitals, the problem must be solved self-consistently. A complete calculation would involve a self-consistent determination of the radial and angular wave functions for the occupied orbitals using a realistic two nucleon interaction. This procedure would be very difficult to carry out, and so calculations for light nuclei generally use a much simpler approach. The radial wave functions are taken a priori as harmonic oscillator functions restricted to a single major shell, and a simple phenomenological two-body interaction is used. An additional single-particle potential is also included to account for the interaction with particles in a lower harmonic oscillator major shell. The Hartree-Fock equations in this approach are (Ke 63)

$$
\langle \alpha | h | \beta \rangle = \langle \alpha | K | \beta \rangle + \sum_{\lambda} \langle \alpha \lambda | V | \beta \lambda \rangle - \langle \alpha \lambda | V | \beta \rangle
$$

$$
\hbar | \alpha \rangle = \varepsilon_{\alpha} | \alpha \rangle,
$$

(23)

where \( h \) is the Hartree-Fock single-particle potential, \( K \) is the additional single-particle potential plus the kinetic energy, \( | \alpha \rangle \) is a single-particle orbital, and \( \varepsilon_{\alpha} \) is the Hartree-Fock single-particle energy. The range of the summation extends over all the nucleons in the last major shell. This type of calculation produces single particle orbitals very similar to the Nilsson wave functions. Hence this restricted-basis
Hartree-Fock theory demonstrates that two body forces can give rise to a deformed one-body potential such as the Nilsson potential, and it provides a means of calculating the nuclear deformation microscopically.

Another approach to the structure of rotational nuclei is provided by the SU$_3$ model (Ha 68). This model is a special case of the many-particle shell model. If particles in harmonic oscillator orbitals interact by means of a quadrupole-quadrupole force, $v(i,j) = r_1^2 r_2^2 P_2(\cos \theta_{ij})$, then the eigenfunctions of the Hamiltonian can be classified approximately according to the representations of the group SU$_3$. This provides a means of picking out the low-lying states of the system without diagonalizing the Hamiltonian on a complete set of shell model basis functions. It is found that within a representation of SU$_3$ the eigenvalues are proportional to $L(L+1)$, where $L$ is the orbital angular momentum. The SU$_3$ model thus provides a microscopic explanation for nuclear rotational structure.

The model is limited in application because it requires a particular form for the residual interaction and because it cannot treat spin-orbit forces, which break the SU$_3$ symmetry. It is most appropriate for the treatment of even-even nuclei, where the spins of the nucleons can couple to $S = 0$, so that $L$ will be the total angular momentum.

Another point of contact between the microscopic and macroscopic aspects of nuclear rotation is the moment of inertia parameter. This collective parameter can be obtained from a microscopic calculation by means of Inglis' semiclassical "cranking" model (In 54). The basic idea of the cranking formula and other similar calculations is to use perturbation theory to determine the increase in energy of the system when it is rotated...
about the x axis with a small angular velocity \( \omega \). If the wave function of the unperturbed state is \( \phi \) and of the rotated state \( \phi_\omega \), then the moment of inertia is given by the relation (Ke 67)
\[
\langle \phi_\omega | H | \phi_\omega \rangle - \langle \phi | H | \phi \rangle = \frac{1}{2} J \omega^2.
\]
This leads to the result for the moment of inertia of a state \( |0\rangle \):
\[
J = 2\hbar^2 \sum_{i \neq 0} \frac{|\langle i | J_\times |0\rangle|^2}{E_i - E_0},
\]
where the index \( i \) runs over all the available intrinsic states of the system, and \( E_i \) is the energy of the state \( |i\rangle \). This formula gives incorrect results when the Nilsson energy levels are used in the denominator, but when residual interactions of the pairing type are included excellent agreement with experiment is obtained for heavy nuclei (Ni 66). The situation is not so clear for light nuclei, where the cranking formula may not be correct (Ke 67). In any event, it is interesting to see that the collective moment of inertia can be obtained from a microscopic theory, at least in some cases.

Although the Bohr-Mottelson unified model has not been derived explicitly from a more fundamental microscopic theory, the consistency of the unified model results with the microscopic Hartree-Fock and cranking models reinforces our confidence in the unified model.
Chapter III Unified Model for Light Odd-Odd Nuclei

This chapter gives a detailed presentation of the collective rotational model formalism, outlined in the previous chapter, for the case of an odd-odd nucleus with axial symmetry.

The model Hamiltonian can be written as

$$ H = T_{\text{rot}} + H_{\text{s.p.}}, $$

where $T_{\text{rot}}$ is the collective rotational energy and $H_{\text{s.p.}}$ is the Hamiltonian for the two extra-core nucleons.

**Single-particle Hamiltonian and Wave Functions**

We will take $H_{\text{s.p.}}$ as the sum of the Nilsson energies for the odd neutron and odd proton plus a residual interaction between the two odd nucleons:

$$ H_{\text{s.p.}} = \epsilon_n + \epsilon_p + V(n,p), $$

where the Nilsson energy $\epsilon$ is given in terms of the Nilsson eigenvalue $E$ by Equation 12 below. The form of the Nilsson Hamiltonian, as discussed in Chapter II, is

$$ h = -\frac{\hbar^2}{2\mu} + \frac{\mu}{2} (\omega_x^2 + \omega_y^2 + \omega_z^2) + C \cdot \hat{s} + D \cdot \hat{t}, $$

where $\mu$ is the reduced mass of the nucleon, $\omega_x$ and $\omega_z$ are the oscillator frequencies in the radial and longitudinal directions, $C$ is the strength of the spin-orbit splitting, and $D$ is the strength of the $\hat{t} \cdot \hat{t}$ term which serves as a correction to the harmonic oscillator potential. In the calculations discussed below, we use the same potential for neutrons and protons, so $h_n = h_p \equiv h.$
The single-particle wave functions can be obtained by diagonalizing the Hamiltonian (3) on a basis consisting of all the harmonic oscillator wave functions with the same principal quantum number \( N \). As mentioned earlier this procedure involves the neglect of terms connecting states differing in \( N \) by 2. An alternative approach, which we will adopt here, has been suggested by Nilsson (Ni 55). We make the transformation

\[
\xi = \left( \frac{\mu \omega_x}{\hbar} \right)^{1/2} x, \quad n = \left( \frac{\mu \omega_y}{\hbar} \right)^{1/2} y, \quad \zeta = \left( \frac{\mu \omega_z}{\hbar} \right)^{1/2} z. \tag{4}
\]

Then for our basis set we use wave functions for which the infinitesimal rotation operator \( \hat{t} \) in the pseudo-space \( (\xi, n, \zeta) \) is a good quantum number \( r \) instead of the orbital angular momentum \( \hat{l} \).

We now introduce parameters \( B \) and \( \omega_0(B) \) (Ne 60) through the equations

\[
\omega_\rho = \omega_0(B)(1 + \frac{1}{2} B) \quad \omega_z = \omega_0(B)(1 - B). \tag{5}
\]

The relationship between \( B \) and the collective deformation parameter \( \beta \) is given by

\[
B = \frac{2c(2-c)}{4-c^2}, \quad \text{with} \quad c = \left( \frac{5}{4\pi} \right)^{1/2} \beta. \tag{6}
\]

The requirement that the nuclear volume be independent of the deformation gives the constraint

\[
\omega_0(B) = \omega_0[1 - \frac{3}{4} B^2 - \frac{1}{4} B^3]^{-1/3}.
\]

Substituting these parameters into the Hamiltonian (3) gives the result

\[
\hbar = \frac{1}{4} \hbar \omega_0(B) \left[ -\frac{d^2}{d\xi^2} + 2\xi^2 - \frac{d^2}{dn^2} + 2n^2 - \frac{d^2}{d\zeta^2} + 2\zeta^2 \right] + \omega_0(B)\left( \frac{4}{5\pi} \right)^{1/2} Bp^2 Y_{20}(\theta, \phi) + C_{\chi}^+ \chi^+ + D_{\xi}^+ \xi^+,
\]

\[
\tag{7}
\]
where we have substituted the pseudo-angular momentum $\hat{l}_c$ for $\hat{l}$ in the last two terms. These terms are included as corrections to the harmonic oscillator potential in order to reproduce the shell-model level scheme at zero deformation. They therefore have only a phenomenological origin, and so the use of $\hat{l}_c$ is just as appropriate as the use of $\hat{l}$.

Diagonalization of this Hamiltonian gives us the Nilsson wave function $\chi^\nu_\Omega$ in the form

$$\chi^\nu_\Omega = \sum_j c^\nu_j \chi_j^\Omega,$$

and for the opposite sign of $\Omega$,

$$\chi^{-\Omega} = \sum_j (-)^{j-1/2} c^\nu_j \chi_j^{-\Omega},$$

where $\nu$ is an index used to distinguish between different Nilsson levels with the same projection $\Omega$, and $\chi_j^\Omega$ is a harmonic oscillator wave function with angular momentum $j$ and projection $\Omega$ in the pseudo-space ($\xi, \eta, \zeta$). The quantum numbers $N$, $\ell$, and $s = \frac{1}{2}$ are to be implicitly associated with the coefficients $c^\nu_j$. To obtain consistent phase relationships, we always use $|N\ell \frac{1}{2} j\Omega>$. For the radial part of the harmonic oscillator wave function, we use the definition of the Laguerre polynomial as given in Morse and Feschbach (Mo 53).

The Nilsson eigenvalues are given by

$$h\chi^\nu_\Omega = E^\nu_\Omega \chi^\nu_\Omega.$$

However, if the deformed potential is due to the two-body interaction between the nucleons, then the total energy of the nucleus is not the sum of the Nilsson eigenvalues for the occupied orbitals, since this would count the two-body interaction twice. We can write the single body potential as

$$V(i) = \sum_j v(i, j).$$
The total energy is

\[ E_0 = \sum_i T_i + \frac{1}{2} \sum_{ij} v(i,j) = \sum_i \left( T_i + \frac{1}{2} V_i \right). \]  \hspace{1cm} (11)

For harmonic oscillator states, \( T = V \), so the total energy is given by (Pr 62)

\[ E_0 = \frac{3}{4} \sum_i E_i - \frac{1}{4} \sum_i \langle c_i \hat{\gamma}_i + D_i \hat{\gamma}_i \rangle = \sum_i \epsilon_i. \]  \hspace{1cm} (12)

This expression is used to calculate the energies of excited states, rather than the Nilsson eigenvalues.

In calculating transition rates and rotational energy terms, we will use operators that depend on the particle angular momentum \( \hat{\ell} \) rather than \( \hat{\ell}_t \). For reasonable values of \( \beta \), it is a good approximation to take \( \hat{\ell} = \frac{1}{2} \hat{\ell}_t \) (Ne 62). Hence no distinction will be made between these two quantities in the present calculation, and the subscript \( t \) will be dropped.

**Rotational Hamiltonian and Wave Function**

The rotational energy is given by (Pr 62)

\[ T_{\text{rot}} = \sum_{\kappa=1}^{3} \frac{R_\kappa^2}{2} + \frac{\hbar^2}{2\kappa} (I_\kappa - R_\kappa)^2, \]  \hspace{1cm} (13)

where \( \hat{I} \) is the total angular momentum, \( \hat{R} \) the core angular momentum, and \( \hat{J} = \hat{J}_n + \hat{J}_p \) the sum of the odd neutron and odd proton angular momenta.

In the axially symmetric case considered here, \( \hat{J}_1 = \hat{J}_2 \equiv \hat{J} \), so we can write

\[ T_{\text{rot}} = \frac{\hbar^2}{2\kappa} \left[ (\hat{I} - \hat{J})^2 - (I_3 - j_3)^2 \right] + \frac{\hbar^2}{2\kappa} (I_3 - j_3). \]  \hspace{1cm} (14)

We will use basis functions for which \( \hat{I}_z \), \( I_3 \), and \( j_3 \) are diagonal with eigenvalues \( I(I + 1) \), \( K \), and \( \Omega \), respectively. Also, as mentioned in Chapter II, \( K = \Omega \) for axially symmetric nuclei. The rotational energy therefore becomes

\[ T_{\text{rot}} = \frac{\hbar^2}{2\kappa} \left[ I(I+1) - 2K^2 \right] - \frac{\hbar^2}{2} \left[ I_+j_- + I_-j_+ \right] + \frac{\hbar^2}{2} j^2, \]
where the usual raising and lowering operators $I_+ = I_{1+}i_2$ and $j_+ = j_{1+}j_2$ have been introduced. The first term gives rise to the familiar rotational band structure. The second is the Coriolis coupling term discussed in Chapter II. The third term, depending only on the particle coordinates, could be incorporated into the single-particle Hamiltonian. However, it seems more logical to treat it on the same footing as the other rotational terms, and we follow this course here.

As previously stated, the wave function $\Psi_{MK}^I(\ell)$ are eigenfunctions of $I^2$, $I_3$, and $I_2$ with eigenvalues $I(I + 1)$, $K$, and $M$ respectively. An appropriate representation for the total wave function is then the product wave function $\Psi_{MK}^I(\ell)\chi^p_\Omega(n)\chi^p_\Omega(p)$ with the constraint $n + p = K$. (We use the definition of Preston (Pr 62) for the $\Psi$ functions).

**Symmetry Requirements**

As mentioned earlier, the wave function in the body-fixed coordinate system must satisfy certain symmetry requirements to ensure that it uniquely determines the wave function in the space-fixed coordinate system. There are 24 different ways of choosing the body-fixed coordinate system, and the resulting body-fixed wave function $\Psi$ must be the same for each of these choices. This can be assured by requiring $\Psi$ to be invariant under the 24 transformations that relabel and reorient the body-fixed axes (BO 52). These 24 transformations can be obtained by repeated application of three transformations $R_1$, $R_2$, and $R_3$, where $R_1$ is a reversal of the 2 and 3 axes, $R_2$ is a $90^\circ$ rotation about the 3 axis, and $R_3$ is a cyclic permutation of the three axes. The effect of these transformations on the collective and single-particle wave functions is discussed by Preston (Pr 62). Extending his result for the one extra-core particle case to
our case with two extra-core particles, we find that the properly symme-
trized and normalized basis function is

\[
\psi(\mathbf{IMKQ}_n \nu \Omega_n \nu_p) = \left( \frac{2I+1}{16\pi^2} \right)^{1/2} \sum \psi_{\mathbf{MK}^\prime \Omega_n \nu_p} \chi_{\mathbf{I}}^{\nu_n \nu_p} \chi_{-\mathbf{I}}^{\nu_n \nu_p} \quad (16)
\]

This form of the basis function also implies some restrictions on
the neutron-proton exchange symmetry and hence on the isobaric spin \( T \)
of the wave function, since the total wave function must be antisymmetric
in space, spin, and isospin coordinates. When the neutron and proton
are in the same orbitals \( (\Omega_n = \Omega_p, \nu_n = \nu_p) \), the wave function is
symmetric with respect to exchange of particles, and so can be labeled
as \( T = 0 \). For the case when both particles are in the same orbital but
with their spin projections opposed \( (\Omega_n = -\Omega_p, \nu_n = \nu_p) \), the wave
function is symmetric for \( I \) odd \( (T = 0) \) and antisymmetric for \( I \) even
\( (T = 1) \). When the neutron and proton occupy different orbitals, both a
symmetric and an antisymmetric wave function can be formed. The wave
functions obtained by diagonalizing the Hamiltonian on the basis func-
tions (12) will be either even or odd under exchange of particles, even
though the basis functions themselves are not, because the Hamiltonian is
even under exchange of neutron and proton.

The symmetry of the basis functions also limits the number of states
which must be included. A glance at (12) shows that \( \psi(-K) = (-)^{I+1} \psi(K) \);
and for \( K = 0 \), \( \psi(-\Omega_n, -\Omega_p) = (-)^{I+1} \psi(\Omega_n, \Omega_p) \). Hence we can obtain all
the independent basis functions in a given major harmonic oscillator
shell by including only positive or zero values of \( K \), and for \( K = 0 \) only
positive values of \( \Omega_n \). This consideration reduces by one half the dimen-
Core-excited (hole) States

For a nucleus with only two nucleons occupying a major harmonic oscillator shell, such as $^{18}_F$ in the s-d shell, the procedure to be followed is now clear - we diagonalize the Hamiltonian (1) on a basis consisting of the rotational bands built on all possible combinations of neutron and proton orbitals in the s-d shell. But for a more complicated nucleus, such as $^{22}_Na$, with 6 particles in the s-d shell outside an $^{16}O$ core, we must take the Pauli principle into consideration. The most straightforward way to do this is to suppose that $A-2$ particles fill the lowest available Nilsson orbitals, so that these orbitals cannot be occupied by the two odd particles. The lowest state available in this picture is illustrated in Fig. 1a, taking the case of one odd particle for simplicity. The lowest orbitals are each occupied by two particles with their spin projections antiparallel, and the valence particle is placed in the lowest unoccupied orbital. Excited states can be formed by raising the valence particle to a higher orbital, as in Fig. 1b. However, excited states of comparable energy can also be formed by raising one of the core particles to the ground state level and coupling it with the valence particle, leaving an uncoupled particle below the Fermi sea, as in Fig. 1c. These core excited or "hole" states are therefore included in the basis set in the present calculation. More complicated states that could be obtained by raising a core particle to a level other than the valence level or raising two or more core particles are not included. The first of these involves breaking a pair of particles coupled to $K = 0$, and the second involves two single-particle energies. It is hoped that
states of these types will lie at high enough energies so that they will have little influence on the low-lying spectrum.

A completely symmetric treatment of the hole states would require us to place all particles in the oscillator shell we are treating on an equal footing. Hence we would write the particle angular momentum as \( \hat{j} = \sum \hat{j}_i \) and the residual interaction as \( v = \sum v(i,j) \), where the sums extend over all the particles in the last unfilled shell. This would of course complicate the calculation considerably, perhaps more so than is justified in view of the simple, phenomenological nature of our model. Accordingly, we have chosen to give up the symmetry of the Hamiltonian with respect to exchange of all the particles in the shell and preserve the simplicity of the model. We therefore take \( \hat{j} = \hat{j}_n + \hat{j}_p \) and \( v = v(n,p) \) where \( \hat{j}_n, \hat{j}_p \), and \( v(n,p) \) operate only on the coordinates of the uncoupled neutron and proton in each basis state.

Off diagonal matrix elements between two hole states or between a hole state and the ground state are not the same as the corresponding matrix elements between particle states because of the different core overlap. We illustrate this point by deriving the formula for the matrix element of a one-body operator between a hole state (Fig. 2a) and the ground state (Fig. 2b) of a three particle system. The Nilsson orbitals available to the particles will be designated by \( \pm \Omega \) and \( \pm \Omega' \). Then we can write the ground state in second-quantized form as

\[
|g, s> = a_\Omega^+ a_{-\Omega}^+ a_{\Omega'}^+ |0> \quad (14)
\]

and the hole state as

\[
|h, > = a_\Omega^+ a_{-\Omega}^+ a_{-\Omega'}^+ |0> . \quad (15)
\]
We assume for definiteness that Ω and Ω' are both positive. Then the matrix element of the operator 0 is

\[ \langle g.s., |0| h. \rangle = \langle a_{\Omega}, a_{-\Omega} |0| a_{\Omega}^\dagger a_{-\Omega}^\dagger \rangle \]

\[ = - \langle a_{-\Omega} |0| a_{-\Omega}' \rangle = -\langle -\Omega |0| -\Omega' \rangle , \]

where \( \langle \Omega' |0| \Omega \rangle \) is the matrix element we would have obtained for the case of two particle states or a particle and the ground state. If we take the opposite sign for either the hole projection or the ground state projection, the sign of the matrix element changes. Hence the general result is

\[ \langle \Omega'(g.s.) |0| \Omega(h) \rangle = - \text{Sgn}(\Omega') \text{Sgn}(\Omega) \langle -\Omega |0| -\Omega' \rangle . \] (16)

where \( \text{Sgn}(\Omega) = +1 \) for \( \Omega > 0 \) and \( -1 \) for \( \Omega < 0 \). (A similar result has been used in a rotational model calculation for Ho^{166} by Motz et al. (Mo 67).)

The same result applies to matrix elements between two hole states, and an analogous result can be derived for a two-body operator by considering it as a product of two one-body operators. We note that there are no matrix elements between hole states and particle excited states, since they differ in the coordinates of more than one particle and therefore cannot be connected by any of the operators in our Hamiltonian.

Core Overlap Factor

In writing the wave function (12), we have assumed that the intrinsic nuclear wave function can be satisfactorily approximated as two particles outside an inert core. However, in a more realistic model, we would expect the core wave function to depend on the single-particle states. If the nucleons occupy orbitals determined by a potential that they themselves generate self-consistently, then when one particle is shifted to another
orbital, all the particles feel a somewhat different potential, and so their wave functions change. In the framework of the Nilsson model, we could interpret this core polarization as a dependence of the nuclear deformation on the orbitals occupied by the last two particles. We therefore expect that the core overlap will be less than one for matrix elements between different single particle states. In general, this core overlap factor will depend on the single particle states involved, but in the present calculations we make the simple approximation of a constant core overlap $Q$ with $Q < 1$. We multiply off diagonal matrix elements by $Q$ if the wave functions differ in one single-particle state and by $0^2$ if they differ in both single-particle states.

Matrix Elements

In order to obtain explicit expressions for the matrix elements of the Hamiltonian, we write (1) in the detailed form

$$H = T_{\text{rot}} + H_{\text{s.p.}}.$$ 

$$= \frac{\hbar^2}{2\hbar} \left[ i^2 - 2I_3 (j_{n+3} + j_{p3}) \right] - I_+ (j_{n+3} + j_{p-}) + I_- (j_{n+3} + j_{p-})$$

$$- j_{n+3}^2 + j_{n+3}^2 + j_{n+3}^2 + j_{n+3}^2 + j_{n+3}^2 + j_{n+3}^2 + j_{n+3}^2 + j_{n+3}^2$$

$$+ \epsilon_n + \epsilon_p + \nu(n,p).$$

(17)

Because of the symmetry of the Hamiltonian with respect to reversal of the direction of the $z$-axis, the matrix elements of (17) on the basis set (12) take the form

$$\langle IM\Omega | \nu \nu | H | \nu \nu \rangle_{n n p p} = \frac{2I+1}{16\pi^2}$$

$$\left( \beta_{MK}^\nu \chi^\nu_{n n p p}, H_{MK}' \chi^\nu_{n n p p} \right)$$

$$+ (-)^{I+1} \left( \beta_{MK}^\nu \chi^\nu_{n n p p}, H_{M-K}^\nu \chi^\nu_{n n p p} \right).$$

(18)
The matrix elements of the diagonal operators have already been discussed. To calculate the matrix elements of the remaining operators we use the following results:

\[
\langle x_j \Omega_n, x_j \Omega_n \rangle^2 = j(j+1)
\]

\[
\langle x_i \Omega_n \chi_{i+1} \Omega_n, x_i \Omega_n \chi_{i+1} \Omega_n \rangle = \left[ \langle j_i \Omega_n \chi_{i+1} \Omega_n, j_i \Omega_n \chi_{i+1} \Omega_n \rangle \right]^{1/2}
\]

\[
\langle x_i \Omega_n \chi_{i+1} \Omega_n, x_i \Omega_n \chi_{i+1} \Omega_n \rangle = \left[ \langle j_i \Omega_n (i \pm K+1), j_i \Omega_n (i \pm K+1) \rangle \right]^{1/2}
\]

It will be noted that the operator \( I_+ \) lowers the projection of the angular momentum on which it operates, and \( I_- \) raises it, so that the operator \( I_{\pm \pi} \) connects states that both have \( K = \Omega \). This is a consequence of the sign reversal in the commutation relations for the operators \( I_1, I_2, \) and \( I_3 \), as compared with the corresponding relations for the projections of an angular momentum operator onto space-fixed axes. The operators \( j_1, j_2, \) and \( j_3 \) obey the usual commutation rules, since \( j \) is defined with respect to the body-fixed coordinate system.

We can now write down expressions for all the matrix elements of (17). The diagonal matrix elements are:

\[
\langle \text{IM}_\Omega \nu \Omega \nu | H | \text{IM}_\Omega \nu \Omega \nu \rangle = \frac{\hbar^2}{2\Omega} [\Omega(I+1) - 2K^2]
\]

\[+ \sum_{n p} c_{n \Omega p}^\nu c_{p \Omega p}^\nu j(j+1) + \sum_{n p} (-)^j j_{\Omega p}^\nu j_{\Omega p}^\nu j(j+1/2) j(j+1/2) \]

\[+ 2\Omega \nu \Omega \nu \nu \nu \]

where \( \epsilon_n \) and \( \epsilon_p \) are the neutron and proton Nilsson energies, and

\[
\langle \text{IM}_\Omega \nu \Omega \nu | v \nu \Omega \nu \nu \nu \rangle \]

is the matrix element of the residual interaction. There is no diagonal matrix element of the Coriolis coupling term, such as the "decoupling" term that arises in odd-even nuclei.
The off-diagonal matrix elements between states with the same value of $K$ are given by

$$
\langle \text{IMK}_n^\Omega \nu \nu | H | \text{IMK'}_n^{\Omega'}\nu'\nu' \rangle
$$

$$
= \frac{\hbar^2}{2\hbar} \sum_j \left( c_{jn}^{p_n} c_{jn'}^{p_n'} \delta_{\Omega',\Omega} \delta_{\nu',\nu} + c_{jn}^{p} c_{jn'}^{p'} \delta_{\Omega',\Omega} \Omega \nu, \Omega' \nu' \right) + \delta_{\nu,\nu'} + \frac{h^2}{2\hbar} \left[ (j_n+\Omega_n)(j_n+\Omega_n+1)(j_n+\Omega_n)(j_n+\Omega_n+1) \right]^{1/2}

\times \delta_{n+1, \Omega_n+1, \Omega_n+1, \Omega_n+1} + (-)^{I+K} \frac{\hbar^2}{2\hbar} \sum_j \left( (-)^{j+1/2} c_{jn}^{p_n} c_{jn'}^{p_n'} \delta_{\Omega',\Omega} \Omega \nu, \Omega' \nu' \right)

\times (j_n+1/2)(j_n+1/2) + \Omega_n \nu, \nu | \Omega' \nu, \nu' \rangle \tag{21}
$$

And finally, the off diagonal elements due to the Coriolis term between states with $\Delta K = 1$ are given by

$$
\langle \text{IMK}_n^\Omega \nu \nu | H | \text{IMK'}_n^{\Omega'}\nu'\nu' \rangle
$$

$$
= \sum_j \left\{ c_{jn}^{p_n} c_{jn'}^{p_n'} \delta_{\Omega',\Omega} \Omega, +1, \Omega +1, \Omega' \nu, \nu' \right\} \left\{ (j_n+\Omega_n)(j_n+\Omega_n+1)(j_n+\Omega_n)(j_n+\Omega_n+1) \right\}^{1/2} \left\{ (j_n+\Omega_n)(j_n+\Omega_n+1) \right\}^{1/2}

\times \delta_{\Omega',\Omega+1} \delta_{K',K+1} + (-)^{I+K} \frac{\hbar^2}{2\hbar} \sum_j \left\{ (-)^{j+1/2} c_{jn}^{p_n} c_{jn'}^{p_n'} \delta_{\Omega',\Omega} \Omega, -1, \Omega +1, \Omega' \nu, \nu' \right\}

\times (j_n+1/2)(j_n+1/2) + \Omega_n \nu, \nu | \Omega' \nu, \nu' \rangle \tag{22}
$$

Expressions similar to (20), (21), and (22) have been given by several authors (Da 63, Mo 67). These expressions are correct for matrix elements between particle excited states, but they must be modified for hole-excited states, and the core-overlap factor must be included, as explained above.
Residual Interaction Matrix Elements

It is convenient to express the matrix elements of the residual interaction in a representation with the particle angular momentum $\vec{J} = \vec{j}_n + \vec{j}_p$ and the isobaric spin $T$ as good quantum numbers. To do this, we first write our wave function in terms of eigenfunctions of $J^2$:

$$|\tilde{J}_1\tilde{J}_2JK\rangle = \sum_{\Omega_1\Omega_2} (\tilde{j}_1\tilde{j}_2| \Omega_1\Omega_2|JK) \chi_{\Omega_1} \chi_{\Omega_2},$$

where

$$|\tilde{j}_1\tilde{j}_2JK\rangle = \sum_{\Omega_1\Omega_2} (\tilde{j}_1\tilde{j}_2| \Omega_1\Omega_2|JK) \chi_{\Omega_1} \chi_{\Omega_2},$$

and $(\tilde{j}_1\tilde{j}_2|\Omega_1\Omega_2|JK)$ is the usual Clebsch-Gordan coefficient. Then we define two-particle wave functions with even or odd symmetry under particle exchange:

$$|j_1j_2JKT\rangle = \frac{1}{\sqrt{12}} [\tilde{|j_1j_2JK\rangle} + (-)^{J+T}|j_2j_1JK\rangle] \quad \text{for } j_1 \neq j_2,$$

$$|jjJKT\rangle = |jjJK\rangle \quad \text{for } J+T \text{ odd},$$

$$|jj\tilde{J}JKT\rangle = 0 \quad \text{for } J + T \text{ even}.$$ (25)

The quantum number $T$ assumes the value 1 for odd functions and 0 for even functions.

Substituting (25) into (23) gives

$$|\tilde{I}MK\tilde{n}_1\tilde{n}_2\tilde{p}_1\tilde{p}_2\rangle = \frac{2I+1}{16\pi^2} \sum_{\tilde{j}_1\tilde{j}_2} \sum_{\Omega_1\Omega_2} \sum_{\tilde{p}_1\tilde{p}_2} \langle \tilde{j}_1\tilde{j}_2| \Omega_1\Omega_2|JK \rangle \tilde{c}_{\Omega_1\Omega_2} \tilde{c}_{\tilde{j}_1\tilde{j}_2\tilde{p}_1\tilde{p}_2} (\tilde{j}_1\tilde{j}_2|\Omega_1\Omega_2|JK)$$

$$\times (1+\delta_{j_1j_2}) \frac{1}{2} \sum_{\tilde{M}\tilde{K}} \sum_{\tilde{n}_1\tilde{n}_2\tilde{p}_1\tilde{p}_2} \langle \tilde{j}_1\tilde{j}_2| \tilde{M}\tilde{K}\rangle + (-)^{I-J} \tilde{\xi}_{\tilde{M}\tilde{K}}\tilde{c}_{\tilde{j}_1\tilde{j}_2\tilde{p}_1\tilde{p}_2} |\tilde{j}_1\tilde{j}_2JKT\rangle.$$

(26)
where we have also used the identity
\[
(j_1 j_2 - \Omega_1 - \Omega_2 | J-K) = (-)^{j_1 + j_2 - J} (j_1 j_2 \Omega_1 \Omega_2 | J K).
\]

We now recall the normalization of the \( \Sigma \) functions,
\[
\langle \Omega^I_{\text{MK}} | \Omega^I_{\text{MK}} \rangle = \frac{8 \pi^2}{2I+1}
\]
and the symmetry with respect to the direction of the 3 axis,
\[
\langle K | v | K \rangle = \langle -K | v | -K \rangle.
\]

This leads to the desired result
\[
\langle \text{IMK}_\Omega \nu_1 \Omega_2 \nu_2 | v | \text{IMK}_\Omega' \nu_1' \Omega_2' \nu_2' \rangle
\]
\[
= \frac{1}{2} \sum_{j_1 j_2 j_1' j_2'} \sum_{J \frac{1}{2}} [(1+\delta_{j_1 j_2})(1+\delta_{j_1' j_2'})]
\]
\[
\times c_{j_1 \Omega_1} c_{j_2 \Omega_2} c_{j_1' \Omega_1'} c_{j_2' \Omega_2'} (j_1 \Omega_1 j_2 \Omega_2 | J K) (j_1' \Omega_1' j_2' \Omega_2' | J K)
\]
\[
\times \langle j_1 j_2 JT | v | j_1' j_2' JT \rangle [1 + (-)^{I-J} \delta_{K0}].
\]

**Static Electromagnetic Moments**

We now derive formulas for calculating the magnetic dipole and electric quadrupole moments from our wave functions. To avoid cumbersome notation, we write our basis functions \( | \text{IMK}_\Omega \nu \Omega_2 \nu_2 \rangle \) in the shorter form
\[
| \text{IMK}_\Omega \nu \Omega_2 \nu_2 \rangle = \left( \frac{2I+1}{16 \pi^2} \right)^{1/2} \left( \frac{I}{\text{MK} X_\beta} + (-)^{I+1} \frac{I}{\text{M-K} X_{-\beta}} \right),
\]
with
\[
X_\beta = X_{\nu_2} X_{\Omega_2} \quad \text{and} \quad X_{-\beta} = X_{-\Omega_2} X_{-\nu_2}.
\]
The index $\beta$ stands for the four indices $n, v, p, \nu$. The nuclear wave function is then

$$|\alpha IM> = \left\{ \frac{2I+1}{16\pi^2} \right\}^{1/2} \sum_{KB} c_{KB}^\alpha \left( I_{MK}^{I} x_{KB} \right) + (-)^{I+1} \left( I_{-MK}^{I} x_{-KB} \right), \quad (32)$$

where $\alpha$ is an index used to distinguish different wave functions with the same angular momentum $I$ and the $c_{KB}^\alpha$ are the coefficients obtained from the matrix diagonalization procedure.

The electromagnetic static moments and transition rates will be calculated in terms of irreducible spherical tensor operators $T_{\lambda\mu}$ defined in the laboratory (space-fixed) coordinate system. We will have to evaluate matrix elements between single-particle wave functions defined in the intrinsic (body-fixed) coordinates. This can be done by means of the transformation (Br 68)

$$T'_{\lambda\mu} = \sum_{\nu} \delta_{\nu}^\lambda (\theta_{I}) T_{\lambda\nu} \quad (33)$$

where $T'_{\lambda\mu}$ is a tensor in the space-fixed system, $T_{\lambda\nu}$ is the same tensor in the body-fixed system, and $\theta_{I}$ are the Euler angles relating the two coordinate systems. Note that the $\theta_{I}$ are also the arguments of the functions in the wave function (30). The matrix elements of a tensor operator are therefore given by

$$<\alpha'^{I^{'}} M^{'} | T'_{\lambda\mu} | \alpha IM> = \frac{(2I+1)(2I'+1)}{16\pi^2} \sum_{KB',K'} c_{KB}^\alpha c_{KB'}^{\alpha'} \times \sum_{\nu} \left( \delta_{M^{'} K'}^{I^{'}} | \delta_{\nu \mu}^\lambda | I_{MK}^{I} \right) (x_{B} | T_{\lambda\nu} | x_{B}) + (-)^{I+1} \left( \delta_{M^{'} K'}^{I^{'}} | \delta_{\nu \mu}^\lambda | I_{-MK}^{I} \right) (x_{B} | T_{\lambda\nu} | x_{-B})$$

$$+ (-)^{I'+1} \left( \delta_{M^{'} (-K')}^{I^{'}} | \delta_{\nu \mu}^\lambda | I_{-MK}^{I} \right) (x_{-B} | T_{\lambda\nu} | x_{B}) + (-)^{I'+1} \left( \delta_{M^{'} (-K')}^{I^{'}} | \delta_{\nu \mu}^\lambda | I_{-MK}^{I} \right) (x_{-B} | T_{\lambda\nu} | x_{-B}) \right]. \quad (34)$$
The integral over three functions is
\[ (E_{I'M'}^{I}, \beta_{\mu} \gamma_{\nu} E_{MK}^{I}) = (\Gamma_{\mu\nu}|I'M')(\Gamma_{K\nu}|I'K'), \] (35)

and the Clebsch-Gordan coefficients satisfy the identity
\[ (\Gamma_{K\nu}|I'K') = (-)^{I+I'-J'}(\Gamma_{K-\nu}|I'-K'). \] (36)

The matrix elements of a tensor operator of rank \( \lambda \) have the symmetry
\[ (x_{\beta}', |T_{\lambda\nu}|x_{\beta}) = (-)^{\lambda}(x_{-\beta}', |T_{\lambda\nu}|x_{-\beta}) \] (37)

Substituting equations (35), (36) and (37) into (34) gives us
\[ \langle a'I'M'|T_{\lambda\nu}|aIM\rangle = \sum_{\beta K} c_{KB} c_{KB} (\Gamma_{\mu\nu}|I'M')(\frac{2I+1}{2I+1}) \]
\[ \times [(\Gamma_{K\nu}|I'K')(x_{\beta}', |T_{\lambda\nu}|x_{\beta}) + (-)^{I+1}(\Gamma_{K-\nu}|I'K')(x_{-\beta}', |T_{\lambda\nu}|x_{-\beta})]. \] (38)

For convenience in computation, we use the Hermitian character of the electromagnetic operators, expressed by
\[ \langle a'I'M'|T_{\lambda\nu}|aIM\rangle = \sum_{\beta K} c_{KB} c_{KB} (\Gamma_{\mu\nu}|I'M')(\frac{2I+1}{2I+1}) \]
\[ \times [(\Gamma_{K\nu}|I'K')(x_{\beta}', |T_{\lambda\nu}|x_{\beta}) + (-)^{I+1}(\Gamma_{K-\nu}|I'K')(x_{-\beta}', |T_{\lambda\nu}|x_{-\beta})]. \] (39)

The prime on the summation indicates that the range of \( K' \) and \( \beta' \) are restricted so that \( K' > K \) and for \( K = K' \), \( \beta' > \beta \).

In calculating the single-particle matrix elements, \( \langle x_{\beta}', |T_{\lambda\nu}|x_{\beta} \rangle \), we use the same procedure for treating the core overlap factor and matrix elements between hole states as in calculating the matrix elements of the Hamiltonian.
We now apply these general results for spherical tensor operators to the static electromagnetic moment operators.

The magnetic moment \( \mu \) is defined as the expectation value in the magnetic substate \( M = I \) of the \( z \) component of the magnetic moment operator \( \hat{m} \), that is,

\[
\mu = \langle \alpha II | m_z | \alpha II \rangle \quad (41)
\]

The operator \( \hat{m} \) in the unified model is composed of components proportional to the angular momentum of the core and to the spin and orbital angular momenta of the odd neutron and proton:

\[
\hat{m} = \mu_o \left[ g_R^+ R + g_{sn}^+ \bar{J}_n + g_{sp}^+ \bar{J}_p + g_{lp}^+ \bar{J}_p \right]^*, \quad (42)
\]

where \( \mu_o \) is the nuclear magneton and the \( g \)'s are the gyromagnetic ratios corresponding to the various angular momenta. We eliminate \( \hat{R} \) by using the relation \( \hat{R} = \hat{J} + \hat{J}_n + \hat{J}_p \), thereby obtaining

\[
\hat{m} = g_R^+ \bar{J} + (g_{sn}^+ - g_R^+) \bar{J}_n + (g_{sp}^+ - g_R^+) \bar{J}_p + (g_{lp}^+ - g_R^+) \bar{J}_p
\]

\[
\equiv g_R^+ \bar{J} + \bar{G}_n + \bar{G}_p \quad (43)
\]

The quantities \( \bar{G}_n \) and \( \bar{G}_p \) are the components of \( \hat{m} \) depending on the neutron and proton coordinates, respectively.

The contributions of the term \( g_R^+ \bar{J} \) can be calculated immediately:

\[
\langle \alpha II | g_R^+ J_z | \alpha II \rangle = g_R^+ \bar{J} \quad (44)
\]

The contributions of \( \bar{G}_n \) and \( \bar{G}_p \) are more complicated. Since these two terms are similar in their effect, we calculate only one of them explicitly. For convenience, we drop the subscripts \( n \) and \( p \), writing

\[
\bar{G} = (g_{sn}^+ - g_R^+) \bar{J}_n + (g_{sp}^+ - g_R^+) \bar{J}_p \quad (45)
\]
The spherical tensor components $\mathcal{J}_\mu$ of a Cartesian vector $\mathcal{G}$ are given by

$$\mathcal{J}_o = G_z \quad \mathcal{J}_{\pm 1} = \mp \frac{1}{\sqrt{2}} (G_x \pm iG_y).$$

Hence the contribution to the magnetic moment of the odd neutron or proton is

$$\langle \alpha II | \hat{\mathcal{J}}_o' | \alpha II \rangle = \sum_{\beta \kappa \delta} c_{\kappa \beta}^\alpha c_{\delta \kappa}^\alpha \langle \Pi \Pi | \mathcal{J}_o \rangle (\mathcal{J}_o \mathcal{J}_o - 1)(\mathcal{J}_o \mathcal{J}_o + 1)
\times \left\{ (\mathcal{J}_o \mathcal{J}_o \mathcal{J}_o \mathcal{J}_o) (\mathcal{J}_o \mathcal{J}_o \mathcal{J}_o \mathcal{J}_o) (\mathcal{J}_o \mathcal{J}_o \mathcal{J}_o \mathcal{J}_o) \right\}.$$

The matrix elements of the spherical components $\mathcal{J}_0$ and $\mathcal{J}_{\pm 1}$ of an angular momentum $J$ can be easily evaluated in a representation $|jm\rangle$ for which $J^2$ and $J_z$ are diagonal. The non-zero matrix elements are simply

$$\langle jm | \mathcal{J}_0 | jm \rangle = m, \quad \langle jm | \mathcal{J}_{\pm 1} | jm \rangle = \pm \frac{1}{\sqrt{2}} [(j \pm 1)(j \mp 1)]^{1/2}.$$

Hence we can calculate the components of $\mathbf{j}_n$ and $\mathbf{j}_p$ in the Nilsson representation used previously,

$$\chi^\nu_\Omega = \sum_j c_{j\Omega}^\nu \chi^\nu_{j\Omega}.$$

But to calculate the components of $\mathbf{\hat{s}}_n$ and $\mathbf{\hat{s}}_p$, it is more convenient to use an $m_L, m_S$ representation,

$$\chi^\nu_\Omega = \sum_{\pm \Lambda \Sigma} a^\nu_{\pm \Lambda \Sigma} |\pm \frac{1}{2} \Lambda \Sigma\rangle, \quad \Lambda + \Sigma = \Omega,$$

where $\Lambda$ and $\Sigma$ are the projections of the orbital angular momentum $\mathbf{j}$ and the spin angular momentum $\mathbf{s} = \frac{1}{2}$. The $a^\nu_{\pm \Lambda \Sigma}$ are related to the $c_{j\Omega}^\nu$ by

$$a^\nu_{\pm \Lambda \Sigma} = \sum_j (\pm \frac{1}{2} \Lambda \Sigma | j\Omega) c_{j\Omega}^\nu.$$
Using these relations, the single particle matrix elements of $\mathcal{H}$ are found to be

$$
\langle \chi^\nu_\Omega | j^i_\omega | \chi^\nu_\Omega \rangle = \delta^\nu_\nu, (g^i_\Lambda - g^i_R)\Omega + \frac{1}{2} (g^i_S - g^i_\Xi) \sum_{\Lambda L} (a^i_{\Lambda L 1/2} a^i_{\Lambda L 1/2} - a^i_{\Lambda L -1/2} a^i_{\Lambda L -1/2})
$$

$$
\langle \chi^\nu_{\Omega+1} | j^i_1 | \chi^\nu_\Omega \rangle = -\frac{1}{\sqrt{2}} (g^i_S - g^i_\Xi) \sum_j c^{i\nu}_j c^{i\nu}_j \bar{\jmath}(j-\Omega)(j+\Omega+1)^{1/2}
$$

$$
+ (g^i_S - g^i_\Xi) \sum_{\Lambda L} a^i_{\Lambda L 1/2} a^i_{\Lambda L -1/2}.
$$

We can now write an expression for the magnetic moment in terms of these quantities:

$$
\mu = g^R I + \sum_{K^\beta K'\beta'} c^\alpha_{K^\beta} c^{\alpha'}_{K'} (I1D | II) \sum_{\nu} \langle \chi^\nu_n | \chi^\nu_n \rangle
$$

$$
\times (\chi^\nu_p | j^i_p | \chi^\nu_p) + (\chi^\nu_p | j^i_p | \chi^\nu_n) + (-)^{I+1} (I1-K\nu | I\nu')
$$

$$
\times (\chi^\nu_n | x^-_\Omega) (\chi^\nu_n | j^i_n | x^-_\Omega) + (\chi^\nu_p | x^-_\Omega) (\chi^\nu_n | j^i_n | x^-_\Omega) \right). (53)
$$

The electric quadrupole moment $Q$ is defined, analogously to $\mu$, as the expectation value of the $z$ projection of the quadrupole moment spherical tensor operator $Q_{20}$ in the state $M = I$:

$$
Q = \langle aII | Q_{20} | aII \rangle (54)
$$

The operator $Q_{2\mu}$ in our model is taken as the sum of a collective part representing the contribution of the deformed core plus a single-particle part for the odd proton. Effective charges are not used in this model, so there is no contribution from the odd neutron, aside from a negligibly small recoil effect, which we ignore here. Hence our expression for $Q_{2\mu}$ is

$$
Q_{2\mu} = Q_{\mu} + \left[ \frac{16\pi}{5} \right] e_r^2 \frac{2\lambda \mu_0}{\lambda^2 (\theta, \phi)}.
$$

(55)
The collective contribution $Q_{\mu}$ is calculated from the classical expression for the charge quadrupole moment of a uniformly charged ellipsoid with deformation $\beta$, volume $\frac{4}{3} \pi R_0^3$, and total charge $Z$:

$$Q_{\mu} = \frac{3(Z-1)e}{4\pi R_0^3} \left[ \frac{16\pi}{5} \right]^{1/2} \int r^3 Y_{2\mu}(\theta, \phi) d^3r. \quad (56)$$

There are no off-diagonal matrix elements of $Q_{\mu}$ in our formalism, so we need to evaluate this integral only for $Q_0$. The result to second order in $\beta$ is (57)

$$Q_0 = \frac{3(Z-1)e}{\sqrt{5\pi}} R_0^2 \beta(1 + 0.168). \quad (57)$$

The matrix elements of the single-particle operator can be calculated by performing the necessary integrals over the proton coordinates, using the $m_{L,S}$ representation for the Nilsson wave functions.

The resulting matrix elements for the collective and single-particle operators in the body-fixed coordinate system are

$$\langle x_{\beta} | Q_{2\nu} | x_{\beta} \rangle = \left( \frac{5}{16\pi} \right)^{1/2} \left\{ Q_0 \delta_{\nu0} \delta_{\beta\beta'} + \frac{2eh}{\mu_0} \langle \Omega_n^v | x_{\Omega_n}^v \rangle \right\} \times \sum_{\ell \Lambda \Sigma} \left[ a_{\ell+2, \Lambda+\Sigma}^v a_{\ell, \Lambda+\Sigma}^p \right] \left( N + \frac{3}{2} \right) \langle \ell 2\Lambda\nu | \ell, \Lambda+\nu \rangle \langle \ell 200 | \ell 0 \rangle$$

$$-a_{\ell+2, \Lambda+\Sigma}^v a_{\ell, \Lambda+\Sigma}^p \left[ \frac{(N-\ell)(N+\ell+3)(2\ell+1)}{2\ell+5} \right]^{1/2} \langle \ell 2\Lambda\nu | \ell+2, \Lambda+\nu \rangle \langle \ell 200 | \ell+2, 0 \rangle$$

$$-a_{\ell-2, \Lambda+\Sigma}^v a_{\ell, \Lambda+\Sigma}^p \left[ \frac{(N-\ell-2)(N+\ell+1)(2\ell+1)}{2\ell+3} \right]^{1/2} \langle \ell 2\Lambda\nu | \ell-2, \Lambda+\nu \rangle \langle \ell 200 | \ell-2, 0 \rangle.$$  \quad (58)

This expression can be substituted into Equation (40) to obtain the quadrupole moment.
Electromagnetic Transition Rates

The electromagnetic transition rates are calculated in terms of the off diagonal matrix elements of the operators used for the static moments. Hence the formalism developed above can be applied with only minor changes.

The probability per unit time of a nucleus in an excited state emitting a photon of energy $\hbar \omega$, angular momentum $\lambda$, and of electric or magnetic type is (Pr 62), in the usual long-wavelength approximation:

$$T(\sigma \lambda) = \frac{8\pi (\lambda+1)}{\lambda (2\lambda+1)!} \frac{1}{\hbar} \left(\frac{\omega}{c}\right)^{2\lambda+1} B(\sigma, \lambda, I \rightarrow I'),$$

where the reduced matrix element $B(\sigma, \lambda, I \rightarrow I')$ depends only on the initial and final nuclear states involved in the transition and not on the energy of the photon. The reduced matrix element can be calculated from the formula (Br 68)

$$B(\sigma, \lambda, I \rightarrow I') = \sum_{\mu, \mu'} |<\alpha' I'M' | T_{\lambda \mu}^\sigma | \alpha IM>|^2,$$

where the spherical tensor $T_{\lambda \mu}$ is the electromagnetic multipole operator of type $\sigma$ (electric or magnetic), multipolarity $\lambda$, and $z$-projection $\mu$.

Using Equation (38), we obtain

$$B(\sigma, \lambda, I \rightarrow I') = \frac{2I+1}{2I'+1} \sum_{M, \mu'} (I\lambda M | I'M')^2$$

$$\times \sum_{\beta' K'K} (I\lambda K | I'K') (x_\beta' | T_{\lambda \mu}^\sigma | x_\beta) + (-)^{I+1} (I\lambda - K | I' K') (x_\beta' | T_{\lambda \mu}^\sigma | x_\beta)^2.$$

The sum over $M$ and $\mu$ is given immediately from the orthogonality condition on the Clebsch-Gordans,

$$\sum_{M, \mu} (I\lambda M | I'M')^2 = \frac{2I'+1}{2I+1}.$$
We therefore obtain
\[
B(\sigma, I \rightarrow I') = \sum_{\beta K' K''} (i \lambda K' \sigma | I' K') (x_{\beta} | T_{\lambda \nu}^{\sigma} | x_{\beta}) 
+ (-)^{I+1} (i \lambda - K | I' K') (x_{\beta} | T_{\lambda \nu}^{\sigma} | x_{-\beta}) \right) |^{2}.
\]

(62)

We will be concerned only with M1 and E2 transition rates, since transition rates for higher multipoles have generally not been measured for the nuclei we will discuss. The M1 transition operator is given in terms of the magnetic moment operator by
\[
M_{10}^{M} = \left( \frac{3}{4\pi} \right)^{1/2} \mu_{z}, \quad T_{1, \pm 1}^{M} = \mp \left( \frac{3}{8\pi} \right)^{1/2} (\nu_{x} \pm i \nu_{y});
\]

(63)

that is, we simply take the spherical tensor components of \( \hat{\mu} \) times the factor \((3/4\pi)^{1/2}\). Similarly, the effective E2 transition operator for transitions between eigenstates of the Hamiltonian is proportional to the quadrupole moment operator,
\[
T_{2 \mu}^{E} = \left( \frac{5}{16\pi} \right)^{1/2} Q_{\mu}.
\]

(64)

The matrix elements of these operators in the intrinsic coordinate system have already been given in Equation (52) and Equation (57). It should be noted that there is no contribution to the M1 transition rate due to the component of \( \hat{\mu} \) given by \( g_{R} \hat{I}_{z} \), since the wave functions are diagonal in the total angular momentum.

Another quantity of physical interest besides the absolute transition rates is the M1-E2 mixing ratio \( \delta \), where \( \delta^{2} \) is the ratio of the E2 transition to the M1 transition rate. Although transition rates for different multipolearities add incoherently, the angular distribution of the radiation does depend on interference terms between different multipolarities, so the sign of \( \delta \) can sometimes be determined experimentally as well as its absolute value. Using our notation and the phase convention of Rose and Brink (Ro 67)
we can express the mixing ratio as

$$\delta(E2/M1) = - \frac{1}{5} \left( \frac{3}{4\pi} \right) \frac{1/2}{\hbar c} \frac{E}{\langle a' I' | T^E_2 | aI \rangle} \frac{\langle a' I' | T^M_1 | aI \rangle}{\langle a' I' | T^M_1 | aI \rangle},$$

(65)

where the reduced matrix element $\langle a' I' | T^\lambda_\lambda | aI \rangle$ is given by

$$\langle a' I' | T^\lambda_\lambda | aI \rangle = \sum_{\mu M'} \langle a' I' | T^\mu_\lambda | aIM \rangle,$$

(66)

$E$ is the energy of the gamma ray, and the transition goes from the state $I$ to the state $I'$. 
In order to use the formalism of the Coriolis model, we must assign numerical values to all the parameters of the model. In this chapter we discuss the methods used to determine these parameters and give the actual numerical values employed in the calculations.

Theoretical parameters can be evaluated by choosing the values that give the best fit to the experimental data that the theory is supposed to explain, and we will use this method to determine the deformation parameter $\beta$ for each nucleus from the experimental energy spectrum. But in many cases there is information available from experiment or from other calculations that suggests an appropriate value for a parameter over a range of nuclei. We will use such information whenever possible to fix our model parameters a priori. This method provides maximum consistency between our calculations and other properties of nuclei besides the ones that we wish to calculate.

**Collective Parameters**

The collective parameters that enter into our calculations are the nuclear deformation $\beta$ and the moment of inertia $I$. The deformation is expected to be different for each nucleus, and so we determine $\beta$ by fitting the experimental energy spectrum for each nucleus. The resultant values are generally found to be consistent with the deformations used in Coriolis model calculations for neighboring odd-even nuclei (Ma 65, Hi 69).

Several theoretical approaches have been used to evaluate the collective moment of inertia. The simple model of rigid rotation implies
The hydrodynamic model of irrotational flow implies \( \mathcal{J}_{\text{irrot}} = \frac{9}{8 \pi} MR_0^2 \beta^2 \). In actual heavy deformed nuclei, typical values of \( \beta \) are between 0.2 and 0.53, which is much larger than the irrotational value. As mentioned earlier, experimental moments of inertia for these nuclei are well reproduced by microscopic models such as the cranking model including a pairing interaction. Several different microscopic models have been used to calculate moments of inertia in s-d shell nuclei (Ke 67), and satisfactory agreement has been obtained for rotational bands in even-even nuclei. However, we find that the Coriolis model energy spectra for odd-odd nuclei in the s-d and f-p shells are quite insensitive to the value of the moment of inertia, so that much the same spectrum is produced for a wide range of this parameter. (This is illustrated for the case of Na\(^{22}\) in Fig. 5.) This implies that our calculations do not provide a test for the validity of microscopic moment of inertia calculations for light odd-odd nuclei, and they do not give any indication of whether the rotations of these nuclei are better represented by rigid rotation or irrotational fluid flow. In view of the insensitivity of the results to the value of \( \beta \), we have somewhat arbitrarily taken \( \hbar^2/2\mathcal{J} = 0.3 \text{ MeV} \) for all s-d nuclei. In the f-p shell, we have used the values suggested by Malik and Scholz (Ma 66) in their calculations for neighboring odd-even nuclei.

**Single-Particle Parameters**

The parameters needed to specify the Nilsson Hamiltonian are the deformation \( \beta \), the oscillator quantum \( \hbar \omega_0 \), the spin-orbit force strength \( g_{s.o.} \), and the angular momentum \( \hbar L \).
parameter $C$, and the $\mathbf{1} \cdot \mathbf{1}$ strength parameter $D$. We follow Nilsson (Ni 55) in determining $h_{\omega_0}$ by requiring that the matter distribution calculated from the single-particle wave functions reproduce the approximate nuclear radius, taken as $R = 1.2A^{1/3}$. This implies $h_{\omega_0} = 41A^{-1/3}$ MeV.

Information on the strength of the spin-orbit force in the s-d shell can be derived from the positions of single-particle states in $^{17}O$ and $^{17}F$ and single-hole states in $^{39}Ca$ and $^{39}K$. We will assume that to a good approximation these states can be regarded as single particles or single holes in the potential generated by the spherical $^{16}O$ or $^{40}Ca$ core. The displacement between the $d_{5/2}$ and $d_{3/2}$ states then gives an indication of the spin-orbit strength at either end of the s-d shell. In $^{17}O$ and $^{17}F$, this splitting is about 5 MeV. In $^{39}Ca$ and $^{40}K$, the $d_{3/2}$ strength measured from neutron and proton pick-up reactions with a $^{40}Ca$ target is spread over several states, but the centroid occurs at about 5.8 MeV (Hi 66). The similarity of these results suggests that the spin-orbit force may be fairly constant in the s-d shell, so we assume an average splitting of 5.5 MeV for the whole s-d shell and require that the Nilsson energies at zero deformation reproduce this splitting. This implies a value of $C = -4.4$ MeV.

The same procedure could be used to evaluate the $\mathbf{1} \cdot \mathbf{1}$ strength parameter $D$, if the position of the s state with respect to the centroid of the d states were also constant throughout the shell. However, this does not seem to be the case. To reproduce the positions of these states in $^{17}O$ and $^{17}F$, we would take $D = 0.4$ MeV, whereas for $^{39}Ca$ and $^{39}K$, we would require $D = -0.33$ MeV (Hi 66). We have performed Coriolis model calculations
with values of $D$ in this range, and for $^{18}$F, $^{22}$Na, and $^{26}$Al we find that the choice of $D$ makes very little difference in the final spectrum obtained, although the value of $\beta$ that gives the best spectrum is sometimes shifted slightly for different values of $D$. We have therefore taken $D = 0$ in our final calculations for these nuclei, as originally suggested by Nilsson (Ni 55). For $^{30}$P the experimental spectrum cannot be well reproduced by the Coriolis model without a large negative value of $D$; we have therefore taken $D = -0.05 \hbar \omega = -0.66$ MeV for $^{30}$P.

In the $f$-$p$ shell, we have adopted the values for $C$ and $D$ suggested by Malik and Scholz in their study of the odd-even nuclei in this region (Ma 66), namely $C = -0.26 \hbar \omega$ and $D = -0.035 \hbar \omega$. This value of $C$ reproduces the observed spin-orbit splitting in $^{41}$Ca, and this value of $D$ is the smallest in absolute value that preserves the shell model level ordering at zero deformation.

For the residual interaction between the neutron and the proton in the $s$-$d$ shell, we use the effective matrix elements calculated by Kuo (Ku 67) from the hard-core Hamada-Johnston potential (Ha 62), which is derived from two-nucleon scattering data. These matrix elements are calculated by reaction-matrix techniques, using a formalism designed to produce appropriate matrix elements for shell-model calculations that use a basis space consisting of $s$-$d$ shell particles outside a closed $^{16}$O core. It should be noted that these matrix elements are intended for use in spherical shell model calculations that treat all $s$-$d$ nucleons as active particles, and there is no a priori justification for using them in a deformed model that includes the interaction only between the last two particles. However, we do in fact find that they provide good agreement with experiment in our
model, at least towards the beginning of the s-d shell. The fact that the same two-body interaction can provide agreement with experiment in both the shell model and the deformed model is one of the interesting results of our calculations.

No corresponding set of matrix elements is available for the f-p shell, so we use a simple phenomenological potential for the two-body interaction that is frequently employed in shell model and restricted basis Hartree-Fock calculations in the s-d shell (El 55, Ke 64), namely a central Yukawa potential with a Rosenfeld exchange mixture (Ro 48). That is

\[ v(n,p) = V_0 \frac{r_1 \cdot r_2}{3} (0.3 + 0.7 \sigma_1 \cdot \sigma_2) \frac{e^{-r_{12}/a}}{r_{12}/a}. \]  

We have taken an interaction strength of \( V_0 = 50 \) MeV and a range of \( a = 1.38f \). Values in this range have generally given good results in shell model and Hartree-Fock calculations. Coriolis model calculations with this potential have been performed in the s-d shell, and the results are found to be similar to results obtained with the Kuo interaction. Matrix elements in j-j coupling have been calculated from the potential (1) with a computer code written by J. Vary (Va 69).

The physical significance of the core overlap or quenching parameter Q has been discussed in Chapter III. The assumption made in these calculations that Q does not depend on the single-particle states involved in the matrix element must be considered only a rough approximation of the true situation, so it has not seemed worthwhile to vary Q in order to obtain the best possible spectrum for each nucleus. Trial calculations have shown that Q must be taken substantially less than one in the s-d shell in order to obtain agreement with experiment, indicating that the wave function
of the nuclear core is strongly dependent on the orbital occupied by the last particles. We have therefore taken $Q = 0.75$ for all calculations in the s-d shell. Hartree-Fock calculations that take into account interactions only between particles in the s-d shell would suggest a value of $Q$ nearly equal to one (Ri 68). However, Coriolis model calculations on odd-even nuclei in the mass 75 region have been performed that incorporate a residual interaction of the pairing type between the core nucleons (Ma 68), and these calculations yield core overlaps in the order of .5 to .75, which suggests that our choice of $Q = .75$ may be justified in terms of a more fundamental theory incorporating residual interactions between the core nucleons. It is also possible that our use of an incomplete core overlap is actually simulating some other nuclear structure effect which our model does not properly take into account, but determining whether this is so is beyond the scope of our model.

In the f-p shell, we have again taken over the values suggested by Malik and Scholz (Ma 66), namely $Q = 1.0$ for the scandium isotopes and $Q = 0.75$ for the manganese isotopes. These values must be considered even more uncertain than the one used in the s-d shell, because very little experimental information is available on the positions of the high-spin states that are strongly influenced by the choice of $Q$.

**Electromagnetic Parameters**

In order to calculate transition rates and static moments, we must assign values to the collective and single-particle gyromagnetic ratios, the intrinsic quadrupole moment $Q_0$, and the effective charges for the neutron and proton.
The collective gyromagnetic ratio is taken as \( g_R = z/A \) (Bo 53), based on the assumption that the spins of the core particles cancel one another so that the core angular momentum is simply the orbital angular momentum of the neutrons and protons. In heavy nuclei, a somewhat smaller value of \( g_R = 0.75 \frac{z}{A} \) gives better agreement with experiment (Pr 62), but for light nuclei, there are so many uncertainties in the wave functions that it does not seem possible to determine whether there is any deviation from this simple theoretical estimate. The orbital angular momentum gyromagnetic ratios for the neutron and proton are \( g_\lambda = 0 \) for the neutron and \( g_\lambda = 1 \) for the proton. The neutron and proton spin gyromagnetic ratios are also given their free-space values, \( g_s = -1.913 \) for the neutron and \( g_s = 2.793 \) for the proton. The spin g-factors must be considered rather uncertain, since the magnetic moment of a nucleon inside a nucleus may not be the same as in empty space. However, in the absence of a detailed theory for this effect, the use of the free-space values seems as good as any alternative.

In the shell model it is generally necessary to assign effective charges to the neutron and proton in order to account for core participation in E2 transitions. The unified model explicitly includes core excitations in the wave function, and so the use of effective neutron and proton charges should not be necessary in this model. Accordingly, we simply take the effective charges equal to the actual charges.

The collective quadrupole moment given by equation (III-56) depends on the deformation parameter \( \beta \) and the nuclear radius \( R_0 \). It should be noted that the value of \( \beta \) obtained by fitting the experimental energy
spectrum will be affected by the values of other parameters, which have been chosen rather arbitrarily in some cases. The nuclear radius, taken as \( R_0 = 1.2A^{1/3} \), must also be considered somewhat uncertain. Accordingly, the absolute collective E2 transition rates predicted by our model may be expected to give only the correct order of magnitude, and relative transition rates within a nucleus will provide a more meaningful test of our model wave functions than will the absolute rates.
Chapter V Applications in the s-d Shell

In this chapter we discuss the application of the Coriolis-coupling model in the s-d shell and present the results of calculations with this model on some self-conjugate odd-odd nuclei in the lower half of the s-d shell.

The first application of the unified rotational model in the s-d shell was motivated by the observation of strongly inhibited M1 transitions in Mg\textsuperscript{25}, which could be naturally explained on the basis of K-selection rules (Li 56). Since then further experimental studies of nuclei in the vicinity of Ne\textsuperscript{20} and Mg\textsuperscript{24} have revealed further evidence for collective behavior, and considerable progress has been made in understanding these nuclei in terms of the same rotational model that has been applied to heavy deformed nuclei.

In heavy deformed nuclei, the single particle orbitals are spread over such a large number of shell model states by the deformation that a shell model or other microscopic calculation is not feasible, and so the unified model provides the only practical theoretical approach to these nuclei. In the s-d shell, on the other hand, it seems to be a good approximation to confine the active nucleons to spherical s-d orbitals, and the number of these is small enough so that various microscopic calculations can be performed. These light nuclei therefore provide a good opportunity to compare the predictions of a unified model with those of other models. Accordingly, it seems appropriate to precede the discussion of the unified model results with a description of other calculations that have been applied to the same nuclei. The physical principles underlying these
other models have been discussed in Chapter II, so we give only a few details pertinent to applications in the s-d shell here.

**Shell Model**

The first shell model calculations with configuration mixing in the s-d shell were performed by Elliott and Flowers (El 55). They allowed the active nucleons outside an $^0_{16}$ core to occupy all possible configurations based on s-d shell orbitals and to interact by means of a phenomenological potential. Because of the rapidly increasing dimension of the basis space as more particles are added, this type of calculation can be performed only for nuclei with a few particles outside the $^0_{16}$ core (or with a few holes in the Ca$^{40}$ core, at the other end of the shell). Calculations on other nuclei have been performed by restricting the active nucleons to a subshell of the s-d shell. In a severely truncated basis set of this type, the two-body force in the shell model Hamiltonian cannot be directly related to a realistic two-nucleon interaction, so it must be determined phenomenologically from the experimental data. This can be done by parameterizing the residual interaction and determining the parameters by a least squares fit to the experimental levels. If the basis space is sufficiently restricted, there are only a few independent two-body matrix elements, and these can be used as parameters. Otherwise, the matrix elements can be calculated from a two-body potential containing adjustable parameters. Calculations of this type have been carried out by several authors (Gl 64, Ar 68, Ha 68a).

The wave functions obtained in these calculations cannot be expected to accurately represent the true nuclear wave functions because of
truncated basis set. Accordingly, it may be necessary to calculate observables using "effective" operators appropriate for the truncated basis. It is generally found that E2 transition rates in light nuclei can be satisfactorily accounted for by the shell model when an effective charge of 3/2 e is used for the proton and 1/2 e for the neutron (Wi 67, Ha 68). In some calculations, effective gyromagnetic ratios are also used in calculating M1 transition rates (Ba 63).

Another type of shell model calculation has recently been performed in which a more fundamental approach is taken (Ha 68a). In these calculations, the whole s-d shell is used as a basis, and the two-body matrix elements are calculated from a realistic two-nucleon potential, using a formalism appropriate for this truncation (Ku 66, Ku 67). These calculations involve in principle no free parameters, and their success in predicting experimental results must be considered a great advance in nuclear structure theory. However, this approach encounters considerable computational difficulties because of the size of the matrices that must be diagonalized as one treats more complicated nuclei, and Na$^{22}$ with six particles in s-d shell is the most complicated case that has been treated in this manner to date. Moreover, effective charges must still be used in calculating E2 transitions. Hence these sophisticated calculations do not eliminate the need for the more phenomenological shell model and unified model as an aid to understanding the properties of s-d shell nuclei. In fact, one of the most interesting features of the shell model with realistic interaction is that the specifically rotational properties of some s-d nuclei, such as rotational band spectra with enhanced interband E2 transitions, are particularly well reproduced.
SU\(_3\) Model

The SU\(_3\) model, discussed earlier in this thesis, has been applied extensively in the s-d shell (Ha 68). In its simplest form, this model applies only to even-even nuclei which have spin angular momentum \(S = 0\). However, for a moderate strength of the spin-orbit force, the model can be applied to odd nuclei, and rotational spectra are produced (El 68). Good qualitative agreement with experiment is obtained in the lower half of the s-d shell, but agreement is poor in the upper half, where the spin-orbit force is apparently more important. The lack of quantitative agreement with experiment suggests that strong representation mixing may be necessary to account for the terms in the Hamiltonian that are not diagonal in the SU\(_3\) representation. Under these circumstances, the SU\(_3\) model loses much of its appeal, since construction of SU\(_3\) basis states is generally not a simple procedure.

No theoretical spectra of odd-odd s-d shell nuclei in terms of the SU\(_3\) model have been reported in the literature. However, Wilsdon has suggested classifications in the SU\(_3\) scheme for some low-lying levels of odd-odd s-d nuclei (Ha 68). The resulting wave function for the ground state of Na\(^{22}\) gives the experimentally observed magnetic moment.

Another model that makes use of the SU\(_3\) classification is the intermediate coupling model of Bouton, Elliott, and Pullen (Bo 67). These authors write the potential energy for an s-d shell nucleus as \(V = \alpha V_0 + x V_1\) with \(V_0\) a two body interaction of the Rosenfeld exchange type with a Yukawa radial dependence plus an \(1^2\) term and \(V_1\) a one-body spin-orbit force. The relative strength of the spin-orbit force with respect to the residual interaction is then given by \(x\). For small \(x\), the wave function is taken as
an L-S coupled state given by the SU^3 model and for large \( x \) it is taken as a j-j coupled state, which diagonalizes the spin-orbit term. The energy in either case is taken as the expectation value of the Hamiltonian for the L-S or j-j coupled wave function. The derivative of the energy with respect to \( x \) is calculated by first-order perturbation theory, and a simple algebraic expression is used to interpolate between the two extremes. The parameters \( x \) and \( \alpha \) are then determined by a least squares fit to the experimental spectrum for each s-d nucleus treated. This technique avoids the difficult problem of diagonalizing the large matrices that would arise in an exact calculation using the same Hamiltonian. However, it has the disadvantage that it provides only the nuclear energy levels and not the wave functions. Agreement with experimental spectra is generally good. The parameter \( \alpha \), which gives the strength of the residual interaction, lies within the range of values expected from the shell-model calculations of Elliott and Flowers, but the values of \( x \) obtained for different nuclei vary quite erratically. The intermediate coupling results will be compared with our unified model calculations below for the cases in which the two overlap.

Hartree-Fock Model

Hartree-Fock calculations have also been applied extensively to the s-d shell nuclei (Ri 68). The usual procedure is to calculate self-consistent Hartree-Fock orbitals using the spherical harmonic oscillator wave functions of the s-d shell as a basis and employing a simple phenomenological two-body force plus the experimental single-particle energies as the Hamiltonian. The resulting single particle energies and wave functions are generally very similar to the Nilsson wave functions. There are,
however, two important differences. The first is that the Hartree-Fock single-particle energies and orbitals depend on the nuclear configuration involved; that is, on which orbitals are occupied and which are empty. This feature is simulated to some extent in the Nilsson model by choosing a different deformation for different nuclei. The second difference is the appearance of a characteristic gap between the occupied and unoccupied orbitals in the Hartree-Fock calculation. This gap cannot of course be predicted by the one-body Nilsson potential. However, the gap between the orbital occupied by the last uncoupled particle in an odd nucleus and the unoccupied orbitals is not very large. Accordingly, there is not much difference between the Hartree-Fock results and the Nilsson results for an appropriate deformation for odd and odd-odd nuclei.

In calculating nuclear wave functions from the Hartree-Fock intrinsic state, two procedures can be employed (Ke 64). The first is to follow the strong-coupling approximation of the unified model, using Hartree-Fock single-particle wave functions instead of Nilsson wave functions. The second is to project a wave function with good angular momentum out of the Hartree-Fock intrinsic wave function. This second procedure completely avoids the use of collective coordinates, and collective parameters such as the moment of inertia. The projected states are found to follow an $I(I + 1)$ rule, at least for low angular momentum, so angular momentum projection does reproduce the rotational character of the spectrum. It can be shown that the unified model wave function for a given intrinsic state may be a good approximation to the projected wave function when the expectation value of the intrinsic angular momentum is large. This condi-
tion is probably satisfied for heavy deformed nuclei, but it may not be adequately fulfilled in the s-d shell (Ri 68). Thus the Hartree-Fock model with angular momentum projection can provide some insight into the microscopic origins of the unified model.

No detailed Hartree-Fock calculations of energy spectra for odd-odd s-d shell nuclei have been reported. However, available Hartree-Fock results do give information about the trend of deformation in the s-d shell, and we will use some of these results in discussing our calculations of specific nuclei.

Other Unified Model Calculations for Odd-Odd Nuclei

The rotational character of the nuclei in the lower half of the s-d shell is well established, and several attempts have been made to understand the properties of odd-odd nuclei in this region in terms of rotational models. But until the recent work of Ascuitto, Bell, and Davidson (As 68), no calculation involving band mixing had been reported. The necessity of including a residual interaction in the unified model Hamiltonian for self-conjugate odd-odd nuclei was demonstrated by Kelson (Ke 64a) who discussed the lowest two states of $^{18}$F, $^{22}$Na, and $^{26}$Al. In the simple rotational model for positive deformation, these nuclei have an odd proton and an odd neutron in the Nilsson orbitals based on $d_{5/2}$ with $\Omega = \frac{1}{2}$, $\Omega = \frac{3}{2}$, and $\Omega = \frac{5}{2}$, respectively. Hence we expect low-lying bands with $K = 0$, when the neutron and proton have opposite projections, and with $K = 1, 3, \text{and } 5$, when they have parallel projections. The rotational model with no residual interaction predicts that the $K = 0$ state will be lower than the $K = 2\Omega$ state by an amount $\Omega h^{2}/\Omega$, as can be shown by evaluating the diagonal matrix elements of the Hamiltonian for these states.
But experimentally the ground states of $^{18}_F$, $^{22}_Na$, and $^{26}_Al$ have spin 1, 3, and 5 respectively. Kelson showed that the experimental energy differences between these two levels for each of these three nuclei can be predicted by including a short range residual interaction between the odd proton and neutron. This calculation was extended to other light odd-odd nuclei by Picard and de Pinho (Pi 66). Their calculation also includes only the lowest two bands and no band mixing. Good agreement was generally obtained for the lowest few states.

These calculations illustrate that the rotational model with a residual interaction can explain some of the features of odd-odd s-d nuclei, but a more complete description of these nuclei requires the inclusion of band mixing due to the Coriolis force. Such a calculation was published by Asciutto, Bell and Davidson (As 68) just as the present work was being completed. These authors treat all odd-odd nuclei above mass 28 in the s-d shell for which there is experimental information available. Since the present work is confined to the self-conjugate odd-odd nuclei in the lower half of the shell, the only common nucleus is $^{30}_F$. We have thought it worthwhile to give our results for $^{30}_F$ since our calculation differs from that of Asciutto et al. in several respects, which we now discuss. The most important differences are that these authors do not include hole excited states in their basis set and do not take the core overlap integral different from unity. Also, their approach to the parameters of the problem is somewhat different from ours. They chose the strength of the residual interaction, the moment of inertia, and the oscillator quantum $\hbar\omega_o$ to produce the best fit for each nucleus, whereas
And finally, their treatment of the Pauli principle is somewhat different. They diagonalize the entire Hamiltonian, including the single-particle Nilsson terms, on a basis of spherical harmonic oscillator functions multiplied by $\delta$ functions. They eliminate from the basis the spherical states which would be occupied by the core particles at the deformation actually used in the calculation. Their model thus contains no single-particle strength in $^{30}\text{P}$ and heavier nuclei, since the $d_{5/2}$ shell would be filled in the Nilsson model at zero deformation for this nucleus. It is difficult to assess the effect of these differences, aside from the obvious fact that our model will produce more states due to the presence of hole-excited configurations. But we feel that these differences make it worthwhile to calculate at least one nucleus using both methods, and so we will present our results for $^{30}\text{P}$ in comparison with those of Asciutto et al.

**Unified Model Results in s-d Shell**

We now discuss the results obtained in the present unified calculations for $^{18}\text{F}$, $^{22}\text{Na}$, $^{26}\text{Al}$, and $^{30}\text{P}$. The Nilsson energy levels used for the s-d shell are illustrated in Fig. 3 for the typical case of $^{22}\text{Na}$. Nilsson energies for other s-d nuclei will differ somewhat from these because of the variation of $\hbar\omega_0$ with $A$ and, in the case of $^{30}\text{P}$, the choice of a different value of $D$, but the diagram of Fig. 3 still serves as an adequate basis for discussion of these nuclei. For convenience, we will refer to the single-particle levels by the projection $\Omega$, writing $\frac{1}{2}$ for the lowest single-particle orbital with $\Omega = \frac{1}{2}$, $\frac{3}{2}$ for the second one, and so on.
The first odd-odd nucleus in the s–d shell is $^{18}F$, consisting of a neutron and a proton outside an $^{16}O$ core. It might be thought that the doubly-magic $^{16}O$ core could not be statically deformed, so that the spherical shell model would provide the best description of $^{18}F$. However, there is considerable evidence that $^{16}O$ is deformed in essentially all of its excited states, so the addition of two more nucleons may well be enough to induce a stable static deformation. An attempt to describe $^{18}F$ as a deformed rotor nucleus therefore seems justified. It is at any rate interesting to determine whether the unified model provides a good description of a nucleus at the edge of the s–d shell. Failure of the model for $^{18}F$ may indicate that this nucleus does not have a well-established static deformation but is instead in a transition region between the spherically symmetric $^{16}O$ and the strongly prolate $^{20}Ne$ nuclei.

A glance at the Nilsson diagram of Fig. 3 shows that for positive deformation the lowest bands will be $K = 0$ and $K = 1$ with the neutron and proton both in the $\frac{1}{2}$ level, coupled antiparallel and parallel, respectively. The odd members of the $K = 0$ band will be $T = 0$ states and will therefore mix with the odd members of the $K = 1$ band because of the Coriolis term. (The even members of these two bands do not mix, since they have opposite exchange symmetry.) The Coriolis term for these two bands is in fact comparable in magnitude to the rotational energy, so that high spin states based on these band will be pushed down considerably from their unperturbed positions.

The Coriolis model results for $^{18}F$ are illustrated in Fig. 4 along with the experimental spectrum and the shell model results of Kuo (Ku 67).
Kuo's spectrum is calculated from the same matrix elements that we have used for our residual interaction. The Coriolis model does not satisfactorily reproduce the experimental spectrum. The spin 5 state and the spin 0(T=1) state are correctly positioned with respect to the spin 1 ground state, but the spin 3 state is pushed down below the ground state by the Coriolis coupling. Also, the displacement between the spin 2(T=1) state and the spin 0(T=1) state, both members of the K = 0 band, is too small. This latter discrepancy is a characteristic feature of our model that will also be found in the other nuclei that we discuss.

The shell model, on the other hand, gives a good account of the low-lying levels in $^6$F, and some of the levels it does not reproduce can be accounted for by a weak-coupling calculation based on excitations of the $^{16}$O core (Ar 67).

These results seem to indicate that the Coriolis model does not provide an appropriate description of $^{18}$F. However, it must be pointed out that this nucleus provides a rather stringent test of the model. The Coriolis matrix elements and rotational energies involved in the calculation of the low-lying spin 1, 3, and 5 states are as large as 8 or 10 MeV, and it is perhaps not surprising that discrepancies of 1 MeV or so exist in the relative positions of these states in view of the simple assumptions underlying the unified model. It is at least interesting to observe that this trio of states with no obvious rotational features is nonetheless predicted, if only approximately, by the rotational model. The poor agreement with experiment may be due to some of the specific assumptions of our model rather than to the general assumptions of collective rotational structure. For instance, the assumption that the deformation is the same
in all the excited states may be a poor approximation for $F_{18}^1$, particularly since this does not seem to be true for $O_{16}^1$.

In view of the failure to obtain agreement with the experimental spectrum, no transition rates or static moments have been calculated for $F_{18}^1$.  

The experimental spectrum of Na$^{22}$ (Wa 68) is illustrated in Fig. 5, along with the Coriolis model result and the shell model result calculated from the matrix elements of Kuo (Ha 68). The low-lying states of this nucleus can immediately be interpreted in terms of a rotational model (Wa 68). The states with spins 3, 4, and 5 at 0, .891, and 1.528 MeV form a $K = 3$ rotational band based on the neutron and proton orbitals with $\Omega = \frac{3}{2}$ coupled with their angular momentum projections parallel. These orbitals can also couple to give $K = 0$, and this band can account for the spin 0($T=1$) state at 0.657 MeV, the spin 1 state at 0.583 MeV, the spin 2($T=1$) state at 1.952 MeV, and the state at 1.984 MeV, assuming a spin of 3 for this state. The spin 1 state at 1.937 MeV could be the first member of a $K = 1$ band based on the $\frac{3}{2}$ and $\frac{1}{2}$ orbitals. This simple interpretation of the low-lying levels is born out quantitatively by the Coriolis band mixing calculation with a realistic residual interaction. However, this calculation also brings out some discrepancies between the rotational picture and the experimental data, which will be discussed below. The shell model reproduces the 3, 4, 5 band rather well, but does not give a good account of the other bands.

The Coriolis model seems to provide a better account of the experimental data for Na$^{22}$ than for the other nuclei studied, so we will give a
more detailed discussion of this nucleus than of the others in order to illustrate some characteristic features of the Coriolis model. In particular, the differences between the full Coriolis-coupling model with band mixing and the simple rotational picture will be emphasized.

Some characteristic aspects of the band-mixed wave functions are illustrated in Figs. 6 and 7. Fig. 6 shows the positions of some unperturbed bands that play an important role in determining the low-lying spectrum. One important feature of the Coriolis model is the strong compression of the rotational bands by the Coriolis mixing between bands with $\Delta K = 1$. The spin 5 state based on the $K = 3$ band, for instance, is pushed down 5 MeV from its unperturbed position. The extent of this mixing is illustrated in Fig. 7, which shows some of the major components of the lowest mixed wave functions. For the ground state $K = 3$ band, the spin 3 state is 78% a pure $K = 3$ state, but the spin 5 state is only 36% $K = 3$. A similar situation prevails for the other bands illustrated. We expect therefore that the rotational model without band mixing will provide an adequate description only of the lowest members of the rotational bands. For high spin states, the rotational band structure is almost completely broken up by the Coriolis coupling, and mixing between bands with $\Delta K > 1$ becomes significant. This illustrates the importance of including the Coriolis term to all orders by performing an exact diagonalization of the Hamiltonian and including all bands within the s-d shell in the basis space.

The effect of varying some of the parameters of the calculation is illustrated in Figs. 8, 9, and 10. Fig. 8 shows the band mixed spectra produced when the moment of inertia parameter $\hbar^2/2I$ is varied between .15 MeV and .45 MeV while the other parameters are held constant. The most
striking feature observed here is that the theoretical results are remarkably insensitive to the moment of inertia. The explanation for this is that a variation in $h^2/2\hbar$ has two opposing effects that approximately cancel each other. On the one hand, an increase in this parameter stretches out the unperturbed rotational bands; but on the other hand, it also increases the Coriolis mixing, which tends to compress the bands. Cancellation of these two effects seems to occur for both $T=0$ and $T=1$ bands to about the same degree. The stability of the spectrum with respect to variation of $h^2/2\hbar$ means that the moment of inertia to be used in a rotational model calculation of an odd-odd nucleus cannot really be determined with any accuracy. Hence, as stated earlier, these calculations do not provide any test of the various schemes proposed for microscopic calculations of moments of inertia. This is in contrast to the situation in heavy deformed nuclei, where band mixing is usually weaker, and the moment of inertia can be determined directly from the experimentally observed rotational bands.

On Fig. 9, the low-lying Coriolis model spectrum of Na\(_{22}\) is given as a function of the deformation parameter $\beta$. For $\beta \geq 0.3$, the lowest spin 5 state moves faster as a function of $\beta$ than any other state, and for all practical purposes the best deformation can be determined by matching the energy of this state to the experimental value. This behavior is also determined by the Coriolis coupling. As we see from the Nilsson diagram (Fig. 3), the single-particle states move further apart as $\beta$ increases. This reduces the Coriolis mixing between adjacent bands, and hence the bands are less compressed.
Fig. 10 illustrates the effect of the core overlap or "quenching" parameter on the spectrum. With no quenching (Q=1), the rotational bands are compressed far too much by the Coriolis term, and agreement with experiment cannot be obtained by varying the other parameters. For large quenching (Q < 1), the results approach the rotational model results with no band mixing. For many of the excited states the effect of a variation in Q is much the same as the effect of a variation in β, since both of these parameters tend to increase or reduce the amount of band mixing. Therefore, our somewhat arbitrary choice of .75 for the value of Q in the s-d shell will influence the equilibrium deformation parameter β obtained in our calculations by fitting the experimental spectrum. It is possible that a different choice of Q or of some of the other parameters would lead to an equally good spectrum at a different deformation. Thus a quantity such as the intrinsic quadrupole moment, which depends on β in our model, cannot be uniquely determined by our calculation.

We now return to our discussion of the experimental Na^{22} spectrum as interpreted by the Coriolis model. As mentioned earlier, the low-lying states based on the K = 0 and K = 3 bands are well reproduced by the model. The only serious discrepancy is that the K = 0, T = 1 band is too compressed, with the spin 2(T=1) level living 300 KeV too low and the spin 4(T=1) level 1 MeV too low. This discrepancy for the K = 0, T = 1 band is found in all four nuclei discussed here.

The spectrum above 2 MeV is more difficult to understand in terms of the rotational model. The first band predicted by the model above the K = 0 and K = 3 bands is a K = 1 band based on orbitals with Ω = \( \frac{3}{2} \)
and \( \frac{1}{2} \), and the experimental spin 1 state at 1.937 MeV, which is the first state not accounted for by the lowest two bands, seems to correspond to the first member of this band. However, a \( K = 2 \) band based on the same orbitals is also predicted in the same energy region, together with the higher spin members of these two bands. This is the origin of the three states predicted by the model between 2 and 3 MeV. However, there are no experimentally observed states that seem to correspond to these states. Between 2 and 4 MeV, the model predicts 11 states, whereas only 5 levels are observed, aside from those known to have negative parity. This sparse density of states is difficult to understand in terms of the rotational model, particularly if the state at 1.937 MeV has been correctly interpreted as the first member of a \( K = 1 \) band. (It is interesting to note that the shell model also predicts too many states in this region). We may conclude that the Coriolis-coupling model gives a good account of the \( ^{22} \text{Na} \) spectrum below 2 MeV but cannot explain the low density of states between 2 and 4 MeV.

The Coriolis model predictions of electromagnetic static moments and transition rates for some low-lying levels of \( ^{22} \text{Na} \) are summarized in Tables I and II along with the experimental results and the shell model predictions for some E2 transitions. The calculated magnetic moments of the first two levels are in good agreement with experiment. (These values are determined mainly by the collective gyromagnetic ratio, given by \( g_R = \frac{Z}{A} = \frac{1}{2} \), since the individual neutron and proton contributions almost cancel.) The interband E2 transitions tend to be predicted 30 to 50\% too slow, whereas the shell model results calculated with effec-
tive charges of $e = \frac{3}{2}e$ and $e_n = \frac{1}{2}e$ are in good agreement with experiment. This discrepancy cannot be considered very significant, however, since these $E_2$ transitions are dominated by the collective contribution, and an increase in $\beta$ of only 10% would raise the classical intrinsic quadrupole moment enough to produce agreement with experiment. The $M1$ transition rates are in fair agreement with experiment, and the $E2/M1$ mixing ratio has the correct sign. One characteristic rotational feature is the strongly inhibited $E2$ transition between the first spin 1 state and the ground state. This transition is primarily $\Delta K = 3$, and the Coriolis model comes much closer to the experimental value than does the shell model. On the whole, the agreement between the Coriolis model predictions and the experimental electromagnetic properties can be considered good.

In summary, the Coriolis-coupling model provides a convincing interpretation of many of the properties of $^{22}$Na. The main discrepancies between theory and experiment are that the density of states above 2 MeV is predicted to be too high and the $K = 0, T = 1$ band is too compressed.

The next nucleus in our series is $^{26}$Al. This nucleus has too many extracore particles to be treated by the "realistic interaction" shell model. The only theoretical spectrum reported in the literature to date seems to be the intermediate coupling result (Bo 67). The Coriolis model result is illustrated together with the experimental spectrum and the intermediate coupling spectrum in Fig. 11.
The ground state spin of $5$ and the presence of a low-lying spin $3$ state suggest a deformation around $\beta = 0.3$, just before the crossing of the $\Omega = \frac{5}{2}^{-}$ and $\Omega = \frac{1}{2}^{-}$ Nilsson levels. This deformation produces low-lying bands with $K = 5$ and $K = 0$ when both odd particles are in the $\frac{5}{2}^{-}$ Nilsson level and $K = 3$ and $K = 2$ when the odd particles are in the $\frac{5}{2}^{-}$ and $\frac{1}{2}^{-}$ levels. It is interesting to note that the first spin $2$ state is separated from the first spin $3$ by $1.4$ MeV. The rotational model with no residual interaction would place the $K = 2$ band below $K = 3$. It is therefore encouraging to see that the realistic interaction used in the present calculation gives nearly the correct separation of the first spin $2$ and spin $3$ states. The great density of states observed above $2$ MeV makes any detailed comparison between theory and experiment impossible. In contrast to the situation in Na$^{22}$, the experimental and theoretical densities of states are in good agreement for Al$^{26}$. In all, the rotational model seems to provide good agreement with experiment for the Al$^{26}$ spectrum except for discrepancies of about $500$ KeV in the positions of the first spin $0(T=1)$ and spin $1$ states.

Unfortunately, however, the observed transition rates do not support this picture of Al$^{26}$. There is a strong E2 transition between the lowest spin $3$ state and the spin $5$ ground state, with $B(E2, 3 \to 5) = 35 e f^{2}$ (Sr 67). The theoretical reduced transition rate is only $3.5 \times 10^{-4} e f^{2}$, since this transition involves $\Delta K = 2$ and is not collectively enhanced in our model. This discrepancy of 5 orders of magnitude strongly suggests that our model does not adequately describe the Al$^{26}$ nucleus, in spite of its good agreement with the experimental spectrum. The most natural interpretation of this phenomenon might be provided by the axially symmetric
rotator model (He 62), since the axial symmetry induces mixing between bands with $\Delta K = 2$. This interpretation is also supported by Hartree-Fock (Ba 65) calculations which suggest that axial symmetry is favored for the $^{24}$Mg nucleus, which forms the $^{26}$Al core in our model.

Another discrepancy is revealed by the observed transition from the spin 1 state at 1.059 MeV to the spin $0^+(T=1)$ state at 0.229 MeV. This is an M1 transition with a strength of 1.84 Weisskopf units (Yo 67). Our model predicts an M1 strength of only .096 Weisskopf units for the transition from the lowest spin 1 state to the spin $0(T=1)$ state but a strength of .97 Weisskopf units for the transition from the second spin 1 state. Although even this second value is too low, it nonetheless suggests that the second calculated spin 1 state should be identified with the lowest experimental spin 1 level, and the experimental counterpart of the first spin 1 state must remain uncertain.

These conclusions exemplify the importance of using other experimental data besides the energy spectrum as a test of a theoretical model.

The intermediate coupling model reproduces the lowest five levels of $^{26}$Al satisfactorily, but the high density of states above 2 MeV is beyond the scope of this calculation, as is the evaluation of transition rates. It would certainly be of interest to see whether some variety of shell model could account for the enhanced E2 transition between the spin 3 and spin 5 levels in this nucleus.

$^{30}$P

Several models have been used to calculate theoretical spectra for $^{30}$P, including the rotational model of Asciutto et al., the intermediate
coupling model, and the shell model with a $^{28}$Si core. These various results are illustrated in Fig. 12 together with the experimental spectrum and our Coriolis-coupling model results.

We find the best fit for $P^{30}$ for a positive deformation of $\beta = .28$, so that the lowest bands will have $K = 0$ and $K = 1$. It is interesting to note that this is similar to the situation in $F^{18}$, where the lowest bands were also $K = 0$ and $K = 1$. However, the structure of the Nilsson states is such that the Coriolis matrix elements between these bands are small, so the high spin states are not pushed down to the same degree as in $F^{18}$, and there is little similarity in the two spectra.

Both Hartree-Fock ($Ri 69$) and Coriolis model calculations for odd-even nuclei ($Hi 69$) indicate that the region around mass 28 to 30 marks a transition from prolate to oblate deformation. Coriolis model calculations for $Al^{29}$ and $P^{31}$ favor positive deformation, whereas negative deformation is favored for $Si^{29}$ and $Si^{31}$ ($Hi 69$). These results suggest that the addition or subtraction of one or two nucleons can be enough to change the sign of the deformation for nuclei in this region, and so it is not surprising that the deformation of $P^{30}$ differs in sign from that of some neighboring odd-even nuclei. Positive deformation is also predicted for $P^{30}$ by Asciutto et al., although some of the other parameters used in this calculation differ from ours.

The Coriolis model is in rather good agreement with the experimental spectrum up to 4 MeV, except for the position of the spin 0(T=1) level, which is predicted 1 MeV too high. The position of this state with respect to the spin 1 ground state is determined primarily by the residual
interaction. The first spin 0(T=1) state was positioned correctly in $^{18}$F and $^{22}$Na. However, it was 600 KeV too high in $^{26}$Al and is now 1 MeV too high in $^{30}$P. This may indicate a systematic variation in the appropriate residual interaction as the mass increases in the s-d shell. One possible explanation for such an effect is that the harmonic oscillator frequency of the wave functions used in calculating the residual interaction matrix elements was chosen for a nucleus with mass $A = 18$. This frequency varies approximately as $A^{-1/3}$, and the difference may be fairly significant for $A = 30$.

In $^{30}$P also, the calculated transition rates fail to support the agreement between theory and experiment for the energy spectrum. The electromagnetic properties of low-lying $^{30}$P levels are summarized in Table III. The spin 2 and spin 3 states at 2.54 and 2.72 MeV have strongly enhanced E2 transitions to the spin 1 ground state, whereas only weak transitions are predicted by the Coriolis model, since these are interband transitions. We would expect the rotational states based on the ground state $K = 1$ band to be the spin 2 and spin 3 states at 1.45 and 1.97 MeV in the rotational model, and these are therefore the only states which can have enhanced E2 transitions to the ground state in our model. This suggests the presence of some other nuclear structure effect that would account for the enhanced transitions from the states between 2.5 and 3 MeV, and so the validity of our interpretation of these states is placed in doubt. As would be expected, the rotational model calculation of Ascuitto et al. also fails to predict these enhanced E2 transitions. No transition rates have been calculated from the shell model or intermediate coupling model. It would be helpful in clarifying this situation to have experimental measurements of absolute transition rates for more states below 2 MeV in $^{30}$P.
Experimental studies in recent years have done much to elucidate the properties of nuclei in the lower half of the $1f-2p$ shell. Considerable progress has also been made in explaining the properties of odd-even nuclei in this region within the framework of both the shell model and the Coriolis coupling model. However, no applications of the Coriolis coupling model have been made to odd-odd nuclei in this region up to now. In this chapter we report the results of such a Coriolis-coupling calculation for the odd-odd $f$-$p$ shell nuclei $^{46}$Sc and $^{56}$Mn.

Shell Model Calculations in the $f$-$p$ Shell

The nuclear $1f_{7/2}$ subshell is pushed down below the other subshells of the $f$-$p$ shell by the spin-orbit force. This suggests the possibility of treating nuclei in this region as an inert Ca core plus extracore particles in pure $1f_{7/2}$ orbitals. Shell model calculations have been performed using this approximation (Mc 64, Gi 65), and in many cases good agreement with experiment is obtained. However, some properties of these nuclei are not reproduced by the $1f_{7/2}$ shell model. In particular, there are generally many more observed low-lying states than are predicted by the model, especially for odd-even nuclei. Also, effective charges as large as 2.5 electron charge units (La 65) are needed for both neutrons and protons in order to account for some E2 transitions.

These discrepancies suggest that a model without the restrictive assumptions of the pure $1f_{7/2}$ shell model may provide a more appropriate framework in which to interpret the properties of the nuclei in this region. The strong E2 transitions indicate that collective degrees of freedom may
be important, and so it is natural to attempt to explain the properties of the $f_{7/2}$ shell nuclei in terms of a unified model. Calculations with this model were first applied to odd-even nuclei in the $1f_{7/2}$ region by Malik and Scholz (Sc 66, Ma 66, Sc 67), and their results successfully interpreted many of the properties of these nuclei that could not be understood in terms of the $f_{7/2}$ shell model.

One of the most interesting features of the Malik and Scholz calculation was its demonstration of the importance of a proper treatment of the Coriolis coupling term in the Hamiltonian for this region of the periodic table. In contrast to the situation for many odd A nuclei in the deformed region of the s-d shell, the ground state spin is not the spin of the lowest member of the rotational band obtained by placing the last odd nucleon in the lowest unoccupied Nilsson orbital. For example, this procedure would give ground state spins of $\frac{1}{2}$ and $\frac{3}{2}$ respectively for Sc$^{43}$ and V$^{51}$, whereas both of these nuclei have observed ground state spins of $\frac{7}{2}$. However, when the band mixing due to the Coriolis term is included, the correct ground state spin is obtained for these nuclei.

In fact, the Coriolis coupling model gives the correct ground state for all the odd-even nuclei in the $f_{7/2}$ shell. Good agreement with experiment is also obtained for electromagnetic static moments and transition rates. In particular, electric quadrupole moments and E2 transition rates are correctly predicted without the use of effective charges, as is required by the shell model.
The success of the Coriolis-coupling model for the odd-even nuclei in the $1f_{7/2}$ region leads us to expect that this model may also provide a good description of the odd-odd nuclei. Because of the large amount of computer time required, a complete survey of the odd-odd nuclei in the $1f_{7/2}$ shell has not seemed worthwhile. Instead, we present results of the Coriolis model calculation for only two nuclei, Sc$^{46}$ and Mn$^{56}$. The first of these is a likely candidate for a deformed model treatment because the spectra of the neighboring odd-even nuclei, Sc$^{45}$ and Sc$^{47}$, are fairly well reproduced by the Coriolis model (Ma 66), but the shell model (Mc 64) cannot account for the observed high density of states below 1 MeV. The calculations on Mn$^{56}$ were performed at the suggestion of Dr. Joseph Comfort of the Wright Nuclear Structure Laboratory at Yale, who recently carried out an experiment on this nucleus. Although Mn$^{56}$ was thus chosen rather arbitrarily from a theoretical point of view, it is interesting to test the Coriolis model in a region where the neutrons are filling the $2p_{3/2}$ shell.

In Fig. 14 the experimental spectrum of Sc$^{46}$ below 2 MeV is illustrated in comparison with the Coriolis model result and the shell model calculation of McCullen, Bayman, and Zamick (Mc 64). In contrast to the situation for the odd Scandium isotopes, the shell model gives a good account of the experimental spectrum below 1 MeV, but the high density of states between 1 and 2 MeV is beyond the scope of this model. The Coriolis model level spectrum below 600 KeV agrees about as well with
experiment as does the shell model, except that the Coriolis model predicts a spin 7 state at 70 KeV that does not seem to have any experimental correspondent. From 600 KeV to 2 MeV, the Coriolis model gives about the right number of states, but no detailed comparison is possible because of the lack of experimental spin assignments.

Only one absolute transition rate between positive parity states of Sc$^{46}$ has been measured. The spin 6 first excited state decays to the spin 4 ground state with $B(E2) = 3.1 \times 10^{-51} e^2 cm^4$ (Fo 68). This value can be reproduced by the $f_{7/2}$ shell model using effective neutron and proton charges with $e_p + e_n = 5.43$ (Fo 68). The Coriolis model with $\beta = .3$ predicts $B(E2) = 16.7 \times 10^{-51} e^2 cm^4$ for this transition. This result comes mainly from the collective contribution to the reduced matrix element and is therefore very sensitive to the choice of $\beta$. For a smaller value of the deformation around $\beta = .2$, good agreement with the experimental transition rate could be obtained without appreciably worsening the agreement with the experimental energy spectrum.

The ground state quadrupole moment $Q$ and magnetic moment $\mu$ have been measured as $Q = .119b$ and $\mu = 3.04$ nm (Pe 62). The Coriolis model predicts these quantities to be $Q = -0.028b$ and $\mu = 1.25$ nm, in rather poor agreement with experiment. The shell model predicts $Q = .036b$ and $\mu = 3.23$ nm (Co 66) using effective charges and gyromagnetic ratios obtained from a least squares fit to the measured transition rates and static moments in several $f_{7/2}$ shell nuclei.

In summary, the Coriolis model provides a satisfactory account of the experimental spectrum of Sc$^{46}$ except for the prediction of a low-lying spin 7 state. The predicted static moments for the ground state
do not agree with experiment for the parameters used, but the enhanced E2 transition from the first excited state can be satisfactorily explained without the use of effective charges. In contrast to the situation for the odd isotopes of Scandium, the present Coriolis model calculations do not establish a clear preference for the deformed model over the spherical shell model.

The experimental and theoretical Coriolis model spectra of Mn\(^{56}\) are illustrated in Fig. 15. The parameters used in the Coriolis calculation are consistent with the values used by Malik and Scholz in their study of the odd Mn isotopes (Ma 66). Agreement between the experimental and theoretical results is rather satisfactory for states below about 500 KeV. For higher states, the lack of experimental spin assignments makes comparison between experiment and theory difficult, but the model does give the observed density of states. Shell model calculations for Mn\(^{56}\) have been performed by J. B. McGrory by taking Ca\(^{48}\) as an inert core and placing the active neutrons in the \(1f_{5/2}, 2p_{3/2}, \) and \(2p_{1/2}\) shells (Co 69). The shell model result agrees with the experimental spectrum about as well as the Coriolis model.

Experimental and Coriolis model relative M1 transition rates are summarized in Table III. Fair agreement with the experimental ratios is obtained for the decay of the 215 KeV spin 2 state and the 486 KeV spin 3 state, but there are many discrepancies for the other decays. It cannot be said that these data support the Coriolis model interpretation of Mn\(^{56}\) with the parameters used here.
these nuclei has two particles in an $\Omega = \frac{1}{2}$ orbital for positive deformation, and the Coriolis interaction for the $K = 0$ and $K = 1$ bands based on these orbitals pushes some odd spin states too far down from their unperturbed positions. In $^{18}$F, the state with spin 3 was pushed down below the experimental ground state, and in $^{42}$Sc states with spin 3 and 5 were pushed down below the ground state.
Chapter VII Summary and Conclusion

The calculations discussed above indicate that the Coriolis model with a realistic residual interaction provides an adequate description of the properties of low-lying states of some self-conjugate odd-odd nuclei in the lower half of the s-d shell. It is perhaps equally interesting that these calculations point out features of the experimental data that apparently cannot be understood in terms of the rotational model as formulated here. Another aspect of these calculations that deserves emphasis is their demonstration of the importance of a correct treatment of band-mixing in applying the rotational model to these nuclei. We now summarize the results obtained for each nucleus in order to form a final assessment of the significance of these calculations.

The Coriolis model does not provide an adequate picture of the properties of $^{18}\text{F}$. In view of the clear evidence for collective behavior in the neighboring nuclei $^{19}\text{F}$ and $^{20}\text{Ne}$, this failure cannot be taken as an indication that $^{18}\text{F}$ is not collective, but it does suggest that a more sophisticated collective model than ours would be necessary to adequately describe this nucleus.

In the case of $^{22}\text{Na}$, on the contrary, the Coriolis model gives a much better account of the experimental data than the shell model. This is accord with a large body of evidence that indicates that nuclei around mass 23 have large stable prolate deformations and exhibit clear rotational properties. The agreement between the Coriolis model and experiment is thus not surprising, and it is perhaps more interesting to compare the Coriolis model interpretation of $^{22}\text{Na}$ with the simple rotational model picture without band mixing. The series of states with spins 3, 4, and 5, although
they do not follow an $I(I + 1)$ rule, seem to suggest a rotational band with a moment of inertia about as large as the rigid-body value. However, the Coriolis model calculations show that this sequence of states is well reproduced for a wide range of the moment of inertia parameter, and very little can be said about the "rigidity" of the Na$^{22}$ nucleus from the point of view of these calculations. This illustrates the importance of performing a complete band-mixing calculation in order to determine the appropriate values of collective parameters.

Al$^{26}$ is another example of a nucleus which is not adequately described by our model. In this case, however, it seems likely that the discrepancy is due not to the assumption of a rotor model but to the restriction to the case of axial symmetry. Unfortunately, relaxing this restriction would sharply increase the difficulty of the calculation, so this hypothesis cannot easily be tested.

The Coriolis model energy spectrum is in excellent agreement with experiment for $P^{30}$ for states up to 4 MeV. But, as in the case of Al$^{26}$, the transition rates predicted by the model do not support this agreement, and we must conclude that there are important nuclear structure effects here that our model does not simulate.

One feature characteristic of all four of these nuclei is that the $T = 1, K = 0$ rotational band is poorly reproduced by the Coriolis model. In each case this band is too compressed by mixing with neighboring bands due to the strong Coriolis coupling. This may indicate that collective parameters describing the nucleus are different in $T = 1$ states than in $T = 0$ states. Such an effect would be outside the scope of our model, which explicitly assumes that the collective parameters are the same in all the low-lying states.
To sum up the situation in the s-d shell, it is clear that the Coriolis model does not fully account for all the experimentally observed properties of any of the four nuclei treated. These calculations are nonetheless of some value in pointing out nuclear phenomena which cannot be understood in terms of this simple model and, in a few cases, suggesting plausible alternative explanations for them. It is obvious from the discussion of the calculations presented here and of other model calculations that our understanding of these nuclei is far from complete, but these calculations may have made some contributions to clarifying the problems involved.

It would be helpful in evaluating the success of the Coriolis model to have experimental measurements of more static moments of odd-odd s-d nuclei. To date only the magnetic moments of the first two levels of Na$^{22}$ have been measured, and these are in agreement with Coriolis model predictions. It would be interesting to see if the same success would be enjoyed in predicting other static moments.

Another interesting question is raised by the observation of more enhanced E2 transitions in P$^{30}$ levels above 2.5 MeV than the Coriolis model can account for. This may indicate a general failure of the rotational picture for high-lying states of light odd-odd nuclei. Measurements of absolute transition rates from states in Na$^{22}$ above 2.5 MeV would be useful in clarifying this situation. If these transitions were also found to be strongly enhanced, this would provide evidence for a more general collective behavior than is provided by the rotational model.
It is somewhat more difficult to evaluate the success of the Coriolis model calculations in the f-p shell. The high density of states makes a meaningful comparison between experimental and theoretical spectra difficult, even when reliable experimental spin and parity assignments have been made. In view of the high cost of performing computer calculations for these nuclei, a thorough investigation of the behavior of the theoretical spectra as a function of the parameters involved did not seem worthwhile, so the results presented may not be the best that could be obtained with the Coriolis model.

In spite of these difficulties, the results obtained indicate that the Coriolis-coupling model may be a useful tool in studying the properties of odd-odd nuclei in the f-p shell. The low-lying energy spectra of $^{46}$Sc and $^{56}$Mn are fairly satisfactorily reproduced and the high density of states above 1 MeV in both nuclei is in accord with experiment. On the basis of presently available experimental data and theoretical results, it does not seem possible to discriminate between the shell model and the Coriolis model as a tool for interpreting properties of these nuclei. However, as more experimental data becomes available, particularly on the spins and parities of levels above 1 MeV, evidence for rotational structure may develop, and the appropriateness of Coriolis model calculations in this region may be borne out.
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Figure Captions

Fig. 1 Structure of different intrinsic states used in Coriolis model calculations.

Fig. 2 Intrinsic states for a three-particle system.

Fig. 3 Nilsson energies as function of deformation $\beta$ for $^{22}\text{Na}$ with $c = -4.4$ MeV, $D = 0$, and $\hbar \omega_0 = 41/A^{1/3}$.

Fig. 4 Comparison of experimental spectrum of $^{18}\text{F}$ (Ma 68a) with Coriolis model spectrum ($\beta = .3$) and shell model spectrum (Ku 67). Experimental states known to have negative parity have been omitted.

Fig. 5 Comparison of experimental spectrum of $^{22}\text{Na}$ (Wa 68) with Coriolis model spectrum ($\beta = .45$) and shell model spectrum (Ha 68). Experimental states known to have negative parity have been omitted.

Fig. 6 Coriolis model spectrum for $^{22}\text{Na}$ with $\beta = .45$. The six columns to the right give the positions of the lowest unperturbed bands that contribute to the band-mixed result.

Fig. 7 Squared amplitudes of some unperturbed states that make large contributions to low-lying levels of $^{22}\text{Na}$ for $\beta = .45$.

Fig. 8 Coriolis model spectrum of $^{22}\text{Na}$ as function of moment of inertia parameter $\hbar^2/2$ for $\beta = .45$, $Q = .75$.

Fig. 9 Coriolis model spectrum of $^{22}\text{Na}$ as function of deformation $\beta$ for $Q = .75$, $\hbar^2/2\Omega = .3$ MeV.
Fig. 10 Coriolis model spectrum of Na$^{22}$ as function of core overlap parameter $Q$ for $\beta = .45$, $h^2/2\hbar = .3$ MeV.

Fig. 11 Comparison of experimental spectrum of Al$^{26}$ (Bi 68) with Coriolis model spectrum ($\beta = .3$) and intermediate coupling model spectrum (Bo 67).

Fig. 12 Experimental and theoretical energy spectra for P$^{30}$:
(a) experimental spectrum (Ha 69); (b) Coriolis model with $\beta = .28$;
(c) rotational model of Ascuitto et al. (As 68); (d) intermediate coupling model (Bo 67); (e) shell model (Gl 64).

Fig. 13 Nilsson energy levels for f-p shell as function of deformation $\beta$ for $h\omega_o = 10.7$ MeV, $C = -.26$ $h\omega_o$, and $D = -.035$ $h\omega_o$.

Fig. 14 Comparison of the experimental spectrum of Sc$^{46}$ (Bo 68) with the Coriolis model result ($\beta = .3$, $h^2/2\hbar = .13$ MeV) and the shell model spectrum (Mc 64). Experimental states known to have negative parity have been omitted.

Fig. 15 Comparison of the experimental spectrum of Mn$^{56}$ (Co 69) with the Coriolis model spectrum ($\beta = .25$, $h^2/2\hbar = .25$ MeV). Dotted lines indicate experimental states that have not been definitely established.
Table I. M1 and E2 transitions between low-lying positive parity states of $^{22}\text{Na}$, $^{26}\text{Al}$, and $^{30}\text{P}$. Shell model results for $^{22}\text{Na}$ are from Ha 68.

<table>
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<th>$E_i$ (MeV)</th>
<th>$E_f$ (MeV)</th>
<th>$I_i$</th>
<th>$I_f$</th>
<th>$B$(M1)(nm$^2$)x10$^4$</th>
<th>$B$(E2)(e$^2$f$^4$)</th>
<th>$\delta$(E2/M1)</th>
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<td>$I_f$</td>
<td>$B$(M1)$ (\text{nm}^2) \times 10^4$</td>
<td>$B$(E2)$ (e^2 f^4)$</td>
<td>(E2/M1)</td>
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Table II. Magnetic moments and electric quadrupole moments for Na\(^{22}\), Al\(^{26}\), and P\(^{30}\). Experimental values for the magnetic moments of the first two states of Na\(^{22}\) are given for comparison with the Coriolis model results. No other static moments have been measured for these nuclei.

<table>
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<th>Magnetic moment (nm)</th>
<th>quadrupole moment (b)</th>
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<td></td>
<td>(MeV)</td>
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Table III  Coriolis Model and Experimental Relative $B(M1)$'s in Mn$^{56}$.

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The United States Atomic Energy Commission provided me with financial support during the course of this work.
Figure 1
Figure 2
Figure 3
Figure 4
Figure 6

\[ E \text{(MeV)} \]

\[ \begin{align*}
\text{Na}^{22} \\
\text{MIXED} & 3/2 & 3/2 & 3/2 & 3/2 & 3/2 & 3/2 & 1/2' & 3/2 & 1/2' & 3/2 & 5/2 & 3/2 & 1/2 \\
\end{align*} \]

\[ \begin{align*}
* \ T & = 1 \\
\Omega_1 & \Omega_2 \\
K & \\
\end{align*} \]
Figure 7

\[ \Omega_1 \Omega_2 \]

\[ K \]

\begin{align*}
I = 3 & \quad E = 0.0 \\
I = 4 & \quad E = 0.860 \\
I = 5 & \quad E = 1.566 \text{ MeV}
\end{align*}

\begin{align*}
I = 0 & \quad E = 0.801 \\
I = 2 & \quad E = 1.650 \\
I = 4 & \quad E = 3.069 \text{ MeV}
\end{align*}

\begin{align*}
I = 1 & \quad E = 0.725 \\
I = 3 & \quad E = 1.936 \\
I = 5 & \quad E = 4.925 \text{ MeV}
\end{align*}
Figure 9

$^{22}$Na

MeV

EXP

$\beta$

$T=1$

Figure 9
Figure 10
Figure 11
Figure 12

*pT = 1*
Figure 14

E (MeV)

Sc$^{46}$

EXP  CORIOLIS  SHELL
Figure 15

$E$ (MeV) vs $Mn^{56}$

EXP vs CORIOLIS