Abstract
The Onset and Breakout of the Stellar Hot CNO Cycle
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During equilibrium hydrogen burning on the main sequence, the gravitational pressure in massive (M > 2 \(M_\odot\)) stars is balanced by the thermal pressure produced by stellar core nuclear reactions in the Carbon-Nitrogen-Oxygen (CNO) cycle:

\[
12C(p, \gamma)^{13}N(\beta^+\nu)^{13}C(p, \gamma)^{14}N(p, \gamma)^{15}O(\beta^+\nu)^{15}N(p, \alpha)^{12}C.
\]

The slowest reaction at main sequence stellar temperatures and densities is the \(^4\!N(p, \gamma)^{15}O\) reaction, which limits the energy generation rate of the cycle and causes the transformation of C and O into N nuclei with the abundance ratio \(^{15}N / ^{14}N = 10^{-5}\), a factor of 100 lower than the observed solar system abundance.

At the higher temperatures characteristic of explosive hydrogen burning in red giants and in novae and supernovae explosions, the \(^{13}N(p, \gamma)^{14}O\) reaction rate exceeds the temperature - independent \(^{13}N(\beta^+\nu)^{13}C\) rate, causing the conversion of the CNO cycle to the Hot CNO cycle:

\[
12C(p, \gamma)^{13}N(p, \gamma)^{14}O(\beta^+\nu)^{14}N(p, \gamma)^{15}O(\beta^+\nu)^{15}N(p, \alpha)^{12}C,
\]

in which the slowest reactions in the cycle are the \(^{14}O\) and \(^{15}O\) beta-decays; this results in an increased energy production rate. Additionally, the \(^{15}N / ^{14}N\) abundance ratio from this cycle can be increased to approximately 1 depending on the temperature and density.

In order to determine the astrophysical sites at which the CNO cycle is converted to the HCNO cycle, and thus to better understand the energy generation and \(^{15}N / ^{14}N\) abundance ratio for these sites, it is necessary to determine the precise temperatures and densities at which the \(^{13}N(p, \gamma)^{14}O\) reaction rate exceeds the \(^{13}N(\beta^+\nu)^{13}C\) rate. At stellar temperatures, the
\[ ^{13}\text{N}(p, \gamma)^{14}\text{O} \] resonant reaction rate depends solely on the properties (energy, total width, and partial-gamma width) of the low energy \((E_{\text{c.m.}} = 540 \text{ keV}, E_x = 5.169 \text{ MeV})\) s-wave resonance in \(^{14}\text{O}\), the first two resonant properties having already been measured. Thus by populating the s-wave resonance in \(^{14}\text{O}\) and measuring the subsequent gamma-decay branching ratio \(\Gamma \gamma / \Gamma\), the reaction rate can be determined. The \(^1\text{H}(^{14}\text{N},^{14}\text{O})n\gamma\) and \(^{14}\text{N}(p, n)^{14}\text{O}_{0,1}\) reactions have been used to determine this gamma branching ratio. The present measurement will be compared with two recent results, and the astrophysical \(^{13}\text{N}(p, \gamma)^{14}\text{O}\) reaction rate will be determined at temperatures and densities typical of explosive hydrogen burning.

At even higher temperatures \((T_g > 5)\), the reaction sequence \(^{15}\text{O}(\alpha, \gamma)^{19}\text{Ne}(p, \gamma)^{20}\text{Na}\) initiates a sequence of rapid proton captures and beta decays, known as the rp process, which can increase nuclear energy generation by a factor of 100 over the HCNO cycle. This reaction sequence also processes CNO seed nuclei out of the HCNO cycle, forming elements with mass > 20. This formation of heavier elements during explosive hydrogen burning may explain the recently observed overabundances of Ne, Na, Mg, and Al isotopes in nova ejecta.

In order to better determine the temperatures and densities where such a breakout occurs, a high-precision measurement of the \(^{20}\text{Ne}(^{3}\text{He}, t)^{20}\text{Na}\) reaction has been made using \(^{20}\text{Ne}\) implanted transmission targets to populate states which may serve as low-energy resonances in the \(^{19}\text{Ne}(p, \gamma)^{20}\text{Na}\) reaction. Excitation energies and upper limits on total widths have been determined for four resonances above the 2.199 MeV proton threshold in \(^{20}\text{Na}\). Utilizing the resonance information from these measurements and two other recent studies, the effect of these resonances on the astrophysical \(^{19}\text{Ne}(p, \gamma)^{20}\text{Na}\) reaction rate will be determined. When combined with a recent study of the \(^{15}\text{O}(\alpha, \gamma)^{19}\text{Ne}\) reaction, a better estimate can be made of the conditions for breakout from the HCNO cycle to the rp process.
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Chapter 1. Introduction

1.1 General Overview of Stellar Hydrogen Burning

One of the fundamental goals of nuclear astrophysics is to determine the rates of stellar thermonuclear reactions. These exothermic reactions, occurring predominantly in the stellar core, involve the fusion of light nuclei into heavier nuclei. Both the transformation of nuclei and the resultant energy release play crucial roles in determining the nature of our universe. The energy release, for example, is responsible for generating not only a star's luminosity, but also the thermal pressure balancing a star's gravitational potential energy, thereby preventing its collapse. Because of this delicate balance keeping stars intact, stellar energy generation is very important to understand.

It is equally important to understand the transformation of nuclei and its associated observable, the relative abundance of the isotopes. Stellar nuclear reactions are thought to be responsible for the synthesis of all heavy elements \((A > 4)\) through thermonuclear fusion reactions in stellar interiors (Bu57); only hydrogen, helium, and a small amount of lithium are produced in the big bang. While the universe as a whole is cooling from the big bang and is characterized by low temperatures which prohibit fusion reactions, these reactions are possible in stellar cores which, due to gravitational pressure, are gradually heating up over time.

In efforts to understand the mechanisms and timescales of stellar energy generation and nucleosynthesis, detailed stellar models have been formulated. In the broadest sense, these numerical models attempt to trace the evolution of a star beginning with its condensation from a nebula, to its main-sequence hydrostatic-hydrogen burning, and finally to its exhaustion of fuel (for low mass stars) or a nova or supernova explosion and gravitational collapse (for heavier stars) (Ib84a). These models require accurate nuclear reaction rates to precisely determine the timescales and resulting nucleosynthesis of the various burning stages in a star's lifecycle. It is
at this point that our astrophysical understanding of stellar evolution and nucleosynthesis relies on information obtained in the nuclear laboratory. No laboratory measurements have been made for a number of stellar reactions, and stellar models in these cases must rely on theoretical estimates which can have uncertainties of many orders of magnitude. Experimental determinations involve either a direct measurement (only possible in some cases) or indirect methods utilizing nuclear structure information about the compound nucleus and residual nuclei. The continuing work of experimentally determining stellar nuclear reaction rates allows the formation of ever-more accurate stellar models.

Nuclear reaction rates are necessary not only for building stellar models, but also for predicting isotopic abundances. Stellar material is introduced into the interstellar medium (ISM) by a variety of mechanisms; nova and supernova explosions are two important examples. Once ejected, this material can either remain in dust clouds in the ISM, condense into meteors and asteroids, or be included in a protostellar nebula and condense into a new stellar system. Knowledge of nuclear reaction rates in stars, combined with models of dispersal mechanisms, allows predictions of isotopic abundances to be made. These predictions can then be compared with observed solar, terrestrial, meteoric, and ISM abundances.

Of all the stellar nuclear reactions responsible for energy generation and nucleosynthesis, those involving hydrogen burning are the most important. Hydrogen constitutes the majority of stellar material (> 70% typically), and thus is the most abundant nuclear fuel: every star burns hydrogen for the majority of its lifetime. Additionally, fusion reactions involving hydrogen have the lowest coulomb barrier and hence need the lowest temperatures to allow barrier penetration to occur on reasonable stellar timescales. Thus hydrogen is the first fuel to be burned when a star collapses from a protostellar nebula.

This hydrogen burning is hydrostatic, quiescent burning, where the thermal pressure generated from the exothermic nuclear fusion reactions maintains an equilibrium with the
gravitational pressure. When such burning occurs, the star is said to be on the main sequence, where it remains for most (typically > 90%) of its lifetime. The two reaction sequences by which hydrogen is hydrostatically burned are the proton-proton (p p) - chain and the carbon-nitrogen-oxygen (CNO) cycle, both of which fuse four hydrogen nuclei into a helium nucleus with the release of ~ 28 MeV of energy per cycle. The p p - chain dominates at temperatures and densities characteristic of the core of lower mass stars: mass $M < 1.5 \, M_\odot$, where $M_\odot = 1$ solar mass = $2 \times 10^{33}$ g; temperature $T_6 \leq 15$, where $T_6 = T (K)/10^6$; and density $\rho = 150$ g/cm$^3$ (Pa86). The CNO cycle occurs at the higher temperatures and densities typical of more massive stars ($M > 2 \, M_\odot$, $20 < T_6 < 100$), these higher temperatures being necessary to balance the increased gravitational pressure (Bu57). Additionally, the CNO cycle requires seed nuclei of carbon, nitrogen, or oxygen as catalysts in order to operate; it will thus occur only in second - (and later - ) generation stars.

In addition to the hydrostatic hydrogen burning described above, explosive, non-equilibrium hydrogen burning can occur in the latter stages of the lifecycle of some stars. This type of burning is characterized by much higher temperatures ($100 < T_6 < 1000$) and densities ($200 < \rho < 10000$ g/cm$^3$) characteristic of red giants, supermassive stars, and nova and supernova explosions, and is important for a number of reasons. The majority of heavy element nucleosynthesis ($A > 4$) is thought to occur in the last, explosive stages of a star's lifecycle: explosive burning determines the ultimate relative abundances of the elements (Ar78), (Tr84), (Ro88). Since the ashes of explosive burning are often explosively released into the ISM at this point in a star's lifecycle, these processes play a determining role in the ISM isotopic abundance distribution. Additionally, explosive hydrogen burning events are not rare astrophysical phenomena. Since most stars occur in binary pairs, sites such as accreting neutron stars and white dwarves are relatively common; these are the precursors for nova and supernova explosions, and are the prime candidates for explosive hydrogen burning. Nova explosions occur on less massive stars, and are thus more common than supernova explosions:
there are estimated to be \( \sim 40 \) nova explosions per year in our galaxy (Cl84).

Finally, much less is known about explosive hydrogen burning than quiescent hydrogen burning, primarily because the former often involves \((p, \gamma)\) proton capture reactions on proton-rich unstable nuclei, requiring radioactive targets or beams to make direct laboratory measurements. Indirect methods are presently necessary to study the residual nuclei in these stellar reactions. Two explosive hydrogen burning reaction sequences of current interest which may be studied with these indirect methods are the Hot CNO cycle and the \( rp \) (rapid proton capture) process, both of which will be discussed in detail in the following sections.

1.2 Quiescent Hydrogen Burning: the CNO cycle

As noted above, the gravitational potential energy in massive \((M > 2 \, M_\odot)\) stars is balanced by the nuclear energy produced during hydrogen burning in the stellar core through the CNO cycle. The reaction sequence is

\[
\begin{align*}
12C(p, \gamma) & \rightarrow 13N(\beta^+\nu) \\
13N(\beta^+\nu) & \rightarrow 13C(p, \gamma) \\
13C(p, \gamma) & \rightarrow 14N(p, \gamma) \\
14N(p, \gamma) & \rightarrow 15O(\beta^+\nu) \\
15O(\beta^+\nu) & \rightarrow 15N(p, \alpha) \\
15N(p, \alpha) & \rightarrow 12C,
\end{align*}
\]

and is shown in Figure 1.1. Four protons are captured by the catalytic seed nucleus and released as a \(^4\text{He}\) nucleus. The energy generation rate of the cycle is limited by its slowest reaction, which at characteristic main-sequence stellar core temperatures of \(2 < T_6 < 80\) and densities of \(\rho \approx 100\, \text{g/cm}^3\) is the \(^{14}N(p, \gamma)\) \(^{15}O\) reaction. The typical timescale of hydrogen burning by this reaction, referred to as the lifetime of protons against this reaction, \(\tau_{^{14}N}^{\prime}\), is 851 years at a temperature \(T_6 = 30\) and density \(\rho = 100\, \text{g/cm}^3\) (Ca62). The next slowest reaction in the cycle at this temperature, the \(^{12}C(p, \gamma)\) \(^{13}N\) reaction, has \(\tau_{^{12}C} = 14.1\) years, down by a factor of 60 from \(\tau_{^{14}N}^{\prime}\). Because the \(^{14}N(p, \gamma)\) \(^{15}O\) reaction is so much slower than the other reactions, the CNO cycle time is approximately equal to \(\tau_{^{14}N}^{\prime}\), and the energy generation rate is proportional to \(\tau_{^{14}N}^{\prime}^{-1}\).

A second consequence of the slow rate of the \(^{14}N(p, \gamma)\) \(^{15}O\) reaction is that C and O
seed nuclei are transformed into $^{14}\text{N}$ nuclei: typically 94% of all the seed nuclei during equilibrium hydrogen burning through the CNO cycle are $^{14}\text{N}$ (Ro88). Additionally, the relative timescales for formation and destruction of $^{15}\text{N}$ and $^{14}\text{N}$ suggest a $^{15}\text{N} / ^{14}\text{N}$ equilibrium abundance ratio of approximately $10^{-5}$, approximately a factor of $10^2$ lower than that observed in the solar system. The relative abundance of $^{14}\text{N}$ and $^{15}\text{N}$ has been a serious problem unexplainable through hydrostatic hydrogen burning in the CNO cycle (Ba80),(Ca77). Clearly some additional processing of the $^{14}\text{N}$ seed nuclei must occur in order to account for the observed ratio.

1.3 Onset of the Hot CNO Cycle

This additional processing can occur at the slightly raised temperatures characteristic of explosive hydrogen burning scenarios during the latter stages of stellar evolution. A higher temperature implies greater proton kinetic energies, resulting in easier barrier penetration and hence rapidly increasing reaction rates; the $\beta^+$ - decay times are, of course, temperature independent. For example, at $T_9 = 0.1$, where $T_9 = T (\text{K})/10^9$, the $^{14}\text{N}(p, \gamma)^{15}\text{O}$ reaction has a timescale $\tau_{14N}$ of 2.2 hours at a hydrogen density of 100 g/cm$^3$, only a factor of 9 larger than the $^{13}\text{N} \beta^+$ - decay ($\tau_{13} = 863$ s).

At the slightly higher temperatures ($T_9 = 0.1 - 0.2$) characteristic of hydrogen burning in red giants, supermassive stars, and novae explosions, three important changes occur. First, the $^{13}\text{N}(p, \gamma)^{14}\text{O}$ reaction rate will exceed the $^{13}\text{N} \beta^+$ - decay rate, resulting in the following sequence of reactions:

$$^{12}\text{C}(p, \gamma)^{13}\text{N}(p, \gamma)^{14}\text{O}(\beta+\nu)^{14}\text{N}(p, \gamma)^{15}\text{O}(\beta+\nu)^{15}\text{N}(p, \alpha)^{12}\text{C}. \quad (1.2)$$

The consequence of this is a change in the abundance of the seed nuclei, as the flow begins to bypass $^{13}\text{C}$. As the temperature continues to increase, the $^{14}\text{N}(p, \gamma)^{15}\text{O}$ reaction rate exceeds the $^{13}\text{N}(p, \gamma)^{14}\text{O}$ reaction rate, making the latter proton-capture rate determine the energy generation of the cycle. Finally, both proton-capture reactions exceed the $^{14}\text{O}(\beta+\nu)^{14}\text{N}$ and
$^{15}\text{O}(\beta^+\nu)^{15}\text{N}$ rates, with 102 and 176 second lifetimes, respectively. When the slower of the two proton capture reactions exceeds the temperature-independent $^{14}\text{O}$ and $^{15}\text{O}\beta^+$-decay rates, the energy generation of the cycle will be limited by these decay rates, and the CNO cycle is converted to the Hot or "β-limited" CNO cycle. The order of these three changes is, however, unknown due to the uncertainty in the $^{13}\text{N}(p,\gamma)^{14}\text{O}$ reaction rate as a function of temperature. There are two important consequences arising from conversion to the Hot CNO cycle. First, the hydrogen burning timescale of the cycle is substantially changed, leading to an increase in energy production by roughly a factor of 3-5. Second, the relative abundance of CNO nuclides changes substantially, because the waiting point in the cycle has been changed from $^{14}\text{N}$ to $^{14}\text{O}$ and $^{15}\text{O}$. Therefore, the Hot CNO cycle can increase the $^{15}\text{N}/^{14}\text{N}$ abundance ratio to ~1 depending on the temperature and the details of the conversion, thereby providing a way to account for the observed solar system abundance (Ba80) through a mixture of CNO and Hot CNO burning products.

In order to determine the astrophysical sites at which the CNO cycle is converted to the Hot CNO cycle, and thus to better understand the energy generation and $^{15}\text{N}/^{14}\text{N}$ abundance ratio for these sites, it is necessary to determine the $^{13}\text{N}(p,\gamma)^{14}\text{O}$ reaction rate as a function of temperature. A direct measurement of this rate would require a radioactive beam or target. At present, a radioactive beam of $^{13}\text{N}$ is under development at Louvain-La-Neuve in Belgium (Da90) but is not yet intense enough (currently $10^8$ particles/s (Ro90), $10^1$ - $10^2$ less than required to obtain a few counts/hr) to make a direct measurement of the $^{13}\text{N}(p,\gamma)^{14}\text{O}$ reaction.

Indirect measurements must therefore be utilized to determine the reaction rate. For temperatures $0.1 < T_9 < 1.0$, the $^{13}\text{N}(p,\gamma)^{14}\text{O}$ stellar reaction rate depends solely on the properties (resonance energy, total width, and partial-gamma width) of the low energy ($E_{\text{c.m.}} = 540$ keV) s-wave resonance in $^{14}\text{O}$ ($E_x = 5.169$ MeV), as will be discussed in Chapter 2. The energy and total width of this first excited-state in $^{14}\text{O}$ have been measured by Chupp et al. (Ch85). The reaction rate can be determined indirectly by populating the $^{14}\text{O}^*(5.169$ MeV)
level and measuring the subsequent gamma-decay branching ratio $\Gamma_{\gamma}/\Gamma$. Two such experiments have recently been concluded, both employing the $^{12}\text{C}(^{3}\text{He}, n)^{14}\text{O}_{1}(\gamma)^{14}\text{O}_{0}$ reaction to measure $\Gamma_{\gamma}/\Gamma$. Fernandez et al. (Fe89) obtained a value of $\Gamma_{\gamma}/\Gamma = 7.2 \pm 3.5 \cdot 10^{-5}$, while Aguer et al. (Ag89) measured $2 \pm 1 \cdot 10^{-4}$, which has been recently revised to an upper limit of $\Gamma_{\gamma}/\Gamma < 3 \cdot 10^{-4}$ (Ro90). The large uncertainties in these values correspond to a factor of 3-uncertainty in the $^{13}\text{N}(p, \gamma)^{14}\text{O}$ reaction rate, which can significantly change the temperature at which this proton capture rate dominates the $^{13}\text{N}(\beta^{+}, \nu)^{13}\text{C}$ rate. Furthermore, both experiments had very poor signal to noise ratios, with a very small number of real n-$\gamma$ coincidences. In an attempt to better determine $\Gamma_{\gamma}/\Gamma$, the $^{1}\text{H}(^{14}\text{N},^{14}\text{O}^{*})n\gamma$ reaction has been used to measure the ratio of $^{14}\text{O}_{1}$ recoils undergoing gamma decay to the $^{14}\text{O}_{0}$ recoils formed directly. This experiment is described in detail in Chapter 3. The branching ratio $\Gamma_{\gamma}/\Gamma$ is then obtained by combining this ratio with the production cross-section ratio $\sigma_{n0}/\sigma_{n1}$, obtained from a measurement of the $^{14}\text{N}(p, n)^{14}\text{O}$ reaction at the same center-of-mass energy described in Chapter 4.

1.4 Breakout of the Hot CNO Cycle

Just as the $^{13}\text{N}(p, \gamma)^{14}\text{O}$ proton capture rate became faster than the competing $^{13}\text{N}(\beta^{+}, \nu)^{13}\text{C}$ rate at $T_{9} = 0.1$ - 0.2, other particle capture reactions begin to have lifetimes smaller than their competing $\beta^{+}$ - decays as stellar temperatures continue to increase. A nova explosion, where temperatures range from $T_{9} = 0.2$ to 0.5, is one site where this can occur. The models of typical nova events (Tr84),(St78) involve the accretion of stellar material ($= 10^{-9} - 10^{-8} \text{ M}_\odot/\text{ yr}$) from a red giant onto the surface of a hot degenerate C-O white dwarf close-binary companion, as shown schematically in Figure 1.2 (Ro88). The temperature of the accreted hydrogen layer increases as more material is added, until it is high enough to ignite hydrogen burning through the HCNO cycle. These exothermic reactions greatly
increase the temperature of the layer; however, since the temperature and pressure of the degenerate white dwarf are decoupled, no expansive cooling (and subsequent lowering of reaction rates) will occur in this layer. Because the reactions are not regulated in the normal fashion, a violent explosion occurs which ejects the remainder of the layer (~ 0.4 M_⊙) into the ISM. Additional explosions of this type may occur as the white dwarf continues to accrete material from its binary companion. This description of the nova event agrees with observations of overabundances of carbon and nitrogen isotopes in nova eject, but cannot explain the recently observed overabundances (up to two orders of magnitude over solar abundances) of Ne, Na, Mg, and Al isotopes (Sn84), (Wi85); see Table 1.1 (Wi86a). Similar enhancements of Ne and Na over solar abundances have been observed in cosmic rays (Wi79) and meteorite samples (Bl69) (Eb79), these possibly representing a galactic equilibrium 22Ne/20Ne ratio greater than the solar value by a factor of 10 to 100. There are two competing theories as to the origin of these overabundances in the so-called "Ne-novae" ejecta. The first contends that the nova event occurs on the surface of a O-Ne-Mg white dwarf; mixture of the base and accreted material during burning and ejection is invoked to explain the anomalous abundances. However, these white dwarves are thought to comprise only ~ 3% (Ib84b) of all white dwarves, the majority being standard C-O core white dwarves. The second explanation of the overabundances involves a nova event on a C-O white dwarf during which the CNO seed nuclei break out of their catalytic roles in the HCNO cycle, undergoing additional processing to reach the mass A>20 region (Wi86a).

How can such processing occur? Since most of the seed nuclei in the HCNO cycle (> 98%) are 14O and 15O, it is natural to consider nuclear reactions involving these two unstable nuclei. Proton capture reactions leading to 15F and 16F are not possible since these nuclei are unstable to proton breakup. Breakouts are possible, however, with alpha-particle induced reactions. The 14O(α,p) 17F(p,γ) 18Ne(β+ν) 18F(p, α) 15O sequence bypasses the 71-second half-life 14O(β+ν) decay and produces 15O at a higher rate than in the HCNO cycle. This speeds up the cycle rather than constituting a breakout: the seed nuclei are still
catalysts. However, a possible breakout may occur if the $^{18}\text{F}(p, \gamma)^{19}\text{Ne}$ reaction competes favorably with the $^{18}\text{F}(p, \alpha)^{15}\text{O}$ reaction; this possibility is presently under investigation (Ch90a).

A more likely breakout sequence, $^{15}\text{O}(\alpha, \gamma)^{19}\text{Ne}(p, \gamma)^{20}\text{Na}$, is shown in Figure 1.1. This sequence must compete against both the $^{15}\text{O}$ and $^{19}\text{Ne}$ $\beta^+$ - decays of 122 and 17 second half-lifes, respectively. Even after this breakout is initiated when the first $\beta^+$ - decay is bypassed, the $^{19}\text{Ne}$ $\beta^+$ - decay can return the seed nuclei back to the HCNO cycle via the sequence $^{19}\text{Ne}(\beta^+\nu)^{19}\text{F}(p, \alpha)^{16}\text{O}(p, \gamma)^{17}\text{F}(p, \gamma)^{18}\text{Ne}(\beta^+\nu)^{18}\text{F}(p, \alpha)^{15}\text{O}$. But once the temperature is such that both $\beta^+$ - decays are bypassed, the seed nuclei will reach the mass A > 20 region and cannot return to the HCNO cycle. The $(p, \alpha)$ reactions which could cycle the seed nuclei back have high negative $Q$-values which prevent these reactions from occurring. If the $^{15}\text{O}(\alpha, \gamma)^{19}\text{Ne}(p, \gamma)^{20}\text{Na}$ is a viable breakout mechanism at nova temperatures and densities, it may be possible to account for the Ne-nova ejecta and the high $^{22}\text{Ne}/^{20}\text{Ne}$ galactic abundance ratio.

There is yet another consequence of such a breakout from the HCNO cycle. A sequence of rapid proton captures and beta decays, the rp process, is initiated once $^{20}\text{Na}$ is reached. This process is analogous to the r-process in which repeated rapid neutron captures are followed by $\beta^-$ - decays, leading to the formation of heavy elements. The reactions in the rp-process continue up to mass 56. Because the $\beta^+$ - decays are now on the order of 1 second, as compared to 100 seconds for the HCNO cycle, the nuclear energy generation rate can be increased by roughly a factor of 100 over the HCNO cycle (Wa81).

Because of these reasons, it is very important to determine the precise temperatures and densities where such breakouts occur. In order to do this, the $^{20}\text{Ne}(^{3}\text{He}, t)^{20}\text{Na}$ reaction has been used to populate states which may serve as low-energy resonances in the $^{19}\text{Ne}(p, \gamma)^{20}\text{Na}$ reaction; see Figure 1.3. This experiment is discussed in Chapter 5. A level diagram with excitation energies and upper limits on total widths has been determined, allowing a
significant reduction in the uncertainty of the \(^{19}\text{Ne}(p, \gamma)^{20}\text{Na}\) reaction rate. When combined with a recent study of the \(^{15}\text{O}(\alpha, \gamma)^{19}\text{Ne}\) reaction (Ma90), a better estimate can be made of the conditions for breakout from the HCNO cycle.
Figure 1.1

The Stellar CNO and Hot CNO cycles, and the rapid-proton capture (rp) process. Stable nuclei are represented by shaded boxes.
Schematic diagram of a nova explosion, where a white dwarf star accretes stellar material from a red giant binary companion (Ro88).

Figure 1.2.
Table 1.1

Mass Fractions of elements observed in nova ejecta compared with solar values (Tr85).

The last column represents all elements heavier than Helium.
<table>
<thead>
<tr>
<th>Element</th>
<th>H</th>
<th>He</th>
<th>C</th>
<th>N</th>
<th>O</th>
<th>Ne</th>
<th>Z&gt;2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solar</td>
<td>0.74</td>
<td>0.24</td>
<td>0.0039</td>
<td>0.00094</td>
<td>0.0088</td>
<td>0.0021</td>
<td>0.019</td>
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<tr>
<td>Nova</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RR Pic</td>
<td>0.53</td>
<td>0.43</td>
<td>0.022</td>
<td>0.0058</td>
<td>0.011</td>
<td>0.039</td>
<td></td>
</tr>
<tr>
<td>HR Del</td>
<td>0.45</td>
<td>0.48</td>
<td>0.027</td>
<td>0.047</td>
<td>0.003</td>
<td>0.077</td>
<td></td>
</tr>
<tr>
<td>T Aur</td>
<td>0.47</td>
<td>0.40</td>
<td>0.079</td>
<td>0.051</td>
<td></td>
<td>0.13</td>
<td></td>
</tr>
<tr>
<td>Cyg 1975</td>
<td>0.49</td>
<td>0.21</td>
<td>0.07</td>
<td>0.075</td>
<td>0.13</td>
<td>0.023</td>
<td>0.30</td>
</tr>
<tr>
<td>Cyg 1978</td>
<td>0.45</td>
<td>0.23</td>
<td>0.047</td>
<td>0.14</td>
<td>0.13</td>
<td>0.0068</td>
<td>0.32</td>
</tr>
<tr>
<td>Cr A 1981</td>
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<td>0.31</td>
<td>0.0046</td>
<td>0.80</td>
<td>0.12</td>
<td>0.17</td>
<td>0.38</td>
</tr>
<tr>
<td>DQ Her</td>
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<td>0.095</td>
<td>0.045</td>
<td>0.23</td>
<td>0.29</td>
<td></td>
<td>0.56</td>
</tr>
<tr>
<td>Aql 1982</td>
<td>0.053</td>
<td>0.085</td>
<td>0.031</td>
<td>0.095</td>
<td>0.061</td>
<td>0.47</td>
<td>0.86</td>
</tr>
</tbody>
</table>
Figure 1.3.
The $^{20}\text{Ne} (^{3}\text{He}, t)^{20}\text{Na}$ reaction used to populate states in $^{20}\text{Na}$ above the $^{19}\text{Ne} + p$ threshold (Aj87). The $^{20}\text{Na}$ levels indicated are from Lamm et al. (La87).
Tentative Level Structure of $^{20}\text{Na}$

$^{20}\text{Ne}(^{3}\text{He, } t)^{20}\text{Na}$

$2.199 \text{ MeV}$

$^{19}\text{Ne} + p$

1.31
0.96
0.77
0.59

0.0 MeV $^{2^+}$

$^{20}\text{Na}$

12 MeV

Threshold States

$-13.906 \text{ MeV}$
Chapter 2. Indirect Determination of Stellar Reaction Rates

2.1 General Considerations

The reaction rate between a pair of particles 1 and 2 is given by the product of their number densities \( N_1 \) and \( N_2 \), their relative velocity \( v \), and the cross-section for the reaction \( \sigma(v) \) as

\[
R_{12} = N_1 N_2 v \sigma(v) ,
\]

(2.1)

where the units are reactions / s \( \cdot \) cm\(^3\). For the calculation of the stellar reaction rates of interest, the particles are part of a non-degenerate, non-relativistic gas in thermodynamic equilibrium at temperature \( T \); they will thus have a Maxwell-Boltzmann velocity distribution given by

\[
N_i(v_i) \, dv_i = N_i \left( \frac{m_i}{2\pi k T} \right)^{3/2} \exp \left( - \frac{m_i v_i^2}{2 k T} \right) \, dv_i ,
\]

(2.2)

where \( m_i \) and \( v_i \) denote the mass and velocity, respectively, of the individual particles. The stellar reaction rate \( R_{12} \) is then given by the integral of this distribution over velocities \( v_1 \) and \( v_2 \), and can be expressed as

\[
R_{12} = N_1 N_2 \langle \sigma \, v \rangle
\]

(2.3)

where

\[
\langle \sigma \, v \rangle = \left( \frac{8}{\pi \mu} \right)^{1/2} (kT)^{-3/2} \int_0^{\infty} E \sigma(E) \exp \left( - \frac{E}{kT} \right) \, dE ,
\]

(2.4)

\( E \) denotes the center of mass energy, and \( \mu \) the reduced mass. The number densities \( N_i \) in Eq. 2.3 are typically replaced with the density of the stellar medium \( \rho \) and the mass fractions \( X_i \) as

\[
N_i = \frac{\rho X_i N_A}{A_i}
\]

(2.5)

where \( m_i = A_i m_u \) and \( N_A \) is Avogadro's number. Furthermore, the lifetime \( \tau_1(2) \) of a heavy nucleus (say particle 1) against hydrogen burning (particle 2) can be expressed as
\[ R_{12} = \left( \frac{dN_1}{dt} \right)_2 = -\frac{N_1}{\tau_1 (2)} \Rightarrow (\tau_1 (2))^{-1} = \frac{R_{12}}{N_1} , \]  

(2.6)

thus making it common to discuss the reaction rate per target particle 1, with units of reactions / s. The rate per heavy nucleus is chosen because heavy nuclei such as \(^{13}\text{N}\) and \(^{19}\text{Ne}\) are so much less abundant than hydrogen in stellar environments. With these considerations, Eq. 2.3 can be rewritten as

\[ \frac{R_{12}}{N_1} = \left( \frac{X_H \rho}{A_H} \right) N_A <\sigma v> , \]  

(2.7)

where \( N_A <\sigma v> \) in (reactions cm\(^3\) / mole s) is a density-independent measure of the reaction rate.

From Eq. 2.4, it is evident that a knowledge of the energy dependence of the cross-section \( \sigma(E) \), which can be measured in the nuclear laboratory, will allow the calculation of the stellar reaction rate as a function of temperature. The energy dependence of the cross-section has two terms which may be immediately factored out. When no resonances are present, the dominant energy dependence for charged-particle reactions at low energies is caused by the Coulomb and centrifugal barrier penetrability term, which is relatively independent of the nuclear physics of the interacting nuclei. The s-wave \( (l = 0) \) case gives the largest contribution of any of the partial waves because the centrifugal barrier is zero. This penetrability, given by

\[ \exp \left( -\frac{E_G}{E} \right)^{1/2} , \quad E_G = (2\pi Z_1 Z_2 \alpha^2) \left( \frac{\mu c^2}{\hbar} \right) , \]  

(2.8)

is factored out of the cross-section. \( E_G \) is referred to as the Gamow energy and has a value of 44.5 MeV for the \(^{13}\text{N}(p, \gamma)^{14}\text{O}\) reaction and 93.0 MeV for the \(^{19}\text{Ne}(p, \gamma)^{20}\text{Na}\) reaction. It should be noted that this factor is often written as \( \exp(-2\pi \eta) \), where \( \eta \) is the Sommerfeld parameter \( Z_1 Z_2 e^2 / \hbar v \). Additionally, the geometrical dependence of the cross section \( \pi \lambda^2 \sim 1/E \) can be factored out, leaving the expression

\[ \sigma(E) = \frac{S(E)}{E} \exp \left( -\frac{E_G}{E} \right)^{1/2} \]  

(2.9)

where the cross-section factor \( S(E) \) represents the nuclear contribution to the cross-section energy dependence. \( S(E) \) has units of energy \( \cdot \) cross-section; for example, the \(^3\text{He}(\alpha, \gamma)^7\text{Li}\)
reaction, important for the pp-chain, has \( S(0) = 0.54 \text{ keV-barn} \) (Pa86). In the non-resonant case, \( S(E) \) is a slowly varying function of energy and is approximated by a constant or a Taylor series expansion, whereas \( \sigma(E) \) still varies rapidly with energy because of the penetrability.

Cross-sections for CNO and heavier nuclei can only be measured directly in the laboratory down to energies of \( \sim 100 \text{ keV} \) because of the high Coulomb barrier (~ few MeV) which must be penetrated. Therefore the factorization given in Eq. 2.9 is especially useful in extrapolating laboratory measured cross-sections down to stellar energies of 10-100 keV (\( T_9 \sim 0.1 - 1.0 \)).

Inserting the factored expression for \( \sigma(E) \) into Eq. 2.4, we get

\[
N_A \langle \sigma \nu \rangle = \left( \frac{8}{\pi \mu} \right)^{1/2} (kT)^{3/2} \int_0^\infty S(E) \exp \left(-\frac{E}{kT} - \left(\frac{E_0}{E}\right)^{1/2}\right) dE . \tag{2.10}
\]

The two exponential terms in the integrand, shown plotted for the \(^{13}\text{N} \left( p, \gamma \right) ^{14}\text{O} \) reaction in Figure 2.1 at \( T_9 = 0.3 \) (\( kT = 26 \text{ keV} \)), greatly restrict the region over which the integral in Eq. 2.10 must be evaluated. The product of the two terms is sharply peaked at an energy \( E_0 \), the "most effective stellar energy", and the shape can be approximated by a gaussian of full width \( \Delta E_0 \). For the \(^{13}\text{N} \left( p, \gamma \right) ^{14}\text{O} \) and \(^{19}\text{Ne} \left( p, \gamma \right) ^{20}\text{Na} \) reactions at \( T_9 = 0.3 \), \( E_0 = 195 \) and 250 keV and \( \Delta E_0 = 164 \) and 185 keV, respectively. The temperature dependence of the most effective energy \( E_0 \) is determined by finding the maximum of the exponential term in the integrand of Eq. 2.10, and of the width \( \Delta E_0 \) by setting the value at \( E_0 \pm \Delta E_0 / 2 \) equal to half the maximum value of the integrand, giving

\[
E_0 = \left( \frac{E_0^{1/2} kT}{2} \right)^{2/3} , \quad \Delta E_0 = 4 \left( \frac{kT E_0}{3} \right)^{1/2} . \tag{2.11}
\]

Note that the integrand in Eq. 2.10 decreases exponentially for energies outside the range \( E_0 \pm \Delta E_0 / 2 \), the so-called "Gamow window": the evaluation of the stellar reaction rate \( N_A \langle \sigma \nu \rangle \) at any temperature therefore depends almost entirely on the behavior of the cross-section factor \( S(E) \) over the Gamow window. This energy window increases in width as the temperature increases: at \( T_9 = 1 \), the highest temperatures for nova explosions, \( E_0 \) and \( (\Delta E_0 / 2) \) are 440 keV and 225 keV, respectively, for the \(^{13}\text{N} \left( p, \gamma \right) ^{14}\text{O} \) reaction, and 557 keV and 253 keV, respectively, for the \(^{19}\text{Ne} \left( p, \gamma \right) ^{20}\text{Na} \) reaction. This shows that the S-factor
needs to be evaluated only for center of mass energies $E < 1$ MeV for both reactions. Figure 2.2 shows a plot of the Gamow window, $E_0 \pm \Delta E_0 / 2$, vs. temperature for the $^{13}$N(p, $\gamma$)$^{14}$O and $^{19}$Ne(p, $\gamma$)$^{20}$Na reactions for $T_\odot = 0.1 - 1.0$.

The presence of a resonance in the compound system is characterized by a sharp peak in $S(E)$ and thus $\sigma(E)$ at some "resonance energy" $E_\tau$ (measured in the center-of-mass frame); without such a resonance, $S(E)$ will be slowly varying over the Gamow window. However, not all resonances have a dramatic effect on the reaction rate: a resonance at energy $E_\tau$ with total width $\Gamma$ will not contribute significantly to the reaction rate at any temperature where $E_\tau$ is more than several $\Gamma$'s outside the Gamow window. Furthermore, a resonance in the Gamow window may fail to contribute significantly if the resonance strength $\omega_\gamma$ is small, or if only partial waves with $l$ - values greater than 2 are possible. For the $^{19}$Ne(p, $\gamma$)$^{20}$Na reaction, an s-wave ($l = 0$) resonance in the Gamow window can, depending on the properties of the resonance, increase the reaction rate by up to 4 orders of magnitude over the non-resonant terms. The resonant and non-resonant contributions to the stellar reaction rate will be discussed in Sections 2.2 and 2.3, respectively.

2.2 Resonant Reaction Rate

A resonance in the cross-section can occur when the entrance channel energy $Q_0 + E_\tau$ corresponds to an excited state at energy $E_\chi$ in the compound system. As mentioned above, the presence of a resonance is characterized by a peak in the cross-section $\sigma(E)$ at some energy $E_\tau$, and can increase the reaction rate by many orders of magnitude over the non-resonant rate. It will be shown that the contribution of such a resonance to the rate may be calculated with the knowledge of the energy, the spin, and the formation and decay partial widths of the resonant level. In order to determine which resonances are in the Gamow window at any given stellar temperature and will thus contribute to the reaction rate, a high-precision measurement of the
level structure in the compound system is necessary. The properties of these contributing 
resonances can then be obtained indirectly, employing standard techniques of nuclear 
spectroscopy with reactions different from the stellar reaction. This section will discuss the 
resonant properties necessary to make such an indirect determination of the resonant reaction 
rate.

Recall from Eq. 2.4 that the dependence of the cross-section on the center-of-mass 
energy $E$ is needed to calculate the reaction rate. The energy dependence of the cross-section of 
a resonant process is given by the Breit-Wigner formula to be

$$\sigma(E) = \pi \left( \frac{\hbar^2}{2\mu E} \right) \omega \frac{\Gamma_a(E) \Gamma_b(E)}{(E - E_r)^2 + \left(\frac{\Gamma}{2}\right)^2}, \tag{2.12}$$

where we define

$E_r$ = resonance energy corresponding to excitation energy $E_X = Q + E_r$

$\Gamma$ = total width of the resonant state

$\Gamma_{a,b}$ = partial width of formation and decay of the resonant state

$J_{1,2,r}$ = spin of incident particles 1 and 2, and resonant level, respectively

and $\omega = (2J_r + 1) / ((2J_1 + 1)(2J_2 + 1))$ (Ro88).

We are concerned with resonances in the stellar proton capture reactions $^{13}\text{N}(p, \gamma)^{14}\text{O}$ and $^{19}\text{Ne}(p, \gamma)^{20}\text{Na}$, where $\Gamma_a = \Gamma_p$ and $\Gamma_b = \Gamma_\gamma$. When the cross-section given by Eq. 2.12 is 
inserted into the reaction rate given by Eq. 2.4, we get

$$N_A \langle \sigma v \rangle = N_A \left( \frac{\hbar}{\pi \mu} \right)^{1/2} (kT)^{3/2} \frac{\hbar^2}{2\mu} \pi \omega \int_0^\infty \frac{\Gamma_p(E) \Gamma_\gamma(E)}{(E - E_r)^2 + \left(\frac{\Gamma}{2}\right)^2} \exp \left( -\frac{E}{kT} \right) dE. \tag{2.13}$$

This is the most general expression for the resonant reaction rate. It can alternatively be cast in 
the form of Eq. 2.10 with the definition of a resonant $S$-factor:

$$S_{\text{res}}(E) = \pi \left( \frac{\hbar^2}{2\mu} \right) \omega \frac{\Gamma_p(E) \Gamma_\gamma(E)}{(E - E_r)^2 + \left(\frac{\Gamma}{2}\right)^2} \exp \left( -\left(\frac{E}{E_r}\right)^{1/2} \right) \tag{2.14}.$$

For sufficiently broad resonances ($\Gamma / E_r \geq 0.05$), the values of $\Gamma_p$ and $\Gamma_\gamma$ vary 
significantly with energy over the Gamow window. This is the case for the $^{13}\text{N}(p, \gamma_1)^{14}\text{O}$
reaction, where the resonance energy of the first state above threshold is 540 keV and the total width is 38 keV. In this case it will be useful to relate the value of the partial widths at any energy to their value at the resonance energy \( E_r \), a quantity measurable in the laboratory. The proton width \( \Gamma_p \) is given by

\[
\Gamma_p(E) = \frac{2\hbar}{R_n} \left( \frac{2E}{\mu} \right)^{1/2} P_l(E, R_n) \Theta_f^2
\]

where \( \Theta_f^2 \) is the proton reduced width, \( R_n \) is the nuclear radius, and \( P_l \) is the \( l \)-wave penetrability given by

\[
P_l(E) = \frac{1}{\Gamma_f^2(k R_n) + G_l^2(k R_n)}, \quad k = \sqrt{2 \mu E / \hbar}
\]

Here, \( k \) is the wavenumber, and \( F_l \) and \( G_l \) are the regular and irregular Coulomb functions.

The spin of the resonant level \( J_r \) sets limits on the \( l \)-transfer of the proton, and thus on the \( l \)-wave penetrability needed for the proton width in Eq. 2.16. It is important to note that \( P_l \) drops by one to several orders of magnitude for an increase in \( l \) by one unit; the approximate dependence in the low energy case given by

\[
P_l(E) \propto \exp \left( -2l(l+1) \left( \frac{\hbar^2}{2 \mu Z_1 Z_2 e^2 R_n} \right)^{1/2} \right)
\]

Therefore the lowest \( l \)-value permitted by angular momentum conservation is the partial wave that dominates the reaction rate. This is especially true for the s-wave \((l = 0)\) capture which occurs in both the \(^{13}\text{N}(p, \gamma)^{14}\text{O}\) and \(^{19}\text{Ne}(p, \gamma)^{20}\text{Na}\) reactions. The low energy dependence of the \( l = 0 \) (s-wave) penetrability is given by the behavior of \( G_0(kR) \sim kR \cdot \exp(-\pi\eta) \) (Ab65). Thus we have

\[
P_0(E) \propto E^{-1/2} \exp \left( -\left(\frac{E_0}{E}\right)^{1/2} \right)
\]

From Eq. 2.15, the energy dependence of the proton width can be expressed with respect to the value at resonance \( \Gamma_p(E_r) \) as

\[
\frac{\Gamma_p(E)}{\Gamma_p(E_r)} \propto \frac{P_l(E)}{P_l(E_r)} \left( \frac{E}{E_r} \right)^{1/2}
\]

The use of the ratios of penetrabilities makes this expression for \( \Gamma_p(E) \) relatively insensitive to the value of the nuclear radius chosen. In the \( l = 0 \) case, this can be written as
\[ \Gamma_p(E) = \Gamma_p(E_r) \frac{\exp\left(-\frac{(E - E_r)^{1/2}}{E_r^{1/2}}\right)}{\exp\left(-\frac{(E - E_r)^{1/2}}{E_r^{1/2}}\right)} . \] (2.20)

For the gamma-width \( \Gamma_\gamma \), the scaling with energy is given by the Weisskopf formula (Bl62), \( \Gamma_\gamma \propto E_\gamma^{2L+1} \), where \( L \) is the multipolarity of the gamma radiation and the energy of the gamma radiation is given by \( E_\gamma = Q_\omega + E \) for transitions to the ground state of \( ^{14}_0 \) or \( ^{20}_4 \) Na. A rough estimate (good only to two orders of magnitude) for \( E1 \) transitions (such as \( ^{14}_0 \) \( \gamma \)-decay) is given by

\[ \Gamma_\gamma(E1) = 0.07 \left( \frac{E_\gamma(\text{MeV})}{E_\gamma(\text{MeV})} \right)^3 A^{2/3} \text{eV} . \] (2.21)

The energy dependence of the partial gamma width can be expressed as

\[ \Gamma_\gamma(E) = \Gamma_\gamma(E_r) \left( \frac{Q_\omega + E}{Q_\omega + E_r} \right)^{2L+1} . \] (2.22)

The resonant rate for the \( ^{13}_2 \)N(p, \( \gamma \))\( ^{14}_0 \) reaction will be calculated from Eq. 2.10, using the resonant S-factor given in Eq. 2.14 and the energy dependence of the proton and gamma widths given in Eqs. 2.20 and 2.22, respectively. It is evident that the resonance parameters necessary for an indirect determination of the resonant reaction rate calculation are the resonance energy \( E_r \), the partial widths \( \Gamma_p \) and \( \Gamma_\gamma \) evaluated at resonance, the total width \( \Gamma \), and the spin \( J_\gamma \) of the resonant level. For the \( ^{13}_2 \)C(p, \( \gamma \))\( ^{14}_0 \) reaction, the only resonance in the Gamow window at nova temperatures (0.1 < \( T_9 \) < 0.5) is the 5.169-MeV level \( ^{14}_0 \); the next highest level is at \( E_x = 5.920 \text{MeV} \), which is not in the Gamow window until well over \( T_9 = 1 \). A measurement of the gamma width of the 5.169 MeV resonance is crucial in the determination of the resonant reaction rate; this is the goal of the experiment described in Chapter 3.

Experimental and theoretical studies of the \( ^{12}_2 \)C(p, \( \gamma \))\( ^{13}_0 \) reaction, the first reaction in the CNO cycle, gives strong evidence for the general reliability of this indirect method of determining stellar reaction rates, and in particular for its application to the \( ^{13}_2 \)N(p, \( \gamma \))\( ^{14}_0 \) reaction. The \( ^{12}_2 \)C(p, \( \gamma \))\( ^{13}_0 \) reaction is dominated at stellar temperatures by resonant proton
capture into the broad (32-keV), $E_{cm} = 425$ keV, $l = 0$ resonance in $^{13}$N, just as the $^{13}$N(p, $\gamma$)$^{14}$O reaction is dominated by capture into the broad (38-keV), 540-keV, $l = 0$ resonance in $^{14}$O. Figure 2.3 shows the excellent agreement over 7 orders of magnitude of the experimentally measured energy dependence of the cross-section of this reaction with a calculation using the formalism developed above (Vo63).

The Breit-Wigner cross-section given by Eq. 2.13 is peaked at energy $E_T$ and drops by more than 90% at energies $E_T \pm 3 \Gamma$ as a result of the term $\Gamma_p \Gamma_\gamma/(E - E_T)^2 + (\Gamma/2)^2)$. This allows a simplification to be made in the case of a narrow resonance $\Gamma \ll E_T$: the partial widths and the Maxwell-Boltzmann distribution term can be pulled out of the energy integral in Eq. 2.13 and evaluated at the resonance energy $E_T$. This may be case for the $^{19}$Ne(p, $\gamma$)$^{20}$Na reaction, where the resonance energy of the first level above the proton threshold is $= 450$ keV, but the total width is thought to be less than 10 - 20 keV based on the analog states in $^{20}$F.

The $^{20}$Ne($^3$He, $t$)$^{20}$Na measurement in Chapter 5 gives the first upper limit on the total width of this state as $< 9$ keV. Evaluating the partial widths and velocity distribution terms in Eq. 2.13 at resonance gives the following expression for the resonance reaction rate (Cl68), (Ro88):

$$N_A(\sigma v) = N_A \left( \frac{2\pi}{\mu kT} \right)^{3/2} \hbar^2 (\omega \gamma)_r \exp \left( \frac{-E_T}{kT} \right)$$

where we have defined the resonance strength $\omega \gamma$ to be

$$\omega \gamma = \omega \frac{\Gamma_p \Gamma_\gamma}{\Gamma}.$$  

(2.24)

If a number of narrow resonances with spacing $\Delta E_T \gg \Gamma$ are present in the Gamow window, the total resonant rate is given by the sum of the rates for each individual resonance:

$$N_A(\sigma v) = N_A \left( \frac{2\pi}{\mu kT} \right)^{3/2} \hbar^2 \sum_r (\omega \gamma)_r \exp \left( \frac{-E_T}{kT} \right).$$

(2.25)

Because of the exponential dependence in Eq. 2.25, the precise determination of the resonance energy $E_T$ is critical for the rate calculation. The resonance energy of the $^{14}$O$_1$ level has been previously measured by Chupp et al. (Ch85) to be $540 \pm 2$ keV. This is not the case with the $^{19}$Ne(p, $\gamma$)$^{20}$Na reaction, however: the 4% uncertainty in the resonance energy of the first level above the $^{19}$Ne + p threshold in $^{20}$Na ($450 \pm 16$ keV, La(89)) contributes an
uncertainty of a factor of 540% to the resonant reaction rate at \( T_9 = 0.1 \), and 85% at \( T_9 = 0.3 \).
Reducing the uncertainty in \( E_T \) to ±5 keV would reduce the resonant rate uncertainties to 80% and 20%, respectively. A measurement with this precision will be discussed in Chapter 5, along with a measurement of the total widths of the contributing resonances necessary to determine whether the narrow resonance expression Eq. 2.25 can be applied to the \(^{19}\text{Ne}(p, \gamma)^{20}\text{Na} \) stellar reaction.

2.3 Non-Resonant Reaction Rate

In the absence of resonant levels in the compound nucleus, the S-factor has a slow variation with energy and can be often be approximated by a constant value \( S(E_Q) \) or a second-order Taylor series expansion in \( E \) over the Gamow window. Two mechanisms contribute to this non-resonant S-factor: the direct capture (DC) process, and reactions on the tails of higher energy resonances which are not specifically included in the sum in Eq. 2.25. The first is a one-step process in which a free particle interacts with the repulsive Coulomb potential and attractive nuclear potential of the second nucleus, causing acceleration and bremsstrahlung and resulting in a capture into a bound state (Ro73),(Ro74). It occurs at all energies since no resonant levels in the compound system are involved. The second process occurs via the strong interaction and results from broad resonances \( \Gamma / E_T \geq 0.05 \) with peak energies \( E_T \) outside of the Gamow window. The large width \( \Gamma \) ensures that reactions can proceed through these intermediate resonant levels even at stellar energies far from \( E_T \). The non-resonant DC process forms the background upon which the resonant capture process occurs in both the \(^{13}\text{N}(p, \gamma)^{14}\text{O} \) and \(^{19}\text{Ne}(p, \gamma)^{20}\text{Na} \) reactions. Additionally, the 38 keV - width of the \(^{14}\text{O}*(5.169-\text{MeV}) \) level and its small resonance strength \( \omega \gamma \) combine to give an interference term between the resonant and the DC reaction rates (Fo75), analoguous to the interference seen in the \(^{12}\text{C}(p, \gamma)^{13}\text{N} \) reaction (Ro74). Thus for the \(^{13}\text{N}(p, \gamma)^{14}\text{O} \) reaction, we can write
where \( S_{\text{res}} \) is given by Eq. 2.14, and the phase of the interference is given by the resonance phase shift

\[
\delta_{\text{res}}(E) = \tan^{-1}\left( \frac{\Gamma_p(E)}{2(E - E_r)} \right).
\]

Because the interference between the resonant and non-resonant terms for the \(^{12}\text{C}(p, \gamma)^{13}\text{N}\) (Ro74) and \(^{13}\text{C}(p, \gamma)^{14}\text{N}\) (Az85) reactions have been measured to be constructive for \( E < E_r \) and destructive for \( E > E_r \), the same phase is assumed for the \(^{13}\text{N}(p, \gamma)^{14}\text{O}\) reaction, agreeing with two recent theoretical models (Fu87), (Ma84). Such an interference term would be important for the \(^{19}\text{Ne}(p, \gamma)^{20}\text{Na}\) reaction only if one of the contributing resonances is broad; this will be determined not to be the case by the measurement described in Chapter 5.

To evaluate the form of the non-resonant reaction rate, for both tail and DC mechanisms, a gaussian approximation to the exponential term in the integrand of Eq. 2.10 is made:

\[
\exp\left( -\frac{E}{kT} - \left( \frac{E_0}{E} \right)^{1/2} \right) = \exp(-\tau) \exp\left( -\left( \frac{E - E_0}{\Delta E_0/2} \right)^2 \right),
\]

where \( \tau = 3E_0/kT \) and \( \exp(-\tau) \) is the maximum value of the exponential term in the integrand. Inserting this approximation in Eq. 2.10, and using \( S(E) = S(E_0) \) over the Gamow window, we get

\[
N_A(\sigma \nu) = N_A\left( \frac{2^{1/2}}{\mu} \right) (kT)^{1/2} \Delta E_0 S(E_0) \exp(-\tau)
\]

A correction for the slight asymmetry of the integrand is made (Fo67) by defining the function \( F(\tau) \) such that

\[
N_A(\sigma \nu) = N_A\left( \frac{2^{1/2}}{\mu} \right) (kT)^{1/2} \Delta E_0 S(E_0) \exp(-\tau) \frac{\sqrt{\pi}}{2} F(\tau).
\]

The parameter \( \tau \) has the value 22.6 and 29.0 for the \(^{13}\text{N}(p, \gamma)^{14}\text{O}\) and \(^{19}\text{Ne}(p, \gamma)^{20}\text{Na}\) reactions, respectively, at \( T_0 = 0.3 \); thus \( 1/\tau \) is a valid expansion parameter. Eq. 2.30 can be expanded in \( \tau \) and set equal to Eq. 2.10 to solve for \( F(\tau) \), giving
A more detailed description of the energy variation of $S(E)$ can be made with a second-order Taylor series expansion for $S(E)$ about zero energy,

$$S(E) = S(0) + S'(0) E + S''(0) \frac{E^2}{2} + \ldots$$  \hspace{1cm} (2.32)$$

Repeating the above procedure with this second order expansion, we get

$$N_A(\sigma v) = N_A \left( \frac{2}{\mu} \right)^{1/2} (kT)^{-3/2} \Delta E_0 S_{\text{eff}}(E_0) \exp(-\tau),$$  \hspace{1cm} (2.33)$$

where the effective S-factor is given by

$$S_{\text{eff}}(E_0) = S(0) \left( 1 + \frac{5}{12\tau} + \frac{S'(0)}{S(0)} \left( E_0 + \frac{35}{36} kT \right) + \frac{S''(0)}{2 S(0)} \left( E_0^2 + \frac{89}{36} E_0 kT \right) + \ldots \right).$$  \hspace{1cm} (2.34)$$

This can be written in terms of temperature as

$$S_{\text{eff}}(E_0) = S(0) \left( 1 + \sum_i a_i T^{i/3} \right)$$  \hspace{1cm} (2.35)$$

where the coefficients $a_i$ will be different for the DC and tail non-resonant processes.

Additionally, the tail process needs a cut-off term so that this contribution is not counted when the resonance is in the Gamow window. This is done by multiplying the tail term by the factor $f_{\text{cutoff}}$, where

$$f_{\text{cutoff}} = \exp \left( -\frac{E_{\tau_1}}{kT} \right),$$  \hspace{1cm} (2.36)$$

and $E_{\tau_1}$ is the first resonance above threshold. The corrected tail rate is given by

$$N_A(\sigma v) = N_A \left( \frac{2}{\mu} \right)^{1/2} (kT)^{-3/2} \Delta E_0 S_{\text{eff}}(E_0) \exp(-\tau) f_{\text{cutoff}}.$$  \hspace{1cm} (2.37)$$

Eqs. 2.33 and 2.37 will be used in Chapter 6 for the non-resonant terms in the $^{13}\text{N}(p, \gamma)^{14}\text{O}$ and $^{19}\text{Ne}(p, \gamma)^{20}\text{Na}$ reaction rates.
Figure 2.1.
The Gamow window for the $^{13}\text{N}(p, \gamma)^{14}\text{O}$ reaction at $T_9 = 0.3$. The product of the Maxwell-Boltzmann velocity distribution and the Coulomb penetrability is sharply peaked at an energy $E_0$, the "most effective energy" for stellar nuclear reactions.
Gamow Window

Integrand Factors

Boltzmann Factor

Penetrability

Product

Energy (keV)

$E_0$
Temperature dependence of the Gamow Window $E_0 \pm \Delta E_0 / 2$ for the $^{13}\text{N}(p, \gamma)^{14}\text{O}$ and $^{19}\text{Ne}(p, \gamma)^{20}\text{Na}$ reactions. A resonance will have an effect on the reaction rate only when it is in the Gamow window.

Figure 2.2.
Gamow Window for the

$^{13}\text{N} (p, \gamma)^{14}\text{O}$ and $^{19}\text{Ne} (p, \gamma)^{20}\text{Na}$ Reactions

Gamow Window Limits (MeV)

Temperature $T_9$
Figure 2.3.

Measured Energy Dependence of the $^{12}\text{C}(p, \gamma)^{13}\text{N}$ cross-section and calculated rate based on the parameters of the $E_{\text{cm}} = 424$-keV resonance in $^{13}\text{N}$; the laboratory energy of this resonance is 460 keV.
Cross section, barns

$0.460 \text{ Mev res.}$

$C^{12}(p, \gamma)N^{13}$

Lab proton energy, Mev
Chapter 3. Measurement of the $^1H\,(^{14}N,\,^{14}O\,*(5.17))\,n\,\gamma$ Reaction

3.1 Introduction

A direct measurement of the $^{13}N(p,\,\gamma)^{14}O$ reaction would require the use of a radioactive ($\tau_{1/2} = 598\,s$) $^{13}N$ beam or target. Development of such a beam has been proposed at TRIUMF (Da85) and pursued at a number of facilities, including Lawrence Livermore National Laboratory (Ma85) and Louvain-la-Neuve (Da90). At present, the highest intensity (10$^8$ particles/s) has been reached at Louvain-la-Neuve. At that facility, a cyclotron is used to produce a high intensity (500 $\mu$A) 30 MeV primary proton beam incident on a graphite ($^{13}C$) pellet, producing $^{13}N$ via the $^{13}C(p,\,n)^{13}N$ reaction. $N_2$ gas is continuously flowed through the target to extract the $^{13}N$, with an efficiency of 80%. The gas flows to an ECR (electron-cyclotron resonance) ion source for ionization (4% efficiency), acceleration (to 9 keV), and injection into a second cyclotron for further acceleration. Their present beam intensity, along with a cross-section of 5 nb at 200 keV (Fe89) and a CH$_2$ target thickness of 1 mg/cm$^2$, would give a $^1H(^{13}N,\,^{14}O)\,\gamma$ counting rate of 5 x 10$^{-5}$ cts/s, a factor of 10 - 100 too low to be practical. A factor of 5 enhancement in beam intensity is anticipated from offline tests of the individual components of the system (Ro90).

Until this happens, however, an indirect measurement of the $^{13}N(p,\,\gamma)^{14}O$ reaction can be made. At novae temperatures ($T_g > 1$), the $^{14}O_1(5.169\,\text{MeV})$ state is the only state in the Gamow window. Hence by the discussion in Section 2.2, the resonant reaction rate depends solely on the energy $E_r$, proton width $\Gamma_p$, total width $\Gamma$, spin $J_r$, and gamma width $\Gamma_{\gamma}$ of this state. The first four of these parameters have been previously measured: Chupp et al. (Ch85) used the $^{14}N(^3\text{He},\,t)^{14}O$ reaction to determine the excitation energy (5.169 ± 0.0018 MeV) and total width (38.1 ± 1.8 keV) of $^{14}O_1$, and the state has a well-established spin and parity of 1$^-$ (Aj81). These are shown in the $^{14}O$ level diagram in Figure 3.1. As
discussed in Section 2.2, an experimental determination of the gamma width is crucial for the determination of the reaction rate since $\omega \gamma = \omega \Gamma_\gamma$. Since theoretical predictions of $\Gamma_\gamma / \Gamma$ are $\leq 10^{-4}$ (See Table 3.1), such a measurement will be near the limit of sensitivity obtainable at most nuclear laboratories.

The three previous indirect measurements of $\Gamma_\gamma / \Gamma$ are listed in Table 3.1. Wang used a triton - $^{14}$O$_o$ recoil coincidence measurement $^{14}$N($^3$He, t)$^{14}$O$_1$$^{14}$O$_o\gamma$ at a beam energy of 33 MeV to set an upper limit of on $\Gamma_\gamma / \Gamma$ (Wa86). In that study, two silicon surface-barrier semiconductor ( Si (SB) ) detectors were used to measure and identify the tritons at 80° in a $\Delta$E-E telescope arrangement. Similarly, the $^{14}$O recoils were detected at the kinematically required angle of 36° with a gas proportional counter, which detected the energy loss over a small gas volume, and a Si (SB) detector, which detected the residual $^{14}$O energy. This arrangement of detectors is referred to as a $\Delta$E - E "telescope" and allows the identification of particles on the basis of their charge and mass. Specifically, the $^{14}$O$_o$'s of interest are separated from the recoils from the dominant $^{13}$N + p decay channel. The kinematics of the reaction allowed = 100% recoil detection efficiency. No coincidence counts were seen due to the low ($^3$He, t) cross-section $\sim 10 \mu$b/sr ; an upper limit of $\Gamma_\gamma / \Gamma < 4.5 \cdot 10^{-4}$ was set based on the 65 % coincidence efficiency of the system, determined from $^{14}$N($^3$He, t)$^{14}$O$_o$ coincidences. With $\Gamma_{tot} = 38.1 \pm 1.8$ keV (Ch85), this corresponds to an upper limit on $\Gamma_\gamma$ of 17 eV. An increase in sensitivity of at least a factor of 10 is needed in light of model predictions for the branching ratio of $(3 - 6) \cdot 10^{-5}$ by, for example, Funck and Langanke (Fu87), Mathews and Dietrich (Ma84), and Descouvemont and Baye (De89), as listed in Table 3.1.

In efforts to obtain this higher sensitivity, the second and third groups attempting an indirect measurement of $\Gamma_\gamma / \Gamma$ used the $^{12}$C($^3$He, n)$^{14}$O reaction to populate the $^{14}$O$_1$ state and measured the 5.169 - MeV decay gamma-rays in coincidence with the neutrons (Ag89), (Fe89). The $^{12}$C($^3$He, n)$^{14}$O reaction has a differential cross-section of 1 mb/sr at 0° (Ad70),
a factor of 100 higher than the $^{14}\text{N}^{(3}\text{He, t)}^{14}\text{O}_1$ cross-section. However, the detection efficiency for the coincidence $\gamma$-rays is much lower than the $\sim$100% recoil detection in the $^{14}\text{N}^{(3}\text{He, t)}^{14}\text{O}_1^{(14}\text{O}_0)\gamma$ reaction. Much experimental effort and elaborate detection systems are thus required to obtain a reasonable $n-\gamma$ coincidence rate. To maximize the $\gamma$ count rate, Fernandez and collaborators (Fe89) placed their 5" x 6" NaI detectors very close (5 cm) to the target, subtending 30% of $4\pi$ sr. This close placement made it necessary to limit the beam current to 10 nA to keep the NaI rates less than 75 kHz in order to avoid signal pileup. A high-density polyethylene shield was fabricated to prevent neutrons from scattering from the NaI detectors to the neutron detector, a major source of real background coincidences in the experiment. A second substantial source of background $n-\gamma$ coincidences was from the $^{13}\text{C}^{(3}\text{He, n}\gamma)^{15}\text{O}^*(5.23-\text{MeV})$ reaction: the $^{15}\text{O}^*(5.23-\text{MeV})$ gamma decay has roughly the same $\gamma$ - ray energy as the $^{14}\text{O}_1(\gamma)^{14}\text{O}_0$ decay, but a 100% gamma branching ratio, a factor of $10^4$ higher. This results in the $^{15}\text{O}\gamma$-rays being a factor of $10^2$ more intense than the 5.169 MeV $^{14}\text{O}\gamma$-rays of interest for a contamination of 1% $^{13}\text{C}$. In an effort to avoid this background, Fernandez et al. reduced the $^{13}\text{C}$ content in their target by using $^{12}\text{C}$ implanted into a high-purity gold backing. Finally, pulse-shape discrimination was used on signals from the liquid-scintillator detector to distinguish neutrons from the high flux of $\gamma$s at 0°. In spite of these efforts, the low number of $n-\gamma$ coincidence events and high background made it necessary to employ elaborate statistical methods to extract a branching ratio of $(7.2 \pm 3.5) \times 10^{-5}$ from the data.

Aguer and collaborators (Ag89) also employed implanted $^{12}\text{C}$ targets in their measurement of the $^{12}\text{C}^{(3}\text{He, n}\gamma)^{14}\text{O}$ reaction. In this case, a 74 - element $4\pi$ hexagonal - crystal BaF$_2$ $\gamma$-detector array ("le chateau de cristal") was employed to detect the 5.169 MeV $\gamma$ - rays, and 7 hexagonal Ne213 liquid scintillator detectors were used to detect neutrons near 0°. Even with such an impressive detection system, there were significant problems with this experiment. The $^{12}\text{C}^{(3}\text{He, n)}^{14}\text{O}_1$ singles rate could not be directly obtained due to a large
neutron background in the time-of-flight spectrum; it was necessary to use the $^{12}\text{C}(^{3}\text{He}, n)^{14}\text{O}$ yield and the previously measured $n_0 / n_1$ ratio (Ad70) to extract the $n_1$ yield. Furthermore, because of this large background, the presence of a neutron - $\gamma$ coincidence peak is questionable. Their preliminary result was a branching ratio of $(2\pm 1) \times 10^{-4}$; this has recently been revised to an upper limit of $3 \cdot 10^{-4}$ (Ro90).

To summarize the results of these previous measurements, low coincidence counting rates and high backgrounds have contributed a $\pm 50\%$ uncertainty in $\Gamma_\gamma / \Gamma$ in spite of the experimental techniques described above. Since the $^{13}\text{N}(p, \gamma)^{14}\text{O}$ resonant reaction rate is linearly dependent on $\Gamma_\gamma$, a more precise and conclusive measurement of the gamma width is needed.

We have made an independent measurement of $\Gamma_\gamma / \Gamma$ using the $^1\text{H}(^{14}\text{N}, ^{14}\text{O}) n \gamma$ reaction, an "inverse kinematics" version (heavy projectile incident on a light target) of the $^{14}\text{N}(p, n)^{14}\text{O}$ reaction. In such a reaction, the heavy products are forward focussed near 0°, allowing efficient detection. At a center of mass energy of 11.67 Mev (175 MeV $^{14}\text{N}$ lab energy), both the $^{14}\text{O}_0$ and $^{14}\text{O}_1$ states are populated by the $^1\text{H}(^{14}\text{N}, ^{14}\text{O})n$ reaction, with the dominant decay channel of $^{14}\text{O}_1$ being $^{13}\text{N} + p$. However, approximately one time in $10^4$, the $^{14}\text{O}_1$ decays via gamma emission, leaving a recoiling $^{14}\text{O}_0$ which may be kinematically separated with a magnetic spectrograph from the $^{14}\text{O}_0$ recoils formed directly. This method was chosen (Wa87) precisely because of the forward focussing: the kinematics (Figure 3.2) require that all of the $^{14}\text{O}$ recoils formed after $\gamma$- emission are within a $\pm 10^\circ$ cone centered at 0°, while the $^{14}\text{O}_0$ recoils formed directly are within a $\pm 30^\circ$ cone about 0°. The large solid angle of the spectrograph then allows an 80% efficiency for detection of the $\gamma$- decayed $^{14}\text{O}$'s and 25% efficiency for the $^{14}\text{O}_0$ 's formed directly; these efficiencies will be discussed further in Section 3.3. Another advantage to the $^1\text{H}(^{14}\text{N}, ^{14}\text{O}) n$ experiment is that it is a singles measurement, thus having an inherently higher counting rate than a coincidence experiment such as the $^{12}\text{C}(^{3}\text{He}, n \gamma)^{14}\text{O}$ reaction.

There are, however, some disadvantages to using the $^1\text{H}(^{14}\text{N}, ^{14}\text{O}) n \gamma$ reaction.
First, the experiment described above does not give a self-contained measurement of $\Gamma_\gamma / \Gamma$. The ratio of the number of $^{14}\text{O}_1$ which undergo $\gamma$-decay to the number of $^{14}\text{O}_0$'s formed directly, determined by this experiment, must be multiplied by the ratio of $^{14}\text{O}_0 / ^{14}\text{O}_1$ formed (the production cross-section ratio $\sigma_0 / \sigma_1$) in order to obtain the branching ratio $\Gamma_\gamma / \Gamma$. This cross-section ratio $\sigma_0 / \sigma_1$ was measured using the $^{14}\text{N}(p, n)^{14}\text{O}$ reaction at the University of Washington FN Tandem facility at the same center of mass energy as the $^{1}\text{H}(^{14}\text{N}, ^{14}\text{O}) n \gamma$ reaction, and will be described in detail in Chapter 4.

The experimental difficulties associated with the $^{1}\text{H}(^{14}\text{N}, ^{14}\text{O}) n \gamma$ reaction primarily concern the usual problems associated with trying to identify and separate ~100 events from a background of $10^7$ events, as well as those associated with the use of a magnetic spectrograph at 0°. The tail of the $^{1}\text{H}(^{14}\text{N}, ^{14}\text{O}) n$ distribution spreads into the kinematically allowed region of the (factor of $10^5$ less intense) $^{14}\text{O}_0$ recoils from $^{1}\text{H}(^{14}\text{N}, ^{14}\text{O}_1) n \gamma$, making the kinematic separation difficult to perform. This separation was further hampered by $^{12}\text{C}(^{14}\text{N}, ^{14}\text{O}_0) ^{12}\text{B}$ reactions which produced $^{14}\text{O}_0$'s of nearly the same rigidity as the $^{14}\text{O}_1(\gamma)^{14}\text{O}_0$'s of interest. Section 3.3 will discuss the methods employed to obtain the best separation possible. The difficulties associated with making a 0° measurement in a magnetic spectrograph will be discussed in Section 3.2.

In addition to the singles measurement described above, two coincidence measurements were also attempted with our experimental setup. The first was a $\gamma$ - $^{14}\text{O}$ recoil coincidence measurement, detecting the 5.169 MeV $\gamma$-rays in an array of eight 5" x 5" NaI detectors and the $^{14}\text{O}_0 \gamma$-decayed recoils in the focal plane detector. However, the placement of the NaI array close to the target, to obtain a high efficiency, resulted in such a large random coincidence rate between $\gamma$-rays and the very intense $^{14}\text{O}_0$ singles that this was not usable in the final analysis. Neutrons from $^{1}\text{H}(^{14}\text{N}, ^{14}\text{O}) n$ contributed an additional background in the NaI array, since no pulse shape discrimination was used in the NaI detectors. Additionally, the predicted coincidence yield for a 10% $\gamma$-ray detection efficiency was determined, after a
measurement of the production cross-section, to be approximately 10 counts, down a factor of 10 from previous estimates. This is to be compared to a background of a few thousand counts. For these reasons, the $\gamma - {^{14}}O$ recoil coincidence measurement was unable to significantly increase the signal to noise ratio in the kinematic separation technique.

A neutron - $^{14}O$ recoil coincidence measurement was also attempted. A liquid scintillator detector placed at $0^\circ$ at a flight path of $= 2$ m detected the (primarily forward focussed) neutrons corresponding to the population of $^{14}O_1$. When combined with the focal plane measurement of the $\gamma$ - decayed $^{14}O_0$'s, this measurement has the potential for producing a self contained measurement of $\Gamma_\gamma / \Gamma$. However, the very high neutron background from the tantalum focal plane beamstop completely overwhelmed the neutrons of interest and prevented the n - $^{14}O$ recoil coincidence measurement from being made.

3.2 Experimental Setup

The 175 MeV $^{14}N$ beam used in the $^1H(^{14}N, {^{14}}O)n\gamma$ experiment was produced at the ATLAS Superconducting Heavy Ion Accelerator facility at Argonne National Laboratory (Bo87.) The system consists of a 9-MV FN Tandem injector accelerator, a 20MV superconducting booster linac, and the 20-MV ATLAS superconducting linac addition. Beams ranging in mass from $^7Li$ to $^{127}I$ with energies up to 25 MeV/u can be obtained. To produce the 175 MeV $^{14}N+7$ beam, a $^{14}N^{-1}$ beam from a sputter ion source is bunched to 1 ns and injected into the low energy end of the FN Tandem. The ions are accelerated to the terminal of the tandem, stripped of their electrons, and accelerated to the high energy end where they emerge with a variety of energies according to their charge state. After being momentum and charge-state analyzed in a 90$^\circ$ magnet, the beam is bunched with a superconducting buncher (to $\sim 400$ ps), chopped, and injected into the booster linac. The superconducting split ring RF resonators comprising the linac were tuned to give an exit energy of 175 MeV to the $^{14}N$
beam, this energy being measured with a time-of-flight system. The pulsed beam consists of 350 ps - wide bursts separated by 82.5 ns, corresponding to a linac frequency of 12.125 MHz. A sweeper was employed to accept only every third beam burst, thus effectively running at a frequency of 4.04 MHz, with 247 ns between bursts. The beam was then steered to the magnetic spectrograph in Target Area III. The maximum beam current obtainable was 1 particle-nA (1 pnA) of $^{14}\text{N}^+7$ ($6 \times 10^9$ particles/s), resulting in a (nearly limiting) 1 kHz focal plane detector counting rate.

In order to accommodate the NaI detector array, a small target chamber was built for this experiment. It consisted of a horizontal 1" diameter beam tubing "cross" which allowed the target ladder to be positioned in the center of the array, with their front surfaces ~4 cm from the target. Approximately 80% of the total solid angle was covered. However, a consequence of this configuration was a relocation of the targets 15 cm upstream from their normal position at the ion-optical object of the Split-Pole Magnetic Spectrograph. This translation of the object resulted (to first order) to a translation of the image, and is evident in a plot of momentum vs. energy of the $^{14}\text{O}$ particles arriving at the focal plane, as discussed in Section 3.3.

Self-supporting polypropylene ( $(\text{CH}_2)_n$ ) targets of 850 $\mu$g/cm$^2$ thickness were used in the experiment as the source of $^1\text{H}$, the plastic material being stretched over standard Al target frames. Carbon targets of 650 $\mu$g/cm$^2$ were used to determine the background from $^{12}\text{C}(^{14}\text{N}, ^{14}\text{O})^{12}\text{B}$ and $^{16}\text{O}(^{14}\text{N}, ^{14}\text{O})^{16}\text{N}$ reactions shown in the kinematics in Figure 3.2. An 830 $\mu$g/cm$^2$ CD$_2$ target was used to obtain a focal plane position vs. momentum calibration with the $^{12}\text{C}(^{14}\text{N}, ^{14}\text{O})^{12}\text{B}$, $^2\text{H}(^{14}\text{N}, ^{15}\text{O})n$, and $^2\text{H}(^{14}\text{N}, ^{14}\text{O})2n$ reactions. This calibration will be discussed in Section 3.3.

The Enge Split Pole magnetic spectrograph in ATLAS Target Area III was used to kinematically separate the $^{14}\text{O}_o$ recoils formed after gamma decay from those formed directly; the setup is shown in Figure 3.3. Particles entering the magnet follow a circular orbit with a radius of curvature based on their momentum/charge (p/q) ratio. The use of two pole pieces with specially shaped edge designs allows for the acceptance of a large solid angle (typically 2
- 5 msr) and for second order focussing in the horizontal momentum - dispersion plane (Sp67). Excellent data-collection rates over a large momentum bite \( \left( \frac{p_{\text{max}}}{p_{\text{min}}} \approx 2.8 \right) \) are possible with the long (60 cm) focal plane and the large solid angle. In our experiment, positioning the spectrograph at 0° and using the 2.4 msr rectangular aperture allows the collection ~ 80% of the \(^{14}\text{O}_1\) recoils and ~ 25% of the \(^{14}\text{O}_0\) recoils formed directly; these percentages will be discussed in Section 3.3 and 4.4. The spectrograph was run with a magnetic field of 10.357 kG, which bent the 175 MeV \(^{14}\text{N}^+7\) beam particles into the faraday cup at the extreme high momentum end of the focal plane.

There were a number of problems associated with a 0° measurement in a magnetic spectrograph that needed to be addressed. First, the forward focussing of the \(^{14}\text{O}\) recoils in the reaction results in a large \( \frac{dE}{d\Theta} \) for these particles \( \approx 12 \text{ MeV / degree} \). This necessitated a careful optical alignment (to within 0.05° of 0°) for the 4-jaw beam collimating slits (located ~ 0.5 m upstream of the targets), the magnet entrance aperture, and the magnet itself. A greater difficulty with the 0° measurement concerns the approximately \( \approx 10^{10} \) \(^{14}\text{N}\) beam particles/s which enter the spectrograph and possibly the focal plane detector. Figure 3.3 shows the tantalum faraday cup used to stop the beam at the extreme high-momentum (high magnetic rigidity) end of the focal plane. The placement at the focal plane minimized the scattering of beam particles from the cup into the detector. A tantalum shield on the faraday cup and a moveable tantalum mask over a portion of the focal plane detector entrance window were used to further minimize the scattering from the faraday cup into the focal plane detector.

The elimination of any "halo" on the beam was additionally necessary to successfully make the 0° measurement. Beam halo is a result of the primary beam particles inelastically scattering off beam line slits, thereby slightly lowering their energy and magnetic rigidity. The usual consequence of this scattering is a slight energy spread in the detected recoil particles. However, there is a much greater problem in a 0° measurement: beam halo particles, with their lower magnetic rigidity, are bent into the focal plane detector instead of into the faraday cup.
The beam was therefore carefully tuned to eliminate halo by minimizing both the beam current on a small (3 mm) aperture at the target position and the count rate in the focal plane detector, while simultaneously maximizing the current on the focal plane faraday cup. After such a procedure, the count rate in the focal plane detector with an empty target frame in the target position and a 1 pnA \( (6 \cdot 10^9 / s ) \) \(^{14}\)N beam dumped in the faraday cup was less than 100 Hz. Particles reaching the focal plane must pass through a 36 mm wide x 11 mm high rectangular spectrograph entrance aperture located 425 ± 5 mm downstream from the targets. This determined the spectrograph opening angles to be ± 2.43 ° (horizontal) and ± 0.74 ° (vertical), and the solid angle subtended by the spectrograph to be 2.4 msr. By restricting the vertical acceptance of the \(^{14}\)O_0's formed directly, this aperture was crucial in allowing the \(^{14}\)O_1 recoils which gamma-decay not to be completely masked by the \(^{14}\)O_0 particles formed directly; this will be discussed in Section 3.3.

The focal plane detector (Figure 3.4) consists of a parallel plate avalanche counter (PPAC) backed by a Bragg curve detector (BCD) (Re88). The PPAC gives both a fast timing signal ( ~ 250 ps) and a position signal with intrinsic resolution of 0.75 mm and typical resolution (with windows and target contributions) of 1.0 - 1.25 mm. Isobutane gas flowing through the PPAC serves as the ionizing gas; windows of 2.5 μm mylar isolate this gas at a pressure of 5 Torr from the spectrograph vacuum and the BCD. The two electrode planes are fabricated of 2 μm aluminized mylar with a spacing of 3.2 mm and a bias of ± 270 V, giving an electric field of 169 kV / m. A wire plane is located between the electrode foils, oriented vertically and parallel to the foils. It consists of 480 Au-plated tungsten wires spaced 1.27 mm apart; pairs of adjacent wires are connected together externally to one leg of a delay line integrated circuit, with 2 ns delay between legs. Particles passing through the PPAC leave a trail of ionized isobutane \((C_4H_{10})\) molecules and electrons before entering and stopping in the BCD. In the high electric field, the electrons accelerate to the anode plane, causing both an electron avalanche at the anode and an induced charge on two or three wires in the wire plane.
The anode signal is used to start the ion time-of-flight (TOF) signal, the stop coming from the sweeper signal (every 247 ns). This TOF signal was used to determine each particle's entrance angle into the detector: the ion's time of flight through the spectrograph is proportional to the horizontal angle at which it enters the spectrograph, $\Theta_X$, which in turn is proportional to the detector entrance angle. The induced charge on the wires travels up to, and propagates to both ends of, the delay line. The time difference between these signals is proportional to the horizontal position of the ionization trail. The 480 ns delay line is divided into two sections of 240 ns to minimize signal loss, with readouts at each of the 4 ends.

The Bragg Curve detector (Re88) gives the total energy $E$, the nuclear charge $Z$, the range $R$, and the entrance angle $\Theta_X$ of the incident particle. The detector has a Frisch grid biased at +1000 V and an anode foil biased at +1100 V, both mounted normal to the incident particles. Freon (CF$_4$) at 100 Torr serves as the ionizing gas. Particles entering the detector leave an ionized trail of Freon molecules and electrons, stopping before they reach the Frisch Grid. The electrons drift through the Frisch grid to the anode plane; the grid allows the total charge deposited on the anode plane (the energy signal $E$) to be independent of the range of the incident particle. $E$ is measured by integrating the anode charge with a long time constant (3 $\mu$s); the resolution is approximately 2%. The $Z$ signal is determined from the height of the Bragg maximum, measured by integrating the anode charge with a short (0.5 $\mu$s) time constant. The resolution is typically $\Delta Z / Z = 1 / 70$ at $Z = 28$. The range and angle of the incident ion both are determined from the drift time, the time delay between the entrance to the detector and the drift of the end of the ionization trail to the anode; see Figure 3.5.

The electronics used to process the signals from the Focal plane detector are shown in Figure 3.6. For the position signal, the four signals from the ends of the split delay line are sent to a fast amplifier (FA) and then to a constant fraction discriminator (CFD), where fast timing signals (NIM negative logic) are generated. A good focal plane event will have signals in only one-half of the delay line, and thus only two of the four position timing signals. After a
suitable delay, the two signals are sent to separate stops of a TDC, the common start being obtained from the PPAC anode signal run through a FA and a CFD. This PPAC anode signal also starts the TOF Time-to-Amplitude Converter (TAC) and the BCD Angle or Range TAC. The beam sweeper signal serves as the stop of the TOF TAC, and the BCD anode signal (routed through a Timing Single Channel Analyzer (TSCA)) serves as the stop of the BCD Angle / Range TAC. The two TAC signals and the BCD Energy and Angle/Range signals are send to ADC's. The strobe for the ADC's and TDC's was the focal plane events signal, generated from the PPAC anode.

A CAMAC-based data acquisition was employed to interface the experimental signals with the computer for on-line data analysis and storage. The system consists of 2 CAMAC crates with a Hensley event handler (He79) connected via a parallel bus to four National Semiconductor microprocessors running in parallel. The parallel processing ensures that the system throughput is I/O limited instead of CPU limited. The multibus connecting the CAMAC crates to the processors has a throughput of 450 kBytes/s (Mo88), corresponding to \( \sim 1 \text{ kHz} \) from the experiment (with 400 bytes / event). However, the data acquisition rate of the experiment was limited by the beam (1 pnA), producing \( \leq 1 \text{ kHz} \) in the focal plane detector. This rate resulted in approximately one 10" 6250 bpi magnetic tape written an hour. The data acquisition software used was DAPHNE (We87, Mo88), which allows for real time collection, display, and manipulation of up to 500 parameters and associated histograms. A user written FORTRAN subroutine allows the determination of the focal plane position from the four position TDC signals. Examples of the histograms employed will be given in Section 3.3.

Section 3.3 Data Analysis and Results

The objective of the \( ^1\text{H}(^{14}\text{N},^{14}\text{O})_n \) experiment is to determine the number of focal plane events corresponding both to those \( ^{14}\text{O}_1 \) which \( \gamma \)-decay, and to those \( ^{14}\text{O}_0 \) which are formed directly. Combinations of the parameters Energy (E), Range (R), position or
momentum (P), time of flight (TOF), and entrance angle (θx) for each event are used to first separate Oxygen focal plane events from Carbon and Nitrogen events and then to separate 14O events from 15O and 16O events. The separation of Oxygen focal plane events from Carbon and Nitrogen events, a "cut" in nuclear charge Z, is obtained by plotting (Energy)² vs Range, (E² ⊗ R) for all focal plane events and drawing a "window" or gate on the 2-dimensional histogram around the Oxygen events; any event with an (E² ⊗ R) value outside of this window is rejected. Figure 3.7 shows a typical E² ⊗ R plot for one of the runs.

The cut in mass, separating the 14O focal plane events from the 15O and 16O events, is obtained by drawing a gate around the 14O events on a 2-dimensional histogram of Position vs. Energy (P ⊗ E) and rejecting events outside this window. Figure 3.8 shows a typical P ⊗ E spectrum. The elliptical shape of the 14O group in the P ⊗ E plot is due to the placement of the focal plane detector slightly in front of the focal plane. Recall that the target position was moved upstream from the spectrograph object position to accommodate the NaI array, resulting in a translation of the focal plane. Particles entering the spectrograph at slightly different angles θ (and therefore different energies), which are bent to the same position on the focal plane, have slightly different positions at our detector location. The effect is noticeable both because the measurement is made at 0° and because dE/dθ is so large (~ 12 MeV / degree) for 1H(14N,14O)n reaction.

A 2-dimensional histogram of Position vs. time of flight, or momentum vs. detector entrance angle, is made for the remaining 14O events and is shown in Figure 3.9. To understand the shape of this distribution, consider the following aspects of the kinematics of the 1H(14N, 14O) n reaction (Figure 3.2). The 14O particles formed directly are emitted within a cone of laboratory opening angle θ3L = ± 30°, centered on the beam axis, while the opening angle for the 14O₁'s is approximately ± 10°. The subscript 3 in θ3L here denotes the 14O particle, and the L denotes the laboratory frame. The recoiling particles are emitted with an isotropic distribution in the azimuthal angle φ3L. Figure 3.2 also shows that for each angle
\( \Theta_{3L} \) (except the maximum angles) there are two energies of emitted \(^{14}\text{O}\) particles. For the \(^{14}\text{O}\)'s formed directly, the maximum angle is \((\Theta_{3L})_{\text{max}} = 2.89^\circ\), corresponding to \(\Theta_{\text{cm}} = 94^\circ\); for the excited state ellipse, the maximum angle is \((\Theta_{3L})_{\text{max}} = 0.97^\circ\), corresponding to \(\Theta_{\text{cm}} = 113^\circ\). There are both "forward" and "backward" solutions of the kinematics for both the \(^{14}\text{O}\)'s formed directly and those formed after \(\gamma\)-decay.

Secondly, a geometrical discussion is necessary to relate the observed distribution, Figure 3.9, to the kinematics, Figure 3.2. It is important to note that the angle plotted in Figure 3.9, \(\Theta_x\), is not the same angle plotted in the kinematics of the reaction, \(\Theta_{3L}\), in Figure 3.2. Rather, \(\Theta_x\) is the projection of the polar angle \(\Theta_{3L}\) onto the horizontal plane, so that

\[
\tan(\Theta_{3L}) \sin(\phi_{3L}) = \tan(\Theta_x) \tag{3.1}
\]

This is shown schematically in Figure 3.10. The two angles \(\Theta_x\) and \(\Theta_{3L}\) are equal at \(\phi_{3L} = 90^\circ\), in the horizontal plane; off the horizontal plane, \(\Theta_{3L} > \Theta_x\). If the entire \(3^\circ\) cone of particles were accepted into the spectrograph, then a plot of lab momentum \(P_{3L} \cos \Theta_x\) would appear as a solid (filled) ellipse for both the \(^{14}\text{O}\)'s formed directly (the "ground state ellipse") and the \(^{14}\text{O}\)'s formed after \(\gamma\)-decay (the "excited state ellipse"). If this were the case, the ground state ellipse would completely cover the excited state ellipse, since the \(^{14}\text{O}\)'s formed directly occur \(\sim 10^5\) times more often than those formed after \(\gamma\)-decay; this would make this measurement impossible. The presence of a rectangular aperture at the entrance of the magnetic spectrograph breaks the \(\phi_{3L}\) symmetry: the horizontal acceptance angle is decreased slightly to \(\pm 2.43^\circ\), but the vertical acceptance is decreased by more than 2/3, to \(0.74^\circ\). This aperture has an 80% acceptance of the excited state group with its opening angle of \(\sim 1^\circ\). However, the aperture permits only \(\sim 25\%\) of the \(^{14}\text{O}\)'s formed directly due to their \(\pm 3^\circ\) opening angle. Those \(^{14}\text{O}\)'s formed directly with a large vertical angle \(\Theta_y\) are not accepted into the spectrograph. The effect of the aperture is shown schematically in Figure 3.10, and has been verified by transforming the \(^{14}\text{N}(p,n)_0 \))\(^{14}\text{O}\) cross-sections, measured at Seattle and described in Chapter 4, into the \(^1\text{H}(^{14}\text{N},^{14}\text{O})\) n laboratory frame and integrating
over the solid angle of the aperture. The transformation shows that the portion of the ground state group passed by the aperture has an annular shape, as observed in the spectrum shown in Figure 3.9, and also determines the precise percentages of the ground state and excited state groups accepted into the spectrograph. Most importantly, the excited state ellipse lies in the central location of the ground state annulus, thus allowing the rare excited state events to be seen along with the intense ground state events. For the typical run shown in Figure 3.9, the statistics are too low to allow the excited state group to be seen; the ground state annulus is visible along with 2 groups from $^{12}\text{C}$ ($^{14}\text{N}, ^{14}\text{O}$) $^{12}\text{B}$. Comparing Figure 3.10 with 3.9, it is evident that the largest angles $\theta_x$ (and thus the largest angles $\Theta_{3L}$ when $\phi_{3L} = 90^\circ$) are cut off of our spectra, dividing our ground state annulus into two "lobes." This is caused by the horizontal opening angle of the spectrograph aperture being $2.43^\circ$ instead of $3^\circ$. This maximum angle determines that the "forward kinematic solution" lobe has a $0^\circ - 60^\circ$ range in $\Theta_{cm}$, while the "backward" lobe has a $125^\circ - 180^\circ$ range in $\Theta_{cm}$.

In the $P_{3L} \otimes \Theta_x$ plot, a software window is drawn around that part of the ground state annulus corresponding to $\Theta_{cm} < 60^\circ$, and to horizontal angles $\Theta_x > 0^\circ$. A second window is drawn inside the ground state annulus which contains the excited state group for horizontal angles $\Theta_x > 0^\circ$. These windows were drawn to include only positive angles $\Theta_x$ in order to avoid a small leakage of $^{15}\text{O}$ events in the $\Theta_x < 0^\circ$ range. As will be discussed later, the use of the full angular range in $\Theta_x$ gives results which are consistent with the use of just the positive $\Theta_x$ angles, but with a larger uncertainty. Only the forward solutions $\Theta_{cm} < 90^\circ$ were included because the $^{14}\text{N} \ (p, n)^{14}\text{O}_{0,1}$ cross sections, used to convert the ATLAS results to an $\Gamma_\gamma / \Gamma$ value, could only be measured out for $\Theta_{cm} < 90^\circ$, as discussed in Chapter 4.

The position spectrum, Figure 3.11.a, is gated on these two windows, giving the momentum spectra for the ground state and excited state events; the latter is shown in Figure 3.11.b. The first step in the identification of the peaks in these spectra is to make a momentum calibration (momentum vs.position channel) of the focal plane. This calibration is determined
from the location of $^{14}\text{O}$ and $^{15}\text{O}$ groups seen in the CD$_2$ target data. The momenta, calculated with a relativistic kinematics program, and observed positions of the calibration groups are listed in Table 3.2, along with the resulting calibration. Two calibrations are shown in Figure 3.12: the magnetic field of the spectrograph was shifted for one run to bring two $^2\text{H}(^{14}\text{N}, ^{15}\text{O}^*)\text{n}$ groups onto the focal plane, aiding in the identification of all of the groups. When combined with our momentum calibration, the identification of peaks in the position spectra 3.11 is made. Groups are seen from the $^{12}\text{C}(^{14}\text{N}, ^{14}\text{O}_0)^{12}\text{B}$ and $^{16}\text{O}(^{14}\text{N}, ^{14}\text{O}_0)^{16}\text{N}$ reactions in Figure 3.11.b, and are listed in Table 3.3 with their momenta and observed and predicted channels. There is little available information concerning the relative strengths expected for populating the $^{12}\text{B}$ and $^{16}\text{N}$ levels with a heavy-ion charge-exchange reaction like the $(^4\text{N}, ^4\text{O})$ reaction. A study of the $^{16}\text{O}(^7\text{Li}, ^7\text{Be})^{16}\text{N}$ reaction at 7 MeV/u (Co84) indicates preferential population for the 0.0, 0.297, 3.96, 5.52, and 5.72 levels in $^{16}\text{N}$. For the $^{12}\text{B}$ states, a study of the $^{12}\text{C}(^{12}\text{C}, ^{12}\text{N})^{12}\text{B}$ reaction at 35 MeV/u indicates preferential population for the 0.0 and 4.52 levels (Wi86b), consistent with our observations. This was confirmed by a study at 30 MeV/u (Wa87) with a $0^\circ$ measurement with a magnetic spectrograph, observing a momentum vs. angle distribution very similar to Figure 3.9.

There are a number of effects which prevent these $^{14}\text{O}$ groups corresponding to residual states in $^{12}\text{B}$ and $^{16}\text{N}$ from being separated at the focal plane. The target thickness (850 $\mu$g/cm$^2$ CH$_2$) contributes most of the width, due to the differing energy losses of the entering $^{14}\text{N}$ and the exiting $^{14}\text{O}$ particles in the target material. Reactions occurring at the front of the target (facing the beam) have no $^{14}\text{N}$ beam energy loss in the target, but the exiting $^{14}\text{O}$'s lose energy through the entire thickness of the target. Alternatively, reactions occurring on the back of the target have the $^{14}\text{N}$ beam losing energy through the entire thickness of the target and no energy loss of the exiting $^{14}\text{O}$'s. For 175-MeV incident $^{14}\text{N}$'s and 155-Mev exiting $^{14}\text{O}$'s, the energy difference of the $^{14}\text{O}$'s leaving the target in the two cases is $\sim$ 800 -
1000 keV (10 - 12 channels). Since the level spacings in $^{16}$N and $^{12}$B are less than 1 MeV, this target thickness effect causes the peaks to overlap.

A second effect smearing out the peaks on the focal plane is caused by the variation of particle energy with angle, as evidenced in Figure 3.9: the magnetic spectrograph essentially views an angular distribution of the $^{16}$N, $^{12}$B, and $^{1}$H groups. Angular distribution effects add essentially no width to the $^{16}$N and $^{12}$B peaks, since $dE/d\Theta$ is very small for both these reactions at $\Theta = 0^\circ - 2.5^\circ$. However, it is the major component of the width of the $^{14}$O$_{0}$ and $^{14}$O$_{1}(\gamma)^{14}$O$_{0}$ groups, since $dE/d\Theta$ is so large (~12 MeV/degree). The result is a spreading of the forward lobe ($\Theta_{cm} < 90^\circ$) of those $^{14}$O$_{0}$ formed directly over 200 channels (17 MeV), and the forward lobe of those $^{14}$O$_{0}$ formed after $\gamma$ - decay over 60 channels (5 MeV.) The intrinsic energy widths of the individual states adds a negligible contribution to the focal plane widths of any of the observed groups. A list of these focal plane width effects for the different states is given in Table 3.4.

A comparison of CH$_{2}$ and C target runs is shown in Figure 3.13 to help determine the events due to reactions involving the hydrogen in the CH$_{2}$ target. Any events corresponding to those $^{14}$O$_{0}$ formed after $\gamma$ - decay will be located on the high momentum shoulder of the $^{12}$B$_{0}$ group in the CH$_{2}$ target runs. The expected shape of the $^{14}$O$_{1}(\gamma)^{14}$O$_{0}$ group on the focal plane, shown in Figure 3.14, is determined by transforming the $^{14}$N (p, n)$_{1}^{14}$O cross-section to the $^{1}$H($^{14}$N, $^{14}$O) n laboratory frame, changing coordinates from ($\Theta_{3L}, \phi_{3L}$) to ($P_{3L}, \Theta_{X}$), and finally integrating over $\Theta_{X}$ to get a $P_{3L}$ distribution. A software gate at $\Theta_{cm} = 40^\circ$ ($\Theta_{3L} = 0.6^\circ, \Theta(p,n) = 39^\circ$) is also shown in Figure 3.14, roughly corresponding to the focal plane position where the $^{14}$O$_{1}(\gamma)^{14}$O$_{0}$ group becomes totally masked by the $^{12}$B$_{0}$ group. This distribution will be discussed in detail in Chapter 4.

The number of events due to $^{14}$O$_{1}$ $\gamma$ - decay is determined by two methods. The first involves appropriately subtracting the carbon target momentum spectrum from the CH$_{2}$ target momentum spectrum. More specifically, the carbon target momentum spectrum shown in
Figure 3.13 is scaled up by the ratio of areas of the $^{12}\text{B}(4.52\text{ MeV})$ group in the CH$_2$ and C target data sets, then shifted up to match the background level of the CH$_2$ target spectrum. This new spectrum is subtracted from the CH$_2$ target spectrum, resulting in the momentum spectrum shown in Figure 3.15. Scaling up by the ratio of the carbon peaks accounts for the different target thickness and integrated beam current for the CH$_2$ and C data sets, and shifting to match the background levels accounts for the differences in the leakage of the ground state annulus in the $P_{3L} \otimes \Theta_x$ plot into the gate on the central region. Note that the hydrogen content of the carbon target is small but non-zero, and hence there is a ground state annulus in the carbon target data set, down in intensity by a factor of $\sim 13$ from the CH$_2$ target. There is a peak in the subtracted spectrum at channels 985-1000 which, according to the momentum calibration, corresponds to the $^1\text{H}(^{14}\text{N}, ^{14}\text{O}_1) n (\gamma ^{14}\text{O}_o$ group. The area of the group is 45 ± 24 counts, the uncertainty due to counting statistics and an uncertainty of ±2 counts/channel in the shift of the Carbon backgrounds (to match the CH$_2$ background) over a 12 channel range. When a similar procedure is carried out with the full angular range in $\Theta_x$ (not just $\Theta_x \geq 0$), the result is 105 ± 40 counts; this is also shown in Figure 3.15. In order to be certain that this peak is due to $^{14}\text{O}_1(\gamma ^{14}\text{O}_o$, it is necessary to understand all of the peaks in the subtracted momentum spectrum.

Two refinements on the above subtraction method may be made to clean up the resulting momentum spectrum. First, small periodic peaks with a spacing of approximately 5 channels are evident on both the ungated, gated, and subtracted position spectra. This effect is due to the discrete wire structure in the PPAC visible when running at high event rates ($> 1 \text{ kHz}$) (Re90). When the event rate is very high, the ionization trail from some incident particles may not produce a strong avalanche to the PPAC wires due to a general space charge effect of the positive ions created in the gas (Kn79). A cubic spline smoothing routine was used to correct the spectrum for this periodic structure; the algorithm preserved the area to better than 1.5%.
A second refinement concerns the different widths for the $^{12}\text{B}$ groups in the CH$_2$ and C target momentum spectra. The different peak widths are due to the different target thicknesses: the 850 $\mu$g/cm$^2$ of CH$_2$ and the 622 $\mu$g/cm$^2$ of C contribute approximately 10 channels (800 keV) and 6 channels (500 keV) width, respectively, to the 20 and 18 channel widths of the $^{12}\text{B}(4.52\text{ MeV})$ state. As discussed above, the different energy loss of $^{14}\text{N}$ and $^{14}\text{O}$ in the targets are the cause of this width on the focal plane. A raw subtraction of two gaussians of different widths results in three peaks of alternating polarity which are labeled as "overshoot" in Figure 3.15. This target thickness effect can be removed by using a gaussian smoothing routine, which symmetrically spreads the counts in any one position channel into a gaussian distribution over $T$ channels. The width $T$ was chosen to be 10 channels to increase the measured 18 channel width of the $^{12}\text{B}(4.52\text{ MeV})$ peak in the C target spectrum to 20 channels in the CH$_2$ target spectrum. Figure 3.16 shows the results of (a) cubic spline and gaussian smoothing both C and CH$_2$ spectra, (b) scaling the C spectrum up to match the CH$_2$ spectrum, and (c) subtracting the two. These two refinements have eliminated a number of smaller peaks, but has not changed the area of the peak located at channel 985-1000.

The remaining peaks in the subtracted momentum spectrum are labeled in Figure 3.16. There are three peaks due to $^{16}\text{O}(^{14}\text{N}, ^{14}\text{O})^{16}\text{N}$ groups located at channels 956, 963, and 1024. Because of the different oxygen contamination amounts in the two targets, and because the subtraction procedure accounts only for the different amounts of carbon, these groups will not be vanish upon subtraction. Part of the peak at channel 957 is due to an intense group of $^{16}\text{O}$ events on the focal plane. The intensity of this group leads to spurious energy signals, such that some of these events fall within the $^{14}\text{O}$ window on the P $\otimes$ E plot. This is evident by a vertical "streaking" on the P $\otimes$ E plot (a range of energy values at one position value) from this $^{16}\text{O}$ group. There is no such $^{16}\text{O}$ streaking between channels 980 and 1005, the region of interest for the $^1\text{H}(^{14}\text{N}, ^{14}\text{O})\gamma$ reaction, but a small group at channel 1005-1010. This was checked by gating on $^{16}\text{O}$ events in the P $\otimes$ E plot, keeping the same gate on the $P_{3L}$ $\otimes$ $\Theta_X$ plot, and looking at the resulting gated position spectrum.
There is a second problem with "streaking", evident in Figure 3.8, this one due to the intense $^{15}$O group which streaks in position (one energy value, many position values) into the $^{14}$O window. When gating simultaneously on $^{14}$O and $^{15}$O in the P $\otimes$ E plot, an examination of the $P_{3L}$ $\otimes$ $\Theta_x$ plot shows that the $^{15}$O events are primarily in the negative $\Theta_x$ region. This was the motivation for considering only the positive $\Theta_x$ angles in the subtracted momentum spectrum in Figure 3.15. The consistency of these two angular ranges is simply determined by comparing the ratio of the number of events in the $^{14}$O$_1(\gamma)^{14}$O$_o$ peak to the number of events in ground state annulus. The results are:

\[
0 \leq \Theta_x \leq 2.43^\circ : \frac{(45 \pm 24)}{(3,896,000 \pm 2000)} = (1.16 \pm 0.62) \cdot 10^{-5} \tag{3.2}
\]

\[-2.43^\circ \leq \Theta_x \leq 2.43^\circ : \frac{(105 \pm 40)}{(7,065,000 \pm 3000)} = (1.48 \pm 0.59) \cdot 10^{-5} \]

The fact that the results with the entire $\Theta_x$ angular range are consistent with that of the positive $\Theta_x$ angular range implies that the $^{15}$O streaking is not a significant problem. Since the full-angular range may contain some contributions from $^{15}$O, the cleaner result from the positive angular range will be used.

The second method of determining the number of $^{14}$O$_1(\gamma)^{14}$O$_o$ events involves fitting the peaks in the $\text{CH}_2$ target momentum spectra and extracting the number of events from the area of the $^1\text{H}(^{14}\text{N}, ^{14}\text{O}_1) n \gamma$ peak. The first constraint on the fit is the location of the peak centroids, obtained from the momentum calibration (Table 3.2.) The next most stringent constraint on the fit is the requirement that the ratio among the areas under the $^{12}$B peaks be the same in both the $\text{CH}_2$ and C targets. In particular, the ratio of areas of the 4.52 and 0.00 MeV levels in $^{12}$B in the $\text{CH}_2$ target should be 2.5, the value obtained from the Carbon target momentum spectrum. This ratio of areas must also be consistent with both the full and positive angular ranges in $\Theta_x$. Consistency also demands that the ratio of $^{14}$O$_1(\gamma)^{14}$O$_o$ events to $^{14}$O$_o$'s formed directly be the same in both targets, for both angular ranges. Thus the ratio of $^{14}$O$_1(\gamma)^{14}$O$_o$ events in the two target data sets must equal the ratio of $^{14}$O$_o$ events, which is
approximately 12.5 for the positive $\Theta_\chi$ angular range and 13.0 for the full $\Theta_\chi$ angular range. Since we are observing roughly 50 - 100 $^{14}\text{O}(\gamma)^{14}\text{O}$ events in the CH$_2$ target spectra, this means that only 4 - 10 such counts should be observed in the Carbon target spectra, too small to be observed; thus this is only a weak constraint on the fit. A stronger constraint concerns the peak widths: the difference in width of the relatively well isolated $^{12}\text{B}(4.52 \text{ MeV})$ peak, 18 and 20 channels in the C and CH$_2$ target spectra, is attributable to different Carbon and CH$_2$ target thicknesses. Since the $^{12}\text{B}_0$ peak is measured to have a width of 16 channels in the C target spectrum, this constrains the width of the $^{12}\text{B}_0$ peak in the CH$_2$ spectrum to be 18 channels.

A quadratic background with an additional exponential tail from the $\Theta_{cm} = 0^\circ$ $^{14}\text{O}$ group was employed for both fits. The two most critical regions for the fit are the $^{12}\text{B}(4.52 \text{ MeV})$ peak and the $^{12}\text{B}_0$ peak with its high momentum - shoulder of $^{14}\text{O}(\gamma)^{14}\text{O}$. The $^{12}\text{B}(4.52 \text{ MeV})$ peak is relatively well isolated (there is a small shoulder due to the $^{12}\text{B}(3.79 \text{ MeV})$ group), allowing reasonably precise determinations of its area and width. This information, along with the constraint on the ratio of areas mentioned above, allows a good fit of the $^{12}\text{B}_0$ group, which has its low - momentum side obscured by the $^{12}\text{B}(0.95 \text{ MeV})$ and $^{16}\text{N}(3.36 \text{ MeV})$ groups. Once the $^{12}\text{B}_0$ group is fit, the area comprising the high - momentum shoulder can be extracted as the difference between the data and the $^{12}\text{B}_0$ peak. Second, the intermediate regions in the momentum spectrum, from channels 945 - 980, contains more than enough $^{12}\text{B}$ and $^{16}\text{N}$ groups to obtain a good fit: there are fifteen $^{16}\text{N}$ levels in this region, with excitation energies ranging from 3.36 to 5.52 MeV, and three $^{12}\text{B}$ levels (0.95, 1.67, and 2.62 MeV.) As mentioned above, the relative strengths of these various levels are not well known. Rough constraints from $^{16}\text{O}(^7\text{Li}, ^7\text{Be})^{16}\text{N}$ (Co84) and $^{12}\text{C}(^{12}\text{C}, ^{12}\text{N})^{12}\text{B}$ (Wi86b) were used both to get a preliminary identification of the peaks in our momentum spectrum, and to roughly set the areas of the gaussian peaks used in the fit.

As a result of the considerations discussed above, a preliminary fit of the momentum spectrum is generated. The peak shapes are taken to be gaussian due to the major contribution
of energy loss effects in determining the peak widths. Variations of the fit parameters to
determine the best value for the area of the $^{14}\text{O}_1(\gamma)^{14}\text{O}_0$ group were made in two ways. First,
the area of the $^{14}\text{O}_1(\gamma)^{14}\text{O}_0$ group was set to zero, and the area of the $^{12}\text{B}_0$ peak was varied
to obtain the minimum $\chi^2$ value of the fit when compared to the data over the high-momentum
side of the $^{12}\text{B}_0$ peak, channels 978 - 1000. The resulting $^{12}\text{B}_0$ peak (plus background) was
subtracted from the data, resulting in a group between channels 985 - 1000. The area of this
peak, $49 \pm 10$, is attributed to $^{14}\text{O}_1(\gamma)^{14}\text{O}_0$ events. The uncertainty is determined from the
maximum and minimum values of the $^{12}\text{B}_0$ peak which increase the $\chi^2$ value by 1.00
(Bev69). The second method approximates the shape of the $^{14}\text{O}_1(\gamma)^{14}\text{O}_0$ peak to be
gaussian, even though this is shown in Figure 3.14 not to be the actual shape, and varies the
areas of both the $^{12}\text{B}_0$ and $^{14}\text{O}_1(\gamma)^{14}\text{O}_0$ peaks to search for the minimum $\chi^2$ value. This
method gave $45 \pm 20$ counts for $^{14}\text{O}_1(\gamma)^{14}\text{O}_0$ peak, consistent with the results of the first
method. A list of fitting attempts and their results is given in Table 3.5, for both the positive
and full $\Theta_x$ angular ranges, and for smoothed and unsmoothed data sets. A representative best
fit to the CH$_2$ spectrum is shown in Figure 3.17. The two methods of fitting the momentum
spectrum and the subtracting the C and CH$_2$ momentum spectra give consistent results for the
ratio of $^{14}\text{O}_0$ particles formed after $\gamma$ - decay to those formed directly. These results will be
used in Section 4.5 to determine the branching ratio of the $^{14}\text{O}^*(5.169\text{-MeV})$ level.
Figure 3.1.
Level diagram for $^{14}\text{O}$ (Aj81) and previously measured parameters for the 540-keV $^{13}\text{N} + p$ resonance (Ch85). Energies are in MeV unless otherwise noted.
$E_{x} = 5168 \pm 1.8 \text{ keV}$
$\Gamma = 38.1 \pm 1.8 \text{ keV}$
$\ell = 0$
$E_{cm} = 540.5 \pm 2 \text{ keV}$

4.628 MeV

Level Structure of $^{14}\text{O}$

$^{13}\text{N} + p$

<table>
<thead>
<tr>
<th>Energy</th>
<th>Quantum Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.590</td>
<td>2$^+$</td>
</tr>
<tr>
<td>6.270</td>
<td>3$^-$</td>
</tr>
<tr>
<td>5.920</td>
<td>0$^+$</td>
</tr>
<tr>
<td>5.169</td>
<td>1$^-$</td>
</tr>
</tbody>
</table>

$0.0 \text{ MeV}$

$^{14}\text{O}$
Table 3.1.
The three previous measurements of $\Gamma_\gamma / \Gamma$
((Wa87), (Fe89), and (Ag89)). Two are
upper limits, and Fernandez et al. (Fe89) has
a measurement with a 50 % uncertainty.
Also listed are several theoretical predictions
for $\Gamma_\gamma$. 
### Experimental Measurements

<table>
<thead>
<tr>
<th>Group</th>
<th>Reaction</th>
<th>$\Gamma_\gamma/\Gamma_{\text{tot}}$</th>
<th>$\Gamma_\gamma$ (eV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wang et al.</td>
<td>$^{14}\text{N}(^{3}\text{He}, t)^{14}\text{O}^*(\gamma)^{14}\text{O}_o$</td>
<td>$\leq 4 \cdot 10^{-4}$</td>
<td>$\leq 17$</td>
</tr>
<tr>
<td>Yale/Princeton 1986</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Wa86)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fernandez et al.</td>
<td>$^{12}\text{C}(^{3}\text{He}, n)^{14}\text{O}^*(\gamma)^{14}\text{O}_o$</td>
<td>$(7.2 \pm 3.5) \cdot 10^{-5}$</td>
<td>$2.7 \pm 1.3$</td>
</tr>
<tr>
<td>Seattle 1989</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Fe89)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Aguer et al.</td>
<td>$^{12}\text{C}(^{3}\text{He}, n)^{14}\text{O}^*(\gamma)^{14}\text{O}_o$</td>
<td>$&lt; 3 \cdot 10^{-4}$ (a)</td>
<td>$&lt; 11.4$</td>
</tr>
<tr>
<td>Strasbourg 1988</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Ag89)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Theoretical Estimates

<table>
<thead>
<tr>
<th></th>
<th>$\Gamma_\gamma$ (eV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mathews and Dietrich (Ma84)</td>
<td>2.44</td>
</tr>
<tr>
<td>Barker (Ba85)</td>
<td>1.2</td>
</tr>
<tr>
<td>Langanke, van Roosmalen, and Fowler (La85)</td>
<td>1.50</td>
</tr>
<tr>
<td>Funck and Langanke (Fu87)</td>
<td>1.8</td>
</tr>
<tr>
<td>Descouvemont and Baye (De89)</td>
<td>4.1</td>
</tr>
</tbody>
</table>

(a) Revised (Ro90) from a value of $(2 \pm 1) \cdot 10^{-4}$ given in (Ag89).
Figure 3.2.

Energy of the emitted $^{14}$O particle as a function of laboratory emission angle for the $^1\text{H}(^{14}\text{N}, ^{14}\text{O})\;\text{n}$ reaction. The $^{14}$O's from the $^1\text{H}(^{14}\text{N}, ^{14}\text{O}_0)\;\text{n}$ reaction are within a cone of ±3° about 0°, while those from $^1\text{H}(^{14}\text{N}, ^{14}\text{O}_1)\;\text{n}$ are within a cone of ±1° about 0°.
Figure 3.3.

Target Chamber, Split Pole Spectrograph, and focal plane detector at Argonne National Laboratory ATLAS facility, used in the $^{1}H(^{14}N, ^{14}O) n$ experiment.
240 nS  4.17 MHz

Target  CH₂ Polypropylene
850 µg/cm²

Focal Plane Detector

Split Pole Spectrograph
at 0°

Neutron Detector

Neutron Detector

Movable Faraday Cup

Movable Shield

Focal Plane Detector
Figure 3.4.

Schematic of the Parallel Plate Avalanche Counter (PPAC) and Bragg Curve Detector (BCD) at the focal plane of the Spectrograph (Re88). The top figure schematically shows the delay line in the PPAC, and both indicate the energy loss of the incident heavy ions as they pass through the detector gas.
Parallel Plate Avalanche Counter

5 Torr ISOBUTANE

Bragg Curve Detector
(100 Torr CF$_4$)

5 cm

energy loss dE/dx

Frisch Grid

Anode

Pressure Foil
Cathode Foil
480 Position Wires
Anode Foil
Pressure Foil
Cathode Foil
Figure 3.5.

Determination of Range and Angle of the incident ions by using the drift time in the Bragg Curve Detector (Re88).
\[ D_j = D - R \cos \Theta \]

\[ R = \text{Range} \quad \Theta = \text{incident angle} \]

\[ D_i (E) = \text{const} - \frac{E^2}{mZ^2} \]

\[ E = \text{Energy} \]
Figure 3.6.

Schematic of the electronics for the $^1\text{H}^{(14\text{N}, 14\text{O}) \, \gamma}$ experiment (Re88).

PA = Preamplifier
FP = Fast Preamplifier
FA = Fast Linear amplifier
MA = Main Linear amplifier
CFD = Constant Fraction Discriminator
TSC = Timing Single Channel Analyzer
TAC = Time to Amplitude Converter
TDC = Time to Digital Converter
ADC = Amplitude to Digital Converter.
Position Timing signals

To ADCs

Diagram showing electrical connections and signals for high and low energy levels.
Figure 3.7.

$(\text{Energy})^2$ vs. Range plot $(E^2 \otimes R)$ for the $^1\text{H}(^{14}\text{N}, ^{14}\text{O})n$ reaction, allowing the separation of oxygen, carbon, and nitrogen groups on the focal plane. The third dimension of the plot is logarithmic in intensity.
Range

ESQ_RANGE  Sum 3,706,270

Energy^2

Carbon  Oxygen  Nitrogen
Figure 3.3.

Position vs. Energy plot (P ∇ E) for the $^{1}H(^{14}N, ^{14}O) n$ reaction, allowing the separation of $^{14}O$, $^{15}O$, and $^{16}O$ groups on the focal plane. The third dimension of the plot is logarithmic in intensity.
Focal Plane Position vs. Time Of Flight plot, equivalent to Momentum vs. Angle ($P_{3L} \otimes \Theta_x$) for the $^{14}O$ particles. The outer annular shaped group is due to $^{14}O$'s formed directly, and two groups from $^{12}C(^{14}N, ^{14}O)^{12}B$ are visible. The group due to $^{14}O_1(\gamma)^{14}O_0$ is inside the annulus but is not intense enough to be seen on this plot. The third dimension of the plot is logarithmic in intensity.
Figure 3.10.
Schematic of the effect of the spectrograph aperture on the focal plane distribution of $^{14}$O$_6$'s formed directly and formed after $\gamma$-decay of the 5.17-MeV level. Without an aperture present, an $E$ vs. $\Theta_x$ plot (or $P_{3L}$ vs. $\Theta_x$ plot) is a filled-in ellipse; with an aperture, the distribution has an annular shape.
Kinematics of $^1\text{H}(^{14}\text{N},^{14}\text{O})\text{n}$

Angle $\Theta_{3L}$

$\tan \theta_X = \tan \Theta_{3L} \sin \phi_{3L}$

$\theta_X = \text{Angle in Horizontal Plane (Detector Angle)}$

Particle Distribution Incident on Detector with no Aperture

Particle Distribution Incident on Detector with Aperture

Aperture excludes outer regions

$\Theta_X$ vs $E$ ellipse is filled in

$\Theta_X$ vs $E$ ellipse is not filled in

Annular Shaped
Figure 3.11.a.

$^{14}\text{O}$ Momentum spectrum for the $^{1}\text{H}(^{14}\text{N}, ^{14}\text{O})\ n$ reaction. The two large peaks at the ends of the spectrum are the $\Theta_{\text{cms}} = 0^\circ$ and $\Theta_{\text{cms}} = 180^\circ$ groups from the $^{1}\text{H}(^{14}\text{N}, ^{14}\text{O})\ n$ reaction.
$^{1}H (^{14}N, ^{14}O_{0})_{n}$  $\Theta_{cms} = 0^\circ$

$^{1}H (^{14}N, ^{14}O_{0})_{n}$  $\Theta_{cms} = 180^\circ$
Figure 3.11.b.

$^{14}\text{O}$ Momentum spectrum for the

$^1\text{H}(^{14}\text{N}, ^{14}\text{O})\ n$ reaction, gated on the

interior of the $^1\text{H}(^{14}\text{N}, ^{14}\text{O}_o)$ annulus in

the Position vs. TOF spectrum, Figure 3.9.
Position Spectrum Gated on the Excited State region of the Position-TOF plot

Counts/Channel

Focal Plane Position (Channel)
Table 3.2.

Calibration of the focal plane for the $^1H(^{14}N, ^{14}O) n$ experiment with momentum spectra taken with CD$_2$ targets at two different magnetic fields.
<table>
<thead>
<tr>
<th>Reaction</th>
<th>Momentum (MeV/c²)</th>
<th>Channel Run060</th>
<th>Channel Run059</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{( ^2)H( ^{14})N,( ^{14})O) 2n} )</td>
<td>2079</td>
<td>1107</td>
<td>916</td>
</tr>
<tr>
<td>( \text{( ^1)H( ^{14})N,( ^{14})O) n (( \Theta_{\text{cms}} = 0^\circ )} )</td>
<td>2091</td>
<td>1140</td>
<td>946</td>
</tr>
<tr>
<td>( \text{( ^{12})C( ^{14})N,( ^{15})O*(6.176))} \text{( ^{11})B} )</td>
<td>2111</td>
<td>1167</td>
<td>967</td>
</tr>
<tr>
<td>( \text{( ^{12})C( ^{14})N,( ^{15})O} \text{( ^{11})B*(2.125)} )</td>
<td>2138</td>
<td>1228</td>
<td>1032</td>
</tr>
<tr>
<td>( \text{( ^{12})C( ^{14})N,( ^{15})O} \text{( ^{11})B}_0 )</td>
<td>2152</td>
<td>1258</td>
<td>1057</td>
</tr>
<tr>
<td>( \text{( ^1)H( ^{14})N,( ^{14})O) n (( \Theta_{\text{cms}} = 180^\circ )} )</td>
<td>1883</td>
<td>744</td>
<td></td>
</tr>
<tr>
<td>( \text{( ^2)H( ^{14})N,( ^{15})O(6.739)) n} )</td>
<td>2187</td>
<td>1121</td>
<td></td>
</tr>
<tr>
<td>( \text{( ^2)H( ^{14})N,( ^{15})O} \text{( ^{15})O n} )</td>
<td>2217</td>
<td>1168</td>
<td></td>
</tr>
</tbody>
</table>

Run059 Calibration: \( P_{3L} \text{ (MeV/c}^2\text{)} = 1580 + 0.5428 \cdot x \)

Run060 Calibration: \( P_{3L} \text{ (MeV/c}^2\text{)} = 1492 + 0.5264 \cdot x \)
Figure 3.12.

Focal Plane Position vs. Momentum calibration, shown for data taken at two magnetic field settings with a CD$_2$ target.
Focal Plane Calibration CD$_2$ Target

Run 060 $B = 10.357$ kG

Momentum = 1492.3 + 0.52643 $\cdot$ Channel

Run 059 $B = 10.871$ kG

Momentum = 1580.1 + 0.5427 $\cdot$ Channel
The momentum and channels (from the calibration) of the $^{12}\text{C}(^{14}\text{N},^{14}\text{O})^{12}\text{B}$, $^{16}\text{O}(^{14}\text{N},^{14}\text{O})^{16}\text{N}$, and $^{1}\text{H}(^{14}\text{N},^{14}\text{O})$ n focal plane groups.

Table 3.3.
<table>
<thead>
<tr>
<th>Reaction</th>
<th>Momentum (MeV/c²)</th>
<th>Channel</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{12}\text{C}(^{14}\text{N},^{14}\text{O})^{12}\text{B}_0$</td>
<td>2009</td>
<td>980</td>
</tr>
<tr>
<td>$^{12}\text{C}(^{14}\text{N},^{14}\text{O})^{12}\text{B}^*(0.953)$</td>
<td>2003</td>
<td>967</td>
</tr>
<tr>
<td>$^{12}\text{C}(^{14}\text{N},^{14}\text{O})^{12}\text{B}^*(1.674)$</td>
<td>1998</td>
<td>957</td>
</tr>
<tr>
<td>$^{12}\text{C}(^{14}\text{N},^{14}\text{O})^{12}\text{B}^*(3.759)$</td>
<td>1982</td>
<td>928</td>
</tr>
<tr>
<td>$^{12}\text{C}(^{14}\text{N},^{14}\text{O})^{12}\text{B}^*(4.518)$</td>
<td>1977</td>
<td>918</td>
</tr>
<tr>
<td>$^{16}\text{O}(^{14}\text{N},^{14}\text{O})^{16}\text{N}_0$</td>
<td>2031</td>
<td>1021</td>
</tr>
<tr>
<td>$^{16}\text{O}(^{14}\text{N},^{14}\text{O})^{16}\text{N}^*(3.96)$</td>
<td>2003</td>
<td>968</td>
</tr>
<tr>
<td>$^{16}\text{O}(^{14}\text{N},^{14}\text{O})^{16}\text{N}^*(5.52)$</td>
<td>1993</td>
<td>948</td>
</tr>
<tr>
<td>$^{1}\text{H}(^{14}\text{N},^{14}\text{O})\text{n (CMS }= 0^\circ)$</td>
<td>2091</td>
<td>1140</td>
</tr>
<tr>
<td>$^{1}\text{H}(^{14}\text{N},^{14}\text{O})\text{n (CMS }= 0^\circ)$</td>
<td>1883</td>
<td>744</td>
</tr>
<tr>
<td>$^{1}\text{H}(^{14}\text{N},^{14}\text{O})\gamma(^{14}\text{O}_0)\text{(CMS }= 0^\circ)$</td>
<td>2020</td>
<td>1000</td>
</tr>
<tr>
<td>$^{1}\text{H}(^{14}\text{N},^{14}\text{O})\gamma(^{14}\text{O}_0)\text{(CMS }= 40^\circ)$</td>
<td>2012</td>
<td>985</td>
</tr>
</tbody>
</table>
Table 3.4.

Contributions of target thickness, intrinsic widths, and angular distribution effects to the widths in channels of three $^{14}\text{O}$ groups in the focal plane position spectra.
<table>
<thead>
<tr>
<th>Effect</th>
<th>Width in channels</th>
</tr>
</thead>
<tbody>
<tr>
<td>Target thickness</td>
<td>10 10 6</td>
</tr>
<tr>
<td>Intrinsic Width of State</td>
<td>4 1 4</td>
</tr>
<tr>
<td>Angular distribution</td>
<td>1 1 15 (a)</td>
</tr>
<tr>
<td>detector resolution</td>
<td>10 10 10</td>
</tr>
<tr>
<td>Total width</td>
<td>15 14 19</td>
</tr>
<tr>
<td>Observed width</td>
<td>19 17 15 - 20</td>
</tr>
</tbody>
</table>

(a) $0 \leq \Theta_{\text{cms}} < 40^\circ$ for this group
Figure 3.13.
A comparison of $^{14}\text{O}$ Momentum spectrum with CH$_2$ and C targets to indicate the reactions from hydrogen in the CH$_2$ target.
Calculated momentum distribution, in arbitrary units, of the $^1\text{H}(^{14}\text{N}, ^{14}\text{O}_0)\ n$ and $^1\text{H}(^{14}\text{N}, ^{14}\text{O}_1)\ n$ groups on the focal plane from a transformation of the measured $^{14}\text{N}(p, n)^{14}\text{O}_{0,1}$ cross-sections into the $^1\text{H}(^{14}\text{N}, ^{14}\text{O})\ n$ laboratory frame, described in Chapter 4. The shape of the momentum distribution arises from the spectrograph acceptance of $0 < \Theta_{\text{cm}} < 60^\circ$. The jagged edges are due to the finite mesh size used in the calculation. Three cases are shown: with no spectrograph aperture (100% acceptance), with the aperture present but no software cut, and with both aperture and software cuts.
No aperture

With aperture

With aperture and software cut
($\Theta_x \geq 0^\circ, \Theta_{cms} \leq 40^\circ$)

Focal Plane Position
Figure 3.15.
Scaling and subtracting the $^{12}\text{C}$-target $^{14}\text{O}$ momentum spectrum from the CH$_2$ target spectrum.
$^{12}\text{B}(4.52\text{ MeV})$ overshoot $^{16}\text{N}(5.52)$ $^{12}\text{B}(0.00\text{ MeV})$ overshoot $^{16}\text{N}(0.0)$ $^{1}\text{H}(^{14}\text{N},^{14}\text{O})\text{n }\gamma$

$0 \leq \Theta_\chi \leq 2.43^\circ$ $45 \pm 24$ Counts

$-2.43^\circ \leq \Theta_\chi \leq 2.43^\circ$ $105 \pm 40$ Counts

Focal Plane Position (Channel -900)
Figure 3.16.
Scaling and subtracting the $^{12}$C-target
smoothed $^{14}$O momentum spectrum from the
smoothed CH$_2$ target spectrum.
Table 3.5.

Results of fitting the momentum spectra in the $^1H(^{14}N, ^{14}O) n$ experiment with three different methods to obtain the area under the $^1H(^{14}N, ^{14}O) n \gamma(^{14}O) \gamma$ peak.
<table>
<thead>
<tr>
<th>Angular Range</th>
<th>$0^\circ \leq \Theta_x \leq 1^\circ$</th>
<th>$-1^\circ \leq \Theta_x \leq 1^\circ$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Spline Smoothing</strong></td>
<td>No</td>
<td>Yes</td>
</tr>
</tbody>
</table>

**Fitting Method**

- Vary area of $^{12}\text{B}_\text{O}$ peak and subtract best fit from data:
  - $49 \pm 5$  $43 \pm 6$  $96 \pm 8$  $84 \pm 8$

- Vary area of $^{12}\text{B}_\text{O}$ and $^{14}\text{O}_\text{I}$ peaks:
  - $45 \pm 15$  $45 \pm 15$  $84 \pm 20$  $52 \pm 10$

- Vary area and widths of $^{12}\text{B}_\text{O}$ and $^{14}\text{O}_\text{I}$ peaks:
  - $59 \pm 15$  $33 \pm 8$  $52 \pm 15$  $52 \pm 10$

- Comparison with Scaling and subtracting the CH$_2$ and C target data:
  - $45 \pm 24$  $105 \pm 40$
Figure 3.17.

Gaussian fits to the $^{14}\text{O}$ momentum spectrum, showing only three groups for clarity: $^{12}\text{C}(^{14}\text{N}, ^{14}\text{O}) ^{12}\text{B}_{0}$, $^{12}\text{C}(^{14}\text{N}, ^{14}\text{O}) ^{12}\text{B}(4.52 \text{ MeV})$, and $^{1}\text{H}(^{14}\text{N}, ^{14}\text{O}_{1}) \gamma(^{14}\text{O}_{0})$. 
Chapter 4. Measurement of the $^{14}\text{N}(p,n)^{14}\text{O}$ Reaction

4.1 Introduction

As discussed in Chapter 3, the $^{1}\text{H}(^{14}\text{N}, ^{14}\text{O})$ experiment performed at ATLAS gives a measurement of the ratio of those $^{14}\text{O}_0$ formed by $\gamma$-decay of $^{14}\text{O}_1$ to those formed directly, integrated over the spectrograph solid angle and appropriate software cuts. To obtain the $\gamma$-branching ratio, $\Gamma_\gamma / \Gamma_{\text{tot}}$, it is necessary to multiply the measured ratio by the number of $^{14}\text{O}_0 / ^{14}\text{O}_1$ formed, the production cross-section ratio $\sigma_0 / \sigma_1$. To be more explicit, we can define

$$N_1 = \text{number of } ^{14}\text{O}_1 \text{ which } \gamma\text{-decay}$$

$$N_0 = \text{number of } ^{14}\text{O}_0 \text{ which are formed directly}$$

$$(d\sigma/d\Omega)_{0,1} = \text{differential cross-sections of the } ^{1}\text{H}(^{14}\text{N}, ^{14}\text{O}_{0,1}) \text{ reactions}$$

$${\Omega}_{0,1} = \text{solid angle of spectrograph and software cuts}$$

for the $^{1}\text{H}(^{14}\text{N}, ^{14}\text{O}_{0,1})$ reactions.

Then the $^{1}\text{H}(^{14}\text{N}, ^{14}\text{O})$ experiment measures $N_1 / N_0$, which is equal to

$$\frac{N_1}{N_0} = \frac{\int_{\Omega_1} (d\sigma/d\Omega_1) \left(\frac{\Gamma_\gamma}{\Gamma}\right) d\Omega}{\int_{\Omega_0} (d\sigma/d\Omega_0) d\Omega} = \left(\frac{\Gamma_\gamma}{\Gamma}\right) \frac{\int_{\Omega_1} (d\sigma/d\Omega_1) d\Omega}{\int_{\Omega_0} (d\sigma/d\Omega_0) d\Omega}$$  \hspace{1cm} (4.1)

Solving for the branching ratio $\Gamma_\gamma / \Gamma$, we find that

$$\left(\frac{\Gamma_\gamma}{\Gamma}\right) = \frac{\left(\frac{N_1}{N_0}\right) \int_{\Omega_0} (d\sigma/d\Omega_0) d\Omega}{\int_{\Omega_1} (d\sigma/d\Omega_1) d\Omega} = \frac{N_1}{N_0} \frac{\sigma_0}{\sigma_1}$$  \hspace{1cm} (4.2)
where we have defined the cross-section ratio \( \sigma_0 / \sigma_1 \) to be

\[
\frac{\sigma_0}{\sigma_1} = \frac{\int_{\Omega_0} \left( \frac{d\sigma}{d\Omega} \right)_0 d\Omega}{\int_{\Omega_1} \left( \frac{d\sigma}{d\Omega} \right)_1 d\Omega}.
\]

The production cross-sections \( (d\sigma/d\Omega)_{0,1} \) were determined by measuring a neutron time of flight (TOF) angular distribution for the \(^{14}\text{N}(p, n)^{14}\text{O}\) reaction at the University of Washington FN-Tandem Accelerator Facility, at the same center of mass energy (11.67 MeV) as the \(^{1}\text{H}(^{14}\text{N}, ^{14}\text{O}) n\) experiment discussed in Chapter 3. These cross-sections were then transformed into the inverse kinematics \(^{1}\text{H}(^{14}\text{N}, ^{14}\text{O}) n\) laboratory frame and integrated over the spectrograph solid angle to get the "normalization" factor \( \sigma_0 / \sigma_1 \); the transformation will be discussed in detail in Section 4.4.

### 4.2 Experimental Setup

The proton beam for the \(^{14}\text{N}(p, n)^{14}\text{O}\) reaction was produced in an RF ion source, bunched at 4.17 MHz, chopped to a width of 1.2 ns, and then injected into and accelerated to 12.5 MeV by an FN-Tandem Van de Graaff accelerator. The resulting pulsed beam was then momentum analyzed with a 90° bending magnet and routed to the target chamber with a steering magnet; the average beam intensity on target was 1\(\mu\)A. Most of the beam passed through the transmission melamine (\(\text{C}_3\text{H}_6\text{N}_6\)) target, out the back of the target chamber, and down a 5 meter lead-lined beam pipe to a heavily shielded beamstop. In any neutron TOF measurement, all sources of neutrons other than the target must be either eliminated or carefully shielded. These primarily include the neutrons generated when beam particles strike the steering magnet aperture, quadrupole magnet aperture, and collimating slits for the present experiment. These beam line elements were shielded with paraffin and LiH, since neutrons
typically transfer more energy per inelastic collision to light elements than to heavy elements. When passing through the target, the beam emittance increased dramatically, resulting in beam particles striking the downstream beam pipe and creating a substantial neutron flux. Even with elaborate shielding, this neutron flux overwhelmed the $^{14}\text{N}(p, n)^{14}\text{O}$ neutrons at detector locations corresponding $\Theta_n = 0^\circ - 15^\circ$, closest to the beam pipe; no cross-section measurements were therefore possible at these angles. This limits the precision of our normalization but does not invalidate it: the contribution of these very forward angles to the overall normalization is small, due to the small size of $\sin\Theta$ in the integration of $\sin\Theta \cdot d\sigma / d\Omega$ over the spectrograph aperture. Furthermore, this contribution can be estimated by fitting the cross-section with Legendre polynomials from $15^\circ - 90^\circ$ and extending the fit smoothly in to $0^\circ$ (Be69). For $\Theta_n > 90^\circ$, the $^{14}\text{N}(p,n)^{14}\text{O}_1$ cross-section decreased rapidly while the background neutron flux (from beam particles striking collimating slits upstream of the target) increased; these angles were therefore also excluded from our measurement. This restriction limited our analysis to determine the branching ratio of the 5.17-MeV level to the use of the $^1\text{H}^{(14}\text{N}, ^{14}\text{O}) n$ events to the forward kinematic solution only ($\Theta_{\text{cm}} < \sim 90^\circ$), as mentioned in Section 3.3.

The targets were $3 \pm 0.5$ mg/cm$^2$ melamine ($\text{C}_3\text{H}_5\text{N}_g$) evaporated onto 40 $\mu$g/cm$^2$ carbon foils. The neutrons from the $^{14}\text{N}(p, n)^{14}\text{O}$ reaction were detected with a 4" diameter x 2" deep cylindrical NE213 liquid scintillator detector located at 1.41 m from the target, at angles ranging from $15^\circ$ to $90^\circ$; see Figure 4.1. The neutrons populating the $^{14}\text{O}_0$ and $^{14}\text{O}_1$ states differ in energy by $\sim 5$ MeV, and their energies change with angle $\Theta_n$. In order to compare the integration of the cross-sections over $\Theta_n$, it is necessary to know the efficiency of the neutron detector as a function of energy. However, Eq. 4.3 shows that it is the ratio of the integral of the two production cross-sections that is needed to normalize the results of the $^1\text{H}^{(14}\text{N}, ^{14}\text{O}) n$ experiment to obtain a value of $\Gamma_\gamma / \Gamma$; therefore only a relative neutron detection efficiency $\varepsilon(E_n)$ is necessary. This was determined by measuring the $^7\text{Li} (p, n)^7\text{Be}$ reaction at $0^\circ$ from 2 - 8 MeV in proton energy and comparing the present yields with previous
absolute cross-section measurements (El72). A neutron of energy $E_n$ entering the detector undergoes a sequence of collisions with the protons in the NE213 organic scintillator material (hydrogen / carbon ratio of 1.213), resulting in a recoil proton energy distribution that is approximately flat over proton energies from 0 to $E_n$. These recoil protons lose energy by raising the molecules in the organic medium to excited electronic states which then slowly ($\tau = 3.7$ ns for NE213) decay (fluoresce) by photon emission to a lower vibrational state built on the ground state (Kn79). As the recoil protons travel in the medium, the light output per unit path length is proportional to their energy loss per unit path length. This light is carried by a light guide at the rear of the detector to a photomultiplier tube, which converts the light signal into a large current pulse in a two stage process. The front surface of the photomultiplier tube is a photocathode, a thin layer of material which absorbs energy of incident photons and transfers energy to electrons within the material. The electrons migrate to the inner surface of the photocathode and escape from the surface. The second step is the amplification of this signal by an electron multiplier structure, with typical gain of $10^6$. This linear signal is now ready to be processed by the electronics used in the experiment.

Figure 4.2 shows a block diagram of the standard NIM electronics modules used for both the $^{14}$N (p, n) $^{14}$O angular distribution measurements and the $^7$Li (p, n)$^7$Be efficiency measurement. The signal from the photomultiplier tube on the neutron detector is fanned out to a CFD, an amplifier, and a pulse shape discrimination (PSD) module. The CFD unit gives a fast timing signal that starts the neutron TOF TAC, the stop coming from the buncher RF signal (240 ns between pulses, each pulse ~ 0.4 ns wide.) The linear amplifier generates the neutron energy signal $E_n$ which is sent to an ADC. The PSD unit allows for the discrimination between neutrons and $\gamma$'s in the liquid scintillator detector based on the rise times of the signals: the response of the scintillator to photons is faster than to neutrons. The PSD module uses a zero-crossing technique to determine the time dependence of the light output corresponding to each input pulse. The zero-crossing of a bipolar pulse, generated from
the input unipolar pulse, must fall within an adjustable window based on the rise time of the
input pulse in order to produce an output logic pulse (Pi89). This output starts the PSD TAC,
the stop coming from the CFD. The TOF TAC and PSD TAC signals are each sent to an ADC.
The beam stop is connected to a current integrator, the output of which is sent to two scalers.
The ADC busy signal is used to gate one of the two scalers to correct for the "dead time"
during the ~100 µs conversion time of the ADC. The XSYS data acquisition system (Go83)
was used to interface the experimental signals with a computer for collection, on-line analysis,
and storage of the data. Examples of the histograms used for on-line analysis are given in
Section 4.3.

For the $^7$Li (p, n)$^7$Be efficiency measurement, a different beamline was used to permit
a cross-section measurement at 0°. This allowed the extraction of the neutron detection
efficiency as a function of neutron energy on the basis of previously measured absolute cross-
sections as a function of energy at 0° (El72). The detector and electronics modules and
settings were unchanged from those used in the $^{14}$N (p, n) $^{14}$O angular distribution
measurement to give the greatest consistency between the efficiency and the angular
distribution measurements. Special care was taken to avoid any change in the threshold of the
CFD which generates the event signals: raising the threshold lowers the sensitivity (and
therefore the efficiency) of the system for neutrons of energy $E_n = 0.5 - 1.5$ MeV.

The target chamber on this beamline consisted of a Pyrex glass tee attached to an
aluminum adapter and then to the beam tube. A two-position target ladder was centered in the
glass tube, and a tantalum foil lined the tube to prevent scattered beam from striking the glass
(Fe89). The targets, 500 µg/cm$^2$ LiF evaporated onto thick (5 mil) Ta backings, stopped the 2
- 8 MeV proton beam without generating many neutrons due to the high coulomb barrier of
protons on Ta. This allowed the liquid scintillator detector to be placed at 0° with respect to the
target, at a distance of 3.85 m. At the lower proton energies $E_p < 3.5$ MeV, the neutrons have
a time of flight from target to detector longer than the 0.5 µs range on the TOF TAC, causing a
wraparound of the TOF spectrum. In order to prevent wraparound and also keep the same TAC setting, it was necessary to move the detector closer to the target (to 1.61 m.). The loss in energy resolution due to the shorter flight path caused no problems for the efficiency measurement.

4.3 Data Analysis and Results

There is a prompt $\gamma$-peak in every TOF TAC spectrum, corresponding to all the $\gamma$-rays produced in the target which arrive simultaneously at the detector regardless of their energy. The delay of the TAC stop signal (the RF buncher signal) was chosen to place this peak at the end of the 500 ns-wide TOF TAC spectrum. The time difference $T$ between a group of neutrons of energy $E_n$ and the prompt $\gamma$-peak is given by

$$T = \left( \frac{L(m)}{1.43 \text{ m}} \right) (E_n)^{-1/2} 103 \text{ ns},$$

(4.4)

where $L$ denotes the flight path. Higher neutron energies $E_n$ correspond to lower excitation energies in $^{14}$O or $^7$Be and to shorter time of flight values. Since we are stopping on the RF signal of the next pulse, higher $E_n$ corresponds to larger $T$ values and higher channels in the TOF spectrum. The slight tails on the TOF peaks are due to the asymmetrical time structure of the beam, contributed by the buncher. The energy resolution $\Delta E_n$ of the TOF system has one contribution from the width of the beam pulses $\delta t$ ($\sim 1.0 - 1.2$ ns), and a second from the active depth of the detector (giving an uncertainty in the path TOF path length) $\delta L$ (10 cm):

$$\frac{\Delta E_n}{E_n} = 2 \left( \frac{\delta T}{T} \right)^2 + \left( \frac{\delta L}{L} \right)^2 \frac{1}{2}$$

(4.5)

where

$$\frac{\delta T}{T} = 0.0097 \frac{\delta T(n)(E_n(MeV))^{1/2}}{1.43 \text{ m}}.$$  (4.6)

For the $^{14}$N (p, n) $^{14}$O measurement, the path length contribution to the energy resolution was $\delta L/L = 0.07$, while the beam pulse width contribution ranged from 0.08 - 0.30 for the least
and most energetic neutrons, respectively; the total neutron energy resolution for these two cases was 0.040 and 2.00 MeV, respectively, more than adequate to separate the neutrons from the $^{14}\text{O}_0$ and $^{14}\text{O}_1$ states.

The separation of neutrons from $\gamma$'s was accomplished by plotting the PSD TAC signal vs. the neutron energy $E_n$, as shown in Figure 4.3. Separate windows are drawn around the $\gamma$'s and around the neutrons. Figure 4.4 shows a typical neutron-gated TOF spectrum at an angle $\Theta_n$ of 28.5° and flight path of 1.41 m. Two groups are seen in this spectrum, from the $^{14}\text{N} \,(p, n) ^{14}\text{O}_0$ and $^{14}\text{N} \,(p, n) ^{14}\text{O}_1$ reactions. The $\gamma$-gated TOF spectrum has the prompt $\gamma$-peak as its only visible structure; the presence of any other structure would indicate a non-linearity or excessive noise in the TOF TAC module. Some prompt gammas do get into the neutron gated-spectrum, as evident from Figure 4.4; however, this peak is far enough from the $^{14}\text{O}_0$ and $^{14}\text{O}_1$ peaks that it causes no problems in determining the yields of these two neutron groups.

The area under each of these neutron peaks corresponds to the raw yield of $n_0$ and $n_1$ neutrons. The raw yield is related to the cross-section by

$$\text{Yield} = N_{tgt} N_{beam} \frac{d\sigma}{d\Omega} \Delta\Omega \varepsilon (E_n)$$

where $N_{tgt}$ is the melamine target thickness in particles/cm$^2$, $N_{beam}$ is the number of beam particles striking the target, $d\sigma/d\Omega$ is the differential cross-section in cm$^2$/sr, $\Delta\Omega$ is the solid angle subtended by the detector in steradians, and $\varepsilon (E_n)$ is the detection efficiency as a function of neutron energy. Putting in typical values, we have

$$\text{Yield} = \left(\frac{\rho t}{3 \text{ mg/cm}^2}\right) \left(\frac{Q(\text{clicks})}{7 \cdot 10^5}\right) \left(\frac{d\sigma}{d\Omega} \text{ mb \ sr}^{-1}\right) \left(\frac{1.43 \text{ m}}{L}\right)^2 \varepsilon (E_n) 3.0 \cdot 10^5 \text{ cts}$$

where $\rho t$ is the target mass thickness, $L$ is the flight path, and $Q$ the integrated charge read from the scaler where 1 click = 200 nC. Table 4.1 gives the raw yield measurements and corresponding cross-section values, uncorrected for detection efficiency, for the 8 angles measured between 15° and 90°.

A relation similar to Eq. 4.7 holds for the $^7\text{Li} \,(p, n)^7\text{Be}$ efficiency measurement, where
now the cross-section is known, and a measurement of the yield gives the efficiency \( \varepsilon (E_n) \) as

\[
\varepsilon (E_n) = \frac{\text{Yield}}{N_{tgt} N_{beam} \frac{d\sigma}{d\Omega} \Delta \Omega}.
\] (4.9)

Inserting typical values for these parameters gives

\[
\varepsilon (E_n) = \frac{\text{Yield}}{\left(\frac{d\sigma}{d\Omega} \text{ (mb sr)} \right) \left(1.61 \text{ m}^2\right)^2 \left(\frac{\rho t}{500 \mu \text{g/cm}^2}\right) \left(\frac{Q(\text{clicks})}{1 \times 10^5}\right) 4600 \text{ cts}}\] (4.10)

Table 4.2 gives the raw efficiency yields, the cross-sections (Ei72), neutron energies calculated from the TOF channel of the \(^7\text{Be}_0\) and \(^7\text{Be}_1\) groups, and the calculated efficiency from Eq. 4.10. Figure 4.5 shows the relative efficiency, normalized to \( \varepsilon (E_n = 1.26 \text{ MeV}) = 1 \), plotted as a function of neutron energy. The smooth curve through the data points is to guide the eye.

The values of \( \varepsilon (E_n) \) for the neutrons measured in the \(^{14}\text{N} (p, n) ^{14}\text{O}\) angular distribution are read off this graph and used to determine the corrected cross-section values, given in the last column of Table 4.1. The relative cross-sections are plotted as a function of neutron angle \( \Theta_n \) in Figure 4.6. The uncertainty in the relative efficiency measurement is approximately 10%, due primarily to the uncertainty of the previously measured \(^7\text{Li} (p, n) ^7\text{Be}\) absolute cross-sections; this then gives a 10% uncertainty to the \(^{14}\text{N} (p, n) ^{14}\text{O}_{0,1}\) cross-sections. This uncertainty is almost negligible compared to the 40 - 50% uncertainty in the number of \(^{14}\text{O}_1(\gamma) ^{14}\text{O}\) events measured in the \(^1\text{H}(^{14}\text{N}, ^{14}\text{O})\) experiment (Section 3.3), and thus contributes very little to the uncertainty in the branching ratio of the \(^{14}\text{O}^*(5.17 \text{ MeV})\) level.

### 4.4 Conversion to the \(^1\text{H}(^{14}\text{N}, ^{14}\text{O})\) Laboratory Frame

In order to normalize the results of the \(^1\text{H}(^{14}\text{N}, ^{14}\text{O})\) experiment to obtain a branching ratio, an evaluation of Eq. 4.3 must be made. Since the integrals in this equation are over the spectrograph solid angle for the \(^1\text{H}(^{14}\text{N}, ^{14}\text{O})\) experiment, the \(^{14}\text{N} (p, n) ^{14}\text{O}_{0,1}\) production cross-sections must be transformed to the inverse kinematics lab frame. This transformation involves a product of Jacobians (Ha63) given by
where we have used

\((\Theta_n, \phi_n) = \text{spherical angles of neutron in } ^{14}\text{N}(p, n) \rightarrow ^{14}\text{O} \text{ laboratory frame} \)

\((\Theta_{\text{cms}}, \phi_{\text{cms}}) = \text{corresponding spherical angles in center of mass frame} \)

\((\Theta_{3L}, \phi_{3L}) = \text{spherical angles of } ^{14}\text{O} \text{ in } \gamma(14\text{N}, 14\text{O}) n \text{ laboratory frame} \)

\((P_{3L}, \Theta_x) = ^{14}\text{O} \text{ momentum and angle in horizontal plane} \)

in \(\gamma(14\text{N}, 14\text{O}) n \text{ lab frame.} \)

The momentum spectrum of the \(^{14}\text{O}\)'s formed directly and by \(^{14}\text{O}_1 \gamma\)-decay can now be determined by integrating the transformed cross-section over angles \(\Theta_x\):

\[
f_{0,1}(P_{3L}) = \int \left( \frac{d\sigma}{d\Omega} \right)_{0,1} (P_{3L}, \Theta_x) \ g_{\text{aperture}}(P_{3L}, \Theta_x) \ g_{\text{soft}}(P_{3L}, \Theta_x) \ \sin\Theta_x \ d\Theta_x \quad (4.12)
\]

where \(g_{\text{aperture}} \) and \(g_{\text{soft}}\) give cuts corresponding to the spectrograph aperture and the software window on the \(P_{3L}\) vs. \(\Theta_x\) plot, respectively. \(g_{\text{aperture}} = 1\) if the particular \((P_{3L}, \Theta_x)\) coordinates correspond to lab angles \((\Theta_L, \phi_L)\) that are passed by the spectrograph aperture, 0 otherwise; \(g_{\text{soft}} = 1\) if the particular \((P_{3L}, \Theta_x)\) coordinates are in the window drawn on the \(P_{3L}\) vs. \(\Theta_x\) plot, 0 otherwise. The functions \(g_{\text{soft}}\) allow a cut to be made on positive \(\Theta_x\) angles \((0^\circ \leq \Theta_x \leq 2.43^\circ)\) in the \(P_{3L}\) vs. \(\Theta_x\) plot. Additionally, \(g_{\text{soft}}\) sets a window on the range in \(P_{3L}\) of the excited state group over the same channels as observed in the \(\gamma(14\text{N}, 14\text{O}) n\) experiment.

A Legendre polynomial fit to the \(^{14}\text{N}(p, n)^{14}\text{O}\) angular distributions, similar to that used by Taddeucci et al. (Ta84) at a \(E_p = 35\) MeV, allows the transformation and subsequent numerical integration of the cross-sections over the spectrograph aperture as given by Eqs. 4.9 and 4.10. The cross-sections were approximated over the range \(0^\circ - 15^\circ\) by a smooth continuation of the Legendre polynomial fit over the \(15^\circ - 90^\circ\) range. Since \(\sin\Theta_x\) is small over \(0^\circ - 15^\circ\), the uncertainty of the extrapolation of the integral of \((d\sigma/d\Omega)_{0,1} \sin\Theta_x\) in Eq.
4.12 is smaller than the uncertainty due to the 10% uncertainty in the values of \((d\sigma/d\Omega)_{0,1}\) over 15° - 90°. The coefficients for the fit are given in Table 4.3, and the fits are shown with the data in Figure 4.6. The high order of the fit, 10th and 11th order Legendre polynomials as used in (Ta84), was necessary to get a smooth curve through the data points and to minimize the \(\chi^2\) value, and does not imply any particular nuclear structure. The integration in Eq. 4.12 was performed for three cases: with no spectrograph aperture (100% acceptance, gaperture and gsoft identically equal to 1), with the aperture present but no software cut, and with both aperture and software cuts. The results are shown in Figure 3.14. The integration was carried out numerically, dividing the \(P_{3L}\) and \(\Theta_x\) ranges into 2048 bins each; this finite mesh size of the integration causes the jagged spikes in Figure 3.14. The integration confirms the discussion in Section 3.3 concerning the effect of the spectrograph on the focal plane spectra. With no aperture, the ground state group completely dominates the excited state group; the presence of the aperture allows both groups to be seen simultaneously. The software cut, corresponding to where the \(^{12}\text{B}_0\) group dominates the excited state group as observed in the \(^1\text{H}(^{14}\text{N}, ^{14}\text{O})\) n experiment, is at a momentum corresponding to \(\Theta_{\text{cms}} = 40°\) for the \(^{14}\text{O}_1(\gamma)^{14}\text{O}_0\) events. The desired cross-section ratio \(\sigma_0 / \sigma_1\) is determined from the ratio of areas under the two momentum groups:

\[
\frac{\sigma_0}{\sigma_1} = \frac{\int f_0(P_{3L}) g_{\text{aperture}}(P_{3L}, \Theta_x) g_{\text{soft}_0}(P_{3L}, \Theta_x) dP_{3L}}{\int f_1(P_{3L}) g_{\text{aperture}}(P_{3L}, \Theta_x) g_{\text{soft}_1}(P_{3L}, \Theta_x) dP_{3L}}
\] (4.13)

The aperture cut omits of 75% of the ground state group and 21% of the excited state group, while the software cut omits 60% of the remaining excited state events. The uncertainty in these ratios are due to the 10% uncertainty in each cross-section measurements, and a small uncertainty in the placement of the software and aperture windows to reproduce the spectra taken in the \(^1\text{H}(^{14}\text{N}, ^{14}\text{O})\) n experiment. The value for the cross-section ratio \(\sigma_0 / \sigma_1\) that will be used for the normalization is 27 ± 4, and is the same for both the positive-\(\Theta_x\) and full-\(\Theta_x\) angular ranges. The use of this normalization to determine the branching ratio is discussed.
4.5 The Gamma-Width of the $^{14}\text{O}^*(5.17\text{-MeV})$ Level

In the notation of Section 4.1, the result of the ATLAS $^1\text{H}(^{14}\text{N}, ^{14}\text{O})\, n$ experiment given in Section 3.4 can be stated as

$$N_1 = \frac{\text{Number of } ^{14}\text{O} \text{ which } \gamma \text{ - decay}}{\text{Number of } ^{14}\text{O} \text{ formed directly}}$$

$$(4.14)\quad \frac{(45 \pm 24)}{(3,896,000 \pm 2000)} = (1.16 \pm 0.62) \times 10^{-5}$$

The normalization from the Seattle $^{14}\text{N} (p, n) ^{14}\text{O}_{0,1}$ experiment is given in Section 4.3 as

$$\frac{\sigma_0}{\sigma_1} = \frac{\int \frac{d\sigma}{d\Omega_0}}{\int \frac{d\sigma}{d\Omega_1}} = 27 \pm 4 \quad (4.15)$$

The Gamma-branching ratio is then given by Eq. 4.2 as

$$\left(\frac{\Gamma_\gamma}{\Gamma}\right) = \frac{N_1}{N_0} \frac{\sigma_0}{\sigma_1} = \left( (1.16 \pm 0.62) \times 10^{-5} \right) (27 \pm 4) = (3.1 \pm 1.7) \times 10^4 \quad (4.16)$$

When combined with the total width of the 5.17-MeV level of $38.1 \pm 1.8$ keV, as measured by Chupp et al. (Ch85), the gamma-width is given by $\Gamma_\gamma = 12 \pm 7$ eV. The two previous results of Fernandez et al. (Fe89) and Aguer et al. (Ag89) and a number of theoretically predicted values of $\Gamma_\gamma$ are given in Table 3.1: the present value for $\Gamma_\gamma$ is the highest measured value, and is consistent with the upper limit of Aguer et al., but is a factor of 4 higher than that of Fernandez et al. All three measurements are consistent with $\Gamma_\gamma = 0$ at the 2-σ level. A weighted mean of the present measurement and that of Fernandez et al. gives $\Gamma_\gamma = 2.9 \pm 1.3$ eV, well within the 1-σ uncertainty of the measurement by Fernandez et al. There is still a 50% uncertainty on this value of $\Gamma_\gamma$ which is used in Chapter 6 to determine the $^{13}\text{N} (p, \gamma) ^{14}\text{O}$
resonant reaction rate with the formalism developed in Section 2.2. The implications of a gamma-width of 12 eV will also be discussed.

It may be possible to make another measurement of the branching ratio with modifications of the $^{14}\text{N}(^{3}\text{He}, \text{t})^{14}\text{O}1(^{14}\text{O})\gamma$ triton - $^{14}\text{O}$ recoil coincidence experiment used by Wang (Wa86) to set an upper limit of $\Gamma_\gamma / \Gamma < 4 \times 10^{-4}$. This study was made at a $^{3}\text{He}$ beam energy of 33 MeV, and observed no coincidence counts and no background counts in one day of running. Using a newly-installed Split-Pole magnetic spectrograph at Yale to detect the recoil $^{14}\text{O}$'s may eliminate the majority of the background recoils which limited the counting rate in the previous measurement. Preliminary work on this experiment is currently in progress.
Figure 4.1.
Experimental setup for the $^{14}\text{N}(p,n)^{14}\text{O}$ experiment at the FN-Tandem Van de Graaff Laboratory at the University of Washington, Seattle.
Proton Beam
1 μA
1.2 nS pulsed
4.17 MHz

Proton Beam
Collimator
Melamine Target
3 mg/cm²

Target Chamber
1.41 m

Lead Lined Beam Tube
Neutron Detector
Ne²¹³ Liquid Scintillator

Parrafin Shielding
Figure 4.2.
Schematic of the electronics for the \( ^{14}\text{N}(p, n)^{14}\text{O} \) and \( ^{7}\text{Li} (p, n)^{7}\text{Be} \)
measurements.
RF = rf-signal from beam sweeper
CFD = Constant Fraction Discriminator
GDG = Gate and Delay Generator
TAC = Time to Amplitude Converter
PSD = Pulse Shape Discrimination Unit
ADC = Amplitude to Digital Converter.
Figure 4.3.

A plot of Pulse Shape Discrimination (PSD) vs. neutron energy $E_n$, allowing the separation of neutrons and $\gamma$'s.
Neutron Pulse Height (Channel)

Pulse Shape Discrimination (PSD) Channel
Gated neutron TOF spectrum showing the $^{14}$N (p, n) $^{14}$O$_{0,1}$ groups at $\Theta_n = 20.6^\circ$, and a flight path of 1.43 m.
Table 4.1.

Yields and cross-sections for the
$^{14}\text{N}(p, n)^{14}\text{O}_{0,1}$ experiment. Includes
correction for detector efficiency given in
Table 4.2 and Figure 4.5.
<table>
<thead>
<tr>
<th>Angle deg.</th>
<th>Integrated Charge (clicks)</th>
<th>dead time</th>
<th>Yield $^{14}$N $(p,n)$ $^{14}$O $_{0}$ (mb/sr)</th>
<th>Corrected Cross-section $^{14}$N $(p,n)$ $^{14}$O $_{0}$ (mb/sr)</th>
<th>Yield $^{14}$N $(p,n)$ $^{14}$O $_{1}$ (mb/sr)</th>
<th>Corrected Cross-section $^{14}$N $(p,n)$ $^{14}$O $_{1}$ (mb/sr)</th>
</tr>
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<tr>
<td>15.0</td>
<td>693000</td>
<td>0.727</td>
<td>141000 ± 1000</td>
<td>0.21</td>
<td>2.3</td>
<td>7400 ± 1000</td>
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<td>20.6</td>
<td>700000</td>
<td>0.706</td>
<td>134000 ± 600</td>
<td>0.21</td>
<td>2.5</td>
<td>9600 ± 600</td>
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<tr>
<td>28.5</td>
<td>700000</td>
<td>0.686</td>
<td>121000 ± 600</td>
<td>0.21</td>
<td>1.9</td>
<td>10700 ± 300</td>
</tr>
<tr>
<td>43.4</td>
<td>700000</td>
<td>0.733</td>
<td>749000 ± 400</td>
<td>0.23</td>
<td>1.1</td>
<td>7100 ± 200</td>
</tr>
<tr>
<td>59.1</td>
<td>700000</td>
<td>0.748</td>
<td>544000 ± 300</td>
<td>0.24</td>
<td>0.75</td>
<td>3700 ± 200</td>
</tr>
<tr>
<td>73.0</td>
<td>700000</td>
<td>0.766</td>
<td>502000 ± 300</td>
<td>0.26</td>
<td>0.64</td>
<td>2500 ± 200</td>
</tr>
<tr>
<td>88.2</td>
<td>700000</td>
<td>0.728</td>
<td>910000 ± 500</td>
<td>0.28</td>
<td>0.54</td>
<td>3800 ± 200</td>
</tr>
</tbody>
</table>

(a) Efficiency $\varepsilon(E_n)$ from $^7$Li $(p,n)^7$Be measurement, uncertainty of 10%.

(b) uncertainty of 10%
Yields and deduced efficiencies from the $^7$Li(p, n)$^7$Be measurement.
<table>
<thead>
<tr>
<th>E_p (MeV)</th>
<th>E_n (MeV)</th>
<th>Flight Path (m ± 3mm)</th>
<th>Yield (7Li (p, n) 7Be_0) (mb/sr)</th>
<th>dσ/dΩ (a)</th>
<th>ε</th>
<th>ε_0</th>
<th>E_n (MeV)</th>
<th>Yield (7Li (p, n) 7Be_1) (mb/sr)</th>
<th>dσ/dΩ (c)</th>
<th>ε</th>
<th>ε_0</th>
</tr>
</thead>
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<tr>
<td>2.4</td>
<td>0.655</td>
<td>1.610</td>
<td>197600 ± 500</td>
<td>100</td>
<td>0.43</td>
<td>0.93</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.6</td>
<td>0.865</td>
<td>&quot;</td>
<td>134500 ± 400</td>
<td>67</td>
<td>0.45</td>
<td>0.97</td>
<td>0.358</td>
<td>1900 ± 100</td>
<td>2.5</td>
<td>0.17</td>
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<td>0.928</td>
<td>&quot;</td>
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<td>3000 ± 100</td>
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<td>0.26</td>
<td>0.59</td>
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<td>1.081</td>
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<td>0.98</td>
<td>0.587</td>
<td>5800 ± 100</td>
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<td>0.43</td>
<td>0.97</td>
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<td>&quot;</td>
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<td>50</td>
<td>0.46</td>
<td>1.00</td>
<td>0.804</td>
<td>6800 ± 100</td>
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<td>0.85</td>
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<td>3.2</td>
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<td>6700 ± 200</td>
<td>4.4</td>
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<td>0.77</td>
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<td>&quot;</td>
<td>89900 ± 400</td>
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<td>0.45</td>
<td>0.96</td>
<td>1.216</td>
<td>7700 ± 200</td>
<td>4.8</td>
<td>0.36</td>
<td>0.80</td>
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<tr>
<td>3.6</td>
<td>1.865</td>
<td>3.848</td>
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<td>0.45</td>
<td>1.00</td>
<td>1.408</td>
<td>1400 ± 50</td>
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<td>4.0</td>
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<td>0.97</td>
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<td>3.791</td>
<td>2400 ± 100</td>
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</table>

(a) dσ/dΩ for 7Li (p, n) 7Be from S.A. Elbakr et al. (El72), uncertainty of ± 10 mb/sr.

(b) normalized to ε (E_n = 1.26) = 1.00, uncertainty of 10 %

(c) uncertainty of ± 0.5 mb/sr.
Figure 4.5.

Efficiency of the neutron detector as a function of neutron energy from the $^7\text{Li}(p, n)^7\text{Be}_{0,1}$ measurement. The smooth line through the data points is to guide the eye.
$^7$Li (p, n)$^7$Be Efficiency Measurement
Figure 4.6.

$^{14}_N (p, n) ^{14}O_{0, 1}$ angular distributions measured from $\Theta_n = 15^\circ$ - $90^\circ$, and Legendre polynomial fits.
$^{14}\text{N}(p, n)^{14}\text{O}$ Cross-section

$d\sigma/d\Omega$

(mb/sr)

$\Theta_n$ (deg)

$^{14}\text{N}(p, n')^{14}\text{O}$ Cross-section

$d\sigma/d\Omega$

(\mu b/sr)

$\Theta_n$ (deg)
Table 4.3.

Coefficients for the Legendre polynomial fits of the $^{14}$N(p, n)$^{14}$O angular distributions. Because of the high order of the polynomial fits, the 12 significant figures shown are necessary.
<table>
<thead>
<tr>
<th>L</th>
<th>$^{14}\text{N} (p, n)^{14}\text{O}_0$ Coefficient for $P_L(\cos \Theta_n)$</th>
<th>$^{14}\text{N} (p, n)^{14}\text{O}_1$ Coefficient for $P_L(\cos \Theta_n)$</th>
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<td>0</td>
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<td>1.4710298422e+3</td>
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<td>2.30165364491e+2</td>
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Chapter 5. Measurement of the $^{20}\text{Ne}(^{3}\text{He}, \, t)^{20}\text{Na}$ Reaction

5.1 Introduction

As mentioned in Chapter 1, the $^{15}\text{O}(\alpha, \, \gamma)^{19}\text{Ne} \, (p, \, \gamma)^{20}\text{Na}$ reaction sequence is a likely breakout mechanism of the Hot CNO cycle, which would process CNO seed nuclei into the mass $A > 20$ region and initiate the rapid proton capture ($\text{rp}$) process, thereby increasing the nuclear energy generation rate by $\sim 100$ over the HCNO cycle (Wa81).

For these reasons, it is very important to determine the precise temperatures and densities where such a breakout occurs. A recent study of the $^{15}\text{O}(\alpha, \, \gamma)^{19}\text{Ne}$ reaction (Ma90) has been made, determining the temperatures at which this alpha-capture reaction rate equals the $^{15}\text{O} \beta^+ - \text{decay rate}$. This initiates the breakout, but the $^{19}\text{Ne}(p, \, \gamma)^{20}\text{Na}$ reaction must be similarly studied to ascertain whether the breakout continues or the seed nuclei return to the Hot CNO cycle. A direct measurement of the $^{19}\text{Ne}(p, \, \gamma)^{20}\text{Na}$ reaction would require the development of a radioactive ($\tau_{1/2} = 17 \text{ s}$) $^{19}\text{Ne}$ beam or target, the short half life making this very difficult. $^{19}\text{Ne}$ beams have been developed and used in medical physics applications at Lawrence Berkeley Laboratory (Ch89) and in parity-violation experiments at Princeton University (Pi84). However, the low intensities ($\sim 0.2 \text{ pA and } \sim 5 \text{ nA, respectively}$) of these beams makes them unsuitable for a direct $^{19}\text{Ne}(p, \, \gamma)^{20}\text{Na}$ measurement.

Recently, a proposal for a $^{19}\text{Ne}$ target has been made using the Princeton radioactive $^{19}\text{Ne}$ beam (Ch90b). This proposal involves alternatively implanting $^{19}\text{Ne}$ into an Al backing to build up an equilibrium abundance of approximately $15 \mu\text{g/cm}^2$, and bombarding the target with protons from a second, high current ($\sim 100 \mu\text{A}$) accelerator. Protons are captured by $^{19}\text{Ne}$, populating states in $^{20}\text{Na}$ through resonant capture and the much weaker non-resonant (direct capture) reaction mechanisms. While the majority of these excited states may proton
decay back to $^{19}\text{Ne}$, a certain fraction will $\gamma$-decay to $^{20}\text{Na}$, which $\beta^+$-decays with a 16% branch to $^{20}\text{Ne}^*(7.42\text{ MeV})$, which subsequently $\alpha$-decays to $^{16}\text{O}$ with $E_\alpha = 2.69\text{ MeV}$. It is proposed to detect these $\beta$-delayed $\alpha$'s with a Si(SB) detector telescope, thereby giving a measure of the $^{19}\text{Ne}(p, \gamma)^{20}\text{Na}$ reaction. Two of the numerous difficulties associated with this proposed experiment are the operation of the Si(SB) detectors in the presence of a "hot" target, with $\sim 10^{10}$ 511-keV annihilation $\gamma$'s per $\beta$-delayed $\alpha$, and the uncertain implantation (depth) profile of the equilibrium radioactive $^{19}\text{Ne}$ target.

Indirect methods to study the $^{19}\text{Ne}(p, \gamma)^{20}\text{Na}$ reaction are presently available. As discussed in Chapter 2, an indirect determination of the resonant reaction rate requires only the values of the energy $E_T$ and strength $\omega \gamma$ of any possible resonances above and near the 2.199 MeV proton threshold in $^{20}\text{Na}$. Until 1987, this region was very poorly understood: only one $^{20}\text{Na}$ state was known for $2 \leq E_x \leq 3\text{ MeV}$, whereas 4 - 6 are expected from comparison with the mirror nucleus $^{20}\text{F}$. With no other experimental information, Wallace and Woosley (Wa81) assumed the level structure of $^{20}\text{Na}$ to be identical to that of $^{20}\text{F}$ in order to calculate the $^{19}\text{Ne}(p, \gamma)^{20}\text{Na}$ resonant reaction rate. The partial widths are also necessary for the rate calculation: the $\gamma$-widths were estimated to be 10 meV, the order of magnitude of $\Gamma_\gamma$ in $^{20}\text{F}$, and the proton reduced widths were assumed to be $\Theta_p^2 = 0.01$, from which the proton partial widths $\Gamma_p$ are calculated using Eq. 2.15. Nine resonances were used in their rate calculation. The most important feature of their calculation was that no $^{19}\text{Ne} + p$ resonances were expected for $E_p < 700\text{ keV}$. Using Figure 2.2, this implies that no resonances will be in the Gamow window for $T_9 < 0.8$, and thus the $^{19}\text{Ne}(p, \gamma)^{20}\text{Na}$ reaction rate will be dominated by non-resonant processes at nova temperatures, which implies that the $^{15}\text{O}(\alpha, \gamma)^{19}\text{Ne}(p, \gamma)^{20}\text{Na}$ reaction will not cause substantial breakout of the Hot CNO cycle at nova temperatures and densities. A subsequent calculation by Langanke et al. (La86) used the Thomas-Ehrman energy shift to correct the $^{20}\text{Na}$ energy levels predicted from $^{20}\text{F}$: the unbound $^{19}\text{Ne}$ plus nucleon levels in $^{20}\text{Na}$ have a different energy shift relative to the particle threshold than do the
bound analog states in $^{20}$F (Th52). The resulting resonance energies were found to be lower than those of Wallace and Woosley, especially for the first s-wave resonance in the $^{19}$Ne + p system. Wallace and Woosley have this state at $E_p = 1340$ keV with a proton width of 6 keV, while Langanke et al. use the Thomas-Erman shift to predict an energy of $E_p = 680$ keV for this state. The significantly lower resonance energy and the predicted 6 keV width makes this level dominate their predicted reaction rate for $0.3 > T_9 > 0.7$. Langanke et al. took the $\gamma$-widths of the states from the $\gamma$-widths in $^{20}$F (when available) or estimated them from systematics, scaling the width according to the multipolarity ($\Gamma_\gamma \propto E_\gamma^{2L+1}$). Seven resonances were used in the rate calculation. In comparison to Wallace and Woosley, Langanke et al. find a greatly enhanced rate for $0.1 < T_9 < 0.3$, where the non-resonant direct capture mechanism was found to dominate the resonant rate, and an increased total rate over the temperature range $0.3 < T_9$ due to the $l = 0$ resonance discussed above.

Since the reaction rate is exponentially dependent on the resonance energy, a measurement of the excitation energies of levels above the proton threshold in $^{20}$Na is the most crucial information necessary for an accurate determination of the $^{19}$Ne(p, $\gamma$)$^{20}$Na reaction rate. In an effort to determine these levels and their energies, Lamm et al. (La87) used the $^{20}$Ne($^3$He, t)$^{20}$Na reaction to populate states in the excitation energy range 2.0 - 3.2 MeV in $^{20}$Na. Figure 1.3 gives a rough level diagram resulting from their study, and the excitation energies are listed in Table 5.1. A $^{20}$Ne gas cell target was employed, and the exiting tritons were momentum-separated by a magnetic spectrograph and detected by a position-sensitive gas ionization detector telescope. Two broad groups with widths 100 and 300 keV were seen in their study at 2.6 MeV and 3.0 MeV, respectively, corresponding to resonance energies of approximately 400 and 800 keV. However, energy loss and straggling effects in the $^{20}$Ne gas and in the 2 $\mu$ Ni cell windows resulted in an energy resolution of 80 keV, preventing these broad groups from being resolved into the 4 - 6 states expected from $^{20}$F. There were additional problems in the separation of deuterons from the strong $^{20}$Ne($^3$He, d)$^{21}$Na
reaction \( \frac{d\sigma}{d\Omega} \approx 10 \text{ mb/sr} \) and tritons from the much less intense \(^{20}\text{Ne}(^{3}\text{He}, \text{t})^{20}\text{Na} \) reaction \( \frac{d\sigma}{d\Omega} \approx 10 \mu\text{b/sr} \). On the basis of the level structure of \(^{20}\text{F} \) and the Thomas-Ehrman shift, the two broad groups were decomposed into individual levels, resulting in a tentative level structure. At that time, our preliminary studies with implanted \(^{20}\text{Ne} \) targets were begun at Princeton; these experiments also suffered from poor deuteron-triton separation, as well as low counting rates due to insufficient target thickness.

A more recent \(^{20}\text{Ne}(^{3}\text{He}, \text{t})^{20}\text{Na} \) measurement has been carried out by Lamm et al. (La90) at \( E(^{3}\text{He}) = 30 \text{ MeV} \), utilizing the same gas cell targets as their previous study and an improved focal plane detector. Kubuno et al. (Ku89) made a similar measurement with a gas cell target at \( E(^{3}\text{He}) = 55 \text{ MeV} \). Because of the gas cell targets, neither of these experiments was able to achieve a resolution better than 60 - 80 keV, or to clearly resolve the individual states. An estimation of the excitation energies of the individual levels was made, with the centroids of the levels being determined to \( \pm 16 - 20 \text{ keV} \). Both groups made \( (^{3}\text{He}, \text{t}) \) angular distribution measurements and compared the results with DWBA predictions in attempts to determine or at least set constraints on the \( J^\pi \) values of these levels, thereby aiding in the \(^{20}\text{F} \) analog assignments. The excitation energies and \( J^\pi \) assignments from these studies are given in Table 5.1, and the triton momentum spectra are shown in Figure 5.1. The group at \( E_x = 2.57 \text{ MeV} \) in (La87) has been withdrawn as a \(^{20}\text{Na} \) level, and is attributed to oxygen target contamination. This changed the location of the first state above threshold to 2.65 MeV, a change in resonance energy from 375 to 450 keV. This is still substantially lower than the predictions of Wallace and Woosley and Langanke et al. The two studies are in rough agreement concerning the number of levels and their resonance energies, but differ in their analog assignments. Because the partial proton and gamma widths were determined from the analog states in \(^{20}\text{F} \), using the method of Langanke et al. (La86), these analog assignments are of critical importance in determining the resonance parameters to be used in the calculation of the stellar reaction rate. However, the largest uncertainty in the reaction rate was still due to the
uncertainty in the resonance energies of the $^{20}$Na levels. The number of levels and their locations must be determined with a high resolution measurement. Even if the level structure resulting from the previous $^{20}$Ne($^{3}$He, t)$^{20}$Na studies is accurate, the ± 16 keV uncertainty (4%) in resonance energy $E_R$ of the first state above threshold contributes an uncertainty of 540% in the resonant reaction rate at $T_9 = 0.1$, and 85% at $T_9 = 0.3$. An additional problem associated with the poor energy resolution of these previous studies is that only a very high upper limit of 60-80 keV can be set on the total widths $\Gamma$ of the $^{20}$Na states. If any of these states are wide, they can contribute to the stellar reaction rate at significantly lower energies than their resonance energies.

In an effort to remove these uncertainties, we have made a high resolution measurement of the $^{20}$Ne($^{3}$He, t)$^{20}$Na reaction employing thin, implanted $^{20}$Ne targets. The energy loss of the $^{3}$He beam in these implanted targets, with 40 μg/cm$^2$ transmission Carbon backing foils, is reduced to 6 keV, down a factor of 10 from the gas cell target studies. This reduces the energy spread of the tritons from the $^{20}$Ne($^{3}$He, t)$^{20}$Na reaction by the same amount, thereby allowing a much better determination of the $^{20}$Na level structure and a precise measurement of their excitation energies and total widths. The disadvantage to using implanted targets is in yield: with a maximum thickness of $= 7$ μg/cm$^2$ $^{20}$Ne, the yield for the $^{20}$Na states above the proton threshold is approximately 1 - 10 Hz with our beam current of 200 nA and detector solid angle of 14 msr. This low yield precluded any measurement of an angular distribution to help determine the $J^\pi$ of the levels, or of particle coincidences to measure the partial widths of the states. The targets will be described in detail in the following section, and the experimental setup for the $^{20}$Ne($^{3}$He, t)$^{20}$Na measurement in Section 5.3.

5.2 Implanted $^{20}$Ne Targets

As discussed above, the use of thin implanted $^{20}$Ne foils allows a high-resolution study of the $^{20}$Ne($^{3}$He, t)$^{20}$Na reaction to be made: straggling and energy loss of the $^{3}$He 's
99

and tritons are reduced by an order of magnitude over gas cell targets. Non-transmission implanted $^{20}\text{Ne}$ targets have been previously produced (Ke77), where $^{20}\text{Ne}$ ions of 5 - 50 keV were implanted into 1.0 mm thick carbon backings. The $^{20}\text{Ne}$ thicknesses achievable were approximately 7 $\mu g/cm^2$. The thick backings, however, make these targets unsuitable for the $(^3\text{He}, \alpha)$ measurement.

Our transmission $^{20}\text{Ne}$ targets were produced at the 300 kV Cockroft-Walton accelerator laboratory at Yale University. $^{20}\text{Ne}^+$ ions were produced in a duoplasmatron ion source and injected into the Cockroft-Walton, where they were accelerated to energies of 15-60 keV. A $30^\circ$ bending magnet momentum-analysed and steered the beam onto the target holder assembly shown in Figure 5.2. The beam passes through a Ta collimator with an 8 mm - diameter aperture, and a second collimator / electron suppressor biased at -300 V, before stopping in a 40 $\mu g/cm^2$ carbon foil mounted on a standard Al target frame. The bias on the second collimator prevents secondary electrons produced in the first collimator from reaching the target and deflects those produced in the target back to the target surface, ensuring an accurate integrated-charge measurement. The entire assembly was mounted on a lucite flange at the end of the beamline, allowing for visual inspection of any damage to the backing foil during implantation. Currents were read off the first collimator and the target frame, and were typically 20 $\mu A$, and 100 - 600 nA, respectively. Secondary electrons increase the target current by $\sim 50\%$ without the suppressor bias. A liquid nitrogen cold trap was used to keep Si-based diffusion-pump oil from diffusing into the beam line and contaminating the target. Typical vacuum pressures were $1 - 3 \times 10^{-6}$ Torr near the target. The beam was rastered over a large area ($\sim 100 \text{ cm}^2$) on the first collimator, the small 8 mm diameter aperture ensuring a uniform distribution on the target. This prevented any "hot spots" which could blister or tear the foils.

Thermal damage to the carbon backing foils and a local saturation of $^{20}\text{Ne}$ were the major difficulties with the implantation. The foil damage is similar to that encountered with
carbon stripping foils used in the terminals of Tandem Van de Graaff accelerators (Ar79): during irradiation, there is a shrinkage of the bombarded area, pulling in material from the surrounding foil regions and forming radial stress lines from the edge of the beam spot to the target frame. This shrinkage of the beam spot area and stress on the outer foil area continues until the bombardment is stopped (the ideal case) or the foil ruptures. Rupture occurs in minutes with a 1 μA current and 50 keV energy, and after a few hours with a 300 nA current. The thermal damage increases as the deposited power/area, the product of beam current and energy divided by beam spot size, during implantation. However, achieving low power by lowering the beam current is not practical, since this can extend the implantation process to an unacceptable duration of several days.

The thermal stress problem can be alleviated by slackening the foils before starting the implantation, thereby allowing the shrinking process to continue for a longer period of time before rupture occurs. A camera flash unit, held ~ 15 cm from the target, was used to slacken the foils: the heat from the flash increases the surface area of the foil, giving it a rough, pebbled appearance. This was sufficient to allow much longer implantations before foil rupture. An attempt was also made to reduce the target thermal stress by cooling the target frame. The heat generated by implantation would ideally be carried by conduction through the foil to the frame, and then to a liquid nitrogen reservoir. Conduction through the thin carbon foils, however, was negligible, making this approach impractical. Additionally, as the target returned to room temperature, there is the possibility of contaminants sticking to the cold foil surface, even when the cooling occurs in vacuum. For these reasons, the cooled target holder was not used.

The initial target production efforts employed only one $^{20}$Ne beam energy for each target, and produced targets of approximately 2 μg/cm$^2$ of $^{20}$Ne regardless of the exposure (duration of implantation) as shown by Rutherford backscattering (RBS) analyses (Figure 5.3). This saturation in $^{20}$Ne content is most likely due to sputtering effects. The sputtering yield of Ne on Ne is ≈ 0.9, meaning that 90% of the incident neon ions striking previously
implanted neon ions are ejected from the surface of the target. Thus, running at one implantation beam energy for an indefinite amount of time will not increase the $^{20}\text{Ne}$ content in the target past ~2 μg/cm$^2$. The $^{20}\text{Ne}$ distributions in carbon were studied with Rutherford backscattering for a range of implantation energies. At 60 keV, for example, the center of the $^{20}\text{Ne}$ distribution is located at a depth of 25 μg/cm$^2$ in the carbon foil, spread over a width of approximately 12 μg/cm$^2$ of carbon; 50, 40, 30, and 20 keV energies correspond to depths of 22, 18, 15, and 12 μg/cm$^2$, respectively.

With the three considerations of thermal damage, $^{20}\text{Ne}$ saturation levels, and implantation time, the best method of packing as much $^{20}\text{Ne}$ into the target as possible was to vary the implantation energy to change the location of the implantation, thereby avoiding saturation, and simultaneously vary the current to keep the power/area constant. The optimal power was found to be 25 mW/cm$^2$, produced for example by a current of 400 nA, an energy of 30 keV, and an implantation area of 0.5 cm$^2$. The procedure followed was to first expose the foil to approximately 1 - 3 μg/cm$^2$ of beam at the highest energies (40 keV) and low current (300 nA), depositing $^{20}\text{Ne}$ near the center of the foil (~18 μg/cm$^2$ depth). Next, the energy was decreased and current increased in two steps for additional implantations, working back towards the front surface of the target. Finally, the target was reversed and the above procedure was repeated. In this manner it was possible to implant ~7.5 μg/cm$^2$ in the 40 μg/cm$^2$ carbon foils. Targets made in this fashion were observed to have a high durability: no outgassing or thermal damage could be measured during light-ion bombardment in the $^{20}\text{Ne}(^3\text{He}, t)^{20}\text{Na}$ reaction at a beam energy of 30 MeV, a current of 200 nA, and a beam spot size of 2 mm (40 mW/cm$^2$). Additionally, the RBS analyses show no evidence for $^{28}\text{Si}$ or heavier contaminants in these targets.

In order to determine the optimal implantation power, a large sample of trial targets were RBS analysed using 0.9 MeV α's from the Yale 1MV Van de Graaff accelerator, detected with a Si(SB) detector at 175°. Because of the poor target chamber and beamline vacuum, unsuppressed current integration, and a crude venting procedure which made the
likelihood of target destruction about 20% for this setup, this analysis was used primarily for trial targets and for some of the thicker targets before their final implantations. The $^{20}\text{Ne}$ content in each of these targets was determined by comparing the yield of the $^{12}\text{C}$ and $^{20}\text{Ne}$ peaks, and using the relation

$$
(pt)_{\text{Ne}} = (pt)_{\text{C}} \frac{N_{\text{Ne}}}{N_{\text{C}}} \frac{\sigma_{\text{Ruth}}}{\sigma_{\text{Ruth}}} 
$$

(5.1)

There is an additional correction concerning the value of $(pt)_C$: a certain percentage of the $^{12}\text{C}$ has been sputtered away during the implantation process. Since the sputtering yield of neon on carbon is $\sim 1$, the amount of Carbon sputtered $(pt)_{\text{C-Sputtered}}$ after exposure to an amount $(pt)_{\text{Ne-Exposed}}$ beam is estimated to be

$$
(pt)_{\text{C-Sputtered}} = (pt)_{\text{Ne-Exposed}} \frac{12}{20} 
$$

(5.2)

Figure 5.3 shows the preliminary RBS, after exposure to 3 $\mu g/cm^2$ and 4 $\mu g/cm^2$ on its two sides, on target "H1". In addition to the carbon group, two $^{20}\text{Ne}$ groups are evident in the backscattering spectrum corresponding to the two implantations. From Eq. 5.2, an exposure to 7 $\mu g/cm^2$ of $^{20}\text{Ne}$ implies that 4.2 $\mu g/cm^2$ of carbon were sputtered from the target, and thus that 35.8 $\mu g/cm^2$ remains. From the yields and Eq. 5.1 we then get each $^{20}\text{Ne}$ group having a thickness of $\sim 2.4 \mu g/cm^2$, giving a total thickness of 4.8 $\mu g/cm^2$ and an implantation efficiency of 68%. We can thus estimate the total $^{20}\text{Ne}$ content of the target after an additional exposure to 4 $\mu g/cm^2$ to be $7.5 \pm 1.0 \mu g/cm^2$.

Another check of the thickness of target "H1" was made with during the $^{20}\text{Ne}(^{3}\text{He},t)^{20}\text{Na}$ measurement by comparing our measured yield with a previously measured cross-section. The differential cross-section for the $^{20}\text{Ne}(^{3}\text{He},t)^{20}\text{Na}^*(1.967 \text{MeV})$ reaction at $10^\circ$ was taken from Lamm et al (La89) to be $29 \pm 9 \mu\text{b/sr}$. The solid angle, integrated charge, and measured yield for this bound state in $^{20}\text{Na}$ during the present experiment give a
$^{20}$Ne target thickness of $6.5 \pm 2 \mu g/cm^2$, consistent with the above determination. A weighted mean of these two methods gives $7.3 \pm 1 \mu g/cm^2$.

5.3 Experimental Setup

This study of the $^{20}$Ne($^3$He, $t$)$^{20}$Na reaction is similar to the two recent studies ((La89) and Ku(89)) in that the reaction products are separated on the basis of their magnetic rigidity (momentum/charge ratio) with a magnetic spectrograph using a position-sensitive detector telescope at the focal plane to identify the particles and measure their position. The $^3$He$^{++}$ beam was produced at the Princeton University AVF cyclotron facility. Since the Q-value for the $^{20}$Ne($^3$He, $t$)$^{20}$Na reaction is -13.99 MeV, a 30 MeV beam energy produces 12 - 16 MeV tritons, allowing a reasonable (~1 MeV) energy range to be focussed onto the focal plane of the magnetic spectrograph and making them easily detectable with the focal plane detector. There were two considerations for the precise choice of beam energy: first, it was chosen to give the tritons a magnetic rigidity different from the deuterons produced by the intense (~ 100 mb/sr) $^{20}$Ne($^3$He, $d$)$^{21}$Na and $^{12}$C($^3$He, $d$)$^{13}$N reactions, and also different from the tritons from the $^{16}$O($^3$He, $t$)$^{16}$F reaction; second, it was chosen to enable the $\alpha$'s corresponding to known $^{19}$F and $^{29}$Si states from the $^{19}$F($^3$He, $\alpha$) and $^{30}$Si($^3$He, $\alpha$) reactions to calibrate the focal plane at the same magnetic field setting used for the $^{20}$Ne($^3$He, $t$)$^{20}$Na reaction. After the 30 MeV $^3$He$^{++}$ ions are extracted from the cyclotron, they are momentum analysed by a 90° magnet and then routed by a switching magnet to the QDDD (Quadrupole-Dipole-Dipole-Dipole) magnetic spectrograph target room.

The target chamber, spectrograph, and focal plane detector (Ko74) are shown in Figure 5.4. Four targets were loaded into the vertical target ladder: a 50 $\mu g/cm^2$ CaF$_2$ on a 10 $\mu g/cm^2$ carbon backing, a 50 $\mu g/cm^2$ SiO target on a 40 $\mu g/cm^2$ carbon backing, an implanted $^{20}$Ne in carbon, and a 40 $\mu g/cm^2$ carbon blank were used to measure the $^{19}$F($^3$He, $\alpha$) $^{18}$F, $^{30}$Si($^3$He, $\alpha$)$^{29}$Si, $^{20}$Ne($^3$He, $t$)$^{20}$Na, and $^{12}$C($^3$He, $d$)$^{13}$N reactions,
The three dipole magnets in the QDDD spectrograph bend the reaction products along circular orbits according to their momentum/charge ratio, while the Quadrupole and Multipole elements reduce aberrations caused by the large acceptance of the device. This spectrograph is ideally suited to our measurement for three reasons. First, it meets the requirements for our high-resolution study by virtue of its large dispersion, \( D = 8.72 \text{ MeV mm / keV} \): particles of energy \( E + \Delta E \) are separated at the focal plane by an amount \( D \Delta E / E \) from particles of energy \( E \). This means, for example, that two triton groups of energy \( \approx 15 \text{ MeV} \) that correspond to \(^{20}\text{Na} \) states separated by only \( 10 \text{ keV} \) in excitation energy are separated by \( 5.8 \text{ mm} \) on the focal plane, a separation easily measured by a number of detectors. The large dispersion naturally limits the range of the energy spectrum that can be focussed on the focal plane at any one magnet field value. However, it is sufficiently large to detect all of the \(^{20}\text{Na} \) states within \( 1 \text{ MeV} \) of the proton threshold at a single magnetic field setting. The dynamic range \( \Delta E / E \) is obtained from the length of the focal plane detector (60 cm) and the dispersion to be \( 1 / 14.5 \), corresponding to \( 0.8 - 1.0 \text{ MeV} \) for the \( 12 - 15 \text{ MeV} \) tritons from the \(^{20}\text{Ne}(^{3}\text{He}, t)^{20}\text{Na} \) reaction. The longer counter has a dynamic range of \( 1 / 8.72 \). The entire level structure of \(^{20}\text{Na} \) from \( 0 \leq E_x \leq 3 \text{ MeV} \) may be measured with three or four magnet settings, corresponding to overlapping bites in energy or momentum. For a Split Pole spectrograph, the dynamic range \( \Delta E / E \approx 1.2 \), increased by a factor of 10 over the QDDD; however, the dispersion is similarly decreased by a factor of 7 (En79.)

Secondly, the QDDD does not sacrifice angular acceptance to gain its high resolution. Its large solid angle \( (\Delta \Omega = 14.7 \text{ msr}) \) allows reasonable count rates even with the low cross-section \( \sim 20 \mu b / sr \) \(^{20}\text{Ne}(^{3}\text{He}, t)^{20}\text{Na} \) reaction and very thin \(^{20}\text{Ne} \) targets. However, to retain the high position resolution at the focal plane, both ion-optical aberrations and the kinematic shift \( (dp/p \cdot d\Theta) \) of the reaction must be corrected for. This correction is provided by the Quadrupole magnet and the Multipole element: the Quadrupole is set to give an intermediate
vertical cross-over at the Multipole element, between the first and second dipole magnets, and the Multipole is adjusted to compensate for the horizontal aberrations and the reaction's kinematic shift. Its location at a vertical crossover means that it has little effect on the vertical focussing. Since the kinematic shift is dependent on the mass of the target and light-ion product, the Quadrupole and Multipole elements can only be set to focus one species of ion at a time. In the present case, tritons from $^{20}\text{Ne}(^{3}\text{He}, t)^{20}\text{Na}$ are focussed while tritons from $^{16}\text{O}(^{3}\text{He}, t)^{16}\text{N}$ and deuterons from $^{20}\text{Ne}(^{3}\text{He}, d)^{21}\text{Na}$ and $^{12}\text{C}(^{3}\text{He}, d)^{13}\text{N}$ are defocussed, giving in wider groups on the focal plane.

The third advantage to using the QDDD is that its focusing properties allow a relative simple focal plane detector to be used. The horizontal magnification of 2.04 and target beam spot diameter of approximately 1 mm suggest that a focal plane position resolution better than 2 mm is unnecessary; a 2 mm resolution is easily achievable by a variety of detectors. Additionally, the vertical cross-over at the focal plane, provided by the shapes of the second and third dipole edges, means that only one-dimensional positional information is required to determine the particle rigidity, further simplifying the detector design.

The focal plane detector is shown in Figure 5.5. Two position-sensitive resistive-wire gas proportional counters form the first sections of the detector, and measure the position and the rate of energy loss of the incident ions. A thick plastic scintillator forms the final section which stops the particles and measures the residual energy signal. 200 Torr propane flowing through the two proportional counters serves as the ionizing gas; a window of 13 µm kapton isolates this gas from the spectrograph vacuum. Three 10 µm aluminized mylar foils serve as ground planes for the high -resistance, 13 µm nichrome anode wires biased to 1500 V. Particles pass through the two proportional counters, leaving a trail of ionized propane ($\text{C}_3\text{H}_8$) molecules and electrons before stopping in the plastic scintillator. In the high electric field (120 kV/m), the electrons accelerate to the anode wire, causing an electron avalanche; the negative charge deposited on the wire is proportional to the initial amount of ionization in the
gas at the location of the initial ionization trail. This charge travels to both ends of the resistive wire. The sum of the charge deposited at the two ends is proportional to the energy loss $\Delta E$ of the particle in the counter, while the amount of charge deposited to each end of the resistive wire is proportional to the distance to the avalanche. The position is thus determined by dividing the charge at one end by the sum of both ends of the wire.

Normal operation of the detector has the first proportional counter giving both the position and energy loss of the particle, the second proportional counter inactive, and the plastic scintillator giving the residual energy of the particles. Particles entering the 0.25"-thick pilot B organic plastic scintillator (Hydrogen/Carbon ratio of 1.1) lose energy by raising the scintillator molecules to excited electronic states which then rapidly ($\tau = 1.8$ ns for pilot B, (Kn79)) decay by photon emission. The total light output for a particle stopping in the scintillator is proportional to its energy when entering the scintillator, the residual energy after passing through the proportional counters. This light is channeled by a light guide at the rear of the detector to a photomultiplier tube, which converts the light signal into a large current pulse. The residual energy $E_{\text{residual}}$ and the energy loss $\Delta E$ allow the identification of particles, specifically the separation of tritons from deuterons, protons, alphas, and background events. This will be discussed in more detail in Section 5.4.

A second measurement of the $^{20}\text{Ne}(^3\text{He},t)^{20}\text{Na}$ reaction employed a slightly different focal plane detector from the one described above. The length was extended to 100 cm with approximately the same position resolution, and a new scintillator was installed with a lightpipe and photomultiplier assembly at each end, allowing the $E_{\text{residual}}$ signals to be relatively independent of position. The hope was to get better separation between deuterons and tritons with the new scintillator, and to detect more of the spectrum with each setting of the spectrograph. The $\Delta E - E_{\text{residual}}$ particle identification plot was substantially cleaner with the new detector, and the position resolution was comparable to the shorter detector. However, the intensity of the $^{20}\text{Ne}(^3\text{He},d)^{21}\text{Na}$ and $^{12}\text{C}(^3\text{He},d)^{13}\text{N}$ reactions still caused difficulties
in the separation of deuterons from tritons, resulting in a $^{12}\text{C}(^{3}\text{He}, \text{d})^{13}\text{N}$ group leaking into the triton position (momentum) spectrum, as will be discussed in Section 5.4. The beam energy was such that this did not obscure any of the groups of interest, and the identification of this group as due to $^{13}\text{N}$ was easily done by measuring a momentum spectrum with a carbon target.

The standard NIM electronics modules used to process the signals from the focal plane detector are shown in Figure 5.6. The signals from both sides of the first proportional counter's anode wire are sent to a preamp and amplifier. The left anode signal is then sent to a CAMAC ADC, which is interfaced to a Data General Computer operating the data acquisition program ACQUIRE (Ko87). The right and left anode signals are summed, giving the $\Delta E$ signal, which is also digitized. The summed signal is also sent to a Timing Single Channel Analyzer (TSCA), which generates a logic pulse correlated in time with the event in the proportional counter. The position is determined by dividing the left anode signal by the sum anode signal in software. The (slow) dynode signal from the photomultiplier tube attached to the scintillator gives the total energy deposited in the scintillator, $E_{\text{residual}}$. This signal is amplified and sent to an ADC and a TSCA, the latter generating a timing signal. The two timing signals are then sent to a coincidence unit, whose overlap signal defines a good focal plane event and is used both to strobe the ADC's and to count events in a scaler. Unsuppressed current is read off the beam stop located in the target chamber, and is sent to a beam current integrator and then to two scalers, one corrected for dead time. ACQUIRE allows one and two dimensional histograms to be displayed in real time, along with gating similar to that described by the XSYS system in Chapter 4.

5.4 Data Analysis and Results

The objective of the $^{20}\text{Ne}(^{3}\text{He}, \text{t})^{20}\text{Na}$ measurement is to determine the excitation energies of the levels in $^{20}\text{Na}$ above the proton threshold at 2.196 MeV. This is done by first
separating the tritons arriving at the focal plane from deuterons, protons, alphas, and background events, then examining the resulting position spectrum. Reactions on $^{30}$SiO and CaF$_2$ targets leading to final states in $^{29}$Si and $^{18}$F with accurately known excitation energies are used to determine the beam energy and a position vs. momentum calibration of the focal plane. This calibration allows the excitation energies of the first four $^{20}$Na excited states above threshold to be measured with a precision of $\pm 5$ keV, as will be discussed below. In order to determine that these levels were in the $^{20}$Na residual nucleus, spectra were measured at three angles, 10°, 15°, and 20°, and the position of the triton groups were compared with the calculated momentum shift with angle.

To identify the particles arriving at the focal plane, the energy loss in the proportional counter, $\Delta E$, is plotted against the energy deposited in the scintillator, $E_{\text{residual}}$. Since $\Delta E \ll E$ for the light ions we are interested in, $E_{\text{residual}} = E - \Delta E \approx E$. The energy loss or specific ionization of the particle along the short path through the proportional counter is related to the mass, nuclear charge $Z$, and total energy $E$ of the ion by (Fu79)

$$\Delta E = \frac{M Z^2}{E},$$

so that a plot of $\Delta E$ vs. $E_{\text{residual}}$ will give distinct groups for deuterons, tritons, and alphas. Both deuteron and triton groups are indicated on the typical $\Delta E$ vs. $E_{\text{residual}}$ plot shown in Figure 5.7; however, the voltage on the anode wire was high enough to put the much larger energy loss of alpha particles off the edge of the plot. Windows are drawn around the tritons and around the deuterons, and the raw position spectrum is gated on each of these windows. The proportional counter gas pressure, anode voltage, anode signal gain, and scintillator signal gain were all adjusted to get the best possible separation between the deuteron and triton groups. Even with this effort, some deuterons from the intense $^{12}$C($^3$He, d)$^{13}$N reaction leak into the triton gate.

Figure 5.8 shows the triton-gated position spectrum measured with the longer 100 cm counter at $\Theta_t = 10^\circ$, $E(^3$He) = 29.7 MeV, and with the QDDD magnetic field set to position
$E_x = 2.6 \text{ MeV}$ in $^{20}\text{Na}$ at the center of the focal plane. Four levels from $^{20}\text{Ne}(^{3}\text{He}, t)^{20}\text{Na}$ above the 2.199-MeV proton threshold are indicated in this figure, along with the 0.424 and 0.720 MeV levels in $^{16}\text{F}$ from $^{16}\text{O}(^{3}\text{He}, t)^{16}\text{F}$ and the $^{13}\text{N}(6.364 \text{ MeV})$ group from the $^{12}\text{C}(^{3}\text{He}, d)^{13}\text{N}$ reaction. Excitation energies are also listed, and will be discussed below.

Figure 5.9 gives the same spectrum as measured with the shorter counter: the same four $^{20}\text{Na}$ states threshold and the $^{13}\text{N}(6.364)$ state are present. By measuring spectra at three angles with the short counter, the identification of these triton groups as coming from the formation of levels in $^{20}\text{Na}$ was confirmed. There is a fifth level seen in the momentum spectrum with the longer counter, Figure 5.8, that may possibly be due to $^{20}\text{Na}$. This group was too low in rigidity to be seen with the 60 - cm counter, and spectra were taken at only one angle with the longer 100-cm counter. In contrast to the results of Lamm et al. (La90), our results show no $^{20}\text{Na}$ levels between the confirmed level at 3.05 MeV and this candidate level at 3.3 MeV. It is necessary to determine the excitation energies and widths of these $^{20}\text{Na}$ levels as accurately and precisely as possible.

The first information obvious from these spectra is that all the resonances are narrow. The focal plane position resolution is 11 ± 1 keV, and the widths for the first three states above threshold are 14 ± 1, 11 ± 1, and 14 ± 1, and thus the intrinsic widths of these states are all less than 9 keV. For the fourth state above threshold, the measured width is 18 ± 2 keV, setting an upper limit of 14 ± 2 keV on its width. These values are obtained from the observed width in channels and the focal plane momentum calibration described below. Because of these small widths, the narrow resonance formula, Eq. 2.25, may be used to calculate the contribution of these resonances to the stellar $^{19}\text{Ne}(p, \gamma)^{20}\text{Na}$ reaction rate.

The relative excitation energies can be very well established (to ± 2 keV) using the spacing of the peaks in Figures 5.8 and 5.9 and the energy/channel or momentum/channel on the focal plane, the slope of the momentum calibration. This slope is relatively insensitive to the beam energy, and is determined by using the $^{19}\text{F}(^{3}\text{He}, \alpha)^{18}\text{F}$ and $^{30}\text{Si}(^{3}\text{He}, \alpha)^{29}\text{Si}$
reactions to be approximately 1.1 keV/channel at the location of the first $^{20}$Na state above threshold. Figures 5.10 a and b show position spectra for these two reactions from the long focal plane counter, with the excitation energies of these levels ($A_f$, $E_r$) indicated next to the peaks. The focal plane position and excitation energies of these groups are listed in Table 5.2. Two things other than targets were altered between the data and the calibration runs: the spectrograph horizontal entrance slits were narrowed down to $1/6$ of the full aperture in order to give the highest possible resolution, and the voltage on the proportional counter anode wire was lowered by $\sim5\%$ to bring the $\alpha$ groups from the ($^3$He, $\alpha$) reactions onto the $\Delta E$ vs. $E_{\text{residual}}$ plot. Neither of these adjustments affect the focal plane momentum calibration. Most importantly, the magnetic spectrograph settings were unchanged for the calibration runs.

A modified version of the program CALIB (Ko74) was used to obtain a momentum calibration of the focal plane from the spectra shown in Figures 5.10. The input consists of the position and excitation energy of the calibration states with uncertainties, the magnetic spectrograph settings, a target description, and the beam energy. Using a least-squares fitting routine, CALIB generates a second-order magnetic rigidity vs. position channel calibration with uncertainties. A list of observed positions of $^{20}$Na groups, from Figures 5.8 and 5.9, is then used with this calibration and the beam energy to generate the excitation energies in $^{20}$Na.

The method described above requires a value of the beam energy. Due to hysteresis effects in the analyzing magnet and the finite size of the image and object slit widths, this can only be crudely estimated ($\sim200$ keV) from the magnetic field setting. This accuracy is sufficient to determine the slope of the momentum calibration and therefore the relative excitation energies of the $^{20}$Na states; however, it is the resonant energies $E_T = E_X - Q_\alpha$, and therefore the absolute $E_X$ values, which are needed for the resonant reaction rate calculation described in Section 2.2. The uncertainty in beam energy $\delta E_b$, as well as uncertainties in target thickness, spectrograph angle, and excitation energy and position of calibration states, contribute to the uncertainty $\delta E_X$ of the absolute excitation energies in $^{20}$Na. There is also an
uncertainty of 7 keV in the $Q_Q$ value, 2.199 ± 0.007 MeV (Aj87). CALIB was modified to determine the contribution of each of these effects to $\delta E_x$ by separately varying each of the above parameters. The results, given in Table 5.3, indicate that the limit on the precision of the $^{20}$Na excitation energies is determined entirely by the precision of the beam energy determination; its contribution is given by

$$\delta E_x (\text{keV}) = 0.6 \delta E_b (\text{keV}) .$$

A beam energy uncertainty of 10 keV would give $\delta E_x = 6$ keV, which from Table 5.3 is a factor of 3 larger than all the other effects combined. If $\delta E_b$ can be reduced to this level, the $^{19}$Ne(p, $\gamma$)$^{20}$Na resonant reaction rate can be determined with a 40% uncertainty due to $\delta E_x$ at $T_9 = 0.3$.

In efforts to reduce $\delta E_b$, CALIB was modified to perform the focal plane calibration as described above using the $^{19}$F($^3$He, $\alpha$) and $^{30}$Si($^3$He, $\alpha$) alpha particle groups over a range of beam energies. The deviation ($\chi^2$) between the predicted and known momenta of the $^{18}$F and $^{29}$Si calibration states is determined for each beam energy; the beam energy that produces the minimum value of $\chi^2$ is then taken as the actual beam energy, and $\delta E_b$ is the change in beam energy necessary to raise the $\chi^2$ value by 1. Because only alpha groups were used, however, this momentum calibration is very insensitive ($\pm 100$ keV) to the chosen beam energy; a much more rigid constraint on the beam energy ($\pm 12 - 16$ keV) is achieved by calibrating with particles of different masses. Thus deuterons from $^{12}$C($^3$He, d)$^{13}$N (6.634), tritons from $^{16}$O($^3$He, t)$^{16}$F*$ (0.424 , 0.720 MeV), and alphas from $^{19}$F($^3$He, $\alpha$) and $^{30}$Si($^3$He, $\alpha$) were used in this manner with a resulting beam energy of 29.740 ± 0.016 MeV and 29.649 ± 0.012 MeV for the Spring 1989 and Fall 1989 runs, respectively. These beam energy uncertainties correspond to $\delta E_x = 7 - 10$ keV.

A second determination of the beam energy was made in which the QDDD magnetic field was changed to focus tritons from the $^{19}$F($^3$He, t)$^{19}$Ne reaction (at $E_x (^{19}$Ne) = 4.6 MeV) and deuterons from the $^{19}$F($^3$He, d)$^{20}$Ne reaction (at $E_x (^{20}$Ne) = 5.6 MeV) onto the focal plane. Using both deuterons and tritons of known rigidity to calibrate the focal plane
with CALIB as described above gives a ± 12 keV determination of the beam energy: the results are 29.755 ± 0.012 MeV and 29.631 ± 0.012 MeV for the Spring and Fall 1989 runs, respectively. Even greater precision can be achieved if a $^{19}$Ne and $^{20}$Ne group have the same position on the focal plane, and thus the same magnetic rigidity. In efforts to do this, the spectrograph was moved to find the angle at which a deuteron and a triton group cross-over on the focal plane. Unfortunately, this occurs only for a limited range in beam energies and angles; the beam energies we used were slightly outside this range.

A weighted average of the two beam energy determinations is made, with resulting beam energies of 29.750 ± 0.010 MeV and 29.640 ± 0.009 MeV for the Spring and Fall 1989 runs, respectively. These beam energies are now used in CALIB with the calibrant states listed in Table 5.2 and the $^{20}$Na states shown in Figure 5.8 and 5.9 to get a momentum calibration and the excitation energies of the $^{20}$Na states; the uncertainties are ± 6 keV for both the Spring and Fall 1989 runs, and the energies are listed in Table 5.4 along with a weighted mean for each of the first four states above the particle threshold. The two experiments are entirely consistent, and give mean values for the excitation energies of 2.646, 2.857, 2.986, and 3.056 MeV, each with an uncertainty of 5 keV. The fifth level seen with the longer counter, if due to $^{20}$Na, has an excitation energy of 3.298 ± 0.005 MeV. The corresponding center of mass resonance energies are 447 ± 5 keV, 658 ± 5 keV, 787 ± 5 keV, and 857 ± 5 keV; for the fifth candidate level the resonance energy is 1099 ± 5 keV; adding in the uncertainty in the $Q_o$ value results in a total uncertainty of 9 keV for the absolute excitation energy of each of these states. The center of mass resonance energies of these levels relative to the 447-keV level are 211 ± 2, 340 ± 2, and 410 ± 2 keV; for the candidate level it is 652 ± 2 keV. The uncertainties are smaller here because the relative excitation energies have no contribution from the beam energy uncertainty or the $Q_o$ uncertainty; see Table 5.3. The limitation of the precision of the resonance energy values is now given by the uncertainty of the $^{20}$Na mass. A comparison of the present results with the previously measured energies is given in Table 5.5. The values of
Lamm et al. (La90) and Kubuno et al. (Ku89), with $\Delta E_x = 16 - 20$ keV, are consistent with the present work for the first four states above threshold. However, Lamm et al. have assigned levels at $E_x = 3.035 \pm 0.015$ and $3.100 \pm 0.014$ MeV to the broad group they measured at $\sim 3$ MeV. The present work with its much higher resolution shows that there is only one level at $E_x = 3.056 \pm 0.009$ MeV, with a width of $14 \pm 2$ keV, or possibly two levels with spacing of less than 10 keV. The assignment of spins, parities, and analog states in $^{20}\text{F}$ will be discussed in detail in Chapter 6, where the present excitation energies will be used to determine the $^{19}\text{Ne}(p, \gamma)^{20}\text{Na}$ resonant reaction rate.
Table 5.1.

Tentative level structure from $^{20}$Ne($^3$He, $t$)$^{20}$Na measurements of Lamm et. al (La89, La90) and Kubuno et al. (Ku89).
20Na Excitation Energies and Analog Assignments
From Previous 20Ne(3He, t)20Na Studies

<table>
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<th>Ex(20Na) (MeV ± keV)</th>
<th></th>
<th>Ex(20F)(b) (MeV)</th>
<th>Jπ</th>
<th>l</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lamm et al. (La90)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Kubuno et al. (Ku89)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lamm et al. (La87)</td>
<td></td>
<td>2.570 ± 20(a)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2.649 ± 16</td>
<td>2.637 ± 15</td>
<td>2.660 ± 40</td>
<td>3.173</td>
<td>1+</td>
</tr>
<tr>
<td></td>
<td>2.836 ± 12</td>
<td>2.842 ± 15</td>
<td>2.880 ± 40</td>
<td>2.966</td>
<td>3+</td>
</tr>
<tr>
<td></td>
<td>2.972 ± 13</td>
<td>2.967 ± 20</td>
<td>2.960 ± 40</td>
<td>2.865</td>
<td>3-</td>
</tr>
<tr>
<td></td>
<td>3.035 ± 15</td>
<td>3.046 ± 20</td>
<td>3.060 ± 40</td>
<td>3.488</td>
<td>1+</td>
</tr>
<tr>
<td></td>
<td>3.100 ± 14</td>
<td>3.160 ± 40</td>
<td>3.526</td>
<td>0+</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>3.324 ± 11</td>
<td>3.302 ± 30</td>
<td>3.587</td>
<td>2+</td>
<td>2</td>
</tr>
</tbody>
</table>

(a) withdrawn as 20Na state in (La89b).

(b) Analogue assignments from Lamm et al. (La90),
    based on Jπ values from Lamm (La89) and Kubuno et al. (Ku89).
Figure 5.1.
$^{20}\text{Ne}(^{3}\text{He}, t)^{20}\text{Na}$ momentum spectra and excitation energies measured by Lamm et. al (La89) and Kubuno et al. (Ku89), both using gas-cell $^{20}\text{Ne}$ targets limiting the energy resolution to 60 - 80 keV.
Figure 5.2.

The target holder assembly for producing transmission $^{20}$Ne targets, showing the Ta collimator with an 8 mm - diameter aperture, the second collimator / electron suppressor biased at -300 V, and the 40 $\mu$g/cm$^2$ carbon foil mounted on a standard Al target frame.
Secondary Electron Supressor

Target Frame

12.7 mm diameter

Collimator

8 mm diameter

Rastered $^{20}\text{Ne}$ Beam

Target Current

To BCI

100 MΩ

Collimator Current

To BCI

10 MΩ

-300 V
Figure 5.3.

$175^\circ$ Rutherford backscattering spectrum of a transmission $^{20}\text{Ne}$ target bombarded with alpha particles of 0.9 MeV, showing two $^{20}\text{Ne}$-implanted regions.
Rutherford Backscattering Spectrum of a $^{20}\text{Ne}$ - Implanted
40 $\mu \text{g/cm}^2$ $^{12}\text{C}$ foil
Figure 5.4.
The target chamber, spectrograph, and focal plane detector at the Princeton University Cyclotron Facility (Ko74).
Figure 5.5.
The 60 cm focal plane detector for the $^{20}$Ne($^3$He, t)$^{20}$Na experiment, consisting of two position-sensitive resistive-wire gas proportional counters, measuring position and energy loss, backed by a thick plastic scintillator measuring residual energy (Ko74).
60 cm QDDE Detector Schematic

PROPEANE

CHARGED PARTICLE

0.3 MIL KAPTON PRESSURE WINDOW

0.5 MIL NICHROME WIRE

0.125 mil ALUMINIZED MYLAR

0.5" x 1.25" LIGHT PIPE PHOTOTUBE

0.25" PILOT B SCINTILLATOR

\( S_{GL} \sim \frac{X}{L} M^2 \)

\( S_{GR} \sim \frac{L-X}{L} M^2 \)

\( S_S \sim \left( \frac{Z}{M} \right)^{2.6} \)
Figure 5.6.

Schematic of the electronics for the $^{20}\text{Ne}(^{3}\text{He}, t)^{20}\text{Na}$ experiment.

AMP = Linear amplifier
CFD = Constant Fraction Discriminator
G+D = Gate and delay generator
PA = Preamplifier
Slow Coin = logical and μs coincidence
SUM = Summing amplifier
Figure 5.7.

$\Delta E$ vs. $E_{\text{residual}}$ plot for focal plane events, allowing the separation of the weak triton groups from the intense deuteron groups.
Figure 5.8.

Triton-gated position spectrum at $\Theta_t = 10^\circ$, $E_x = 2.6$ MeV in $^{20}\text{Na}$, with the 100-cm long focal plane detector, showing four $^{20}\text{Na}$ groups above the $^{19}\text{Ne} + p$ threshold at 2.199 MeV. The excitation energies shown are determined from the focal plane calibration for this run as discussed in the Section 5.4.
$^{20}\text{Ne}(^{3}\text{He}, t)^{20}\text{Na}$

$E_x^{(20}\text{Na})$ (MeV)

- 3.053
- 2.982
- 2.854
- 2.647

$^{12}\text{C}(^{3}\text{He}, d)^{13}\text{N}(6.36)$ leakage

$^{16}\text{F}$ (0.720 MeV)

$^{16}\text{F}$ (0.424 MeV)

Counts/Channel vs. Triton Momentum (Channel)
Figure 5.9.

Triton-gated position spectrum at $\Theta_t = 10^\circ$, $E_x = 2.6$ MeV in $^{20}\text{Na}$, with the 60-cm long focal plane detector, showing the same four $^{20}\text{Na}$ resonances as in Figure 5.8.
$^{20}\text{Ne}(^{3}\text{He}, t)^{20}\text{Na}$

$E_x^{^{20}\text{Na}}$ (MeV)

Counts/Channel

Triton Momentum (Channel)

- 2.861
- 2.645
- 3.058
- 2.990

$^{12}\text{C}(^{3}\text{He}, d)^{13}\text{N}(6.36)$ leakage
Figure 5.10.
Position spectra for the $^{19}$F($^3$He, $\alpha$) $^{18}$F and $^{30}$Si($^3$He, $\alpha$)$^{29}$Si reactions, used for a Position vs. Momentum calibration of the focal plane, at $\Theta = 10^\circ$ with the 100 - cm long counter.
Table 5.2.

Position and excitation energies of
$^{19}\text{F}(^{3}\text{He}, \alpha)^{18}\text{F}$ and $^{30}\text{Si}(^{3}\text{He}, \alpha)^{29}\text{Si}$
groups and resulting momentum calibration
of the focal plane.
<table>
<thead>
<tr>
<th>Channel</th>
<th>$\sigma_X$</th>
<th>$E_X$ (MeV)</th>
<th>$\sigma_{E_X}$ (MeV)</th>
<th>Channel</th>
<th>$\sigma_X$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{19}$F($^3$He, $\alpha$)$^{18}$F Fall</td>
<td>0.6</td>
<td>1.70081</td>
<td>0.00018</td>
<td>632.9</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td>0.5</td>
<td>2.10061</td>
<td>0.00010</td>
<td>536.8</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td>0.5</td>
<td>2.52335</td>
<td>0.00018</td>
<td>436.9</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td>0.5</td>
<td>3.06184</td>
<td>0.00018</td>
<td>313.5</td>
<td>0.5</td>
</tr>
<tr>
<td>856.6</td>
<td>0.5</td>
<td>3.13387</td>
<td>0.00015</td>
<td>536.8</td>
<td>0.5</td>
</tr>
<tr>
<td>738.8</td>
<td>0.5</td>
<td>3.35820</td>
<td>0.00100</td>
<td>569.2</td>
<td>0.5</td>
</tr>
<tr>
<td>588.4</td>
<td>0.5</td>
<td>3.56184</td>
<td>0.00018</td>
<td>509.7</td>
<td>0.5</td>
</tr>
<tr>
<td>569.2</td>
<td>0.5</td>
<td>3.13387</td>
<td>0.00015</td>
<td>536.8</td>
<td>0.5</td>
</tr>
<tr>
<td>509.7</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$^{30}$Si ($^3$He, $\alpha$)$^{29}$Si

<table>
<thead>
<tr>
<th>Fall</th>
<th>0.5</th>
<th>1.2733</th>
<th>0.0001</th>
</tr>
</thead>
<tbody>
<tr>
<td>1118.7</td>
<td>0.5</td>
<td>2.0282</td>
<td>0.0003</td>
</tr>
<tr>
<td>776.8</td>
<td>0.5</td>
<td>2.4526</td>
<td>0.0002</td>
</tr>
<tr>
<td>596.4</td>
<td>0.5</td>
<td>3.0671</td>
<td>0.0003</td>
</tr>
<tr>
<td>445.6</td>
<td>0.5</td>
<td>3.6235</td>
<td>0.0005</td>
</tr>
</tbody>
</table>

$^{12}$C($^3$He,$d$)$^{13}$N

<table>
<thead>
<tr>
<th>Fall</th>
<th>0.8</th>
<th>6.364</th>
<th>0.009</th>
</tr>
</thead>
<tbody>
<tr>
<td>1057.8</td>
<td>0.8</td>
<td>7.155</td>
<td>0.005</td>
</tr>
<tr>
<td>718.2</td>
<td>0.5</td>
<td>299.2</td>
<td>0.5</td>
</tr>
</tbody>
</table>

$^{16}$O($^3$He,$t$)$^{16}$F

<table>
<thead>
<tr>
<th>Fall</th>
<th>0.5</th>
<th>0.424</th>
<th>0.005</th>
</tr>
</thead>
<tbody>
<tr>
<td>1536.4</td>
<td>0.5</td>
<td>0.720</td>
<td>0.004</td>
</tr>
<tr>
<td>1203.0</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Resulting calibrations $\rho$ (cm) vs. channel $X$ and $B\rho$ (kG•cm) vs. $X$:

**Spring:**

$\rho$ (cm) = 91.22 + 5.030e-03 X + 4.845e-08 X^2

$B\rho$ (kG•cm) = 893.4 + 4.927e-02 X + 4.746e-07 X^2

**Fall:**

$\rho$ (cm) = 88.91 + 4.062e-03 X - 5.366e-08 X^2

$B\rho$ (kG•cm) = 876.9 + 4.006e-02 X - 5.292e-07 X^2
Contributions of beam energy, target thickness, spectrograph angle, position and excitation energies of calibrant peaks, and position of $^{20}$Na peaks to the uncertainty of the absolute excitation energies in $^{20}$Na.

Table 5.3.
<table>
<thead>
<tr>
<th>Effect</th>
<th>Uncertainty</th>
<th>Contribution to $\sigma_{E_x}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam Energy</td>
<td>$\sigma_{E_x} = 0.6 \sigma_{E_b}$ (keV)</td>
<td>6 keV</td>
</tr>
<tr>
<td>Target thickness</td>
<td>$\sigma_{E_x} = 0.1 \sigma_{\rho_t}$ (ug/cm$^2$)</td>
<td>0.2 keV</td>
</tr>
<tr>
<td>Angle of Spectrograph</td>
<td>$\sigma_{E_x} = 0.1 \sigma_{\Theta}$ (degrees)</td>
<td>1.5 keV</td>
</tr>
<tr>
<td>Centroids of $^{20}$Na peaks</td>
<td></td>
<td>0.6 keV</td>
</tr>
<tr>
<td>Centroids of Calibration peaks</td>
<td></td>
<td>$\sim$0.2 keV</td>
</tr>
<tr>
<td>Q-values of Calibration peaks</td>
<td></td>
<td>$\sim$0.4 keV</td>
</tr>
<tr>
<td>Total without $E_b$ uncertainty</td>
<td></td>
<td>1.7 keV</td>
</tr>
<tr>
<td>Total uncertainty</td>
<td></td>
<td>6.1 keV</td>
</tr>
</tbody>
</table>
Table 5.4.

Excitation energies of the $^{20}$Na states from the present $^{20}$Ne($^3$He, t)$^{20}$Na measurement.
Excitation Energies in $^{20}\text{Na}$ (a)

<table>
<thead>
<tr>
<th>Channel</th>
<th>$E_x$</th>
<th>Channel</th>
<th>$E_x^{(a)}$</th>
<th>$E_x$</th>
<th>$E_{\text{res}}^{(b)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spring</td>
<td>668.7</td>
<td>Spring</td>
<td>2.6453 ± 6</td>
<td>950.9</td>
<td>2.646 ± 5</td>
</tr>
<tr>
<td></td>
<td>2.6453 ± 6</td>
<td>Fall</td>
<td>2.6473 ± 6</td>
<td>2.646 ± 5</td>
<td>447 ± 9</td>
</tr>
<tr>
<td>499.0</td>
<td>2.8607 ± 6</td>
<td>Fall</td>
<td>748.4</td>
<td>2.8537 ± 6</td>
<td>2.857 ± 5</td>
</tr>
<tr>
<td>395.6</td>
<td>2.9904 ± 6</td>
<td>Fall</td>
<td>627.1</td>
<td>2.9816 ± 6</td>
<td>2.986 ± 5</td>
</tr>
<tr>
<td>341.3</td>
<td>3.0580 ± 6</td>
<td>Fall</td>
<td>560.4</td>
<td>3.0531 ± 6</td>
<td>3.056 ± 5</td>
</tr>
<tr>
<td></td>
<td>338.0</td>
<td>(3.298 ± 5)</td>
<td>(3.298 ± 5)</td>
<td>(1099 ± 9)</td>
<td>(857 ± 9)</td>
</tr>
</tbody>
</table>

(a) $E_x$ in MeV ± keV, based on the momentum calibration given in Table 5.2.

(b) $Q_0 = 2.199 ± 0.007$ MeV for the $^{19}\text{Ne}(p, \gamma)^{20}\text{Na}$ reaction (Aj87)

(c) Not confirmed as a $^{20}\text{Na}$ level.
Table 5.5.

Comparison of present and previous $^{20}\text{Na}$ excitation energies of Lamm et al. (La90) and Kubuno et al. (Ku89).
Comparison of $^{20}$Na Excitation Energies (a)

<table>
<thead>
<tr>
<th>Present (b)</th>
<th>Lamm et al. (La90)</th>
<th>Kubuno et al. (Ku89)</th>
<th>Lamm et al. (La87)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.646 ± 5</td>
<td>2.649± 16</td>
<td>2.637 ± 15</td>
<td>2.660± 40</td>
</tr>
<tr>
<td>2.857 ± 5</td>
<td>2.836± 12</td>
<td>2.842 ± 15</td>
<td>2.880± 40</td>
</tr>
<tr>
<td>2.986 ± 5</td>
<td>2.972± 13</td>
<td>2.967 ± 20</td>
<td>2.960± 40</td>
</tr>
<tr>
<td>3.056 ± 5</td>
<td>3.035 ± 15</td>
<td>3.046 ± 20</td>
<td>3.060 ± 40</td>
</tr>
<tr>
<td></td>
<td>3.100 ± 14</td>
<td></td>
<td>3.160 ± 40</td>
</tr>
<tr>
<td>(3.298 ± 5) (c)</td>
<td>3.324 ± 11</td>
<td>3.302 ± 30</td>
<td></td>
</tr>
</tbody>
</table>

(a) Excitation energies given in MeV ± keV

(b) Experimental uncertainty for the present work; with the 7-keV uncertainty in the mass of $^{20}$Na, these uncertainties become 9 keV. The relative excitation energies have an uncertainty of ± 2 keV.

(c) Not confirmed as a $^{20}$Na level.

(d) withdrawn as $^{20}$Na state in (La90).
Chapter 6. The Onset and Breakout of the Hot CNO Cycle

6.1 The Stellar $^{13}\text{N}(p, \gamma)^{14}\text{O}$ Reaction Rate

As discussed in Section 1.3, the conversion of the CNO cycle to the Hot CNO cycle results in an increased stellar energy generation rate and a substantial change in the relative abundances of the CNO nuclides. The goal of the $^1\text{H}(^{14}\text{N}, ^{14}\text{O})\gamma$ experiment is to measure the gamma-width of the $^{14}\text{O}^*(5.17\text{-MeV})$ level and thereby determine the $^{13}\text{N}(p, \gamma)^{14}\text{O}$ reaction rate as a function of temperature. A comparison can then be made with the $^{14}\text{N}(p, \gamma)^{15}\text{O}$ rate and the $^{13}\text{N}, ^{14}\text{O},$ and $^{15}\text{O} \beta^+$ decay rates to determine the precise temperatures and densities at which the conversion of the CNO cycle to the Hot CNO cycle takes place.

As discussed in Section 2.3, the total S-factor for the $^{13}\text{N}(p, \gamma)^{14}\text{O}$ reaction is given by Eq. 2.26 as the sum of the resonant S-factor, the non-resonant DC S-factor, and a third term due to their interference. The temperature dependence of the Gamow window, Figure 2.2, indicates that the resonant S-factor needs to be evaluated only for center-of-mass energies $E < 1$ MeV. Once the total S-factor is determined over this energy range, Eq. 2.10 can be used to determine the total reaction rate as a function of stellar temperature.

The expression for the resonant S-factor given by Eq. 2.14 will be used with the properties of the broad $^{14}\text{O}^*(5.169\text{-MeV})$ level to calculate the resonant reaction rate; no other resonances contribute significantly to the reaction rate at temperatures $0.1 \leq T_9 \leq 1.0$. The resonance parameters necessary for this indirect rate determination are the resonance energy ($E_r = 540 \pm 2$ keV), the total width and partial proton width ($\Gamma = \Gamma_p(E_r) = 38.1 \pm 1.8$ keV), the partial gamma-width ($\Gamma_\gamma = 2.9 \pm 1.3$ eV), and the spin $J_r = 1$. Figure 6.1 shows a plot of $S_{\text{res}}(E)$ for $E \leq 1$ MeV. The uncertainty in $S_{\text{res}}$ arises almost completely from the uncertainty in $\Gamma_\gamma$. 
The DC S-factor for the $^{13}$N(p, $\gamma$)$^{14}$O reaction is obtained from a measurement of the direct capture cross-section of the $^{13}$C(p, $\gamma$)$^{14}$N reaction to the 2.313-MeV, T=1, $J^\pi = 0^+$ first-excited state in $^{14}$N, the analog to the ground state of $^{14}$O (Az85). The $^{13}$C(p, $\gamma$)$^{14}$N cross-section cannot be similarly used for an estimate of the $^{13}$N(p, $\gamma$)$^{14}$O resonant cross-section because the E1 $\gamma$ - decay from the $^{14}$N*(8.062-MeV, T=1, $J^\pi = 1^-$) level, the analog of the $^{14}$O*(5.17-MeV) state, to the $^{14}$N*(2.313-MeV, T=1, $J^\pi = 0^+$) level is isospin forbidden. The theoretical $^{13}$C(p, $\gamma$)$^{14}$N direct capture cross-section is obtained from the measured DC cross-section and spectroscopic factor, and is proportional to the square of the integral of the product of the radial wavefunction and the E1 multipole operator (Ro73). It may thus be scaled to the $^{13}$N(p, $\gamma$)$^{14}$O reaction by a ratio of radial integrals and the Clebsch-Gordan coefficients (Fe89) to get

$$S_{\text{DC}}(E) = 3.68 \times 10^{-4} \exp\left(- 1.088 \times 10^{-3} E(\text{keV})\right) \text{ MeV - barns} \quad (6.1)$$

$S_{\text{DC}}$ is shown plotted with the $S_{\text{res}}$ in Figure 6.1, and is lower than $S_{\text{res}}$ by 1 and 4 orders of magnitude at the lowest energies and at the resonance energy, respectively.

The interference term between the DC and resonant S factors is given by Eq. 2.26, with the phase of the interference being given by the resonant phase shift in Eq. 2.27. As discussed in Section 2.3, the phase of the interference is constructive for energies below resonance and destructive above. The three terms are then summed to give the total S-factor which is also plotted in Figure 6.1. The uncertainty in $S_{\text{tot}}$, shown as the shaded region in Figure 6.1, arises almost completely from the $\sim 50$% uncertainty in $\Gamma_\gamma$.

The total S-factor is used in Eq. 2.10 to determine the total stellar reaction rate $N_A \langle \sigma v \rangle$ as a function of temperature for the $^{13}$N(p, $\gamma$)$^{14}$O reaction, shown plotted in Figure 6.2 and tabulated in Table 6.1 for 0.08 $\leq T_9 \leq 1.0$. The $^{14}$N(p, $\gamma$)$^{15}$O and $^{12}$C(p, $\gamma$)$^{13}$N reactions are also plotted in Figure 6.2, using rates taken from the compilation of Fowler et al. (Fo75). It is evident that the $^{13}$N(p, $\gamma$)$^{14}$O reaction is the slowest proton-capture reaction in the CNO cycle for temperatures 0.14 $\leq T_9 < 0.63$. 


In order to determine the characteristics of the onset of the Hot CNO cycle, the density-independent rate \( N_A <\sigma v> \) for the \(^{13}\text{N}(p, \gamma)^{14}\text{O} \) reaction is converted into a lifetime against destruction by hydrogen burning per \(^{13}\text{N} \) particle with the use of Eqs. 2.6 and 2.7,

\[
\tau_{^{13}\text{N}(H)} = \frac{N_{^{13}\text{N}}}{R_{^{13}\text{N}H}} = \left( \frac{A_H}{\rho X_H} \right) (N_A <\sigma v>)^{-1} ;
\]

(6.2)
a similar relation holds for the \(^{14}\text{N}(p, \gamma)^{15}\text{O} \) and \(^{12}\text{C}(p, \gamma)^{13}\text{N} \) reactions. These three lifetimes are then compared with the \(^{13}\text{N} \), \(^{14}\text{O} \), and \(^{15}\text{O} \beta^+ - \) decay lifetimes of 863, 102, and 176 s, respectively, in Figures 6.3 and 6.4 for two characteristic stellar densities. Figure 6.3 is plotted with a hydrogen density of \( \rho X_H = 100 \text{ g/cm}^3 \), typical of hydrogen burning in the core of a main-sequence star, while Figure 6.4 is plotted with a density of 5000 \text{ g/cm}^3 and hydrogen mass fraction \( X_H = 0.77 \), typical of nova explosions (Wi86a). For main-sequence densities, the lifetime of \(^{13}\text{N} \) burning via the \(^{13}\text{N}(p, \gamma)^{14}\text{O} \) reaction, \( \tau_{^{13}\text{N}} \), becomes shorter than the \(^{13}\text{N} \beta^+ - \) decay lifetime at \( T_9 = 0.11 \), at which point the cycle proceeds predominantly through \(^{14}\text{O} \) rather than \(^{13}\text{C} \) in the reaction sequence

\[
^{12}\text{C}(p, \gamma)^{13}\text{N}(p, \gamma)^{14}\text{O}(\beta^+\nu)^{14}\text{N}(p, \gamma)^{15}\text{O}(\beta^+\nu)^{15}\text{N}(p, \alpha)^{12}\text{C}.
\]

(6.3)

For temperatures above \( T_9 = 0.13 - 0.14 \), the proton capture reaction lifetimes become smaller than the sum of the \(^{14}\text{O} \) and \(^{15}\text{O} \beta^+ - \) decay lifetimes, \( \tau_{^{14}} + \tau_{^{15}} \). This is the onset of the Hot CNO cycle, during which the energy generation rate is limited by the \(^{14}\text{O} \) and \(^{15}\text{O} \beta^+ - \) decay rates. The \(^{13}\text{N}(p, \gamma)^{14}\text{O} \) reaction is 2.25 times faster than the \(^{13}\text{N} \beta^+ - \) decay at this temperature, meaning that 70% of the mass flow is through \(^{14}\text{O} \) rather than \(^{13}\text{C} \). By \( T_9 = 0.15 \) and 0.20, \(^{13}\text{N}(p, \gamma)^{14}\text{O} \) is a factor of 10 and 100 faster than the \(^{13}\text{N}(\beta^+\nu) \), respectively, resulting in only 10% and 1% of the flow going through \(^{13}\text{C} \). The two temperature ranges discussed here are typically reached in rapid succession, since the temperature is often increasing exponentially with time in explosive hydrogen burning events.

The onset of the Hot CNO cycle occurs at lower temperatures for the higher densities typical of nova explosions. Here, the mass flow through \(^{14}\text{O} \) begins at \( T_9 = 0.09 \), where the \(^{13}\text{N}(p, \gamma) \) and \(^{13}\text{N}(\beta^+\nu) \) have equal rates. The rate of the slowest proton capture reaction in
the sequence in Eq. 6.3, $^{14}\text{N}(p, \gamma)^{15}\text{O}$, exceeds the $^{14}\text{O}$ and $^{15}\text{O}$ $\beta^+$ - decay rates at $T_9 = 0.10$, at which temperature the $^{13}\text{N}(p, \gamma)^{14}\text{O}$ reaction is already 6 times faster than the $^{13}\text{N} \beta^+$ - decay rate. This results in 85% of the flow going through $^{14}\text{O}$ at the very onset of the Hot CNO cycle, increased to more than 99% when the temperature reaches $T_9 = 0.14$. It is evident that the Hot CNO cycle will be operating at even the lowest nova temperatures of $T_9 = 0.10$. This results in $^{14}\text{O}$ and $^{15}\text{O}$ being the most abundant nova CNO nuclides, and their $\beta^-$ decay products $^{14}\text{N}$ and $^{15}\text{N}$ should be enhanced in material ejected from nova explosions (Wi86a). Observations by Williams and Gallagher (Wi79) of elemental nitrogen enhancements (as well as carbon and oxygen enhancements) in novae ejecta of a factor of ~10-100 over solar abundances is consistent with a mixing of carbon-oxygen white dwarf material into the hydrogen accretion layer, resulting in explosive burning via the Hot CNO cycle and the production of $^{14}\text{N}$ and $^{15}\text{N}$.

For the general case, the onset of the Hot CNO cycle is found by equating the sum of the $^{14}\text{O}$ and $^{15}\text{O}$ $\beta^-$ decay rates and the lower of the two proton capture rates to get the density - temperature relationship

$$\rho(T) = \frac{A_H}{(\tau_{14} + \tau_{15}) X_H (N_A (\sigma v)(T))_{\text{min}}}.$$  \hspace{1cm} (6.4)

The resulting $\rho$ - $T$ curve is plotted in Figure 6.5. If the density and temperature of the stellar environment fall above this curve on the $\rho$ $\otimes$ $T$ plot, then the Hot CNO cycle operates. The adopted value of $\Gamma_\gamma$ of 2.9 eV in the present work indicates an only slightly less restrictive environment (lower temperatures) for the operation of the Hot CNO cycle at all stellar densities than that of Fernandez et al. (Fe89) ($\Gamma_\gamma = 2.6 \pm 1.3$ eV), and a significantly less restrictive environment than that resulting from the theoretical predictions of Funck and Langanke (Fu87) ($\Gamma_\gamma = 1.8$ eV) and Barker (Ba85) ($\Gamma_\gamma = 1.2$ eV). Similar to the conclusions of Mathews and Dietrich (Ma84), the present results indicate that the Hot CNO cycle is turned on at an earlier stage of a nova explosion. Additionally, it may effect the late stages of evolution of supermassive stars (low-density and high temperatures), where an earlier ignition of the Hot
CNO cycle could generate sufficient energy to prevent the collapse into a black hole (Fu82).

6.2. The Stellar $^{19}\text{Ne}(p, \gamma)^{20}\text{Na}$ Reaction rate

The total rate for the $^{19}\text{Ne}(p, \gamma)^{20}\text{Na}$ stellar reaction is a sum of the non-resonant direct capture rate and the resonant rate, where the latter is expected to dominate for the temperatures of interest, $0.1 \leq T_9 \leq 1.0$. Because $\Gamma \ll E_\gamma$ for the $^{20}\text{Na}$ states as determined by the $^{20}\text{Ne}(^{3}\text{He}, t)^{20}\text{Na}$ measurement in Chapter 5, the narrow resonance formalism may be used to calculate the resonant $^{19}\text{Ne}(p, \gamma)^{20}\text{Na}$ reaction rate. Additionally, because the spacing of the levels is much greater than their widths, $\Delta E_x \gg \Gamma$, the contributions from the individual resonances may simply be summed together as in Eq. 2.25 to give the total resonance rate.

The resonance parameters necessary for this calculation are the resonance energies $E_\gamma$ and the resonance strengths $\omega \gamma$ of the $^{20}\text{Na}$ levels. Because of the exponential dependence of the resonant rate on the resonance energies, these are the most important parameters for the rate calculation. The new values presented in Section 5.4 represent a decrease in the uncertainty of the reaction rate by a factor of 6 - 10 at the lowest temperatures over the previous calculation of Lamm et al. (La90), based on measurements of excitation energies with 3 - 4 times larger uncertainties.

Lacking any measurements of the proton or gamma widths in $^{20}\text{Na}$ at the present time, it is necessary to use information from the analog states in $^{20}\text{F}$ to estimate the resonance strengths $\omega \gamma$. The first information necessary to make analog assignments and to determine possible $l$ - transfers in the $^{19}\text{Ne} + p$ reaction is the spins and parities of the four observed $^{20}\text{Na}$ resonances. Since the ground state of $^{19}\text{Ne}$ has $J^\pi = (1/2)^+$, a $^{20}\text{Na}$ resonance with $J^\pi = 1^+$ can correspond to an $l = 0$ proton capture and thus can have a tremendous effect on the resonant reaction rate, especially when it is in the Gamow window. For example, the penetrability and thus the proton width for an $l = 2$ transfer is $10^3$ times smaller than for an
$l = 0$ transfer for the $^{19}\text{Ne}(p, \gamma)^{20}\text{Na}$ reaction, for the same reduced width $\Theta_l^2$. However, the importance of the $l$-transfer decreases as the resonance energy increases. If $E_r$ is greater than a few hundred keV, the partial gamma-width of the state is usually smaller than the partial proton-width, and therefore the resonance strength is determined by the gamma width.

It can be argued that this is the case for the $^{20}\text{Na}$ states above the proton threshold. The gamma widths for the resonances in $^{20}\text{Na}$ can be taken from the gamma-widths of the analog $^{20}\text{F}$ states, appropriately scaled by the gamma energy as in Eq. 2.22. The analog assignments are, at present, still tentative. However, for all the possible analog states in $^{20}\text{F}$, the gamma widths have values of $\sim 0.001 - 0.01$ eV. This suggests that $\Gamma_\gamma \ll \Gamma_p$ will hold for any $^{20}\text{Na}$ levels above the $^{19}\text{Ne} + p$ threshold which have a proton width greater than 1 eV, and thus that $\omega_\gamma = \omega \Gamma_\gamma$ for these resonances. Hence the determination of the gamma-widths is more important than the proton widths for the calculation of the $^{19}\text{Ne}(p, \gamma)^{20}\text{Na}$ resonant reaction rate.

The spins and parities of the resonances in $^{20}\text{Na}$ are taken from the comparisons made by Lamm et al. (La90) and Kubuno et al. (Ku89) of $^{20}\text{Ne}(^3\text{He}, t)^{20}\text{Na}$ angular distribution measurements and DWBA calculations as discussed in Section 5.1. It should be pointed out that the $^{20}\text{Ne}(^3\text{He}, t)^{20}\text{Na}$ reaction is not very $l$-transfer-selective, and that DWBA calculations for this reaction are much more dependent on the excitation energies than on the $l$-transfers (Sh90). Furthermore, Lamm et al. do not resolve the third and fourth levels, and there is an additional level assigned at $E_x = 3.100 \pm 14$ keV that is not seen in the present work with higher energy resolution. This makes their extraction of these two angular distributions somewhat suspect. The assignments made on the basis of these studies are: $J^\pi = 1^+$ for the 447-keV resonance; $3^+$ for the 658-keV resonance; ($3^+$) for the 787-keV resonance; and $1^+$ for the 857-keV resonance. On the basis of these $J^\pi$ values, tentative analog assignments have been made to states in $^{20}\text{F}$. The analogs of the 658-keV and 787-keV resonances are the 2.966 and 2.865 MeV levels in $^{20}\text{F}$, respectively. Since these are expected to be $l = 2$ and 3 proton
captures in $^{19}\text{Ne} + p$, respectively, and since they are not in the $^{19}\text{Ne}(p, \gamma)$ Gamow window until $T_9 > 0.76$ and $T_9 > 0.98$, respectively, the contribution of these resonances to the reaction rate is small for $0.1 < T_9 < 1.0$. The analog state of the $J^\pi = 1^+$ resonance at $E_x = 857 \text{ keV}$, the fourth $^{20}\text{Na}$ level above the $^{19}\text{Ne} + p$ threshold, is the $3.488-\text{MeV}$, $J^\pi = 1^+$ level in $^{20}\text{F}$. This level is not in the Gamow window until $T_9 > 1.1$, and hence will contribute little to the reaction rate at nova temperatures in spite of its $l = 0$ nature. The unconfirmed level from this study at $E_x = 3.298 \pm 0.009 \text{ MeV}$ ($E_{\text{res}} = 1099 \pm 9 \text{ keV}$) has no contribution to the resonant reaction rate at nova temperatures.

This is not the case for the $J^\pi = 1^+$, 447-keV resonance in $^{20}\text{Na}$, which is in the Gamow window for temperatures $T_9 > 0.44$. It has been argued by Lamm et al. (La90) that the analog of the 447-keV state is the $3.173-\text{MeV}$, $J^\pi = 1^+$ level in $^{20}\text{F}$. The spin and parity of this $^{20}\text{F}$ state are determined from measurements of the $^{19}\text{F}(d,p)^{20}\text{F}$ (Fo74), $^{18}\text{O}(^3\text{He},p)^{20}\text{F}$ (Me76), and $^{14}\text{N}(^7\text{Li},p)^{20}\text{F}$ (Fo77) reactions; however, its detailed nuclear structure is uncertain at present. It is argued that this state in $^{20}\text{F}$ cannot be explained in terms of the sd-shell; rather that it is the lowest lying $2\hbar \omega J^\pi = 1^+$ core-excited intruder state of the $(1p)^2 (2s1d)^6$ configuration space with little or no mixing with the $0\hbar \omega J^\pi = 1^+ (2s1d)^4$ state (Me76),(Ai87). The $l = 0$ strength of such an intruder state is expected to be very small, in agreement with an angular distribution measurement of the $^{19}\text{F}(d,p)^{20}\text{F} *(3.173-\text{MeV})$ reaction (Fo74). It can then be argued that the $l = 2$ strength will dominate the $l = 0$ strength for $^{19}\text{Ne}$ proton capture through the analog $^{20}\text{Na}$ resonance at $E_x = 447 \text{ keV}$. Note that this is only important if the proton width of this state is calculated to be smaller than the gamma width, in which case the approximation $\omega \gamma = \omega \Gamma_\gamma$ cannot be used. This calculation is done using Eqs. 2.15 and 2.16 with the spectroscopic factors of the $^{20}\text{Na}$ states being taken from the single particle spectroscopic factors $\Theta_n^2$ in the analog $^{20}\text{F}$ states from $^{19}\text{F}(d,p)^{20}\text{F}$ (Fo74), with the results given in Table 6.2.

The gamma-widths, which determine the resonance strengths for the $^{19}\text{Ne}(p, \gamma)^{20}\text{Na}$
reaction, can also be taken from the analog states in $^{20}\text{F}$. For the $J^\pi = 3^+$, 658-keV resonance, the $\gamma$-decay of the analog 2.966-MeV $^{20}\text{F}$ level is predominantly M1 to the 0.822-MeV level; for the $J^\pi = 1^+$, 857-keV resonance, the $\gamma$-decay of the analog 3.488-MeV $^{20}\text{F}$ level is predominantly M1 to the ground state of $^{20}\text{F}$ (Aj87). The gamma widths inferred from lifetime measurements of these states are 0.0109 and 0.0149 eV, respectively. The lifetimes of the $^{20}\text{F}$ analog states of the 447-keV and 787-keV $^{19}\text{Ne} + p$ resonances are unknown, and therefore their gamma widths must be calculated from systematics. It is known (Aj87) that the 3.173-MeV $^{20}\text{F}$ level, the analog to the 447-keV resonance, decays > 95% by an E1 transition to the $J^\pi = 1^-$, 0.984 MeV level, and that the 2.865-MeV $^{20}\text{F}$ level, the analog to the 787-keV resonance, decays = 100% via E1 to the $J^\pi = 2^+$ ground state of $^{20}\text{F}$. Calculations of $\Gamma_\gamma$ for these $^{20}\text{F}$ states have been performed by Langanke et al. (La86) and Lamm et al. (La90). The corresponding $^{20}\text{Na}$ gamma widths are given in Table 6.2. There is roughly an order of magnitude uncertainty on these $\gamma$-widths; even so, they are small enough to allow the approximation $\omega \gamma = \omega \Gamma_\gamma$ to be used. A summary of all the resonance parameters is given in Table 6.2.

The resonant reaction rate can now be calculated using Eq. 2.25 with the resonance parameters given in Table 6.2 for the first four levels above the proton threshold in $^{20}\text{Na}$. Figure 6.6 plots the individual contributions from the 447-, 658-, 787-, and 857-keV resonances and their sum. It is evident that the resonant rate is completely determined by the 447-keV resonance for temperatures $0.1 < T_\odot < 0.9$; the other resonances begin to contribute only at higher temperatures. The uncertainty in the resonant reaction rate over this temperature range, shown as the shaded region in Figure 6.6, is almost completely due to the order of magnitude uncertainty in $\Gamma_\gamma$ for the 447-keV resonance.

The non-resonant direct capture rate is taken from the calculation of Langanke et al. (La86), with direct capture proceeding to the ground state ($J^\pi = 2^+$) and the excited states at
0.59 MeV (3⁺), 0.77 MeV (4⁺), and 0.96 MeV (1⁺). It can be expressed in the form discussed in Section 2.3 as

\[ N_A (\sigma v)_{DC} = 1.713 \times 10^6 T_9^{2/3} \exp\left(-19.38 T_9^{-1/3}\right) \cdot (1 + 2.12 \times 10^{-2} T_9^{1/3} + 1.30 \times 10^{-1} T_9^{2/3} + 1.95 \times 10^{-2} T_9 + 3.86 \times 10^{-2} T_9^{4/3} + 1.47 \times 10^{-2} T_9^{5/3}) \] (6.5)

The direct capture process is the only non-resonant process which contributes significantly to the reaction rate. The small estimated total width of 0.170 eV for the 447-keV resonance in Lamm et al. (La90) makes capture into the tail for this resonance at low (~100 keV) energies negligible. Capture into the tail of the 857-keV resonance still occurs, but is reduced to an order of magnitude lower than the DC rate for \( T_9 < 1 \), and can therefore be neglected. Furthermore, interference effects between the resonant and non-resonant terms do not need to be considered because of the small widths and large spacings of the \(^{20}\)Na levels.

The non-resonant DC rate is plotted as a function of temperature in Figure 6.7, along with the resonant reaction rate and their sum, giving the total reaction rate for the \(^{19}\)Ne(p, γ)^{20}\)Na reaction. The DC rate dominates the total rate for the temperatures 0.1 < \( T_9 < 0.2 \), and the resonant rate dominates for all higher temperatures. A tabulation of the rate is presented in Table 6.3. As is true for the resonant reaction rate, the uncertainty in the total reaction rate is almost completely due to the order of magnitude uncertainty in \( \Gamma_\gamma \) for the 447-keV resonance, making this partial width the most important of the unknown resonance parameters to measure.

A comparison can now be made between the \(^{19}\)Ne(p, γ)^{20}\)Na reaction rate and the rate of the 17 s \(^{19}\)Ne β⁺ - decay, using the relation

\[ \rho X_H N_A(\sigma v)(T)/A_H = (\tau_{19})^{-1} \] (6.6)

Figure 6.8 has a plot of the ratio of the proton capture rate and the \(^{19}\)Ne(β⁺v) decay rate as a function of temperature with a typical nova density of 5000 gm/cm³ and hydrogen mass fraction of 0.77 (Wi86a). In this case, the two rates are equal at temperatures \( T_9 = 0.25 \pm 0.03 \). Figure 6.8 also includes the ratio of the \(^{15}\)O(α, γ)^{19}\)Ne reaction rate
(Ma90) and the cycle time for the Hot CNO cycle, \( \tau_{14} + \tau_{15} \), where a helium mass fraction of \( X_{\text{He}} = 0.20 \) is assumed. The uncertainty in the \( ^{15}\text{O}(\alpha, \gamma)^{19}\text{Ne} \) reaction rate is almost completely due to the uncertainty of \( \Gamma_\alpha \) for the 4033-keV resonance in \( ^{19}\text{Ne} \), the first state above the \( ^{15}\text{O} + \alpha \) threshold in \( ^{19}\text{Ne} \). The ratio of these latter two rates is equal to 1.0 at temperatures \( T_9 = 0.35 \pm 0.05 \). It is evident from Figure 6.8 that when the temperature is hot enough for the \( ^{15}\text{O}(\alpha, \gamma)^{19}\text{Ne} \) rate to initiate a breakout, \( T_9 = 0.35 \), the \( ^{19}\text{Ne}(p, \gamma)^{20}\text{Na} \) rate already is faster than the \( ^{19}\text{Ne}(\beta^+, \nu) \) decay rate by a factor of \( \sim 200 \), with an uncertainty of an order of magnitude. This indicates that only 0.05% - 5% of the \( ^{19}\text{Ne} \) nuclei will be returned to the Hot CNO cycle rather than continue on the \( ^{15}\text{O}(\alpha, \gamma)^{19}\text{Ne}(p, \gamma)^{20}\text{Na} \) breakout sequence, clearly indicating that the \( ^{15}\text{O}(\alpha, \gamma)^{19}\text{Ne} \) reaction rate is the critical rate for determining when a breakout occurs. It is important to know the rates of both reactions as accurately as possible, however, in order to better determine the amount and timescale for the processing of CNO nuclides out of the Hot CNO cycle. This requires a measurement of the gamma width of the 447-keV resonance in \( ^{20}\text{Na} \), and the alpha width of the 4033-keV resonance in \( ^{19}\text{Ne} \).

In the general case, Eq. 6.6 is solved for the density \( \rho \), giving the density - temperature relationship plotted in Figure 6.9. Also included in this figure is the analogous relationship between the \( ^{15}\text{O}(\alpha, \gamma)^{19}\text{Ne} \) reaction rate (Ma90) and the cycle time for the Hot CNO cycle, \( \tau_{14} + \tau_{15} \). Again, it is evident that when the temperature and density of the stellar environment are such that the breakout is initiated with the \( ^{15}\text{O}(\alpha, \gamma)^{19}\text{Ne} \) reaction, the breakout will continue with the \( ^{19}\text{Ne}(p, \gamma)^{20}\text{Na} \) reaction, and thus the seed nuclei are processed into the mass \( A > 20 \) region where they cannot return to the Hot CNO cycle. Typical nova temperatures and densities are also indicated in Figure 6.9, which shows that the \( ^{15}\text{O}(\alpha, \gamma)^{19}\text{Ne}(p, \gamma)^{20}\text{Na} \) is a viable breakout mechanism for the hottest and densest nova explosions. This may account for the Ne-nova ejecta and the high \( ^{22}\text{Ne}/^{20}\text{Ne} \) galactic abundance ratio. In order to determine the viability of this explanation, a measurement of this alpha-particle width for the 4033-keV state in \( ^{19}\text{Ne} \) is necessary to reduce the uncertainty in the
15O(α, γ)19Ne reaction rate.

6.3. Summary of the Current Determinations of the 13N(p, γ)14O and 19Ne(p, γ)20Na Reaction Rates

The experimental measurements of the 1H(14N, 14O)n and 14N (p, n)14O reactions presented in Chapters 3 and 4, respectively, are used to determine a value of 12 ± 7 eV for the gamma-width of the 5.17-MeV level in 14O, consistent with the upper limit of Aguer et al. (Γγ < 12 eV), but a factor of 4 higher than the measurement of Fernandez et al. (2.6 ± 1.3 eV). Both the present measurement and that of Fernandez et al. are consistent with Γγ = 0 at the 2-σ level, and taking a weighted mean gives Γγ = 2.9 ± 1.3 eV. This still has a 50% uncertainty, and is well within the 1-σ uncertainty of the measurement by Fernandez et al. Using this value for the gamma width to calculate the 13N(p, γ)14O stellar reaction rate, it is found in Section 6.1 that

1. The 13N(p, γ)14O reaction is the slowest proton-capture reaction in the CNO cycle for temperatures 0.14 < T9 < 0.63;
2. At main-sequence densities, the 13N(p, γ)14O reaction becomes faster than the 13N β+ - decay at T9 = 0.11, and the onset of the Hot CNO cycle occurs for temperatures T9 > 0.14, where the 13N(p, γ)14O reaction is already 5 times faster than the 13N β+ - decay rate;
3. At nova densities, the mass flow through 14O begins at T9 = 0.09, and the onset of the Hot CNO cycle occurs at T9 = 0.10, where the 13N(p, γ)14O reaction is already 6 times faster than the 13N β+ - decay rate;
4. The adopted value of Γγ of 2.9 eV in the present work indicates a significantly less restrictive environment (lower densities at temperatures T9 > 0.1) for the operation of the Hot CNO cycle than that resulting from the theoretical predictions of Funck and
Langanke (Fu87) \((\Gamma_\gamma = 1.8 \, \text{eV})\) and Barker (Ba85) \((\Gamma_\gamma = 1.2 \, \text{eV})\).

The factor of 4 difference in \(\Gamma_\gamma\) between the present work and that of Fernandez et al., and the fact that both are consistent with \(\Gamma_\gamma = 0\) at the 2-\(\sigma\) level, suggest that an additional measurement of this gamma width is necessary. The implications of a 12 eV gamma width are, in a list similar to that given above,

1. the \(^{13}\text{N}(p, \gamma)^{14}\text{O}\) reaction is the slowest proton-capture reaction in the CNO cycle for temperatures \(0.18 < T_9 < 0.47\);

2. at main-sequence densities, the \(^{13}\text{N}(p, \gamma)^{14}\text{O}\) reaction becomes faster than the \(^{13}\text{N}\) \(\beta^+\) - decay at \(T_9 = 0.11\), and the onset of the Hot CNO cycle occurs for temperatures above \(T_9 = 0.13\), where the \(^{13}\text{N}(p, \gamma)^{14}\text{O}\) reaction is already 14 times faster than the \(^{13}\text{N}\) \(\beta^+\) - decay rate;

3. at nova densities, the mass flow through \(^{14}\text{O}\) begins at \(T_9 = 0.08\), and the onset of the Hot CNO cycle occurs at \(T_9 = 0.10\), where the \(^{13}\text{N}(p, \gamma)^{14}\text{O}\) reaction is already 19 times faster than the \(^{13}\text{N}\) \(\beta^+\) - decay rate;

4. A much less restrictive environment for the operation of the Hot CNO cycle results from \(\Gamma_\gamma = 12 \, \text{eV}\) than that of all previous experimental and theoretical estimates.

It may be possible to make another measurement of the branching ratio \(\Gamma_\gamma / \Gamma\) with modifications of the \(^{14}\text{N}(^3\text{He}, t)^{14}\text{O}_1(1^{14}\text{O}_0)\gamma\) triton-\(^{14}\text{O}_0\) recoil coincidence experiment used by Wang (Wa86) to set an upper limit of \(\Gamma_\gamma < 17 \, \text{eV}\). Using a newly-installed Split-Pole magnetic spectrograph at Yale to detect the recoil \(^{14}\text{O}_0\)'s may eliminate the majority of the background recoils which previously limited the counting rate. Preliminary work on this experiment is currently in progress.

The experimental measurement of the \(^{20}\text{Ne}(^3\text{He}, t)^{20}\text{Na}\) reaction presented in Chapter 5 is used to determine the excitation energies and upper limits on the total widths of resonances in \(^{20}\text{Na}\) above the 2.199-MeV \(^{19}\text{Ne} + p\) threshold. The present measurement determines that \(\Gamma \propto E_\gamma\) for the \(^{20}\text{Na}\) resonances, therefore allowing the use of the narrow resonance formalism
to calculate the resonant \(^{19}\text{Ne}(p, \gamma)^{20}\text{Na}\) reaction rate. It is also determined that the spacing of the levels is much greater than their widths, \(\Delta E_r \gg \Gamma\), and hence the contributions from the individual resonances may simply be summed together to give the total resonance rate.

Finally, the excitation energies are measured with an uncertainty of 5 keV in this experiment, as compared to \(\pm 16\) keV in the study of Lamm et al. (La90); adding in the uncertainty in the \(Q_\sigma\) value results in a total uncertainty of 9 keV for the absolute excitation energy of each of these resonances. The center of mass resonance energies relative to the first level above threshold are determined to \(\pm 2\) keV. The limitation of the precision of the resonance energy values is now given by the uncertainty of the \(^{20}\text{Na}\) mass. The uncertainty in \(E_r\) in the study of Lamm et al. (La90) contributes an uncertainty of 540\% in the resonant reaction rate at \(T_\rho = 0.1\), and 85\% at \(T_\rho = 0.3\); the present results, \(\delta E_r = 9\) keV, reduce the uncertainty to 180\% uncertainty at \(T_\rho = 0.1\), and 40\% uncertainty at \(T_\rho = 0.3\). The \(^{19}\text{Ne}(p, \gamma)^{20}\text{Na}\) proton capture rate and the \(^{19}\text{Ne}(\beta^+\nu)\) decay rate are found at nova densities to be equal at \(T_\rho = 0.25 \pm 0.03\), as described in Section 6.2 Since the \(^{15}\text{O}(\alpha, \gamma)^{19}\text{Ne}\) reaction initiates a breakout at \(T_\rho = 0.35 \pm 0.05\), where the \(^{19}\text{Ne}(p, \gamma)^{20}\text{Na}\) rate already is faster than the \(^{19}\text{Ne}(\beta^+\nu)\) decay rate by a factor of \(\sim 200\), this indicates that only \(\sim 0.5\%\) of the \(^{19}\text{Ne}\) nuclei will be returned to the Hot CNO cycle rather than continue on the \(^{15}\text{O}(\alpha, \gamma)^{19}\text{Ne}(p, \gamma)^{20}\text{Na}\) breakout sequence.

Therefore the \(^{15}\text{O}(\alpha, \gamma)^{19}\text{Ne}(p, \gamma)^{20}\text{Na}\) is a viable breakout mechanism for the hottest and densest nova explosions and therefore may account for the Ne-nova ejecta and the high \(^{22}\text{Ne}/^{20}\text{Ne}\) galactic abundance ratio; furthermore, the \(^{15}\text{O}(\alpha, \gamma)^{19}\text{Ne}\) reaction rate is the critical rate for determining when a breakout occurs. A measurement of the alpha width of the 4033-keV resonance in \(^{19}\text{Ne}\), and the gamma width of the 447-keV resonance in \(^{20}\text{Na}\), is necessary to better determine the precise amount and timescale for the processing of CNO nuclides out of the Hot CNO cycle.
Figure 6.1.

The resonant, non-resonant direct capture, and total Astrophysical S-Factors for $E_{cm} \leq 1$ MeV for the $^{13}\text{N}(p, \gamma)^{14}\text{O}$ reaction, using $\Gamma_{\gamma} = 2.9 \pm 1.3$ eV for the 540-keV resonance in $^{14}\text{O}$. The shaded region shows the 1-$\sigma$ uncertainty in $S_{tot}(E)$. 
Astrophysical S-Factor for the $^{13}\text{N} \left( p, \gamma \right) ^{14}\text{O}$ reaction
Figure 6.2.
Total stellar reaction rate $N_A \langle \sigma v \rangle$ as a function of temperature for the $^{13}\text{N}(p, \gamma)^{14}\text{O}$ reaction from the present work, and the $^{14}\text{N}(p, \gamma)^{15}\text{O}$, and $^{12}\text{C}(p, \gamma)^{13}\text{N}$ reactions taken from the compilation of Fowler et al. (Fo75).
$^{13}\text{N} (p, \gamma)^{14}\text{O}$ Stellar Reaction Rate
The total stellar reaction rate $N_A \langle \sigma v \rangle$ as a function of temperature for the $^{13}\text{N}(p, \gamma)^{14}\text{O}$ reaction, in cm$^3$/mole s. The $^{12}\text{C}(p, \gamma)^{13}\text{N}$ and $^{14}\text{N}(p, \gamma)^{15}\text{O}$ reaction rates are shown for comparison.
Reaction Rates $N_A <\sigma v>$ in cm$^3$/mole s

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Figure 6.3.

Comparison of $^{13}\text{N}(p, \gamma)^{14}\text{O}$, $^{14}\text{N}(p, \gamma)^{15}\text{O}$, and $^{12}\text{C}(p, \gamma)^{13}\text{N}$ reaction rates to the $^{13}\text{N}$, $^{14}\text{O}$, and $^{15}\text{O} \beta^+$- decay rates as a function of temperature, for a stellar hydrogen density of $\rho \times X_\text{H} = 100 \text{ g/cm}^3$ typical of main sequence burning.
Lifetimes of $^{12}$C, $^{13}$N, and $^{14}$N against Hydrogen Burning at $\rho X_H = 100 \text{ gm/cm}^3$
Figure 6.4.

Comparison of $^{13}\text{N}(p, \gamma)^{14}\text{O}$, $^{14}\text{N}(p, \gamma)^{15}\text{O}$, and $^{12}\text{C}(p, \gamma)^{13}\text{N}$ rates to the $^{13}\text{N}$, $^{14}\text{O}$, and $^{15}\text{O}$ $\beta^+$- decay rates as a function of temperature, for a stellar density of $\rho = 5000 \text{ g/cm}^3$ and hydrogen mass fraction of $X_H = 0.77$ typical of nova explosions.
Lifetimes of $^{12}$C, $^{13}$N, and $^{14}$N against Hydrogen Burning at $\rho = 5000$ gm/cm$^3$
Figure 6.5.

Density and temperature range where the Hot CNO cycle operates, defined by requiring the slowest proton capture rate \((^{13}N(p, \gamma)^{14}O\) or \(^{14}N(p, \gamma)^{15}O\) ) to be faster than the sum of the \(^{14}O\) and \(^{15}O\) \(\beta^+\) - decay lifetimes,

\[\tau_{14} + \tau_{15}.\]
Density and Temperatures for the Onset of the Hot CNO Cycle

Temperature $T_9$

- Novae
- Hot CNO Cycle
- CNO Cycle
Summary of the resonance parameters necessary for an indirect determination of the $^{19}\text{Ne}(p, \gamma)^{20}\text{Na}$ reaction rate as a function of temperature. The resonance energies are taken from the present work.

Table 6.2.
Resonance Parameters for the $^{19}$Ne(p, $\gamma$)$^{20}$Na reaction

<table>
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<tr>
<th>Ex($^{20}$Na) (MeV)</th>
<th>$E_{\text{res}}$ (MeV)</th>
<th>Ex($^{20}$F) (a) (MeV)</th>
<th>$J^\pi$</th>
<th>$I$</th>
<th>$\Theta_i^2$ (b)</th>
<th>$\Gamma_p$ (eV)</th>
<th>L</th>
<th>$E_\gamma$($^{20}$F) (MeV)</th>
<th>$E_\gamma$($^{20}$Na) (MeV)</th>
<th>$\Gamma_\gamma$($^{20}$F) (c) (eV)</th>
<th>$\Gamma_\gamma$($^{20}$Na) (d) (eV)</th>
<th>$\omega_\gamma$ (eV)</th>
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</table>

(a) Analogue assignments from Lamm et al. (La90), based on $J^\pi$ values from Lamm (La89) and Kubuno et al. (Ku89).

(b) $\Theta_i^2$ from $^{19}$F($^3$He, d)$^{20}$F (Fo74) spectroscopic factors ($\Theta_n^2$)

(c) $\Gamma_\gamma$($^{20}$F) from lifetimes (Aj87)

(d) $\Gamma_\gamma$($^{20}$Na) scaled from $\Gamma_\gamma$($^{20}$F) values or calculated from systematics by Langanke et al. (La86) and Lamm et al. (La90).
Figure 6.6.

The $^{19}$Ne(p, $\gamma$)$^{20}$Na resonant reaction rate as a function of temperature for $0.1 < T_9 < 1.0$. The 447-keV resonance dominates the rate over this temperature range.
$^{19}\text{Ne} (p, \gamma)^{20}\text{Na}$ Resonant Reaction Rate

Reaction Rate $N_A\langle \sigma v \rangle$ cm$^3$/mole s

Temperature $T_9$
Figure 6.7.

The resonant, non-resonant direct capture, and total reaction rate for the $^{19}\text{Ne}(p, \gamma)^{20}\text{Na}$ reaction. The DC rate dominates for $T_9 < 0.20$, and the resonant rate for higher temperatures.
$^{19}\text{Ne} (p, \gamma)^{20}\text{Na}$ Total Reaction Rate
Table 6.3.

The resonant, non-resonant direct capture, and total $^{19}\text{Ne}(p, \gamma)^{20}\text{Na}$ reaction rates as a function of temperature, in cm$^3$/mole s. Also tabulated is the $^{15}\text{O} (\alpha, \gamma)^{19}\text{Ne}$ reaction rate (Ma90).
### Reaction Rates $N_A<\sigma v>$ in cm$^3$/mole s

<table>
<thead>
<tr>
<th>$^19$Ne(p, $\gamma$)$^{20}$Na</th>
<th>$^15$O($\alpha$, $\gamma$)$^{19}$Ne</th>
</tr>
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Figure 6.8.

Ratio of the $^{19}\text{Ne}(p, \gamma)^{20}\text{Na}$ and $^{19}\text{Ne}(\beta^+\nu)$ rates, and ratio of $^{15}\text{O}(\alpha, \gamma)^{19}\text{Ne}$ and cycle time for the Hot CNO cycle, as a function of temperature, for a stellar density of $\rho = 5000 \text{ g/cm}^3$ and mass fractions of $X_\text{H} = 0.77$ and $X_\text{He} = 0.20$, typical of nova explosions.
Ratio of Proton Capture and Beta-decay Rates

$^{19}\text{Ne}(p,\gamma)^{20}\text{Na}$

$^{15}\text{O}(\alpha,\gamma)^{19}\text{Ne}$

Temperature $T_9$
Figure 6.9.

Density and temperature range where a breakout of the Hot CNO cycle occurs, defined by requiring the $^{19}\text{Ne}(p, \gamma)^{20}\text{Na}$ rate to be faster than the $^{19}\text{Ne}(\beta^+\nu)$ rate, and the $^{15}\text{O}(\alpha, \gamma)^{19}\text{Ne}$ rate to be faster than the sum of the $^{14}\text{O}$ and $^{15}\text{O}$ $\beta^+$ - decay lifetimes,

$\tau_{14} + \tau_{15}$.
Density and Temperatures for Breakout from the Hot CNO Cycle
References

(Ab65) M. Abramowitz, I.A. Stegun, Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables. Dover, New York, 1965, 538.


(Bu57) E.M. Burbridge, G.R. Burbridge, W.A. Fowler, F. Hoyle, Rev. Mod. Phys. 29 (1957) 547.


(Ca77) G.R. Caughlan, CNO-Isotopes in Astrophysics, Reidel, Netherlands, 1977, 121.


(Ch90a) A.E. Champagne, private communication.


(Fu79) H.W. Fulbright, J.R. Erskine, NIM 162 (1979) 355.
(Fu82) G.M. Fuller, S.E. Woosley, T.A. Weaver, Bull. AAS 14 (1982) 947.
(Re90) K.E. Rehm, private communication.

(Ro90) C. Rolfs, private communication.

(Sh90) R. Sherr, preprint, Princeton University, 1990.


(Wa87) T.F. Wang, private communication.


