ABSTRACT

ELASTIC SCATTERING OF

$^{14}\text{C} + ^{12}\text{C}$, $^{14}\text{C} + ^{16}\text{O}$, and $^{14}\text{C} + ^{18}\text{O}$

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We have carried out highly detailed studies of the elastic scattering of $^{14}\text{C}$ from $^{12}\text{C}$, $^{16}\text{O}$, and $^{18}\text{O}$ in a search for resonant phenomena in heavy-ion systems of non-zero isospin and zero spin. Angular distributions from $^{14}\text{C} + ^{12}\text{C}$ and $^{14}\text{C} + ^{16}\text{O}$ scattering were measured at $^{14}\text{C}$ bombarding energies ranging from 20 to 40.3 MeV in 0.35 MeV steps. $^{14}\text{C} + ^{18}\text{O}$ angular distributions were measured at $^{14}\text{C}$ energies from 20 to 30 MeV in 0.40 MeV steps and from 22.5 to 32.5 MeV in 2.5 MeV steps.

The great variety of behavior which characterizes heavy-ion scattering is clearly illustrated by our data. The $^{14}\text{C} + ^{12}\text{C}$ and $^{14}\text{C} + ^{16}\text{O}$ systems both exhibit marked gross structure in their excitation functions and deep oscillatory structure with a large backward angle rise in their angular distributions. They are in sharp contrast with the $^{14}\text{C} + ^{18}\text{O}$ system, where structure is not apparent, and cross-sections fall steeply at larger angles and higher energies. However, only in the $^{14}\text{C} + ^{12}\text{C}$ system are the excitation functions strongly fragmented by intermediate width structure. The anomalous appearance of the $^{14}\text{C} + ^{12}\text{C}$ angular distributions near 17.5 MeV (cm) does suggest the existence of a resonant state (tentatively identified as $f=10$ by phase shift analysis).
In our analysis, we point out that the remarkable qualitative differences among these systems may be related to open-channel systematics. We find that general characteristics of the $^{14}\text{C}+^{18}\text{O}$ data are reproduced with a strongly absorbing optical potential model, contrary to the situation in the other two systems, where a specific mechanism, such as elastic transfer, must be invoked to account for the backward rise. We have pursued analysis with a one-step DWBA elastic transfer model which was quite successful at low energies. Its underprediction of the backward rise at higher energies provides further evidence of the importance of multi-step processes in two-nucleon transfer reactions.

We have also applied recent group theoretical models of heavy-ion scattering to our data. The single-channel models involving the $\text{SO}(3,2)$ and $\text{SO}(3,1)$ dynamical symmetries were found to be somewhat more flexible than standard optical models, but were still not able to provide an acceptable gross description of the $^{14}\text{C}+^{12}\text{C}$ and $^{14}\text{C}+^{16}\text{O}$ data. However, a two-channel $\text{SO}(3,1)$ dynamical symmetry model in which the elastic transfer amplitude is obtained as the second channel has provided very good fits to individual highly structured angular distributions and better overall reproduction of the datasets than the other models which we have studied.
ELASTIC SCATTERING OF

$^{14}_{C} + ^{12}_{C}, \; ^{14}_{C} + ^{16}_{O}, \; \text{and} \; ^{14}_{C} + ^{18}_{O}$

A Dissertation

Presented to the Faculty of the Graduate School
of
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Doctor of Philosophy

Stephen Michael Sterbenz

December 1987
To Dad, Mom,

Chris, Ron, and John
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CHAPTER I

INTRODUCTION

The scattering of heavy ions involves a complex interplay of reaction dynamics and nuclear structure which leads to a very broad range of observable phenomena, as even the briefest review of experimental data will confirm. Even elastic scattering, that ever-present and most basic of scattering processes, presents us with a fascinating diversity of behavior from system to system and as a function of bombarding energy. That this is so should not surprise us since the nuclear many-body problem, with its manifold degrees of freedom and strong fundamental interactions, must be the essence of any process which results in an overlapping of wave functions of complex nuclei. In fact, when we view elastic scattering in this light we realize that the very reemergence of elastic channel flux from such interactions is in principle as complex and interesting a process as any other nuclear reaction, and is one whose study can provide us with valuable information about the very fundamental underlying problem.

The great variety of behavior seen in heavy-ion scattering is well demonstrated in Fig. 1.1 which depicts 90° c.m. excitation functions for some light symmetric or nearly symmetric systems which are the "classics" of the field. This great variety, in spite of much experimental and theoretical inquiry over the past 25 years, still defies a complete understanding today, as no single model has been completely successful in explaining these scattering data. The $^{12}$C+$^{12}$C system is well known for its sharp dinuclear resonances (Br61) in the vicinity of
FIGURE 1.1. 90° Excitation functions for five heavy-ion systems which exemplify the diverseness of heavy-ion scattering behavior.
the Coulomb barrier, which is just below the scale limit in the figure. At higher energies, this system exhibits pronounced structures of intermediate width (a few hundred keV) which are superimposed on a background of wide structures (easier to see upon energy-averaging). These structures have been interpreted in terms of a double resonance mechanism involving inelastic excitation of one or both carbon nuclei (Im68, Pa77), but other mechanisms such as compound elastic fluctuations are known to play a role as well (Sh74).

At still higher energies, the intermediate width structure begins to decrease, but gross structure several MeV wide remains. It is generally thought that these broad structures are a non-resonant phenomenon associated with amplitudes arising from a narrow 1-window centered on the grazing partial wave appropriate to the bombarding energy. This has been linked with the surface transparency of these systems, which results when few other open channels are available for decay of the entrance channel.

The situation in $^{12}\text{C}+^{16}\text{O}$ is somewhat similar to that in $^{12}\text{C}+^{12}\text{C}$ but less dramatic. The excitation function shows intermediate width resonant structure superimposed on a background of gross structures. $^{16}\text{O}+^{16}\text{O}$ shows the diffraction-like gross structure very clearly with relatively less pronounced intermediate width structure. A very tightly bound nucleus, $^{16}\text{O}$ has a very high first excited state and is thus perhaps less able to participate in double resonance formation than $^{12}\text{C}$. $^{18}\text{O}+^{16}\text{O}$ shows even less intermediate width structure than $^{16}\text{O}+^{16}\text{O}$, while in $^{18}\text{O}+^{18}\text{O}$ even the gross structure is damped out and the high energy cross-section has fallen off dramatically compared with the other
systems. This featureless behavior is characteristic of strongly absorb-
ed systems, which generally have a relatively large number of open exit
channels well matched in angular momentum (i.e., with favorable Q-
values). The fact that $^{18}$O has a fairly low first excited state ($a 2^+$ at
1.98 MeV) and that its two valence neutrons are not tightly bound may
therefore account for the washing out of resonant structure in its
elastic scattering.

The many-body problem which must be solved in order to understand
heavy ion interactions in detail is far too complex to attack in a
general and microscopic way. However, many different theoretical ap-
proaches to the problem have made much progress in elucidating general
features of elastic and "quasielastic" scattering (inelastic and trans-
fer processes), some of which were just mentioned above. These also
include general studies of the effective internuclear potential,
phenomenological optical potential models with various embellishments,
parametrized phase-shift analysis, compound nuclear fluctuation
analysis, and numerous coupled-channel approaches. Also, group theoreti-
cal models have proved useful in classifying resonant states in some
systems (Er81). For all of these models, it has become clear that what
is needed on the experimental side are data of a more global kind in
energy and angle, as it is often easy to describe a single excitation
function or angular distribution in terms of one model or another, only
to find little correlation between theory and experiment when other
measurements on the system become available. At this point in the his-
tory of heavy-ion physics it is no longer appropriate simply to choose a
system and measure cross-sections at isolated energies or angles.
Detector and data analysis technology now make it possible to do systematic studies, and in scattering measurements this means gathering high angular-resolution data over a wide angular range and in relatively fine energy steps, so that the detailed dependence of the cross-sections on energy and angle can be known. Indeed, very complete measurements have been done for some of the classic resonant systems (such as $^{12}\text{C}+^{12}\text{C}$ and $^{12}\text{C}+^{16}\text{O}$ for example), but there remains a great deal of unbroken ground.

In this work, we have sought to extend systematic elastic scattering studies to three systems which, because of their similarities to and differences from each other and previously studied systems, seemed likely to show us some interesting physics, and provide further information on the physical mechanisms underlying heavy ion scattering. The systems are $^{14}\text{C}+^{12}\text{C}$, $^{14}\text{C}+^{16}\text{O}$, and $^{14}\text{C}+^{18}\text{O}$, all of which involve $^{14}\text{C}$ (as the beam, in fact), are spinless, have nonzero isospin, and involve either alpha-nuclei or nuclei two neutrons away from alpha-nuclei. $^{12}\text{C}$, $^{16}\text{O}$, and $^{18}\text{O}$ are old friends of course. $^{14}\text{C}$ brings in two additional neutrons but is very tightly bound. It turns out that all three systems show remarkably different behavior, with $^{14}\text{C}+^{12}\text{C}$ exhibiting in its angular distribution the most pronounced oscillatory structure (with a strong rise at backward angles) and $^{14}\text{C}+^{18}\text{O}$ the least.

Our original aim was to search the completely unexplored $^{14}\text{C}+^{18}\text{O}$ system for molecular resonances, so that it would then be possible, at least in principle, to observe among such states electric dipole transitions, predicted to be enhanced by current group theoretical models, but forbidden to first order by an isospin selection rule in self-conjugate ($T=0$) systems such as $^{12}\text{C}+^{12}\text{C}$ and $^{12}\text{C}+^{16}\text{O}$ (however, evidence for
electric quadrupole transitions among $^{12}\text{C} + ^{12}\text{C}$ resonances exists (Na81)). The high T of $^{14}\text{C} + ^{18}\text{O}$ (2 units) and current interest in the structure of $^{18}\text{O}$ itself at Yale made that system seem a desirable candidate for study, in spite of certain negative systematics for resonances in systems involving $^{18}\text{O}$ which will be discussed in chapter IV. However, in the process of gathering data on the $^{14}\text{C} + ^{18}\text{O}$ system, we have collected data on several other systems by virtue of the very heterogeneous composition of metallic oxide targets (especially when they are placed under beam in typical scattering chambers for some time)! The elastic data on $^{14}\text{C} + ^{12}\text{C}$ and $^{14}\text{C} + ^{16}\text{O}$ were singled out among these for analysis as the most important for a more general study of heavy ion scattering and also as the greatest in range and quality. Before this study, only fragmentary $^{14}\text{C} + ^{16}\text{O}$ data existed (Be78, Ko81, Sc75), while the $^{14}\text{C} + ^{12}\text{C}$ system had received more substantial attention, along with $^{14}\text{C} + ^{14}\text{C}$ (Ko80, Ko85). However, our extensive and finely spaced (160 keV c.m.) low energy angular distribution data extend and are complementary to those of the earlier studies.

In chapter II of this thesis, general theoretical ideas relevant to heavy-ion scattering will be presented, with an emphasis on the physical signatures with which these ideas are associated. A number of models which we will attempt to apply to our data will be discussed. These range from time-honored standards to some very new ones which hold promise for the future of scattering studies. Chapter III describes the considerable experimental effort which was expended in gathering and reducing data. A full appreciation of the wonders of nuclear physics is surely denied one who would blithely flip through this chapter. In chapter IV the analysis of the data in terms of model
calculations and systematics is presented. This work is largely an application of the ideas given here and in chapter II. Finally, chapter V summarizes our conclusions and points the way toward further study.
CHAPTER II

GENERAL DESCRIPTION OF HEAVY ION SCATTERING

2.1 Preliminary Remarks

Collisions between heavy-ions above Coulomb barrier energies typically involve high angular momenta and wavelengths much smaller than nuclear dimensions, and, the long-range part of the heavy-ion interaction, which dominates elastic scattering at small angles, is the well-understood Coulomb interaction. Thus, it is not obviously unreasonable to expect that a classical picture in which the colliding nuclei follow well-defined trajectories under the influence of a partly-Coulombic two-body potential would have some validity in the description of heavy-ion scattering. However, it is crucial to recognize at the outset of any investigation of these phenomena that such a description explicitly ignores the composite nature of complex nuclei; objects with internal structure whose many degrees of freedom are anything but frozen. Even at near-barrier scattering energies collisions of light heavy-ions may result in a compound nucleus which is very highly excited simply by virtue of binding energy differences (e.g. 22.7 MeV for $^{30}$Si formed by $^{14}$C+$^{16}$O).

The composite nature of nuclei is responsible for the strong absorption, and hence the depletion, of elastic channel flux at higher energies which generally characterizes heavy ion interactions, and which is clearly evidenced in the figure of chapter I. Although the average effect of absorption can be modelled semi-classically using somewhat
less transparent notions such as complex trajectories and turning points in the WKB formalism (Kn74), it is clear that in detail the problem of heavy ion scattering is in its essence quantum mechanical, involving as it does the structure of nuclei. It is also true that the advantages of a quantum mechanical approach persist when we consider the near-barrier energy regime, where, even though absorption is weaker, we cannot take well-localized trajectories very seriously because of the slowing down of the colliding ions by their mutual Coulomb repulsion. In this situation, the center of mass DeBroglie wavelength for smaller impact parameters becomes comparable to the approach distance, and many partial waves contribute to the cross-section at all scattering angles. Also, the simple operational fact is that classical and semi-classical theories have generally not been so successful in reproducing heavy-ion scattering data as quantum mechanical ones. Thus, our policy here will be to pursue a number of quantum mechanical approaches in the analysis of our elastic scattering data, but before we discuss them, we will review some of the relevant classical ideas, which are in fact important for a pictorial understanding of heavy-ion scattering processes, and which give us a sort of vocabulary with which to discuss more realistic models.

2.2 Rutherford Orbits.

Let us first consider the Rutherford scattering of two ions of charges $Z_1$ and $Z_2$, (where we adopt the subscript convention 1-projectile and 2-target). For most radial distances the potential between them is well-approximated by the point-charge Coulomb potential

$$V(r_{12}) = \frac{Z_1 Z_2 e^2}{r_{12}}$$  \hspace{1cm} 2.2.1
The classical trajectory of either particle relative to the center of mass of the system is a hyperbola for which the impact parameter $b$ is related to the center of mass scattering angle $\theta$ by

$$b = \eta \cot(\theta/2)$$  \hspace{1cm} 2.2.2

and the distance of closest approach of the two particles for a given $\theta$ is

$$d = \eta \left(1 + \csc(\theta/2)\right)$$  \hspace{1cm} 2.2.3

where $\eta$ is the Sommerfeld parameter, a dimensionless number which measures the strength of the interaction, and $\kappa$ is the wave number of either particle relative to the center of mass. In terms relevant to nuclear physics, $\kappa$ and $\eta$ are easily obtained using the following expressions:

$$\kappa = \frac{1}{\hbar c} \left(2(\mu c^2)E_{cm}\right)^{1/2} \text{ (fm}^{-1}\text{)}$$  \hspace{1cm} 2.2.4

and

$$\eta = \frac{Z_1 Z_2 e^2}{\mu \hbar^2 \kappa} = Z_1 Z_2 \alpha \frac{(\mu c^2/2E_{cm})^{1/2}}{\hbar^2 \kappa}$$

with $\hbar c = 197.33$ MeV-fm, $\alpha = e^2/\hbar c = 1/137.036$, $\mu c^2 = 939$ MeV times the reduced mass $\mu$ of the system in amu, and $E_{cm}$ the total kinetic energy (in MeV) in the c.m. frame. Of course, $E_{cm} = \left(m_2/m_1 + m_1\right)E_{lab}$ and $\mu = m_2m_1/(m_2+m_1)$.

Since the nuclear interaction is of finite range, it makes sense to define a "grazing trajectory" in classical scattering. This is a trajectory whose distance of closest approach is equal to the separation of the nuclear centroids when the scattering nuclei just graze one another (as far as the strong interaction is concerned). If the grazing separation is given by the usual
\[ R = r_0 (A_1 + A_2) \]  \hspace{1cm} 2.2.5

then by setting \( d = R \) we can solve for the scattering angle of the grazing trajectory, \( \theta_R \), called the Rutherford grazing angle

\[ \theta_R = 2 \sin^{-1} \left( \frac{R \kappa / \eta - 1}{2} \right) \]  \hspace{1cm} 2.2.6

For trajectories with \( \theta < \theta_R \), the scattering is purely Rutherford, while \( \theta > \theta_R \) trajectories will bring the nuclear interactions into play (classically). We see that if \( R \kappa / \eta < 2 \), there is no \( \theta_R \); i.e. the particles' centroids never come as close to each other as \( R \). Actually, the condition \( R \kappa / \eta < 2 \) is equivalent to the condition

\[ E_{cm} < \frac{Z_1 Z_2 e^2}{R} \quad \text{(using } E_{cm} = \frac{\hbar \kappa^2}{2 \mu}). \]  \hspace{1cm} 2.2.7

This makes sense, as the RHS is just the Coulomb barrier energy, \( E_C \).

We can also define a semiclassical orbital angular momentum \( \hbar l \) corresponding to a given impact parameter \( b \):

\[ \hbar (l(l+1))^{1/2} = \hbar (l+1/2) = \kappa b \hbar \]  \hspace{1cm} 2.2.9

Thus, for a grazing trajectory, we have

\[ l_g = \frac{\kappa R (1 - 2m)}{\kappa R} - 1/2 \]  \hspace{1cm} 2.2.10

or, using 2.2.6,

\[ \frac{l_g}{\kappa R} = \kappa R (1 - 2m)^{1/2} - 1/2 \]  \hspace{1cm} 2.2.11

This is the semiclassical estimate of the grazing partial wave.

In table 2.1, we summarize, for the systems under study in this work, the values of the above classical and semiclassical parameters. The energies given represent the low and high energy limits of our experiments for each of the three systems. All of the parameters to the right of \( R \) depend on \( R \) and hence on our choice of \( r_0 = 1.50 \text{ fm} \). This radius constant gives \( l_g \) values which are reasonably close to those deduced from optical model calculations, but the nuclear radius does
TABLE 2.1

Interaction Parameters for the Systems under Study

Energies are in MeV, Distance units are fm or fm⁻¹, and we take $r_0 = 1.50$ fm.

<table>
<thead>
<tr>
<th>SYSTEM</th>
<th>$E_{14C}$</th>
<th>$E_{cm}$</th>
<th>$\eta$</th>
<th>$\kappa$</th>
<th>$\lambda$</th>
<th>$d_0$</th>
<th>$R$</th>
<th>$\theta_R$</th>
<th>$V_c$</th>
<th>$\xi_g$</th>
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</thead>
<tbody>
<tr>
<td>$^{14}_C + ^{12}_C$</td>
<td>20.0</td>
<td>9.23</td>
<td>4.74</td>
<td>1.69</td>
<td>3.72</td>
<td>5.62</td>
<td>7.05</td>
<td>83°</td>
<td>7.35</td>
<td>4.9</td>
</tr>
<tr>
<td>$^{14}_C + ^{12}_C$</td>
<td>40.3</td>
<td>18.6</td>
<td>3.34</td>
<td>2.40</td>
<td>2.62</td>
<td>2.79</td>
<td>7.05</td>
<td>29°</td>
<td>7.35</td>
<td>12.6</td>
</tr>
<tr>
<td>$^{14}_C + ^{16}_O$</td>
<td>20.0</td>
<td>10.67</td>
<td>6.32</td>
<td>1.95</td>
<td>3.22</td>
<td>6.48</td>
<td>7.40</td>
<td>103°</td>
<td>9.35</td>
<td>4.6</td>
</tr>
<tr>
<td>$^{14}_C + ^{16}_O$</td>
<td>40.3</td>
<td>21.49</td>
<td>4.46</td>
<td>2.77</td>
<td>2.27</td>
<td>3.22</td>
<td>7.40</td>
<td>32°</td>
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<td>14.9</td>
</tr>
<tr>
<td>$^{14}_C + ^{18}_O$</td>
<td>20.0</td>
<td>11.25</td>
<td>6.32</td>
<td>2.06</td>
<td>3.05</td>
<td>6.14</td>
<td>8.55</td>
<td>87°</td>
<td>9.16</td>
<td>6.2</td>
</tr>
<tr>
<td>$^{14}_C + ^{18}_O$</td>
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<td>18.28</td>
<td>4.96</td>
<td>2.62</td>
<td>2.39</td>
<td>3.78</td>
<td>8.55</td>
<td>39°</td>
<td>9.16</td>
<td>13.5</td>
</tr>
</tbody>
</table>
depend on the way in which it is measured. Our point of view is that $r_0$
should be such that the strong interaction radii of the scattering
nuclei overlap slightly, and this gives a value which is somewhat higher
than nuclear radii usually determined from, say, electron scattering.
Values for $r_0$ up to 1.68 fm have been found in some studies (Ch73).

2.3 Classical Orbits More Generally.

Cross-sections for potentials other than the Coulomb one are easily
calculated classically. Unfortunately, but, perhaps not surprisingly,
such calculations have not been successful in reproducing detailed
heavy-ion scattering data very well. However, there is still a qualita-
tive, pictorial value in the classical treatment which can be useful if
we do not take it too seriously. With this in mind we will consider now
a two-particle system with reduced mass $\mu$, center of mass energy $E$, and
impact parameter $b$, which is governed by a classical central potential
interaction $V(r)$.

The relationship between the scattering angle and the impact
parameter is called the classical deflection function, denoted by $\theta(b)$,
and is given by

$$\theta(b) = \pi - 2 \int_{r_{\text{crit}}}^{\infty} \frac{b}{r^2} \left[ 1 - \frac{V(r)}{E} - \frac{b}{r^2} \right] \frac{dV}{dr}, \quad 2.3.1$$

where $r_{\text{crit}}$ is the largest value of $r$ for which the expression in the
integrand vanishes (this is the distance of closest approach - the
classical turning point). The deflection function may in general have any value between \(-\infty\) and \(\pi\), unlike the physical scattering angle \(\theta\), which is confined between 0 and \(\pi\). The relation between them can be expressed as

\[ \theta(b) = a\theta - 2\pi n \]  

where \(a = \pm 1\) and \(n\) is a positive integer or zero. For a given value of \(\theta\), the value of \(a\) and \(n\) are chosen such that \(0 < \theta < \pi\).

If the force between the particles is always repulsive (so that \(V(r)\) decreases monotonically) then the integral in 2.3.1 is never larger than \(\pi/2\) and \(0 < \theta < \pi\) as we would expect. However, if \(V(r)\) does not decrease monotonically, the integral can have any positive value and \(\theta\) can therefore have any negative value. Physically, this arises from the orbiting which may occur (for suitable \(E\) and \(b\) that is) when the potential gives rise to attractive forces for some range in \(r\). A negative deflection function corresponds to an attractive potential, with \(n\) in eqn. 2.3.2 giving the number of orbits which occur during the scattering of the two particles. We see then that for an arbitrary potential \(V(r)\) there can be many impact parameters, or, equivalently, angular momenta (since \(L=bp\)) which lead to the same physical scattering angle.

The deflection function is useful in obtaining scattering cross-sections for classical interactions. Let \(N\) be the number of beam particles per unit area incident on a target during a fixed period of time. All of the beam particles within an element of area \(bd\phi\) (where \(\phi\) is just the azimuthal angle around the beam direction) are scattered in the direction \((\theta,\phi)\) into a cone of solid angle \(d\phi d\theta \sin \theta\). The number of these particles is just \(Nbd\phi\). However, this number is also \(N(d\sigma/d\Omega)d\Omega\) by the definition of \(d\sigma/d\Omega\), so since \(\Omega=d\phi d\theta \sin \theta\), we have that
If \( b(\theta) \), the inverse of the deflection function, is multi-valued (as is the situation when orbiting occurs) then this expression becomes a sum (with a term for each value of \( b \)). It is also interesting to note that 2.3.3 and 2.3.1 imply that all potentials of infinite range give divergent classical cross-sections at \( \theta=0 \) (Ne66).

The Rutherford scattering cross-section is easily obtained from 2.3.3 using the deflection function we already know from 2.2.2. It is just

\[
\frac{d\sigma}{d\Omega} = \frac{n^2 \csc^4 \theta/2}{4\kappa^2} = \left( \frac{Z_1 Z_2 e^2}{4E_{\text{cm}}} \right)^2 \frac{1}{\sin^4 \theta/2},
\]

the same as the quantum mechanical result. A short-range attractive potential meant to simulate the nuclear force classically can also be added, the deflection function obtained and the cross-section evaluated in a similar manner. Fig. 2.1a illustrates schematically the form of a typical potential

\[
V(r) = Z_1 Z_2 e^2 + V_N
\]

with \( V_N \) some tens of MeV deep and something like a Woods-Saxon function in shape. Clearly the attractive interior region results in a large modification of the deflection function from Coulomb scattering. Classical trajectories for potentials of this type are illustrated in Figs. 2.1b and 2.1c. We see that for large \( b \) the scattering is essentially pure Rutherford, but with closer approach ("grazing trajectories") the orbit is pulled inward by the nuclear force. If the impact parameter is smaller and the kinetic energy less than a certain critical value, the system can find itself with an inter-nuclear radius
such that Coulomb and nuclear forces combine to provide just the centrifugal force necessary for nearly circular motion; this causes "spiral" or "orbiting". If b is smaller still, the nuclear attraction may be sufficient to pull the orbit directly to a negative deflection (a "plunging trajectory"). In Fig. 2.1b we see three trajectories of different impact parameters for which the same scattering angle results. We see that trajectory 3 has a negative deflection (so $a = -1$ in 2.3.2). Such trajectories are often referred to as "far side" trajectories, while those which experience positive deflection are termed "near side". All three trajectories in the figure contribute to the cross-section at the physical scattering angle $\theta$. The deflection function for this "typical" potential is shown in fig. 2.2.

The impact parameters $b_1$, $b_2$, and $b_3$ in figure 2.2 correspond to the trajectories of fig. 2.1b. When $b=b_x$ (again referring to fig 2.2), $d\theta/db=0$, and $d\sigma/d\Omega$ is singular according to 2.3.3. This condition leads to "rainbow scattering" (in analogy with optical rainbows) and this particular one is sometimes referred to as the Coulomb rainbow. At $b=b_y$, $\sin\theta$ vanishes so that $d\sigma/d\Omega$ is again singular, and "glory scattering" results. Forward glory scattering is lost in the Coulomb divergence at zero scattering angle, but $\theta=\pi$ glory effects are in principle visible (glories from sunlight on water mist are in fact well known in meteorology). The singularity in $\theta$ at $b_0$ corresponds to orbiting. If the kinetic energy is too large for orbiting, the singularity at $b_0$ becomes simply a minimum, and this turning point gives rise to a second rainbow (sometimes called the "nuclear rainbow"). In heavy-ion scattering the
FIGURE 2.1 a) Schematic drawing of a classical approximation to the nuclear potential $V(r) = \frac{Z_1 Z_2 e^2}{r} + V_N$.

FIGURE 2.1 b) Classical scattering trajectories for three different impact parameters when the potential has the form of a) above. The dashed lines indicate pure Rutherford scattering trajectories. Note that the final scattering angle is the same for all three trajectories. Trajectory 3 originates on the "far-side" of the nucleus.

FIGURE 2.1 c) The same as b), but with impact parameters chosen sequentially smaller so as to demonstrate the transition from "skimming" to "orbiting" to "plunging" trajectories. The "plunging" (innermost) trajectory is actually a far-side trajectory because of its negative deflection angle.
FIGURE 2.2 The deflection function for a potential of the form depicted in fig. 2.1a. Impact parameters $b_1$, $b_2$, and $b_3$ correspond to the three trajectories of fig. 2.1b.
wavelike nature of the particles has the effect of "smearing out" classical trajectories and removing singularities in $\Theta$. Some authors have cited evidence for this kind of wave-modified nuclear rainbow (Go74), Coulomb rainbow (Da73), and glory (Br68) scattering in nuclear systems; especially, but not exclusively, in systems involving alpha-particle projectiles. However, others, notably Frahn (Fr74), and separately, Fuller (Fu75) have concluded that these effects are generally not relevant to heavy-ion scattering; the physical argument being that plunging trajectories require large transparency for interior partial waves (small impact parameters) if they are to survive, and this is a condition which colliding heavy ions do not respect. The consensus today seems to be that the general features of scattering cross-sections are better understood in terms of diffraction theories, where the crucial effect of absorptive modification of the Coulomb scattering is taken into account.

Semiclassical calculations of the deflection function using optical potentials have generally found that the negative branch of $\Theta(b)$ disappears when absorption is suitably taken into account (Ha74); the more interesting classical effects then simply vanish. This effect has been associated (Ge74) with the reflection of flux by the strongly absorbing potential which is required in quantum mechanical optical models to fit scattering data. This is not inconsistent, since in quantum scattering the unitarity of the $S$-matrix demands that there be no absorption without reflection as well. Therefore, the net potential $V_N$ becomes only weakly attractive and the deflection function does not develop the deep pockets which are necessary if rainbow and glory effects are to manifest themselves.
2.4 Diffraction Scattering.

Diffraction theories for the scattering of complex nuclei have distinct physical advantages over classical scattering ones in that they provide a wave-mechanical description which includes the strong absorption known to characterize these processes. In this picture, the target nucleus is imagined as a perfectly absorbing sphere which diffracts the projectile's waves at its surface. The Coulomb field can be thought of as a diverging lens of focal length $b_R \cot \theta_R$ (Fr72) which refracts the incoming waves and causes the illumination of the diffracting black nucleus by what is effectively a virtual point source, as fig. 2.3 illustrates. The general condition for diffraction is that the grazing angular momentum parameter be greater than one:

$$\Lambda = \frac{L}{\hbar} = \left( \frac{\ell (\ell + 1)}{\ell + 1} \right)^{1/2} = \kappa b_R$$  \hspace{1cm} 2.4.1

If the coulomb interaction is quite strong, the distance of the effective point source from the target nucleus is fairly short, giving rise to a Fresnel diffraction pattern. A relatively weak Coulomb field, on the other hand, results in Fraunhofer scattering, where the source of illumination may be regarded as infinitely far away (in fact, this is clearly the Coulomb-free limit). Quantitatively, we may define a pattern parameter $p$ in analogy with classical optics such that

$$p << 1 \text{ means Fraunhofer scattering and }$$  \hspace{1cm} 2.4.2

$$p \geq 1 \text{ means Fresnel scattering.}$$

Classically, $p = \kappa a^2 / d$, where "a" is the radius of the absorber, "d" is the distance of the source from the absorber, and $\kappa a^2$ is the Rayleigh length; the length of the absorber's shadow. For nuclear scattering then, we may take
FIGURE 2.3 This diagram illustrates the effect of the Coulomb field of the nucleus in simulating that of a diverging lens of focal length $R \cot \theta_R$ on scattered particle trajectories.
It is relevant now to return briefly to the question of whether or not classical trajectories are appropriate for heavy-ion scattering. In order for classical motion to obtain, the wavepacket which describes a particle's motion must contain many wavelengths and must be small compared with its distance of closest approach to the scattering center (otherwise it will spread dramatically and unclassically upon scattering). The condition is that \( \lambda \ll d_0 \), which is \( \frac{2\pi}{\kappa} \ll \frac{2\pi}{\kappa} \) using 2.2.3, or just

\[ \eta \gg \pi \]

2.4.4

At energies above the barrier we know we will get absorption; hence simple classical motion is not possible. It is, however, not unreasonable to consider a picture involving "absorbed classical trajectories" as long as

\[ \lambda \ll R \text{ or equivalently, } \eta H \gg \pi \]

2.4.5

where

\[ H = \frac{E}{E_C} = \frac{\kappa R}{2\eta} \]

2.4.6

It is possible to obtain a very useful overview of nuclear scattering by constructing a diagram in which \( \eta \) is plotted against \( H \) for various systems and energies. The resulting "diffraction diagram" is divided into a number of regions defined by the conditions 2.4.1, 2.4.2, and 2.4.4. The relations between the diffraction parameters \( \Lambda \) and \( p \) and the diagram variables \( \eta \) and \( H \) are

\[ \Lambda = 2\eta \left( H(H - 1) \right)^{\frac{1}{2}} \]

2.4.7

and

\[ p = 2\eta \left( 1 - (2H - 1)^{-2} \right) \]

Note that \( p \) and \( \Lambda \) go to zero as \( H \geq 1 \) and only make physical sense when \( H > 1 \). Fig. 2.4 shows the different regions of the diffraction diagram,
FIGURE 2.4 The diffraction diagram and its zones. Here log\(\eta\) is plotted against log\(H\). The hatched line which runs horizontally to the left limit is from the sub-barrier semi-classical condition 2.4.4. The other hatched line borders come from 2.4.1 and 2.4.2. All \(p\)-constant and \(\Lambda\)-constant lines come together and climb nearly vertically when log\(H\) approaches zero. The dotted line \(\eta=\alpha\) indicates the lower limit for relativistic charged-particle scattering. (from Frahn 1975).
SEMICALSSICAL
NON-DIFFRACTIVE SCATTERING

FRESNEL
DIFFRACTIVE SCATTERING

QUANTAL
NON-DIFFRACTIVE
\[ \Pi = \alpha \gamma \] Scattering

TRANSITION REGION

FRAUNHOFER
DIFFRACTIVE SCATTERING

below above
Coulomb barrier

\( \log \Pi \)

\( \log H \)
FIGURE 2.5 The diffraction diagram divided into areas appropriate for heavy-ion experiments. The triangular regions correspond to lab energies of 1, 3, and 10 MeV per nucleon with the lower right corner representing $^6\text{Li}+^6\text{Li}$, the upper left corner $^{238}\text{U}+^{238}\text{U}$ and the lower left $^6\text{Li}+^{238}\text{U}$. 
and in fig. 2.5 we see where most heavy-ion scattering experiments fall. As indicated in fig. 2.5, the experiments discussed in this work fall in a region which is perhaps best described as "extremely transitional".

The Fresnel diffraction theory is capable of explaining a number of salient features of heavy-ion scattering in the region of the diffraction diagram where it is applicable. The simplest theory of this kind (Fr71) assumes a sharp-edged, black nucleus and yields cross-sections of the form

\[ \frac{d\sigma}{d\Omega}(\theta) = \frac{1}{2} \int_{0}^{\infty} \frac{dD}{d\Omega}(\theta, \omega) \left[ \frac{1}{2} - C(\omega)^2 + \frac{1}{2} - S(\omega)^2 \right] d\omega \]

where C and S are the Fresnel integrals and \( \omega \) is given by

\[ \omega = \left( \frac{\theta - \theta_R}{\Lambda / \pi \sin \theta_R} \right)^{1/2} \]

\( \theta_R \) is the Rutherford grazing angle defined in 2.2.6. This oversimplified model has only one parameter, an angular momentum cutoff \( \Lambda \), but is still able to describe qualitatively the forward-angle scattering in many heavy ion systems; especially ones involving projectiles and targets very asymmetric in mass. In fig. 2.6 we see a very typical Fresnel-diffractive cross-section. More sophisticated diffractive models which include the effects of surface diffuseness and real nuclear phase shifts have even been able to reproduce some heavy-ion cross-sections quantitatively (Va76, for example). The Strong-Absorption Model (SAM) of Frahn and Venter, its generalizations, and further work by Fuller are good examples (and are excellently covered in Fr84 and Fu75). These models involved a parametrization of the nuclear S-matrix by a cleverly chosen function of a continuous angular momentum variable (analogous to \( \ell + 1/2 \)) such that expressions for the cross-section could be obtained in a closed form. These expressions directly display the diffractive nature.
FIGURE 2.6  Simple Fresnel diffraction model applied to forward angle scattering of $^{16}\text{O}$ on $^{207}\text{Pb}$ at $E_{\text{lab}}=166$ MeV. $\Lambda=86.5$, $\eta=32.05$, $\theta_R=40.9^\circ$ (data Ke62, calculation Fr66). The Rutherford grazing, $\theta_R$, is indicated.
of the scattering process. We will not consider them in detail here because of the simple fact that no such model has proved to be adequate for the description of the more interesting heavy-ion systems - those which exhibit rapidly changing cross-section structure as a function of energy such as the $^{14}_C^{12}_C$ and $^{14}_C^{16}_O$ systems under study in this work. However, it is very much worth summarizing the physical traits which diffractive models have in common, as they constitute a reasonable first approximation to the general features of heavy-ion scattering:

(i) The cross-sections have a "quarter-point" property, i.e.

$$\frac{d\sigma(\theta)}{d\Omega} \frac{d\sigma_R(\theta)}{d\Omega} = 1 \text{ at } \theta = \theta_R$$

(ii) In the "illuminated region" ($\theta < \theta_R$), the cross-section experiences an increase above the Rutherford value which can go as high as 50% for $\theta$ around $\theta_R - (3 \sin \theta_R/\Lambda)$.

(iii) In the "shadow region" ($\theta > \theta_R$), the cross-section compared with Rutherford scattering decreases by two or more orders of magnitude, and

(iv) From more detailed models such as the SAM, highly oscillatory and periodic structure in the shadow region can be obtained as an interference effect between near and far-side diffracted waves. This constitutes a modulation of the Fresnel-diffraction "envelope" by regular Fraunhofer-type oscillations which strongly calls to mind the example of two-slit diffraction familiar to us in classical optics.
2.5 S-Matrix Approaches.

Quantum mechanical descriptions of heavy-ion interactions have generally followed one of two different paths. One tries to find a scattering amplitude either by solving the Schrodinger equation with a microscopically or phenomenologically determined potential, or proceeds directly from a model-generated or empirically derived S-matrix to calculate the amplitude in a partial-wave-expanded form. The two paths are not equivalent to the extent that the general inverse scattering problem (i.e. given the scattering amplitude, find the potential) is not exactly solvable. Each method has its advantages. For example, the potential approach is in some sense more "satisfying" because one feels that the potential confers a physical understanding of the interaction responsible for the scattering. On the other hand, it is important to keep in mind that there is fundamental concern about the validity of any nuclear model which relies on the collapsing of the complex many-body interaction into a mere two-body potential, even when that "effective potential" is determined microscopically, or when many potentials are coupled in a multi-channel analysis.

Thus, one turns to methods such as phase-shift analyses or parametrized S-matrix models in order to bypass the sticky problem of finding effective potentials which reproduce data. At least the S-matrix gives us what is, in principle, a "complete" picture of scattering in that it summarizes, in terms of the asymptotic wave functions, all of the information which is experimentally known to us (if it yields the correct cross-section that is) and has, in its \( \ell \)-basis representation, a fairly clear physical interpretation. Of course, the numerical problem of determining even the S-matrix from the cross-section is fraught with
ambiguities. Also it must be recognized that, finally, a thorough understanding of any process does require knowledge of the underlying interactions (unless we prefer to believe in magic!), and, in typical heavy-ion scattering problems, such knowledge may not be easy to extract from the S-matrix itself. Nevertheless, no one can doubt that S-matrix approaches have added greatly to our understanding of heavy-ion interactions; so, bearing these warnings in mind, we now consider the S-matrix formalism in some detail.

Formally, one defines the S-matrix as the operator

$$ S = \lim_{t_f \to \infty} U(t_f, t_i), \quad \text{where } U(t_f, t_i) \text{ is the propagator} $$

corresponding to the Hamiltonian, $H$, for a given scattering problem, and $i$ and $f$ represent all the quantum numbers of the initial and final states, respectively. The probability that an initial state $|\psi_i>$ develops into the final state $|\psi_f>$ under the influence of $H$ is just $|\langle \psi_f | S | \psi_i \rangle|^2$. Since angular momentum and energy are conserved during the scattering, $H$ is diagonal in the common eigenbasis of $E$ and $J$, and so is $S$, as it is a function of $H$. If all particles are spinless, the orbital angular momentum, $\ell$, is also conserved, and so we may label $S_{fi}$ with $\ell$ and $E$ explicitly.

The cross-section for scattering from $|\psi_i>$ into $|\psi_f>$ is given by

$$ \frac{d\sigma_{if}(\theta, E)}{d\Omega} = \frac{v_f}{v_i} |f_{fi}(\theta, E)|^2 $$

where $v_i$ and $v_f$ are the initial and final state asymptotic relative velocities, $\theta$ is the center of mass scattering angle, $E$ is the center of mass energy and $f_{fi}(\theta, E)$ is the scattering amplitude, given by the asymptotic form of the outgoing scattered wave
(we have assumed here a two-body final state with the scattering caused by a central potential \( V(r) \)). The representation of \( |\psi_f> \) in the \( l \)-basis is the familiar partial wave expansion in terms of Legendre polynomials, which results in the identification

\[
f_{fi}(\theta,E) = \frac{i\kappa_f^z}{2\kappa_f} \left(\frac{v_f}{v_i}\right)^\frac{1}{2} \sum_{l} \frac{(2l+1)}{(2\pi^2)} (\delta_{fi} - S_{fi}(E)) P_l(\cos \theta) \]

(with the restriction that no particles have intrinsic spin).

Since the Coulomb interaction is always present in heavy-ion scattering it is useful to include it in the formalism explicitly. Of course, the Coulomb potential contributes a complication to the scattering problem because it does not vanish faster than \( 1/r \). This means that charged particles are never free from the influence of their mutual interaction, and so, even at large \( r \), solutions to the Schrodinger equation require distorted waves and 2.5.2 must be modified accordingly.

\[
|\psi_f> = \delta_{fi} \psi_{\text{distorted plane wave}} + f_{fi} \psi_{\text{distorted spherical wave}}
\]

The distorted spherical waves which multiply \( f_{fi} \) are the radial part of the asymptotic Coulomb wave functions:

\[
\frac{1}{r} \exp(\kappa_f r - \eta_f \ln 2\kappa_f r)
\]

with \( \eta_f \) the Sommerfeld parameter for the state \( f \). A partial wave expansion of these "not-quite-free-particle" wave functions results in the same identification of \( f_{fi}(\theta,E) \) as in 2.5.3; however, we may cast it in a somewhat more useful form by factoring the S-matrix into a Coulomb part, \( S_C \), and a "nuclear" part, \( S_N \), where "nuclear" refers to anything other than point-charge Coulomb interactions. In order to do this, we...
note that the solution of the Schrodinger equation for pure Coulomb scattering gives the S-matrix

\[ S_{C, \ell}(E) = e^{2i\sigma_\ell} = \frac{\Gamma(\ell + 1 + i\eta)}{\Gamma(\ell + 1 - i\eta)} \]

where the \( \sigma_\ell = \text{arg}\Gamma(\ell+1+i\eta) \) are the Coulomb phase shifts (they are zero for \( \eta=0 \) as we would expect). The defining relation for \( S_N \) is then

\[ S_{fi, \ell}(E) = S_{Nfi, \ell}(E) \cdot S_{Cf, \ell}(E) = S_{Nfi, \ell}(E) e^{2i\sigma_\ell} \]

with \( \sigma_\ell \) evaluated using \( \eta_f \) - the Sommerfeld parameter appropriate for the scattered waves.

If we introduce 2.5.6 into 2.5.3 and factor out \( e^{2i\sigma_\ell} \), the sum over \( \ell \) breaks into two terms:

\[ f_{fi}(\theta, E) = \frac{1}{2\kappa_i} \left( \frac{\eta_f}{\eta_f} \right) \sqrt{2(\ell + 1)} \sum_{\ell} P_\ell(\cos\theta)[(\delta_{fi} - S_{Nfi, \ell}(E))] e^{2i\sigma_\ell} \]

However, from 2.5.3 applied to pure Coulomb scattering we see that the second term is simply the Coulomb scattering amplitude, for which the sum over \( \ell \) can be evaluated in closed form:

\[ f_{Cf}(\theta, E) = -\frac{2i\eta_f}{2\kappa_i \sin^2\theta/2} \]

(Here \( \kappa = \kappa_i \) and from now on variables without subscripts refer to the initial state). Thus, using 2.5.1 we may write the cross-section in terms of the nuclear S-matrix as

\[ \frac{d\sigma_{fi}(\theta, E)}{d\Omega} = |\delta_{fi} f_{Cf}(\theta, E) + 1 \sum_{\ell}(2\ell + 1) e^{2i\sigma_\ell} (\delta_{fi} - S_{Nfi, \ell}(E)) P_\ell(\cos\theta)|^2 \]

By using the orthogonality of the \( P_\ell \)'s we can also obtain a simple expression for the individual angle-integrated reaction cross-sections:
\[
\sigma_{fi}(E) = \int \frac{d\sigma_{fi}(E) d\Omega}{d\Omega} = \pi \Sigma (2l+1) \frac{\delta_{fi} - S_{Nfi,l}}{\kappa^2}.
\]
2.5.10

If \( i=f \), this is valid only when \( \eta=0 \) because of the divergence of \( f_c(\theta) \) at \( \theta=0 \) in charged-particle elastic scattering. This expression exhibits the very noteworthy property that the contributions of different partial waves to the total cross-section do not interfere with each other — a comforting conclusion in view of angular momentum conservation.

The unitarity of the S-matrix follows from conservation of probability; a condition which of course a propagator must satisfy. Thus 
\[
\Sigma S \Sigma^{\dagger} = \delta_{\alpha\beta}.
\]
Since \( S \) is diagonal in \( E \) and \( l \) (if all particles are spinless) this implies that
\[
\Sigma_f |S_{fi,l}(E)|^2 = 1.
\]
2.5.11
For convenience, let us from now on write simply \( S \) for the elastic \( S \)-matrix, \( S_{ii,l}(E) \). We may bring \( S \) outside of the sum in 2.5.11 and use 2.5.10 to conclude that the total \( l \)-wave reaction cross-section,
\[
\sigma_r,l(E) = \Sigma_f \sigma_{fi,l}(E),
\]
is given by
\[
\sigma_r,l(E) = \pi \frac{(2l+1)}{\kappa^2} (1 - |S|_l^2).
\]
2.5.12
We may define the "reflection coefficient" \( \eta \) and the "nuclear" phase shift \( \delta \) for elastic scattering with the relations
\[
\eta \equiv |S_{N,l}| \quad \text{and} \quad \eta \equiv \frac{2i\delta}{2\sigma},
\]
2.5.13
Since \( S \equiv S_{N,l} \), we see that \( \eta = |S|_l \) also. Now, solving 2.5.12 for \( \eta \), we find that
\[ \eta_l = \left[ 1 - \kappa^2 \sigma_{x, l} / \pi (2l+1) \right]^{1/2} \] 2.5.14

or, using 2.5.10

\[ \eta_l = \left[ 1 - \Sigma \left| S_{Nf, l} \right|^2 \right]^{1/2} \]

These relations, which are easily generalized for scattering with spin, demonstrate that the reflection coefficient for each partial wave decreases whenever any reaction cross-section experiences an increase. This has a profound effect on the elastic scattering of heavy ions, since by 2.5.9 the cross-section is given by a polynomial in \( \eta_l \),

\[ \frac{d\sigma_{\ell}}{d\Omega} = \left| f_C(\theta,E) + i \Sigma (2\ell+1) \sqrt{1 - \eta_l^2} e^{2i\ell} P_\ell(\cos \theta) \right|^2, \] 2.5.15

and we know that numerous reaction channels open when the Coulomb barrier is surmounted.

This demonstrates that elastic and nonelastic processes in quantum scattering are intimately linked by the unitarity of the \( S \)-matrix; each one is in a sense the "shadow" of the other. If \( \eta_l \) is much less than unity then that \( \ell \)-wave is "strongly absorbed" and flux appears in \( \sigma_{x, l} \).

At the same time, a term with a large coefficient is contributed to the partial wave sum in the elastic scattering amplitude (this "shadow scattering" is very directly exhibited when there is no \( f_C \) in the problem and we can integrate \( d\sigma/d\Omega \) for the total cross-section). On the other hand, reflected partial waves, for which \( \eta_l = 1 \), do not contribute to the reaction cross-section at all, while there may still be a nuclear contribution to the elastic scattering by virtue of a nonzero phase shift \( \delta_\ell \).
2.6 Parametrizations of the S-matrix: simple models.

The essential feature of absorption of the lower partial waves in heavy-ion elastic scattering was recognized almost as soon as data became available in the middle-1950's, with the scattering of $^{14}\text{N}+^{14}\text{N}$ (Re56). Blair had proposed (Bl54) an S-matrix model for alpha-particle on nucleus scattering which, though it made the rather radical assumption that $S_{N,\ell}$ was simply zero for $\ell<\ell_{g}$ and unity otherwise, proved useful in the analysis of these early data. It clearly embodied, in the crudest possible form, the strong absorption which is characteristic of the nucleus-nucleus S-matrix, and which all subsequent descriptions have retained in one form or another.

For scattering angles smaller than $\theta_{R}$, this "sharp-cutoff" model reproduces quite well some of the gross features of heavy-ion scattering data which were mentioned earlier in connection with diffraction scattering. In particular, sharp-cutoff cross-sections generally have the quarter-point property - to a good approximation - and exhibit Fresnel-like oscillations above and below the Rutherford value when $\theta<\theta_{R}$, including a distinct rise before the rapid descent begins. Generally, it was found by adjusting $\ell_{g}$ to get agreement with forward-angle data that the model gave consistent predictions for nuclear interaction radii via semiclassical prescriptions such as 2.2.10. Zucker (Zu60), for example, reported a value for $r_{0}$ of 1.46 fm for systems involving heavy targets using this method. However, the sharp-cutoff picture turned out not to be very realistic for large-angle scattering, as typical calculated cross-sections have periodic, unphysically large, oscillatory structures, and they do not show, on average, a rapid enough decline just beyond the grazing angle to be consistent with measurements. It is also
interesting to note, however, that cross-sections well in excess of the Rutherford value are sometimes found for extreme large-angle scattering in this model, as we will soon see, while more realistic models are often hard-pressed to reproduce the strong "backward rise" which is sometimes seen in the data. We will return to discuss some interesting aspects of the regular backward-hemisphere structure in the Blair model in the next section.

Following shortly after the Blair model, somewhat more refined and flexible S-matrix parameterizations began to appear. A five-parameter version which can easily reproduce the diffractive features of "normal" angular distributions was proposed by McIntyre (Mc69). In this model, the nuclear S-matrix is given by

$$\eta_\ell = \left[ 1 + \exp\left( \frac{\ell - \ell_0}{\Lambda} \right) \right]^{-1}$$
$$\delta_\ell = \delta_0 \left[ 1 + \exp\left( \frac{\ell - \ell_p}{\Lambda} \right) \right]^{-1}$$

2.6.1

Both the modulus of $S$ and its phase are Woods-Saxon in terms of functional dependence on $\ell$. Models such as this one, in which $\eta_\ell$ smoothly changes from zero to one in the vicinity of $\ell_0$, generally go under the heading of "smooth-cutoff models", and the "strong absorption profile" of $\eta_\ell$ which they possess has turned out to be a general characteristic of all successful heavy-ion scattering models. In fig. 2.7 we compare a smooth-cutoff calculation with data on $^{16}\text{O} + ^{28}\text{Si}$ at 26.2 MeV (cm) of Braun-Munzinger, Gai et al. (Ga86). This system is well known for its "anomalous" large-angle oscillations, but the remainder of the angular distribution is fairly exemplary of "normal" diffractive scattering. The calculation represents a compromise in that we have sacrificed reproduction of the very deep minimum at $120^0$ in order to come within an order of magnitude of the last two (relatively) enormous
FIGURE 2.7 McIntyre smooth cutoff calculation compared with $^{16}_{\text{O}} + ^{28}_{\text{Si}}$ data of Braun-Munzinger (Ga86) at 26.2 MeV (cm).

$l_g - l_p = 15.9$, $\Delta - \Delta_p = .75$, and $\delta_0 = 90^0$. 
26.2 MeV $^{16}$O + $^{28}$Si
maxima. The model parameters here are $l_g = l_p = 15.9$, $\Delta_p = .75$, and $\delta_0 = 90$. That they result from a fit "by hand" demonstrates an important advantage inherent in the $S$-matrix models (even if the fit here is not so spectacular): namely, that computation time is much shorter than in potential-based models, where the Schrodinger equation must be integrated numerically, making fitting more time-consuming and difficult.

In the figure we see that the general Fresnel pattern, including the steep decline beyond $\theta_R$, emerges naturally, and the Fraunhofer modulation seems to follow the pattern of oscillations in the large-angle region, while it misses the smaller structures in the middle range. It is not possible to change the model parameters from the quoted values very much without adversely affecting the fit. The parameter $l_g$ is apparently the most important of them; the position of the main Fresnel maximum and the period of the Fraunhofer oscillations depend primarily on it alone. On the other hand, $l_p$ has to "follow along" more or less with $l_g$ or the cross-section either becomes too large at medium angles ($l_p < l_g$) or develops bizarre structures ($l_p > l_g$).

The diffuseness also are fairly well tied together in that small values of either $\Delta$ or $\Delta_p$ result in large sharp-cutoff-like oscillations at intermediate angles. If they are taken much larger than .75, the oscillations largely damp out and the average cross-section becomes too small at very large angles (although this would be acceptable for a more "normal" system than $^{16}O + ^{28}Si$). The nuclear-phase scale-parameter $\delta_0$ also affects the size of the oscillations and the average level of the cross-section for $\theta > \theta_R$, but a larger $\delta_0$ gives larger oscillations and a generally lower cross-section, while $\delta_0 = 0$ results in almost no
oscillations and too large an average cross-section if the A's are also large (if not, then we simply get back the sharp-cutoff model). In this example, we were perhaps fortunate to find that the value of \( \ell \) which reproduced the pattern of backward-angle oscillations also happened to locate the Fresnel maximum in roughly (though perhaps not exactly) the right place. We note that in fact \( \eta = 11.0 \) for this system at 26.2 MeV (cm) and so, choosing \( r_o = 1.50 \) fm, we find that, semiclassically, \( L = 15.2 \) and \( \theta_R = 72^\circ \); which is about 10° too high relative to the quarter point prescription.

The \(^{16}O^{28}\)Si system is a difficult one with which to begin a study of heavy-ion scattering, but it is a useful example for two reasons. First, it illustrates the sort of striking behavior which is often seen in low-energy, lighter heavy-ion elastic scattering and which, not withstanding extensive experimental and theoretical work over the years (as evidenced, e.g. in Sh83), is still not ready to be catalogued in that file of "well understood phenomena". Second, our simple exercise with the McIntyre model is illustrative of a point which proves to be valid after a much more general analysis: namely, that the unusually large and oscillatory backward angle cross-sections sometimes seen in these systems represent a departure from smooth strong-absorption behavior. This is not to say that the strong-absorption profile of the S-matrix is catastrophically altered, but rather that it experiences modifications in the form of kinks or spikes near \( \ell \) or excursions away from zero for somewhat lower partial waves, as the energy-dependent opening or closing of real and virtual reaction channels, or the occurrence of resonances, results in increased absorption or reflection.
for a given partial wave. Realistic heavy-ion scattering models must attempt somehow to take these all-important effects into account.

2.7 A Note on the Qualitative Form of Angular Distributions.

Since angular distributions for the scattering of heavy-ions sometimes exhibit beautiful but complicated and highly energy-dependent structure, it is natural to ask whether a merely qualitative survey of the most obvious features of these "information-rich" quantities can lead us to any reasonably sound conclusions about the system in question. What, for example, does the number and placement of local minima and maxima tell us? Such questions about the information content of scattering measurements are especially relevant when we attempt to transcend the model-imposed limitations of simple S-matrix parametrizations or the potential approach, and use the full power of the S-matrix method in phase-shift analysis, where we may perform partly constrained or even completely unconstrained searches in phase-shift space in order to fit data.

In quasi-elastic processes, the analog of the strong absorption profile in the elastic scattering S-matrix is the clustering of the non zero \( |S_{f1,2}| \) in a fairly narrow "window" in \( l \) centering on the grazing value. Lower partial waves go to more drastic nuclear rearrangements such as the formation of a compound nucleus, while higher partial waves are beyond the reach of the nuclear interaction and do not contribute either. Taken with the absence of interfering Coulomb amplitude here, this leads via 2.5.9 to an expression for \( d\sigma/d\Omega \) which may be dominated by a single \( P_{2}^{2} \), thus furnishing us with the interpretation that the
grazing $l$-value may be determined merely by comparing the cross-section with a particular $P^2_l$.

However, the situation for elastic scattering is much more complicated. Referring to 2.5.15 and assuming strong absorption for $S^l$, we see that in addition to $f^l_{C}$, we must also include in the scattering amplitude all the $P^2_l$'s with $l \leq l_g$ (i.e. where $\eta^l_0 < 1$). Thus it is difficult to see by inspection how any correspondence between $l_g$ or $l$-wave resonances and the structure in the angular distribution can come out of this expression. Nevertheless, as the following figures show, the correspondence does apparently persist to a reasonable approximation.

In figures 2.8 - 2.11 we display the results of some simple model calculations for the elastic scattering of $^{14}C + ^{16}O$ at a $^{14}C$ bombarding energy of 39.95 MeV (or 21.31 MeV cm); comparing them with our data and with expressions involving the squares of Legendre polynomials. We show the data simply to maintain a reasonable perspective on the degree to which any of the simplest models can reproduce actual measurements, and not to suggest that we can really understand them in this way. The figures here represent a summary of conclusions based on more extensive numerical work using sharp-cutoff, smooth-cutoff, and arbitrarily chosen S-matrices in an attempt to understand what qualitative features of elastic scattering angular distributions can be linked with the approximate "shape" of the S-matrix.

In fig. 2.8a we have plotted the Blair model cross-section for $l_g = 15$, which is the semiclassical grazing $l$ at this energy. There is, at least, a reasonable correspondence between the pattern of maxima in the calculation and in the data, even though the two cross-sections are grossly different in detail. The important thing to note, however, is
FIGURE 2.8 a) Cross-section for $^{14}\text{C}+^{16}\text{O}$ elastic scattering at 39.95 MeV lab calculated from the $\ell_g=15$ sharp-cutoff model (solid line) and compared with our data (points) and $[(P_{1s}-P_{1s})/2]^2$ (broken line).

FIGURE 2.8 b) The same as 2.8 a) except that here the cross-section is calculated in the McIntyre smooth-cutoff model with $\ell=\ell_g=15.5$, $\Delta=\Delta_p=0.6$, and $\delta_0=45^\circ$. 
\( ^{14}\text{C} + ^{16}\text{O} \) SHARP CUTOFF

\[
\frac{\sigma}{\sigma_{\text{RUTH}}} \quad \theta_{\text{CM}}
\]

\( ^{14}\text{C} + ^{16}\text{O} \) SMOOTH CUTOFF

\[
\frac{\sigma}{\sigma_{\text{RUTH}}} \quad \theta_{\text{CM}}
\]
that the expression \(\left(\frac{P_{16} - P_{15}}{2}\right)^2\) has maxima and minima which correspond to those in the calculation with extremely high precision for the angles from the first "bump" after the Fresnel maximum all the way to 180°. A \(P_{15}\) by itself has one structure too few, while a \(P_{16}\), although it has the correct number of maxima, has them somewhat out of phase with the sharp-cutoff calculation at angles forward of 90°.

Fig 2.8b is similar to 2.8a, but here we have used a McIntyre smooth-cutoff S-matrix with \(g=15.5\). This value gives an S-matrix profile which is closest to that of \(g=15\) Blair model and, in addition, results in an angular distribution with the same number of maxima. The other parameters, which are given in the caption, were chosen in accordance with our earlier studies of this model. Their values are not terribly critical here, since, as was remarked earlier, the period of the Fraunhofer-type oscillations is determined primarily by \(g\) as long as the other parameters are such that the oscillations exist. The solid lines in figures 2.9a and 2.9b specify the chosen S-matrix. The expression \(\left(\frac{P_{16} - P_{15}}{2}\right)^2\) yields better overall reproduction of the angular positions of the extrema in figure 2.8b than any single \(P_a^2\), though it is clearly not so precise as in the sharp-cutoff comparison.

Having found this apparent regularity in these simple, well behaved models, can we generally conclude that the pattern of minima and maxima alone allow a determination of \(g\)? The answer turns out to be no, but the deviations from this rule, with S-matrices that are a less smooth variation on the strong absorption theme, are quite interesting in themselves.
Let us consider, for example, the effect of a resonance on the elastic cross-section. We may decompose $S_{N,\ell}$ into a sum of complex amplitudes, each arising from some microscopic process (nuclear reactions, formation of quasibound states, etc.). Taking $\ell$ to be slightly smaller than $\ell_{\text{g}}$, so that $S_{N,\ell}$ is still reasonably large, suppose that one of the terms, $S_{R,\ell'}$, has a particularly large modulus and a phase such that it contributes destructively to the modulus of the sum, $\eta_{\ell}$. If $S_{N,\ell}$ becomes resonant at some energy, turning by $\pi$ in the complex plane while the sum of the other amplitudes (the "background" amplitude $S_{BG,\ell}$) changes only slowly, it will then contribute constructively to $\eta_{\ell}$ and cause a "spike" in $\eta_{\ell}$ at the resonant $\ell$. Admittedly, this is a rather idealized example for heavy-ions, where the often strong energy dependence of $S_{BG,\ell}$ and the rapid onset of strong absorption in the grazing region spoil the "formal resonance" picture of an $S_{N,\ell}$ tracing a neat circle in the complex plane. However, the basic idea is physically reasonable: resonant-amplitudes may give rise to deviations in the profile of the S-matrix from a smooth, strongly absorbed form.

In fig. 2.9a, the dotted line indicates such deviations. We have arbitrarily increased the value of $\eta_{14}$ by a factor of ten from its McIntyre value, $\delta_{14}$ being left fixed. This has the effect of substantially decreasing the contribution of the $\ell=14$ term to the sum in 2.5.15. The cross-section which results is most interesting. It is plotted in fig. 2.10a against a pure $P_{14}^2$, and its extrema clearly follow those of the Legendre polynomial for angles greater than 90°. It also shows a very large rise at backward angles while maintaining the deep minimum around 70° which the data require - a feature which neither of the simple models considered so far possesses. As a further exercise, we
FIGURE 2.9 a) The moduli, \( \eta_{\ell} \), of the smooth-cutoff S-matrix elements used for the cross-section in fig. 2.8b are plotted against \( \ell \). The dashed line indicates the modified profile for \( |S_{\ell}| \) used to calculate the cross-section in fig. 2.10a.

FIGURE 2.9 b) The nuclear phases used in the S-matrix for the cross-section in fig. 2.8b are plotted in degrees against \( \ell \). Here the dashed line indicates the modified profile for \( \delta_{\ell} \) used to calculate the cross-section in fig 2.10b.
PROFILE OF $S$-MATRIX IN SMOOTH CUTOFF MODEL

$L = 14$ SPIKE

PROFILE OF NUCLEAR PHASE IN SMOOTH CUTOFF MODEL

$L = 14$ SPIKE

$\ell$-Value
FIGURE 2.10 a) The 39.95 MeV $^{14}\text{C}^{16}\text{O}$ cross-section calculated with the smooth-cutoff S-matrix containing a spike in $\eta_j$ at $\ell=14$ which is shown in fig 2.9a (dotted line). This is compared with data and with $P_{14}^2$.

FIGURE 2.10 b) The same as 2.10a except that the cross-section is calculated from an S-matrix without the spike in $\eta_j$, but with a spike in $\delta_j$ at $\ell=14$. (Shown in fig 2.9b with a dotted line).
have also considered the effect of a sudden large change in the phase angle $\delta_{14}$ on the cross-section. We use the smooth McIntyre reflection coefficients so as not to mix effects, and take $\delta_{14}=90^0$ as shown by the dotted line in fig. 9b. Figure 10b shows the result. We see that the cross-sections average level is grossly different from that of fig. 10a; yet the extrema correspond quite well with those of a $P_{14}$ as far forward as $40^0$.

Finally, we stretch this idea to its limits (albeit unphysical) by putting an $\eta_{12}=1$ spike in an $l=15$ sharp-cutoff S-matrix. Remarkably, as fig. 2.11a shows, the resulting cross-section has extrema which, in the backward-angle hemisphere, closely follow those of a $P_{12}^2$! It is also worth nothing that in the forward hemisphere, the pattern of extrema is essentially the same as that in the plain $l=15$ sharp-cutoff cross-section shown in fig. 8a.

It is possible to gain a little insight into the origin of the simpler regularities analytically through some simplifying approximations and the use of a summation identity for the Legendre polynomials. Let us consider the cross-section in the sharp-cutoff model, which from 2.5.15 is just

$$\frac{d\sigma_{el}}{d\Omega} = |f_C(\theta,E) + \frac{1}{2\pi} \sum_{l=0}^{2l+1} e^{2i\phi_l} P_l(\cos \theta) |^2 . \ 2.7.1$$

Although $|f_C(\theta,E)|$ is a monotonically decreasing function of $\theta$, the complex quantity $f_C(\theta,E)$ contributes an interfering phase, so it is convenient for now to consider only the backward-angle part of the cross-section, where $f_C$ is very small compared with the nuclear amplitude and may be ignored. Thus,
Now, there is a summation identity for the $P^*_l$'s given by

$$\sum_{l=0}^{n} (x-z)^2 P^*_l(x)P^*_l(z) = (l+1)[P^*_{l+1}(x)P^*_l(z) - P^*_l(x)P^*_{l+1}(z)], \quad 2.7.3$$

where $x$ and $z$ may even be complex. If we choose $z=0$, $x=\cos\theta$, and $n=\ell$, this becomes

$$\sum_{l=0}^{\ell} (2l+1)P^*_l(\cos\theta) = (\ell+1)[P^*_{\ell+1}(\cos\theta) - P^*_\ell(\cos\theta)].$$

If we now make the seemingly ridiculous assumption that the Coulomb phases $\sigma^*_l$ in 2.7.2 are also zero, we obtain, simply

$$\frac{d\sigma_{el}}{d\Omega} = \frac{(\ell+1)^2 [P^*_{\ell+1}(\cos\theta) - P^*_\ell(\cos\theta)]^2, \quad \theta > 90^\circ}{4\kappa^2 \cos^2\theta}, \quad 2.7.4$$

which would justify the comparisons of fig. 2.8. (The $\cos^2\theta$ denominator does not contribute any extrema in this expression, nor does it cause a divergence at $\theta=\pi/2$, as the numerator always goes to zero faster there).

This result is interesting, but certain puzzles remain. First, we note that the forward-angle extrema, except for the very first maxima, are very well reproduced in fig. 2.8a by $(P^*_{\ell+1} - P^*_\ell)^2$, so it is not really necessary to use the small $f_C$ approximation. More significantly, the $\sigma^*_l=0$ approximation also proves to be unnecessary, and it is difficult to understand why. In figure 2.11b, where $e^{-2i\sigma^*_l}$ for this system is plotted in the complex plane for $\ell$ from 0 to 25 (the starred point being $e^{-2i\sigma^*_{25}}$), we see that the phases change significantly over the range of $\ell$ and large destructive interference occurs in the sum of 2.7.4. In addition, it is found numerically that replacing $\sigma^*_l$ by random numbers from 0 to $2\pi$ results in the complete destruction of the
FIGURE 2.11 a) Here the 39.95 MeV $^{14}$C+$^{16}$O cross-section is calculated with the sharp cutoff $S$-matrix used for fig. 2.8a but with $\eta_{12}$=1. We compare this with a $P_{12}^2$.

FIGURE 2.11 b) This shows the Coulomb part of the total $S$-matrix, $e^{-2i\sigma_l}$, used in figures 2.8 through 2.11a.
COULOMB PHASES FOR 21.31 MeV (CM) $^{14}$C $^{16}$O
correlation of extrema, and so the success of the $\sigma_{l=0}$ approximation does not arise solely from Legendre polynomial properties.Apparently, the form of the Coulomb phases is also important.

The picture which emerges from these examples is that the conventional wisdom in studies of heavy-ion elastic scattering, although perhaps only carried over from reactions studies where the cross-section has a far simpler mathematical form, is really not far wrong. The conclusion we reach is that the qualitative form of angular distributions does indeed yield valuable information on the physics of heavy-ion elastic scattering, giving us, at the very least, a good idea of the grazing partial wave, and possibly even allowing an identification of resonant spins if we have independent grounds for concluding that a given interaction is "on resonance". However, it is just as important to keep in mind that a multiparameter search in phase shift space, which is much more capable of reproducing data quantitatively than any model-dependent description, will frequently yield a number of different S-matrices which all produce fits of excellent quality. In other words, there is often more than one way to get $N$ maxima in an angular distribution. Therefore, we should be very careful when we attempt to identify resonant structure in this way. Showing that the data have the qualitative form of a $P^2_{l}$ is interesting, but it is certainly not convincing proof of the existence of resonant structure, as even the simple examples considered here indicate.

A real world situation in which this discussion becomes relevant is in our own data. The $^{14}C+^{12}C$ system, for example, displays an interesting anomaly around 17.5 MeV (CM), where the number of extrema in the angular distributions becomes abruptly smaller than it had been at
slightly lower energies (i.e., where $l_g$ was lower), only to resume its normal upward trend at higher energies. Such behavior is strongly suggestive of resonant structure, but a positive identification must await measurements of correlated structure in other (non-elastic) exit channels. We will see that phase shift analysis performed on the data in this anomalous region also suggests resonant resonant structure. However, the ambiguities to which such analysis is prone render a definitive identification of resonances impossible here, even though the data could be fitted superbly.

2.8 Phase Shift Analysis.

By performing unconstrained or only partially constrained searches in the space of $S$-matrix elements it is possible to obtain calculated cross-sections which reproduce data with extremely high accuracy. In fact, the quantitative success of such "phase shift analyses" has not been equalled in any other approach to heavy-ion scattering. Admittedly, this is not surprising, since the expansion of the scattering amplitude in a series of Legendre polynomials would permit us to fit any function if we used enough terms. However, we know that the finite range of nuclear forces must limit the number of partial waves which are important (given that we use the complete Coulomb amplitude), so whether or not phase shift analysis is really useful depends on the extent to which one can extract the $S$-matrix elements unambiguously. Unfortunately, after a great deal of study this is still very much in question in heavy-ion scattering.

The experimentalist, seeking to make some physical sense out of his intricately structured data, is immediately drawn to phase shift analysis as an alternative to the disheartening prospect of tedious
ductive work toward a more "fundamental" understanding of the problem which, in the end may not be sufficient to justify the effort. He may decide that the more deductive approach of phase shift analysis is preferable, in as much as the S-matrix, if he can deduce it uniquely, is a model-independent distillation of all the information derivable from quantum-scattering measurements. Even though the S-matrix itself finally requires an interpretation, knowledge of it would certainly constitute great progress, as it provides a succinct description of cross-section measurements in terms of physically meaningful reflection coefficients and phase angles which can be easily compared with predictions of current models. In addition, phase shift analyses enables one to identify resonant states and extract partial widths directly. This method has proved to be extremely useful in nuclear scattering with simple probes such as nucleons, pions, or even light ions; yet, its applicability to heavy-ion scattering remains a point of contention because of the complications arising from strong absorption. In any event, we may find comfort in the hope that the structure of the S-matrix will one day be understood in terms of microscopic interactions, fundamental nuclear symmetries, or both, and go on to address the practical questions of phase shift analyses.

As we already noted, the key factor in the success or failure of analyses by S-matrix fitting is the accuracy with which the $S_x$'s can be determined from data. A number of fitting techniques ranging from $\chi^2$-gradient searches to very fast and efficient Newton iteration techniques (Va86) have been employed to obtain highly accurate fits to data (i.e. fits going through all the error bars), but a consistent finding of researchers in the field, this writer included, is that the detailed form of the S-matrix resulting from such procedures depends quite
strongly on the initial "guess". This does not mean that one sees radical departures from the standard strong absorption profile, but rather that the "fine" structure of the S-matrix, which we would like to use to discriminate among models or to identify resonant structure, becomes ambiguous. Such ambiguities are not, however, to be confused with the discrete S-matrix ambiguities which are well known in the phase shift analysis of nuclear reactions, which are easily explained as follows.

Suppose that the scattering amplitude, \( f(\theta) \), is expressed in the usual partial-wave expanded form with \( \ell_{\text{max}} + 1 \) terms (or \( (\ell_{\text{max}} + 2)/2 \) terms if the beam and target are identical). This expression is a polynomial in \( x = \cos \theta \) up to the order \( \ell_{\text{max}} \) which may be decomposed into \( \ell_{\text{max}} \) factors of the form \( (a_i - x) \), where \( a_i \) are the roots of the polynomial. If any one of the \( a_i \) is replaced by its complex conjugate, the same cross section results (\( \sigma = f \cdot f^* \)). Thus, by virtue of there being \( 2^{\ell_{\text{max}}} \) sets of such \( a_i \), there are also \( 2^{\ell_{\text{max}}} \) sets of S-matrix elements which yield exactly identical cross-sections. This is one situation, however, where the Coulomb amplitude which is required for elastic scattering actually comes to our assistance. Since \( f_C(\theta) \) has no cutoff in \( \ell \) and may be summed analytically, it effectively adds a great deal of information to the cross-section, and it has been shown (Ch81) that if a sufficient number (\( -2(\ell_{\text{max}} + 1) \)) of data points are known with effectively infinite precision then there are no ambiguities in the elastic scattering S-matrices.

The ambiguities which are most problematic in phase shift analysis, from a practical point of view, are "continuous" ones which arise from the existence of uncertainties in the measured cross-sections. Recent work by Onno van Roosmalen in this Laboratory (Va86), in which these
ambiguities are studied with reference to the error matrix for extracted phase shifts, indicates that unless data are extremely complete in angular range, the existence of experimental uncertainties of the order typically encountered in heavy-ion experiments allows many sets of equally superb fits to data with S-matrices which are rather different in detail regardless of the amount of "structure" in the data. This work also emphasizes that very forward-angle cross-sections are critically important in fixing the overall form of the S-matrix, and that measurements should not focus on structure at extreme backward-angles at the expense of ignoring the region of the small-angle Fresnel oscillations. Of course, forward angle elastic scattering measurements anywhere near the beam are quite difficult, and normalization errors alone can contribute substantially to the ambiguities problem. Thus our hope of being able to conduct model-independent analyses of heavy-ion scattering data dims considerably upon a close inspection of the requirements of the problem. We will return to these matters in our actual data analysis in chapter IV.

2.9 Heavy-Ion Potentials and the Optical Model.

As we have emphasized throughout, the many-body problem which one would need to solve in detail in order to fully understand the interactions of heavy-ions is presently quite intractable mathematically. In microscopic approaches to the scattering problem, not only do we lack a calculable field theory for the strong interactions at low energies, but even with our fairly extensive phenomenological knowledge of nucleon-nucleon interactions, we are unable to overcome the many difficulties arising from the immersion of the nucleons in nuclear matter (which include three-body and Pauli exclusion effects, and absorption, to
name the most notable). Thus, the scattering problem has generally been treated with the simplifying assumption that the interactions of two complex nuclei may be represented by a one-body potential which has a real part to describe refraction and an imaginary part to take into account the absorption into reaction channels. These "optical" potentials, or at least their real parts, have been given a general form by reasonably fundamental approaches such as folding integral models, where phenomenological nucleon-nucleon interactions are folded with nuclear density distributions to yield an effective one-body potential. Phenomenological analysis guided by these forms has achieved fairly detailed reproduction of actual cross-section measurements in nucleon and light-ion scattering as well. However, on nuclei, and sometimes in heavy-ion scattering, the situation in heavy-ion scattering is complicated somewhat by (1) another ambiguity problem, this time in the potentials themselves, and (2) the existence of sharp energy-dependent structure. To some extent, at least, these two problems counteract each other.

The simplest optical calculations involve numerical solution of the Schrodinger equation with a two-body nuclear potential of Woods-Saxon form for both real and imaginary parts. The Coulomb potential is usually taken to be that of a point-charge interacting with a uniformly charged sphere, but the scattering cross-section generally depends very weakly on the radius parameter \( R_C \), so the basic optical model has effectively a six-parameter potential of the form

\[
V(r) = V_{\text{Coul}}(r) + \frac{\hbar^2}{2\mu r^2} \ell(\ell+1) + \frac{-V}{1+\exp[(r-R_1)/a_1]} + \frac{-iW}{1+\exp[(r-R_1)/a_1]}.
\]

2.9.1
Strong absorption, as characterized by the profile of the reflection coefficients, emerges naturally from this model when $W \geq V$. Now, much is often made of the fact that very strong absorption can give rise to radius vs. well-depth ambiguities in optical potentials, as was first noted by Igo in the analysis of alpha plus nucleus scattering (Ig59). The idea is that the "hiding" of the nuclear interior caused by the small mean free path associated with strong absorption makes the overall shape of angular distributions relatively insensitive to the shape of the potential inside a reasonably defined "strong absorption radius" (sometimes approximated by the semiclassical relation

$$\kappa R_{SA} = (\Lambda + \eta)^{1/2} + \eta).$$

2.9.2

The exponential tails of the real and imaginary parts of the nuclear potential in 2.9.1 are clearly left invariant by transformations of $V$ and $R$ such that

$$V \exp(R/a) = \text{constant}.\quad 2.9.3$$

These transformations generate the so-called Igo ambiguities.

In heavy-ion scattering, however, it is quite clear that such ambiguities hold only when cross-sections are particularly featureless. In our studies of the problems, and elsewhere (Sh83), it has been found that although the small-angle Fresnel-dominated part of heavy-ion angular distributions pays some respect to the invariances of 2.9.3, strong Fraunhofer structure at middle and backward angles does not, especially when $\eta$ is fairly small (as at energies well above the Coulomb barrier). Thus our conclusion is that Igo ambiguities are not a problem when structure is present in heavy-ion cross-sections. This does not mean, however, that we are free from approximate ambiguities of a more complex nature. In fact, we have found that very similar fits to data with oscillatory structure are sometimes (but not always) possible with
different sets of optical parameters not related by 2.9.2. In particular, we saw that very large differences in well depths are possible if one is willing to consider somewhat strange geometries (such as very large or small radii or diffusenesses close to zero). Maher et al. (Ma69) have reported that discrete sets of ambiguous potentials can even be generated by varying only V and W. In addition, it is commonly found that optical potentials which have only a fraction of the depth suggested by folding model calculations can reproduce some heavy-ion data quite well on average (and may in fact be necessary).

In view of such difficulties, it is certainly unwise to put too much stock in well depths from optical fits, especially if only one angular distribution is being analyzed. It may be useful to use the resulting optical potential to generate distorted waves for a DWBA analysis of transfer reactions, for example, but as a means of understanding the elastic scattering itself this kind of limited-scope analysis is probably not very informative. If, on the other hand, one has at his disposal a large set of data spanning a wide range of energies and angles new possibilities open up, along with another set of problems.

The operative philosophical question when plentiful data are available becomes "what do we want to learn from optical analyses?" The answer depends on the data themselves. A heavy-ion system with relatively featureless cross-sections may be reasonably well-described by a simple optical potential without much trouble from ambiguities as long as many angular distributions have been measured and fitted simultaneously and the energy-dependence of the parameters kept to a minimum (for example, a linear increase of W with energy is customary and necessary in order to keep oscillatory structure from becoming excessive at
higher energies). The optical potential which results from such analysis may then be regarded as having some validity for the purpose of comparison with microscopic calculations. Besides this, the successful fitting of the data within the limited freedom of the optical model demonstrates that the system in question, in spite of its many-body nature, behaves, on average, as a relatively simple one whose scattering properties arise from its geometry alone.

Systems showing much structure, especially when that structure is strongly energy-dependent, give us a different aim in optical analyses. Here we know from the outset that the complexity of such a large set of data will overdetermine the potential unless we resort to freeing the optical parameters for each angular distribution or excitation function (which may still not be enough). However, this policy certainly would be of no use, since we would wind up both mired in ambiguities and confounded at the very prospect of having to find some physics in the vast collection of oddly varying parameters. The viable alternative is simply to allow the overdetermination and sacrifice the quality of the fit. The aim is then to use the simple optical description as a first approximation - a basic framework on which to build more sophisticated models of heavy-ion scattering.
2.10 Potential-Based analyses: Variations on the Optical Theme and the Inclusion of Elastic Transfer Effects.

As we have already noted, heavy-ion data which exhibit pronounced energy-dependeant structure are generally not describable in detail by a simple optical model. This is particularly true when this structure is accompanied by a significant rise in the cross-sections at backward angles - a distinctive feature of lighter, tightly bound, and nearly symmetric systems such as $^{14}\text{C} + ^{12}\text{C}$ or $^{14}\text{C} + ^{16}\text{O}$.

However, it has been found that "surface-transparent" optical potentials, which have a nonzero real component reaching beyond the range of the imaginary part, are generally better able to reproduce oscillatory structure than "strongly absorbing" forms with imaginary geometries that are equivalent to or more extended than the real ones, though they are often still not adequate to describe data in detail. Since the frequency of the oscillations in calculated angular distributions increases with increasing energy (reflecting the change in $E_{g}$, really) excitation functions typically show gross structure (2-4 MeV) as cross section extrema pass through a fixed angle. Hence, gross structure in heavy-ion excitation functions is often linked with surface transparency (e.g. K085).

Surface transparency may be obtained from a standard optical model by using a relatively shallow imaginary well (such that $V/W \geq 2$) and a smaller imaginary radius or diffuseness than that of the real potential. A frequently used modification of the standard optical model involves multiplying the imaginary potential by an $\ell$-dependent factor which cuts off absorption near the surface (Ch70). This usually takes the form
\[ W(r) = W(r)(1 + \exp(\frac{\Delta - L_c}{2r^4})) \]

with \( L_c \leq \ell \). However, it has also been noted (Go75) that this correction is not very different from using a very small \( a_1 \) (it is possible to have transparency even when \( R_1 > R_0 \) if \( a_1 / a_0 \) is quite large). In principle, surface transparency could also be included explicitly by taking a positive absolute sign for the imaginary surface absorption term which most optical codes can include in the optical potential. This term is usually taken as the derivative of a Woods-Saxon function and so can contribute up to three new parameters to the model; it is typically used when additional surface absorption is required (its absolute sign is then negative).

A more substantial improvement in the optical description of heavy-ion cross-sections pronounced angular oscillations (especially at very backward angles, as in \( ^{28}\text{Si}^{16}\text{O} \)), has been shown to be possible with parity-dependent potentials (De78, Sh83). These usually take the form

\[ V_{\text{Nuclear}}(r, \ell) = V(r)[1 + C_\ell(-)^\ell] + \iota W(r)[1 + C_1(-)^\ell] \]

where \( V(r) \) and \( W(r) \) have their standard optical form. A similar idea has also been attempted in an S-matrix approach where a parity dependent term is attached to the reflection coefficients, producing odd-even staggering. However, these descriptions are not identical, as parity-dependence in the potential does not produce consistent staggering in the S-matrix (Ma79).

The use of parity-dependence in effective one-body potentials is motivated by various microscopic approaches to the nuclear scattering problem which have in common the inclusion of exchange effects. It is
well known that standard folding models - which do not yield parity-dependent potentials - suffer from an inability to take the Pauli exclusion principle properly into account. However, it has been shown in resonating group calculations that Pauli exclusion gives rise to Majorana exchange of core-clusters in light-ion scattering ($\alpha$, $^3$He, and protons on $^{16}$O for example (Le77)), resulting in a $(-j^2)$ term in the effective potential. Core-exchange implicit in the LCNO (linear combination of nuclear orbitals) formalism of Von Oertzen (Bo72, Vo75) also leads to an equivalent potential with this simple parity dependence, and even predicts a phase rule based on the statistics of the exchanged core in a large number of lighter, nearly symmetric systems at energies near $V_C$. Its success is demonstrated in figure 2.12.

It is clear that space-exchange involving wave functions of identical cores in these models is the essential ingredient for parity-dependence in the effective potential, but there is another approach to the underlying physics of this problem which brings the study of elastic scattering and transfer reactions together in a very interesting and intuitively appealing way, and which does not employ parity-dependence explicitly. The idea here is that the elastic scattering of non-identical nuclei can be viewed microscopically as a composite process involving specific and very simple reaction channels which are asymptotically indistinguishable; namely: ordinary elastic scattering to the angle $\theta$ (for which an optical description is assumed adequate), and elastic transfer of valence nucleons between identical cores leading to a scattering angle $\pi-\theta$ (this may be calculated in some standard approach
FIGURE 2.12 Predictions of the LCNO description of core-exchange processes compared with data. The oscillations in these angular distributions obey a phase rule which reflects the statistics of the core in each system. Proceeding from top to bottom in the figure, the cores are $^{12}$C (boson), $^{13}$C (fermion), $^{13}$C, $^{12}$C, $^{14}$N (boson), and $^{15}$N (fermion) (Vo75).
BOSON AND FERMION INTERFERENCE EFFECTS IN ELASTIC SCATTERING OF NONIDENTICAL NUCLEI

- $^{12}\text{C} - ^{13}\text{C}$, $E_{\text{c.m.}} = 7.8\text{ MeV}$
- $^{12}\text{C} - ^{14}\text{C}$, $E_{\text{c.m.}} = 7.8\text{ MeV}$

- $^{14}\text{N} - ^{13}\text{C}$, $E_{\text{c.m.}} = 9.9\text{ MeV}$
- $^{14}\text{N} - ^{12}\text{C}$, $E_{\text{c.m.}} = 9.6\text{ MeV}$

- $^{14}\text{N} - ^{16}\text{O}$, $E_{\text{c.m.}} = 13.3\text{ MeV}$
- $^{15}\text{N} - ^{16}\text{O}$, $E_{\text{c.m.}} = 13.0\text{ MeV}$
to rearrangement reactions - such as DWBA). The process is illustrated in figure 2.13 for $^{14}\text{C}+^{12}\text{C}$ scattering.

Elastic transfer is physically equivalent to the core-exchange process which gives rise to parity-dependent optical potentials. It results in a partial symmetrizing of the scattering amplitude, thus increasing the large-angle cross-section, and may also introduce oscillatory structure by its interference with the ordinary elastic scattering. Reflection makes it clear, however, that we may now have a very complicated model, as in principle we would have to consider simultaneous elastic transfers arising from all possible core choices. If we are to do simple but realistic calculations with this idea, then, we must be able to isolate a particular elastic transfer channel which can reasonably be expected to dominate all the others.

We can do this most reliably in systems in which the smaller of the constituent nuclei is well-bound, and is therefore more likely to retain its integrity as a core. If, taking that nucleus as a core in the larger nucleus leaves a relatively simple configuration of valence particles to be transferred, then the system is a good candidate for testing the elastic transfer idea with a single transfer channel. The systems which we have studied experimentally all satisfy this criterion, and for this reason, a very substantial part of our analysis work will involve elastic transfer amplitudes. The $^{14}\text{C}+^{12}\text{C}$ system involves the elastic transfer of two $p_{1/2}$ neutrons between closed $p_{3/2}$ subshell $^{12}\text{C}$ cores. In $^{14}\text{C}+^{16}\text{O}$, the transferred particles are two $p_{1/2}$ protons and the core is $^{14}\text{C}$, which has a closed $p$-shell for neutrons and a closed $p_{3/2}$ subshell for protons. Finally, in $^{14}\text{C}+^{18}\text{O}$ we would have the elastic transfer of
FIGURE 2.13 A schematic depiction of the 2n elastic transfer in $^{14}\text{C} + ^{12}\text{C}$. Note the equivalence of the transfer process with the interchanging of the $^{12}\text{C}$ cores.
A. SCATTERING

\[ f_{sc}(\theta) \]

\[ \frac{d\sigma}{d\Omega} = \left| f_{sc}(\theta) + f_{tr}(\pi-\theta) \right|^2 \]

B. TRANSFER

\[ f_{tr}(\pi-\theta) \]
an alpha-particle between $^{14}\text{C}$ cores. This seemed a very interesting possibility in view of the recent discussion of alpha-clustering in $^{18}_0$ (Ga83, Ga87, Ha87).

A great advantage of the elastic transfer hypothesis is that it is straightforward to implement in realistic calculations using the very fast and sophisticated computer codes which have been designed to calculate elastic scattering in the optical model and to perform full-recoil DWBA calculations for transfer reactions. Scattering amplitudes for the two processes may thus be calculated separately and the cross-section very simply obtained from their coherent sum:

$$\frac{\text{d}\sigma}{\text{d}\Omega} = |f_{\text{elastic}}(\theta) + f_{\text{transfer}}(\pi-\theta)|^2 \quad \text{2.10.3}$$

In addition, since transfer amplitudes calculated via DWBA are to be scaled by independent spectroscopic amplitudes, which reflect the parentage of the states involved, we have in principle a way of extracting nuclear structure information from elastic scattering measurements.

There are some difficult problems, however, which need to be carefully addressed if conclusions based on this model are to be credible. Foremost among them is the fact that the issue of choosing an optical potential to describe the elastic scattering and to generate the distorted waves for a DWBA calculation becomes moot. After all, we cannot measure the "bare" elastic scattering and so determine this potential. Rather, we must attempt to find a potential which reproduces the data only when the elastic transfer is actually included. Thus, we are confronted with yet another kind of ambiguity. In fact, a similar situation exists even in the conventional DWBA analysis of transfer reactions.
themselves, where, as pointed out by Ascuitto (As84), the use of optical potentials obtained phenomenologically from elastic scattering in both the entrance and exit channels effectively double-counts the strong-coupling influence of the transfer on the relative motion of the nuclei. This, of course, is only a problem if there is strong-coupling. We will return to these problems in the analysis of our data, where they can be addressed more specifically.

2.11 Dynamical Symmetry.

As a final stop in our admittedly selective tour through the physics of heavy-ion scattering we consider a new approach to the problem, currently under development in this laboratory (Al86d), in which algebraic methods are used to specify the form of the S-matrix. Since spectrum-generating algebras have been very successfully applied to the problem of nuclear bound states, it is of great interest to see whether or not there are also simple symmetries of the Hamiltonian which are important in the continuum.

The general method in algebraic approaches, as exemplified by the interacting boson model (IBM) for low-lying nuclear bound states, involves expressing the nuclear-interaction Hamiltonian, $H$, as a polynomial, usually limited to second-order, in the generators of a compact Lie group $G$ ($U(6)$ for the IBM). The coefficients of the polynomial are model parameters which may be chosen for the system in question. Dynamical symmetries are said to exist when this expression can be further limited to terms which are Casimir invariants of $G$ or of
any member of a chain of its subgroups. In such a situation, eigenvalues of $H$ can be obtained analytically from the expectation values of the Casimir operators, and so, the bound states problem is solved in the manner of second quantization, without direct reference to a Schrödinger equation in configuration space.

In scattering, the problem is complicated by two major difficulties; namely: the passage to the continuum, and the circumstance that the final states exist in a region free of the very interactions which we seek to describe with a non-trivial dynamical symmetry. However, these difficulties have recently been overcome; the first by employing non-compact groups, which possess continuous and infinite-dimensional unitary representations, and the second by a technique called "Euclidean connection". This technique provides an algebraic means of expanding scattering states characterized by a given "dynamical group" in terms of asymptotic states characterized by the appropriate "asymptotic group". Since the asymptotic symmetries include spatial translations as well as rotations, the asymptotic group contains at least $E(3)$, the Euclidean group in three dimensions, as $E(3)$ has both the angular momentum operators $L_i$ and the linear momentum operators $P_i$ as its generators; hence the name "Euclidean connection".

The technical details of this group-theoretical scattering model are well-covered in references Al86a,b, Fr86, Ia86a,b, Wu85, and Wu86. For present purposes it is sufficient to concern ourselves only with specific formulations within the general framework of the model, which have focussed on the dynamical groups $SO(3,1)$ and $SO(3,2)$ and their connection with the asymptotic groups $E(3)$ and $E(3) \times E(2)$ respectively.
Both formulations are capable of describing "modified-Coulomb" interactions which may be particularly relevant in heavy-ion scattering. In either situation, one obtains the S-matrix by solving recursion relations for the expansion coefficients of dynamical group eigenstates in terms of incoming-wave and outgoing-wave, free-particle, eigenstates. In particular, with SO(3,2) as the dynamical group we have

\[ \text{SO}(3,2) \ (E(2) \times E(3)) \]

\[ |\omega, l, m, v > = A_{l,v}(k) |-k, l, m, v > + B_{l,v}(k) |+k, l, m, v > \]

\[ \text{incoming} \quad \text{outgoing} \quad 2.11.1 \]

where \( k \) is the cm wavevector, \( v \) an interaction strength which may depend on \( l \) and \( m \) the angular momentum and its z-projection, and \( \omega(\omega+3) \) the eigenvalue of the scalar quadratic Casimir invariant of SO(3,2). In the continuous, unitary representation of SO(3,2), \( \omega = -3/2 + i f(k) \) and \( f \) may be any real function of \( k \). The S-matrix is then given by

\[ S_{l}(k) = (-)^{l+1} \frac{B_{l,v}(k)}{A_{l,v}(k)} \quad 2.11.2 \]

and the ratio \( B_{l,v}(k)/A_{l,v}(k) \) of the complex outgoing and incoming flux coefficients is found from the recursion relations which result when SO(3,2) shift-operators for \( l \) and \( v \) and their Euclidean-connected counterparts are applied to the left and right sides of 2.11.1 respectively (A186). For example, the recursion relation for \( R_{l,v} = B_{l,v}/A_{l,v} \) which is obtained when both \( l \) and \( v \) are shifted by +1 is

\[ R_{l+1,v+1}(k) = \frac{-l + 3/2 + v + if(k)}{l + 3/2 + v - if(k)} R_{l,v}(k) \quad 2.11.3 \]

The set of four such relations resulting from shifting \( l \) and \( v \) by plus or minus one may then be solved using the factorial property of the gamma function. The result is
where \( v \), the interaction strength, is to be chosen empirically and \( f(k) \) determined from the relation (if it is known) between the Hamiltonian and scalar quadratic Casimir invariant of SO(3,2). We note that if we desire that \( S^{(k)} \) approach the Coulomb S-matrix in the limit of large \( \ell \), as is required in heavy-ion scattering, then we may identify \( f(k) \) as \( \eta \) (the Sommerfeld parameter), put \( \Delta(k) = 2\ln 2\eta \), and demand that \( \lim_{\ell \to \infty} v = 1/2 \)

(the equivalence is then easily established using the identity

\[ \Gamma(z)\Gamma(z+1/2) = 2^{1-2z}/\pi \Gamma(2z) . \]

In applying the SO(3,2) model to our data we have kept to the foregoing conditions, taking for \( v \) a four-parameter Woods-Saxon form which provides for non-Coulombic interactions at small \( \ell \)

\[ v = \frac{v_R + iv_i}{1 + \exp(\ell - \ell_0/\Delta)} + 1 . \]

The S-matrices given by 2.11.4 are unitary only if \( v \) is purely real, and so the introduction of an imaginary component in \( v \) allows one to describe scattering in the presence of absorption, which is of course necessary for heavy-ions. It is also important to note that these S-matrices satisfy the unitarity bound as long as

\[ \text{Im } v^2 < 0 . \]

This condition must also be respected.

Our work with this model has involved a systematic search over the model parameters \( v_R, v_i, \ell_0, \) and \( \Delta \) in an attempt not only to fit data, but also to understand the model space itself, as the physical meaning
of the parameters is not really clear from first principles. We present the results of these analyses in chapter IV.

The foregoing ideas provide a means of generating an S-matrix from a given dynamical group, but they do not, in general, tell us what interaction in the space-basis Schrodinger equation corresponds to such an S-matrix. In fact, the only known exception to this rule in three-dimensional scattering is the Coulomb interaction, which has been known to have the dynamical symmetry SO(3,1) for some time (Pa26). The current application of dynamical symmetry to the scattering problem via the Euclidean connection has shown (by methods analogous to those described above) that all problems with SO(3,1) symmetry have S-matrices given by

\[ S_\ell(k) = \frac{\Gamma(\ell + 1 + \text{i}f(k))}{\Gamma(\ell + 1 - \text{i}f(k))} e^{i\phi(k)}. \]  

Again one can easily recover pure Coulomb scattering (by putting \( f(k)=\eta \) and setting \( \phi(k) \) to zero), but here there is an important difference; the scalar quadratic Casimir invariant of SO(3,1) can be expressed as

\[ C = A^2 - L^2, \] 

where \( h\hat{\alpha}_m \) is the Lenz vector and \( \hat{L} \) the angular momentum, which together constitute the six generators of SO(3,1). The SO(3,1)-symmetric Hamiltonian is then simply given by

\[ H = \frac{\alpha^2Z_1Z_2 \mu c^2}{2(C-1)} \] 

and the correspondence between the interaction and the dynamical symmetry is clearly established. However, one may also consider non-Coulombic interactions with S-matrices of the SO(3,1) form by using a
more general $f(k)$ in 2.11.7. Since it is still desirable to obtain the Coulomb S-matrix in the large-$l$ limit, Woods-Saxon forms such as

$$f(k) = \frac{v_R + iv_i}{1 + \exp((l-l_0)/\Delta)} + \eta \quad 2.11.10$$

are typically used (De86). A continuing aim of research in this field is to determine what potentials correspond to such "modified-Coulomb" versions of these models - or whether potential equivalents exist at all.

A most intriguing aspect of the group-theoretical scattering models presently under development is that they are amenable to generalization to multi-channel problems (Ia86a). Certainly it is fascinating to consider that a single, relatively simple group symmetry may be sufficient to specify the form of S-matrices for any number of different exit channels in complex heavy-ion reactions. An algebraic description of the coupled-channels problem would have the clear advantage over those involving numerical solution of coupled Schrodinger equations in being much more efficient computationally, since the S-matrices would be given in closed form. Obviously this would be extremely helpful when fitting data, but it also reminds us that by merely invoking a group symmetry we cannot expect to answer all of our questions about heavy-ion scattering. There are still interactions whose form must be determined empirically for now, but which we would like to understand, finally, in a more fundamental way. The algebraic approach therefore does not offer final answers, but surely holds great promise as a vehicle for progress in this direction.
In the present work, since our emphasis has been on elastic scattering measurements, we have not been able to apply the dynamical symmetry coupled-channels model to a multi-channel problem directly. However, it is worth pointing out that the formalism may have a useful application in the description of elastic transfer processes. For example, a simple two-channel model for spinless particles using SO(3,1) as the dynamical symmetry yields S-matrices of the form (Ia86b)

\[ S^\text{elastic}_\ell = S^+\ell + S^-\ell \quad \text{and} \quad S^\text{non-elastic}_\ell = S^+\ell - S^-\ell \]

with \( S^+\ell = \frac{\Gamma(\ell+1+iv+iu)}{\Gamma(\ell+1-iv-iu)} \) and \( S^-\ell = \frac{\Gamma(\ell+1+iv-iu)}{\Gamma(\ell+1-iv+iu)} \). 2.11.11

The "elastic interaction" \( v \) can be given the form 2.11.10, while \( u \), which clearly induces the non-elastic process, has been taken to be the \( \ell \)-derivative of \( v \) (Sh6), but with its own real and imaginary scale parameters \( u_R \) and \( u_I \). This choice essentially localizes the non-elastic process in the nuclear surface region, as is physically resonable for simple rearrangement reactions. Using this model to include elastic transfer is then straightforward. The S-matrices above may be used to calculate \( f^\text{elastic}(\theta) \) and \( f^\text{transfer}(\pi-\theta) \), and the cross-section may then be obtained from 2.10.3 as before.
CHAPTER III

THE EXPERIMENTS

3.1 Overview.

Elastic scattering measurements are very straightforward in principle. The cross-sections are reasonably large and the kinematics, at typical tandem Van de Graaff energies, are generally favorable for detection with high resolution by current-generation charged particle detectors. These convenient circumstances make feasible the projects of broad scope and high detail which are needed for a better understanding of heavy-ion interactions. However, it is important to bear in mind that in elastic scattering measurements one still faces the usual difficulties involved in high-resolution particle-detection experiments. For example, detector geometry must be fixed with extreme precision, especially when the size of the program requires that large aperture position sensitive detectors be placed close enough to the target that they subtend a large range of scattering angles. Also, the separation of the data of interest from the many possible beam and target-constituent reactions is often not a trivial affair. Finally, the sheer labor involved in processing large volumes of data for which extensive off-line sorting is required is very much more than a simple inconvenience, computer based analysis notwithstanding.

In this work, we had undertaken measurements of $^{14}\text{C}+^{18}\text{O}$ elastic scattering differential cross-sections over an extensive energy-angle
grid, our aim being to obtain a cross-section surface which could be examined globally or conveniently "sliced" into individual angular distributions or excitation functions. Our early efforts involved $^{18}_0$ beam from Yale's MP-1 tandem Van de Graaff accelerator and self-supporting carbon targets highly enriched (~90%) in $^{14}_C$. However, this method was abandoned as too dangerous after the fatigue-induced bursting of a very thin and necessarily fragile ionization chamber window resulted in the destruction of one such target inside the scattering chamber. Several months of exhausting effort were then required to decontaminate the chamber and its associated beam-line and hardware. The natural alternative was to invert beam and target, and so we carried our experiments to the tandem Van de Graaff facility at Brookhaven National Laboratory, where excellent beams of $^{14}_C$ were routinely available.

Our first experiment employed two independently-operated counters, each comprising a gas ionization chamber for measurement of $\frac{dE}{dx}$ ("$\Delta E$") and a solid-state detector for simultaneous measurement of both residual energy ("$E$") and position in the scattering plane ("$X$" - essentially the scattering angle). Both counters detected scattered and recoil particles, and together subtended a large enough angular range to measure an entire angular distribution with a single angular setting. Collecting angular distributions in a single run was advantageous not only because it saved time, enabling us to collect more data, but also because it greatly reduced the difficulty of normalization. In this experimental configuration, energy vs. angle kinematics and charge identification via
$\Delta E$ would allow us to separate $^{14}\text{C}+^{18}\text{O}$ scattering from most other reactions. However, elastic scattering with other isotopes of oxygen would be problematic; so we needed pure targets. Unfortunately, isotopic abundance measurements at Yale after this experiment showed that only 15% of the oxygen in the target was $^{18}\text{O}$, the rest being $^{16}\text{O}$ (the why's will be taken up below). Although scattered-particle $^{14}\text{C}+^{18}\text{O}$ data could be resolved from $^{14}\text{C}+^{16}\text{O}$, the recoil-particle data, which yield the backward-angle cross-sections, could not.

After this, there ensued a lengthy attempt to obtain $^{18}\text{O}$ targets of sufficient purity. We considered gas targets briefly, but determined that adequately high angular resolution would not be possible with such targets unless we moved our detectors far away and sacrificed a great deal of efficiency (hence, data). Finally, the best solid targets which we were able to obtain were SiO, about 70% enriched in $^{18}\text{O}$. As this was still not sufficient for a clean experiment, we decided to accept a moderate sacrifice in time and efficiency and employ a kinematical coincidence technique, using a single $\Delta E$-$E$-$X$ counter in coincidence with a bare $E$-$X$ detector. This now required three angular settings per energy, but could provide us with enough kinematical information to differentiate $^{14}\text{C}+^{18}\text{O}$ scattering from the many contaminant reactions present.

Our coincidence experiment was quite successful in that we could, indeed, resolve $^{18}\text{O}$ from $^{16}\text{O}$ scattering, but discovering then the lack of pronounced structure and the relatively small backward-angle cross-sections in $^{14}\text{C}+^{18}\text{O}$, we realized that the $^{14}\text{C}+^{16}\text{O}$ data of the previous experiment were, indeed, effectively uncontaminated and very much worth
analyzing. Other reactions seen in the earlier experiment also appeared to be analyzable upon a closer inspection. Of these, the $^{14}\text{C}+^{12}\text{C}$ scattering which resulted from carbon build-up on the target was found to be the most prominent of the clearly resolved processes. Various reactions involving the Be target component contributed large fluxes of carbon and oxygen ions to our data set on event tape, and $^{14}\text{C}+\text{Si}$ elastic scattering was also clearly visible at forward angles. However, the $^{14}\text{C}+^{12}\text{C}$ data were better resolved, more complete, and more interesting in the context of heavy-ion scattering studies than any of these; so they and the $^{14}\text{C}+^{16}\text{O}$ data have been the focus of our analysis in the earlier experiment. Of course, the other data are available on tape should interest in them justify their extraction in the future.

3.2 The Detector Systems.

The primary detectors in both experiments were position-sensitive E-ΔE telescopes built at Yale to a design from Heidelberg by Moshe Gai and Joe Cimino. These counters are functionally divided into two independent parts, the first a Frisch-grid gas ionization chamber, and the second an Ortec P-series solid state position-sensitive detector (PSD), which is placed within the volume of the ionization chamber gas.

Figure 3.1 is a schematic view of the ionization chamber, which consists of three essential elements: an anode, a cathode, and a Frisch grid. The anode and cathode serve to define a uniform field which sweeps apart the positive and negative charge carriers created by the passage of charged particles through the working gas. Only the anode of this detector is actually used for charge collection. Thus, we work entirely
FIGURE 3.1 Schematic views of one of the ionization chambers with solid state PSD. The operating pressure was 20 torr of P-10 gas and the voltage settings were as shown.
with the relatively fast electron signal which is good for timing, (electron drift velocities are about $10^3$ times larger than those of the positive ions, and the electron collection time is only a few microseconds). However, the effective charge induced on the anode of a simple parallel plate ionization chamber depends on the distance, $Y$, between the anode and the point at which the ionization took place, through

$$q_{\text{induced}} = e \left( \frac{Y}{D} \right), \quad 3.2.1$$

where $D$ is the anode-cathode distance and $e$ the electron charge. In order to eliminate this unwanted dependence of the $\Delta E$ signal on the vertical location of the track, one may interpose a grid of wires kept at a fixed positive voltage between the anode and the region in which ionization occurs. This "Frisch grid" effectively shields the anode from the charge carriers below it, and thereby guarantees that $Y$ in 3.2.1 is always just the distance between the grid and the anode, regardless of the location of the ionization track.

The "optimal" electrode voltages shown in figure 3.1 were determined empirically from a study of the energy resolution, breakdown threshold, and signal amplitude characteristics of the detector. We also found that a large capacitance preamplifier, such as an Ortec model 142, was necessary to process the $\Delta E$ signals. These ionization chambers and their electrodes were machined from solid aluminum and highly polished on all interior surfaces. Their Frisch grid was made from .8 mil phosphor-bronze alloy wire, wound with spring tension onto a rectangular holder-frame (spaced 20 to the inch) and then fixed to the grid frame with epoxy. This ensured that the grid was quite flat; an important
consideration since the grid-anode spacing was only .40". In practice, the FWHM $\Delta E$ resolution of our ionization chambers (for tandem-energy heavy-ions ranged from about 13% down to about 9% (of which at least 4-5% arises from unavoidable straggling)) depending on their state of development. The larger figure is appropriate for the earlier experiment, and the smaller figure for the later one, the change reflecting modifications in the anode's internal connections (which had always been subject to problems, as future users of these detectors should keep in mind when troubleshooting them). This energy resolution is quite adequate to separate completely the different charge species of heavy-ions with $Z$ up to about ten at energies above the Bragg maximum, where $\frac{dE}{dX}$ is roughly proportional to $\frac{MZ^2}{E}$. It is not, however, adequate to discriminate masses.

The ionization chambers were run with P-10 gas (90% Argon 10% Methane) at a pressure of 20 torr. A manostat-controlled gas handling system of our own design and construction was used to maintain the system at constant pressure. Fresh gas flowed through the detectors in series after being cleaned of any possible gas system pump oil contamination in a copper coil cooled to -40° C (a refrigerated alcohol bath was used for this purpose). Finally, the gas was vented to the atmosphere via the gas system pump. The pressure of the working gas was chosen to be large enough to give us $\Delta E$ signals of sufficient amplitude for positive detection and good resolution, and yet small enough (1) that the gas could be contained with the thinnest leak-proof entrance foil which we could make, and (2) that the particles of interest would lose
only a small fraction of their total kinetic energy in passing through the gas. The latter condition was very important for the total energy resolution of the experiment, as the solid state detector was typically capable of energy resolution of 1.5-2%. Our decision was to match in hardware the gains of the energy signals from the PSD and the ionization chamber and add them in order to obtain a total energy signal accurately reflected the reaction kinematics, at least to the extent that the unrecoverable energy losses in the target and window could be kept small. It was then clearly advantageous to make the fraction of the particle energy left in the poorer-resolution \( \Delta E \) detector as small as was practicable. However, it should be mentioned that the method of adding \( \Delta E \) and \( E' \) signals to obtain total \( E \) for a given particle event results in an effective improvement of 5% or so over the separately measured \( \Delta E \) resolution by virtue of the elimination of straggling fluctuations (the energy that one detector does not see, the other does).

The effective thickness of the ionization chamber gas for purposes of ion stopping power calculations is given by

\[
\rho(\text{gm/cm}^2) = \frac{P A 273.15 D}{760 22400 T}
\]

where \( P \) is the pressure in torr, \( A \) the molecular weight, \( T \) the temperature in °K and \( D \) the path length in cm. For our detectors, \( D=8.0 \) cm; so with \( T=290^\circ K \), we find a P10 gas thickness of 333\( \mu \)g/cm\(^2\) for our counters. According to the stopping power code IRMA, this yields a \( \Delta E \) range of \( E \)

2.2% (40 MeV \(^{14}\text{C}\)) to 23.9% (10 MeV \(^{18}\text{O}\)) for the particles of interest in our experiments.
In order to minimize unrecoverable energy losses which would make
kinematical analysis of the reactions more difficult, we used extremely
thin entrance window foils of polypropylene (chemically, \((\text{CH}_2)^n\)) which
were manually stretched from 1 mil commercially supplied foil down to a
thickness of 100\(\mu\)g/cm\(^2\) (verified by weighing using a Mettler balance).
These foils are thin enough to show the rainbow-like colors characteristic of thin film interference. They may even be stretched down to 70
\(\mu\)g/cm\(^2\), where their interference color becomes a more uniform magenta,
but our experience has been that foils so thin are much more subject to
breakage and pinhole leaks. Foils of 100 \(\mu\)g/cm\(^2\) are much stronger, and
only slightly less uniform in spite of their variegated appearance
(their actual thickness is at least 3\(\mu\), which is six full wavelengths of
blue-green light and less than five wavelengths of red light). The 100
\(\mu\)g/cm\(^2\) foils have a \(\Delta E\) range of 1.1\% (40 MeV \(^{14}\)C) to 13.2\% (10 MeV \(^{18}\)O)
and contribute straggling, as estimated using IRMA, at least 4-6\% of the
\(\Delta E\), which is not significant compared with the overall 2\% resolution of
the PSD ionization-chamber system.

Ortec P-series PSD’s with a rectangular active area of 47x8 mm were
the solid-state detectors used in conjunction with the ionization cham-
bers to furnish measurements of \(E’\) and \(X\). Somewhat different from
surface-barrier detectors in principle, they consist of a rectangular
slab of extremely uniform n-type silicon which has on its front side a
very thin p-type surface layer created by direct implantation of boron
ions. An evaporated aluminum contact covering the back surface of the
silicon slab is electrically connected to a microdot connector located
centrally on the detector-housing, and the short sides of the boron-implanted layer are connected to microdot connectors at either end. The p-type layer is resistive and essentially dead as far as charge collection is concerned (thus contributing a small energy loss), but its interface with the silicon base constitutes a p-n junction from which, on biasing, a depletion layer extends into the silicon.

Relative positive bias applied to the aluminum contact sweeps electrons resulting from ionization in the depletion layer to the back and holes to the p-type layer in the front. The electron signal is simply proportional to \( E' \), and in our experiments, was processed by an Ortec model 124 pre-amp with a modified bias resistor of 10 MΩ. However, hole signals may be taken from either end of the resistive p-layer, which, with the distributed capacitance of the depletion layer, forms an RC transmission line having the property that signals are attenuated in proportion to the distance they travel. When normalized by \( E' \), these "p"-signals yield a measurement of \( X \), the horizontal distance of the ion's impact from that end of the resistive layer from which the signal is taken.

In our experiments, we grounded the forward-angle end of the resistive layer to the ionization chamber and took the P-signal from the backward-angle end, employing an Ortec model 125 pre-amp at virtual ground (no bias). We did this to insure that the large-amplitude signals would suffer the greatest attenuation, but in fact we still lost the first few millimeters of position sensitivity because of the very small final amplitude of any signals having to travel the entire distance
across the detector. This problem was solved before our second experi-
ment by the insertion of a 10MΩ resistor between the chamber-ground and
the microdot connector at the high-energy end of the detector. The
additional resistance, which was about 10% of the total resistance
across the implanted layer, had the effect of shifting the P-signal
amplitude upward by a constant amount; yet it did not adversely affect
the position linearity.

The maximum rated bias for these detectors is 200 V, and this gives
a minimum depletion depth of 100μ. However, we found that in our modest-
energy heavy-ion experiments, half this value was sufficient for fully
efficient charge collection. Keeping the bias down and cooling the PSD’s
with a Peltier element placed on the back wall of the ionization chamber
(the chamber wall in turn being water-cooled) enabled us to achieve sub-
microamp leakage currents and energy resolution of 1.5-1.8%, depending
on the detector and details of its installation. The forward detector
(serial number 21-681C) in earlier use had been subjected to higher
rates and hence more radiation damage than its backward counterpart (21-
642E). It drew about twice the leakage current (.7-1.3 μA cooled) and
had slightly poorer energy resolution. The position resolution of both
detectors was very similar - about .5 mm.

3.3 Experiment I: Apparatus.

Figure 3.2 illustrates the experimental arrangement for the $^{14}\text{C}+^{16}\text{O}$
and $^{14}\text{C}+^{12}\text{C}$ experiment in the 30" Ortec scattering chamber at
Brookhaven’s tandem laboratory. The "forward" detector covered
FIGURE 3.2 Experimental arrangement for the $^{14}\text{C}+^{16}\text{O}$ and $^{14}\text{C}+^{12}\text{C}$ experiment.
SINGLES EXPERIMENTS $^{16}O(^{14}C,^{14}C)^{16}O$ and $^{12}C(^{14}C,^{14}C)^{12}C$

TARGET 1: $30 \mu g/cm^2$ Be
$6 \mu g/cm^2$ O
$\approx 8-40 \mu g/cm^2$ C

TARGET 2: $100 \mu g/cm^2$ Au

ANGULAR RANGES
FORWARD $20^\circ \leq \theta \leq 32.5^\circ$
BACKWARD $34^\circ \leq \theta \leq 49^\circ$
laboratory angles from 20° to 32.5°, acquiring data at the most forward and backward center-of-mass scattering angles, while the "backward" detector subtended the range 34° to 49°, and provided the bridge between the two parts of the angular distribution measured by the forward detector. The forward and backward detector's PSD's were located 175 mm and 147 mm from the center of the scattering chamber, respectively. This geometry was dictated by the kinematics of $^{14}\text{C} + ^{18}\text{O}$ scattering. In the $^{14}\text{C} + ^{18}\text{O}$ system, the laboratory scattering angle of scattered $^{14}\text{C}$ becomes larger than that of the corresponding recoil $^{18}\text{O}$ nuclei at 48.2°. Going up to 49° in the backward detector thus brings about a small overlap of recoil and scattered particle cross-section measurements in the center-of-mass. After this angular limit was selected, the backward detector was moved as close to the target as possible without putting it in the way of the beam. Then, the forward detector was placed so that there would be a small overlap between its range and that of the backward detector. In practice, however, the loss of position-sensitivity in the first few millimeters of the PSD's resulted in a loss of this overlap; thus, the ranges quoted here are the actual sensitive ranges of the detector. Count-rate limitations prevented us from moving the forward detector closer to the target and looking at more forward angles.

The "turnaround" angles for $^{14}\text{C}$ on $^{16}\text{O}$ and $^{14}\text{C}$ on $^{12}\text{C}$ are only 46.8° and 42.6°, respectively; thus, we had ample recoil and scattered particle overlap in the backward detector for the systems. In fact, the overlap was so large in the $^{14}\text{C} + ^{12}\text{C}$ data, (almost 20° cm) that it turned out to be very useful for angular calibration. We found that in order for the strong oscillatory structure in the overlapping parts to
coincide precisely (and also to align with the not-quite-overlapping forward detector data), the location of the beam spot had to be .55 mm upstream and 1 mm to the right (looking downstream) of the geometrical center of the scattering chamber. Thus, not only did we solve the usual problem of determining where beam optics and target ladder geometry put the beam spot, we also obtained a sensitive calibration of the absolute scattering angle. The detector angles had been surveyed very carefully telescopically with reference to the baseplate angles of the Ortec chamber, readable to .05°. Assuming that the indicated repositioning of the beam spot implies an average shift in laboratory angle from these values of +.22° for the forward detector and -.42° for the backward detector. The accuracy of our angular calibration is therefore determined by the sensitivity of the "structure-matching" method in establishing these shifts. We would estimate it at .1° or slightly better. However, the angular resolution of our detectors depended primarily on uncertainties arising from a 2x2 mm beam spot, .5 mm PSD resolution, and multiple scattering in the target and entrance window. These effects, added in quadrature, typically yield an angular resolution of .40° (FWHM). This was quite adequate for our purposes, as the angular acceptance for individual data points was always at least this large.

Since the PSD's provided a continuous position scale, it was convenient for both angular calibration and data integration to "mark" the position spectra by placing a 13-slice brass mask over the detector faces. Thin bars separating the slices cast sharp shadows in the position spectra. These shadows are then used to calibrate the angle scale
for the position spectra and to delimit the angular acceptance for each of 13 data points (or 26 if we choose to split slices). The average slice widths are 1.0° (forward) and 1.3° (backward), but in both detectors we have made one slice 50% larger and one 50% smaller than the remainder so that all slices may be identified unambiguously.

3.4 Experiment I: Beam, Target, and Normalization.

$^{14}$C beams of 20-40 nA intensity were obtained from the MP-7 tandem Van de Graaff accelerator at Brookhaven using a solid cone-type sputter source. Proceeding from 20 MeV in 350 keV steps, we measured 59 angular distributions in all, reaching 40.3 MeV finally. The time and energy ordering of the runs was the same. Collimation of the beam was done with pairs of vertical and horizontal slits approximately a meter apart, the last set just outside the scattering chamber. The downstream settings were 20x20 mm and the upstream ones 1.5x1.5 mm, which gave us a square spot about 2 mm on a side. Larger anti-slit scattering collimators were also required inside the chamber.

There were actually two targets used in this experiment, as figure 3.2 illustrates. The primary target was a foil of oxidized beryllium (Be and BeO) approximately 35 μg/cm$^2$ thick about which more will be said below. The secondary target was of gold, about 100 μg/cm$^2$ thick, and was used for relative normalization from one angular distribution to the next. Symmetric monitors (50 mm$^2$ Ortec surface barrier detectors) were placed at the rear of the scattering chamber so that they could see only this target. These detectors monitored of the right-left steering of the beam (which was found to be extremely stable during the experiment) and
provided a run-to-run estimate of the relative number of beam particles incident on the two targets via the $1/E_{cm}^2$ energy-dependence of the strictly Rutherford $^{16}$C+Au scattering.

As was mentioned earlier, it was intended that the primary target in this experiment would be highly enriched in $^{18}$O, but in fact it was not. A number of these targets had been made for us at the University of Pennsylvania using focussed light source oxidation of pre-mounted beryllium foils in an $^{18}$O atmosphere, supposedly 98.7% enriched. It is interesting to note that even though the heated portion of the foil changes from metallic in appearance to whitish and translucent as it is oxidized, the oxidation is actually only partial. Too much oxidation invariably results in loss of strength and breakage.

After the first experiment at Brookhaven, definitive elemental abundance tests were performed at Yale in our own 30" Ortec chamber with beams of 15 MeV $^{16}$O from the MPI tandem Van de Graaf accelerator. A single Ortec surface barrier detector (50 mm$^2$ and <1% resolution) was mounted with a 1" slit collimator 25 cm from the target to be tested. Our idea here was to obtain precise abundances by exploring the very deep Mott scattering minimum near 41° at this sub-barrier energy. Its presence would clearly differentiate $^{16}$O+$^{16}$O scattering from all other contaminants. Absolute abundances for all of the target constituents could also be estimated using the beam current integration (BCI).

We tested an identical twin of the target actually used in the Brookhaven experiment and observed the striking minimum in the $^{16}$O+$^{16}$O cross-section near 41°, shown clearly in the spectra of figure 3.3. With
FIGURE 3.3 Energy spectra from the 15 MeV $^{16}$O-beam target composition tests at Yale. The horizontal scale for each spectrum is given in arbitrary but consistent energy units (histogram bin number). The vertical scale is proportional to counts but normalized to the peak from $^9$Be ions (recoiling from a Coulomb scattering with $^{16}$O). Laboratory scattering angles are indicated in each spectrum. The Mott (identical particle Coulomb) scattering cross-section for $^{16}$O here is given in the lower right figure. Note the striking decrease in the $^{16}$O+$^{16}$O cross-section at the Mott minimum, $41^\circ$. 
this positive identification of $^{16}\text{O}$, we found an $^{16}\text{O}/^{18}\text{O}$ particle ratio of 5.9±0.2 or 85.5%. Absolute thicknesses in $\mu\text{g/cm}^2$ were as follows: (1)$^{18}\text{O}$, 1.0±0.2, (2)$^{16}\text{O}$, 5.0±0.5, (3)$^9\text{Be}$, 29.0±2.0, (4)$^5\text{Si}$, ≤0.5, and (5)$^w$ or $^\text{Ta}$, <0.1. With such a target, the full energy width in the experiment would have been 170 keV for 20 MeV $^{14}\text{C}$ beam and 115 keV for 40.3 MeV $^{14}\text{C}$ beam (these are total energy losses in the laboratory). However, the high level of pump oil contamination in the scattering chamber at Brookhaven apparently resulted in the deposition of as much as 40 $\mu\text{g/cm}^2$ of carbon (98.9% of which is $^{12}\text{C}$) on the target during the course of the experiment. This brought the total energy-loss up to 250 keV at 40.3 MeV, but provided a quite suitable target for studying $^{14}\text{C}+^{12}\text{C}$.

3.5 Experiment I: Electronic and software logic.

Electronic processing of signals from the pre-amplifiers was done in the control room of the BNL tandem laboratory using NIM-standard modules almost exclusively of Ortec manufacture, the one exception being BNL-built analog-divide units. The arrangement was as shown in figure 3.4, and its important features may be summarized as follows. (1)Triple coincidences were required among $\Delta E$, $E'$ and $P$ (with pile-up rejection on $E'$) in order to generate a logic trigger for the data acquisition interface of the Sigma VII computer, which handled data processing online. (2)True position signals, $X$, were obtained by analog division of $P$ by $E'$. (3)The total energy, $E$, was obtained by summing $E'$ and an attenuated $\Delta E$ (for this we used non-adjustable attenuator boxes installed on the sum module's $\Delta E$ input connector). (4)The gain of the $\Delta E$ amplifier had been adjusted so that $E'$ and the attenuated $\Delta E$ shared the same absolute
FIGURE 3.4 A schematic diagram of the electronic logic for either of the ΔE-E-X detectors in Experiment I.
energy scale. This "gain-matching" procedure involves setting the $\Delta E$ gain with gas in the ionization chambers so as to bring the summed $E$ signal to the same voltage as was seen when there was no gas in the ionization chambers. A gold target was placed on the target ladder to provide a convenient source of relatively monoenergetic scattered particles when this was required. (5) Final events consisted of $\Delta E-E-X$ triples from either detector. These were digitized in 1024-channel ADC's and written on tape when the computer interface received its logic trigger from the coincidence module. The digitized $\Delta E$ was of course not attenuated, as we wanted it to be as large as possible for the best possible digital resolution.

The Brookhaven tandem laboratory's data acquisition system is quite powerful and very easy to use, but was not adequately flexible to allow us to do online, high-resolution analysis of the specific type required. Thus, while running, we had to be content with somewhat nebulous views of $\Delta E-E$ spectra and unsorted, grossly-binned ones in $E$ vs. $X$. This was enough only to tell us that our data made sense; the real analyses were necessarily done offline at Yale. In fact, the BNL computer system was also suffering from a system malfunction, unrecognized at the time, which caused a regular cycling of the identities of the device outputs as written on event tape. This problem, fortunately, was discovered and easily corrected for in our offline analyses.
3.6 Event Tape Replays.

Event tapes were replayed on our IBM 4341 computer in this laboratory with a Fortran event routine written specifically for this purpose, the basic operation of which was relatively straightforward. We began by forming, for each run, two 128x128 histograms of $\Delta E$ vs. $E$; one for the forward detector (EDEF) and one for the backward detector (EDEB). Examples are shown in figures 3.5 and 3.6. The distinctive curves in these histograms give $\frac{dE}{dx}$ for the various charge species as they pass through the ionization chambers, and may be gated to isolate a particular element. Our event routine allowed for two gates on each $\Delta E-E$ histogram, and its logic was such that if the coordinates of the $\Delta E-E$ pair from a given event triple $(\Delta E-E-X)$ fell within one of these gates, another two-dimensional histogram was incremented by one count at coordinates specified by the $E-X$ pair of that triple. Thus, we were able to generate a separate energy vs. scattering angle histogram for two different charge species in each detector. The resulting spectra were 127x256 bin objects ($E$ and $X$ respectively) for which the degree of compression from the 1024-bin ADC data and the zero offset are adjustable by the event routine for best range and resolution.

Of course, in practice, we chose to gate the carbon and oxygen lines in EDEF and EDEB, and thus generate four $E$ vs. $X$ histograms which contained kinematically resolved data on the reactions of interest. The four histograms were: (1) carbon-gated forward (EXCF), (2) oxygen-gated forward (EXOF), (3) carbon-gated backward (EXCB) and (4) oxygen-gated
FIGURE 3.5 Forward angle $\Delta E$-$E$ histogram (EDEF) for run 61 data, where the $^{14}\text{C}$ beam energy was 36.1 MeV. The $\Delta E$-$E$ gates are also shown.

The size of a point in this figure and all such figures is proportional to $(N-\text{min})/(\text{max}-\text{min})$ as long as $\text{min} \leq N \leq \text{max}$ (where $N$ = number of counts, $\text{min}$ = minimum cut-off, and $\text{max}$ = saturation point). No point is plotted if $N<\text{min}$, the largest point is plotted when $N>\text{max}$, and there are sixteen levels of size. Here we have set max=700 counts and min=2 counts in order to show the $\Delta E$-$E$ regions more clearly. Some points within the carbon gate actually represent as many as 10000 counts. In subsequent figures, min and max are chosen to enhance clarity, but min is never greater than 2, and is 1 unless specifically noted.

FIGURE 3.6 Backward angle $\Delta E$-$E$ histogram (EDEB) for run 61 data. Gates are also indicated. Min=2
Individual reactions were positively identified by plotting their properly calibrated energy vs. angle kinematics over all of the histograms at several experimental energies. One of these kinematical comparisons was done for run 61, and its EXCF part is shown in figure 3.7. It was through this analysis that we first discovered the poor $^{18}_0$ enrichment of our target.

Knowing the identities of the reactions from their kinematics in the $E$ vs. $X$ histograms, we then proceeded to gate and project the loci of interest onto the $X$-axis. Thus we obtained, finally, one dimensional histograms which could be integrated between the shadows left by the PSD masks to obtain yields for angular distributions. When statistics were adequately large as in the higher energy $^{14}_C + ^{12}_C$ data, we split the slices, integrating between the PSD mask shadows and points lying halfway between them. Figures 3.8 and 3.9 contain examples of the gates, and in figure 3.11 we show all of the $X$-projections for run 61.

In order to minimize discretization errors in selecting slice boundaries, we rebinned the projection histograms on a denser mesh (1024 bins instead of 256). This was particularly important when the slices were split. The bin numbers corresponding to the centroids of the mask shadows in each histogram were found for each run by inspection and used as input to a routine which performed the yield integrations and calculated the cross-sections.

Since forward recoil-particle laboratory scattering angles correspond to backward center-of-mass scattering angles, angular
FIGURE 3.7 Forward angle carbon-gated E-X histogram (EXCF) for run 61 data. Min-2. Kinematical loci for various interactions are also plotted and labelled by target particle (the detected particle is always a carbon ion here). A star indicates inelastic excitation of the target particle. Rearrangement reactions are fully labelled in the usual way. The PSD mask shadows are clearly visible as gaps in the data at constant X.

FIGURE 3.8 Backward angle carbon-gated E-X histogram (EXCB) for run 61 data. Min-2. Also shown are the gates for the X-projections from which final yields were obtained. The reaction $^{12}\text{C}(^{14}\text{C},^{13}\text{C})^{13}\text{C}$ is quite prominent here.

FIGURE 3.9 Backward angle oxygen-gated E-X histograms (EXOB) for run 61 data. The gate for the X-projection is shown.

FIGURE 3.10 Forward angle oxygen-gated E-X histogram (EXOF) for run 61 data. The locus of recoiling $^{16}\text{O}$ ions is indicted. The data in the lower left corner arise from reactions of $^{14}\text{C}$ with $^9\text{Be}(^9\text{Be}(^{14}\text{C},^\alpha)^4\text{He})$, which proceed via three-body kinematics.
FIGURE 3.11 X-projections for the $^{14}$C+$^{12}$C angular distribution measurement of run 61. The projections are labelled by their parent histograms. Here, the number of counts in each position (scattering angle) bin is plotted against bin number. The corresponding laboratory scattering angle ranges are indicated in each plot. Positions of the mask shadows used for angular calibration and yield integration were obtained from these projections after rebinning them on a mesh four times denser (the resolution is the same - this simply amounts to splitting bins to avoid discretization errors).
distributions for $^{14}\text{C}+^{16}\text{O}$ in the center-of-mass frame were assembled from X-projections in the histograms EXCF, EXCB, EXOB and EXOF in that angular order. For $^{14}\text{C}+^{12}\text{C}$, all of the data had to come from EXCF and EXCB since we could not distinguish $^{14}\text{C}$ from $^{12}\text{C}$ using $\Delta E$-$E$ gating. Fortunately, though, the kinematics were such that recoil $^{12}\text{C}$ data were resolvable from scattered $^{14}\text{C}$ data in the $E$ vs. $X$ histograms everywhere except at very forward angles. We therefore did not use the very forward-most part of EXCF for the $^{12}\text{C}$-recoil data. The center-of-mass angular order of the X-projections for $^{14}\text{C}+^{12}\text{C}$ scattering was then EXCF($^{14}\text{C}$), EXCB($^{14}\text{C}$), EXCB($^{12}\text{C}$), and EXCF($^{12}\text{C}$).

Other interactions do not substantially interfere with those of interest here thanks to (1) the tight binding of $^{14}\text{C}$, $^{12}\text{C}$, and $^{16}\text{O}$, (2) the minimal presence of $^{18}\text{O}$, and (3) the favorable kinematics of elastic scattering for heavier target species. Attesting to this is the fact that the overlapping parts of the elastic scattering angular distributions are generally in very impressive agreement; yet they have a completely different kinematical origin (scattered and recoil). Only in very deep minima where one is most sensitive to background contributed by the few reactions whose $E$ vs. $\theta$ loci do cross those of the interactions of interest is there occasional disagreement larger than the statistical uncertainty. At the "crossing angles" these reactions (for example, $^{16}\text{O}(^{14}\text{C},^{12}\text{C})^{18}\text{O}$ ($Q=-.93$) and $^{16}\text{O}(^{14}\text{C},^{13}\text{C})^{17}\text{O}$ ($Q=-4.03$) were usually too small in cross-section to be clearly visible in the histograms, and thus did not contribute significant contamination unless the elastic cross-section was very small. The only prominent reaction not involving $^9\text{Be}$ which was observed this experiment was $^{12}\text{C}(^{14}\text{C},^{13}\text{C})^{13}\text{C}$
(Q=-3.23 MeV). This appears very strongly at higher energies (as in fig 3.8) but was always well-separated from the elastic scattering. Inelastic scattering involving $^{14}\text{C}, ^{12}\text{C}, \text{or } ^{16}\text{O}$, because of the high excitation energies involved, was generally lost in the lower-energy smear resulting from the $^9\text{Be}$ reactions, as were most of the non-$\text{Be}$ transfer reactions (expect in EXOB, where kinematics prohibits any beryllium-reaction contribution). It is also worth noting that almost all of the oxygen flux in the forward detector is contributed by $^9\text{Be}(^{14}\text{C}, ^n\text{He})^8\text{He}$ or $^{12}\text{C}(^{14}\text{C}, ^{18}\text{O})^8\text{Be}$ reactions, which are reasonably energy favored and proceed by multi-body kinematics, thus "filling-in" an entire region of E vs. X in EXOF (figure 3.10). Fortunately, none of these reactions can interfere with the backward angle $^{14}\text{C}+^{16}\text{O}$ scattering. Our conclusion, finally, was that with careful gating the elastic scattering data were free from any significant contamination from other reactions.

We discovered only two experimental problems in this analysis, and both were easily solved. First, it turned out in retrospect that the proper $\Delta E$ attenuator for the summed E signal in the backward detector's electronics had not been installed. Thus, the $\Delta E$ gain-matching was way off in the backward detector and the total E-resolution poor (the $\Delta E$ signal in the sum was much too large). This problem was solved in the offline processing by subtracting $\alpha \Delta E$ from E, where $\alpha$ was a fixed gain-matching constant, chosen so as to maximize the total E resolution. This actually worked as well in the end as the hardware gain-matching in the forward detector. The second problem was that the hardware-divided X-signals occasionally suffered from energy-dependent nonlinearity which
caused the angular calibration of the X-projections to shift. The solution here was simply to establish bin locations for all the mask shadows in every run and recalibrate each time. Actually, this would have had to be done several times anyway, as amplifier gains had to be adjusted periodically as the beam energy was changed.

3.7 Experiment I: Extraction of the Cross-Sections.

Since we integrated our data across PSD slices or half-slices, we viewed each slice as a separate detector and extracted laboratory cross-sections according to

\[
\frac{d\sigma(\theta_i)}{d\Omega} = \frac{Y_i}{\Omega_i N_t N_b}
\]

where \( Y_i \) is the yield in slice \( i \), \( \Omega_i \) the solid angle, \( \theta_i \) the laboratory scattering angle, \( N_b \) the number of beam particles incident on the target, and \( N_t \) the number of target particles per cm\(^2\). Passage to the center of mass is easily accomplished using the appropriate recoil or scattered-particle solid-angle Jacobian, and the data are finally displayed and tabulated as ratios to the Rutherford cross-section. Uncertainties plotted in all the figures are purely statistical in origin.

Overall normalization of the data was provided by external fixed points, generally, so the detector absolute solid angle was not critical. However relative variations in the solid angles of slices within a given detector were determined from a comparison of \(^{14}\text{C} + \text{Au}\) Rutherford scattering data with the known theoretical cross-section. In this determination we used the data of run 62, a 36.1 MeV \(^{14}\text{C}\) on gold-target test.
run which had been carried out explicitly for this purpose. Since high count-rates in this run led to a large dead-time difference between the detectors, however, forward vs. backward detector normalization was done using the very flat $^{14}$C+Si data in the 21.1 MeV run 15 (a production run for $^{14}$C+$^{12}$C and $^{14}$C+$^{16}$O). The absolute scale for solid angle was fixed by taking the average solid angle of a slice in the backward detector to be that obtained from the surveyed detector geometry alone ($1.04 \times 10^{-3}$ sr).

Since $N_t$ was fixed for $^{16}$O but variable for $^{12}$C, we had to employ different overall normalizations for the two data sets. To fix the normalization for the $^{14}$C+$^{16}$O data, we used the forward angle data of run 8 at 20 MeV, the lowest beam energy in the experiment. The ratio to the Rutherford cross-section exhibited a plateau here; so we took it to be unity in the slice most forward in angle. From this we determined $N_b N_t$ in run 8. It was then possible to find $N_b N_t$ for all other runs using the yields of the secondary target monitors with the relation

\[
(N_b N_t)(x) = \frac{(N_b N_t)(8)}{Y_{mon}(8) E^2(8)} Y_{mon}(x) E^2(x) .
\]

A number of useful consistency checks for all of our experimental information can be performed with various separate estimates for $N_t$ and $N_b$. For example, $N_t$ can be obtained from the abundance experiment and $N_b$ can be estimated from either run time and beam current or nominal monitor geometry and estimated secondary target thickness. The product $N_b N_t$ was always within 15% of the value which we used in the analysis ($1.295 \times 10^{31}$/cm$^2$ in run 8).
The normalization of the $^{14}\text{C} + ^{12}\text{C}$ data was done with reference to the $77^\circ$ excitation function of Konnerth et al. (Ko85). Their data were taken with a larger energy stepsize and range, and the $77^\circ$ excitation function was most forward in angle, but the lower energy $77^\circ$ data are adequately smooth to allow us to define fixed points in our $^{14}\text{C} + ^{12}\text{C}$ data from them. We use their points nearest 20 and 26.3 MeV, (performing a small local interpolation in energy and angle) and normalize our data to theirs by adjusting $N_b N_t$. We know $N_b(\theta)/N_b(8)$ from 3.7.2, and so we may normalize the entire $^{14}\text{C} + ^{12}\text{C}$ dataset using the assumption that $^{12}\text{C}$ target growth is linear in time, the slope determined by the two fixed points. The time, energy, and run number ordering of all runs was the same, so the target thickness was taken to be proportional to run order (bad runs were always short, so run number is not as accurate). This assumption results in a forward excitation function which is very smooth except at 27.35 MeV where it shows a sudden but explainable drop. The target had been repositioned vertically just before this run, and so the effective $^{12}\text{C}$ target thickness was less, as the beam spot was now slightly higher on the target. The solution, then, was to assume linear $^{12}\text{C}$ growth with a discontinuity at 27.35 MeV sufficient to make the forwardmost ($43.8^\circ$) excitation function smooth there. Finally, we should note that if one takes the $^{16}\text{O}$ target thickness to be $5\mu\text{g/cm}^2$, we find with this assumption that the $^{12}\text{C}$ target thickness ranged from 8 to $40\mu\text{g/cm}^2$ over the course of the measurement.
3.8 The \(^{14}\text{C} + ^{16}\text{O}\) Data.

Figures 3.12 and 3.13 display the \(^{14}\text{C} + ^{16}\text{O}\) angular distributions and excitation functions plotted as ratios to the Rutherford cross-section against center-of-mass scattering angle. Following these, in figure 3.14, is a cross-section surface-plot which summarizes the entire dataset.

The construction of surface plots merits a detailed explanation, as they are displayed and discussed throughout our analysis work, the subject of the next chapter. The procedure is as follows. We begin with a set of center of mass angular distributions expressed as ratios to the appropriate Rutherford scattering cross-section. If the angular distributions are calculated from a model, we obtain smooth curves for a surface plot by using a fine angular mesh in the calculation. However, smooth curves for our measured angular distributions are obtained from our data by interpolation with cubic splines (this is conveniently accomplished using the function SMOOTH in SPEAKEZ and provides essentially perfect fits). To enhance the three-dimensional quality of the surface plots, we "fill in" the set of angular distributions by inserting two additional curves between each pair of original ones. These are obtained simply by linear interpolation. Finally, the cross-section surface is generated by plotting all of the angular distributions on a broken logarithmic scale where the scale shift between successive plots is small and the lower energy curves "shadow" the higher energy ones. (It is as though each curve represented the upper boundary of an opaque white sheet). At regular energy-intervals, the angular distributions are labelled by the corresponding center of mass scattering energy, the label tick marks located at the Y-location of the first (lowest-angle)
point in the curve. This results in a slightly nonuniform energy scale in the measured surface-plots because of forward angle excitation function effects. In all of our surface plots here, the "local" logarithmic scale for a given angular distribution is the same - there are about nine decades between the upper and lower borders of the plots. The size of the scale shift between curves and the overall offset is sometimes adjusted to make the surface fit within the plot boundaries, but this adjustment is generally not necessary except in surfaces where the cross-sections are very small at backward angles.

In the surface-plot of figure 3.14 we see at a glance the essential characteristics of our $^{14}\text{C} + ^{16}\text{O}$ data. The low energy angular distributions are quite smooth, with cross-sections falling monotonically as a function of angle. Broad, rolling structure begins to be evident around 12.5 MeV (center-of-mass). At 15.5 MeV and above, the angular distributions have very pronounced oscillatory structure and a significant backward-angle rise. Alternating crests and troughs can also be seen at fixed angles in the surface quite clearly. The same structures manifest themselves as 3-4 MeV-wide undulations in the excitation functions, which display only a modest amount of finer-scale structure.
Scattering. Elastic scattering cross-sections here and throughout this work are expressed as ratios to the corresponding Rutherford cross-section and displayed on a broken logarithmic scale. Individual angular distributions are labelled by the corresponding center of mass scattering energy. The uncertainties shown here are purely statistical in origin. Lines connecting the data points are provided (when necessary) to guide the eye.

FIGURE 3.13 Center of mass excitation functions for $^{12}$C+$^{16}$O elastic scattering. These are the same data points shown in figure 3.12; they are merely viewed in a different way. Each excitation function is labelled by the corresponding center of mass scattering angle (Note: it is a special property of nonrelativistic elastic scattering that the transformation from laboratory to center of mass angle is independent of the scattering energy).

FIGURE 3.14 Cross-section surface for $^{12}$C+$^{16}$O elastic scattering. The procedure for its construction from our data is discussed in the text. The ordinate is center of mass scattering angle. The vertical axis is labelled by the center of mass energy corresponding to the curve opposite the ticks marks, but the abscissa is actually $\log(\sigma/\sigma_R)$ for each individual curve.
$^{14}$C $^{16}$O ANGULAR DISTRIBUTIONS

$\sigma/\sigma_{\text{RUTH}}$

$\theta_{\text{CM}}$

30 40 50 60 70 80 90 100 110 120 130 140
$^{14}$C + $^{16}$O EXCITATION FUNCTIONS

$\sigma / \sigma_{\text{RUTH}}$ vs. CM ENERGY (MeV)
$^{14}$C $^{16}$O EXCITATION FUNCTIONS

CM ENERGY (MeV)
3.9 The $^{14}\text{C} + ^{12}\text{C}$ Data.

Individual angular distributions and excitation functions for $^{14}\text{C} + ^{12}\text{C}$ elastic scattering are shown in figures 3.15 and 3.16. The data are also shown as a surface in figure 3.17 with the same relative scale as that of figure 3.14. We use the same relative scale on all of the surfaces in this work.

The first obvious feature of these data is the very large scale of their angular structure and the backward rise, both of which are much more pronounced than in $^{14}\text{C} + ^{16}\text{O}$ even if we correct for the lower barrier energy. There are fewer extrema in the angular distributions, reflecting the lower grazing partial wave involved, and the alternating crest-trough structure is less clear, while the large angle minima are more static in angle. Correspondingly, the excitation functions do not have the same clean appearance as do those for $^{14}\text{C} + ^{16}\text{O}$. Rather, there is plentiful 500-800 keV intermediate-width structure superimposed on less obvious, broad undulations of 2-3 MeV width. The 95.9° excitation function provides an especially good example of such structure.

Of particular interest is the 17-18 MeV energy region. The larger-angle excitation functions here are all deeply structured, and one can see in the surface that this is the only place where there is a break in the second major "ridge". There is no such break in the first ridge but the cross-section is generally small until very large angles. Also, if we look at individual angular distributions in this region, one sees the anomalous decrease in the number of oscillations which was mentioned in section 2.7.
FIGURE 3.15 Center of mass angular distributions for $^{14}\text{C}+^{12}\text{C}$ elastic scattering.

FIGURE 3.16 Center of mass excitation functions for $^{14}\text{C}+^{12}\text{C}$ elastic scattering.

FIGURE 3.17 Cross-section surface for $^{14}\text{C}+^{12}\text{C}$ elastic scattering.
$^{14}$C $^{12}$C ANGULAR DISTRIBUTIONS
$^{14}\text{C} + ^{12}\text{C}$ EXCITATION FUNCTIONS

$\sigma/\sigma_{\text{th}}$

CM ENERGY (MeV)
$^{14}\text{C} + ^{12}\text{C}$ EXCITATION FUNCTIONS

$\sigma / \sigma_{\text{RUTH}}$

CM ENERGY (MeV)
$^{14}C + ^{12}C$ EXCITATION FUNCTIONS

$\sigma/\sigma_{\text{RUTH}}$

CM ENERGY (MeV)
3.10 Experiment II: Apparatus.

It was necessary in this experiment to abandon one of the ionization chambers in favor of a bare PSD so that the low-energy, backward-scattered coincidence counterparts of forward-scattered particles could be detected with reasonably high resolution. This becomes clear when one considers that, with a 20 MeV $^{14}$C beam, the recoiling $^{18}$O nuclei left behind by $^{14}$C ions scattered to 18° (our forward-angle limit) have a kinetic energy of only 1.5 MeV! Such particles would not be able to give us $\Delta E$ signals adequate for processing without giving up most of their energy and suffering excessive multiple scattering. The resulting degradation of resolution in energy and angle would then have meant the loss of the critical kinematical information we need.

The arrangement of hardware for this experiment, also done in the 30° Ortec chamber at BNL, is shown in figure 3.18. Here the $\Delta E$-E-X counter is to be thought of as the primary detector. It performed the same function as either of the counters in the first experiment, measuring both recoil and scattered-particle cross-sections and remaining forward in lab angle to see higher-energy particles which it could process with high resolution. For all of the data in this experiment, scattering angles were defined only with reference to the position scale in this primary detector.

The bare E-X detector, on the other hand, was present strictly for the purpose of imposing logical constraints on the data. It provided us with the additional kinematical information which we needed to discriminate the $^{14}$C+$^{18}$O scattering from all other reactions. This "coincidence" detector was Peltier-cooled and mounted on a water-cooled
FIGURE 3.18 Experimental arrangement for the $^{16}_{\text{C}}^{18}_{\text{O}}$ experiment.
COINCIDENCE EXPERIMENT $^{18}_O(^{14}_C,^{14}_C)^{18}_O$

Primary Detector Angular Range

<table>
<thead>
<tr>
<th>Angular Settings</th>
<th>Primary</th>
<th>Coincidence</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>18.0°-33.3°</td>
<td>71.6°-93.0°</td>
</tr>
<tr>
<td>B</td>
<td>18.0°-33.3°</td>
<td>58.0°-79.4°</td>
</tr>
<tr>
<td>C</td>
<td>33.0°-48.3°</td>
<td>47.1°-68.5°</td>
</tr>
</tbody>
</table>

Target: 9 $\mu$g/cm² Si
5 $\mu$g/cm² $^{18}_O$
2 $\mu$g/cm² $^{16}_O$
4 $\mu$g/cm² C
1.5 $\mu$g/cm² Ta
aluminum block. We also installed a liquid nitrogen cold-finger in the scattering chamber to prevent excessive hydrocarbon buildup on to the cold, bare detector.

The distances of the PSD's from the target were chosen for the best compromise between angular resolution and angular range: 166.5 mm and 121.0 mm for primary and coincidence PSD's respectively. In figures 3.19 and 3.20 we display, separately, the angle vs. angle kinematics for the scattered and recoil particles most prominent in this experiment. The three dashed rectangles in the figures indicate the angular ranges of the primary and coincidence detectors at the three settings necessary to obtain full $^{14}\text{C} + ^{18}\text{O}$ angular distributions. In setting C (rectangle C), where the primary detector covers relatively backward angles, the coincidence detector is able to cover the coincidence angles for both $^{14}\text{C}$ and $^{18}\text{O}$ at once. However, when the primary detector covers forward angles, two different settings for the coincidence detector are required in order to cover the angular ranges of the very backward-scattered $^{14}\text{C}$ (setting A) and recoiling $^{18}\text{O}$ (setting B). Angular distributions for $^{14}\text{C} + ^{18}\text{O}$ may therefore be assembled from five primary-detector X-projections in the following center-of-mass angular order (where S stands for $^{14}\text{C}$ and R for $^{18}\text{O}$ in the primary detectors): B(S), C(S), C(R), B(R), A(R).

The only changes in this experiment, relative to experiment I, involving the counters themselves were the installation of the position-to-ground resistors on both PSD's (which allowed us to use the full sensitive width of the detectors), the improvement of electrical connec-
FIGURE 3.19 Coincidence angle vs. primary angle kinematics for the elastic scattering of $^{14}\text{C}$ from various target constituents, as labelled. These kinematics apply when a carbon ion is scattered into the primary detector. The three rectangles A, B and C delimit the kinematical regions spanned by our detectors in the corresponding angular settings. Note that only settings B and C contribute data on the $^{14}\text{C} + 1\text{E0}$ scattering in this situation.

FIGURE 3.20 Again, coincidence angle vs. primary angle kinematics for elastic scattering involving $^{14}\text{C}$, but this time with carbon ions scattered into the coincidence detector. Here, it is primarily settings A and C which subtend the angular ranges of interest for $^{14}\text{C} + 1\text{E0}$ scattering. The kinematics here correspond to backward angle scattering in the center of mass system, while those in figure 3.19 correspond to forward hemisphere scattering. The division here is natural, as we are able to use $\Delta E$-E information to determine what kind of ion strikes the primary detector.
KINEMATICS FOR 30 MeV $^{14}$C BEAM

\[ \theta_c \]

$^{18}$O*: 1.98 MeV

$^{12}$C recoil

$^{28}$Si

$^{18}$O

$^{12}$C

$^{16}$O
KINEMATICS FOR 30 MeV $^{14}$C BEAM

$\theta_c$

$\theta_p$

$^{16}$O $1.98 \text{ MeV}$

$^{18}$O

$^{28}$Si
tions within the ionization chambers, and the replacement of the 13-slice masks with only three slim bars for angular calibration. The latter modification was necessary because interfering shadows from 13-slice detectors in coincidence would make solid angle estimation very difficult, if not impossible. Even as it was, solid-angles had to be calculated by Monte Carlo simulation (which we discuss in section 3.14). Since we were aware that the coincidence requirement made detector geometry very critical, we spent much time prior to the experiment to insure that all detector elements were aligned with sub-millimeter precision. Good beam collimation was achieved with the same slit settings as before, and again, anti-slit scattering collimators were required and installed inside the scattering chamber.

3.11 Experiment II: Beam, Target, and Normalization.

Here again we used a $^{14}$C-beam from Brookhaven's MP-7 tandem Van de Graaff, but time-economy forced a slightly larger energy step and lesser energy range than in experiment I. We measured first from 20 MeV to 32.5 MeV in gross 2.5 MeV steps, and then returned to 20.4 MeV, and proceeded upward to 30 MeV in 400 keV steps.

As for the targets used in experiment II, it should be pointed out that continuing attempts at fabricating hot-light oxidized $^{18}$O foils involving both L.Csihas of U. Penn and the author had been somewhat more successful. We achieved up to 43% $^{18}$O-enrichment, but even these were not adequate to permit consideration of non-coincidence measurements with them. It was clear that such targets are always subject to contamination by $^{16}$O, in spite of every reasonable precaution in storage.
(under vacuum with dessication) and manufacture. It is possible that the Be foils themselves had been partially oxidized as they were evaporated, in as much as Be is known to be a good getter for oxygen at $10^{-5}$ torr and below. The continuing exchange of target-$^{18}$O with $^{16}$O in residual air could also contribute to the problem. In this connection, it has been suggested that humidity could be the critical factor with the $^{16}$O exchange occurring through an H$_2$O intermediary.

The target manufacturing efforts continued with the production of $^{18}$O enriched Al$_2$O$_3$ foils for us by J. Heagney of Micromatter Inc. (Seattle, WA). These were fabricated by anodizing aluminum foil in 98% enriched $^{18}$O-water and subsequently etching away the unoxidized Al metal. According to $^{16}$O-beam abundance measurements however, these beautiful, glass-like foils were in fact only ~50% enriched in $^{18}$O.

A third attempt at pure $^{18}$O targets was also carried out for us by Micromatter staff; 98.7% $^{18}$O-enriched silicon oxide, provided to Edith Fehr of this laboratory by the Weizmann Institute (Rehovoth, Israel), was evaporated from an electron-beam-heated tantalum boat onto a thin carbon backing. The highest enrichment attained here was 73%, again not adequate for a non-coincidence experiment, but the best we were able to produce. Since oxygen compounds with elements of higher Z such as V$_2$O$_5$ or WO$_3$ would only load our forward angle detectors with excessive Coulomb scattering, we settled upon a coincidence experiment with one of these SiO targets.

As a test, a few of these targets had been produced with very rapid evaporation from a very hot Ta boat. The $^{16}$O-beam abundance test on
these revealed that they bore a slight, but very stable, Ta contamination which proved to be very useful for normalization purposes. The Ta contamination was not sufficient to overwhelm our forward detectors with high count rates, and did not even appear in our coincidence data, while a monitor detector placed on the back wall of the Ortec chamber could always detect $^{14}\text{C}+\text{Ta}$ scattering, and so enable us to extract $N_b$ for any run. We chose one of these targets for our coincidence experiment. The areal density of the constituents of this target, in $\mu\text{g/cm}^2$, were as follows: (1)Si, 8.6±2, (2)$^{18}\text{O}$, 4.60±10, (3)$^{16}\text{O}$, 2.00±10, (4)C, 3.90±10, (5)Ta, 1.60±10. This gives an $^{18}\text{O}/^{16}\text{O}$ enrichment, by particle, of 67.0%. Interestingly, the oxygen/silicon particle-ratio here is only 1.23, which means that the silicon oxide compound, at least after evaporation, is mostly SiO and SiO$_2$ (the SiO targets evaporated more slowly generally had oxygen/silicon particle ratios around 1.08 together with slightly higher enrichment and no Ta contamination).

The energy width of this target, for 20 MeV $^{14}\text{C}$ beam, was about 90 keV prior to carbon build up. By the end of the experiment, however, substantial buildup had occurred, and the total energy-loss was estimated to be approximately 200 keV for 30 MeV $^{14}\text{C}$ beam. The target was mounted in the scattering chamber with its normal at an angle of 21° relative to the beam, facing slightly in the direction of the coincidence detector.

Overall normalization in this experiment was provided by knowledge of $N_c$ for $^{18}\text{O}$ and Ta from the abundance experiment. Relative normalization of the ninety useful runs was provided using a 50 mm$^2$ Ortec surface barrier detector at 20.5°, which easily resolved the peak from $^{14}\text{C}+\text{Ta}$.
scattering and thus provided $N_b$ in the same way as the secondary target monitors in the first experiment. The overall error in our normalization not more than 10%.

3.12 Experiment II: Electronic and Software Logic.

As one can see in figure 3.21, where the signals from the primary and coincidence detectors are labelled by a "1" and a "2" respectively, we employed more or less the same electronic hardware here as in the first experiment. The notable difference in this experiment, of course, is that we require coincidences between the two counters, and not triple or double coincidences of signals from a given one.

We arranged the electronics so that coincidences within two-microseconds between $E_1$ and $E_2$ would trigger the data acquisition interface and enable the output in all of the linearizing modules (Ortec 442 linear gate stretchers). The interface would then read all the ADC's, store their contents, and periodically dump them to the computer for storage on event tape. Early in the experiment, five output devices were stored for each event: $\Delta E$, $E_1$, $P_1$, $E_2$, and $P_2$. However, at 21.2 MeV we changed the system to record septuples: $\Delta E$, $E_1$, $P_1$, $X_1$, $E_2$, $P_2$, $X_2$.

Accidental coincidences between $E_1$ and $E_2$ would occasionally result in storage of invalid events, but these were always detectable during event tape replays and were easily eliminated. A more serious problem was that some of the ADC discriminators had been set too high early in the experiment and good events with very low-energy coincidence particles were lost. This problem was responsible for the loss of extreme
FIGURE 3.21 Schematic diagram of the electronic logic for the $^{14}\text{C}+^{18}\text{O}$ coincidence experiment.
forward angle points in earlier runs (visible as gaps in the cross-section surface of figure 3.30), but, fortunately, was solved before the 21.6 MeV run.

Analysis of the coincidence data required a much more powerful and flexible event processing and display capability than did the singles data of the first experiment. Once again, Brookhaven's online data acquisition system was not useful for our analyses although it permitted us to manage the experiment itself (monitoring the effect of gain adjustments and so forth).

3.13 Experiment II: Event Tape Replays.

Although each of the three angular settings required a different event routine - because of the different kinematical situations which it involved - the event tape analyses at Yale involved essentially the same procedures as were employed in the singles experiment. The general theme was that we gated on the desired charge species in a $\Delta E-E$ histogram (here "EDE") and then filled other histograms according to data from various combinations of the output devices (e.g. $E_1$ vs. $X_1$, $X_2$ vs. $X_2$, or mass2 vs. mass1). Sometimes these histograms were gated further, but finally, projections to one-dimensional histograms on the $X_1$-axis were required to yield angular distributions. The monitor data did not require event processing and were not stored on event tape. The yield for $^{14}$C scattering on tantalum was found by integrating the appropriate peak in complete spectra recorded on tape with other online spectra at
the end of each run. (The secondary target monitors in the first experiment had been treated similarly). The monitor spectra were the only online spectra that were used in our analysis.

The event routines used to replay the data at Yale allowed for very flexible scaling and binning so that we could take better advantage of the 1024-bin ADC resolution with histograms of manageable size. Here we used 128×128-bin arrays exclusively.

The energy resolution of our detectors was such that the smallest useful compression factor was 2, but values of 3, 4 or 5 were more common, as we often needed to view a large energy-window in the ADC spectrum, but not the complete spectrum. When it was necessary to focus on a very narrow energy-window and yet be able to see at once all the data of interest, we had to provide for straightening of the kinematical loci in the E vs. X analyzers. This was done with an event-routine transformation of the form \(E_s = E + \alpha X\), where \(E_s\) is the "straightened" energy and \(\alpha\) an adjustable parameter chosen to adjust the slope of the locus. Its effect is apparent in the very expanded E vs. X spectrum of figure 3.27b.

Position scaling was relatively straightforward. We required no ADC windows, but chose compression factors such that the range of the relevant position data filled as many of the 128 bins as possible. A more complicated issue was whether to use the pre-divided X or raw P data in our event processing, since we could alternatively obtain excellent position information by dividing the P-data by \(E'\) in software (\(E'\) was in turn obtained from the total E output stored on tape by subtracting \(\Delta E\) times an empirically determined gain-matching factor). It turned
out that both methods had their uses, at different energies and different angular settings.

The logical gating sequences required in the event routines to isolate $^{14}$C+$^{18}$O elastic scattering from all other interactions at the three angular settings are given in table 3.1. This table also contains lists of the histograms created in the analyses. Following this, in figures 3.22-3.27, we display examples of typical spectra, some details concerning which are given in their captions. A particular technique which we used frequently and which maybe worth explaining here is mass-gating, which was found to be essential in eliminating background from the inelastic and transfer channels of $^{14}$C+Si, as is evident from figures 3.24 and 3.26.

By conservation of momentum alone it can be shown that ejectile and residual particle masses in a reaction A(B,C)D are given by

$$M_C = \frac{E_{B,B} \sin^2 \theta_D}{E_C \sin^2 (\theta_C + \theta_D)}$$

and

$$M_D = \frac{E_{B,B} \sin^2 \theta_C}{E_D \sin^2 (\theta_C + \theta_D)}$$

3.14.1

Since all the quantities on the right-hand sides are either known to us a priori or are measured in each event, we can extract masses for both primary and coincidence particles and construct a histogram of $M_D$ vs. $M_C$ which can be used for subsequent mass gating.

3.14 Experiment II: Extraction of the Cross-Sections.

With the normalization in this experiment well-determined using the Tantalum monitor, the only remaining question involved in obtaining
TABLE 3.1

Summary of Event Logic and histograms used in offline analysis of coincidence experiment data.

Three event routines were used - one for each angular setting. Shorthand labels for the various histograms here correspond to those used in the figures. The identities of the event descriptors used in each histogram are given in brackets following the histogram label. X1 and X2 refer to generic position data, not to X-type or P-type data specifically.

Key: \( X \rightarrow Y \) Histogram \( Y \) is produced by active event-routine gating in histogram \( X \). The symbol \( O \rightarrow \) denotes oxygen ion \( \Delta E-E \) gating, while \( C \rightarrow \) denotes carbon ion \( \Delta E-E \) gating.

\( Y \rightarrow Z \) One-dimensional histogram \( Z \) is obtained by projecting a gated region in histogram \( Y \) on its \( X_1 \)-axis. This is done outside of the event routines.

**SETTING A**

\[
\begin{align*}
\text{EDE } & [\Delta E-E1] \quad O \rightarrow \text{OEX1}[E_1-X_1] \rightarrow \text{OXX}[X_2-X_1] \rightarrow A(R)[X_1] \\
\text{EDE } & [\Delta E-E1] \quad O \rightarrow \text{OEX2}[E_2-X_2]
\end{align*}
\]

**SETTING B**

\[
\begin{align*}
\text{EDE } & [\Delta E-E1] \quad C \rightarrow \text{CEX1}[E_1-X_1] \rightarrow \text{CXX}[X_2-X_1] \rightarrow \text{CEXG}[E_1-X_1] \rightarrow B(S)[X_1] \\
\text{EDE } & [\Delta E-E1] \quad O \rightarrow \text{CEX2}[E_2-X_2] \\
\text{EDE } & [\Delta E-E1] \quad O \rightarrow \text{OEX1}[E_1-X_1] \rightarrow \text{OXX}[X_2-X_1] \rightarrow B(R)[X_1] \\
\text{EDE } & [\Delta E-E1] \quad O \rightarrow \text{OEX2}[E_2-X_2]
\end{align*}
\]

**SETTING C**

\[
\begin{align*}
\text{EDE } & [\Delta E-E1] \quad C \rightarrow \text{MSS1}[Mass2-Mass1] \rightarrow \text{CXX}[X_2-X_1] \rightarrow C(S)[X_1] \\
\text{EDE } & [\Delta E-E1] \quad C \rightarrow \text{CEX1}[E_1-X_1] \\
\text{EDE } & [\Delta E-E1] \quad C \rightarrow \text{CEX2}[E_2-X_2] \\
\text{EDE } & [\Delta E-E1] \quad O \rightarrow \text{MSS2}[Mass1-Mass2] \rightarrow \text{OXX}[X_2-X_1] \rightarrow C(R)[X_1] \\
\text{EDE } & [\Delta E-E1] \quad O \rightarrow \text{OEX1}[E_1-X_1] \\
\text{EDE } & [\Delta E-E1] \quad O \rightarrow \text{OEX2}[E_2-X_2]
\end{align*}
\]
FIGURE 3.22 The following series of histograms exhibits data from three runs (one for each angular setting, A, B, and C) at a $^{14}\text{C}$ bombarding energy of 28.4 MeV. Min-1 everywhere. This particular figure displays the $\Delta E$-El histogram (EDE) at angular setting C. Gates for carbon and oxygen ions are indicated.

FIGURE 3.23 Carbon-gated El vs. Xl histograms (CEX1) for setting C ($33.0^\circ \leq \theta_{lab} \geq 48.3^\circ$). Elastic scattering of $^{14}\text{C}$ on Si dominates this spectrum (the target was primarily SiO). Non-elastic processes involving Si lie on loci nearly parallel to that of the elastic scattering, and clearly contaminate the $^{14}\text{C}+^{18}\text{O}$ data here. As the following figures demonstrate, however, this is easily eliminated using the coincidence particle information and mass-gating techniques. Loci for the important processes here are labelled by target particle.
Carbon-gated X2 vs. X1 histogram (CXX) for setting C without mass-gating. In this spectrum, too, the $^{14}\text{C}+^{18}\text{O}$ data are contaminated by reactions involving Si (the kinematical locus of one such reaction is discernable below that for the $^{14}\text{C}+^{16}\text{O}$ elastic scattering in the far left quarter. This locus extends across the spectrum into the $^{14}\text{C}+^{18}\text{O}$ data in the right quarter). Note that the three-bar PSD masks cast shadows which form a rectangular grid across this spectrum. (The X2 shadows are less distinct because of the lower resolution in the coincidence detector - a result of the comparatively low energy of the coincidence particles and the smaller detector-target distance). Knowledge of both primary and coincidence particle scattering angles allowed us to construct mass-spectra for gating, as explained in the text.
CXX  C(S)

X1

X2

\begin{itemize}
\item Si
\item ^{16}\text{O}
\item ^{16}\text{O}
\item ^{12}\text{C}
\end{itemize}

^{12}\text{C} \text{ recoil}
FIGURE 3.25 Carbon-gated Mass2 vs. Mass1 histogram (MSS1) for setting C. The mass axes here are scaled in arbitrary units; approximate mass values are indicated explicitly. The mass gate used to produce the histogram of figure 3.26 is also shown. The poorer resolution of energy and angle measurements on the coincidence particles results in "tail" of higher apparent mass. Since mass resolution is not critical, we have used generous gates here to assure inclusion of all the \(^{14}\text{C}^{18}\text{O}\) data. A small fraction of the Si scattering is included in the gate as well. Mass gating was required only for the setting C kinematics region. Other techniques were employed for setting A and B data, as is shown in table 3.1.
same data as in figure 3.24. The data of interest are now free of contamination from $^{14}$C+Si interactions. Since the angular resolution is sufficient to allow us to discriminate between $^{18}$O and $^{16}$O ions, the $^{14}$C+$^{18}$O elastic yield as a function of $X_1$ may be obtained from this spectrum simply by drawing a gate around the kinematic locus of the $^{14}$C+$^{18}$O scattering and projecting it into a one dimensional $X_1$-histogram. This particular $X_1$-projection is of the C(S) type (i.e. setting $C$, "scattered"), for which the center of mass scattering angle increases with increasing $X_1$.

Mass gating here provides extremely clean $X_2$ vs. $X_1$ histograms. Testifying to this is the fact that the weak "line" which appears under the $^{14}$C+$^{16}$O data is still present even when the Si region is completely excluded from the mass-gate. The kinematical locus of these data is perfectly consistent with that of $^{14}$C+$^{18}$O inelastic scattering to the first excited state in $^{18}$O ($1.98$ MeV $2^+$). Without mass-gating, these data would be totally obscured by Si reactions, as we saw in figure 3.24 (Note: the $X_2$ scaling in figure 3.24 is slightly different from that used here. This is of no consequence).
FIGURE 3.27 $^{14}$C+$^{18}$O angular distributions are assembled from five one-dimensional X1-yield histograms, as explained in the text. Figures 3.22-3.26 illustrate the procedure used to find one of three - C(S). Two-dimensional histograms from which the other four are projected, analogous to the CXX histogram in figure 3.26, are shown here. The event logic used in their construction is outlined in table 3.1. Gates used for the actual projections are shown in all of these examples. The direction of increasing cm scattering angle is also indicated.

a) Oxygen-gated X2 vs. X1 histogram (OXX) for setting C. The $\theta_c$ vs. $\theta_p$ kinematics permit unambiguous identification of $^{16}$O and $^{18}$O ions. Structure is evident in the $^{14}$C+$^{16}$O data here. C(R) is the X1-projection from this histogram.

b) Carbon-gated, El-X1-gated, X2-X1-gated histogram in El vs. X1 (CEXG) for setting B. The X1-projection, B(S), contains the far forward angle yields. (18.0°<$\theta_{lab}$<33.3°). The slope of the kinematical locus for $^{14}$C+$^{18}$O scattering has been made close to zero so that we could fully take advantage of the 1024-bin ADC resolution.

c) Oxygen-gated El-X1-gated X2 vs. X1 histogram (OXX) for setting B. The X1-projection is B(R). The setting B runs were intended primarily for the forward angle, B(S) data and were fairly short. Therefore, the projection B(R), which contains only 24 counts, is used to obtain only a single data point with a 16° cm angular acceptance.

d) Oxygen-gated El-X1-gated, X2 vs. X1 histogram (OXX) for setting A. The X1-projection from this histogram contains the far backward yields and is split into four angular points; we trade angular resolution for better statistics.
cross-sections from our detector yields was that of the effective solid angle of the primary detector operated in the coincidence mode. Necessary considerations were as follows: (1) We had planar detectors relatively close to a target with a square 2x2 mm beam spot. (2) Each detector had a three bar mask; so particles striking a bar on either detector eliminate a coincidence event. This causes a shadow in the position spectrum of both detectors. (3) Detector geometry was established with respect to the kinematics for $^{14}$C+$^{18}$O scattering, and this necessarily entailed a loss of coincidence efficiency at some angular settings of up to 50%; this efficiency is also angle-dependent (the kinematical image of the primary detector in the plane of the coincidence detector is wedge-shaped and is wider than the sensitive area of the PSD).

All of these effects were taken into account simultaneously, in the specific context of $^{18}$C+$^{18}$O kinematics, using a Monte Carlo simulation. The effective solid angles derived in $10^7$-event simulations are shown in figure 3.28 for each of the five kinematical situations. The effect of the finite beam spot size on the angular resolution of the primary detector, estimated by $\frac{1}{2}\sigma^2$, was about $.2^0$, but again PSD resolution and multiple scattering increase this to about $.4^0$.

It should be emphasized that the more usual empirical method of determining solid angles by test-scattering in a system for which the cross-section is known would have been very difficult here. We would have needed to study a system with the same kinematics as $^{14}$C+$^{18}$O, and this requires the same mass ratio. We could not use $^{14}$C+$^{18}$O Rutherford scattering, as going low enough in energy to make certain that the
FIGURE 3.28 Monte Carlo simulation results for the effective solid angle of the primary detector. Since coincidence efficiency depends on kinematics, a simulation had to be run for each of the five projections. Each simulation consisted of $10^7$ events, of which $2-5 \times 10^5$ resulted in coincidences. Here the rectangular face of the primary detector is divided into fifty slices of equal width and the effective solid angle of each slice in sr is plotted against slice number. The primary detector mask shadows are seen here as relatively sharp minima, while those from the coincidence detector mask are much broader. In projection C(R), the shadows almost coincide.
MONTE CARLO SOLID ANGLES

\[ \Omega \]

\[ \Omega \]

\[ \Omega \]

A(R)

X1 BIN
scattering was purely Coulombic would have made the backward-scattered coincidence particles undetectable.

Final integration of the primary X-projections for yields was done with twelve-slice binning in the projections B(S), C(S) and C(R). This choice places the shadows of the primary detector mask on slice borders and fixes our analysis acceptance at 1.275°. The tantalum normalization provides good continuity between the different data sets. Use of the solid angles obtained from the Monte Carlo simulation suppresses artificial nonuniformities which would otherwise exist in the angular distributions because of angle-dependent coincidence efficiency.

Statistics were generally poor in the A(R) and B(R) projections because the backward angle cross-sections are very small. We decided, therefore, to widen the angular acceptance for data points from these projections. Yields from A(R) were integrated to obtain four equally spaced data points, each with an angular acceptance of 3.1°. The B(R) projection was used to obtain only a single point with an angular acceptance of 8.0°.

3.15 The \(^{14}\text{C} + ^{18}\text{O}\) Data.

The \(^{14}\text{C}+^{18}\text{O}\) elastic scattering cross-sections, normalized to Rutherford scattering, are shown in angular distribution form in figure 3.29, then as a cross-section surface in figure 3.30. They are generally featureless and drop rapidly with increasing energy, but are not entirely without structure. At higher energies we do see, for example, the development of structure in the 80-110° region and a deep minimum around 100-110°. Also, in many of the angular distributions there is a modest
upturn in the far backward region which appears to correlate with some excitation function structure as well. The overall form of these data would seem to reflect very strong absorption in the $^{14}\text{C}+^{18}\text{O}$ system, and stands in marked contrast to the strongly oscillatory structure and dramatic backward-angle rise which we have observed in the $^{14}\text{C}+^{12}\text{C}$ and $^{14}\text{C}+^{16}\text{O}$ systems at comparable center of mass energies.
FIGURE 3.29 Center of mass angular distributions for $^{14}$C+$^{18}$O scattering. Lines serving to guide the eye connect data points obtained from a common X1-projection. The first projections, in order of $\theta_{\text{cm}}$, are B(S), C(S), and C(R). Each yields twelve data points. The projection A(R), which contains the far backward angle data, yields only four points. The single point from the B(R) projection stands alone. This point should be viewed as an average cross-section measurement ($113.4^\circ < \theta_{\text{cm}} < 129.4^\circ$) rather than a point in the angular distribution equivalent to the others.
$^{14}\text{C} + ^{16}\text{O}$ ANGULAR DISTRIBUTIONS
$^{14}C + ^{16}O$ ANGULAR DISTRIBUTIONS
FIGURE 3.30 Cross-section surface for $^{14}\text{C}+^{18}\text{O}$ elastic scattering. Curves are plotted for laboratory scattering energies from 20.0 to 32.5 MeV in .10 MeV intervals. Using this energy mesh involves placing three interpolated curves between each pair of measured ones, except at the highest two energies (the energy interval there was 2.5 MeV (1.4 MeV cm)).
CHAPTER IV

ANALYSIS

4.1 Qualitative Discussion of the Data and Heavy-Ion Scattering Systematics.

The current state of development of theoretical models for heavy-ion scattering is such that we cannot hope to obtain a reasonable description of the full energy and angle-dependence of scattering cross-sections without using guidance from a qualitative examination of the data. We must attempt to build into the theories specific physical processes which would be expected to have a bearing on salient qualitative features of the data, such as the presence or absence of oscillatory structure, or a rise in the Rutherford-normalized cross-section at backward angles, to name two of the most important. It is relevant, therefore, to discuss the qualitative features of our data and how these features fit into the larger scheme of things in heavy-ion scattering systematics before we present the results of our quantitative analyses.

Let us first summarize the notable qualitative features of our three datasets.

In the $^{14}\text{C} + ^{12}\text{C}$ and $^{14}\text{C} + ^{16}\text{O}$ systems we have found both gross and intermediate-width structure, but the narrower structure is far more prominent in the $^{14}\text{C} + ^{12}\text{C}$ data, while the gross structure is much clearer
and more uniform in the $^{14}\mathrm{C}+^{16}\mathrm{O}$ data. A few MeV above the Coulomb barrier, both systems exhibit angular distributions with deep oscillations and a large increase in the Rutherford-normalized cross-section at backward angles. That this structure is seen to begin relatively lower down in the $^{14}\mathrm{C}+^{12}\mathrm{C}$ surface may reflect the difference in Coulomb barrier energies.

The cross-section surfaces for the two systems both give the overall appearance of having a degree of symmetry relative to a scattering angle somewhat less than $90^\circ$. However, the large-angle side of the surfaces does not in either situation have the same continuously shifting locus of maxima which we see on the forward side. Instead, there are numerous "breaks" where, as a function of energy, minima give way to maxima (and vice versa) after a several-MeV interval in which their location had remained relatively static in angle. These qualitative features of the cross-section surfaces constitute a kind of signature which it was our aim to reproduce with model calculations.

Structure in the $^{14}\mathrm{C}+^{18}\mathrm{O}$ system, although not entirely lacking, is much more subtle, and the backward angle rise is comparatively small. Since the Coulomb barrier would be expected to be lower there than in the $^{14}\mathrm{C}+^{16}\mathrm{O}$ system, the contrast at these low energies is even more remarkable.

Precisely why differences such as this exist in heavy-ion scattering is still an open question, but our data lend support to the conjecture that Q-value or "open-channel" systematics play a very important role. The early belief that narrower structure (<1 MeV) was limited to a few "special" systems involving light alpha-nuclei (nuclei with
N-Z-even) has now given way to the view that such structure is a fairly widespread feature of heavy-ion scattering. Its occurrence is neither limited to light systems (as was demonstrated by the remarkable discovery of intermediate-width structure in the compound nucleus $^{56}_{\text{Ni}}$ (Be79, 81), via $^{28}_{\text{Si}}$+$^{28}_{\text{Si}}$ scattering), nor to alpha-nuclei (as evidenced by our own data).

For lighter systems, a consistent finding is that the occurrence of structure is inversely correlated with the existence of a large number of available exit channels which are well-matched in angular momentum with the entrance channel. Effectively, this amounts to having Q-values which are not prohibitively negative. In figure 4.1 we display the Q-values for rearrangement reactions leading to the ground states of various nuclei from each of the three entrance channels we have studied experimentally. Clearly, the $^{14}_{\text{C}}$+$^{18}_{\text{O}}$ system, which exhibits the least structure, has the largest number of Q-favored exit channels. The $^{14}_{\text{C}}$+$^{12}_{\text{C}}$ system has the fewest open channels and the most structure, while $^{14}_{\text{C}}$+$^{16}_{\text{O}}$ scattering is intermediate in both respects. Including inelastic excitations here would make the difference even greater.

These Q-value systematics may also be applied successfully to the systems whose excitation functions are depicted in figure 1.1. Proceeding from $^{12}_{\text{C}}$+$^{12}_{\text{C}}$ down to $^{18}_{\text{O}}$+$^{18}_{\text{O}}$, the number of available exit channels in these systems also increases as the complexity and scale of their cross-section structure decreases.

For purposes of establishing systematic features of heavy-ion scattering in these light systems, it is useful to compare the qualitative behavior of the systems in figure 1.1 with that of the system of
FIGURE 4.1 Q-values for rearrangement reactions to other nuclear ground states in each of the three systems $^{14}\text{C}+^{12}\text{C}$, $^{14}\text{C}+^{16}\text{O}$, and $^{14}\text{C}+^{18}\text{O}$. The p, n, d, α, .. emission channels are labelled.

This figure provides a qualitative sense of the number of exit channels open to these systems as a function of available or center of mass energy.
Comparison of Ground State Q-Valves

- $^{12}$C + $^{14}$C
- $^{14}$C + $^{16}$O
- $^{14}$C + $^{18}$O
our study and others. We begin by observing that none of our systems can match the $^{12}\text{C}^{12}\text{C}$ system for general richness of structure. However, our $^{14}\text{C}^{12}\text{C}$ excitation function data are rugged enough to be considered comparable to those of $^{12}\text{C}^{16}\text{O}$. The $^{14}\text{C}^{16}\text{O}$ system bears a qualitative resemblance to the $^{16}\text{O}^{16}\text{O}$ one, with well-defined gross structure and a moderate amount of fragmentation. Recent data on $^{14}\text{C}^{14}\text{C}$ scattering (Ko80, 85) indicate that in terms of structure in excitation functions that system is also comparable to $^{16}\text{O}^{16}\text{O}$. A great many other heavy-ion systems possess well-developed gross structure, but display little or no structure of the irregular, narrow kind. This is true of the $^{16}\text{O}^{18}\text{O}$ system depicted in figure 1.1, and in many well-bound odd N or odd Z systems ($^{14}\text{N}^{14}\text{N}$, for example). Finally, the largely featureless cross-sections of the $^{14}\text{C}^{18}\text{O}$ system, which drop steeply with increasing energy and angle, are analogous to what is seen in $^{18}\text{O}^{18}\text{O}$ scattering. It is interesting that, except in the last instance, $^{14}\text{C}$ and $^{16}\text{O}$ seem to play a qualitatively similar role in these structure systematics.

Pure alpha-particle composition is clearly not a strict prerequisite for the occurrence of either gross or intermediate width structure, as the data on systems involving $^{14}\text{C}$ clearly show. However, there are no known instances where the addition of neutrons to systems with N≠Z actually enhances structure seen in heavy-ion scattering. That the extra neutrons in $^{14}\text{C}$ do not cause the same degree of damping of structure brought about by those in $^{18}\text{O}$ is possibly explained with the Q-value systematics. It should be emphasized that the matter is not a simple question of binding energies, since the binding energy per nucleon for $^{18}\text{O}$ (7.7 MeV) is actually greater than that of $^{14}\text{C}$ (7.52 MeV) (for
comparison, the values for $^{12}\text{C}$ and doubly-magic $^{16}\text{O}$ are 7.68 and 7.98 MeV/nucleon respectively).

One notable difference between $^{18}\text{O}$ and each of the other nuclei in our experiments is that the excitation energy of its first excited state is relatively small. The lowest excited state in $^{18}\text{O}$ is a $J^\pi=2^+$ state at 1.98 MeV. First excited states for $^{12}\text{C}$, $^{14}\text{C}$, and $^{16}\text{O}$ are at 4.44 MeV ($2^+$), 6.09 MeV ($1^-$), and 6.05 MeV ($0^+$) respectively. Thus, what may be an important consideration in structure systematics is the shell structure of these nuclei, as perhaps it is the binding energy of valence nucleons that is critical. This is certainly true in the double-resonance picture, where the trapping of flux by temporary excitation of low lying excited states leads to resonant structure in the elastic cross-section.

It is also noteworthy that heavier systems which exhibit structure, such as $^{16}\text{O}+^{28}\text{Si}$ and $^{28}\text{Si}+^{28}\text{Si}$ are problematic for the Q-value systematics. The absolute number of open channels there is actually greater than one finds in the strongly absorbed $^{18}\text{O}+^{18}\text{O}$ system by virtue of the sheer number of particles involved. However, it is also found that the addition of neutrons to these systems increases the number of available channels, and at the same time virtually eliminates the very striking structure (Be84) (in fact, the presence of additional neutrons also has the effect of depressing backward-angle cross-sections considerably). Nevertheless, it is clear that merely counting open channels does not provide an infallible predictor of structure either.
4.2 Physical Interpretation of Structure Systematics.

We have seen that heavy-ion angular distributions frequently exhibit Fraunhofer-type oscillations with a period reflecting the absorption of flux in partial waves with $l \leq g$. Thus, it follows that, as bombarding energy changes and the grazing partial wave shifts, the extrema in the angular distributions also shift. The angular movement of the extrema as a function of energy then gives rise to gross undulations in excitation functions. We see, then, that the occurrence of gross-structure is actually linked with the existence of oscillations in the angular distributions.

If many channels, well-matched in angular momentum with the entrance channel, are available to receive flux, then even surface flux can be absorbed easily if it is within reach of the nuclear potential at all. This strong absorption of surface flux has the effect of diminishing the size of the undulations in angular distributions, and, consequently, removing structure from excitation functions. Surface transparency, which arises from the inability of grazing flux to find an avenue of escape other than the elastic channel, is therefore crucial to the existence of gross structure.

The fundamental origin of intermediate-width structure is, as we pointed out in the beginning, much more puzzling. In view of the above discussion of gross-structure, it is clear, operationally, at least, that the irregular, fragmented appearance of some heavy-ion excitation functions results from rapid energy-dependent changes in the oscillatory pattern of angular distributions - changes which have nothing to do with
the smooth shifting of $\ell$. This is very well exemplified by the 17-18 MeV region of our $^{14}\text{C}+^{12}\text{C}$ data.

From a coupled-channels perspective, it can be argued that since many interfering nonelastic processes can contribute to elastic scattering by strong coupling, the more such processes there are, the greater the probability that no single amplitude can undergo a resonant change and affect the cross-section noticeably. The $^{14}\text{C}+^{12}\text{C}$ system has relatively few available exit channels compared with the other systems, and may therefore be quite sensitive to changes in particular amplitudes. One such amplitude is that arising from two-neutron elastic transfer, especially at lower energies where few reactions but this one, with its zero Q-value, would be expected to be important. This could also be said for two-proton elastic transfer in the $^{14}\text{C}+^{16}\text{O}$ system.

Since the elastic transfer process is physically equivalent to the exchange of identical cores, it is not surprising that it tends to symmetrize the nuclear part of the scattering amplitude and cause an increase in the cross-section at backward angles. However, in principle, it can also contribute structure; either directly through interference with the "bare-potential" elastic scattering, or indirectly through the coupling of higher order processes (e.g. multiple exchanges).

Other channels whose effect on the elastic channel would be expected to be particularly significant in $^{14}\text{C}+^{12}\text{C}$ scattering include inelastic excitations to the lowest-lying states in $^{12}\text{C}$ and $^{14}\text{C}$ and the one-neutron transfer reaction to the symmetric $^{13}\text{C}+^{13}\text{C}$ exit channel ($Q=-3.23$ MeV). The latter reaction was very prominent in our higher-energy
data, as one can see in figure 3.8. However, inelastic $^{14}\text{C}+^{12}\text{C}$ scattering was generally lost in the scattering from $^9\text{Be}$ in our target because of the large negative Q-values which excitation of $^{12}\text{C}$ or $^{14}\text{C}$ requires.

In analogy with the situation in the $^{12}\text{C}+^{12}\text{C}$ system, the possibility of a double resonance mechanism's giving rise to intermediate-width structure in $^{14}\text{C}+^{12}\text{C}$ scattering is also present. However, the higher excitation energy of the first excited state of $^{14}\text{C}$ may inhibit the excitation which is necessary to drop the dinuclear system from a broad virtual resonance in the entrance channel into a quasibound state in the internuclear potential well. The inability of $^{16}\text{C}$ to participate as $^{12}\text{C}$ does in this process may account for the relative lack of intermediate width structure reported in the $^{14}\text{C}+^{14}\text{C}$ system, and the somewhat smoother form of our $^{14}\text{C}+^{12}\text{C}$ excitation functions compared with those of $^{12}\text{C}+^{12}\text{C}$. One could also make a similar argument for $^{16}\text{O}$, with its highly lying first excited state, and then go on to group the systems as follows: $^{14}\text{C}+^{12}\text{C}$ with $^{16}\text{O}+^{12}\text{C}$, and $^{14}\text{C}+^{16}\text{O}$ with $^{14}\text{C}+^{14}\text{C}$ and $^{16}\text{O}+^{16}\text{O}$.

The double resonance model would not necessarily predict resonant structure in a system such as $^{14}\text{C}+^{18}\text{O}$ in spite of $^{18}\text{O}$'s very accessible first excited state. This is because a prerequisite for its validity is that absorption out of the elastic channel be small, so that the width for the decay of the quasibound resonances is also small (Fi72). The $^{14}\text{C}+^{18}\text{O}$ system is too strongly absorbed to show gross structure clearly; so intermediate-width structure would therefore not be expected.

The arguments above are tempting, but admittedly rather speculative. However, it is noteworthy that the general idea of dinuclear resonance formation in heavy-ion scattering has received support from
deformed shell model calculations in $^{24}$Mg (Le75). These calculations show a secondary minimum in the potential energy surface at very large deformations, indicating the quasistability of extended, molecule-like nuclear systems (at least in the $^{12}$C+$^{12}$C system). From our point of view, it would be most interesting to see what such calculations would find in the systems $^{26}$Mg, $^{28}$Mg, $^{28}$Si, $^{30}$Si, and $^{32}$S.

Before we move on to a quantitative analyses we should point out that statistical fluctuations arising from the population and decay of many overlapping levels in the compound nucleus also cannot be ruled out as a possible origin of the narrower structure in our $^{14}$C+$^{12}$C and $^{14}$C+$^{16}$O data. An auto-correlation analysis on our elastic-scattering data was not attempted here, as it was felt that a more definitive analysis using cross-correlation methods could be performed once inelastic scattering and reaction data become available for these systems in this energy range. However, experience in many heavy-ion systems (such as $^{12}$C+$^{12}$C, $^{12}$C+$^{16}$O, $^{24}$Mg+$^{24}$Mg, and $^{28}$Si+$^{28}$Si to name but a few) has demonstrated that the degree of correlation of intermediate width structure among different channels generally exceeds that which would be expected from purely statistical phenomena. We cannot state positively that this is so here, but the width and magnitude of the structure in the $^{14}$C+$^{12}$C data is such as to make this a very reasonable contention.
4.3 AN OPTICAL ANALYSIS.

4.3a Overview.

The nuclear optical model is of course not intended to provide a thorough and detailed description of nuclear scattering when strong energy-dependence of the cross-sections is present. Forcing it to do so necessarily involves substantial "embroidery" on the basis potential which is frequently not clear as far as its physical interpretation is concerned. Therefore, in our optical analysis we have generally employed unadorned potentials of standard Woods-Saxon form, although some experimentation with explicit surface-transparency terms and parity-dependent potentials was also tried in order to determine whether a major qualitative improvement in the appearance of cross-section surfaces could be obtained. The results in these efforts were not positive enough to justify further study.

The computer program used in all of our work involving optical and DWBA calculations was PTOLEMY (Ma76, Pi76); its 1981 VM/CMS version is currently implemented on the IBM 4341 computer in this laboratory. This code provides superior speed, accuracy, flexibility, and freedom from computational problems compared with other similar programs currently available. Source code for PTOLEMY is not available; thus it is not susceptible to modifications, but very reliable.

PTOLEMY finds the radial wave function, \( f_2(r) \), for two-body scattering states by numerical integration of the Schrödinger equation:

\[
\left( -\frac{d}{dr^2} + \frac{\ell(\ell+1)}{r^2} + \frac{2\mu}{\hbar^2} V(r) - k^2 \right) f_2(r) = 0. \tag{4.3.1}
\]
The total S-matrix (nuclear plus Coulomb) is found from $f_\epsilon(x)$ using the asymptotic relation

$$\lim_{r \to \infty} f_\epsilon(x) = 1/2 \left[(1 + S_\epsilon) F_\epsilon(\eta, kr) + i(1 - S_\epsilon) G_\epsilon(\eta, kr)\right]$$

where $F_\epsilon$ and $G_\epsilon$ are the regular and irregular Coulomb wave functions.

Cross-sections are then calculated from the S-matrix in the usual way.

The potential, $V(r)$, takes the standard Woods-Saxon form with parameters which may be given up to quadratic dependence on energy. We decided, however, to limit the energy variability to a linear dependence on the absorptive strength, $W$ as is customary in the literature. $V(r)$ may also contain a spin-orbit interaction (which we obviously did not need) and a surface-absorption term in the form of a Woods-Saxon derivative. The sign of this term could be changed to allow for additional surface transparency rather than absorption.

For the Coulomb part of $V(r)$ we chose to take the point plus uniform charged sphere form with radius parameters $r_{c0}=1.35$ fm. Varying this parameter was never found to have a significant effect on the cross-section calculations, and so it was not included among the fitted parameters. It should be noted that, all of the radius parameters discussed in this work are understood to be related to the corresponding physical radius by the relation 2.2.5.

For optical model calculation, PTOLEMY is also provided with a number of sophisticated fitting routines. After trials with different ones we decided that the default fitter LMCHOL, which is guided by numerically-computed derivatives of the potential, is the most effective and well-behaved for heavy-ion scattering data.
4.3b $^{14}\text{C}^{18}\text{O}$ Optical Analysis.

The lack of deeply oscillatory, strongly energy-dependent structure, and the small backward-angle cross-sections in the $^{14}\text{C}^{18}\text{O}$ system are signatures of strong absorption which the optical model can reproduce without difficulty. By fitting six angular distributions simultaneously using the output from a fit to a single, low energy, angular distribution as a starting point, we were able to find a strongly absorbing optical potential which yields cross-sections consistent with the overall form of our data. The fits are shown in figure 4.2 together with the full cross-section surface calculated with this potential, the parameters for which are given in the caption.

The angular distributions calculated with this potential fit the data very well at forward angles and reproduce their angular slope quite well overall. The modest backward-angle rise and uneven oscillatory structure above $90^\circ$ is not reproduced - and we were not able to reproduce those features with an optical parameter set which yields reasonable angular distributions over the entire dataset. This problem is much more serious in the $^{14}\text{C}^{16}\text{O}$ and $^{14}\text{C}^{12}\text{C}$ data, as we shall see.

It is interesting that, in a technical sense, this potential is surface transparent, as $|V(r)|$ is significantly larger than $|W(r)|$ at large internuclear separations $r$. However, the enormous depth of the imaginary potential compared with the real potential renders the precise geometry of the real potential less significant here. Because of the potential ambiguities, in fact, we cannot claim that the potential parameters found here have clear physical implications except that the system is strongly absorbed. A number of other potentials with similar,
but less physically reasonable parameters were found to reproduce the slope and energy-dependent magnitude of the measured cross-sections quite well. The cross-sections found from these potentials were not identical to those shown in figure 4.2, but were close enough that we did not have a strong preference for one over the other, except in terms of the reasonableness of the potential parameters themselves. We were able to use this potential to quantify the importance of elastic transfer in this system, which we discuss in section 4.5.

4.3c $^{14}_C+^{16}_O$ and $^{14}_C+^{12}_C$ Optical Analyses.

We discuss the optical analyses of the $^{14}_C+^{16}_O$ and $^{14}_C+^{12}_C$ data together because the problems which we encountered in the two situations were similar, and we addressed them in a similar way. Our success in reproducing even crude features of these systems' behavior with the optical model was very limited in both instances.

We began our optical fitting effort by exploring the model space systematically, considering variations on well-known potential parameter sets in order to be able to anticipate the model's behavior in more general circumstances. This provided information which was useful in determining fitting strategies. An example is given in figure 4.3, where the potential of Reilly et al. (Re73) for $^{12}_C+^{12}_C$ scattering is applied to nonidentical-particle $^{14}_C+^{12}_C$ scattering and the effect of varying the diffuseness of the imaginary well is investigated. The Woods-Saxon parameters are given in the caption.
FIGURE 4.2 a) Fits to six $^{14}\text{C}+^{18}\text{O}$ angular distributions spanning our experimental energy range. The potential parameters are
\[ R_0 = 1.56, \quad R_{10} = 1.18, \quad a_1 = .55, \quad a_2 = .50, \quad V = 7.05 \text{ and} \]
\[ W = 186.2 + .551 \times E_{\text{cm}} \text{ (thus its range was 192.4-196.3 MeV).} \]
We note that here and in subsequent references radius parameters are defined through equation 2.2.5 and given in units of fm. Also, the Coulomb radius $R_{C0}$ will be assumed to be 1.35 fm. The well depths always have units of MeV and the sign convention is as is given in equation 2.9.1. Data are indicated by points. Each calculated curve (solid line) in this and similar figures is labelled with the appropriate cm energy. The relevant measured cross-section surface is also provided above for comparison.

FIGURE 4.2 b) A cross-section surface calculated from the $^{14}\text{C}+^{18}\text{O}$ potential. As in the surface plots for the data, the vertical axis in all calculated surfaces plots is simultaneously an energy scale and a logarithmic scale of cross-section normalized to Rutherford scattering. The energy scale is very linear in the calculated surface plots because the calculations are continued forward to a scattering angle of 1°, where excitation functions are always flat (and equal to unity). The vertical dotted lines in all surface plots indicate the maximum angular range of data in the corresponding experiment, and the energy range is exactly the same as that of the experiment.
FIGURE 4.3 a) Energy vs Angle cross-section surface calculated for $^{14}\text{C}+^{12}\text{C}$ scattering using the Reilly potential: $R_{x0}=1.35$, $R_{10}=1.40$, $a_{x}=a_{1}=0.35$, $V=14.0$, $W=0.4+0.1E_{cm}$ (range is 1.32 to 2.25 MeV).

FIGURE 4.3 b) A cross-section surface for $^{14}\text{C}+^{12}\text{C}$ scattering using the Reilly potential at 40 MeV lab (18.46 MeV cm) but with $a_{1}$ varied linearly from 0.01 to 0.7. The scattering energy is constant.
The Reilly potential is not appropriate for $^{14}\text{C}+^{12}\text{C}$ scattering but the energy vs. angle cross-section (surface in part a) of figure 4.3 usefully illustrates one form of "typical" optical behavior. The angular distributions contain deep undulations which move forward smoothly in angle as the energy is increased, thus increasing the number of extrema, as is demanded by the increasing value of $l_g$. Excitation functions from this potential have smooth, broad (~3 MeV) undulations, easily seen in the surface-plot by sighting along a locus of constant center-of-mass scattering angle (the transformation from center-of-mass to laboratory scattering is independent of energy in nonrelativistic elastic scattering).

As we noted earlier, this behavior is characteristic of potentials which are weakly absorbing in the surface region. However, it is notable that the Reilly potential and many of the optical potentials which we obtained by fitting satisfy the surface-transparency criterion only because the ratio $V/W$ is large. Such potentials are perhaps better described as "weakly absorbing" than "surface-transparent", because the real potential is deeper than the imaginary potential everywhere; not just in the surface region. This reminds us that the optical model potentials which yield a given angular distribution or excitation function are only ambiguously related to the "actual" two-body effective potential for which the concept of "surface transparency" is really meaningful physically. We should also point out, however, that simple potential transformations of the form 2.9.3 (the Igo invariance) do not preserve the behavior of the cross-sections in any potentials which we found to be of relevance to our data.
The optical potential of Bohlen et al. (Bo72) represents a more realistic description of the $^{14}$C-$^{12}$C data at low energies, where the average cross-section is well-reproduced but the oscillatory structure is not. At higher energies, although there is much oscillatory structure, the shapes of the calculated angular distributions are very different from those of the data, and the average cross-section is underpredicted as well. The potential itself is depicted in figure 4.4 and is clearly surface transparent. A cross-section surface calculated from it is shown in figure 4.5a, while figure 4.5b demonstrates the effect of varying the real potential depth when all of the other potential parameters and the scattering energy are fixed.

Taken together, figures 4.5a and b clearly illustrate that the depth of oscillatory angular distribution structure increases both with increasing energy and real potential depth, V. Since one can obtain qualitatively similar behavior by rescaling both V and W, the imaginary well depth, by a factor, it appears that the ratio V/W is actually more critical than the absolute values of V and W themselves. Thus it is often found that W must be allowed to grow with energy if oscillatory structure is not to become excessive.

As is evident in figures 4.1-4.5, the model-space for standard optical potentials is quite large and complex. It is, in fact, difficult to isolate specific attributes contributed by an individual potential parameter both because of the existence of ambiguities and because the cross-sections depend strongly on the choice of all the other parameters as well. However, after performing very extensive searches through the model-space "by hand" and with PTOLEMY's fitting routine, we determined
FIGURE 4.4 Real (solid line) and imaginary (broken line) potentials of Bohlen et al. (Bo72) for $^{14}_C^{12}$C scattering at near-barrier energies (<10 MeV cm). The Woods-saxon parameters are $R_{r0} = 1.33$, $R_{r10} = 1.35$, $a_r = .46$, $a_i = .30$, $V = 17.0$ and $W = .70$. This potential exhibits slight surface-transparency.
FIGURE 4.5 a) Cross-section surface for $^{14}$C+$^{12}$C scattering calculated with the potential of figure 4.4. The potential parameters here do not contain any energy dependence.

FIGURE 4.5 b) Cross-section surface calculated from the potential of figure 4.4, but with the $^{14}$C bombarding energy fixed at 30 MeV (13.85 MeV cm) and V varied linearly from 15.0 to 133.0 MeV. Oscillatory structure clearly deepens as V is increased. It is also interesting that the "ridges" formed by the local maxima have a constant slope in the V vs $\theta$ plot. Similar ridges in the E vs $\theta$ plot of part a) of this figure and figure 4.3b have a slope which steepens with increasing energy. The result is that the number of extrema in the angular distribution is much more sensitive to energy than to the value of V.
that the large backward-angle rise and deep oscillatory structure seen in the $^{14}\text{C} + ^{12}\text{C}$ and $^{14}\text{C} + ^{16}\text{O}$ data were not describable simultaneously with standard optical model calculations. Our efforts with parity-dependent potentials and potentials which included an explicit surface-transparency term also did not yield qualitatively good fits. Even ignoring the backward-angle rise, we were not able to obtain cross-section surfaces which display the alternating crest-trough structure except at very large scattering angles, beyond the range of our data. (Some of this structure is evident in the far-backward region of the surface-plot in figure 4.3a).

The degree of difficulty which we experienced in attempting to reproduce our data with optical calculations was strongly dependent on the form of the angular distributions we sought to fit. For example, $^{14}\text{C} + ^{16}\text{O}$ data at low and intermediate scattering energies were actually amenable to relatively detailed fitting, as figure 4.6 demonstrates for the structured intermediate energy data. Here the absence of the backward-angle rise (or perhaps it is only the lack of data at more backward angles) permitted us to obtain a very good fit to five angular distributions consecutive in energy. However, as was typical, this potential yielded very poor fits to the structure at higher energies and to the normalization at lower energies. Excellent fits could also be obtained for the very smooth low-energy $^{14}\text{C} + ^{16}\text{O}$ data, but the potentials thus found again did not yield good detailed fits at higher energies.

The particular potential used in figure 4.6 (whose parameters are given in the caption, as will henceforth be our convention) is another example of the weakly absorbing type, exhibiting surface transparency.
FIGURE 4.6 Good fits to $^{14}C + ^{16}O$ elastic scattering data at five consecutive experimental energies. The potential is given by the following parameters: $R_r = 1.00$, $R_i = 1.30$, $a_r = .574$, $a_i = .351$, $V = 140.9$ and $W = -3.97 + 0.718 \times E_{\text{cm}}$ ($W = 6.37$ at 14.4 MeV cm). Because of the rapid change of structure with energy in the data, this potential does not yield good fits at energies differing from these by more than 1 MeV or so.
because \( V/W \) is very large and in spite of the fact that the imaginary radius parameter, \( R_{10} \), is larger than the real radius parameter, \( R_{r0} \) (the larger real diffuseness, \( a_r \), is also a contributing factor here).

Unlike the low-energy \(^{14}\text{C}+^{16}\text{O}\) data, \(^{14}\text{C}+^{12}\text{C}\) data at low energies were not reproducible in detail at all by optical calculations, even when we completely disregarded the quality of the fits at higher energies. The difficulty is the deep undulation between \(75^\circ \) and \(100^\circ\), which is apparently the low-energy precursor to the pronounced oscillatory structure and backward-angle rise seen at higher energies in both systems. That those features could not be reproduced simultaneously with optical calculations suggests that the optical model lacks a degree of freedom which is very important in these systems. This conclusion motivated our attempt to include elastic transfer in the description of our data, which is discussed in section 4.5.

Figure 4.7 is an illustration of the "choice" between oscillatory structure and backward rise that one must make in fitting the higher energy \(^{14}\text{C}+^{12}\text{C}\) or \(^{14}\text{C}+^{16}\text{O}\) data. The calculated curve in part a) of the figure constitutes a reasonable reproduction of the Fraunhofer diffractive structure up to about \(100^\circ\), when the frequency of the oscillations in the measured angular distribution abruptly increases and the average cross-section begins to rise. At about \(100^\circ\) the fit is very poor. The potential which yielded the curve shown here did not fit even the forward-angle data at energies significantly different from 16 MeV (cm).

In part b) of figure 4.7 we show an example of a calculation which reproduces the backward-angle rise but not the oscillatory structure in the same angular distribution. It bears noting that in both examples,
FIGURE 4.7 $^{14}\text{C} + ^{12}\text{C}$ scattering at 16.34 MeV cm. This figure illustrates the choice we have in fitting our higher energy data. Part a) shows a good fit to the forward angle Fraunhofer structure with the potential $R_{r0}=1.515$, $R_{i0}=1.560$, $a_r=.357$, $a_i=.0046$, $V=9.03$, and $W=2.31$.

Part b) shows a structureless fit which reproduces the backward rise. The potential here is given by $R_{r0}=1.533$, $R_{i0}=1.501$, $a_r=.196$, $a_i=.0182$, $V=39.20$, and $W=73.18$. 

the imaginary diffuseness, \( a_4 \), is very small. The potential used for figure 4.7b is also not generally descriptive of the \( ^{14}\text{C}+^{12}\text{C} \) data over a wide energy range. However, we found that a similar potential was able to reproduce the overall behavior of the \( ^{14}\text{C}+^{16}\text{O} \) dataset apart from oscillatory structure. Sample fits to six angular distributions which nearly span the energy range of the experiment are shown as part a) of figure 4.8, and the corresponding cross-section surface as part b). Again, \( a_4 \) is really too small to take seriously; the cutoff in absorption could not really be that sharp. Problems such as this clearly demonstrate basic inadequacies in the optical description of these reactions.

Having learned from the foregoing analysis that even a qualitative reproduction of our \( ^{14}\text{C}+^{16}\text{O} \) and \( ^{14}\text{C}+^{12}\text{C} \) data was not possible within the framework of a standard optical model, we turned to the question of what route to take in further analysis. It seemed desirable to include in our calculations an additional degree of freedom which could be provided by elastic transfer; however, the implementation of the idea described in section 2.10 required that we have a suitably accurate "pure" elastic scattering amplitude \( f_{\text{elastic}}(\theta) \) with which to begin. Since the undulations in the \( ^{14}\text{C}+^{12}\text{C} \) angular distributions generally become narrower beyond 90°-100°, where the backward rise begins, it seemed reasonable that the two interfering amplitudes \( f_{\text{elastic}}(\theta) \) and \( f_{\text{transfer}}(\pi-\theta) \) might carry structure with different angular periodicities. The actual periodicities observed in different parts of the angular distributions would then reflect the dominance of one amplitude over the other.
FIGURE 4.8 a) A relatively structureless fit to the $^{14}\text{C}+^{16}\text{O}$ data which shows a backward-angle rise. The potential parameters are

$R_0=1.307$, $R_{10}=1.397$, $a_x=.253$, $a_y=.0013$, $V=305.23$, and $W=183.36 - 6.633 \times E_{\text{cm}}$.

FIGURE 4.8 b) Cross-section surface calculated with the potential of part a). $W$ ranges from 112.61 at 10.67 MeV down to 40.80 at 21.49 MeV. Since $W$ decreases with increasing energy here, we see oscillatory structure appear and deepen at the higher energies.
It was clearly of interest to find potentials for \(^{14}\text{C}^{12}\text{C}\) and \(^{14}\text{C}^{16}\text{O}\) which could be used to generate a realistic \(f_{\text{elastic}}(\theta)\), reproducing the forward-angle cross-sections with their Fraunhofer-diffractive structure after the fashion of the calculation shown in figure 4.7a. The backward rise would simply be ignored temporarily, on the assumption that it would be furnished by the transfer amplitude.

Unfortunately, we were not able to find optical potentials which reproduced the forward angle data in detail. Even here, the energy-dependence of the cross-sections was apparently too strong. In the end, however, we found strongly absorbing optical potentials which yielded cross-sections relatively free of structure but with reasonably good "average" forward-angle behavior. What we mean by this is best understood by studying figures 4.9 and 4.10. Part a) of each of those figures displays a comparison between calculated and measured angular distributions at six widely-spaced energies for \(^{14}\text{C}^{12}\text{C}\) and \(^{14}\text{C}^{16}\text{O}\) scattering respectively. In parts b, calculated cross-section surfaces are shown.

We will refer to these potentials henceforth as our "bare" optical potentials. In addition to providing \(f_{\text{elastic}}\), they will be used in the elastic transfer calculation to generate distorted waves for the DWBA calculations of \(f_{\text{transfer}}\). These potentials are reasonable for these purposes in that their use does not "double count" effects such as the backward rise which elastic transfer is expected to introduce. However, their disadvantage is that they are relatively strongly absorbing, and so place the burden of bringing about structure entirely on the transfer amplitude. this, however, may be quite reasonable for the lower energy data.
FIGURE 4.9 a) Bare optical potential fits to $^{14}\text{C}+^{12}\text{C}$ angular distributions at six energies spanning the range of our data. The potential parameters are $R_{r0}=1.33$, $R_{i0}=1.35$, $a_r=.46$, $a_i=.30$, $V=17.0$, and $W=18.6 - .290 \times E_{\text{cm}} \ (13.20 < W < 15.92)$. $W$ decreases slightly with increasing energy. This enhances the shallow oscillatory structure, and is important for overall normalization.

FIGURE 4.9 b) Surface-plot calculated from the potential of part a).
FIGURE 4.10a) Bare optical potential fits for $^{14}\text{C} + ^{16}\text{O}$ data. The potential parameters are $R_{r0} = 1.370$, $R_{\rho 0} = 1.206$, $a_r = .495$, $a_\rho = .389$, $v = 9.295$, and $W = -82.0 + 8.2843\times E_{\text{cm}}$. The range of $W$ over the entire dataset is then 6.37 MeV to 96.05 MeV. This potential is therefore weakly absorbing at low energies and very strongly absorbing at high energies.

FIGURE 4.10b) Surface-plot calculated from the potential of part a).
The bare potential for the $^{14}\text{C}+^{12}\text{C}$ system is essentially the Bohlen one with a larger and slightly energy-dependent imaginary well-depth. Figure 4.11 shows the $r$-dependence of this potential evaluated at a $^{14}\text{C}$ bombarding energy of 20 MeV (9.23 MeV cm). The bare potential for the $^{14}\text{C}+^{16}\text{O}$ system is very strongly energy dependent. In fact, at low energies, where $W<V$, it is even surface transparent. In figure 4.12, we show the $r$-dependence of this potential for a $^{14}\text{C}$ energy of 30 MeV (16.0 MeV cm), where it is very strongly absorbing.

4.3d Other Optical Model Calculations.

In view of the failure of the standard optical potentials to provide us with even a qualitatively correct image of our data except in the $^{14}\text{C}+^{18}\text{O}$ system, we felt it important to pursue a number of less restrictive optical approaches. The point was to see if immediate improvement in the quality of fits could be obtained using only simple energy dependences in the parameter sets or more general potential forms.

Our conclusions were that, while it was possible to gain some flexibility with these approaches, this flexibility was simply not such as to warrant the additional complications which they entailed. Allowing energy dependence in the various optical parameters, for example, introduced degrees of freedom which tended to confuse numerical fitting algorithms, sending them often to unphysical regions in the parameter space with only little compensation in terms of fit quality. In the end, such an approach then borders on sheer numerology.
FIGURE 4.11 Bare potential for $^{14}\text{C}+^{12}\text{C}$ system as a function of $r$ at 9.23 MeV cm scattering energy. The solid line is the real potential and the dashed one the imaginary potential.
$^{14}_{\text{C}} + ^{12}_{\text{C}}$ BARE POTENTIAL

$V = 17 \quad W = 15.9 \quad A = 0.46 \quad A_{\text{I}} = 0.3 \quad R_{0} = 1.33 \quad R_{\text{IO}} = 1.35$
FIGURE 4.12 Bare potential for $^{14}\text{C}+^{16}\text{O}$ system as a function of $r$ at 16.0 MeV cm scattering energy. The potential is clearly strongly absorbing at this energy. The solid line is the real potential and the dashed one the imaginary potential.
Another approach which we attempted involved the inclusion of an explicit imaginary surface term. The Woods-Saxon derivative form which is available in PTOLEMY can be added to the imaginary potential with positive net sign, thus reducing the depth of the imaginary potential in a certain neighborhood of radii. Since this term is parameterized by a depth, radius, and diffuseness of its own, this neighborhood may be located anywhere. However, it is only physically reasonable to locate it near the surface, where we have some reason to believe that flux may not be strongly absorbed.

In practice, we found that explicit surface transparency included in this way did not significantly improve the optical description of our data. In fact, our experience is that the changes in the cross-section brought about by the presence of this term can be fairly well simulated simply by reducing the radius or diffuseness of the imaginary well.

An example of the effect of the explicit surface-transparency term is given in figure 4.13, where we demonstrate its effect on the shape of the Bohlen potential and on the resulting cross-section surface (these are to be compared with figures 4.4 and 4.5a). The structure seen in the angular region above 140° is somewhat interesting (it recalls the large-angle behavior of the Reilly potential), but we were not able to bring it into the region of our data.

The final investigation in our optical analysis program was a brief study of the effect of allowing the optical potential to depend on the parity of the particular partial wave. Our efforts in this direction were hampered, however, by our inability to do fitting with such potentials, as the CMS version of PTOLEMY does not incorporate calculation of
FIGURE 4.13 a) Potential of figure 4.4 with explicit surface-transparency term in the imaginary potential. This is a derivative of a Woods-Saxon form with a depth of -5.0 MeV, a radius parameter of 1.10 fm, and a diffuseness of .30. The solid line is the real potential and the dashed one the imaginary potential.

FIGURE 4.13 b) Cross-section surface calculated using the potential of part a). This is to be compared with figure 4.5b. The additional surface transparency introduces new structure and increases the cross-section at intermediate and backward angles, but not to an extent which is irreproducible with a standard optical potential (without the surface term).
cross-sections from parity-dependent potentials as an option. Instead, we computed cross-sections from parity-dependent potentials of the form 2.10.2 using the following algorithm. PTOLEMY is run twice: once with the potential which is required for odd partial-waves and again with the even partial-wave potential (of course, by 2.10.2, the only difference between the two potentials is the size of W and V). We save the nuclear S-matrix elements from both calculations and select only the odd-\( \ell \) elements from one and the even-\( \ell \) elements from the other in the final calculation of the cross-sections. All of this was automated so that calculations in the end involved nothing more than specifying the potential parameters in each situation. However it bears emphasis that we did not actually fit data here.

The results of these efforts were quite interesting, as even a relatively modest parity-dependence introduces new and unusual structure, especially at backward angles. Although the S-matrices did not appear to exhibit regular odd-even staggering in the values of their reflection coefficients or phase-shifts, some unbalancing of the odd and even partial-wave cancellations is apparently resonsible for these phenomena.

Figure 4.14 contains examples of +6\% (\( C_+ = C_1^- .06 \) in 2.10.2) and -6\% (\( C_- = C_1^- -.06 \)) parity-dependence applied to the bare potential for \(^{14}\text{C} + ^{12}\text{C}\). Clear differences in structure are found above 100° in cm scattering angle among both surfaces shown here and the one from the original potential illustrated in figure 4.9b. It does not appear that the behavior of the cross-sections in the +6\% calculation is simply related to that in the -6\% calculations, although distinct qualitative
FIGURE 4.14 a) Cross-section surface calculated from the bare potential for $^{14}\text{C}+^{12}\text{C}$ scattering with a +6% parity dependence (the even partial-wave potential is greater than the original potential by 6%).

FIGURE 4.14 b) Cross-section surface calculated from $^{14}\text{C}+^{12}\text{C}$ bare potential with -6% parity dependence (the signs of the parity-dependent differences from the original potential are reversed compared with part a).
similarities are evident. Also, it is interesting that the increased depth of structure in both of the parity-dependent calculations is not accompanied by a decline in overall backward-angle cross-section, as is common with a standard optical model. This effect we also find in the elastic transfer calculations. However, a much larger parity dependence than the 6% used here would be required to reproduce the full scale of the backward rise, and this would have a significant effect on the cross-sections even at relatively forward angles. Thus, pursuing this kind of analysis further would require our being able to fit using parity-dependent potentials. This should soon be possible using an updated version of PTOLEMY.

One other observation worth making here is that the parity-dependent potentials introduce extremely fine small amplitude oscillatory structure into the excitation functions. This structure is in fact quite apparent in the surfaces of figure 4.14. We do not understand its origin, and, as far as we are aware, it has not been noted before. It may simply reflect the fact that calculations on such a fine energy grid have not been done with parity-dependence before.
4.4a Elastic Transfer Analyses.

Analyses involving all three sets of our elastic scattering data were performed with an elastic transfer model of the type described in section 2.10. The expression for the cross-section, 2.10.3, requires that we obtain two scattering amplitudes - one, $f_{\text{elastic}}$ meant to describe the "pure elastic" part of the reaction, and the other, $f_{\text{transfer}}$, intended to describe the Q=0 rearrangement reaction $A(B,A)B$ as though it could be distinguished from the elastic scattering. This method is advantageous in that it allows us to employ powerful existing codes to obtain the necessary amplitudes.

In all of our work, the transfer amplitudes were obtained within the distorted-wave-Born-approximation (DWBA) formalism used commonly in the analysis of nuclear rearrangement reactions. Again PTOLEMY was best-suited to our needs as it was originally intended for direct-reaction calculations involving heavy-ions in particular (in contrast with other well known DWBA codes such as DWUCK, which are primarily intended for light-ion reactions).

In a first-order DWBA formalism, a nuclear rearrangement reaction is essentially a coupled two-channel problem in which the potential inducing the transition between the channels is treated as a perturbation on the optical potentials inducing elastic scattering in either one. For a reaction $A(a,b)B$ we may define relative coordinates

\[
\vec{r}_\alpha = \vec{r}_a - \vec{r}_A \quad \text{and} \quad \vec{r}_\beta = \vec{r}_b - \vec{r}_B
\]

so that the incoming and outgoing "distorted" waves which describe elastic scattering in the entrance channel, $\alpha$, and exit channel, $\beta$, can be written simply as $\chi^{(+)}_\alpha(\vec{k}_\alpha, \vec{r}_a)$ and $\chi^{(-)}_\beta(\vec{k}_\beta, \vec{r}_b)$. Since the potential
inducing the transition is assumed to be weak, the channels are effectively decoupled if only \( x_\alpha \) and \( x_\beta \) are sought. Therefore, these wave functions may be obtained as solutions to a two-body potential scattering problem simply by integration. Optical potentials required to generate \( x_\alpha \) and \( x_\beta \) are usually determined from fits to elastic scattering data. The DWBA scattering amplitude then has the form

\[
f_{\text{DWBA}} = \frac{J \mu_\alpha \mu_\beta}{2\pi \hbar^2} \langle k_\alpha \rangle \langle k_\beta \rangle \int \int d^3 r_\alpha d^3 r_\beta \left( x_\alpha^{(-)}(\vec{r}_\alpha) \right)^* \left( x_\beta^{(+)}(\vec{r}_\beta) \right) \langle \psi_\beta | V | \psi_\alpha \rangle x_\alpha^{(+)}(\vec{r}_\alpha) x_\beta^{(-)}(\vec{r}_\beta)
\]

where \( V \) is the potential which induces the transition from \( \alpha \) to \( \beta \) and \( J \) is the Jacobian of the transformation to the relative coordinates. The nuclear matrix element here is a function only of \( \vec{r}_\alpha \) and \( \vec{r}_\beta \), all other internal variables being integrated over. It contains all of the nuclear structure and angular momentum information pertinent to the reaction.

In practice, the matrix element (form factor) in 4.4.2 is expanded in a multipole series and the six dimensional radial integral evaluated numerically one partial-wave at a time. The calculation is rather involved, but computational techniques for the purpose are now relatively standard and have been effectively implemented in computer codes such as PTOLEMY. Since our aim here is not to explore direct reaction theory per se, we will not discuss these techniques in detail. Thorough treatments of the application of distorted wave methods to nuclear reactions have been given by Austern (Au69) and by Satchler (Sa85). A short and interesting review of DWBA theory for multi-step processes has also been given by Ascuitto (As84).
In PTOLEMY, the evaluation of the form factor for n-nucleon transfer is carried out in a "full recoil" coordinate system as a three-body problem involving two internally inert cores and an internally inert n-nucleon cluster. For V, the interaction which induces the transfer, the program uses the potential that binds the cluster either to the projectile or to the target (which results in "prior" or "post" representations depending on whether the reaction is pickup or stripping). In elastic transfer this choice is unimportant because the cores are identical; we always specify the same potential at target and projectile vertices. Coulomb corrections to the interaction V are also available in PTOLEMY, but we found that their use made little difference in our calculations.

The inputs to the program which are of primary physical concern are (1) the identity of the reaction, (2) the optical potentials to be used for elastic scattering in the entrance and exit channels, (3) the geometries of the Woods-Saxon potentials used for binding the n-nucleon cluster to the cores, (4) the number of nodes desired in the resulting bound-state wave functions and (5) their orbital angular momenta (which effectively fix the l-value of the transfer). Given only a radius parameter and a diffuseness for V at a given vertex, PTOLEMY fits the potential depth such that the bound-state wave function with the desired number of nodes has a binding energy equal to the known separation energy of the n-nucleon cluster from the core (the program contains the entire 1975 Oak Ridge Atomic Mass Adjustment).

Calculations for the elastic transfer analysis were carried out in three independent steps. First, we used PTOLEMY to calculate the DWBA
transfer amplitude, \( f_{\text{DWBA}} \), for the elastic transfer reaction of interest. Then we used the optical scattering capability of PTOLEMY to obtain the corresponding elastic scattering amplitude, \( f_{\text{OPT}} \). Typically, the optical potential used in this step was the same "bare-potential" as was used to generate the distorted waves in the transfer amplitude calculation. Finally, in a separate program, the cross-section was obtained from the stored amplitudes using the relation

\[
\frac{d\sigma}{d\Omega} = |f_{\text{elastic}}(\theta) + e^{i\phi} \sqrt{S_{\text{ET}}} f_{\text{transfer}}(\pi-\theta)|^2 \quad 4.4.3
\]

and the results were compared with the data.

The complex phase factor in 4.4.3 was usually fixed at +1, but we felt it important to investigate the effect of varying the phase \( \phi \), as it has been pointed out by others (As86, Fr84) that the relative phase involved in the superposition of the two amplitudes is not necessarily well-defined. The factor \( \sqrt{S_{\text{ET}}} \) in 4.4.3 is an overall normalization factor for the elastic transfer amplitude which we varied in order to fit the scale of the backward-angle rise in the data. Since the optical potentials and the bound-state well-geometries were fixed beforehand, the two factors in 4.4.3 finally constituted the only variable parameters in this analysis (we investigated the effect of varying the well geometries, but in the end used standard values).

The parameter \( S_{\text{ET}} \) is defined in such a way as to suggest an analogy with the spectroscopic factor of ordinary DWBA-based direct-reaction spectroscopy. However, such interpretation must be approached with caution.
In standard DWBA analyses, the spectroscopic factor, $S$, is defined as the scale factor by which the DWBA estimate of a given direct reaction cross-section differs from the measured cross-section:

$$\frac{d\sigma}{d\Omega}(\theta) = S \frac{d\sigma_{DWBA}}{d\Omega}(\theta) .$$

Defined in this way, $S$ is proportional to the square of an overlap integral involving the initial and final states of the target nucleus in the reaction. In particular, when certain trivial dependences are removed, the spectroscopic amplitude $\sqrt{S}$ in n-nucleon transfer may be compared directly with the theoretically calculated matrix element of an n-body creation operator between the initial and final states. (The spectroscopic factor, $S_{ET}$, which we have defined here is equivalent to the combination $<C>^2 S$ often seen in direct reaction spectroscopy. $<C>$ is an isospin Clebsch-Gordon coefficient which is equal to unity in the $^{14}_C + ^{12}_C$ and $^{14}_C + ^{18}_O$ systems, and $1/3$ in the $^{14}_C + ^{16}_O$ system. The statistical spin weighting factor necessary for stripping reactions does not appear in elastic transfer because $J_f = J_i$: in our systems all nuclei are spinless.

From one perspective, then, the determination of a spectroscopic factor, $S_{ET}$, for the elastic transfer contribution to elastic scattering provides us with a means of extracting spectroscopic information from such measurements, and can thus become a valuable tool. However, another viewpoint is that it is the complicated reaction dynamics involved in elastic scattering of heavy-ions - rather than nuclear structure per se - which determines the relative importance of elastic transfer in various systems. This point is especially relevant in multi-nucleon...
transfer reactions, where the standard one-step DWBA is known to be inadequate in reproducing the overall scale of transfer cross-sections (it usually gives answers much too small). Hence, by this argument, $S_{ET}$ should be used more as a gauge of the strength of the mechanisms which produce elastic transfer than as a true "spectroscopic factor".

Nevertheless, many authors have extracted spectroscopic factors from low-energy heavy-ion elastic scattering data using various models of elastic transfer, and the results sometimes agree quite well with those found in DWBA analyses of the corresponding light-ion reactions. (A useful compilation of elastic transfer spectroscopic factors has been given by Von Oertzen (Vo75)). As an example, an elastic transfer analysis of $^{12}\text{C}+^{13}\text{C}$ scattering (at 15 MeV $^{12}\text{C}$ bombarding energy) by Chua (Ch74), yielded a spectroscopic factor in the energy range .51-.71, while DWBA analysis of data on the reaction $^{14}\text{C}(d,t)^{12}\text{C}$ yielded a spectroscopic factor ($<C>^2S$) of .48 (D78). Bohlen and Von Oertzen (Bo72) obtained a value of .81 for the $^{12}\text{C}+^{13}\text{C}$ elastic transfer spectroscopic factor. This was obtained from an LNCO model, where multiple exchanges of the transferred nucleons are allowed (Vo75).

One must keep in mind, however, that in elastic transfer analysis a somewhat arbitrary choice must always be made as to what features of the cross-sections will be ascribed to the direct part of the elastic transfer component. Unavoidably, the analysis therefore suffers from serious ambiguity which can have a significant effect on $S_{ET}$. In addition, since the theoretical description of even ordinary transfer processes, especially those involving more than one nucleon, is far from
complete at this time, we cannot be confident that a particular calculation provides a reliable scale for elastic transfer processes.

4.4.b Elastic Transfer in $^{14}$C+$^{12}$C.

The first of the three systems considered in this analysis was the $^{14}$C+$^{12}$C one, where the elastic transfer process is the two-neutron stripping reaction $^{12}$C($^{14}$C,$^{12}$C)$^{14}$C. We recall that, in these data, even the angular distribution at 20 MeV (9.23 MeV cm) exhibited a deep undulation between 70° and 110° which could not be fitted optically while reproduction of the smooth forward angle data was preserved. However, the DWBA-based elastic transfer calculations provided a very good description of the low energy angular distributions even with the nominal model parameters which we have assumed. The higher-energy data, however, are not so well reproduced.

As input for both entrance and exit channel distorted wave potentials we used the bare optical potential given in figure 4.9a. The geometry of the Woods-Saxon potential for the two-neutron bound state was fixed with a radius parameter $R_{BS0}$=1.40 and a diffuseness $a_{BS}$=.65 - standard values for light nuclei. The two stripped p$_{1/2}$ neutrons were assumed to be coupled to $l$=0 relative to either of the $^{12}$C-cores, and the number of nodes in their wave function determined by the usual shell-model prescription:

$$2n+l = \sum_{1} Q_{i}$$

4.4.5

Here, $n$ is the number of nodes in the bound-state wave function exclusive of the origin and infinity, $l$ is the orbital angular momentum
of the bound-state relative to the core, and $Q$ is the number of shell model quanta associated with each of the constituent particles. For p-shell particles, $Q=1$, and therefore $n=1$ here. The same bound-state wave functions were used at the projectile and the target vertices.

The results of our calculations for the lower energy data are quite impressive, especially when one considers that we did no fitting. Figure 4.15 displays a comparison between the 9.23 MeV (cm) data and a calculation where $e^{i\phi}$ and $S_{ET}$ were both set to 1.0. It is interesting that this value of $S_{ET}$ is actually the best one for the low energy data; the amplitude of the undulatory structure is approximately proportional to $S_{ET}$ and a value of 1.0 reproduces the scale of this structure quite well (we were sensitive here to changes on the order of .1). The effect of varying the phase $\phi$ was to cause the positions of the maxima and minima to shift in angle. We found, in fact, that when $\phi=70^\circ$, the oscillatory structure in the calculations at low energies is very precisely in phase with that of the data. Clear phase-rule behavior can be discerned in the calculated angular distributions shown in figure 4.16 where calculations in which $e^{i\phi} = -1$ are compared with those using $\phi=0$. Except at the highest energy shown there, the oscillations in the $\phi=0$ curves are close to $180^\circ$ out of phase with those in the $\phi=180^\circ$ curves. This is reminiscent of the core-statistics dependence of the phase of angular distribution oscillations in the LCNO model (examples of which were shown in figure 2.12). Thus, the value of the phase $\phi$ may be related to the boson statistics of the $^{12}$C cores in the present situation.
FIGURE 4.15 Comparison between $^{14}\text{C} + ^{12}\text{C}$ angular distribution at 9.23 MeV and elastic transfer calculation. Here $e^{i\phi} = 1$ and $S_{ET} = 1.0$. 
FIGURE 4.16 Comparison between elastic transfer calculations of six $^{14}\text{C}^{+}^{12}\text{C}$ angular distributions with phase factors $e^{i\phi}$ of +1 (solid line) and -1 (broken line). Phase rule behavior is evident.
In figure 4.17a, calculated cross-sections with $S_{ET}=1.0$ and $e^{i\phi} = 1$ are compared with selected data over the energy range of our experiment. Figure 4.17b displays the corresponding cross-section surface. The backward-angle rise present in the calculations is reasonably consistent with what is found in the data up to about 15 MeV. However, a spectroscopic factor on the order of ten would be required to reproduce the backward rise in the highest energy data, and this would bring about ridiculously large oscillations in the lower energy data. This probably reflects a fundamental shortcoming in the one-step DWBA description of two-nucleon transfer reactions and we will return to discuss it below.

Neither details of the structure in the higher energy data not even the qualitative form of the energy-dependence are well-described by this elastic transfer model, although some features of the cross-section surface are encouraging. There is, for example, some of the alternating crest-trough structure in the calculated surface, but again, this structure is confirmed to the far backward-angle region (which is actually the far forward-angle region for the transfer reaction itself), not reaching into the angular range in which we have data. Also, angular distribution extrema in the calculated surface do not shift very much in angle as a function of energy. This feature - which would be desirable if a bit less extreme - is a reflection of the slow energy-dependence of direct reaction cross-sections. Nearly all of the structure in figure 4.17b except for the small "ridges" visible in the forward-angle region arises directly from the DWBA-calculated elastic transfer component of the scattering amplitude. Thus, the greatest drawback of this analysis may actually lie in the determination of the such potentials. Perhaps if
FIGURE 4.17a $^{14}\text{C} + ^{12}\text{C}$ angular distributions calculated with elastic transfer model ($e^{1\phi} = 1$ and $S_{ET} = 1.0$) compared with data at energies spanning the range of our experiment.

FIGURE 4.17b Cross-section surface calculated using the elastic transfer model of part a).
we had been able to find bare potentials which were consistent with the forward angle cross-sections and yet preserved some "standard" gross structure elsewhere, we would have been able to obtain a more realistic-looking "interference pattern" from the two amplitudes making up the surface.

We tried to react to this problem by recomputing the direct scattering amplitude using the bare optical potential as before, but allowing for the coupling of the 4.44 MeV $2^+$ in $^{12}$C to the elastic channel. This was motivated by the observation that the sudden increase in structure visible in the data-surface near 12 MeV could well be related to the opening of this channel. The calculations were done using the deformed optical model coupled-channel capability of PTOLEMY. We assumed, in this analysis, a value of .5 for the deformation parameter ($\beta_2$) of $^{12}$C. Combining the coupled-elastic amplitude with the same elastic transfer amplitude as before, and using a phase of +1 and a spectroscopic amplitude, $\sqrt{S_{ET}}$, of 1.0, we obtained the angular distributions shown in figure 4.18. The effect of inelastic-channel coupling on the bare potential-scattering is quite minimal, and so we do not obtain significant structure.

4.4c Elastic Transfer in $^{14}$C+$^{16}$O.

The elastic transfer reaction appropriate for our $^{14}$C+$^{16}$O data was of the two-proton pickup type: $^{16}$O($^{14}$C,$^{16}$O)$^{14}$C. Again we used a bare optical potential (figure 4.10a) in both the direct scattering and transfer amplitude calculations. The specifications of the bound-state wave functions are exactly the same as before, since the transferred
FIGURE 4.18 Comparison between $^{14}$C+$^{12}$C data and angular distributions calculated with elastic transfer model using coupled-elastic direct scattering amplitude from coupled-channels calculation (inelastic excitation of 4.44 MeV $2^+$ in $^{12}$C). Phase factor is +1 and $S_{ET}$=1.0.
protons are also $p_{1/2}$ particles. We assumed that they are coupled to $\lambda=0$ and again specify 1 node in the bound-state wave function. Taking the phase factor $e^{i\phi}$ to be +1 once again, we adjusted the spectroscopic amplitude $\sqrt{S_{ET}}$ so that the lowest energy data were well-described. This procedure yielded a value $.7$ for $\sqrt{S_{ET}} (S_{ET}=.5)$. A comparison of some angular distributions with data and the cross-section surface are shown in figure 4.19a and b.

Examining figure 4.19, we see that the DWBA-based elastic transfer model exhibits the same overall behavior here as in the $^{14}\text{C}+^{12}\text{C}$ analysis. The interesting structure is confined to very backward angles, and there is only a small energy-dependent shift in the angular locations of the extrema that lie within the angular range of our data. Once again, a very large spectroscopic factor would be required in order to obtain average backward-angle cross-sections which are consistent with the higher energy data (an $S_{ET}$ on the order of 5 would suffice). On average, the transfer cross-section by itself was only about 1/3 as large as was found in the $^{14}\text{C}+^{12}\text{C}$ calculations. Thus, two proton elastic transfer at all energies is considerably weaker than two neutron - in absolute terms. The difference may well reflect the much higher separation energy of the two protons in this system (22.3 MeV) as compared with that of the two neutrons in the $^{14}\text{C}+^{12}\text{C}$ system.

4.4d Elastic Transfer in $^{14}\text{C}+^{18}\text{O}$.

The relatively small backward-angle rise observed in $^{14}\text{C}+^{18}\text{O}$ scattering would seem to indicate that elastic transfer is a priori much less important. However, we believed that it would be useful to quantify
FIGURE 4.19a $^{14}$C+$^{16}$O angular distributions calculated with elastic transfer model ($e^{i\phi}+1$ and $S_{ET}=0.5$) at energies spanning the range of our experiment of our experiment.

FIGURE 4.19b Cross-section surface calculated using the elastic transfer model of part a)
this by extracting a spectroscopic factor for elastic transfer in the \( ^{14}\text{C}^{+^{18}\text{O}} \) system as well.

The relevant transfer process here is the alpha-particle pickup reaction \( ^{18}\text{O}^{(^{14}\text{C},^{18}\text{O})^{14}\text{C}} \). The bare potential used for both direct and transfer amplitude calculations was the strongly absorbing potential obtained in our optical analysis, parameters for which are given in the caption of figure 4.2a. That figure makes it quite clear that this potential constitutes a much better description of the data to begin with than did either of the bare potentials used in \( ^{14}\text{C}^{+^{12}\text{C}} \) or \( ^{14}\text{C}^{+^{16}\text{O}} \) calculations.

The geometry of the bound-state potential well was specified with the same parameter values as always: \( R_{\text{BSO}}=1.40 \) and \( a_{\text{BS}}=.65 \). Assuming that the total orbital angular momentum of the alpha-particle relative to the core was zero and that the constituent particles were two \( ^{1}\text{p}_{1/2} \) protons and two \( ^{1}\text{d}_{5/2} \) neutrons (which carry two quanta each) we determined that a wave function with three nodes was appropriate (by 4.4.5).

The results of the calculations were quite interesting. As shown in figure 4.20, the average magnitude of the backward rise is reproduced at all experimental energies, and the cross-section surface is similar in its gross features to the ones which we have obtained for the \( ^{14}\text{C}^{+^{12}\text{C}} \) and \( ^{14}\text{C}^{+^{16}\text{O}} \) systems with this model. However, the interesting difference is that here we have used a spectroscopic amplitude, \( /S_{\text{ET}} \), of only .1. This means that we have suppressed the transfer contribution to the scattering cross-section by a factor of 10.
FIGURE 4.20a $^{14}\text{C} + ^{18}\text{O}$ angular distributions calculated with the elastic transfer model ($e^{i\phi} = 1$ and $S_{ET} = 0.01$) compared with data.

FIGURE 4.20b Cross-section surface calculated using the elastic transfer model of part a)
The reason that this suppression must be so large, however, is not merely that the experimental cross-section at large angles is fairly small, although that condition plays a significant role. Rather, we find in addition that the DWBA cross-section for the $^{18}_0(^{14}_0, ^{18}_0)^{14}_0$ reaction is itself very large, typically exceeding by an order of magnitude cross-sections calculated for the $^{16}_0(^{14}_0, ^{16}_0)^{14}_0$ and $^{12}_C(^{14}_0, ^{12}_C)^{14}_0$ reactions. This is possibly an artifact of the very low (6.2 MeV) separation energy of an alpha-particle from $^{18}_0$ as compared with that of two neutrons from $^{14}_0$ (13.1 MeV) or two protons from $^{16}_0$ (22.3 MeV). The larger rms radius of the bound-state wave function resulting from the requirement of two additional nodes would also be expected contribute to the difference. We recognize, though, that whatever the source of the difference, a "true" spectroscopic factor of .001 would not be realistic for this system, as alpha-clustering is known to be of considerable importance in $^{18}_0$.

4.4e Discussion of the Results.

By explicitly including an elastic transfer mechanism in our calculations, we have been able to reproduce certain gross features of the scattering behavior of all three experimentally studied systems. In particular, we find that it is possible at last to generate angular distributions which possess both oscillatory structure and a large backward rise - characteristic features of the $^{14}_0^{12}_C$ and $^{14}_0^{16}_O$ data. However, most of the oscillatory structure which appears in these calculations arises from structure in the transfer cross-section alone, the final cross-section surfaces appearing as though they were the
result of modulating the relatively smooth bare potential surfaces with the transfer cross-section. Transfer cross-sections obtained from conventional DWBA calculations typically exhibit structure which changes very slowly with energy (as does structure in the single-nucleon transfer reactions to which such calculations have been so successfully applied). The calculated cross-section surfaces bear this out. Their appearance is too uniform because the constituent angular distributions lack the energy-dependent structure which characterizes the $^{14}\text{C}+^{12}\text{C}$ and $^{14}\text{C}+^{16}\text{O}$ data.

The structure which is missing from the calculated surfaces would appear to be of the Fraunhofer-diffractive sort; we need undulations whose frequency reflects the dominant partial wave at a given scattering energy. Thus, a dramatic improvement in the degree of qualitative similarity between measured and calculated cross-section surfaces for the two lighter systems probably requires the introduction of structure in $f_{\text{elastic}}$. Including coupling to an inelastic channel in the $^{14}\text{C}+^{12}\text{C}$ system was of no avail. Thus, future work should perhaps focus on finding bare potentials or bare scattering amplitudes (if a coupled-channels approach is used) which better describe the forward-angle $^{14}\text{C}+^{12}\text{C}$ and $^{14}\text{C}+^{16}\text{O}$ data.

The $^{14}\text{C}+^{18}\text{O}$ data are finally well-described by the potentials obtained in our optical analysis. The weak backward-angle oscillatory structure and modest backward rise seen in these data are qualitatively reproducible with the addition of elastic transfer, but only when the transfer effects are suppressed very drastically with a spectroscopic
factor far too small to be realistic as a spectroscopic indicator of the
degree of alpha-clustering in $^{18}$O.

The latter point brings us back to the important question of
whether or not elastic transfer can be used as a reliable spectroscopic
tool. The serious scaling difficulties which we face in the higher
energy $^{14}$C+$^{12}$C and $^{14}$C+$^{16}$O data would appear to be another negative
indicator. However, our problems here are perhaps not as much a
condition to be associated with elastic transfer as a reflection of the
inadequacy of our particular model in describing the complex reaction
dynamics of multi-nucleon transfer. It is true that the additional
ambiguity which arises from the presence of a direct elastic scattering
amplitude is possibly a serious problem in spectroscopic analysis using
elastic transfer. However, our understanding of even ordinary (i.e. non-
elastic) transfer processes which involve more than one nucleon is
presently far from complete. Even with far more sophisticated models
than the one-step or "simultaneous-transfer" DWBA which we have used
here, one and two order-of-magnitude discrepancies are frequently
encountered in the analysis of multi-nucleon transfer data. Such
difficulties overwhelm those associated with ambiguities in choosing the
correct bare potential (or the direct scattering amplitude) in elastic
transfer models, and may well account for much of the trouble in our
analysis.

Unphysically large spectroscopic amplitudes are needed to obtain
large-angle cross-sections consistent with those of our high-energy
$^{14}$C+$^{12}$C and $^{14}$C+$^{16}$O data. At the same time, the low-energy data are
reproduced relatively well with fairly modest values of $\sqrt{S_{ET}}$. Therefore,
if in fact elastic transfer is responsible for the backward-angle rise observed in these systems, it is clear that other mechanisms besides simultaneous two-nucleon transfer must make increasingly important contributions to the elastic transfer cross-section as energy increases. In order to account for the full energy-dependence of the spectroscopic factors, the contribution of these other processes at the highest energies needs to become about ten times larger than that of the simultaneous transfer process.

It is interesting and perhaps very relevant that DWBA calculations in which two-nucleon transfer reactions are treated as two-step or "sequential" transfer processes have predicted cross-sections which may exceed those calculated with simultaneous transfer models by as much as a factor of ten in light nuclei (Ka75). Unfortunately, a definitive demonstration of the relative importance of sequential transfer in an experimental situation has been hampered by the circumstance that the angular shape of sequential and simultaneous transfer cross-sections is generally predicted to be very similar.

It has been observed that when the intermediate Q-values in a two-step transfer reaction differ greatly, the angular shapes predicted by simultaneous and sequential transfer models may be indistinguishable. In a sequential transfer process, two-neutrons elastic transfer in the $^{14}\text{C} + ^{12}\text{C}$ system would proceed through a $^{13}\text{C} + ^{13}\text{C}$ configuration with intermediate Q-values of ±3.23 MeV, while the $^{14}\text{C} + ^{16}\text{O}$ reaction would proceed through $^{15}\text{N} + ^{15}\text{N}$ with intermediate Q-values of ±1.91 MeV. The intermediate Q-values are probably not sufficiently large to enable a
clear distinction to be made - especially in view of the many ambiguities which affect elastic transfer analysis.

As we have noted, the particular elastic transfer model which we have used in the analysis presented in this section does not provide an adequate description of the $^{14}\text{C}+^{12}\text{C}$ and $^{14}\text{C}+^{16}\text{O}$ data at high energies and seems to overestimate the transfer contribution to $^{14}\text{C}+^{18}\text{O}$ scattering everywhere. However, this one-step DWBA-based model is able to reproduce the low-energy data in the two lighter systems very well. In addition, the spectroscopic factors extracted from the low-energy analysis have reasonable values ($S_{ET}=1.0$ and $.5$ in the $^{14}\text{C}+^{12}\text{C}$ and $^{12}\text{C}+^{16}\text{O}$ systems respectively). This is not to claim that these values are necessarily meaningful in a true spectroscopic sense (i.e. in being comparable to an overlap integral). However, it may be that this simple model contains the essential "physics" of these heavy-ion interactions at low energies.

Perhaps this is true only near Coulomb barrier energies because elastic transfer is one of the relatively few open channels there, and the absence of strong coupling with other channels renders the entire process simple enough for the one-step DWBA to provide an adequate description. Even sequential elastic transfer may be relatively hindered in these systems at low scattering energies because of the less optimal $l$-matching and negative $Q$-values involved in the transitions to the intermediate states.

It is interesting that if the value of $1.0$ for $S_{ET}$ which was extracted from the low-energy $^{14}\text{C}+^{12}\text{C}$ data were accurate in a true spectroscopic sense, then a comparison with other experimental results
may indicate the existence of some degree of pair correlation between the valence neutrons of $^{14}$C. In particular, DWBA analyses of data on the reactions $^{13}$C(d,t)$^{12}$C and $^{13}$C(d,t)$^{14}$C have yielded spectroscopic factors of .48 (Da78) and .59 (Aj81) respectively. As we noted earlier, Chua (Ch74) also found a spectroscopic factor of .61 for the elastic transfer in $^{13}$C+$^{12}$C scattering using a DWBA-based model similar to the one used here. (Actually if we had used a bound state potential radius parameter of only 1.25, as did Chua, our $S_{ET}$ would have been 1.4). These results imply a relative enhancement in the $^{12}$C+2n parentage of $^{14}$C as compared with the $^{13}$C+1n parentage, while the latter is consistent with the $^{12}$C+1n parentage of $^{13}$C. Considerations such as these may be relevant to the discussion of superfluid-like coupled-pair tunnelling in sub-barrier interactions (a nuclear Josephson effect (Di70a, 70b)). To date, however, such effects have not been observed in light nuclei.
4.5 PHASE SHIFT ANALYSIS.

The techniques of phase shift analysis were not emphasized in our work because it became apparent very quickly that the ever-present problem of ambiguities in the extracted phase shifts would not allow a clear identification of resonant structure. We found that although our datasets were comprehensive, still the individual angular distributions were too restricted in angular range to permit the extremely accurate analyses which identification of resonant structure in the midst of strong absorption requires. According to a recent study of Van Roosmalen (Va86), knowledge of the small angle cross-sections is particularly important reducing such ambiguities; measurement to angles even further forward than the Fresnel maximum is highly desirable. Such measurements are generally very difficult and would not have been possible with our apparatus.

In view of the comparatively featureless character of our $^{14}$C+$^{18}$O data, phase shift analyses in search of resonant structure would clearly have been futile. The $^{14}$C+$^{12}$C and $^{14}$C+$^{16}$O data sets both would have been of more interest for phase shift analyses, but the results from our first efforts with our $^{14}$C+$^{12}$C data were sufficiently discouraging to lead us to drop this approach. If more forward angle-data on either of these systems becomes available to complement our data, phase shift analyses may well prove useful, particularly in conjunction with theoretical studies involving the group-theoretical S-matrix models now being developed for heavy-ion scattering (some aspects of which we discussed in chapter II. The next section is devoted to their application to our data).
Our work here then focusses on the $^{14}$C+$^{12}$C system, where first we were interested in determining whether the imposition of smoothness constraints would enable us to circumvent the ambiguities problem and yet obtain good fits too data. Our second interest was the investigation of the "anomalous" region (~17.5 MeV) of the data, where the loss of an extremum in the angular distributions compared with those at lower energy suggests the existence of resonant structure.

The phase-shift fitting routine which was used in this analysis employs an extremely fast and efficient algorithm for model-independent (unconstrained) extraction of the S-matrix from heavy-ion scattering data. Developed recently by van Roosmalen (Va87), the algorithm is a Newton-Raphson iteration for the S-matrix which converges quite rapidly because angular distributions in heavy-ion scattering are dominated by the nuclear-Coulomb cross-term, $2\text{Re}[f_N^*(\theta)f_C(\theta)]$, and so are approximately linear in the $S_{l,n}$. The linear equations by which one finds successive iterates for the S-matrix are solved by a least squares method and the corresponding error matrix extracted. The fitting programs themselves were executed on an IBM PCAT personal computer, and were found to be both accurate and fast, a typical iteration requiring one minute and convergence usually being obtained in five iterations or less.

In the first analysis, the 17 angular distributions lowest in energy (20.00-25.60 MeV $^{14}$C beam, 9.23-11.82 MeV cm) were subjected to fitting with a linearity constraint on the energy dependence of the $S_{l,n}$. Ten sets of eight angular distributions consecutive in energy were
fitted with S-matrix elements whose variation with energy was constrained to be linear. The fourth angular distribution from the lower energy end in each set was tabulated as the fitting proceeded, moving from lower to higher energy. The starting input for a given set contained the output from the set next lowest in average energy except in the first set, where a sharp cutoff S-matrix was used. In this way, S-matrices were tabulated for ten angular distributions from 21.05 to 24.20 MeV.

As is shown in figure 4.21, the fits were of high quality, but the phase shifts show no remarkable features. The smooth transition of the grazing partial wave from \( \ell=6 \) to \( \ell=7 \) is evident in figure 4.21b, but the grazing partial waves do not exhibit resonance-like structure. This was not a great disappointment, however, as it was clear from the relatively weak energy dependence of the low energy cross-sections that, a priori, resonances were unlikely. The point of our exercise here was to establish that our smoothing-method could be implemented with reasonable results so that we could use it to study the more interesting structure in the higher energy data. Unfortunately, it was found that the energy dependence of the cross-sections - even at energies only slightly higher than those considered here - was too strong to allow the high quality of fits which a phase shift analysis must produce to be meaningful.

The second phase of the analysis involved unconstrained phase shift fitting where the initial estimate for the S-matrix at one energy was provided by the final fitted S-matrix from the previous one (a sharp cutoff or smooth cutoff S-matrix was used as input for the fit to the lowest energy data). We fitted the angular distributions measured at ten
FIGURE 4.21a $^{14}$C+$^{12}$C angular distribution at 11.01 MeV and fit from phase shift analysis by van Roosmalen (unpublished, 1986).

FIGURE 4.21b S-matrix elements from low energy phase shift analysis plotted in the complex plane as a function of energy. The locus of each $S_\ell$ is labelled by the corresponding $\ell$-value.
a) $^{14}\text{C}+^{12}\text{C}$

\[ \sigma/\sigma_{\text{Ruth}} \]

\[ 10^0 \quad 10^{-1} \quad 10^{-2} \]

\[ 0 \quad 20 \quad 40 \quad 60 \quad 80 \quad 100 \quad 120 \quad 140 \quad 160 \quad 180 \]

$b) ^{14}\text{C}+^{12}\text{C}$ Phase Shift Analysis $9.2 \leq E_{\text{CM}} \leq 12.6$ MeV

\[ \text{Im}(S_{2,N}) \quad \text{Re}(S_{2,N}) \]
consecutive experimental energies starting at 35.05 MeV (16.18 MeV cm) and ending at 38.20 MeV (17.63 MeV cm). This set of data begins in the "normal" region where we see seven distinct maxima between 40° and 140° and ends in the anomalous region, where only six maxima exist.

The quality of the resulting unconstrained fits is excellent, with a $\chi^2$ per point generally less than three and at times very close to one. Three examples which span the energy range of the fitted data are shown in figure 4.21. However, as is characteristic of unconstrained phase shift analyses, the S-matrix elements which were finally obtained depended quite strongly on the initial input guess, and sets of S-matrix elements which differed dramatically in phase, $\delta_2^l$, and by up to 20% in modulus, $\eta_2^l$, could be extracted from a singular angular distribution, the resulting calculated cross-sections being equally excellent as fits to our data. We found that no two fits were exactly alike, but the flexibility allowed by our experimental uncertainties did not permit distinction among them.

It is, nevertheless, interesting to examine the results of one "run" of ten angular distribution fits which showed particularly smooth behavior. The curves in figure 4.22 were calculated from these results. In figure 4.23 the reflection coefficients obtained from this series of fits are displayed, and in figure 4.24, we show full Argand diagrams for the $S_2^l$'s with $l=6$ to $l=17$.

The S-matrices from the fits clearly display a general strong-absorption profile, but in figure 4.24 we can see much jagged structure in the $\eta_2^l$'s. It is also interesting that the lowest partial waves are only rarely completely absorbed (only $\eta_5$ at higher energies is nearly
FIGURE 4.22 Fits to $^{14}\text{C}^{12}\text{C}$ data from phase shift analysis at three energies spanning the anomalous region.
PHASE SHIFT ANALYSIS FITS TO $^{14}\text{C} + ^{12}\text{C}$ DATA
FIGURE 4.23 Moduli of the S-matrix elements extracted from phase shift analysis on anomalous region $^{14}_C + ^{12}_C$ data. The $\eta_\ell$ curves are ordered in increasing energy in the vertical direction and the vertical scale is shifted by .25 for each successive curve. The broken line indicates an ordinate value of zero for the corresponding curve.
PSA RESULTS FOR 16.18-17.63 MEV (CM) DATA
FIGURE 4.24 Argand plots of S-matrix elements from phase shift analysis for $\ell$-6 to 17.
zero) and some of the higher partial waves are not completely reflected. In fact, the $\eta_\ell$ values both below and above the grazing $\ell$, which is evidently between 10 and 11 here, are actually quite important for the details of the fit. They cannot be set, respectively, to zero or one without dire consequences as far as the $\chi^2$ for the fit is concerned.

The angular distributions here are not easy to compare with a $P^2_\ell$ because of the variable period of the middle and backward undulations. However, we should point out that a $P^2_{12}$ has seven extrema between $40^\circ$ and $140^\circ$, and a $P^2_{10}$ has six extrema in that range. It is interesting that the profile of the $\eta_\ell$'s in the vicinity of $\ell_g$ is smoother for "normal" angular distributions (lower energies here) than it is for the anomalous ones. At higher energies, we see "spikes" develop in the $\eta_\ell$ at $\ell$=10 and 12, which could also be interpreted, perhaps, as "dips" at $\ell$=9 and 11, and even at $\ell$=13. This behavior also is reminiscent of the "odd-even staggering", which has frequently been associated with large backward-angle cross-sections in heavy-ion scattering. Indeed, in figure 4.22 the backward rise is more pronounced at 17.63 MeV, where the staggering is greatest.

Another interesting aspect of this set of fits is that small, resonance-like loops are present in some of the Argand diagrams for near-grazing $\ell$-values. As examples, both $S_{10}$ and $S_{12}$ execute not-quite-complete counterclockwise turns (the sense is not evident in figure 4.24), a half-turn requiring about three energy-steps. This would yield a width of about 500 keV for corresponding resonant states - if present.

We must urge caution in the interpretation of these results in as much as $S_{14}$ also follows a circular trajectory in the correct resonant
sense. It must also be added that other series of fits showed much less orderly behavior and lacked such features (although \( \eta_2 \)-staggering appeared consistently). The presence of strong absorption and phase shift ambiguities unfortunately prohibit any definite conclusions - at least for now. We look forward to forward-angle scattering studies which may be done in the near future in this laboratory. It is hoped that the nature of this possibly resonant structure could then be clarified through more complete phase shift analyses.
4.6 DYNAMICAL SYMMETRY ANALYSIS.

4.6a Overview.

Group theoretical or "dynamical symmetry" models, examples of which were discussed in chapter II, are presently undergoing rapid development. They have now been generalized to describe many-channel problems with non-zero exit channel spins (Sh86b) and single-channel problems involving spin 1/2 projectiles (De86b). Some of these models have already been used to achieve impressive results (for example, in $^{16}\text{O}+^{24}\text{Mg}$ elastic and inelastic scattering, and alpha particle transfer reactions $^{24}\text{Mg}(^{16}\text{O},^{12}\text{C})^{28}\text{Si}$ to the residual ground and first excited states). In the present work, however, we have applied the spinless boson dynamical symmetry models described in chapter II to our own data with a dual purpose in mind. Our foremost aim was to attempt to determine whether the symmetries which these models contain are in fact present in complex nuclear interactions. Thus, we wanted to use the models to explain our data, paying attention to both quantitative and qualitative aspects of each of the three data sets. A second important aim was simply to explore the new model spaces themselves. We hoped that results from this effort would prove useful in the current ongoing theoretical study of the $\ell$-dependent form of the interaction strength. In our analysis these strengths have been one of the standard Woods-Saxon forms of equations 2.11.5, 2.11.10, and 2.11.11. These particular phenomenological forms may eventually be supplanted by others which are better motivated from first principles, but the present collections of parameters and datasets should prove useful in achieving that aim.
One of the great advantages of the group-theoretical approaches in nuclear physics is that they lend themselves to more straightforward calculation. Typically, as in the interacting boson model, one is able to replace differential equations with algebraic ones. In dynamical symmetry models also, the S-matrix is obtained not by integrating the Schrödinger equation, as in the optical model, but by simple calculation from analytical forms such as 2.11.5.

Because of this calculational convenience of the dynamical symmetry models, much of the fitting reported here was done "by hand". S-matrices, angular distributions, and entire cross-section surfaces were calculated quickly on an IBM 4341 computer using Fortran programs which required very little development time. Immediate comparison with data was made feasible with the interactive graphics capability of SPEAKEZ. Cross-section surfaces were drawn, as always, using noninteractive PLT10 graphics software. We also performed detailed fitting to data with Fortran routines provided by B. Shao (also implemented on an IBM 4341). An interesting comparison is the following. A cross-section surface consisting of fifty-nine angular distributions evaluated at 180 angles when calculated using an optical model and the PTOLEMY code requires about ten minutes of CPU time on an IBM 4341. A DWBA transfer reaction calculation of the same scope requires about one hour. A cross-section surface calculated using the SO(3,2) dynamical model requires less than one minute!
4.6b General Studies on the SO(3,2) Model.

Scanning the model space of \( v_R, v_I, \ell_0, \) and \( \Delta \) in the SO(3,2) dynamical symmetry model (equation 2.11.4) and studying both cross-sections and S-matrices reveals that the critical-\( \ell \) parameter, \( \ell_0, \) and the diffuseness, \( \Delta, \) are closely related to the corresponding parameters of the smooth cutoff model of McIntyre in terms of their effect on the S-matrix and the resulting angular distributions. For example, \( \ell_0 \) is approximately the grazing \( \ell \)-value (as determined from the profile of the \( \eta_{\ell} \)'s) and \( \Delta \) more or less determines the width of the grazing region in \( \ell \)-space. Thus, \( \ell_0 \) is directly related to the number of oscillations observed in the angular distributions, and \( \Delta \) controls their depth. Large \( \Delta \)-values (\( \approx 1.5 \)) generally render angular distributions featureless, while small values (\( \leq 0.2 \)) are reflected in deep oscillatory structure.

The real and imaginary strength parameters, \( v_R \) and \( v_I \), are related to the shapes of the angular distributions in a more complicated way. They do not correspond directly to optical potential well depths, although it can be argued that a qualitative similarity is present; it appears that the absolute magnitude of these parameters is not as important as their ratio. A not unreliable rule of thumb is that the larger the ratio \( R_{IR} = |v_I|/|v_R| \), the deeper the predicted oscillatory structure and the lower the predicted average cross-section in the forward hemisphere (however, if \( R_{IR} \) is very large or very small, very irregular structure may result, as we shall demonstrate). As noted in chapter II, \( v_I \) and \( v_R \) must be of opposite sign in order to preserve the unitary bound, and \( v_I \) must be non-zero if we are to include absorption, as heavy-ion scattering requires.
In the process of exploring the SO(3,2) dynamical symmetry model space while fitting our data, we discovered that the model admits of approximate ambiguities of a continuous nature which involve a number of distinct invariances. The simplest of these is found in continuous transformations of $\Delta$ and $R_{IR}$ when they are changed in the same direction. If one is changed arbitrarily, a value for the other may be found which leaves the average slope and the absolute magnitude of the angular distribution invariant. The invariance may apply to the angular distribution as a whole if $\Delta$ is large or $l_0$ very small so that there is little structure, but in general it is rather crude.

Other, more complicated invariances are also found in some parts of the model space. For example, if, once again, $\Delta$ is large (so that the model angular distributions show little structure) but $R_{IR}$ is fixed, simultaneous variation of $l_0$, the overall scale of the interaction strengths, and $\Delta$, either pairwise or all together, can result in sets of very similar predicted angular distributions. However, when $\Delta$ is not large, oscillatory structure, whose form is critically $l_0$-dependent, limits the invariance to transformations involving $\Delta$ and the scale of the strengths. Our impression is that $R_{IR}$ and $l_0$ are less ambiguous in general than are $\Delta$ and the overall scale of $\nu_R$ and $\nu_I$.

Another point worth remarking upon is that all of the algebraic scattering models proposed thus far differ very markedly from the optical ones in not automatically incorporating an energy dependence of the grazing partial wave. In current realizations of dynamical symmetry models, the only explicit energy dependence is through $f(k)$. Therefore, one must always choose $l_0$ in an explicitly energy-dependent way, as one
would have to do in a smooth-cutoff model in order to be able to obtain a grazing partial wave consistent with the scattering energy. Having to fit data over a wide energy range such as ours, it is clearly necessary to choose some form of reasonable energy dependence for \( l_0 \). Our approach to the problem has been one of the following: (1) We fit all parameters, including \( l_0 \), to the data at a given energy, and then solve for \( R \) in 2.2.10, the semiclassical expression for \( \ell \), by taking \( \ell = l_0 \). The parameter \( l_0 \) at all other energies is then determined from 2.2.10. Or (2) We simply interpolate, linearly, between \( l_0 \) values fitted to data near the low and high energy extremes of our datasets.

Characteristic features of angular distributions calculated with the \( SO(3,2) \) model are illustrated in figure 4.25 for \(^{14}C+^{16}O\) scattering within the energy range of our experiment. The model parameters for the cross-section surface - which is shown in part a) of the figure - were obtained by fitting forward angle data at 21.49 MeV cm (the highest energy) and then linearly interpolating \( l_0 \) between a low-energy (10.66 MeV cm) value of 6.0 and the high-energy value of 13.0. The low-energy value for \( l_0 \) was chosen to reproduce the slope of the angular distribution at 10.66 MeV, but the overall normalization is too large at lower energies. Other parameters are given in the figure caption.

Except at forward angles, angular distribution structure in figure 4.25 a) does not reproduce that of the data even qualitatively, as there are too few oscillations at high energies, the backward cross-sections are not large enough, and the middle-angle cross-sections are not too large. However, the surface has an overall appearance which is more evocative of our data than any other calculation thus far. We see here a
FIGURE 4.25a SO(3,2) dynamical symmetry model cross-section surface for $^{14}\text{C}+^{16}\text{O}$ scattering in the energy range of our experiment. Model parameters are $v_R = 4.0, v_I = -5.07, \Delta = .033$ and $\lambda_0$ interpolated linearly between 6.0 and 13.0.

FIGURE 4.25b Dynamical symmetry cross-section surface calculated using parameters of part a) except that $\Delta$ is now .3.
series of criss-crossing "ridges" which have an approximate symmetry axis near 110°. The criss-crossing pattern gives rise to the alternating crest-trough excitation function structure which has not been reproducible using the optical model, with or without the elastic transfer amplitude. This structure is not merely an artifact of the extremely small diffuseness parameter $\Delta$, as is demonstrated in part b) of figure 4.25 where $\Delta$ has been arbitrarily increased to .3. The "smoothing" effect of an increased $\Delta$ is very evident. It is not evident but also true and characteristic that the average cross-section - up to about 90° - is actually larger in the larger-$\Delta$ calculation, although the backward angle cross-section is clearly smaller.

The relative suddenness of the changes in structure in the small-$\Delta$ surface as a function of energy is easily explained and illustrative of a point. They occur because the small value of $\Delta$ causes the Woods-Saxon form for the interaction strength to behave as a step function. The strength $v(l)$ is then either $v_R + v_1 + 1/2$ or just 1/2 depending on whether $l$ is less than or greater than the current value of $l_0(E)$. The interesting point is that the ratio of complex-valued gamma functions which yields the S-matrix in this model does not itself seem to introduce complicated $l$-dependence in the reflection coefficients, $\eta_L$. This quite general feature of the model is demonstrated in figure 4.26, in which the $\eta_L$'s from the small-$\Delta$ calculation for figure 4.25a are plotted against $l$. This behavior reflects the step-function behavior of the interaction strength.

Although odd-even staggering and the complicated $l$-dependences are not found for $\eta_L$ in the SO(3,2) model, it is found that both $\eta_L$ and $\delta_L$
FIGURE 4.26 Values of $\eta_2$ from the calculation of figure 4.25a.

In this figure the $\eta_2$ curves are ordered in increasing energy in the vertical direction.
may exhibit very unusual energy dependent behavior when the ratio $R_{IR}$ is very large or very small ($>10$ or $<1/10$). In the "normal" situation, where $R_{IR}$ is close to unity, the phase behavior of the model is very standard, even if the strength parameters $v_I$ and $v_R$ are very large (in the hundreds). The imaginary part of $S_\ell$ is then never far from zero as $|S_\ell|$ makes its journey from 1 to 0 as energy increases and $\ell_0(E)$ overtakes $\ell$. This situation is clearly depicted in figures 4.31 and 4.32, where we display the phase behavior of our dynamical symmetry fit to the $^{14}C+^{18}O$ data (to be discussed in the next subsection).

If $R_{IR}$ is very large or small, on the other hand, a number of curious things happen. For partial waves whose $\ell$-values is few units less than $\ell_0$, the modulus of $|S_\ell|$ ($-\eta_\ell$) is never smaller than a few tenths (absorption is incomplete) and the complex $S_\ell$ itself traces very neat, slowly turning, clockwise spirals around the origin as energy increases. For $\ell$ in the vicinity of $\ell_0(E)$, $S_\ell$ spirals around the origin much more rapidly in the counter clockwise sense until that value of $\ell$ lies somewhat below $\ell_0(E)$. At such energies, the motion then becomes slower and clockwise, as it was for the lower-$\ell$ S-matrix elements. Only at low energies do we recover some normal (but trivial) phase behavior: When $\ell$ exceeds $\ell_0(E)$ by a few units, $S_\ell\approx 1+0i$ (i.e., the partial wave is entirely reflected and also suffers no phase shift). (Note: in this discussion and in all plots involving the complex-valued S-matrix elements, we are actually considering the "nuclear" part of the S-matrix only. That is, we are investigating the behavior of $S_N,\ell$ defined by $S_\ell=N_{\ell}S_\ell^{,N}$, where $S_\ell$ is given by one of the model-defining expressions
2.11.4, 2.11.7, or 2.11.11, and $S_{\ell,C}$ is the point-charge Coulomb scattering S-matrix.

This odd behavior of the S-matrix when $R_{IR}$ is large was discovered by fitting data. With an automated fitting program we were able to obtain a very good fit to the highest energy $^{14}\text{C}+^{12}\text{C}$ data (18.60 MeV cm), but only at the cost of allowing enormous $v_R$ and $v_I$ values (112.9 and -1172 respectively) and an $R_{IR}$ of 10.3. In figure 4.27a we display a comparison of the 18.60 MeV data with the corresponding calculated angular distribution. The fit is good, but part b) of this figure is much more interesting. There we show the cross-section surface calculated from the SO(3,2) model with the fitted parameters (with $l_0$ linearly interpolated between 6.0 and the fitted value of 14.01) changes so rapidly with energy that it was necessary for the sake of clarity in the figure to calculate angular distributions on almost twice as fine an energy grid as usual. Figure 4.28 depicts the bizarre behavior of the individual $S_{\ell,s}$ as functions of energy for $\ell$=5 through 16. Interestingly, $\eta_2$ is still very smooth and well behaved as a function of $\ell$.

Very extensive investigation subsequent to this discovery consistently yielded the result that the sole determinant of "strange vs. normal" phase behavior in the model is the value of $R_{IR}$. However, the origin of the random-looking structure seen in the surface plot of figure 4.27b is not just the large $|v_I|/|v_R|$ ratio. Apparently, the very large absolute magnitude of $v_I$ and $v_R$ is also a contributing factor, as the cross-section surfaces appear somewhat more orderly when $|v_R|$ and $|v_I|$ are both less than 100 even if $R_{IR}$ is ten or more and the phase
FIGURE 4.27a Comparison of SO(3,2) model calculation with 18.60 MeV $^{14}$C+$^{12}$C data. Model parameters are $v_R=112.9$, $v_I=-1172.2$, $\omega_0=14.01$ and $\Delta=0.297$

FIGURE 4.27b Cross-section surface calculated from parameters given in part a). Here $\omega_0$ is linearly interpolated from 6.0 to 14.01.
$^{14}\text{C} + ^{12}\text{C}$ SO(3,2) DYNAMICAL SYMMETRY FIT

$\theta_{\text{CM}}$

$\log_{10}(\sigma/\sigma_{\text{Roth}})$

18.60 MeV

$^{14}\text{C} + ^{12}\text{C}$

CM ENERGY (MeV)

$\theta_{\text{CM}}$
FIGURE 4.28 Argand plots for $S_\ell$ from calculation of figure 4.27b for $\ell=5$ to 16. Although not indicated in the figure, the sense of motion of $S_\ell$ in the complex plane is clockwise for small $\ell$ where the motion is slow. For $\ell \geq \ell_g$ the motion is clockwise and rapid.
behavior is "strange" (henceforth, "type II", while "normal" will be called "type I").

Unfortunately, no clear explanation of the origin of type II behavior has emerged as yet. Our first attempt to understand it was through an approximate expression for the nuclear phase shift in the SO(3,2) model derived recently by Amado and Sparrow (Am86); namely:

\[
\delta_\ell = -\frac{n v(v+1)}{2\ell(\ell+1)}
\]

where \(v\) is the complex interaction strength given by 2.11.5. We considered this expression analytically in several limits and found that it correctly predicts both type I behavior when \(v_I \sim v_R\) and type II behavior when \(v_I \gg v_R\). However, the expression fails when \(v_R \gg v_I\), predicting counter clockwise motion of \(S_\ell\) when \(\ell \ll \ell_0\) (i.e. a \(\delta_\ell\) which steadily increases with increasing energy). This opposite-sense motion is not seen in exact model calculations. The expression 4.6.1 also runs into difficulties when \(\ell > \ell_0\). These results discouraged us from experimenting further with approximate expressions tending to trivialize the complicated behavior of the model.

The resonance-like character of type II phase behavior prompted an examination (by F.Iachello and B.Shao (Ia86)) of the Regge pole structure of the SO(3,2) model. In parallel with that effort, we examined the pole structure of the model in the complex E-plane, our aim being to find poles of \(S_\ell\) near the real axis which would permit an interpretation of type II phase behavior in terms of resonant interactions. Poles of \(S_\ell\) in the complex E-plane are relatively easy to find in this model because they arise only when the gamma functions in the numerator of equation
2.11.4 become infinite, and this, of course, happens only when their arguments are negative integers. Thus, a discreet infinity of E-plane poles is associated with each \( l \) in each of the two gamma function arguments in the numerator of 2.11.4. Individual poles are easily found by equating a given argument with a negative integer and solving for \( E \).

Neither the Regge pole nor the E-plane pole investigation yielded definitive answers, however, the simple finding was that no clear correlation exists between the location of the S-matrix poles and the occurrence of type I or type II behavior. The lack of systematic differences is evident in figure 4.29, where the pole structure of the SO(3,2) model with a type I parameter (part a) is compared with a type II parameter set result (part b). The poles have an accumulation point at the origin in both situations and are never very close to the region of the physical scattering energy (and there are no poles farther from the origin than the farthest ones shown here). In each plot we have included poles arising from negative integer arguments in the range -1 to -100 for each value of \( l \) from zero through 14 (1500 poles in all, most of which are extremely close to the origin).

4.6c Fitting Data with the SO(3,2) Dynamical Symmetry Models.

The two very different types of phase behavior which we have observed in the SO(3,2) model presents us with a dilemma in the fitting of data. The SO(3,2) model is apparently capable of reproducing individual angular distributions anywhere in our dataset. In contrast with the optical model, it is able to incorporate both deep oscillatory structure and a large backward angle rise at once. However, our conclusion after
FIGURE 4.29a Locations of poles in the complex E-plane for a calculation with type I phase behavior. Model parameters here are \( v_R = 10 \), \( v_I = -30 \), \( \Delta = 0.3 \) and \( \ell_0 = 10 \).

FIGURE 4.29b Locations of poles in the complex E-plane for a calculation with type II phase behavior. Model parameters here are the same as those used in the surface calculation of figure 4.27b.
COMPLEX E-PLANE POLES IN SO(3,2) MODEL

a)

b)
extensive fitting work is that a price must be paid for this flexibility. Invariably we found that good fits to our higher energy $^{14}\text{C}+^{12}\text{C}$ and $^{14}\text{C}+^{16}\text{O}$ data require large values of $R_{IR}$ (hence type II phase behavior), and large absolute values of $v_R$ and $v_I$. The effect of these conditions on cross-section surfaces is an undesirable lack of regularity as a function of energy. Besides that, type II phase behavior is, perhaps, physically unreasonable by itself (we recall the large values of $\eta$ even for small $l$).

Our other alternative is simply to require type I phase behavior and that the sizes of $v_R$ and $v_I$ be restricted to a reasonable value (such as fifty) which is small enough to insure that the model is stable and the energy dependence of the cross-sections is not too erratic. This means sacrificing the ability to fit the more highly structured angular distributions in detail, but allows us to obtain smooth surfaces which successfully characterize some of the gross features of our data. We have chosen the latter alternative as a better overall approach to data-fitting, and have also tried to keep arbitrary energy dependences in the resulting parameter sets to a minimum.

Of the three systems which we have studied experimentally, the $^{14}\text{C}+^{18}\text{O}$ system was again the most amenable to fitting - as is not surprising. The relatively featureless form of the measured angular distributions enabled us to obtain an SO(3,2) model parameters set with which the average scattering behavior of the $^{14}\text{C}+^{18}\text{O}$ system can be described quite well, as shown in figure 4.30a (the model parameters are given in the caption, as is our custom). The rise at backward angles, even though not large, is, however, simply not reproducible with
FIGURE 4.30a  SO(3,2) dynamical symmetry model fits to $^{14}$C+$^{18}$O angular distributions at six energies which span the range of our experiment. The model parameters are $v_R=5.0$, $v_I=-10.1$, $\Delta=1.3$ and $l_0$ is given by the semiclassical prescription with $r_0=1.64$ (here $8.12 \leq l_0 \leq 15.62$).

FIGURE 4.30b  Cross-section surface for $^{14}$C+$^{18}$O scattering calculated using model parameters given in part a).
a well-behaved $SO(3,2)$ model. The corresponding cross-section surface is shown in figure 4.30b. Figures 4.31 and 4.32 then present the values of $\eta_2$ and some of the $S_4$'s involved in the surface calculation. The $S$-matrix elements here exhibit very ordinary type I behavior.

The $^{14}\text{C}+^{16}\text{O}$ system presented quite a different problem. In figure 4.25a we have already shown one approach to this system; namely: a fit to the higher-energy forward-angle data. This fit has interesting qualitative features, but is quantitatively poor. Individual angular distributions in the surface plot are similar in form to those of a sharp cutoff model because $\Delta$ is so small. The larger-$\Delta$ calculation of figure 4.25b still shows the qualitatively desirable crest-trough structure and partial symmetry, but its angular distributions are on the order of fifty times too small at backward angles.

It was not possible to reproduce the deep oscillatory structure and large rise of these data without invoking type II phase behavior and a large value of $v_1$. However, we were able to do considerably better in obtaining a "bare" elastic scattering cross-section (which lacks the backward rise) than was possible using the standard optical model. The result is shown as a cross-section surface in figure 4.33a. In this calculation, we have allowed $\Delta$ to become energy-dependent, its value varying linearly from 1.88 at 10.66 MeV to 1.3 at 21.49 MeV. It is also notable that the semiclassical prescription for $l_0$ was not applicable here. Good fits at low energies required a larger $l_0$-value than would be given by 2.2.10 using $r_0$ obtained from the higher energy $l_0$-value. Figures 4.34 a,b and c show comparisons of calculated angular
FIGURE 4.31 Modulus of the $S$-matrix elements obtained for the surface calculation of figure 4.30b. Again, the curves are ordered in increasing energy in the vertical direction. The relatively large value of $\Delta$ in this calculation is reflected in the large $I$-width on the grazing region.
C14+018 SO(3,2) DYNAMICAL SYMMETRY REFLECTION COEFS.
FIGURE 4.32 Argand plots of $S_\ell$ obtained for the surface calculation of figure 4.30b for $\ell$ from 7 to 18. These display typical type I ("normal") phase behavior.
FIGURE 4.33a  Cross-section surface for $^{14}$C+$^{16}$O scattering calculated from SO(3,2) model with parameters $v_R=7.6$, $v_I=-19.8$, $\Delta$ linearly interpolated from 1.88 to 1.30, and $\ell_0$ also linearly interpolated from 4.04 to 12.30. Backward angle cross-section is sacrificed here in favor of more accurate reproduction of the forward angle structure.

FIGURE 4.33b  $^{14}$C+$^{16}$O cross-section surface from SO(3,2) model with parameters $v_R=5.0$, $v_I=-15.0$, $\Delta=6$, and $\ell_0$ given by the semiclassical prescription with $r_0=1.54$ ($5.10 \leq \ell_0 \leq 15.40$). This surface has the correct number of extrema at high energies and better reproduces the backward angle cross-sections.
FIGURE 4.34a,b,c Comparisons of angular distributions from the calculation of figure 4.33a with $^{14}\text{C} + ^{16}\text{O}$ data at 10.66 MeV, 14.03 MeV, and 21.49 MeV respectively.

FIGURE 4.34d Comparison of $^{14}\text{C} + ^{16}\text{O}$ angular distribution at 21.49 MeV from calculation of figure 4.33b with data.
SO(3,2) MODEL FITS TO $^{14}\text{C} + ^{16}\text{O}$ DATA

a) 10.66 MeV

b) 14.03 MeV

c) 21.49 MeV
d) 21.49 MeV
distributions with data. The fits are good except in that the large-angle cross-sections are both too small and miss an oscillation at high energies (the latter problem is a consequence of the non-constant period of the undulations in the data).

Yet another type of fit to the $^{14}\text{C}+^{16}\text{O}$ data is shown in figure 4.33b. As follows from the comparison with an angular distribution in figure 4.34d, this fit still underpredicts the large-angle cross-section, but does much better there than the previous one, while its behavior at forward and intermediate angles is much worse. The surface was calculated using a semiclassical prescription for $l_0$ (with $l_0$ at 21.49 MeV determined by the fit), and has the correct number of high energy angular distribution maxima. Both surfaces in figure 4.33 are reminiscent of optical model results.

The $^{14}\text{C}+^{12}\text{C}$ data were similarly difficult to fit using $SO(3,2)$ dynamical symmetry. Once again, we found parameters sets with which we could reproduce forward-angle structure and standard optical-type oscillatory behavior, but good quantitative fits were not found for the highly structured high energy data. However, we did find a physically reasonable set of parameters from which we obtained what is probably the best qualitative reproduction of the general pattern of this system's energy-dependent structure that we have found in this work. The cross-section surface corresponding to this parameters set is given in figure 4.35a, and a set of the calculated angular distributions is compared with data in figure 4.35b.

It is apparently the small value of $R_{IR}(.31)$ that is primarily responsible for the large, nearly adequate backward rise which appears
FIGURE 4.35a Comparisons of $^{14}\text{C}+^{12}\text{C}$ angular distributions calculated in the SO(3,2) dynamical symmetry model with data at six energies which span the range of the experiment. The model parameters are $v_R=7.27$, $v_I=-2.25$, $\Delta=.21$ and $l_0$ determined semiclassically with $r_0=1.70$ ($6.87\leq l_0 \leq 14.98$).

FIGURE 4.35b Cross-section surface for $^{14}\text{C}+^{12}\text{C}$ scattering calculated in the SO(3,2) model using the parameters of part a)
in the highest-energy calculations. However, $R_{IR}$ is not so small that type I phase behavior is lost. The backward rise in the calculations is much too large in the 11 to 14 MeV energy range, while at higher energies the forward-angle cross-sections are clearly overestimated. It is also shown in figure 4.35b that the deep minimum near 90° in the intermediate energy data (~15 MeV) is not reproduced at all in this calculation.

That these problems could not be solved simultaneously in the "well-behaved" SO(3,2) model is an indication that some other critical mechanism (such as elastic transfer for example) is missing from the model. Nevertheless, the cross-section surface of figure 4.35a has the criss-crossing pattern of ridges ("alternating crest-trough structure") and the backward angle rise which are the qualitative signatures of the dataset. It is also notable that the angular distributions of the calculation roughly agree with those of the data in the number of undulations they exhibit at a given energy. Finally, it is interesting that the data at the very lowest energies are well reproduced here. This had not been possible in our optical analysis.

4.6d Elastic Transfer with a Dynamical Symmetry Model.

The SO(3,2) model which we have just discussed and the simpler SO(3,1) model which is specified by equations 2.11.7 and 2.11.10 both share with the optical potential approach a basic problem with respect to our data. They are simply not able to describe even individual angular distributions from the higher energy end of our $^{14}_C + ^{12}_C$ and $^{14}_C + ^{16}_O$ datasets with any degree of quantitative accuracy (unless 'of
curse we allow possibly unphysical parts of the model spaces into consideration, as we saw earlier). This failure motivates an exploration of the two-channel $SO(3,1)$ model given by 2.11.11 as a means of gaining further flexibility in fitting data with a large backward rise by using the model' "reaction channel" to describe an elastic transfer process. As was pointed out in section 2.11, cross-sections in this elastic transfer model are easily obtained from the "elastic" and "transfer" amplitudes calculated from the model's $s_{\text{elastic}}$ and $s_{\text{non-elastic}}$ according to the usual prescription, 2.10.3. We simply use the elastic channel quantum numbers everywhere.

The encouraging result of our application of this multichannel model to data-fitting was that we could indeed reproduce any single angular distribution from the $^{14}\text{C}+^{12}\text{C}$ or $^{14}\text{C}+^{16}\text{O}$ datasets in reasonably good quantitative detail. However, we find that when a set of parameters chosen so as to reproduce a particular angular distribution was used in a surface calculation, the result was not as impressive. Apparently, we would require a complex energy dependence in the six parameters of this model in order to reproduce a full cross-section surface with satisfactory accuracy.

Figures 4.36 through 4.39 summarize these conclusions. Figure 4.36 displays a comparison of a fitted cross-section with $^{14}\text{C}+^{12}\text{C}$ data at the highest and lowest experimental energies. The parameterset used here was determined by fitting the high energy data only; this parameterset also yielded reasonable results for the low energy data. Nevertheless, the corresponding cross-section surface, shown in figure 4.37, makes it clear that we have not found a general fit. Somewhat better results are
FIGURE 4.36a Fit to $^{14}\text{C}+^{12}\text{C}$ angular distribution at 18.60 MeV using SO(3,1) coupled-channel elastic transfer model. The model parameters are $v_R=4.04$, $v_I=-.709$, $u_R=3.59$, $u_I=.497$, $\Delta=.341$ and $\ell_0=14.19$. From this value of $\ell_0$, the semiclassical value for $r_0$ is 1.63.

FIGURE 4.36b Comparison of dynamical symmetry plus elastic transfer model calculation (using the same parameters as in part a) with $^{14}\text{C}+^{12}\text{C}$ angular distribution at 9.23 MeV ($\ell_0=6.22$ by the semiclassical prescription here).
FITS TO $^{14}$C $^{12}$C DATA WITH SO(3,1) COUPLED-CHANNEL ELASTIC TRANSFER MODEL

(a) 18.60 MeV

(b) 9.23 MeV
FIGURE 4.37 Cross-section surface for $^{14}\text{C}+^{12}\text{C}$ scattering calculated from the SO(3,1) coupled-channel elastic transfer model with the parameters from figure 4.36a.
FIGURE 4.38 Cross-section surface for $^{14}\text{C}+^{16}\text{O}$ scattering calculated using the SO(3,1) coupled-channel elastic transfer model. The model parameters are $v_R=37.5$, $v_I=-15.5$, $u_R=14.1$, $u_I=5.45$, $\Delta=0.433$ and $l_0$ is given by the semiclassical expression with $r_0=1.44$ (3.64 $\leq l_0 \leq 14.09$).
CM ENERGY (MeV)

θ

CM ENERGY (MeV)

θ

C.14 O16 WITH ELASTIC TRANSFER BY NUC-CHANNEL S(3.1)
FIGURE 4.39 Surface plot of differences between our $^{14}\text{C}+^{16}\text{O}$ data and cross-sections predicted by the model used in figure 4.38.
SURFACE OF DIFFERENCES FOR $^{14}\text{C} + ^{16}\text{O}$
evident in the cross-section surface of figure 4.38 for $^{14}$C+$^{16}$O scattering. The parametser set shown there was also found by fitting only the high energy data.

Instead of showing comparisons of single angular distributions here, we provide in figure 4.39 a surface plot illustrating the difference between the calculated cross-section and the measured ones (the figure actually displays $\log_{10}\left(\frac{\sigma_{\text{exp}}}{\sigma_{\text{R}}}\right) - \log_{10}\left(\frac{\sigma_{\text{theory}}}{\sigma_{\text{R}}}\right)$). If our fits were perfect, this surface would be flat, while smooth-fitted surfaces such as those from our bare optical potential yield a residuum which looks just like the original data. We can see, then, the this fit is far from perfect. The tall "spikes' indicate that there are particular problems in reproducing the deep minima in the data at low and intermediate energies from $70^\circ$ to $90^\circ$. High, sharp ridges at very backward angles reflect the fact that deep oscillations in the calculated cross-sections are out of phase with those of the data in that region. However, the number of extrema is correct. This method of studying surfaces of differences was not exploited more generally in our analysis because our fits were simply not good enough for such a detailed comparison to be meaningful. When we are able to worry about fine detail on such surfaces, we will have made tremendous progress indeed!
CHAPTER V

CONCLUSIONS

As we stressed in the beginning of this work, the systems which we have studied have much in common; yet their scattering behavior is strikingly different. This, however, is not an unusual state of affairs in heavy-ion scattering.

Many years of effort have made it clear that no trivial argument based on systematics or mere nuclear geometry can account for such differences entirely. The final answers appear to be bound up in the nature of the nucleon-nucleon interaction itself, and in the complications associated with the quantum many-body problem of nuclear matter. Thus, understanding the mechanisms which determine the scattering behavior of a given system is a matter of great importance for nuclear physics in general, and is the primary aim of current studies of heavy-ion scattering.

The present state of development of quantitative models for such scattering does not permit us to make realistic predictions regarding the precise form of structure in the cross-sections. In fact, the full energy and angle-dependence of heavy-ion cross-sections appears to be too stringent a constraint for theoretical model calculations attempted so far. Except in phase-shift analyses, the model spaces are always vastly overdetermined by the data.

In the $^{14}\text{C}+^{18}\text{O}$ system, however, we have been able to obtain a reasonably good description of our data with both optical potential and
dynamical symmetry S-matrix approaches - except for the modest backward angle cross-section rise whose origin may lie in a weak elastic alpha-transfer amplitude. No evidence for resonant structure was found in these relatively featureless data. It is perhaps significant that the $^{14}\text{C}+^{18}\text{O}$ system has many more open exit channels than the other two, as Q-value systematics for light nuclear scattering have consistently indicated a correlation between the appearance of structure and the existence of relatively few open exit channels well-matched in angular momentum.

The $^{14}\text{C}+^{12}\text{C}$ and $^{14}\text{C}+^{16}\text{O}$ systems exhibit structure which has not proved to be amenable to even a gross description by standard optical potential scattering. For example, we have found that the backward-angle rise in the ratio to the Rutherford scattering cross-section in the higher energy data on these systems simply appears to be outside the physically reasonable model spaces. We were not successful even in our search for a "bare" potential meant to describe the energy-dependence of the forward and intermediate-angle structure alone. In our elastic transfer calculations we had to use, instead, potentials which merely lead to featureless angular distributions providing only a very crude approximation to the data. Apparently, it would be necessary to introduce rather arbitrary (and therefore undesirable) energy dependence in the optical potential parameters in order to alleviate this problem. Further work with parity-dependent potentials would possibly prove useful in improving the basic description which the optical model can provide, particularly in the backward angle region.

The large backward rise in the $^{14}\text{C}+^{12}\text{C}$ and $^{14}\text{C}+^{16}\text{O}$ angular distributions is apparently an indication of exchange effects which tends
to symmetrize the scattering amplitudes in those systems. Our attempt to
describe this effect with an elastic nucleon-transfer model met with
success in the lower energy regime, particularly in $^{14}\text{C}+^{12}\text{C}$ scattering,
but considerably underestimated the transfer contribution to the cross-
section at higher energies. This underprediction may be related to the
inadequacy of our one-step DWBA description of multi-nucleon transfer
processes. It does not take into account, for example, the sequential
transfer mechanism which has been shown to be of great importance in
ordinary (i.e. non-elastic) two-nucleon transfer reactions.

It is clear that future work on the elastic transfer hypothesis in
these systems should focus on two things. First, a more realistic
description of the transfer process itself is required; this could
perhaps be achieved using coupled-channel Born Approximation theory in
which multi-step processes can be explicitly included within the DWBA
formalism. Second, the direct-scattering amplitude must be made to bear
more of the burden of reproducing the Fraunhofer structure in the
forward-angle data. Our coupled-channel calculation of $f_{\text{elastic}}$ for the
$^{14}\text{C}+^{12}\text{C}$ system (which included inelastic excitation of the $2^+$ in $^{12}\text{C}$)
was an attempt at remedying this situation. However, our bare potential
was apparently such that the coupling had a negligible effect on the
very featureless $f_{\text{elastic}}$. We need a better bare potential with which to
begin.

The only analysis involving the intermediate width structure seen
in our data was the phase shift one performed on $^{14}\text{C}+^{12}\text{C}$ data in the
region of the observed angular distribution anomaly ($\sim 17$ MeV). The
behavior of the angular distributions, excitation functions and the
extracted phase-shifts in that region is consistent with an $\ell=10$
resonant interpretation, but the ambiguity of phase shift analysis makes it impossible to come to a definite conclusion on the matter. If our data were supplemented with more forward angle measurements, however, it might indeed be possible to verify this suggestion.

Finally, we have found that the dynamical symmetry models provide a somewhat more flexible description of the data than does the standard optical model, and yet they also are unable to reproduce large backward-angle cross-sections unless we consider an unphysical (or at least very strange) part of the model space. We can produce excellent fits to data by using a coupled-channels form of the SO(3,1) model to include elastic transfer (or perhaps it would be more correct in this situation to label it "an indistinguishable reaction channel"). However, we are then still unable to calculate a cross-section surface which bears an adequate resemblance to the data, either quantitative or qualitative, without introducing arbitrary energy dependence in the model parameters.

The data which we have collected on three heavy-ion systems provide theoretical nuclear physics with substantial challenges which are presently far from met. Clearly, the theoretical task now at hand is very difficult. It is our hope, however, that these datasets will prove to be of substantial importance in guiding theoretical efforts toward a better understanding of complex nuclear interactions. Even now, though it is not wholly explicable, the form of the data is fascinating - and even beautiful.
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