THE SPECTRUM OF $^{205}\text{Pb}$ AS EXCITED BY THE (d,p) REACTION

BY

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The excitation spectrum of $^{205}$Pb, heretofore largely unknown, has been examined by means of the $^{204}$Pb ($d,p$) $^{205}$Pb reaction. The principal purpose of this study was to determine the extent to which the concepts of the simple shell model would be useful in describing the spectrum of this nucleus three holes removed from a closed shell. Data were obtained at two incident energies, 13.0 and 20.0 MeV, using the Yale MP Tandem and the multigap spectrograph. Two types of states can be formed in this reaction: 1) low-lying ($\leq 2.57$ MeV excitation) states consisting of three neutron-holes in the N=5 major shell (the one closing with the 126th neutron), and 2) higher-lying one-particle four-hole states with the neutron in the N=6 major shell outside the four-hole $^{204}$Pb core. Compared to the higher-lying "particle" states, the "hole" states are only very weakly excited with less than half (15) the known low-lying states of $^{205}$Pb excited above the $\frac{Z_{fb}}{s}$ limit of this study. The extracted spectroscopic factors for the "hole" states were compared to the predications of three residual interaction calculations: phenomenological, realistic-Kuo-Brown series, and realistic-BCS. Only the last calculation remains uncontradicted by the present results.

Unlike the nearly closed N=5 major shell, the N=6 major shell in $^{204}$Pb should be empty and the present work has established 110 states in $^{205}$Pb ($E_{\gamma \max} = 50\text{MeV}/\text{sr} - 2000\text{MeV}/\text{sr}$) between 2.57 and 5.62 MeV which are believed to be all fragments of the N=6 orbitals. With the data at two incident energies providing additional $E_{\gamma}$ discrimination, spectroscopic factors could be assigned for most of the states. The resulting distribution of shell strengths, concentrated more or less narrowly about the centroids (as was also the case with the "hole" states), confirms that the simple shell-model potential has not been overwhelmed by the residual interaction. However, the only theoretical prediction for this spectrum, in terms of the particle-vibration model, is found to give poor results. The systematics of the particle states' spectrum in $^{205}$Pb, $^{207}$Pb, and $^{209}$Pb suggest a more complicated description than will be necessary for $^{205}$Pb.

These data were analyzed by comparing the $^{204}$Pb ($d,p$) $^{205}$Pb angular distributions to those of the $^{208}$Pb ($d,p$) $^{209}$Pb reaction performed at the same incident energies. This would avoid the presumed uncertainties of the DWBA reaction calculation. A re-analysis of previous $^{208}$Pb ($d,p$) $^{209}$Pb data, on which estimates of the DWBA's reliability were based, leads to the conclusion that such estimates were too pessimistic. Improved agreement between theory and data is achieved through a more correct treatment of the incoming deuteron and the outgoing proton channels.
ACKNOWLEDGEMENTS

It is a privilege to acknowledge the many people associated with the completion of this dissertation. In particular I wish to extend special thanks to my research director, Professor Charles Bockelman, and to Dr. William Callender for their patient and continuous encouragement in the difficult beginnings of this work. Their fruitful suggestions during the analysis of the data and finally their careful reading and constructive criticism of the manuscript deserve deep appreciation. More than what I have learned from them, I value their friendship. I am also particularly indebted to Dr. Dennis Kovar for it was he who supervised the collection of a large part of the experimental data. Although diverted to other studies, he has remained enthusiastic and enlightening during the course of this project.

I would also like to thank Professor Allan Bromley for his support and interest in my work. His advice and help will be remembered.

In addition to Drs. Callender and Kovar, my colleagues Lance McVay and William Metz were extremely generous of their time and effort during the experimental data acquisition.

The photographic plates were scanned almost entirely by Mrs. Bernadette Kennedy whose accuracy and unfailing good humor are fondly recalled. Mrs. Muriel Wright continued the scanning of the remaining portions of the data as carefully as her predecessor.

The accelerator staff under the direction of the indefatigable Kenzo Sato labored hard to provide useful deuteron beams for these experiments.
Equally diligent has been Mrs. Hannah Novak who has prepared the figures in this thesis. Thanks are also extended to Dr. Martin Sachs for his very helpful advice with the computer phase of the data reduction.

Most importantly of all, I acknowledge my gratitude to my wife, Roberta, who not understanding, understood.

Finally, I express my appreciation to the United States Atomic Energy Commission whose financial support has made this research possible.
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I. INTRODUCTION

A. THE NUCLEAR STRUCTURE PROBLEM

With the advent of a satisfactory nuclear shell model in 1949 (MA49), the study of nuclear structure phenomena received an immense simplification. As in the case of atomic structure physics, similar nuclear behavior in diverse mass regions could now be predicted as a consequence of the degree of shell occupancy. The eigenstates of the simple shell model serve as the basis for more complex calculations whose aim is to reproduce those experimental data which indicate a breakdown of the first-order theory. Prominent examples of such deviations occur in the deformed regions where energy levels are now the eigenstates of a potential with a non-spherical $T_2$ term (NI55). Departures from expected shell behavior also appear most significantly in the excitation spectra of doubly-closed-shell nuclei, some of whose excited states (1$^-$ and 3$^-$ especially) must be explained as linear superpositions of many shell-model configurations. Indeed such is the coherence of these latter combinations that the states are more readily interpreted by a collective model of nuclear phenomena. This model emphasizes the correlated motion of many nucleons rather than the single particle motion which is more explicit in the simple shell model. Between these two extremes other descriptions of the nuclear structure potential have been developed. From the viewpoint of the simple shell model, such theories include the presence of the so called "residual interaction", namely that part of the summed internucleon force which is not adequately approximated by a central field. Chapter Two contains a detailed review of the different approaches to the residual interaction as they have been applied in the lead region.

Not to be lost sight of, however, is the great success of the simple
shell model around the doubly magic $^{208}_{82}\text{Pb}_{126}$. As Bromley and Weneser have made clear, such a validation of the simple theory is a fundamental watershed for further research in this mass region (BW68). A comprehensive study of the experimental data which emphasize the shell structure prevalent in the lead region was reported by N. Stein at the Montreal Conference (ST69). Two of his figures are especially pertinent. The first, Figure 1-1, illustrates the low-lying spectra of the four odd nuclei immediately adjacent to $^{208}\text{Pb}$. These states are excited by pick-up and stripping reactions which respectively populate the one-hole and one-particle states in the closed shell. The numbers in parentheses are spectroscopic factors indicating the degree to which the experimental level exhausts the predicted shell model excitation strength. It is seen that in 18 of the 24 cases one experimental state takes up at least ninety per cent of the expected strength, while only one level has less than fifty per cent. By comparison, for 7 of 12 cases in the doubly magic $^{48}_{20}\text{Ca}_{28}$ core, one state has ninety per cent of the model strength while two states have less than half the predicted strength (ME72).

A second important feature of the lead region is shown in Figure 1-2 which depicts the proton elastic excitation functions taken on mass $A\approx 208$ nuclei. The observed resonance structure is associated with the formation of isobaric analog states in the compound system. As further explained in Chapter Three, the analog states are quantum mechanically identical with their parents except for the interchange of a neutron with a proton. This displaces the analog states by the Coulomb-energy difference, leaving relative spacings unchanged. Hence, the resonances seen here reflect the neutron single-particle states in the $N=6$ major shell, and in particular reflect those eigenstates of the parent nucleus formed
The observed ordering of the single-particle and single-hole orbits of the $^{208}_{82}$Pb$^{126}$ core. For the neutron, the particle orbits constitute the $N = 6$ major shell, and the hole orbits are in the $N = 5$ major shell. The numbers in parentheses are the experimentally determined spectroscopic factors.
SINGLE PARTICLE STATES

MeV

Pb\textsuperscript{209}

\begin{align*}
\text{(0.95)} & \quad 3d_{3/2} \\
\text{(0.95)} & \quad 2g_{7/2} \\
\text{(0.95)} & \quad 4s_{1/2} \\
\text{(0.95)} & \quad 3d_{5/2} \\
\text{(0.5-0.9)} & \quad 1j_{15/2} \\
\text{(1.0)} & \quad 1i_{11/2} \\
\text{(0.95)} & \quad 2g_{9/2}
\end{align*}

\begin{align*}
\text{(0.95)} & \quad 1i_{13/2} \\
\text{(0.90)} & \quad 3p_{3/2}^{-1} \\
\text{(0.95)} & \quad 2f_{5/2}^{-1} \\
\text{(1.0)} & \quad 3p_{1/2}^{-1}
\end{align*}

\begin{align*}
\text{Bi\textsuperscript{209}}
\end{align*}

SINGLE HOLE STATES

MeV

\begin{align*}
\text{(0.65)} & \quad 1h_{9/2}^{-1} \\
\text{(0.80)} & \quad 2f_{7/2}^{-1} \\
\text{(1.0)} & \quad 1i_{13/2}^{-1} \\
\text{(1.0)} & \quad 3p_{3/2}^{-1} \\
\text{(1.0)} & \quad 2f_{5/2}^{-1} \\
\text{(1.0)} & \quad 3p_{1/2}^{-1}
\end{align*}

\begin{align*}
\text{(0.35)} & \quad 2g_{7/2}^{-1} \\
\text{(0.65)} & \quad 2d_{5/2}^{-1} \\
\text{(0.90)} & \quad 1h_{11/2}^{-1} \\
\text{(1.0)} & \quad 2d_{3/2}^{-1} \\
\text{(0.95)} & \quad 3s_{1/2}^{-1}
\end{align*}

\begin{align*}
\text{Pb\textsuperscript{207}}
\end{align*}

\begin{align*}
\text{Tl\textsuperscript{207}}
\end{align*}
Proton elastic excitation functions from six targets in the lead region showing the isobaric analog resonances. (ST69)
by coupling a neutron to the ground state of a particular target. The fact that the shapes of these excitation functions are strikingly similar suggest the existence of a common single-particle potential for these targets binding the different shell-model orbits at about the same energies throughout the mass region.

It should be realized that the data of these elastic scattering experiments and the data from a (d,p) reaction are sampling the same nuclear structure amplitudes. In fact, the (d,p) reaction is more suitable for the measurement of single-(neutron) particle strength than is the resonance experiment. Being unbound states of a finite width a continuum of other states, the resonant states interfere with each other and with the non-resonant background in the excitation amplitude. Disentangling the contribution of closely-lying states would be a taxing if not impossible task. In contrast the (d,p) experiment has a simpler reaction mechanism. This then was the principal impetus for the present work: 1) experimentally to resolve the gross structure of the $^{204}$Pb ( ,p ) excitation function by means of the finer resolution (d,p) experiment; and 2) theoretically to relate the observed spectrum of $^{205}$Pb to available residual-interaction calculations.

In addition to the single-particle shell structure, Stein also noted the collective excitations common to the lead region nuclei. Unlike the single-particle levels which occur at the same binding energies, the collective states appear at nearly the same excitation energies in the different nuclei. As such these levels are likely to involve the highly collective motion of many nucleons and would not be sensitive to the addition or removal of individual nucleons. The best example of this structure is the octupole vibration at 2.6 Mev in the even nuclei. In
the odd nuclei, the 3^{-} strength is spread over a multiplet of states constructed by coupling the odd particle to the octupole state of the even core. For example, in ^{207}\text{Pb} at 2.625 \text{MeV} and 2.664 \text{MeV} of excitation there is a doublet with spins 5/2^{+} and 7/2^{+} respectively. The enhanced gamma decay rates of these states is suggestive of a collective structure with the configuration* (HA 71)

\[ |^{208}\text{Pb}_{2.62 \text{ MeV}} \otimes (3p_{1/2}^{-1}) \rangle \text{J}\Pi = 5/2^{+}, 7/2^{+} \]

Taken together, the systematics of both shell and collective phenomena prompted Stein to speak of a "persistent lead core" with characteristics independent of the exact number of holes and particles in the doubly-closed ^{208}\text{Pb} core. This independence is modified to an extent by the low-lying shape vibrations present in the non-magic even nuclei. By coupling the neutron particle states to the even parity vibrations, multiplets can be generated. The multiplets are the basis of the analog resonances shown in Figure 1-3. These data are from the inelastic proton scattering to the first 2^{+} states of ^{206}\text{Pb} and ^{204}\text{Pb}. The interpretation of these results is similar to those of Figure 1-2 except that each resonance now includes a whole multiplet of states based on coupling a particular particle state to the 2^{+} state. Again, the close resemblance of the two

* In such configurations v stands for a neutron and π a proton. The quantum numbers n l j refer to a particular shell-model orbit with the exponent denoting the number of particles in that orbit. A negative exponent implies holes in the closed shell.
The proton inelastic excitations to the $2^+$ states of $^{206}\text{Pb}$ and $^{204}\text{Pb}$. The positions of the resonances are shifted by the quadrupole excitation energy. (ST 69)
excitation functions indicates that, insofar as these two nuclei are concerned, such a coupling produces much the same spectrum. An important difference from Figure I-2 is that a pure vibration multiplet in the odd nucleus would be completely orthogonal to a pure single-particle state. While such multiplets may mix with the single-particle configurations by the mechanism of the residual interaction, Figure I-3 offers no clue to the degree of that mixing. Because of the different angular-momentum possibilities involved, the particle-vibration model offers the hope of generating quite a number of states in the odd nucleus' spectrum. In fact the only prediction of the $^{205}$Pb single-particle states is based on this model and is discussed more fully in Chapter Two.

The appeal of a model which generates more than a few states can be understood from Figure I-4. The figure records the proton spectrum from our $E_d = 20.0$ MeV ($d,p$) experiment on $^{204}$Pb as detected at a lab angle of $50^\circ$. These proton groups represent at least 125 levels in $^{205}$Pb, most of them heretofore unknown. Below $2.57$ MeV of excitation only a few (15) groups are excited with a measurable* cross section (≈0.002 mb/sr). Above $2.57$ MeV a great many more states are excited with a much higher total cross section. The qualitative difference between these two regions of the spectrum has a simpler explanation within the framework of the shell model. The $^{204}$Pb ground state has four holes in the N=5 major shell. According to Figure I-1, the lowest cost in energy requires that the first two neutrons be removed from the $3p_{1/2}$ subshell and the second

* Actually this figure represents two separate runs of 10,200 and 735 micro-Coulombs. In the former, designed to observe very low yield levels, the count rate was $2 \times 10^5$/mb. It is believed that a yield of 40 counts would certainly not have escaped detection.
The \((d,p)\) spectrum of \(^{205}\text{Pb}\) at the 20.0 MeV incident energy. See the footnote, page 5, for the accumulated charge.
$^{204}\text{Pb}(d,p)^{205}\text{Pb}$

$E_d = 20.0$ MeV

$\theta_{\text{lab}} = 50.0^\circ$
two from the $2f_{5/2}$ orbital, i.e., $^{208}\text{Pb} \otimes (3p_{1/2}^{-2}) (2f_{5/2}^{-2}) \to 0^+$. Thus in the unperturbed approximation only two transfers to the $N=5$ major shell should be allowed, that to the completely empty $3p_{1/2}$ orbital and to the one-third empty $2f_{5/2}$. As the residual interaction is turned on, this simple analysis will begin to fail. However, it is stated now that the simple picture holds true to a large extent. Chapter Five will quote values which indicate that the $3p_{1/2}$ orbital is about four-fifths empty and the $2f_{5/2}$ orbital about one-fourth empty. For the $3p_{3/2}$ orbital which is the next lowest in energy, the residual interaction empties by about fifteen per cent what is a full sub-shell in zero order. The remaining orbitals in the $N=5$ major shell are at least ninety per cent full in $^{204}\text{Pb}$ and thus cannot accommodate a large neutron-transfer strength.

Once the 2.57 Mev energy is reached in $^{205}\text{Pb}$, it develops that the neutron can populate one of the orbits in the $N=6$ major shell. In zero order these orbits in $^{204}\text{Pb}$ are expected to be completely empty and can therefore be reached by the full sum-rule strength. Thus Figure I-4 displays proton groups from two types of states: those which have one-particle four-hole configurations and those with three-hole configurations. Hereafter, the former will be called "particle states" and the latter "hole states". Major shells have the same parity for all their subshells save that of the highest spin member pulled down by the spin-orbit term in the shell potential. In this nucleus, the $N=5$ subshells have negative parity (except for the positive parity $1j_{13/2}$) and the $N=6$ subshells have positive parity (except for the $1j_{15/2}$). In general then, the two types of configurations will not admix with each other. Compelling evidence will be presented in Chapter Five that practically all the excitation strength above 2.57 MeV in Figure I-4 belongs to the one-particle four-
hole configurations, that is to the positive-parity particle states. At this time a cursory examination of Figure I-4 reveals a definite clumping of cross section around certain energies, for example, 2.70, 4.0, 4.50, and 5.20 MeV if it can be established (and it will be) that these clumps correspond to the fragmentation of a given shell-model orbital, then the entire (d,p) spectrum of $^{205}$Pb will be a beautiful example of the persistence of the shell-model potential in a nucleus three holes removed from a closed shell.

Much more is known about the low-lying spectrum of $^{205}$Pb than about the higher-lying levels. The next section presents the $^{205}$Pb nucleus as it has been experimentally studied in the past. The discussion of the theoretical papers will be deferred until Chapter Two.
B. AN ORIENTATION: THE NUCLEUS $^{205}\text{Pb}$

The nucleus $^{205}_{82}\text{Pb}_{123}$ is unstable. In order to study its excitation spectrum some transmutation process is required. By far the most extensive study of $^{205}\text{Pb}$ has been accomplished with the artificially created isotope $^{205}_{83}\text{Bi}_{122}$ (HA72, RV71). With a 15.3 day half life, this isotope decays to $^{205}\text{Pb}$ via electron capture or, more infrequently, by positron emission. The studies have measured the maximum decay energy to be $2703 \pm 6$ kev and thus no state above this energy can be reached in $^{205}\text{Pb}$ by this reaction. Furthermore, $^{205}\text{Bi}$ has a $9/2^-$ ground state which means that only decays to relatively high-spin states ($\geq 7/2$) are likely to be observed. Subsequent to their population, such states decay to lower levels until the ground state is reached. The referenced papers observed these decays in both singles and coincidence modes and also measured the accompanying electron-conversion spectrum. Through quite painstaking analysis, these groups have placed 130 transitions among 26 definite levels in $^{205}\text{Pb}$, of which all but 5 are assigned a definite spin and parity. These assignments are given in Table 1-1.

Two other important pieces of information are to be had from the $^{205}\text{Bi}$ studies. The first is that the two highest $9/2^+$ states populated in $^{205}\text{Pb}$ have the lowest log $ft$ values ($\approx 6.4$) while the remaining transition rates (e.g. 703 kev $7/2^-$ population) have values clustered about 9.0. The two low log $ft$ values are consistent with these states being $2g_9/2$ single particle fragments coupled to the $^{204}\text{Pb}$ core. Since $^{205}\text{Bi}$ is an $h_{9/2}$ proton coupled to the same core, the first-forbidden selection rules ($\Delta I=1$, parity change) are followed nicely. Conversely, the unexpectedly low population of the 703 kev state (which should go by an allowed Gamow-Teller transition, log $ft \approx 3.0$) can be explained by con-
<table>
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<td></td>
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</tr>
<tr>
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<td>5/2−</td>
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</table>

Y = A level seen only in the present $^{204}$Pb(d,p)$^{205}$Pb study.

* Spin assignment questioned in favor of 9/2^+. 

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sidering the 703 kev state to be made up of configurations like \((p_{3/2}^-, f_{5/2}^{-2})_{7/2}^-\) which would require the \(h_{9/2}\) proton to drop into the \(p_{3/2}\) hole in the core, making this a much more inhibited transition (i.e. second forbidden axial vector). Insofar as the \((d,p)\) experiment is concerned, the two \(9/2^+\) states should be strongly populated while the 703 kev \(7/2^-\) state should be seen only weakly, if at all. The data of Figure I-4 confirm this, as the analysis will show, lending further support to the belief that the one-particle-four-hole configurations in \(^{205}\text{Pb}\) begin to occur above 2.57 MeV of excitation.

The second point of information derives from the fact that the \(^{205}\text{Bi}\) decay studies provide branching ratios for the decays out of the observed levels. In fact the three states mentioned above have branching ratios indicative of some collectivity in their structures. This conclusion will be more fully developed in Chapter Five, but for now it is noted that the \(^{205}\text{Bi}\) experiment can provide clues to the structure of the low lying states in addition to assigning spins and parities.

A complementary study of the low-lying spectrum of \(^{205}\text{Pb}\) utilized the \(^{204}\text{Pb}(n,\gamma)\) reaction with thermal incident neutrons [JV67]. The absorption of slow neutrons by the \(0^+\) \(^{204}\text{Pb}\) ground state necessarily results in \(1/2^+\) capture states. The gamma decay of these states in turn proceeds preferentially by the lowest electromagnetic multipole, \(E1\). Hence, only the lower spin states of \(^{205}\text{Pb}\) can be directly populated. A total of 40 gamma energies were seen. These energies did not display a continuous spectrum, but rather divided into two distinct groups. Sixteen were between 3572 and 6731 kev of energy while the other 24 were between 227 and 2117 kev. There was a pronounced gap in the gamma ray spectrum between 2117 and 3572 kev of energy. Jurney et alremarked that their apparatus
should have detected transitions in that gap if they existed even with very weak strength. The entire spectrum could be placed by assuming that the higher-energy gamma rays directly populated low-lying states in $^{205}\text{Pb}$. The lower-energy gamma rays could then be placed as transitions between the directly-populated states or from the directly populated to other low-lying states of $^{205}\text{Pb}$ known from the $^{205}\text{Bi}$ study. The resulting level scheme of $^{205}\text{Pb}$ included only those states below 2.64 MeV of excitation except for one level at 3.16 MeV. The "hole" in the gamma ray spectrum meant that except for one, no state in $^{205}\text{Pb}$ was seen between 2.64 and the neutron threshold energy, 6.73 MeV [JV67]. Yet it is obvious from Figure 1-4 that there are many states in $^{205}\text{Pb}$ in that energy range. The conclusion is that none of these states can be directly populated via an E1 transition from the $\frac{1}{2}^+$ capture state. In terms of the shell model this would mean that there are no $3p_{1/2}$ or $3p_{3/2}$ fragments above 2.64 MeV of excitation (with one possible exception). Such an explanation is consistent with the more inclusive belief that above 2.57 MeV there are mostly positive-parity configurations which would have to be reached via the much weaker E2 decay mode.

Another complementary set of experiments populating $^{205}\text{Pb}$ consists of the $^{206}\text{Pb}(d,t)$ reaction at $E_d = 20.0$ MeV (TB69), and the $^{204}\text{Pb}(d,p)$ reaction at $E_d = 13.3$ MeV (BH67). As mentioned earlier, the $(d,p)$ reaction populates holes in the $^{204}\text{Pb}$ core ($N=5$ major shell orbitals) or else subshells in the empty $N=6$ major shell. The $(d,t)$ reaction on the other hand pulls a neutron out of an $N=5$ subshell and thus would not be expected to excite the one-particle four-hole states seen in the $(d,p)$ reaction. In fact as listed in Table I-1, the $(d,t)$ reaction excites only ten states in $^{205}\text{Pb}$. The same analysis can also be applied to the $(p,d)$ reaction on
\(^{206}\text{pb}\) (YA 68) which populates almost the same states as does the \((d,t)\) reaction. The earlier \((d,p)\) reaction of Bjørregaard et al. was repeated in the present study. The previous work excited 27 states in \(^{205}\text{pb}\), of which 20 were in the particle region up to a maximum energy \(E_x = 4.13\) MeV. The present work with its better energy resolution and its lower cross-section sensitivity at \(E_d = 20\) MeV largely supersedes this earlier experiment. This will become more evident in the forthcoming chapters.

Finally there is also the \(^{207}\text{pb}\) \((p,t)\) \(^{205}\text{pb}\) reaction which has been done with quite poor resolution (220 keV, RM 67). As this reference explains the \((p,t)\) reaction at \(E_p = 40\) MeV favors the higher-momentum transfers \((l \geq 3)\) over the \(L = 0\) transition. In fact this reaction does populate relatively strongly states in \(^{205}\text{pb}\) at approximately 1.0, 2.6, 3.5, and 4.0 MeV of excitation. These are the energies at which higher-momentum states in \(^{205}\text{pb}\) are seen in the \((d,p)\) reaction. Chapter Five will show these to be the energies of the \(1i\, 3/2\), \(2g\, 9/2\), \(1i\, 11/2\), and the \(1j\, 15/2\) fragments. A closer comparison of the two experiments must await a better-energy-resolution investigation of the \(^{207}\text{pb}\) \((p,t)\) reaction (LA 73).

This completes the experimental background information for \(^{205}\text{pb}\). In the next section a sub-theme of this dissertation is introduced, namely the reaction mechanism of the \((d,p)\) experiment in the lead region.
C. SERENDIPITY: THE (d,p) REACTION ON THE Pb NUCLEI.

This study was primarily intended as a nuclear structure investigation of the $^{205}_{\text{Pb}}$ spectrum. It had been expected that only minor attention would be devoted to the mechanism of the (d,p) reaction. The belief was that the $^{208}_{\text{Pb}} (d,p) ^{209}_{\text{Pb}}$ angular distributions would be the same as those for $^{204}_{\text{Pb}} (d,p) ^{205}_{\text{Pb}}$. Therefore, the $^{208}_{\text{Pb}} (d,p)$ reactions was performed at the same incident energies as the $^{204}_{\text{Pb}} (d,p)$ experiment, 13.0 and 20.0 Mev. However, several authors have commented on the relatively poor quality of the theoretical fits to the experimentally obtained (d,p) angular distributions in the lead region (MA 69, SA71, PP 68). Moreover, the fits seem to worsen as the incident deuteron energy is increased above the Coulomb barrier ($E_d = 12$ MeV), contrary to what is expected for a direct reaction. The present data were obtained at two incident energies near and several Mev above the Coulomb barrier. The pattern of deterioration in the conventional theoretical fits again recurred. Considerable improvement in the fits was obtained for the higher-energy data when modifications were made in the traditional optical-model parameter sets used for this (d,p) reaction analysis. The change in the deuteron parameter set is based on a new model for the deuteron distorted waves (JS 70). The change in the proton set was based on the realization that this set was derived near a resonance in the elastic channel, a feature incompatible with the postulates of the optical model. A complete discussion of the theoretical origins of these conclusions is contained in Chapter Three.

In summary, this dissertation analyzes the (d,p) spectrum of $^{205}_{\text{Pb}}$ as it is excited at two incident energies. The study was motivated principally by the data of the analog $^{204}_{\text{Pb}}(p,p)$ excitation function.
(ST 69) which indicated that the simple central shell potential was persisting to a surprising degree in the one-particle four-hole configurations of $^{205}$Pb. It was seen in Figure I-4 that many such states are excited in the ($d, p$) reaction. This leads to further questions on the type of residual interaction which will generate the observed distribution of these states while still preserving the relative simplicity of the low-lying spectrum. Chapter Two examines in detail the different nuclear structure calculations applied to $^{205}$Pb, only one of which specifically includes the higher-lying configurations. As noted just above, Chapter Three discusses the revisions required for the analysis of the ($d, p$) reaction in the lead region. The predictions of the improved reaction calculation are to be compared with the raw data of the experiments as properly reduced. In view of the fact that for most of the particle-state region no previous data exist on the location or number of states, the success of the reduction phase of this analysis cannot be taken for granted. The role of Chapter Four is to prove that the method of data reduction adopted was successful in extracting positions and yields of the many groups of Figure I-4. Chapter Five presents the results of the analysis and relates these results to the theoretical models discussed in Chapter Two. The final chapter offers the conclusions to be had from this work and suggest other directions for related research.
II. THE SHELL MODEL AND THE RESIDUAL INTERACTION

INTRODUCTION

The usefulness of the shell model in nuclear physics is now beyond question. What is open to serious discussion, in light of the hard core present in the bare nucleon-nucleon interaction, is how the nucleus is able to survive, much less mimic the shell structure understandably present in atomic spectra. For the interior closed shells the Pauli exclusion principle provides most of the answer as explained in section A. For nucleons in the outermost unfilled shell this is not so, and it is no longer possible to regard a nucleon as existing entirely within a single subshell. To a lesser extent this is also true even of the last nucleons of a closed-shell nucleus. This aspect of the non-central or residual interaction is the subject of section B. In particular this section presents the BCS calculation of Harvey and Clement (HC 71) for the low-lying negative parity states in $^{205}$Pb. Although it was not so regarded at the time, it now appears to be the most successful calculation for the quantitative population of the low-lying states via the (d,p) reaction. The last two sections, C and D, consider three explicit approaches to the residual interaction as manifested in other calculations for the $^{205}$Pb spectrum. These are the pure-phenomenological interaction, the realistic, and the semi-phenomenological interactions. Except for one (RA 71) all these methods deal exclusively with the hole states. This is unfortunate from the point of view of a (d,p) reaction, since it is the higher-lying particle states which are more strongly populated. Nonetheless, as will be developed in the next chapter, the cross section for populating either a hole or a particle state in $^{205}$Pb depends critically on the particular structure of the $^{204}$Pb ground state wavefunction. It would be expected then that
those calculations which successfully describe the hole-state population would also offer a hope of predicting the particle-state \((d,p)\) spectrum.
A. THE ORIGIN OF THE SHELL MODEL

First seriously introduced in 1949 (MA 49) the nuclear shell model was a concept whose empirical friends had finally overcome its theoretical enemies. The impediments to shell structure in nuclear physics were indeed formidable. Unlike the centralized long-ranged Coulomb force which produces the electron shells in atomic physics, the nuclear force is short-ranged with no obvious center to the resultant field. Furthermore, as would be learned later from 300 Mev nucleon-nucleon scattering (JA 51), there is an infinitely repulsive hard core in the bare nucleon-nucleon force for a less than 0.4 fm separation. This hard core has diverging matrix elements and thus independent particles approaching this separation distance would be scattered far out of their shell orbits. In order for the shell model to work it becomes necessary for the wave-function for nucleons in a relative S state go to zero within this core, while sufficiently outside the core have the nuclear force bind the nucleons more or less the way the shell model predicts (BR 67).

Fortunately for the shell model the existence of the hard core was not known while evidence for the appearance of magic numbers was accumulating. Indicative of shell structure, these magic numbers are 2, 8, 20, 28, 50, 82, and 126. The basic evidence is that the binding energy of a magic-number nucleon is markedly greater than that of the next nucleon. Other experimental data supportive of shell structure can be found in the literature (GS 68). Recognizing the need for a shell model, Mayer and Jensen predicted the correct magic numbers by introducing a spin-orbit term in addition to the central field potential. The basic shell model potential, $V_{SM}$ can then be written
Originally the harmonic oscillator shape was used for $V_c(r)$, but this is unrealistic since this potential has only bound states whereas nuclei also possess a continuum spectrum. By sacrificing analyticity, one commonly uses the Woods-Saxon potential with a shape

$$V_{SM}(r) = V_{Central}(r) + \frac{i\cdot\sigma}{\gamma} V_{S.O.}(r) \quad (2.1)$$

$$V_c(r) = V_0 \left(1 + \exp\left(\frac{r - R}{a}\right)\right)^{-1} \quad (2.2)$$

$$V_{S.O.} = \frac{1}{\gamma} \frac{\partial V(r)}{\partial r}$$

Here $R$ is the nuclear radius (proportional to the cube root of the mass number), $a$ is the diffuseness, and $\gamma$ is such that the spin-orbit strength is approximately thirty times that of the Thomas value (MJ 55). Since even-even nuclei invariably have zero spin, it was further postulated that nucleons fill shells in time-reversed pairs. That is each nucleon of the pair has its angular momentum anti-parallel to the other member resulting in zero net angular momentum. For the adjacent odd nuclei, the ground-state spin and parity are determined by the last unpaired particle. It is the confirmation of this last property which represents the greatest success of the shell model. Except for the non-spherical deformed regions and isolated pairing effects (PR 62), the odd-mass nuclearground
states and often their low-lying spectra generally follow the spin predictions of this extremely simple shell model.

Granting its success, the simple shell potential cannot be expected to represent accurately the summed two-body nucleon-nucleon interaction. In other words, the truncation implied by equation (2.1) is much too severe. This can be seen by writing the nuclear Hamiltonian as a sum of kinetic plus potential energies for the constituent particles

\[ H = \sum_{i=1}^{A} T_i + \sum_{i<j}^{A} V(\vec{r}_i, \vec{r}_j) \]  \hspace{1cm} (2.3)

Here \( V(\vec{r}_i, \vec{r}_j) \) is the inter-nucleon interaction. The extreme shell model makes the approximation

\[ \sum_{i=1}^{A} V_{SM}(\vec{r}_i) - \sum_{i<j}^{A} V(\vec{r}_i, \vec{r}_j) \approx 0 \]  \hspace{1cm} (2.4)

which equates a summed long-ranged potential to a summed short-ranged potential. In fact, the right-hand side of equation (2.4) is not zero and this equation may be taken as a formal definition of the residual interaction

\[ V_{\text{Res}} = \sum_{i<j}^{A} V_{\text{res}}(\vec{r}_i, \vec{r}_j) = \sum_{i}^{A} V_{SM}(r_i) - \sum_{i<j}^{A} (\vec{r}_i, \vec{r}_j) \]  \hspace{1cm} (2.5)
It is now necessary to discover how the neglect of the residual interaction is compatible with the success of the simple shell model.

The shell model succeeds because, like its Hartree-Fock atomic analogue, it is self-consistent. The healing source in the nucleus is the Pauli exclusion principle. For those nucleons embedded deeply in the nucleus, there are no nearby unfilled orbits into which pairs of nucleons can be scattered. The hard core part of the residual interaction is accommodated by requiring that the relative motion function of two nucleons go to zero within the core radius ($r_c = 0.4\text{fm}$). A modified long-ranged potential can be constructed which converges to the usual long-ranged residual interaction for separation distances sufficiently removed from the core boundary. This method, originally proposed by Moszkowski and Scott (MS 60), is used as a justification for certain (realistic) residual-interaction calculations to be discussed in section D of this chapter. The important point here is that the effects of the hard core are suppressed except for those particles at the top of the Fermi sea.

In a closed shell nucleus, the major energy gap to the next shell inhibits to a large extent important deviations from the simple shell picture, but even for these nuclei, including the doubly magic $^{208}\text{Pb}$, some corrections are necessary (KO 71). It must be remembered, according to the discussion of Figure I-1, that in the lead mass region the simple shell potential is the dominant term.

Since it is expected that the residual interaction, $V_{\text{Res}}$, will have both long-ranging and short-ranging components, it is most useful to make a multipole decomposition of the individual $V_{\text{res}}(\hat{r}_i, \hat{r}_j)$
\[ V_{\text{res}}(\vec{r}_i, \vec{r}_j) = \sum_{l=2}^{\infty} \sum_{m=-l}^{l} f_1(\vec{r}_i, \vec{r}_j) \frac{4\pi}{2l+1} Y_{1m}(\theta_{ij}) Y_{1m}(\theta_{ij}) \]

\[ = \sum_{l=2}^{\infty} f_1(\vec{r}_i, \vec{r}_j) P_{1} \left( \cos \theta_{ij} \right) \tag{2.6} \]

The monopole and dipole terms (density oscillation and giant resonance) are omitted because they are not expected to alter the low-energy spectrum. By regarding the angular shape of \( P_{1}(\cos \theta_{ij}) \), shown in Figure II-1, it is seen that a given multipole of the potential is most effective within an angle \( 1/1 \), or a distance \( R/1 \) where \( R \) is the nuclear radius. Hence the low multipoles of equation (2.6) correlate pairs of particles over a long range while the short-ranged correlations are induced by the higher-order multipoles. The ultimate short-ranged interaction is the delta function force. By examining its decomposition

\[ \delta(\vec{r}_i, \vec{r}_j) = \sum_{l=1}^{\infty} \delta(r_i, r_j) \frac{2l+1}{2\pi r_i^2} P_{1} \left( \cos \theta_{ij} \right) \tag{2.7} \]

it can be seen that the most important components of this force are indeed the higher-order multipoles, those which correlate particles within a short range. The spectrum generated by this force will be discussed in section B.
The angular dependence of the $\ell^{th}$ multipole of the residual interaction. If $a$ is in radius, the term drops to half its initial value at an angle $1/\ell$. 
THE \( \ell \)th MULTIPLE OF THE RESIDUAL INTERACTION

\[ \theta_{12} \]
An obvious question is which correlation is more important. A partial answer is given by the $^{170}$ nucleus which has a quadrupole moment almost equal to that of an odd-proton nucleus instead of an odd-neutron nucleus. The explanation is that the odd neutron has polarized the core via the long ranged $P_2$ component of the residual interaction. Since it is energetically most favorable for the next neutron to overlap its motion as much as possible with the motion of the previous nucleus, it might be expected that a permanent deformation is achieved in all but closed-shell nuclei. Experimentally, it is known that the deformations do not set in until near the middle of filling a shell. It is clear that the higher multipoles are asserting themselves, scattering the extra-core pairs of nucleons into different time-reversed orbits. The resultant distribution then becomes spherical. As more of a shell's degenerate orbits are filled, however, the Pauli Principle inhibits the action of the short-ranged force until sphericity can no longer be maintained. This happens in the middle of a shell, the region of least degeneracy for pair scattering. The total effect is somewhat paradoxical; for a closed shell the Pauli Principle preserves the concept of a central spherical field while in the middle of the last shell, it destroys the ability to maintain sphericity.

Since $^{204}_{82}\text{Pb}_{122}$ is non-magic in its neutron number, it is clearly necessary to include properly the short-ranged correlations between the neutron holes. This is done most successfully in the BCS approximation described in the following section. Essentially, this method insures that, insofar as pairing effects are concerned, the ground-state wavefunction has been correctly calculated. For the odd mass $^{205}_{82}\text{Pb}$ it is also necessary to account for the longer-ranged correlations which, as was seen for $^{170}$,
produce core polarization, thus perturbing the central field. The last two sections of this chapter review the calculations which have treated this effect in $^{205}$Pb.
B. THE PAIRING FORCE AND BCS THEORY

In the $^{209}$Pb study of Kovar (KO 71), the nature of the residual interaction could be inferred by examining the first excited state of $^{208}$Pb, the strongly collective $3^{-}$ octupole vibration. Located at 2.62 MeV excitation, there is no way that the position and transition strength of this level can be made compatible with the existence of a sharp Fermi surface in the $^{208}$Pb ground state, (CP 60). In other words the ground state must have 2-particle 2-hole correlations in order to be "predisposed to collectivity" (LA 69). Such correlations are introduced (GS 66) by the Random-Phase Approximation (RPA). Since a similar octupole vibration exists in $^{204}$Pb at the same excitation energy, a final understanding of the $^{205}$Pb spectrum will have to incorporate RPA effects. Such a calculation has not yet been attempted and therefore the considerations of the RPA method will not be discussed here. It is noted however, that correlations are built on the concept of a sharp Fermi surface which is not present in the four-neutron-hole $^{204}$Pb ground state. To recover the sharp occupation surface upon which correlations can be built, it is necessary to go over into the quasi-particle representation of the ground state wavefunction. This subject will now be treated in detail.

As previously mentioned, an implicit postulate of the shell model is that nucleons pair off in time-reversed orbits. In 1950 Maria Mayer (MA 50) justified this postulate by calculating the two-particle spectrum generated by a delta-force interaction, equation (2.7). The resulting depression of the $J=0$ state, illustrated in Figure II-2, anticipated by seven years the development of the theory of superconductivity by Bardeen, Cooper, and Schrieffer (BC 57). The common denominator of all BCS applications is a fundamental principle of quantum mechanics: should there
On the left, the spectrum generated by a δ function interaction. On the right, the energy gap to the first excited state in the pairing model (Equation 2.11).
\[
\begin{align*}
\{ & J = 6 \} & \quad \{ & J \neq 0, s = 2 \\
\{ & J = 4 \} & & \\
\{ & J = 2 \} & & \\
\end{align*}
\]

8-force

Pairing force
exist a degeneracy in one or more of the eigenvalues of a Hamiltonian, any interaction which can split that degeneracy invariably results in an eigenstate with a lower energy than any of the degenerate levels. In fact, the stronger the interaction compared to the spacing of the eigenvalues, the more depressed the lowest state will become. Such a result is familiar in perturbation theory, but it should be emphasized in advance that the BCS solution cannot be obtained from a perturbation expansion.

The use of the second quantization formalism is most helpful in understanding the BCS method. Particle creation and annihilation operators, $a_{jm}^+$ and $a_{jm}$ respectively, are defined along with a vacuum state $|\rangle$ such that

$$a_{jm}|\rangle = 0$$  \hspace{1cm} (2.8)

The subscript $m$ refers to the azimuthal angular momentum eigenvalue while $j$ includes all remaining quantum numbers in addition to the total angular momentum of the particle. By repeated application of the creation operators any eigenstate may be obtained. Since nucleons are Fermi particles, the operators follow the usual anti-commutation rules:

$$\{a_{jm}^+, a_{jm'}\} = \delta_{jj'} \delta_{mm'}$$  \hspace{1cm} (2.9)

and

$$\{a_{jm}^+, a_{jm'}^+\} = \{a_{jm}, a_{jm'}\} = 0$$
In particular, the number operator, which has an eigenvalue equal to the total number of particles present in an eigenstate, is given by

\[ N_{\text{op}} = \sum_{jm} a_{jm}^+ a_{jm} \]  

(2.10)

As indicated earlier, the pairing force is not strong enough to scatter particles across a major shell gap (or else there would be no shell model). It is strong enough, however, to scatter particles within a major shell among the different subshells such that substantial configuration mixing will result. Unfortunately, there is no known exact treatment of pairing for several particles in more than two subshells. Before proceeding to the many-particle approximate solution, it is very worthwhile to consider two exactly soluble models of the pairing force. The solutions are for \( N \) particles in one degenerate shell and for two particles in several degenerate shells. The complete derivations are available in Lane's text (LA 69). Only the results will be presented here.

For \( N \) particles in one degenerate shell, the energy spectrum is written in terms of the seniority quantum number \( s \) and the pair degeneracy \( \Omega \). The seniority of a given state is just the number of particles not paired to zero net angular momentum while the pair degeneracy is one-half the degeneracy of the shell itself, that is \( \Omega = (2j + 1)/2 \). In that case the pairing energy of a given state is

\[ E_p (N, s) = - \frac{G}{4} (N - s)(2\Omega - N - s + 2) \]  

(2.11)
where $G$ is the pairing strength parameter.

This equation contains some important features which carry over into the approximate solution. The distance between the ground state and the first excited state, i.e. the separation of the $s = 0$ and the $s = 2$ states is always $G\Omega$ and is independent of the number of particles present, $N$. For $s \ll \Omega$ the distance between successive excited states is approximately equal which is also true independent of $N$. This equation contains the origin of the energy gap in even-even nuclei and it also explains the similarity of the low-lying spectra of the even lead and tin mass nuclei.

The second soluble problem involves two particles in several sub-shells with energies, $\epsilon_\alpha$

The eigenvalue equation to be solved in this case is

$$ (2\epsilon_\alpha - E_n) a_\alpha - \sum_\beta G_\beta a_\beta = 0 $$

(2.12)

Here the energy $E_n$ is the $n^{th}$ eigenvalue, $\Omega_\beta$ is the pairing degeneracy of the $\beta$ eigenstate ($=$ $(2j_\beta + 1)/2$) and $a_\beta$ the coefficient of the pair state.

Dividing by $(2\epsilon_\alpha - E_n)$ and summing over $\alpha$ one obtains

$$ \sum_\alpha \frac{G_\alpha}{2\epsilon_\alpha - E_n} = 1 $$

(2.13)

This equation has the graphical solution shown in Figure II-3. For something approaching realism the eigen-energies of the figure have the same spacings as do the single hole states of $^{207}$Pb (see Figure I-1). In more
II-3 The graphical solution to the eigenvalue Equation (2.13). The curves are the r. h. s. and where they intercept the line $Y = 1$ yields the eigenvalue $E_n$.

Note that $E_0$ is depressed below the lowest single hole energy.
realistic calculations the parameter $G$ is no longer a constant but depends on the angular momentum of a given configuration but this was neglected for the present purposes. It can be seen from the figure that six distinct eigenvalues result one of which is depressed below the lowest single hole energy. Such an eigenvalue corresponds to the ground state wavefunction.

$$\psi_0 = (\sum_{\alpha} a_\alpha) \sum_\beta \frac{G_{\alpha\beta}}{2\varepsilon_\beta - E_0} |j_\beta^m, j_{\beta-m}$$

(2.14)

This result demonstrates the correlation induced by the pairing force. All the configurations in this sum are occupied with a coherent (in phase) positive amplitude since $E_0 < 2\varepsilon_\beta$ for all $\beta$.

The effect is a smearing of the Fermi surface whose diffuseness is now a function of the value of the strength parameter $G$ and of the spacings of the single hole energies. The trend is shown in Figures II-4.

The two solutions just discussed represent the only known analytic solutions of the pairing force problem. The difficulty is that the pairing force tends to correlate particles in time reversed orbits scattered among different subshells; the stronger single particle potential opposes this correlation and is largely able to decouple, but not completely, the different subshells. A prohibitive diagonalization can be avoided for the general case by giving up the requirement of strict number conservation in the Hamiltonian. This is not too serious an objection because as Equation (2.11) indicates, the exact low-lying spectrum is insensitive to the precise number of particles present. If this method is adopted then the general solution to the pairing problem is immediately avail-
The amplitudes $a_j (E_0)$ giving the population of the $j^{th}$ pair of orbitals. For the strongest value of $G$, all orbitals are occupied with nearly equal probability.
AMPLITUDES $a_j (E_0)$

$2f_{5/2} \quad l_{13/2} \quad 3p_{1/2} \quad 3p_{3/2} \quad 2f_{7/2} \quad 1h_{9/2}$

$G = 2.00 \text{ MeV}$

$G = 0.20 \text{ MeV}$

$G = 0.05 \text{ MeV}$

ENERGY
able from the theory of an electron gas (BC 57). The vacuum state is now the BCS vacuum:

\[ |\text{BCS}\rangle = \prod_{k \geq 0} (U_k + V_k a^+_k a^-_k)|\rangle. \quad (2.15) \]

where \( k \) describes all quantum numbers with \(-k\) being the time-reversed \( k^{th} \) orbital. It can be readily ascertained that \( |\text{BCS}\rangle \) is not an eigenstate of the number operator (Equation (2.10)). By inspection \( V_k^2 \) is the probability of a pair of particles being in the \( k^{th} \) state. Setting \( U_k^2 + V_k^2 = 1 \) makes \( U_k \) the probability that the \( k^{th} \) pair of orbits is empty.

The solution to the pairing problem is obtained by knowing the \( U_k \) and \( V_k \) in terms of the single particle energies \( \varepsilon_k \) of the relevant subshells.

The procedure is to define a Hamiltonian \( H' = H - \lambda N_{\text{op}} \) where \( H \) is the true nuclear Hamiltonian (single particle plus pairing energies) and \( \lambda \) functions as a Lagrange multiplier. The modified Hamiltonian is then varied subject to the condition that it yield a minimum of energy for the BCS vacuum where that vacuum has a number expectation value equal to the number of nucleons in the given nucleus. Summarizing:

\[ \frac{\partial}{\partial N} \langle \text{BCS}|H'|\text{BCS}\rangle = 0 \quad (2.16) \]

\[ \langle \text{BCS}|N_{\text{op}}|\text{BCS}\rangle = N \]

The complete derivation worked out by the present author is omitted for
typographical reasons, but the results of the equation are (BC 57)

\[
U^2_k = \frac{1}{2} \left\{ 1 + \frac{\varepsilon_k - \lambda}{(\varepsilon_k - \lambda)^2 + \Delta^2} \right\}
\]

\[
V^2_k = \frac{1}{2} \left\{ 1 - \frac{\varepsilon_k - \lambda}{(\varepsilon_k - \lambda)^2 + \Delta^2} \right\}
\]

\[
\Delta^2 \equiv G \sum_k U_k V_k
\]

(2.17)

The Fermi surface of the vacuum state, i.e., \( V_k \) vs. \( \varepsilon \) is illustrated in Figure II-5 where \( \lambda \) is the point of inflection of the curve. Since the surface is now smeared it is no longer convenient to work with particle operators. Instead one defines quasi-particle operators which create partially filled and partially empty single particle orbits. These operators have \(|BCS>\) as their vacuum state and are given by the relations:

\[
\alpha^+_k = U_k a_k^+ - V_k a_k
\]

(2.18)

\[
\alpha^+_k = U_k a_k^+ + V_k a_k
\]

It is easily shown that the above transformation is canonical, meaning that the \( \alpha_k \) obey Fermi anti-commutation rules as do the \( a_k \). The excited states of nuclei in this approximation are of the form
The BCS occupation probability, $V_j$, for the $j^{th}$ pair.
\[ V_j^2 = \frac{1}{2} \left[ 1 - \frac{\epsilon_j - \lambda}{\sqrt{(\epsilon_j - \lambda)^2 + \Delta^2}} \right] \]
\[ |\psi> = \alpha^+ \alpha^+ | \text{BCS}> \]

By using equations (2.17) the energy of this state is shown to be

\[ E = \sqrt{\epsilon_{k1}^2 + \Delta_{k1}^2} + \sqrt{\epsilon_{k2}^2 + \Delta_{k2}^2} \quad (2.19) \]

The energy gap of the earlier analytic solutions now re-appears since it requires a minimum of \(2\Delta\) in energy to break a pair. Similarly the energy of an odd A nucleus may be found by considering it to be a one quasiparticle configuration outside the BCS ground state.

One of the first systematic applications of the BCS solutions was performed by Kisslinger and Sorenson (KS 63) for both the lead and tin regions. The short-ranged part of the residual interaction was thus treated in the quasi-particle approximation, while the long-ranged part was considered weak enough to be calculated in a perturbation expansion. Although they obtained good fits to the B(E2) transitions strength, the observed excitation energy of the first \(2^+\) state could not be reproduced.

Somewhat similar results were also reported by the group at Pittsburgh (CB 66) who used a different approach. Instead of treating \(G\), the pairing strength, as an empirical parameter, they used a realistic nucleon-nucleon potential derived by Tabakin. (TA 64). This non-local potential fits the S, P, and D wave phase shifts for scattering energies up to 320 Mev. The residual interaction given by the Tabakin potential
mixes different two quasi-particle states. Again, the energy gap to the first excited state is predicted incorrectly in the lead nuclei although there is more success in the tin nuclei. They ascribe their failure in the lead region as a consequence of having only the $N=5$ major shell orbitals (the negative parity hole states) as allowable configuration. By appropriately simulating $2p-2h$ (two-particle two-hole) components representing the partial occupation of the $N=6$ major shell orbits (the positive parity particle states) they were able to obtain the correct binding energy of the even lead ground states. (CB 68).

The last reported study of the $^{204}_{\text{Pb}}$ nucleus was made by Harvey and Clement (HC 71). The BCS Method was inverted and then improved by including particle-hole correlations in the excited states as a perturbation. By inverting it is meant that the single hole energies taken from $^{207}_{\text{Pb}}$ were considered to be one-quasi-particle energies rather than shell model energies, $\varepsilon_k$. The inversion procedure is suggested as being more consistent (GR 66). By these methods they were able to obtain improved results for all the lighter lead isotopes, both even and odd. In fact, they conclude that the lead region is more amenable to BCS theory than the nickel or tin isotopes. Directly relevant to the $^{204}_{\text{Pb}}(d,p)^{205}_{\text{Pb}}$ experiment, Harvey and Clement have predicted spectroscopic factors for the population of the low-lying states of $^{205}_{\text{Pb}}$. When published these authors noted a lack of agreement between their values and those measured by the previous $(d,p)$ study of Bjerregaard (BH 67). However, the results of the present experiments, to be presented in Chapter Five, warrant a change in such an interpretation. In particular, the now observed distribution of the $2f_{7/2}$ spectroscopic strength confirms the trend predicted by Harvey and Clement.
This concludes the discussion of the pairing force as it has been applied in the lead nuclei. The next section deals with the particle-vibration model calculation of the positive parity states' fragmentation. In contrast to the pairing force, this model is concerned with the effects of core polarization which has been seen to arise from the longer-ranged multipoles of the residual interaction.
C. PARTICLE-VIBRATION COUPLING

The particle-vibration coupling scheme assumes that the persistent collective vibrations present in the even-even nucleus are not greatly disturbed by the addition of one particle to form the odd A+1 nucleus. Therefore, the nuclear Hamiltonian can be usefully decomposed into three terms.

\[ H_{A+1} = H_A + H_p + H_{\text{int}} \] (2.20)

The two Hamiltonians, the even core \( H_A \), and the odd particle \( H_p \), define two subspaces of eigenstates: \( |A\rangle M \rangle \) and \( |a\langle j \rangle m \rangle \) where A and a distinguish all but the angular momentum quantum numbers. The eigenstates of \( H_{A+1} \) are then obtained from the direct product space of these two subspaces upon diagonalizing the coupling term \( H_{\text{int}} \).

The concept of particle-vibration coupling dates back two decades to Rainwater (RA 50), Bohr and Mottelson (BM 53, BO 52) and to deShalit (DS 50). In their classic papers on nuclear collective motion, Bohr and Mottelson considered the specific form of \( H_A \) to be that given by the liquid drop model whose eigenstates are the phonon vibrations.

\[ |A\rangle M \rangle = |\lambda \rangle N \rangle M \rangle \] (2.21)

for which \( \lambda \) represents the multipole order and parity of the vibration, \( N \) is the number of phonons of that order present, and \( I \) is the resultant angular momentum. By expanding the interaction in terms of the deformation parameters \( \alpha_{\lambda \mu} \), the coupling term has the explicit form
The radial function \( k(r_p) \) couples the radial parts of the single particle functions and can be taken as a constant or set equal to the first derivative of the single particle potential. More importantly, the \( \alpha_{\lambda \mu} \) carry the amplitude of the vibration and contain second quantization operators which can couple only those core states which differ by one phonon. It is then quite easy to calculate the matrix elements of \( H_{\text{int}} \) between the members of the direct product space. The goal is then to derive the coefficients \( B \) in the expansion of the eigenstates of \( H_{A+1} \). This expansion is

\[
| (A+1)J (M+m) > = \sum_{\lambda, N, I, j} B_{\lambda, N, I, j}^{A+1} | \lambda N M; j m; J (M+m) >
\]  

(2.23)

For these coefficients two courses are open. If the matrix elements of \( H_{\text{int}} \) are sufficiently small (the exact criterion is given by Arima and Hamamoto (AM 71)), the \( B \)'s can be obtained by simple first order perturbation theory assuming that the energy denominators therein are large. This constitutes what is known as weak coupling. Otherwise, an intermediate coupling scheme must be used, meaning that the complete matrix of \( H_{A+1} \) between the states of Equation (2.23) must be diagonalized to determine the \( B \)'s. The weak coupling model will be considered first.
The outstanding example of a weak coupling structure is the septuplet of states in $^{209}$Bi with the configuration

$$\left( ^{208}\text{Pb} \ (3^-) \times (1h_{9/2}) \right)^{J^\pi = 3/2^+ \ 15/2^+}$$

The complete septuplet is seen in inelastic proton scattering, but only the $l_n = 6$ members are seen in the ($^3\text{He}, d$) reaction on $^{208}\text{pb}$, and then only weakly. As Arima and Hamamoto point out, this latter small strength confirms the weakness of $H_{\text{int}}$. In the limit that $H_{\text{int}}$ goes to zero, there will be no transfer cross section to the members of the multiplet. This is because the transfer reaction populates particles coupled to the unexcited target ground states. It is only as $H_{\text{int}}$ admixes these configurations with the particle plus excited core configurations of the pure multiplet that there will be any transfer strength to that multiplet. For the $j_2$ spin member of a multiplet, the admixed amplitude $\varepsilon$ is (MO 67):

$$\varepsilon = \frac{\langle 1h_{9/2} \times 3^- \ j_2 \ |H_{\text{int}}| j_2 \rangle}{\varepsilon_{j_2} - \varepsilon_{1h_{9/2}} - \hbar \omega (3^-)} \quad (2.24)$$

and the spectroscopic strength will be $\varepsilon^2$.

The above expression has some important consequences insofar as the applicability of weak coupling to the transfer spectrum is concerned. Since weak coupling is a perturbation expansion, only very weakly ($\varepsilon << 1$) populated states are consistent with the model. Also since the energy
denominator must be large, the single particle fragment $j_2$ is displaced far from its unpreturbed shell model position and thus its form factor in the reaction amplitude is more difficult to determine (see Chapter Three). Thus the dual contradiction: a) the stronger a fragment is excited in a transfer reaction the less likely the weak coupling limit is satisfied, and b) the weaker the fragment the more uncertain the form factor becomes in the reaction calculation making the derived spectroscopic factors questionable.

These comments apply with even more force when couplings based on the quadrupole vibrations of the core are considered. The quadrupole energy is about one MeV in the lead nuclei and thus comparable to the spacings the shell model orbitals. In fact since the vibration is now of positive parity the coupling based on this vibration will have the same parity as the particle state being coupled. Hence there is almost no possibility that the perturbation expansion of the weak coupling model will converge in view of the small energy denominators arising in Equation (2.24).

It is with the next model to be discussed, the intermediate coupling model, that the only theoretical calculation of the positive parity spectrum of $^{205}$Pb has been performed. (RA 71). The strength of the coupling term $H_{\text{int}}$ is given by the parameter $\eta$ which is related to the amplitude of $\alpha_{\chi \mu}$

$$\eta = \left(\frac{5}{16\pi}\right)^{\frac{3}{2}} \frac{k}{(\hbar \omega c)^{\frac{1}{2}}}$$

(2.25)
Since the hydrodynamic model does not give good values for the deformability parameter C, \( \eta \) is regarded as a free parameter varied to best fit the observed spectrum. Using a value of \( \eta = 0.30 \), Rao has calculated the particle states' spectrum shown in figure II-6. The agreement between this spectrum and that of Figure I-4 is very poor. The excitation energy predicted by Rao for the start of the particle states' spectrum, 0.90 MeV, differs badly from the observed 2.57 MeV of excitation. More discussion of this predicted spectrum is contained in Chapter Five, but for now it can be said that the calculation is quite unsuccessful.

Rao predicts only about half the number of states in \(^{205}\text{Pb}\) while 110 levels are actually observed. A simple way to expand the theoretical spectrum is to extend the basis of \( H_A \) by including two phonon core states in addition to the one phonon state. Such a procedure was adopted by Mukherjee, et al. (MC 68) who claimed that this resulted in a better theoretical spectrum for \(^{207}\text{Pb}\). Further study with the \(^{206}\text{Pb} \langle d,p \rangle\) reaction (MC 70), however, tends to invalidate this conclusion. The fundamental problem is that the low-lying spectra of the even lead isotopes are not well described by the vibrational model. Such a model predicts a triad of two phonon states with spins of 0\(^+\), 2\(^+\), and 4\(^+\) at twice the energy of the first 2\(^+\) level. From the data shown in Figure II-7 this is seen not to be the case. The low-lying levels of the even lead nuclei appear to be quite anharmonic. The deviations from harmonicity, can be expressed in terms of an admixing of the vibrational levels. That is, the first 2\(^+\) level is not a pure one phonon state but rather a sum of one and two phonon configurations. Similarly the ground state and the 0\(^+\) member of the triad are mixtures of zero and two phonon configurations. The basis states of \( H_A \) are now written as
The particle states' distribution in $^{205}_{\text{pb}}$ as calculated by Rao with the intermediate coupling model.
CALCULATED $^{205}$Pb EVEN PARITY SPECTRUM (Rao)

<table>
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<th>Excitation Energy (MeV)</th>
<th>S9/2+</th>
<th>S7/2+</th>
<th>S5/2+</th>
<th>S3/2+</th>
<th>S1/2+</th>
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<td></td>
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<td></td>
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</tr>
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<td>0.04</td>
<td>0.04</td>
<td>0.06</td>
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</tr>
</tbody>
</table>
The low-lying vibrational states of $^{206}\text{Pb}$ and $^{204}\text{Pb}$. The $4^+$ state should appear at twice the energy of the $2^+$ state in the pure phonon model.
ANHARMONICITY IN THE EVEN Pb SPECTRA

<table>
<thead>
<tr>
<th>$E_x$</th>
<th>$J^\pi$</th>
<th>$E_x$</th>
<th>$J^\pi$</th>
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<th>$J^\pi$</th>
<th>$E_x$</th>
<th>$J^\pi$</th>
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<th>$J^\pi$</th>
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<tr>
<td>1.489</td>
<td>(4⁺)</td>
<td>1.383</td>
<td>4⁺</td>
<td>1.274</td>
<td>4⁺</td>
<td>1.168</td>
<td>0⁺</td>
<td>1.091</td>
<td>(4⁺)</td>
</tr>
<tr>
<td>1.027</td>
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<td>0.961</td>
<td>2⁺</td>
<td>0.899</td>
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<td>0⁺</td>
<td>0.0</td>
<td>0⁺</td>
<td>0.0</td>
<td>0⁺</td>
</tr>
</tbody>
</table>

$^{200}$Pb | $^{202}$Pb | $^{204}$Pb | $^{206}$Pb | $^{210}$Pb
\[ |0^+_{\text{g.s.}} = \alpha_1 |\lambda = 0, 0^+ + \alpha_2 |\lambda = 2, 0^+ \]

\[ |2^+_{1} = \beta_1 |\lambda = 1, 2^+ + \beta_2 |\lambda = 2, 2^+ \]

etc.

\[ (\alpha_1^2 + \alpha_2^2 = \beta_1^2 + \beta_2^2 = 1) \]

At this point, we can offer an explanation for the failures of the intermediate coupling model in the lead region. Clearly if the actual vibrations are admixtures of pure phonon states, than the effect of the coupling term \( H_{\text{int}} \) is more complicated than has been admitted. For example, the ground state and the second \( 0^+ \) state can be connected and, additionally, an interference term arises when the first and second \( 2^+ \) states are connected. The triad of two phonon states would also be split in this extended vibrational model. It is interesting to note that there is no such difficulty with multiplets built on the octupole vibration. In that case the first \( 3^- \) state is only allowed to mix with the second \( 3^- \) state for which the energy separation is so large that the pure phonon approximation probably works quite well.

The reliance on the concept of a phonon number can be completely avoided by using a general particle-core interaction first proposed by deShalit (DS 61). Although this type of calculation has not yet been attempted in the lead region, several authors have had success in des-
crivbing the structure of lighter mass nuclei such as $^{63}\text{Cu}$, $^{55}\text{Fe}$, and $^{51}\text{Ti}$ (PT 70, LA 70). It would be interesting to discover whether the deShalit interaction or the extended vibrational model proposed above are capable of interpreting the odd mass spectra in the lead nuclei.

In the last section of this chapter, two more examples of nuclear structure calculations are reviewed. Their predicted spectroscopic factors are, however, confined to the hole state region of $^{205}\text{pb}$. 
D. THE PHENOMENOLOGICAL AND THE REALISTIC INTERACTIONS

The phenomenological approach regards the exact residual interaction as either unknown or much too difficult to be mathematically tractable. Therefore, a mathematically convenient parameterization is often substituted. For example, in considering the lighter lead isotopes, Miranda (MI 67) chose a two body potential with the form

\[ V(r_{ij}) = V_0 V_{se}(r_{ij}) \left( P_{se} + \alpha P_{to} \right) \]

Where:

\[ V_{se}(r_{ij}) = -32.5 \exp \left( -\frac{r_{ij}^2}{(1.85 \text{ fm})^2} \right) \text{ MeV} \]

The projection operators \( P_{se} \) and \( P_{to} \), with eigenvalues 0 or 1, select those two-hole configurations which are either singlet even parity or triplet odd parity, the only allowed channels for fermi particles. The parameter \( \alpha \) is the ratio of triplet odd to singlet even strength. The strength and shape of \( V_{se}(r_{ij}) \) are such that with \( \alpha = 0 \) and \( V_0 = 1 \) then \( V(r_{ij}) \) will bind a two nucleon system at zero energy. The effects of core polarization are supposed to be simulated in this calculation by modifying the multipole decomposition of \( V_{se}(r_{ij}) \)

\[ V_{SE}(r_{ij}) = \sum_{k=0}^{\infty} R_k(r_{ij}) P_k(\cos \Theta_{ij}) + \sum_{k=0}^{\infty} \sum_{a=1}^{3} (1+\alpha \delta_{ka}) P_k(\cos \Theta_{ij}) R_k(r_{ij}) \]

(2.27)
The renormalization of the dipole, quadrupole and octupole terms by the $x_a$ ($x_a << 1$ according to Miranda) is induced by couplings to the excited core states. In particular for the $^{205}$Pb calculation, Miranda took as the basis states all possible j-j coupled neutron configurations among the holes $3p_{1/2}$, $3p_{3/2}$, $2f_{5/2}$, $2f_{7/2}$, and the $1i_{13/2}$. The $1h_{9/2}$ orbital was omitted to save computational time since it was believed that this particular state would not affect those levels predicted below 1.5 Mev of excitation. The completed matrix was diagonalized with $V_0$, $\alpha$, and the three $x_a$'s as free parameters. Similar calculations were performed on $^{204}$Pb, $^{206}$Pb, and $^{207}$Pb. In addition to energy levels, spectroscopic factors for the population of the low-lying hole states in $^{205}$Pb by the (d,p) reaction were also computed. These results are tabulated in Chapter Five where they will be compared with the present experimental results.

In contrast to the phenomenological interaction the realistic approach endeavors to reproduce experimental structure data by employing an interaction potential derived from the fit to nucleon-nucleon scattering data. There are no free parameters, and the only variation allowed is in choosing the number of shell model configurations to be used as a basis. This approach was initially avoided because of the hard core present in the scattering potential leads to diverging matrix elements in the structure calculation. However, by following the separation method of Moszkowski and Scott (MS 60), Kallio and Koltveit were able to obtain useful results for the $^{16}$O spectrum (KK 64). The method was extended by Kuo and Brown (KB 68) and applied in the $^{210}$Pb, $^{210}$Bi and $^{210}$Po nuclei by Kuo and Herling (KH 72), and to the lighter (A<208) lead isotopes by McGrory (MG 73). Using the Hamada-Johnston potential described in their papers, these authors constructed matrix elements $V_{\text{eff}}(abcd)$ representing the interactions illus-
trated in Figure II-8. Diagram A represents only particle-particle interactions; diagram B includes the virtual excitation of a particle-hole pair in the core; diagram C includes 2p-2h excitations of the core. Kuo and Herling found that for $^{210}$Pb orders A and B were most successful while the inclusion of C worsened the agreement with experiment. Following this result McGrory used diagrams A and B to calculate the $^{205}$Pb spectrum as well as (d,p) spectroscopic factors. He found it necessary to use only .75 of diagram B's contribution in order to reproduce the experimental spectrum of the lighter lead isotopes (Ex. < 2.0 MeV). Hamilton has concluded (HA 72) that the resulting $^{205}$Pb spectrum is in much better overall agreement with the data of the $^{205}$Bi decay study than the spectrum produced by Miranda. The main difference is the greater number of negative parity states provided by the McGrory calculation which are lacking in the Miranda prediction. Despite this agreement some caution is expressed here in accepting the Kuo-Brown model. Vary (VA 71) has concluded that orders higher than C of Figure II-8 cancel out parts of the B and C contributions. In view of McGrory's reduction factor of .75 and in view of the fact that the inclusion of the C diagram in the Kuo-Herling work worsened the predicted spectrum, it seems likely that Vary is correct. If the Kuo-Brown series can be shown to converge within a reasonable order, however, it will represent an important advance in the understanding of the residual interaction.

This concludes the discussion of the nuclear structure calculations. The principle aim of the present work is to test these theories insofar as they predict the $^{205}$Pb spectroscopic factors. Before this can be done, the data from the reaction must be properly interpreted. Such is the concern of the following chapter.
Schematic representation of the Kuo-Brown series describing core polarization by extra-core nucleons.
KUO-BROWN SERIES

A

B

C
III. THE DIRECT REACTION MECHANISM

INTRODUCTION

In the original conception of this work it had been thought that it would be neither necessary nor desirable to rely heavily on the prediction of the direct reaction theory. It has been believed that the theory is only accurate to about twenty per cent at best because of the approximations inherent within the model (MF 69). Such a margin of error comes from a review of the fits to \(^{208}\text{Pb}(d,p)^{209}\text{Pb}\) single particle angular distributions as obtained with deuteron energies from 8.0 to 24.8 Mev. For the analysis of the present data it was thus felt that it would be sensible to bypass the direct-reaction calculation, using instead the \(^{209}\text{Pb}\) angular distributions as templates for the \(^{205}\text{Pb}\) angular distributions, both obtained at the same incident deuteron energy. There are two problems with such a procedure. The first is that there are no low-lying states in \(^{209}\text{Pb}\) comparable to those in \(^{205}\text{Pb}\), a reflection simply of the fact that \(^{208}\text{Pb}\) has a closed N=5 major shell while \(^{204}\text{Pb}\) does not. Hence, the analysis of the low-lying states would have to proceed along the conventional lines. The second difficulty associated with the bypass method arises once the actual spectrum of \(^{205}\text{Pb}\) is at hand. Then it is seen, as Chapter Five will show, that the particle states in \(^{205}\text{Pb}\) are bound more deeply than in \(^{209}\text{Pb}\) thus necessitating a Q-value correction to the \(^{209}\text{Pb}\) differential cross-sections. Such a correction would have to be calculated with the usual distorted wave method.

For both the low-lying and the higher lying states in \(^{205}\text{Pb}\) a complete bypassing of the direct reaction calculation is not possible. Fortunately, it has been learned that the above mentioned margin of error is too pessimistic. A systematic analysis of earlier \(^{208}\text{Pb}(d,p)^{209}\text{Pb}\) data
suggests that with the proper relative motion functions, the distorted wave method is capable of predicting uniform results at nearly all incident deuteron energies considered. Furthermore, there is a quite simple explanation for the limited range of incident energies at which anomalous spectroscopic information is extracted. As a groundwork for the present results, a more thorough than anticipated review of the direct reaction theory is presented in this chapter. Section A discusses the bypass procedure and the necessary Q-value corrections for $^{205}$Pb. A formal derivation of the distorted wave method is contained in section B with a special stress on the approximations and restrictions presumed by the method. Sections C and D treat the deuteron and the proton channels separately, laying the foundation for the re-analysis of the earlier $^{208}$Pb(d,p) data which is presented in section E. The success of the re-analysis in turn allows the $^{205}$Pb Q-value corrections and $^{205}$Pb low-lying angular distributions to be calculated with an added measure of confidence.
A. THE THEORETICAL NORMALIZATION OF THE CROSS-SECTION

The keystone of all nuclear structure studies involving direct reactions is the spectroscopic factor $S_{\lambda j}^{BA}$. As will be further explained later, this factor is basically the overlap of a state $\lambda j$ in the final nucleus B with the ground state of the target nucleus A. The strength of an observed transition is a product of this factor and an intrinsic cross-section $\sigma_{\lambda j}^{DW}(E, Q, \Theta)$ which carries the dynamical dependencies of the transfer and which is, to a large extent, independent of the specific nuclei involved. Here the intrinsic cross-section is written so as to make explicit its dependence on the incoming energy $E$, the $Q$-value of the reaction, and the angle $\Theta$ at which the outgoing particle emerges. At first sight, it would seem advantageous to compare the cross sections of the $^{205}$Pb transition directly with those of the $^{209}$Pb transition. That is, the $^{208}$Pb(d,p)$^{209}$Pb angular distributions would serve as templates for the analysis of the corresponding particle transfers to $^{205}$Pb. Furthermore, since the spectroscopic factor is a dynamically independent quantity, both reactions in this study were performed at two incident energies, 13.0 and 20.0 Mev. Such a repetition facilitated the assignment of $\lambda$ values since the intrinsic cross section changes with energy quite differently depending on which $\lambda$ value is being considered. A table to this effect is contained in Chapter Five.

In principle, the above bypassing of the reaction calculation substitutes an uncertainty of a nuclear structure calculation for the presumably larger uncertainty of the direct reaction calculation. For example, the calculation of Vary and Ginocchio (VG 70), which includes RPA correlations, predicts that the positive parity states of $^{209}$Pb should have spectroscopic factors within a few per cent of unity. Thus the ex-
Experimental angular distributions of the single-particle states of $^{209}\text{Pb}$ carry a cross section representing unit spectroscopic strength for the particular transfer involved. To first order then the spectroscopic factor for a single-particle fragment in $^{205}\text{Pb}$ is simply the ratio of its experimental cross section to that of the corresponding $^{209}\text{Pb}$ cross section.

The qualifying phrase "first order" is necessary once the actual spectrum of $^{205}\text{Pb}$ becomes available. A complete bypassing of the calculation of $\sigma^{\text{DW}}_{\ell j}$ is then seen not to be possible. Essentially the particle fragments in $^{205}\text{Pb}$ are bound more strongly than are the single-particle states of $^{209}\text{Pb}$. This stronger binding has interesting physical implications but for the present purpose it modifies the $\sigma^{\text{DW}}_{\ell j} (E, Q, \Theta)$ because of the Q-value dependence. This dependence is directly attributable to a change in the outgoing proton's energy and introduces a correction factor to the previous ratio of experimental cross sections. The spectroscopic factor for a fragment in $^{205}\text{Pb}$ is now the ratio of experimental yields corrected by the number $G_{\ell j} (E, Q, Q')$ given by

$$G_{\ell j} (E, Q, Q') = \frac{\sigma^{\text{DW}}_{\ell j} (209) (E, Q, \Theta)}{\sigma^{\text{DW}}_{\ell j} (205) (E, Q, \Theta)}$$

(3.1)

where $Q'$ is the Q-value for a strong state in $^{209}\text{Pb}$ and Q is that for a fragment of the same spin and parity in $^{205}\text{Pb}$. Rather than a knowledge of the absolute magnitude of $\sigma^{\text{DW}}_{\ell j}$ equation (3.1) now requires that the dependence of the intrinsic cross section on the Q-value be correctly calculated. This might be assumed but it cannot be experimentally checked. What can
be done is to check the behavior of $\sigma_{2j}^{DW}$ against the changes in the incident energy $E$. A graph to this effect has been prepared by MacFarlane and is shown in Figure III-1. (MF 69). He makes the following pertinent comments: A) the variations from one experiment to another diminish at energies below the Coulomb barrier; b) the absolute values vary up to 25% from the presumed spectroscopic limit of unity; c) the relative strength within one experiment deviate somewhat less than the absolute strengths from one experiment to another.

It is noted that the spectroscopic factor fluctuations represented in Figure III-1 are encountered when both the deuteron's and the protons energies are varied. While it is true that Equation (3.1) requires only that the dependence on the proton energy be correctly calculated there is no reason to suspect that the discrepancies in Figure III-1 are due solely to a poor knowledge of the deuteron's relative motion function. Were matters to be left standing at this point, there might indeed be a question as how best to proceed with the $^{205}$Pb angular distribution analysis. In fact, we will now show that a combination of problems has affected the analysis represented in Figure III-1. Furthermore, that when properly used, the distorted-wave method yields consistent spectroscopic factors and quite impressive fits to the experimental angular distributions.
A summary of the $^{209}$Pb single particle spectroscopic factors as obtained at various incident deuteron energies. The implication of this graph, namely that the DW theory becomes unreliable at higher incident energies, would be further reinforced if the 18.7 MeV values (JD 69) were the physical realistic ones discussed in on page 75. Section E, however, will revise the analysis on which this graph is based.
B. THE FORMAL THEORY OF THE REACTION MECHANISM

The \( (d,p) \) neutron transfer reaction can be represented as

\[
A + d \rightarrow B + p
\]  

(3.2)

where \( A \) is the target and \( B \) the residual nucleus. The task is to predict the differential cross-section \( \sigma^{th}(\theta) \) of the outgoing proton groups for a given incident energy of deuterons. This will be first accomplished by writing a solution which assumes a knowledge of the exact wave function \( \psi_d(+) \). Such a solution will then be rendered practical by making approximations to \( \psi_d(+) \), and then by approximating the resulting expression (AU 70).

The total wavefunction, \( \psi_d(+) \) satisfies the Schroedinger Equation:

\[
H\psi_d(+) = E\psi_d(+) \tag{3.3}
\]

where \( E \) is the system energy, \( H \) the Hamiltonian, and where the superscript \( (+) \) indicates outgoing spherical waves in all open channels and the subscript \( d \) denotes incoming spherical waves in the elastic deuteron channel only. The Hamiltonian can be written in the exit channel as

\[
H = T_p + H_B + V_{pA} + V_{pn} \tag{3.4}
\]

for which \( H_B \) represents the internal structure of the residual nucleus, \( T_p \) the kinetic energy of the outgoing proton, and \( (V_{pA} + V_{pn}) \) the interaction of the proton with the residual nucleus. For further convenience
an arbitrary function, $U_p(r_B)$, is now introduced whose argument is the magnitude of the center of mass vector $r_B = r_p - \sum_{i}^{A+1} r_i$

The function $U_p$ in turn defines eigenfunctions according to the Equation

$$(T_p + U_p - E) \chi_p(+) = 0$$  \hspace{1cm} (3.5)$$

The formal solution to the scattering problem, as first derived by Gell-Mann and Goldberger (GG 53), solves for the transition matrix $T$

$$T = \langle \chi_p(-) | V_pA + V_{pn} - U_p | \psi_d(+) \rangle$$  \hspace{1cm} (3.6)$$

where $\sigma^{\psi}(\theta)$ is proportional to $|T|^2$, and $\psi_p$, $\psi_B$ represent the internal structures of the proton and the residual nucleus. Except for $U_p$, $\psi_p$ and $\chi_p(-)$ (where the negative superscript implies time-reversal), all the terms of Equation (3.6) are unknown and must be approximated by applicable models.

The first model, the direct-interaction model, replaces the exact $\psi_d(+) )$ by a sum of two-body wavefunctions

$$\psi_d(+) = \sum_{\gamma_1 \gamma_2} \psi_{1\gamma_1} \psi_{2\gamma_2} \xi_{\gamma_1}^*(r_{\gamma_2})$$  \hspace{1cm} (3.7)$$
The functions $\psi$ represent the internal motions of the two interacting particles while $\xi_\gamma$ describes their relative motion function. The channel index $\gamma$ subsumes all pertinent quantum numbers. Such a severe truncation of $\psi_d^{(+)}$ demands that $\xi_\gamma$ be a slowly varying function of energy. Furthermore, since there could be transitions into and out of the included two-body channels via the excluded channels of $\psi_d^{(+)}$, the functions $\xi_\gamma$ and all the operators in the space defined by Equation (3.7) must be constructed to reflect this neglected flux. The required relative motion functions $\xi_\gamma$ are therefore obtained by using optical potentials whose imaginary parts describe the flux generated by the excluded channels, and whose energy-averaging basis allows no resonance behavior in the relative motion function $\xi_\gamma$. For the direct-interaction model, then, the transition amplitude becomes

$$T^{\text{DI}} = \langle \chi_p^{(-)} \psi_B \psi_p | V_{pn} + V_{pn} - U_p | \sum_\gamma \psi_{1\gamma} \psi_{2\gamma} \xi_\gamma (r_\gamma) \rangle$$

(3.8)

This expression is exact to the extent $\xi_\gamma$ compensates for the truncation of $\psi_d^{(+)}$ but it is independent of both $U_p$ and $\chi_p^{(-)}$.

Although the exact Equation (3.8) is not physically useful since there is no prescription for the truncation of the series $\gamma$ in Equation (3.7) nor is there any suggestion how such truncation would affect to the accuracy of Equation (3.8). Obviously, the summation must include the elastic deuteron channel denoted by the index $\gamma = \alpha$ for which
Two further approximations are now made and these constitute the Distorted-Wave Method. The first approximation produces the appropriate entrance channel function $\xi_\alpha$ by an optical model $U_d(r_\alpha)$ which fits the elastic deuteron scattering angular distribution

$$\xi_\alpha(r_\alpha) \approx \chi_d^{(+)}(r_\alpha) \quad (3.10)$$

The second approximation simply ignores the second line of Equation (3.9) and obtains for the transition amplitude

$$T^{DW} = \langle \chi_p^{(-)} \psi_B \psi_p | V_{pA} + V_{pn} - U_p | \psi_d \psi_A \chi_d^{(+)} \rangle \quad (3.11)$$

The first approximation, Equation (3.10), is justified by noting that in the asymptotic region $\xi_\alpha$ and $\chi_d^{(+)}$ should be identical, there being no nuclear effects to couple the elastically scattered deuterons into other channels of $\psi_d^{(+)}$. The fit to the elastic data assures this identity.
It will be shown later that the break-up of the deuteron in the nuclear region requires a modification of Equation (3.10). The validity of the second approximation, the neglect of the second term of Equation (3.9), is not obvious and depends critically on the choice of $U_p$, the optical potential in the exit channel. This can be seen by considering the elastic proton channel, $\gamma = \delta$, amplitude explicitly, viz.

$$\int d^3 r_\delta \chi_p^{(-)}(r_\delta) \{<\psi_B^\dagger \psi_p | V_{pA} + V_{pn}| \psi_B \psi_p> - U_p(r_\delta)\} \xi_\delta(r_\delta)$$

Since there is no internal rearrangement, the elastic matrix element term is significant. The term represents the scattering of the outgoing proton by the residual nucleus and is approximated by choosing $U_p$ to fit the elastic scattering of protons by nucleus B. The remaining terms of the second line of Equation (3.9) must be negligible for the DW method to be correct. This is not the case for certain two-step processes for which the residual state $\psi_B$ can be reached by different competitive routes. Such effects have recently been established (MA 73) for the (d,p) reaction in the rare-earth region as illustrated in Figure III-2. In this region, the amplitude of the two-step route, consisting of an inelastic excitation followed by stripping onto the now excited target, becomes competitive with the direct transition amplitude. Restated according to Equation (3.8), the term representing the excited target in $\psi_d^{(+)}$, i.e.,

$$<\chi_p^{\dagger} \psi_B^\dagger \psi_p(r_B) | V_{pn} - V_{pA} - U_p | \psi_d \psi_A^* \xi^*(r_d)>$$
III-2 Figures representing direct reaction amplitudes. In the upper left appears the usual one-step transition amplitude. The lower left diagram represents a target excitation followed by a transfer. The upper right diagrams shows the full-fledged Coupled Channels Born Approximation (CCBA) in which one-step and two-step processes are considered along with excitation and de-excitation of the target and residual nuclei. The CCBA is accurate to all orders in the inelastic channel whereas the perturbation expansion, represented in the lower right, is only approximate. See (MA 73) and references therein for further detail.
Direct (DWBA)

Core Excitation

CCBA

Perturbation Method
is appreciable with respect to the elastic channel term. Instead of the Equation (3.9), the \((d,p)\) reaction performed in the rare earth region requires the solution of a system of coupled equations. The situation in the spherical lead region is not as conducive to second order processes. There are no low-lying strongly collective rotational levels, but instead there exists a very collective octupole state (~35 w.u.) \((ZP \, 68)\) at 2.62 Mev of excitation. The inelastic excitation and subsequent de-excitation of this level in either the incoming or the outgoing channel could interfere with the direct-transfer amplitude but this has not yet been proved \((AS \, 73)\). In this context it is pointed out that there is one transfer to one of the low-lying states in \(^{205}\text{Pb}\) which cannot be fit by the usual reaction calculation. This failure is discussed at length in Chapter Five. This one possible case aside, it will be assumed hereafter that two-step processes do not ordinarily take place in the \((d,p)\) reaction in the lead region.

The second approximation of the DW method has a corollary which states

\[
\langle \chi_p^- \psi_B \psi_p^- | V_{pA} - U_p \psi_d \psi_A \chi_d^+ \rangle \approx 0 \quad (3.12)
\]

The limits to this corollary can be obtained by noting that \(V_{pA}\) has both elastic and inelastic matrix elements. The latter are likely to be small involving complicated overlaps of rearranged nucleons, while the former are hopefully cancelled by the elastic term \(U_p\). It has been said that the complex potential \(U_p\) cannot completely cancel out the matrix elements of \(V_{pA}\) since \(V_{pA}\) derives from a Hermitian Hamiltonian. This is an erroneous
conclusion since the truncation effected by Equation (3.7) re-defines the Hamiltonian to be complex also in order to account for flux to the excluded channels. It is clear that since the imaginary terms in the usual optical potentials are significant compared to the magnitudes of the real terms, a proper cancellation presumes a complex $V_{pA}$. The cancellation does depend upon the validity of the optical model because the optical potential $U_p$ simulates elastic scattering by the residual nucleus $B$, and not $A$. Since it is implicit in the optical model that there be little distinction in the scattering by adjacent nuclei, the DW method is correct only insofar as the optical model is applicable at the energy and in the mass region under consideration. This point will be discussed later in connection with the existence of isobaric analog resonances in the lead region. On the basis of the preceding considerations, it is now apparent that at these or any other resonance energies the DW method is not reliable since the resonances are not allowed within the framework of the optical model.

If the assumptions of the DW method are appropriate, then the transition amplitude can be written explicitly as

$$T_{dp} = \int d^3r_d \int d^3r_p \chi_p(-)(r_p, k_p)\langle \psi_{Bp} | V_{pn} | \psi_{A_d} \rangle \chi_d^{(+)}(r_d, k_d)$$

(3.13)

This amplitude is seen to be composed of two parts: a nuclear matrix element and an integral which is principally a dynamical function of the distorted waves. The former contains the interesting physical information. The matrix element $\langle \psi_{Bp} | V_{np} | \psi_{A_d} \rangle$, includes an overlap integral of the
target A and the residual nucleus B which produces a wavefunction with the coordinates of the captured neutron. This function together with the proton wavefunction is then connected to the deuteron wavefunction via the potential $V_{np}$. Expressed in terms of the target and the residual momenta, $J_A$ and $J_B$, the result is

$$<\psi_B\psi_p|V_{np}^m|\psi_A\psi_d> = \sum_j J_B^j M_B^j |J_A^j; M_A^j, m_j> <\psi_p^B|V_{np}^m|\psi_d>$$

(3.14)

where:

$$J = J_B - J_A$$

and

$$m = M_B - M_A$$

The bound neutron function $\psi_{BA}^j$, commonly called the form factor, carries the superscript $BA$ to specifically denote the stripping origin of this function and that it is not directly related to the eigenstate of a single particle potential. A further reduction of Equation (3.14) is possible using an explicit representation of the deuteron's and the captured neutron's wavefunctions. The usual approach to the deuteron wavefunction notes that the Schroedinger Equation for $\psi_d$ is

$$V_{np}\psi_d = (\Delta^2 - \gamma^2) \psi_d$$

(3.15)

where $\frac{\gamma^2\hbar}{m_d}$ = binding energy of the deuteron

The function $\psi_d$ is assumed to be a symmetric S state only (which effectively ignores the tensor part of $V_{np}$), and the radial dependence is given by the Hulthen function for which Equation (3.15) becomes
Another reduction is obtained by noting that the range of $V_{np}$ is short compared to the range of significant change in the functions $\chi_d^{(+)}$ and $\chi_p^{(-)}$. The zero-range approximation replaces $D(r_{np})$ by a delta function, meaning physically that the proton is emitted immediately at the point at which the deuteron was absorbed. The advantage of the zero-range approximation is that the six-dimensional integral of Equation (3.13) is reduced to a three-dimensional integral by the zero range function

$$D(r_{np}) \approx D_0 \delta(r_p - \frac{A}{B} r_d)$$  \hspace{1cm} (3.17)$$

The representation of the form factor $\phi_{ij}^{BA}$ is not so straightforward. It must be approximated since $\psi_B$ is generally unknown. Its asymptotic form can be inferred by recalling that it is bound by the binding energy difference $E_B - E_A$, and hence,

$$\phi_{ij}^{BA}(r) \approx \frac{e^{-\gamma r}}{r}$$  \hspace{1cm} (3.18)$$

(where $\gamma^2 = \frac{2m_n}{\hbar^2} (E_B - E_A)$)
At this point, the assumption is made that the form factor can be adequately represented by the product of a spectroscopic factor \( (S_{BA}^{XJ}) \) and an eigenstate of a shell potential whose depth is adjusted to obtain the correct asymptotic decay of Equation (3.18) i.e.,

\[
\phi_{\lambda j}^{BA} (r) = \left( S_{\lambda j}^{BA} \right)^{1/2} R_{n\lambda j} (r) \tag{3.19}
\]

The shell model wavefunction \( R_{n\lambda j} (r) \) contains the radial node number \( n \) as well as the orbital angular momentum value \( \lambda (\lambda = j \pm \frac{1}{2}) \) which is even or odd depending on the parity of the form factor. The spectroscopic factor has a simple nuclear structure interpretation subject to one important condition. Provided that the interior radial dependence of \( \phi_{\lambda j}^{BA} \) is reflected by \( R_{n\lambda j} (r) \), the spectroscopic factor \( S_{\lambda j}^{BA} \) signifies the degree to which the final state \( B \) resembles the target state \( A \) coupled to a neutron in a shell-model orbital specified by the quantum numbers \( n\lambda j \). For the archetypal reaction, that of stripping into a nearly empty shell-model state outside of a doubly-closed core, Equation (3.19) is a very good approximation. For target nuclei removed from a closed shell and with residual nuclei having fragmented shell states, Equation (3.19) becomes less correct since the proper asymptotic behaviour of \( R \) does not guarantee a valid interior behavior. The limits to the separation energy procedure of Equation (3.19) are obtained by noting the extent to which the binding energies of the various states of \( B \) with the same \( \lambda j \) fall near the binding energy predicted by a normal shell potential (\( \approx 50 \text{ Mev deep} \)). For those states satisfying such an equality, the separation energy prescription is consis-
tent and comparisons of the spectroscopic factors are physically meaningful. But for the states a few Mev removed from the shell model energy it is likely that Equation (3.19) fails to describe properly the form factor. More complicated calculations for the form factor are then mandated and the simple interpretation of $S^{BA}_{\lambda_j}$ is lost (PS 65).

With the means of calculating the form factor it becomes possible to obtain an expression for the differential cross section. The derivation is complicated by mundane details of angular momentum coupling, coordinate transformation and anti-symmetrization and thus only the final result is presented here. For a spin-zero target only one $\ell$ value will conserve spin and parity thus allowing for a very simple equality:

$$\sigma^{\text{th}}(\theta) = N_d S^{BA}_{\lambda_j} \sigma^{\text{DW}}(E, Q, \theta)$$

(3.20)

Here the constant $N_d$ relates to the normalization of the deuteron's wave-function ($N_d = 1.5$), and $\sigma^{\text{DW}}$ is the result of the coordinate integration given in Equation (3.13). The calculation of $\sigma^{\text{DW}}$ is done by a computer; in the present case, the code DWUCK (KU 69) was used throughout the angular-distribution analysis.

This completes the formal theory of the reaction mechanism. Uncertainties in the form factor have been alluded too but none were directly expressed concerning the calculation of the relative motion functions $x_d$ and $x_p$. This aspect of the direct reaction calculation is dealt with in the following two sections for which the incoming deuteron channel is treated first.
C. THE DEUTERON BREAK-UP MODEL

The principal difficulty with the DW method as applied to a (d,p), (d,n), or their time-reversed reactions is chillingly simple. The method represents the first term in what is likely to be a non-converging series for the exact transition amplitude (GD 66). Such a deficiency should be disastrous but the overwhelming experimental evidence (AU 70) demonstrates a surprising if not embarrassing success for the method. Therefore, the problem with the simple DW approximations is not necessarily to produce a better reaction theory, but to understand the present reaction mechanism well enough to discover why it is so useful.

The above discussion can be definitely perceived by regarding Equations (3.7) and (3.10) of the previous section in the light of the weak deuteron binding energy (= 2.22 MeV). These equations state the deuteron remains bound despite the fact that the nuclear field of the target can clearly dissociate or distort the deuteron bound (Hulthen) wavefunction. Apparently a correct approach to deuteron reactions ought to account for the three-body aspects of the motion. Yet the standard DW method does no such thing and instead uses the two-body approximation, Equation (3.7). Even worse, the internal structure of the deuteron is assumed to be totally undisturbed by the target's nuclear field, while the relative motion function, which should include both bound and unbound deuterons, is simply extrapolated from a knowledge of the asymptotic elastically scattered (bound) deuterons. Ignored in this formulation is the break-up of the deuteron and the subsequent capture of the now unbound neutron. It has been argued (BA 67, TG 66) that this last process is unimportant since the elastic channel is the dominant part of $|\psi_d^{(+)}\rangle$ and the continuum deuterons are insignificant in comparison. The exact treatment of the continuum
deuterons shows on the contrary that they can be important and must be retained in the calculation. It is this exact calculation first done by Johnson and Soper (JS 70) and more rigorously later by Bloudin and Levin (BL 72) which resolves the dilemma of how the ostensibly poor DW method predicts generally good results.

The complete derivation of the deuteron break-up model is a lengthy and involved three-body calculation. However, the remarkable end result is that the exact transition amplitude in this model has precisely the same form as the ordinary DW amplitude in Equation (3.13). That is, an exact treatment of the continuum deuterons in $\psi_d^{(+)}$ allows the substitution

$$|\psi_d^{(+)}\rangle \rightarrow |\psi_A\psi_d \chi_d^{(+)}\rangle$$

(3.20)

where as before $\psi_A$ and $\psi_d$ represent the internal ground states of the target and the deuteron and $\chi_d^{(+)}$ is a relative motion function which can be generated by a modified optical potential, $\bar{U}_d$. The single important change from the usual DW calculation is emphasized by the bar superscripts. The potential $\bar{U}_d$ and the function $\chi_d^{(+)}$ are not directly connected with the deuteron elastic scattering angular distribution. Rather the function $\chi_d^{(+)}$ specifically relates to the behavior of the deuteron in the vicinity of the target during the stripping transfer. This function is part of a set of three coupled equations with similar functions for the elastic scattering and for the proton stripping, $(d,n)$ reaction. At this point it is certainly not obvious why the deuteron ground-state wavefunction, $\psi_d$, can be retained in Equation (3.20). With equal certainty, it is also not obvious why the two body relative motion function $\chi_d^{(+)}$
describes both bound and unbound relative motion in $\psi_d^{(+)}$. These two facets of Equation (3.20) are not intuitive because: a) the motivation for the introduction of $\psi_d$ was the bound state Schrödinger equation for the deuteron

$$V_{np} \psi_d = (\Delta^2 - \gamma^2) \psi_d$$  \hspace{1cm} (3.15)

which of course does not hold for the deuteron continuum states and b) even if the use of a two-body relative motion function $\chi_d^{(+)}$, is admitted there is no reason why it should correspond in the nuclear region to the usual relative motion function $\chi_d^{(+)}$ which satisfies the deuteron elastic scattering differential cross section.

An insight into the deuteron break-up model can be obtained from the Gell-Mann Goldberger equation for which it is seen that only a partial knowledge of $\psi_d^{(+)}$ is required, viz.

$$V_{np} |\psi_d^{(+)}>$$

Now if it is assumed that $V_{np}$ is a potential separable in the neutron and the proton coordinates, then the following can be shown to be true (BL 71)

$$V_{np} |\psi_d^{(+)}> = V_{np} |\psi_d \chi_d^{(+)}>$$  \hspace{1cm} (3.21)
The interaction $V_{np}$ is seen to behave like a projection operator in that it projects out only the two-body parts of $\psi_d^\dagger$ with the deuteron in an undistorted ground state and that these two bodies have a simple relative motion function, $\chi_d^\dagger$. It is noted that the only model dependent part to this result is the fact that $V_{np}$ be separable. This property is in accord with neutron-proton scattering and triton binding energy calculations. (MI 65).

It is now more clear why the simple DW method has heretofore been so valuable. Insofar as the relative motion function $\chi_d^\dagger$ corresponds to $\chi_d^\dagger$, the ordinary DW calculation has already included exactly(!) the distortion and break-up of the deuteron in the nuclear field of the target. Errors in the differential cross section predicted by the old DW matrix element will be visible only if $\chi_d^\dagger$ differs significantly from $\chi_d^\dagger$, and sometimes (accidentally) not even then. In general the determining factor is the incident energy of the deuteron. The break-up discussed here is induced by the nuclear field of the target and not by the Coulomb field. Hence for the deuteron reactions conducted below the Coulomb barrier, nuclear induced deuteron break-up is not an important consideration. On the other hand, the higher the incident energy of the deuteron in relation to the Coulomb barrier, the more necessary it is to use the function $\chi_d^\dagger$, especially if spectroscopic values are being extracted. The improvements arising from the deuteron break-up model have been observed in widely-separated mass regions ($A \approx 16, 50, 208$) (PR 72, HJ 71, SA 71). Thus it would seem that this model represents an important step forward in the understanding of the reaction mechanism. In fact, the present work will show additional evidence of the success of the break-up model in section E of this chapter. This section will now conclude with a dis-
discussion of the effective distorted wave $\chi_d^{(+)}$, as originally obtained by Johnson and his co-workers.

The effective distorted wave can be generated by the usual optical model Equation (3.5) where instead of using a potential $U_d$ which fits the deuteron elastic scattering angular distribution, one use a potential $U_d$ defined by

$$U_d(R) = \frac{1}{D_0} \int_0^R d\rho \left[ U_n(R + \frac{1}{2}\rho) + U_p(R - \frac{1}{2}\rho) \right] V_{np} \psi_d(r) \tag{3.22}$$

where $D_0$ has been previously defined in Equation (3.17), $R$ is the deuteron center-of-mass, and $r$ is the relative coordinate of the neutron-proton separation in the deuteron center-of-mass, and most importantly, $U_n$ and $U_p$ are neutron and proton optical potentials which fit the respective elastic scattering at one half the incident energy of the deuteron. In general, (e.g. BG 69) $U_n$ and $U_p$ have the same geometrical (radii and diffuseness) values, but somewhat different depths. The main effect of averaging them over the short ranged $V_{np} \psi_d$ is to give a break-up potential with depths about equal to the sum of the depths of $U_p$ and $U_n$ and a geometry about the same as that for either a proton or a neutron. While Equation (3.22) could be put into a standard DW code, Harvey and Johnson have developed a short prescription which avoids the full integration and which is claimed by them to be almost as accurate. That is, the break-up potential can now be defined as (HJ 71)
\[ \bar{V} = V_p + V_n \] (real depth)

\[ \bar{W} = (W_p + W_n) \frac{a^R}{\bar{a}^R} \] (imaginary depth) \hspace{1cm} (3.23)

\[ \bar{a}^R, I = a^R, I + 0.04 \] (real, imaginary difuseness)

\[ \bar{r}^R, I = r^R, I \] (real, imaginary radius)

The break-up potential defined by Equation (3.23) will not fit the deuteron elastic scattering angular distribution. While this means that \( U_d \) gives an incorrect asymptotic relative-motion function, such an error is inconsequential, since asymptotically the form factor is zero. Instead, the break-up potential describes more correctly the motion of the deuteron near the nucleus where the form factor is non-zero. This is the important region for the reaction calculation and it explains why the break-up potential is successful. A comparison of such a potential with a more conventional potential for 23 Mev deuterons on iron is shown in Figure III-3a. According to Harvey and Johnson the most significant difference between the conventional and the break-up potentials is the decreased radial extent of the imaginary absorption term \( W(r) \) from that of the conventional value. The reason \( r^I \) is so large is that it must simulate the dissociated deuterons being removed from the elastic channel. However, the exact three-body calculations show \( \frac{\chi_d}{(+) \chi_d} \) to already have these components. Therefore, \( W(r) \) need not be so effective. The angular distributions predicted by these two potentials are shown in Figure III-3b. The
A comparison of a conventional deuteron optical potential and that prescribed by the adiabatic break-up model of Johnson and Soper. For ease of viewing, the imaginary terms are inverted in sign. The actual parameters are:

(Conventional) \( V = 105 \text{ MeV}, W = 15 \text{ MeV}, r^R = 1.02 \text{ fm}, a^R = 0.86 \text{ fm}, r^I = 1.42 \text{ fm}, a^I = 0.65 \text{ fm}; \) (break-up) \( V = 97 \text{ MeV}, W = 20 \text{ MeV}, r^R = 1.25 \text{ fm}, a^R = 0.69 \text{ fm}, r^I = 1.25 \text{ fm}, a^I = 0.50 \text{ fm}. \)
III-3b  Fits to the $^{56}\text{Fe}(d,p)^{55}\text{Fe}$ experimental angular distribution to the $1/2^-$ state at .417 MeV. The failure of the conventional model was originally interpreted as $j-$dependence in the reaction mechanism.
$^5_{44}$Fe(d,p)$^{55}$Fe

$E_d = 23.0$ MeV

$E_x = 0.417$ MeV  $1/2^-$

$\frac{d\sigma}{d\Omega}$ (Arbitrary units)

$\theta_{c.m.}$
reaction data are for the $\ell_n = 1$ transfer populating the 0.417 Mev $1/2^-$ state in $^{55}$Fe. (YO 67). The same proton parameters were used in each case (SA 67). It is obvious that the deuteron-break-up potential provides the better fit to the data. A good fit to the data can be obtained using the conventional parameters only at the expense of artificially damping the interior contributions to the integrand of Equation (3.13). Similar improvements will be shown for the $^{208}$Pb(d,p)$^{209}$Pb single particle angular distributions taken in the range 17.0 to 24.8 Mev. Before that is done, it is first necessary to re-consider the treatment of the outgoing proton channel. This is the concern of the next section.
D. THE EFFECT OF THE ANALOG RESONANCES IN THE PROTON CHANNEL

As was emphasized in section B, the DW method has an inescapable dependence on the optical model for the provision of correct relative motion functions. This continues to be the case for the deuteron break-up model since it is now required to have good optical potentials for both the neutron and the proton at half the energy of the incident deuteron. The crucial role of $U_p$, the elastic potential for the outgoing protons, has already been examined in the light of Equation (3.9). The central feature of $X_d$ and $X_p$ is that they display the essential character of the direct reaction mechanism, namely: these reactions proceed at high energies where many open channels are available and the reaction proceeds in such a short time that only a few degrees of freedom are excited. The opposite to this is the compound nucleus reaction. Such reactions take place at lower energies where the number of open channels is greatly reduced. At certain energies, for either the incoming or the outgoing particle, the exterior wavefunction at the nuclear surface may match well the ingoing or outgoing part of the eigenfunction in the compound system. The prominent signature of a compound nuclear reaction is then the display of a resonance "bump" in the excitation function for a given reaction, indicating that certain energies are more preferred than others for the formation, or decay, of a compound system. It is clear that the ordinary optical model which presumes many open channels in the compound system and averages over an energy interval in that system, cannot produce a relative motion function which will reflect such an excitation function.

Generally it is thought that this last restriction has no bearing for most direct reactions. As stated above, for energies at which direct re-
actions are possible, there are an extremely large number of states in
the compound system. Were an actual compound nucleus to be formed in
the continuum, there would be many nearby overlapping states through which
it would quickly decay. Hence, it is generally believed that the contrib-
ution of long-lived compound nucleus reactions is small in comparison
to the direct amplitude. There is one important exception to this rule,
however, and that is the existence of isobaric analog resonances in the
proton channel.

The isobaric spin formalism was originally introduced by Heisenberg
(HE 32) as only a mathematical convenience for distinguishing the proton
and the newly discovered neutron. Completely paralleling the Pauli in-
trinsic spin formalism, he defined isospin operators, \( t \) and \( t_z \), and their
eigenstates for which \( t \) has an eigenvalue of \( \frac{1}{2} \) and \( t_z = \frac{1}{2} \) and \( -\frac{1}{2} \) for the
neutron and proton respectively. It was not until years later that stud-
ies in light mirror nuclei revealed the charge independence of
the nuclear force, and hence that isospin was a physically useful con-
cept in nuclear structure theory. The usefulness is of course tempered
by the presence of the Coulomb field which in light nuclei is not suffi-
cient to mix strongly well separated states of different isospin. It
was thought that isospin could not be a good quantum number in heavy nu-
clei since there the non-saturating Coulomb field achieves a strength
comparable to the nuclear field. The error in this belief became appar-
ent when Anderson and Wong (AW 61) accidentally discovered the analog
state in \(^{51}\text{Cr}\) corresponding to the ground state of \(^{51}\text{V}\) as observed in a
\((p,n)\) reaction. The realization then dawned that especially in heavy nu-
clei, the neutron excess overcomes the isospin impurities in the core.
Furthermore, not only do the low-lying states have good isospin, \((T,T_z)\)
but the higher lying \( T > \) isobaric analogs \((T + 1, T_z)\) do not mix appreciably with the extremely dense background of \( T < \) states.

Analog resonances are also seen in proton elastic scattering excitation functions, the difference being that the analog state is now in the compound system rather than in the residual nucleus as in the \((p,n)\) reaction. It should be realized that such resonances, schematically \(|(pA)T,T_z-1>\) are closely related to the low-lying states excited in a neutron stripping reaction, \(|(nA)T,T_z>\); the analogs differ only in the Coulomb energy difference of the last proton (plus the small neutron-proton mass difference) and the projection of the isospin \( T_z \). This has already been mentioned in Chapter One. Shown again in Figure III-4 are three excitation functions taken in the lead region (SC 69). It is noticed that the resonance structure also occurs in the odd target case \(^{207}\text{Pb}\). Here the proton is in a single-particle analog state in the compound system \((^{208}\text{Bi})\), but because the odd core has a \( \frac{1}{2}^- \) spin, the \(^{207}\text{Pb}(p,p)\) excitation function actually represents the population of doublets in the compound nucleus. This can be most clearly seen at 15.0 MeV incident proton energy where the doublet pattern corresponds to the formation of a \( 4^- \) and a \( 5^- \) state in the compound nucleus.

The question now arises as to the role of isobaric analog states as they affect a transfer reaction. From the above discussion it can be realized that such effects are strongly energy dependent, becoming important near those energies relating to the formation of analog states in the compound nucleus, that is 15.0 to 17.5 Mev proton energy for the lead region. Away from these analog resonances the simple interpretation of the reaction mechanism should once again prevail. An exact calculation described below bears this out. Most importantly, even if the outgoing pro-
The proton elastic excitation functions of $^{206, 207, 208}_{\text{Pb}}$ in the resonance region. The splitting of the analog states in $^{208}_{\text{Bi}}$ can best be seen in the first resonance on $^{207}_{\text{Pb}}$. 
EXCITATION ENERGY IN Pb\(^{209}\) (MeV)

Pb\(^{208}\)(p,p)Pb\(^{208}\) g.s. (O\(^+\))
\(\theta_{\text{LAB}} = 165^\circ\)

d\sigma/dQ (mb/\text{sr})

PROTON LABORATORY ENERGY (MeV)

EXCITATION ENERGY IN Pb\(^{208}\) (MeV)

Pb\(^{207}\)(p,p)Pb\(^{207}\) g.s.(\nu 2\(^-\))
\(\theta_{\text{LAB}} = 165^\circ\)

d\sigma/dQ (mb/\text{sr})

PROTON LABORATORY ENERGY (MeV)

EXCITATION ENERGY IN Pb\(^{207}\) (MeV)

Pb\(^{206}\)(p,p)Pb\(^{206}\) g.s. (O\(^+\))
\(\theta_{\text{LAB}} = 165^\circ\)

d\sigma/dQ (mb/\text{sr})

PROTON LABORATORY ENERGY (MeV)
tons from a (d,p) reaction are beyond the last analog resonance of Figure III-4, their relative motion function cannot be calculated from an optical potential parameter set derived near an analog resonance energy. The fact is that such an optical parameter set does have an extensive use (MP 67, MC 70, WS 69). This particular set was obtained by F. Perey in 1964 from 17.0 MeV elastic proton scattering on $^{208}$Pb (PE 64). It is an historical fact then that this study antedates the discovery of the analog resonance in lead at this same incident energy (SC 69). Table III-1 lists this set along with another proton parameter set derived by Satchler who fitted the angular distributions and polarizations of the $^{208}$Pb(p,p) scattering at 19.0, 20.0, 25.0, and 30.0 MeV. (SA 71); also listed are the parameters of a conventional deuteron optical potential (YN 59), and those of a deuteron break-up potential (SA 71). The main difference between the Perey set and the proton set of Satchler is in the depth of the imaginary surface absorption term. The lower value of the Perey set (=7.5 Mev) can easily be explained by noting that the resonance serves to increase the proton flux at this energy over the non-resonant (optical) background. Hence, less than a normal amount of absorption is required at this incident energy. The use of the Perey set has a particularly deleterious effect on the $\lambda_{\pi} = 0$ transfer to the $4s_{\frac{3}{2}}$ state in $^{209}$Pb as will be shown in section E of this chapter.

None of the conclusions drawn in this dissertation are based on data for which the outgoing protons were at a resonance energy. The protons from the higher-lying particle states fell below and above the resonance region at 13.0 and 20.0 MeV incident energy, respectively. Similarly, the data from the low-lying states taken at 20.0 MeV were far above the resonance region; it is only the low-lying states as excited at 13.0 MeV
<table>
<thead>
<tr>
<th>Source</th>
<th>Particle</th>
<th>$V_0$ (MeV)</th>
<th>$r_0$ (fm)</th>
<th>$a_0$ (fm)</th>
<th>$W'_0$ (MeV)</th>
<th>$r'_0$ (fm)</th>
<th>$a'$ (fm)</th>
<th>$V_{s.o.}$ (MeV)</th>
<th>$r_{s.o.}$ (fm)</th>
<th>$a_{s.o.}$ (fm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(YN59)</td>
<td>d</td>
<td>100.0</td>
<td>1.14</td>
<td>.89</td>
<td>13.8</td>
<td>1.33</td>
<td>.75</td>
<td>0.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(SA71)</td>
<td>d</td>
<td>112.0</td>
<td>1.25</td>
<td>0.682</td>
<td>19.4</td>
<td>1.25</td>
<td>.783</td>
<td>6</td>
<td>1.12</td>
<td>0.47</td>
</tr>
<tr>
<td>(PE63)</td>
<td>p</td>
<td>52.0</td>
<td>1.25</td>
<td>0.65</td>
<td>7.5</td>
<td>1.25</td>
<td>.76</td>
<td>0.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(SA71)</td>
<td>p</td>
<td>52.0$^b$</td>
<td>1.25</td>
<td>0.65</td>
<td>10.0</td>
<td>1.25</td>
<td>0.76</td>
<td>6.0</td>
<td>1.12</td>
<td>0.47</td>
</tr>
</tbody>
</table>

$^a$) Potentials are of a Woods-Saxon shape with radius $r_0 A^{1/3}$ and diffuseness $a_0$. The term $V_r$ represents a real volume well; $V_{s.o.}$ is a surface type spin-orbit potential and $W'_0$ is an imaginary surface absorption term. For the sake of consistency with the original sources, the surface terms are given without the factor of 4.0 required by DWUCK.

$^b$) Actually Satchler gave the formula $V_0 (\text{MeV}) = 58.4 - 0.3 E_p$ for the real depth of the proton well. The value 52.0 represents a convenient average for the range of proton energies covered. In the worst case the average is less than 2.0 MeV away from the formula value and this does not make for significant changes in the predicted differential cross sections.
deuteron energy which may be affected by the resonances.* Although the angular distributions for the low-lying states at 13.0 MeV incident energy are presented in Chapter Five, the analysis of these data were not used to support any inferences made from the 20.0 MeV data. The reason for this caution can be understood by considering Figure III-5. This figure depicts the $^{206}$Pb(d,p) excitation function as observed at a lab angle of 165°. (SC 68) The energy scale corresponds to that of the outgoing protons† and the arrows denote the positions of the analog resonance energies for the elastic proton scattering excitation function. There is a pronounced resonance behavior in these (d,p) excitation function not at any particular incident deuteron energy but rather whenever the outgoing proton corresponds to a resonance in the elastic channel. Similar data exist for a variety of (p,d) and (d,p) excitation functions taken on different lead isotopes. (HA 67, BA 68, AB 68, SC 68) It is clear that if such resonance structure were to continue to appear at a more forward angle, then the extracted spectroscopic information would be in serious error depending on precisely what the outgoing proton energy was for the given reaction. These anomalies in the reaction excitation functions have been studied by Tamura and Coker (TC 69) in the context of an exact shell model approach to nuclear reactions (MM 67); that is, the analog continuum states are treated explicitly in the reaction calculation. As was the case with the deuteron break-up model, this approach leads to a set

* The $^{204}$Pb(d,p) $^{205}$Pb Q.g.s. = 4.510 MeV (ND 66). ($E_d=13$ MeV, the first particle state in $^{205}$Pb gives an outgoing proton energy of 14.72 MeV (c.m.) as compared with a 29/2 resonance of 14.86 MeV in the p + $^{208}$Pb experiment.

† The $^{206}$Pb(d,p) $^{207}$Pb Q.g.s. = 4.508 MeV (ND 66).
The $^{206}_{\text{Pb}}(d,p)^{207}_{\text{Pb}}$ excitation functions ($Q_{\text{g.s.}} = 4.508$ MeV) as observed at $\theta_{\text{Lab}} = 165^\circ$. Note the correlation of the anomalies with the proton resonance.
Pb$^{206}$ (d,p) Pb$^{207}$

G.S.(1/2$^-$)

0.57 MeV(5/2$^-$)

0.89 MeV(3/2$^-$)

$(\sigma_\text{d}/d\Omega)$ arbitrary units

(PROTON CHANNEL ENERGY)$_{c.m.}$ IN MeV
of complex equations but produces a relatively simple, modified transition amplitude in place of the direct-reaction amplitude of Equation (3.13). The modified transition amplitude is:

\[ T^{TC}(d,p) = <\chi_p(-) \phi_{lj}^BA | V_{np} | \chi_d(+) > - \frac{i}{2} S^{R\lambda}_{pp} \exp(2i\delta_p) <p_{lj}^BA | V_{np} | \chi_d(+) > \]

(3.24)

The first term in this expression is the usual reaction amplitude, while the second term includes a resonating amplitude contribution from the analog state. The relative-motion function \( Q_p \) is proportional to the purely outgoing part of the distorted wave \( \chi_p(-) \) and the background matrix element, \( \exp(2i\delta_p) \), can also be derived from \( \chi_p(-) \). The quantity \( S^{R\lambda}_{pp} \) resonates the second term of Equation (3.24) and is given by the usual parameters (WB 68) of the analog resonance:

\[ S^{R\lambda}_{pp} = -i\Gamma^\lambda_p \frac{E-E_R + \Gamma^\lambda_T/2}{(E-E_R + \Gamma^\lambda_T/2)^2} \]

(3.25)

The Equation (3.24) can be extended to treat overlapping resonances, which is the situation in the lead region. Theoretical fits to the \(^{207}\text{Pb}(d,p)^{208}\text{Pb} \) excitation function data are shown in Figure III-6. The authors note that the deuteron optical potential is not well defined at

* To be concise, the distorted waves \( \chi_p \), \( Q_p \), and \( \chi_d \) implicitly contain the intrinsic functions \( \psi_p \) and \( \psi_d \).
The $^{207}\text{Pb}(d,p)^{208}\text{Pb}$ excitation function ($Q_{\text{g.s.}} = 5.152$ MeV) as fitted by the Tamura-Coker modified reaction amplitude, Equation (3.24). The alternate fits are generated by two possible choices of the deuteron optical potential as given in the reference (TC 69).
$^{207}\text{Pb}(d,p)^{208}\text{Pb}$

$S_A = 2.8$

$S_B = 1.7$

Cross section in $\mu b/sr$

90°

125°

169°

Deuteron energy (LAB)

$d_{5/2}$

$s_{1/2}$

$g_{7/2}$
the sub-coulomb energies of the data and they thus have included two possible sets of deuteron parameters. One of these, set A, appears to produce a reasonably good fit to the anomaly, especially at the 90° observation angle at which the data show a 25% peak-to-valley resonance. Thus a spectroscopic factor extracted at 11.5 MeV incident deuteron energy would be that much lower than one extracted at 12.0 MeV deuteron energy, assuming that the anomaly persisted to the stripping peak angle (approximately 70° for this reaction, Q_{g.s.} = 5.15 MeV). This data represents the most forward angle at which the isobaric analog resonance is seen to affect the reaction. As a matter of fact, no data have been reported showing that the anomaly does not persist at higher incident energies at which the stripping peaks move to more forward angles. In section E of this chapter we note certain data which may indicate indeed that the resonance has given rise to erroneous spectroscopic factors extracted from certain \text{^{208}Pb(d,p)^{209}Pb} angular distributions obtained at E_d = 17.0 MeV (Q_{g.s.} = 1.720 MeV).

In summary, it can be said that there are two distinct problems connected with the existence of isobaric analogs in the proton channel. The first is that for experiments conducted wherein the protons corresponded to a resonance of the elastic channel, there is a distinct possibility that a final-state reaction is interfering with the direct transition amplitude. The second problem is that even for those experiments conducted outside the resonance region, the analysis is suspect if the Perey 17.0 MeV proton optical parameter set was used. In the former instance it may be necessary to use the modified Tamura-Coker transition amplitude in order to recover useful spectroscopic information. In the latter analysis a
simple refit to the data using the corrected (Satchler) proton parameter set should suffice. The re-analysis of earlier $^{208}\text{Pb}(d,p)^{209}\text{Pb}$ single particle angular distributions will now be presented in section E.
E. THE RE-ANALYSIS OF EARLIER $^{208}\text{Pb}(d,p)^{209}\text{Pb}$ DATA

Concerned with the reliability of the reaction calculation, the previous three sections of this chapter have critically examined the DW method. The focus has been on the foundations of the method, the substitution of the product $\left(\begin{array}{cc} a_{\ell j} & b_{\ell j} \\ c_{\ell j} & d_{\ell j} \end{array}\right) R_{\ell j}(r)$ for the form factor, and the validity of the relative motion functions $x_d$ and $x_p$. It is believed here that the fluctuation in the single-particle spectroscopic factors depicted in Figure III-1 can be traced to the use of incorrect relative motion functions and not to an overall inaccuracy of the direct-reaction calculation. Specifically, the errors involved are: a) ignoring the possibility of a resonance reaction should the outgoing proton correspond to an isobaric analog resonance in the elastic channel; b) using the Perey 17.0 MeV proton parameter set when that energy corresponds to a (non-optical) resonance energy; c) not being cognizant of the deuteron break-up model and the induced changes in $x_d$. It is clear from sections C and D that these three points do not apply at incident energies below the Coulomb barrier and hence the convergence to unity in the spectroscopic factors at these incident energies.

We present in this section a re-analysis of earlier $^{208}\text{Pb}(d,p)^{209}\text{Pb}$ data taken at incident energies of 17.0, 18.7, 20.0, and 24.8 MeV. The 17.0 MeV data actually comes from the $^{206}\text{Pb}(d,p)$ study of Moyer et. al. (MC 70) in which there was a 2% $^{208}\text{Pb}$ admixture in the $^{206}\text{Pb}$ target. These authors did not report angular distributions for the $^{209}\text{Pb}$ single-particle states because the statistical errors on the angular distributions were too great. Spectroscopic factors were however quoted. Experimental shapes were given for the single-particle fragments of $^{207}\text{Pb}$ and these were fit by Moyer using the conventional deuteron parameter set of Table III-1 and
the Perey 17.0 MeV proton parameter set. The 18.7 MeV study was made by Jeans et al. (JD 69), who derived their own deuteron optical-parameter set for this incident energy. When used in conjunction with a global parameter set derived by Perey (PE 63) very poor fits to the data resulted, as well as unacceptably low (30%) spectroscopic factors. This group then did a strange thing. Not believing their own deuteron optical parameter set they arbitrarily deepened its imaginary absorption term to a highly unusual value of 40.0 MeV, about twice what is conventionally used. The resulting shapes and spectroscopic factors were then much more reasonable and it is these particular spectroscopic factors which Macfarlane used for his graph, Figure III-1. While Jeans et al. offer the success of their unorthodox procedure as evidence of its credibility, it is believed here that this tampering with parameters is not in the spirit of the optical model. In fact the difficulty with their original analysis lay not so much with the deuteron parameters (although the deuteron break-up model had not been used) but rather the main problem lay with the proton parameter set. This set had an imaginary absorption term of 17.5 MeV which was not obtained from elastic proton scattering on lead but had been taken from a global formula:

\[ W_d = 3A^{1/3} \quad (3.26) \]

This global formula in turn was the result of a series of elastic proton scattering measurements at \( E_p = 17.0 \) MeV on nuclei with masses 48 to 197. In his next work, however, Perey (PE 64) specifically points out that the
global set is not effective for $^{208}\text{Pb}$ at 17.0 MeV and this second paper is the origin of the Perey 17.0 MeV proton parameter set for the lead region. Thus it was inconsistent of Jeans et. al. to have chosen the global set when it had been shown that this set did not work in the lead region. Such an inconsistency severely undermines their justification for tampering with the deuteron parameter set which had been shown by them to give a good fit to the deuteron elastic angular distribution.

The 20.0 MeV data were taken by Kovar as part of a previous dissertation(KO 71) and were originally fit with the conventional deuteron parameter set of Table III-1 and the Perey 17.0 MeV proton parameter set. The data have recently been re-analyzed using the Satchler proton parameter set. Finally the 24.8 MeV data were taken by Muehllehner et al. (MP 69) with the conventional deuteron parameter set and along with a proton parameter set which actually gives about the same results of as the Perey 17.0 MeV potential. In contrast to the other studies, these authors used a neutron radius of 1.20 fm and a finite-range factor (PS 64) of 1.25, about twice what is normally used. The combination of these two changes increases the extracted spectroscopic factors by about 20% over what they would be if zero-range and $r_n=1.25 fm$ had been used.

In Figures III-7, 8, 9, and 10 we present the original and the re-analyzed fits to the 17.0, 18.7, 20.0, and the 24.8 MeV (d,p) angular distributions. The deuteron break-up potential and the corrected proton potential of Satchler were used throughout this re-analysis. The original and the re-calculated spectroscopic factors are listed in Table III-2. For further comparison we also show in Figure III-11 a reproduction of the original Jeans figure. There the light curves correspond to those predicted by the physical meaningful deuteron potential, the bold lines to
The main fragments in the $^{206}\text{Pb}(d,p)^{207}\text{Pb}$ reaction with their angular distributions as originally fitted and as now re-analyzed. At 17.0 MeV incident energy, only the $2g_{9/2}$ and $1i_{11/2}$ groups are outside the resonance region.
$^{206}\text{Pb}(d,p)^{207}\text{Pb}$

$E_d = 17.0$ MeV

Re-analyzed DW fit

Original DW fit

$E_x = 4.627$

$l_n = 0$

$4s_{1/2}$

$E_x = 4.389$

$l_n = 2$

$3d_{5/2}$

$E_x = 5.219$

$l_n = 2$

$3d_{3/2}$

$E_x = 2.728$

$l_n = 4$

$2g_{9/2}$

$E_x = 5.130$

$l_n = 4$

$2g_{7/2}$

$E_x = 3.510$

$l_n = 6$

$1d_{3/2}$

$E_x = 4.115$

$l_n = 7$

$1f_{5/2}$

$d\sigma/d\Omega$ (Arbitrary Units)

$\theta_{c.m.}$

0°  50°  100°  150°
The single-particle angular distributions of the $^{208}_{\text{Pb}}(d,p)$ $^{209}_{\text{Pb}}$ reaction at 18.7 MeV incident energy.
$^{208}\text{Pb} (d, p) ^{209}\text{Pb}$

$E_d = 18.7$ MeV

--- Re-analyzed DW fit

--- Original DW fit

\begin{align*}
E_x &= 2.033, \quad l_n = 0, \quad 4s_{1/2} \\
E_x &= 1.565, \quad l_n = 2, \quad 3d_{5/2} \\
E_x &= 2.492, \quad l_n = 4, \quad 2g_{7/2} \\
E_x &= 0.799, \quad l_n = 6, \quad 1i_{11/2} \\
E_x &= 1.424, \quad l_n = 7, \quad 1f_{15/2}
\end{align*}
III-9  The single-particle angular distributions of the $^{208}\text{Pb}(d,p)$ $^{209}\text{Pb}$ reaction at 20.0 MeV incident energy.
$^{208}\text{Pb}(d, p)^{209}\text{Pb}$

$E_d = 20.0 \text{ MeV}$

- Re-analyzed DW fit
- Original DW fit

$E_x = 2.033$
$l_n = 0$
$4s_{1/2}$

$E_x = 1.565$
$l_n = 2$
$3d_{5/2}$

$E_x = 2.492$
$l_n = 4$
$2g_{7/2}$

$E_x = 0.799$
$l_n = 6$
$i_{11/2}$

$E_x = 1.424$
$l_n = 7$
$i_{15/2}$

$d\sigma/d\Omega \ (\text{mb}/\text{sr})$

$\theta_{\text{c.m.}}$
The single-particle angular distributions of the $^{208}\text{Pb}(d,p)^{209}\text{Pb}$ reaction at 24.8 MeV incident energy.
$^{208}\text{Pb}(d,p)^{209}\text{Pb}$

$E_d = 24.8 \text{ MeV}$

- Re-analyzed DW fit
- Original DW fit

$E_x = 2.033, \ell_n = 0, 4s_{1/2}$

$E_x = 1.565, \ell_n = 2, 3d_{5/2}$

$E_x = 2.537, \ell_n = 2, 3d_{3/2}$

$E_x = 0.0, \ell_n = 4, 2g_{9/2}$

$E_x = 2.492, \ell_n = 4, 2g_{7/2}$

$E_x = 0.799, \ell_n = 6, 1d_{5/2}$

$E_x = 1.424, \ell_n = 7, 1f_{15/2}$

$d\sigma/d\Omega \ (\text{mb/sr})$

$\theta_{\text{c.m.}}$
Table III-2. Spectroscopic Factors from Earlier $^{208}\text{Pb}(d,p)^{209}$ Experiments

<table>
<thead>
<tr>
<th>n(\ell j)</th>
<th>17.0 MeV(^{a)})</th>
<th>18.7 MeV(^{b)})</th>
<th>20.0 MeV(^{c)})</th>
<th>24.8 MeV(^{d)})</th>
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<td>Orig</td>
<td>Refit</td>
<td>Orig</td>
<td>Refit</td>
</tr>
<tr>
<td>$2g_{9/2}$</td>
<td>.68</td>
<td>.92</td>
<td>.66</td>
<td>.92</td>
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</tr>
<tr>
<td>$1i_{11/2}$</td>
<td>.81</td>
<td>1.10</td>
<td>.75</td>
<td>1.10</td>
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<tr>
<td>$1j_{15/2}$</td>
<td>.39(^f))</td>
<td>.57(^f))</td>
<td>.71</td>
<td>.78</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>$3d_{5/2}$</td>
<td>.65(^f))</td>
<td>.75(^f))</td>
<td>.62</td>
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</tr>
<tr>
<td>$4s_{1/2}$</td>
<td>.52(^f))</td>
<td>.56(^f))</td>
<td>.70</td>
<td>.93</td>
</tr>
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</tr>
<tr>
<td>$2g_{7/2}$</td>
<td>.93(^f))</td>
<td>1.05(^f))</td>
<td>.81</td>
<td>.92</td>
</tr>
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</tr>
<tr>
<td>$3d_{3/2}$</td>
<td>1.13(^f))</td>
<td>1.23(^f))</td>
<td>.88</td>
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</table>

\(^{a)}\) Ref. MC70.
\(^{b)}\) Ref. JD69.
\(^{c)}\) Ref. KO71. The numbers in parentheses refer to the fits obtained using conventional deuteron parameters and the Perey 17.0 MeV proton optical parameter set.
\(^{d)}\) Ref. MP69.
\(^{e)}\) Unacceptable fit to the experimental shape makes this value questionable.
\(^{f)}\) The outgoing protons from these residual states are within the elastic proton resonance region \(\sim 15.0 \rightarrow 17.5\) MeV.
The original Jeans et al. data presentation for their 18.7 MeV $^{208}\text{Pb}(d,p)$ reaction. The bold lines are those generated by the artificial deuteron potential, the light continuous curves are from the physically meaningful deuteron potential, and the broken lines are from a second artificial potential.
the artificial potential, and the broken lines to a second artificial deuteron potential which was not as successful as the one already mentioned. (It should be noted that there is no criterion for choosing an artificial optical potential as opposed to the deuteron break-up potential which has a physical basis).

Ignoring for a moment the $1j_{15/2}$ transfer shapes, it can be seen that the re-calculated fits are at least as good as and generally better than those obtained in the original analyses. In particular the 20.0 MeV data are fit extremely well. Almost the same degree of improvement is also to be had for the 18.7 and the 24.8 MeV angular distributions. Additionally the artificial Jeans deuteron optical parameter set was used to generate those shapes given in Figure III-11 and a close comparison revealed these artificial shapes to be no better than those predicted by this re-analysis. It can also be seen that in all cases there is a marked damping of the original oscillations originally predicted for the $4s_{1/2}$ transfer shapes.

By running all possible combinations of the optical potentials, it can be determined that this improvement derives almost entirely from the use of the new proton parameter set. Similarly it can be found that the higher the $l_n$ transfer, the more sensitive is the predicted shape to the choice of the deuteron potential.

Not only are the fits to the angular distributions improved, but the resulting spectroscopic factors of the re-analysis show a remarkable consistency, the main exception being the 17.0 MeV spectroscopic factors for the $1j_{15/2}$, $3d_{5/2}$, and the $4s_{1/2}$ single particle states. For the same in-

* Muehllehner et al. in their paper reported data for the $^{208}$Pb(d,p) reaction at 20.1MeV incident energy. Kovar has found that while his experimental shapes at 20.0MeV agree with the 20.1MeV data, there is a 20-30% discrepancy in the absolute magnitudes for the $2g_{9/2}$ and the $1j_{11/2}$ transfer cross sections. If this error persisted in the 24.8MeV data, it would act to increase the spectroscopic factors by that amount. We thus do not regard the poor $2g_{9/2}$ spectroscopic factor at 24.8MeV as being significant.
incident energy, the $2g_{7/2}$ angular distribution in $^{207}$Pb is very poorly fit in the forward angular range. What these particular transfers have in common is that the outgoing protons fall within the analog resonance region of the elastic channel, 15.0 to 17.5 MeV. In this regard the discussion in section D of this chapter has mentioned the modified transition amplitude of Tamura and Coker, Equation (3.24) as being the proper means of extracting spectroscopic factors. A priori, there seems to be no way of knowing how the modified transition amplitude will affect either the magnitude or the shape of the angular distribution, since the final result comes about by an interference term. That is, the modified angular distribution may or may not resemble the ordinary prediction. This could explain why only the $2g_{7/2}$ shape seems particularly distorted while the other transfer shapes in the resonance regions are not appreciably disturbed. In contrast, the $2g_{9/2}$ and the $1i_{11/2}$ transfers at this energy have outgoing protons with energies exceeding 17.8 MeV. The fit and the spectroscopic factor of the $2g_{9/2}$ transfer are notably improved and the same might have been said for the $1i_{11/2}$ angular distribution, were it not for the data point at 60°. Neglecting then the five highest lying states at 17.0 MeV incident energy, the re-analysis has provided very good fits to the angular distributions with extracted spectroscopic factors consistently near the expected value of unity. When these results are placed along side the equally consistent results of the sub-Coulomb measurements it becomes clear that new limits can be set on the reliability of the DW method, at least for this mass region and this reaction. Specifically, the extracted spectroscopic factors should be accurate to about 15% and the quality of the angular distribution should approach the degree evident here for the $^{209}$Pb single particle states. There are two qualifications to this conclusion which, however, are not terribly limiting.
The first qualification has to do with the form factor. As has been discussed in section B, the shell-model wave function ceases to be a good approximation for orbital fragments displaced far from their unperturbed position in a shell-model potential. The second qualification has to do with the general problem of momentum mis-matching in the transfer reaction (SB 67). Briefly stated, the elastic scattering is sensitive to only a limited number of partial waves in the partial wave decomposition of the distorted waves $\chi_d$ and $\chi_p$. In conjunction with this fact, a transfer reaction at a given incident energy and for a particular $Q$-value has a classically most favored ("matched") momentum transfer depending on the particular masses involved in the reaction. At the above incident energies, the $^{208}\text{Pb}(d,p)$ matching value of the momentum transfer is about 3. This means that the further away a given momentum transfer is from the value of 3, then the more mis-matched the transfer becomes. In that event, the less-well-determined partial waves become more important in the reaction calculation. Both of these comments apply to the $l_n = 7$, $1j_{15/2}$ transfer. Its energy in $^{209}\text{Pb}$ is displaced far from its expected shell-model position. It is also the most mis-matched of all the single-particle transitions considered. For these reasons it is not unexpected that the $1j_{15/2}$ angular distributions fitted here are the least well reproduced.†

The above qualifications aside, the results of the re-analysis satisfy the concern expressed initially in this chapter over the ability of

† It might also be noted that Moyer et. al. find the $1j_{15/2}$ strength in $^{207}\text{Pb}$ to be 40% greater than it is in $^{209}\text{Pb}$, which seems unlikely. It will be seen that the $1j_{15/2}$ strength in $^{205}\text{Pb}$ is only 50% of what it is in $^{209}\text{Pb}$. It is very possible then that the resonance reaction may be affecting this transfer or else there may be some lower $l_n$ doublet admixture in this angular distribution.
the direct-reaction calculation to predict Q-value corrections for the more bound single-particle states in $^{205}$Pb. Such a capability could not be assumed in view of the fluctuations in the $^{209}$Pb spectroscopic factors as obtained in a series of earlier experiments. Upon a reconsideration of the treatment of both the deuteron and the proton channels it has been seen that these fluctuations are largely spurious and are not a true guide to the reliability of the distorted wave approach to the reaction mechanism.
IV THE DATA ACQUISITION AND REDUCTION

A. THE EXPERIMENTAL APPARATUS

The spectrum of $^{205}\text{Pb}$ from 0.0 to 6.7 MeV of excitation was observed by means of the $^{204}\text{pb}(d,p)$ transfer reaction. Energies of 13.0 and 20.0 MeV were imparted to the incident deuterons by the Yale MP-1 Tandem Van de Graaff accelerator, during a series of experiments. Over the angular range 5.0° to 167.5° the Yale Multigap Spectrograph momentum analyzed and detected with nuclear emulsion photographic plates those proton groups emanating from the reaction. As illustrated in Figure IV-1, the multigap consists of toroid with its center at the target position and in which 5/8" gaps appear at equal (7.5°) angular intervals, twelve gaps in one quadrant of the scattering plane and eleven in a diagonally opposite quadrant. The toroid can be rotated such that the initial gap in the first quadrant is at $0°$, $5.0°$, or $8.75°$ with respect to the incoming beam. These toroidal positions and their associated gap angles are termed carousel positions, numbers 1, 2, and 3 respectively. With the latter two carousel positions the incident beam can be dumped externally for charge collection. Such a capability is especially crucial for a deuteron experiment since it has been found that an unacceptable gamma ray and neutron produced background develop on the photographic plates whenever the deuterons are stopped internally. The WNSL Internal Report #49 (K0 70) and previous dissertations (K0 71, ME 72) contain a summary of the technical details common to all multigap experiments. Such details will for the most part be omitted here.

The various exposures analyzed in this study are listed in Table IV-1. The plate log run number refers to the order of each exposure within the entire gamut of multigap experiments as contained in the WNSL log book #106.
Different views of the Yale Multigap Spectrograph. The top view is looking down upon the toroid for which the gaps are in the carousel #1 position. Outgoing reaction products are momentum analyzed simultaneously in the 23 vertical gaps and recorded on photographic plates as is the middle side view. A factor of 2.5 in energy can be accommodated along the entire plate at one magnetic field setting.
YALE MULTIGAP SPECTROGRAPH
<table>
<thead>
<tr>
<th>MG run #</th>
<th>Ed (MeV)</th>
<th>Length of Exposure</th>
<th>Proton Groups</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>13</td>
<td>20.0</td>
<td>4,000 C</td>
<td>Ex ≥ 0.0</td>
<td>Test run; only $\theta_{\text{lab}} = 20^\circ, 50^\circ, 72.5^\circ$ and $115^\circ$ exposed</td>
</tr>
<tr>
<td>16</td>
<td>20.0</td>
<td>735 C</td>
<td>Ex ≥ 2.50</td>
<td>About half the gaps unusable due to poor resolution</td>
</tr>
<tr>
<td>21</td>
<td>20.0</td>
<td>4,000 C</td>
<td>Ex ≥ 0.0</td>
<td>Poor resolution, only ground state and first excited state yields useful.</td>
</tr>
<tr>
<td>25</td>
<td>20.0</td>
<td>10,200 C</td>
<td>Ex ≥ 0.0</td>
<td>Resolution somewhat better, particle states generally over-exposed, hole-state exposure good.</td>
</tr>
<tr>
<td>28</td>
<td>13.0</td>
<td>2,000 C</td>
<td>Ex ≥ 0.0</td>
<td>Only first quadrant exposed ($5^\circ$ to $87.5^\circ$), only about half the hole states have sufficient statistics Resolution 10 keV</td>
</tr>
<tr>
<td>39</td>
<td>20.0</td>
<td>735 C</td>
<td>Ex ≥ 2.50</td>
<td>Best 20.0 MeV data but only $\theta_{\text{lab}} = 12.5^\circ$ to $57.5^\circ$ exposed</td>
</tr>
<tr>
<td>101</td>
<td>13.0</td>
<td>4,500 C</td>
<td>Ex ≥ 2.50</td>
<td>Only intermediate angle (carousel pos. #3) exposure ever made This and the following runs have about 7 keV resolution.</td>
</tr>
<tr>
<td>103a</td>
<td>13.0</td>
<td>600 C</td>
<td>Ex ≥ 0.0</td>
<td>Carousel position #3 run with $^{208}\text{pb}$ target (&quot;template&quot;).</td>
</tr>
<tr>
<td>102</td>
<td>13.0</td>
<td>2,600 C</td>
<td>Ex ≥ 2.50</td>
<td>Carousel position #2 run used to normalize this series with run 28.</td>
</tr>
<tr>
<td>103b</td>
<td>13.0</td>
<td>600 C</td>
<td>Ex ≥ 0.0</td>
<td>Carousel position #2 run used to complete template exposure.</td>
</tr>
</tbody>
</table>

* The field of the multigap was set such that only those proton groups coming from residual levels higher than the listed excitation energy would strike the photographic plate. The excitation energy 2.57 MeV corresponds to the onset of the particle states.
In order that the data acquisition procedure not appear overly haphazard, it should be pointed out that this study constituted the maiden experiment for the Yale multigap group. A few of the forward angle gaps were not yet in the final focus position (KO 70) at the start of these exposures, thus contributing to some loss of resolution. More significantly, in the initial stages of experimentation it was uncertain as to how sensitive the photographic plates (Ilford K(5)) would be to 20 MeV protons. Aluminum absorbers about 0.6 mm were placed immediately in front of the plates, mainly for the purpose of shielding the plates from the elastic and inelastic deuterons, but thick enough such that the protons would be slowed down enough to deposit a greater amount of energy in the emulsion. This would result in brighter tracks. It is known now that these absorbers were thicker than necessary; further they induced multiple scattering in the transient protons, resulting in a deformed group shape on the plate. As the comments in Table IV-1 indicate, the first four exposures were hindered to a greater or lesser degree by these inevitable start-up problems. In fact run #39 was primarily viewed as a test exposure for the forward angle gaps and its extremely fine resolution ($E/\Delta E \approx 1700$) made regrettable the fact that more gaps were not simultaneously exposed. Finally it is pointed out that the present author is not the original parent of this study although he has carried out all the data analysis of the different runs.

The photographic plates were scanned manually in histograms of 0.5 mm. distance per channel. The data sheets were transcribed onto standard IBM cards appropriately serialized. As such the raw data was now in a format compatible with a series of computer programs designed especially for multigap data reduction. This process will now be described in section B.
B. THE DATA REDUCTION PROCEDURE

In the raw spectrum it is first necessary to separate those proton groups corresponding to excited states in $^{205}$Pb from those groups arising from contaminant elements introduced during target fabrication. As described elsewhere, the targets were made from 99.7% enriched $^{204}$Pb and evaporated onto 10 $\mu$g/cm$^2$ Carbon foils (KO 71). The contaminant separation was accomplished most usefully by the program ALLPLOT of whose output Figure I-4 is a sample. The program essentially compensates for the residual nucleus' recoil energy which, of course, increases as the light outgoing particle is emitted at a larger angle. By adding the recoil energy to the energy of the light particle, the Q-value or excitation energy for the given group can be derived. The recoil energy is also a function of the constituent masses of the reaction, in addition to depending on the angle. It is these dependencies which enable the contaminant lines to be identified. To do this the program receives as input the masses of the particles participating in the important reaction ($^{204}$Pb($c,p$)$^{205}$Pb) as well as that reaction's ground state Q-value. It is according to these parameters and along with the detection angle, incident deuteron energy and the magnetic field that the program calculates excitation energies and plots the spectrum. Plots such as Figure I-4 are then compared for the spectrum as observed at increasing proton angle. In these plots, the groups corresponding to the $^{204}$Pb($d,p$)$^{205}$Pb reaction will line up (by construction) at the same excitation energies for all angles Groups from elements lighter than $^{204}$Pb will shift toward higher excitation energies for increasing angle since their actual recoil energies are larger than those which were calculated using $^{204}$Pb as the initial mass. Similarly groups from heavier elements would shift to lower energies.
The raw spectrum data of every angle at both energies were first analyzed in the above manner. Not unexpectedly, $^{12}$C, $^{13}$C, and $^{16}$O (from oxidation) gave the strongest contaminant lines. Also seen were peaks from $^{14}$N, $^{28}$Si, $^{32}$S, $^{35}$Cl, $^{37}$Cl, $^{40}$Ca, and $^{56}$Fe contaminants. These ten isotopes accounted for all identified contaminant peaks. Other elements including $^{23}$Na, $^{27}$Al, and $^{24}$Mg were searched for but no evidence for their presence was seen. It will be recalled from Table IV-1 that in order to acquire sufficient statistics for the hole-state population at 20 MeV incidental energy, 10,200 μC of charge was collected in one exposure. It was only at this level of accumulated charge that the last five contaminants listed gave peaks of size comparable to those from $^{205}$Pb. The lighter impurities, however, had many levels which completely obscured several groups from $^{205}$Pb as the contaminant line moved down the spectrum. This problem can be alleviated by taking exposures at two carousel positions. The light contaminant groups move sufficiently fast that instead of there being a 15° gap in a group's angular distribution from an obscured angle, there would only be 7.5° gap.

Germane to the above discussion is the method by which the detected particle's energy is calculated. The basic equation for a particle of energy $E$, mass $m$ and charge $q$ travelling in a uniform transverse magnetic field is

$$E = mc^2 \left(1 + \frac{qB \rho}{mc^2} \right)^{\frac{1}{2}} - 1$$

(4.1)

where $\rho$ is the radius of curvature of the resulting circular trajectory.
To each $p$ there corresponds a unique linear distance along the focal surface at which a particle is incident. The relationship has been established for each gap in the form of a polynomial expansion with characteristic coefficients (KO 70). The "distance", $D$, of an isolated peak is somewhat arbitrary even apart from an absolute scale, but it has been determined (ZI 55) that the least variable position of the peak shape is the third height, that is, the distance on the from edge of one-third the height of the best triangular fit to the group shape. Since these data were counted in 0.5 mm channels, about 0.2 mm uncertainty is assigned to the energy levels on this basis corresponding to $\pm 2$ or $\pm 3$ kev for the 13 and 20 MeV experiments respectively.

Of more concern to the obtaining of accurate energies is the reliability of the calibration curves ($p$ vs. $D$) for each focal surface and also a knowledge of the correct value for the magnetic field. It is known that the strength of the magnetic field varies from gap to gap in the toroid and even throughout one gap, which is a consequence of the finite size of the permeability of iron. Hence, even in one gap, protons with different energies will experience stronger or weaker fields depending on their trajectories. This means that in Equation (4.1) there is an uncertainty in $E$ arising from a dual uncertainty in the product $Bp$. The usual approach to this difficulty has been the use of the "effective field" procedures which has been described at length in an earlier dissertation (CO 68) Essentially the dual error, in $B$ and $p$ for each gap, is lumped entirely into the parameter $B$. Proton groups coming from known energy levels in the residual nucleus have outgoing energies known exactly from simple kinematics. Their radii of curvature in the gap are computed assuming the calibration to be perfect. The slightly differing fields $B$
which are then calculated from Equation (4.1) reflect the discrepancies in both the magnetic field and the calibration curve. The average of these fields is then the best parametrization of these discrepancies and this average is the "effective field" for the gap.

Unfortunately there are no previously known levels in $^{205}$Pb above 2.607 MeV of excitation. Use was made therefore of groups emanating from impurities in the target. In general, this is not a desirable approach since the distributions of these contaminants in the target are unknown. For example there is a visible build-up of carbon deposited on the front face of the target. Since there is already carbon backing the differential energy loss effect will give rise to broadened peak shapes for the outgoing proton groups. A chronological ordering of the data also shows significant increase in the amount of $^{14}$N, $^{16}$O, and $^{32}$S, accumulating presumably on the surface of the target. Such groups also show wider peaks than those from $^{205}$Pb, thus introducing an error in their third height energy. Another difficulty is that in the contaminant spectra observed, energy levels are known only to about ±7 keV, limiting the effective field to this error. The combination of these uncertainties leads to the assignment of an average, a ±10 keV error in the determination of the absolute energy levels in any one gap.

It should be remembered that the above comments only apply to absolute energy determinations. Far less subject to uncertainty are the relative energy differentials, that is the difference in energy values computed from Equation (4.1) for groups of given separations in $\rho$. On the basis of numerical calculations it is found that only ±3 keV uncertainty is expected for the relative energy between the first and last of the particle states observed, given an assumed error of 5 parts in 10,000.
for the effective field.* This systematic error in the relative energy will decrease for two groups which are closer together in distance. The procedure adopted then was to choose an isolated group near the center of the spectrum (actually $E_x = 4.590$ MeV) and compute the energies of all other groups relative to this level. From this discussion the variation in relative energy spacings for two groups as computed in all gaps should be less than $\pm 1$ kev, for most cases, insofar as uncertainties in the effective field are concerned. The comparison and identification of corresponding groups in the spectrum of different gaps is made on the basis of these relative energy differentials and is now expected to be constant within the error of third-height determinations. For isolated peaks the relative energy spacings are found to be constant within $\pm 2$ kev for the $13$ MeV data and $\pm 3$ kev for the $20$ MeV data, just the error expected. For peaks which are overlapping, the best third height determination is not always easily accomplished manually. Use was made therefore, of the spectrum decomposition program AUTOFIT which will now be described. (SE 65).

Central to the success of the program AUTOFIT is the fact that to first order the group shapes obtained along the focal surface of a given gap are invariant when expressed in histograms of counts vs. energy interval. Aberration and magnification effects introduce corrections to this invariance, but over a large interval (4 MeV of excitation) the shape invariance has been found to be quite good in the present study. Thus this program accepts as an input a discrete reference peak generally based on the shape of an isolated group in the spectrum. All other groups, in-

* These calculations simply used Equation (4.1). The maximum and minimum values of $\rho$ encountered were used to calculate a relative energy. The systematic energy error is then determined according to the uncertainty in $B$. 
cluding those partially overlapping, are then analyzed according to this basic shape. As many as twenty peaks can be varied, in position and magnitude, in order to fit best a selected region of the raw spectrum. All the spectrum data were analyzed with this program and Figure IV-2 illustrates one particular fit near 4.5 MeV of excitation. The overlapping lines show one decomposition while elsewhere the data points are connected as a visual guide. The program takes into account the variation in solid angle along the plate (MC 73) and transforms the data into the center of mass frame. As such, the predicted peak sums constitute a relative cross section. Errors are also computed for these sums, and for the predicted excitation energies, reflecting the goodness of the calculated fit. These errors are generated internally by the program and are useful as guides to the relative quality of one decompositions vs. another. They are not the assigned experimental errors.

Both the hole and the particle states' spectra were reduced in this manner. The major difference in the two regions, of course, is that for the particle region there is no prior check on the number of states and their positions in contrast to those of the hole states. This was the major obstacle to be overcome in the data analysis: to be able to decompose consistently and satisfactorily and without previous knowledge of the excited levels of $^{205}$Pb those regions of the particle spectrum in which it is obvious that several overlapping groups are contributing.

To attain this objective the spectrum data of each observation angle at both energies were analyzed independently. Extensive modifications were also made in the method of data input to the program AUTOFIT so as to take advantage of the light pen-CRT display and keyboard function interactive package available on this laboratory's IBM 360/44 computer facility. (MA73)
A sample of the decomposition accomplished by the peak analysis program AUTOFIT for one of the 20.0 MeV spectra. The overlapping lines around 4.5 MeV of excitation are those produced by the program, while elsewhere the data points are connected to guide the eye. For comparison, see Figures V-5 and V-6.
$^{204}$Pb(d,p)$^{205}$Pb

$E_D = 20.0$ MeV

$\theta_{\text{lab}} = 50^\circ$
The reference peak, background levels and the initial guesses of the number and positions of the peaks can now be obtained and changed via this package. For purposes of deriving the best reference peak, the rather dense region near 4.5 MeV of excitation (see Figure I-4 and IV-2) was first considered for each gap. The spectrum data of each angle were found to require their own individual reference peaks, most likely because the second-order focusing fringe fields vary from gap to gap. A subjective appraisal of the quality of the fits, in terms of the deviation of the summed resolved spectrum from the actual raw-data spectrum, closely correlated with the size of the computed errors mentioned above. The final fit for a given region was found to be quite independent of the initial guess of a group's position, but very dependent on the number of peaks chosen. Too few peaks caused structure in the raw spectrum to be averaged out; too many peaks guessed at and the program would predict a degenerate pair, one of positive sum and the other negative, adding up to the raw-data values. Overall it has been found that in more than ninety per cent of the data channels, the predicted spectrum formed from only one reference shape was within statistical error of the raw spectrum data. More importantly, the relative energy differentials of those groups representing ninety per cent of the observed cross section were constant within the limits previously mentioned, for all observation angles at both energies. That is, consistent excitation energies were obtained throughout the data reduction for the same number of levels. The constancy of this level number is particularly gratifying when it is realized that since the spectrograph's resolution is relative, the energy resolution of the 13.0 MeV spectrum is about fifty per cent greater than that of the 20 MeV spectrum i.e., 7 kev as opposed to 12 kev. The 20 MeV spectrum data were analyzed first; in
no case was it found necessary to introduce additional levels in order to fit the 13 MeV data. It is only once this internal consistency has been ascertained that confidence can be had in the extracted angular distributions. These are considered in the next chapter. What is to be emphasized here is that although the experimental spectrum of $^{205}$Pb is at first sight complicated, it is not so dense that it cannot be successfully decomposed. It is only after much more of the output represented by Figure IV-2 has been examined that this fact can be really appreciated. Each of the states to be reported in the next chapter has had its excitation energy measured in over fifty observations of the spectra and all these determinations agree within the total expected error of the third-heights of the corresponding proton groups.
C. THE CROSS SECTION NORMALIZATION

Since absolute spectroscopic information is desired, it is necessary to convert the relative cross sections of the extracted angular distributions into absolute strengths. A direct measurement of a cross section assumes an accurate assessment of the target's thickness and a precise calibration of the charge collection system. To circumvent these requirements an indirect approach is often used wherein the yields of two successive exposures are compared, one with a theoretically known (Rutherford) cross section and the other at the experiment's incident energy. In this fashion the unknown target thickness and the unknown charge collection efficiency divide out in the ratio to Rutherford value. An implicit postulate of this method is that the target be sufficiently uniform so that the necessary refocusing of the incident beam between the two energies does not result in the beam striking different effective target thickness. Because such a target uniformity cannot be guaranteed, this indirect approach has been avoided.

Another way has been devised to obtain simultaneously properly normalized cross sections as well as template for the angular distribution at different momentum transfers. This was accomplished in the last 13MeV of runs ('s 101 - 103) listed in Table IV-1. As with all runs, a solid state monitor was placed in the scattering plane at an observation angle of 75° to the incident beam direction. Successive exposures were then made with the $^{204}$Pb and the $^{208}$Pb targets in the order listed. Using the results of recent (UD 72) absolute cross section measurements in the lead region, the elastic deuteron yields detected in the monitor were compared for these exposures. The ratio of these numbers was then the normalization factor relating the strengths of the proton groups which had been
recorded on the photographic plates. That is, with a method independent of target thickness or charge; collection irregularities, the angular distributions of the proton groups from Pb\(^{205}\) were now completely normalized to the strengths of the corresponding groups from Pb\(^{209}\). Denoting by \(\sigma_{do}(204)\) the elastic differential cross section for deuterons on a \(^{204}\)pb target of thickness \(t_p(204)\), one has for the scattering yield \(N_{do}(204)\)

\[ N_{do}(204) = \sigma_{do}(204) \times t_p(204) \times C(204) \, d\Omega_{mon}. \]  

(4.1a)

and

\[ N_{do}(208) = \sigma_{do}(208) \times t_p(208) \times C(208) \, d\Omega_{mon}. \]  

(4.1b)

where

\[ d\Omega_{mon} = \text{solid angle of monitor} \]

with similar motivation for the \(^{208}\)Pb exposure and where the number \(C\) is the number of deuterons incident in a given exposure. At the same time as deuterons are being collected, a yield of \(N_{lj}\) protons is being recorded on the photographic plates according to the equations:

\[ N_{lj}(205) = S_{lj}^{205} \sigma_{lj}^{DW}(205) \times t_p(204) \times C(204) \, d\Omega_{MG} \]  

(4.2a)

\[ N_{lj}(209) = S_{lj}^{209} \sigma_{lj}^{DW}(209) \times t_p(208) \times C(208) \, d\Omega_{MG} \]  

(4.2b)
In the limit that the Q values for the transfers to $^{205}\text{Pb}$ and $^{209}\text{Pb}$ residual levels are equal, the direct reaction cross sections $\sigma_{\lambda_j}^{\text{DW}}(205)$ and $\sigma_{\lambda_j}^{\text{DW}}(209)$ will be equal. The corrections to this limit have been discussed in the previous chapter and will be omitted here. In that case, the ratio of the proton yields is

$$\frac{N_{\lambda j}(205)}{N_{\lambda j}(209)} = \frac{N_{do}(204) \sigma_{do}(208)}{N_{do}(208) \sigma_{do}(204)} \frac{s^{205}_{\lambda j}}{s^{209}_{\lambda j}}$$

(4.3)

where Equations (4.1) have been used to substitute for the product $t_0 \times C$. The ratio of the elastic deuteron cross sections is $1.02 \pm 2\%$ according to the reference quoted. With the use of this number the Equation (4.3) gives the spectroscopic factors for the population of the particle states in $^{205}\text{Pb}$ relative to those in $^{209}\text{Pb}$, which with the exception of the $1j_{15/2}$ state are all close to unity. In this sense, absolute spectroscopic factors for the single-particle states of $^{205}\text{Pb}$ have been obtained.

The 20.0 MeV cross section normalization was obtained in a more standard manner. The target thickness was measured with an $\alpha$ source thickness gauge (BB 65) and also calculated from the (Rutherford) yield of a 4.0 MeV elastic proton scattering experiment. The two measurements agreed quite well with one another and the target is believed to be $225 \ \mu\text{g/cm}^2 \pm 7\%$. Since the target is at $45^\circ$ to the incident beam, a factor of $\sqrt{2}$ must be used in the yield calculation. The programs ALLPLOT and AUTOFIT automatically re-normalize the histogram yields to what these yields would be at a constant solid angle of $3.69 \times 10^{-4} \text{ sr.}$ for all channels.* An acci-

* This is the solid angle on the plate for $90^\circ$ deflection.
dental breakage required the use of another target for the 13.0 MeV runs. This target, whose thickness was measured only with the $\alpha$ gauge, was $180\mu g/cm^2$. It is believed that the 20.0 MeV absolute cross sections are accurate to about 15% and those at 13.0 MeV somewhat worse at 18%. This would apply only to well isolated groups. Overlapping peaks, as will be seen in the next chapter, carry an additional uncertainty.
V. THE (d,p) SPECTRUM OF $^{205}$Pb: POSITIONS, MOMENTA, AND SPECTROSCOPIC STRENGTHS

A. THE EXPERIMENTAL SPECTRUM BELOW 2.57 MeV OF EXCITATION

Heretofore, almost all the experimental and theoretical investigations of the excitation spectrum of $^{205}$Pb have been confined to these levels below 2.57 MeV of excitation, which are expected to be three neutron-hole configurations. For reference, the known spectrum of $^{205}$Pb previously listed in Table I-1 is presented again in this chapter. The theoretical papers discussed in Chapter Two have predicted three sets of (d,p) spectroscopic factors to certain of these levels. Since the present work is sensitive to even very weakly excited states (i.e., those with differential cross sections as low as $2 \mu$b/sr), this study constitutes an important test of the different approaches to the residual interaction in $^{205}$Pb.

The computation of a spectroscopic factor is preceded by the assignment of an angular-momentum-transfer value to the given transition. In this respect the analysis of the hole-state region has been greatly simplified, since for most of the states listed in Table I-1, the spins and parities are already established or at least very delimited. For a transition to one of these states only one angular momentum value will conserve spin and parity. Hence if a proton group corresponds to a known hole state level, its angular distribution is expected to be reproduced according to the uniquely-allowed transfer quantum numbers. A sample of the (d,p) spectrum at one observation angle is shown in Figure V-1. The yield represents 10,200 $\mu$C accumulated charge of 20.0 MeV incident energy deuterons. The triplet of states above 2.57 MeV of excitation is included here to emphasize the qualitatively different modes of excitation avail-
The $^{204}\text{Pb}(d,p)^{205}\text{Pb}$ spectrum of $^{205}\text{Pb}$ at 20.0 Mev incident energy. The numbered groups correspond to $^{205}\text{Pb}$ residual states. The other groups are from contaminants. For the relative yield of the strongest groups, see Figure I-4.
$^204\text{Pb}(d,p)^{205}\text{Pb}$

$E_d = 20.0 \text{ MeV}$

$\theta_{\text{lab}} = 50^\circ$

$10,200 \mu\text{C}$

EXCITATION ENERGY (MeV)

COUNTS PER $\Delta E_x$
able in this (d,p) reaction. The stripped neutron is able to populate a given subshell only to the extent that the subshell is unoccupied in the ground state wavefunction of $^{204}$Pb. It is immediately seen that apart from the first two groups shown in Figure V-1, there remains only a very diminished amplitude for the stripped neutron filling a hole in $^{204}$Pb to make a three neutron-hole configuration in $^{205}$Pb. Once the 2.57 MeV excitation energy point is reached, the neutron is able to populate the N=6 positive parity subshells thus forming a one-particle four-hole state in $^{205}$Pb. Since these subshells are expected to be nearly empty in $^{204}$Pb, the positive parity states are populated with a much greater cross section than are the low-lying negative parity states.

As it turns out none of the states represented in Figure V-1 are unknown. Thus there should be no doubt as to which angular momentum value to use in the direct reaction calculation. The hole-state experimental angular distributions obtained in this work are shown in Figures V-2 and V-3, representing the 13.0 and the 20.0 MeV data, respectively. Shown also are the predicted fits to these data as calculated by the computer program DWUCK (KU 69). Table V-1 lists the optical parameters employed in these calculations. The 13.0 MeV sets, first given by Perey (PP 63) for the deuterons and also Perey (PP 66) for the protons, has been shown by Casten et al. (CC 73) to give an acceptable fit to the differential cross sections and polarizations of the (d,p) reaction on $^{208}$Pb and $^{209}$Pb at 12.3 and 15.0 MeV incident energy. The 20.0 MeV parameter sets, of

* Only the strong triplet of states at 2.6 MeV of excitation were studied by Casten. The polarization data established all three of these states as being $2g_{9/2}$ in agreement with the $^{205}$Bi studies. There has been older assertions that one of these states might be a $2g_{7/2}$ fragment (LT 67) but the issue is no longer in doubt.
Fits to the strongest $^{204}$Pb($d,p)^{205}$Pb hole-state angular distributions at $E_d = 13.0$ Mev. Even with 2000 $\mu$C of charge, the other groups are too weakly excited to have meaningful angular distributions.
\[ ^{204}\text{Pb}(d,p)^{205}\text{Pb} \]
\[ E_d = 13.0 \text{ MeV} \]

\[ \frac{d\sigma}{d\Omega} \text{ (mb/sr)} \]

\[ \theta_{\text{c.m.}} \]

(a) \[ E_x = 0.0023 \]
\[ l_n = 1 \]
\[ 3p_{1/2} \]

(b) \[ E_x = 0.262 \]
\[ l_n = 1 \]
\[ 3p_{3/2} \]

(c) \[ E_x = 0.576 \]
\[ l_n = 1 \]
\[ 3p_{3/2} \]

(d) \[ E_x = 0.999 \]
\[ l_n = 1 \]
\[ (3p_{1/2}) \]

(e) \[ E_x = 1.014 \]
\[ l_n = 6 \]
\[ 11^{13/2} \]

(f) \[ E_x = 1.764 \]
\[ l_n = 3 \]
\[ S_{7/2} = 0.010 \]
V-3  Fits to all the observed $^{204}_{\text{Pb}}(d,p)^{205}_{\text{Pb}}$ hole-state angular distributions at $E_d = 20.0$ MeV. These data are from the 10,200 $\mu$C exposure.
$^{204}\text{Pb}(d,p)^{205}\text{Pb}$

$E_d = 20.0$ MeV
$^{204}\text{Pb}(d,p)^{205}\text{Pb}$

$E_d = 20.0$ MeV

$E_x = 1.044$

$l_n = 3$

$2f_{7/2}$

$E_x = 1.265$

$l_n = 3$

$2f_{5/2}$

$E_x = 1.374$

$l_n = 1$

$(3p_{3/2})$

$E_x = 1.614$

$l_n = 3$

$2f_{7/2}$

$E_x = 1.764$

$l_n = 3$

$S_{7/2} = 0.021$

$E_x = 1.854$

$l_n = 6$

$I_{13/2}$

$E_x = 2.252$

$l_n = 4$

$(2g_{9/2})$

$E_x = 2.354$

$l_n = 1$

$3p_{1/2}$

$\sigma/d\Omega$ (mb/sr)

$\theta_{c.m.}$
Table V-1. Optical parameters used in distorted wave analysis.

<table>
<thead>
<tr>
<th>Incident Energy (MeV)</th>
<th>Particle</th>
<th>( V_0 ) (MeV)</th>
<th>( r_0 ) (fm)</th>
<th>( a ) (fm)</th>
<th>( W' ) (MeV)</th>
<th>( r'_0 ) (fm)</th>
<th>( a' ) (fm)</th>
<th>( V_{so} ) (MeV)</th>
<th>( r_{so} ) (fm)</th>
<th>( a_{so} ) (fm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>13.0</td>
<td>d</td>
<td>106.0</td>
<td>1.096</td>
<td>.81</td>
<td>82.0</td>
<td>1.34</td>
<td>.68</td>
<td>7.0</td>
<td>1.15</td>
<td>.81</td>
</tr>
<tr>
<td></td>
<td>p</td>
<td>58.0</td>
<td>1.25</td>
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<td>54.0</td>
<td>1.25</td>
<td>.47</td>
<td>7.5</td>
<td>1.25</td>
<td>.65</td>
</tr>
<tr>
<td>20</td>
<td>d</td>
<td>112.0</td>
<td>1.25</td>
<td>.682</td>
<td>77.6</td>
<td>1.25</td>
<td>.783</td>
<td>6.0</td>
<td>1.12</td>
<td>.47</td>
</tr>
<tr>
<td></td>
<td>p</td>
<td>52.0</td>
<td>1.25</td>
<td>.65</td>
<td>40.0</td>
<td>1.25</td>
<td>.76</td>
<td>6.0</td>
<td>1.12</td>
<td>.47</td>
</tr>
<tr>
<td>13 &amp; 20.0</td>
<td>n</td>
<td></td>
<td>1.25</td>
<td>.65</td>
<td></td>
<td></td>
<td></td>
<td>((\lambda=25))</td>
<td>((\lambda=25))</td>
<td></td>
</tr>
</tbody>
</table>

a) Optical potentials are of a Woods-Saxon form with \( V_0 \) a real volume term of radius \( r_0 \) \( A^{1/3} \) and diffuseness \( a \); \( W' \) is an imaginary surface absorption where the value given here includes the factor of 4.0 required by DWUCK. \( V_{so} \) is a real surface spin-orbit potential.

b) Depth adjusted until the binding well reproduced the given level's experimental separation energy.

c) Spin-orbit term on form factor potential equal to \( \lambda \) times the Thomas term.
course, are those which have been shown in Chapter Three to give very good fits to the single particle angular distributions of $^{209}\text{Pb}$. It is seen from Figure V-3 that with one prominent exception (Figure V-3k), the experimental angular distributions obtained at 20.0 MeV are fitted reasonably well. The 13.0 MeV shapes are not as acceptably reproduced in general with the 1.764 MeV state again being badly fit. Believed by us to be a special case, further comment on this state will be momentarily deferred. For the $l_n = 1$ transfers (e.g. that to the $3/2^-$ state at 0.262 MeV of excitation), there is a pronounced dip in the experimental shape between 27.5° and 42.5° which is not reproduced. Several other parameter sets were tried in an effort to generate such a dip, but none were successful. Bjerregaard et. al. (BH 67) had a similar difficulty in fitting this same group at 13.3 MeV incident energy. They were led to use a deuteron optical parameter set with an uncharacteristically deep real well ($V_r = -133.4$ MeV) which predicted an only marginally improved fit. It is noted that the protons of this group are emerging near the $2g_{7/2} - 3d_{3/2}$ analog resonance. The effect of this energy coincidence was discussed at length in Chapter Three where it was indicated that a modified transitions amplitude, Equation (3.24) should be used to incorporate the effect of the resonance. At 20.0 MeV incident energy the outgoing protons are far above the resonance region and for this reason the analysis of the higher-energy hole-state data is thought to be more correct. Table V-2 contains the spectroscopic factors extracted from the two sets of experiments in the present work; also listed are the results of Bjerregaard and the spectroscopic factor predictions of Miranda (MI 67), Harvey and Clement (HC 71), and McGrory (MG 73). Before comparing the predictions
<table>
<thead>
<tr>
<th>#</th>
<th>Exp. Ex. ¹ ²</th>
<th>n(&gt;j²</th>
<th>Exp. Spect. Factors</th>
<th>Theoretical Values</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(MeV)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>13.3³</td>
<td>13.0²</td>
<td>20.0⁵</td>
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<tr>
<td>1a</td>
<td>0.0</td>
<td>2f⁵/₂</td>
<td>.37</td>
<td>.26</td>
<td>.27</td>
</tr>
<tr>
<td>1b</td>
<td>.0023</td>
<td>3p₁/₂</td>
<td>.70</td>
<td>.61</td>
<td>.83</td>
</tr>
<tr>
<td>2</td>
<td>.262</td>
<td>3p₃/₂</td>
<td>.15</td>
<td>.12</td>
<td>.15</td>
</tr>
<tr>
<td>3</td>
<td>.576</td>
<td>2f⁷/₂</td>
<td>.006</td>
<td>.007</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>.761</td>
<td>2f⁵/₂</td>
<td>.018</td>
<td>.014</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>.803</td>
<td>3p₃/₂</td>
<td>.003</td>
<td>.004</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>.999</td>
<td>(3p₁/₂</td>
<td>*</td>
<td>.034</td>
<td>.042</td>
</tr>
<tr>
<td>7</td>
<td>1.014</td>
<td>11₁₃/₂</td>
<td>.093</td>
<td>.045</td>
<td>.040</td>
</tr>
<tr>
<td>8</td>
<td>1.044</td>
<td>2f⁷/₂</td>
<td>.004¹⁰</td>
<td></td>
<td>1.02</td>
</tr>
<tr>
<td>9</td>
<td>1.265</td>
<td>2f⁵/₂</td>
<td>.004</td>
<td></td>
<td>1.34</td>
</tr>
<tr>
<td>10</td>
<td>1.374</td>
<td>3p₃/₂</td>
<td>.004</td>
<td></td>
<td>1.24</td>
</tr>
<tr>
<td>11</td>
<td>1.615</td>
<td>2f⁷/₂</td>
<td>.004¹⁰</td>
<td></td>
<td>1.63</td>
</tr>
<tr>
<td>12</td>
<td>1.764</td>
<td>2f⁷/₂</td>
<td>.013</td>
<td>.010</td>
<td>.021¹⁰</td>
</tr>
<tr>
<td>13</td>
<td>1.842</td>
<td>11₁₃/₂</td>
<td>.004</td>
<td></td>
<td>1.93</td>
</tr>
<tr>
<td>14</td>
<td>2.252</td>
<td>2g⁹/₂</td>
<td>.01</td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>2.354</td>
<td>(3p₁/₂</td>
<td>.006¹⁰</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* Bjerregaard et al. assigned all the strength of this doublet to the 11₁₃/₂ member.
† Harvey predicts that the total 2f⁷/₂ strength equals .029 with the calculated 1.82 MeV level taking the most strength.

1) HA72, RV71.
2) Quantum numbers for transfer used in DW calculation.
3) BH 67.
4) Present work, ±18%.
5) Present work, ±15%.
6) MI67.
7) MG73.
8) HC71.
9) Level probably absent
10) Angular distributions poorly fit.
with the experimental results, it is necessary to focus attention on certain of the fits shown in Figures V-2 and V-3.

\[ \text{Ex. (MeV)} = 0.0, \frac{5}{2}^{-}; 0.0023, \frac{1}{2}^{-} \text{ (Figures V-2a and V-3a)} \]

The energy separation of this doublet makes it impossible to resolve the members at either incident energy. The total yield will then be a sum of an \( l_n = 3 \) and an \( l_n = 1 \) angular shape with each distribution weighted by the spectroscopic factor to the given member. Unfortunately, the total yield at both incident energies, 13.0 and 20.0 MeV is dominated by the \( l_n = 1 \) transfer, making the extraction of the \( 2f_{5/2} \) spectroscopic factor more uncertain. Surprisingly it is the \( 3p_{1/2} \) spectroscopic factor which disagrees between the two energies; \( S_{1/2}(13 \text{ MeV}) = .61 \), \( S_{1/2}(20.0 \text{ MeV}) = .83 \). While the extent of the disagreement (25%) is within the absolute error limits quoted in Chapter Four for the cross sections, attempting to invoke such a large error will render the particle-state spectroscopic factors at 20.0 MeV 25% below what they are at 13.0 MeV. In section C, it will be stated now, the particle-state spectroscopic factors as derived at 13.0 and 20.0 MeV are in very good agreement implying that the normalization of the raw data has been accomplished with greater accuracy that the assigned experimental errors would indicate. While other explanations are possible for the disagreement in the \( 3p_{1/2} \) spectroscopic factor as extracted at 13.0 and 20.0 MeV, the fact that the outgoing proton from the 13.0 MeV group is at the analog resonance may have interfered with the direct transition.
Ex. = .262, 3/2- (Figures V-2b and V-3b)

The transfers to this state and to the 0.0023 MeV state are by $l_n = 1$ orbital angular momentum values. Since the respective Q-values differ by only .26 MeV the shapes of the two angular distributions should be about the same. Thus the good fit to the .262 MeV 3/2- shape at 20.0 MeV lends substance to the decomposition of the ground state doublet (Figure V-3a) at this incident energy. On the other hand the poorer fit to the 13.0 MeV .262 MeV distribution makes the 13.0 MeV decomposition of the ground state doublet less reliable (Figure V-2a).

Ex. = 0.999, (1/2-); 1.014, 13/2+ (Figures V-2d, V-2e, and V-3f)

These two states were not resolved in the Bjerregaard work, and that group mistakenly considered only the 13/2+ member to be present. According to the optical parameters which they used, an $l_n = 6$ transfer should have a flat angular distribution. The data obtained in this work clearly shows such an analysis to be erroneous. The doublet is cleanly resolved in this 13.0 MeV experiment and partially resolved in the 20.0 MeV experiment. In both angular distributions, the $l_n = 1$ member is clearly present. If the 13.0 MeV shapes are summed together the result is the flat angular distribution misinterpreted by Bjerregaard et al.

Ex. = 1.764, 7/2- (Figures V-2f and V-3k)

Both the 13.0 and the 20.0 MeV angular distributions from this state are not reproduced. In particular, the calculated shape at 20.0 MeV incident energy is simply unacceptable. The predicted forward angle pattern
is completely out of phase with the observed experimental shape. Contaminant interference and possible doublet admixture (with a different $l_n$ value) were considered as sources of error but these were ruled out. Also rejected was the possibility of a failure of the optical model with regard to the $l_n = 3$ angular distribution. It can be seen that the experimental shape of the 1.761 5/2$^-$ distribution is quite well reproduced (Figure V-3d). Rather it is thought here that the problem has to specifically do with the 2f$_{7/2}$ hole state itself. For example the fits to the other 7/2$^-$ states at 1.044 and 1.614 MeV (Figures V-3g and V-3j) are also poor although in these cases the data are not as convincing as for the 1.764 MeV angular distribution. It was mentioned in Chapter Two that Harvey and Clement have predicted that the 2f$_{7/2}$ should be more fragmented than the lower-lying hole states. The spectroscopic factors listed in Table V-2 for this orbital confirm such a prediction. Similarly in the $^{207}$Pb nucleus Hamamoto has predicted that the main 2f$_{7/2}$ strength should be split via an admixture with a particle-vibration configuration, viz. (HA 70)

$$
\left( ^{208}\text{Pb}^{3-}_{2.62} \otimes 1f^{-1}_{13/2} \right) 7/2^-$
$$

for which an analogous scheme exists for couplings based on the 3$^-$ state in $^{206}$Pb. In fact a recent $^{208}$Pb(p,$\gamma$)$^{207}$Pb experiment (SR72) has reported a higher lying 7/2$^-$ state which they believe is based on the above particle-vibration configuration. The main 2f$_{7/2}$ strength in $^{207}$Pb is located at 2.34 MeV of excitation. In the (d,p) transfer to this
state as seen by Moyer et al. (MC 70) the experimental angular distribution also could not be fit as is shown in Figure V-4. In fact, the pattern shown by the $^{207}$Pb 2f$_{7/2}$ angular distribution resembles to an extent the pattern shown by the $^{205}$Pb 2f$_{7/2}$ angular distribution. Assuming then that the problem is restricted to the 2f$_{7/2}$ hole-state fragments, two non-exclusive explanations can be advanced for the failure of the direct reaction calculation. One is that the form factor is not being well described by the shell model wavefunction substitution, Equation (3.19); the second explanation is that inelastic processes involving the excitation and deexcitation of the 3$^-$ state are interfering with the direct transition amplitude. Both of these explanations would apply especially if there were particle-vibration (3$^-$) admixtures in the main 2f$_{7/2}$ form factor.

\[
\text{Ex.} = 2.252, \ (9/2^+) \ \text{(Figure V-3m)}
\]

This state has an angular distribution which is reproduced nicely by an $l_n = 4$ shape. It has been assigned a 7/2$^+$ spin and parity by Hamilton (HA 72) in their $^{205}$Bi decay study. Although such an assignment is consistent with an $l_n = 4$ transfer, we contend here that a 9/2$^+$ spin is more likely to be correct. The assignment in the $^{205}$Bi study is based solely on one weak transition which a careful reading of the paper shows to be of somewhat dubious existence. The energy of this transition is such that it cannot be resolved from a much stronger line in the singles mode of detection. Its existence has been inferred on the basis of the data in one of the coincidence gates (the 1002.0 kev gate) but in another gate (688.5 kev) where the transition should appear with equal or greater intensity, it is not seen at all. If this transition does not exist then
The main $2f_{7/2}$ angular distribution in the $^{206}\text{Pb}(d,p)^{207}\text{Pb}$ reaction at $E_d = 17.0$ MeV.
$^{206}\text{Pb} (d, p) ^{207}\text{Pb}$

$E_d = 17.0 \text{ MeV}$

$\theta_{\text{c.m.}}$

$d\sigma/d\Omega$ (µb/sr)

$\text{Ex} = 2.339$

$2f_{7/2}$
the 2.252 MeV state could either be $9/2^+$ or $7/2^+$. The $9/2^+$ value is preferred since this choice is consistent with the population of the state via a $2g_{9/2}$ admixture .45 MeV away from its centroid, rather than by a $2g_{7/2}$ admixture 3.07 MeV away from its centroid. The distribution of particle state strength to be shown in section C makes the former mode much more plausible.

The remaining states of Table V-2 are not especially significant, except for the .703 MeV $7/2^-$ level which is not populated in these (d,p) reactions, nor in the (d,t) and (p,d) experiments on $^{206}$Pb. No evidence of this state was seen in our work and the spectroscopic value quoted in Table V-2 for this state represents an upper limit based on our lowest detectable peak cross section (2μb/sr). The lack of direct reaction population to the .703 MeV state will be discussed further in the next section. At this point it is noted that there is a further uncertainty to be attached to all the spectroscopic factors listed in Table V-2. It develops that the binding energies of the hole states in $^{205}$Pb (and in $^{207}$Pb also) are not consistent with a shell potential 50 MeV deep; a somewhat shallower well must be used ($\sim$43 MeV). In fact this is part of a general lead binding energy problem to be developed in section D. It is noted that the use of too shallow a well leads to incorrect normalization of the form factor (it becomes too high) in the reaction calculation. This in turn make the computed spectroscopic factors too low by an amount difficult to determine, but which would certainly bear on any comparison with theory. Thus forewarned, the interpretations of the hole-state spectroscopic factors are now given in Section B.
B. THE DISCUSSION OF THE HOLE STATE SPECTRUM

Entwined with, if not impeded by, reaction mechanism considerations, the hole state spectroscopic factors can be given interpretation only on a very qualitative basis. Even without the form factor problem mentioned at the conclusion of section A, it appears the Miranda's phenomenological residual interaction does not at all come close to producing the actual spectrum. In only three instances are his spectroscopic values in good agreement with experiment: that to the $5/2^-$ ground state, that to the $5/2^-$ 1.265 state, and that to the .703 MeV 7/2$^-$ level for which an upper limit is placed on the spectroscopic factor. Miranda and also Rao (RA 71) have calculated this last level to be based primarily (>$90\%$) on a $2f_{5/2}$ neutron coupled to the first excited $2^+$ state of the core. As such the state would not be expected to be populated by stripping onto the ground state of $^{204}$Pb in a one step reaction. It would be only as the 1.764 7/2$^-$ main fragment admixes with the .703 MeV level that any spectroscopic amplitude would be possible. According to Miranda such admixture is very small, in line with the experimental results presented here. The collective nature of the .703 MeV state is also underscored by the $^{205}$Bi decay work. By a branching ratio of 99.9$\%$ the .703 MeV state E2 decays to the 5/2$^-$ ground state, almost completely eschewing an E2 decay to the .262 MeV 3/2$^-$ level. Such an enhanced transition can be simply explained by considering the $2^+$ excited core to make the E2 transition to the unexcited core, leaving the $2f_{5/2}$ odd neutron as a spectator.

As far as the other two calculations are concerned, those of McGrory and of Harvey and Clement, they do not give as detailed a set of predictions as does Miranda. McGrory seems to place the major $2f_{7/2}$ strength at .703 MeV, instead of 1.764 MeV where it actually occurs. It should be
pointed out that the Kuo-Brown interaction used by McGrory was renormalized in the first order term to best fit the level structures of $^{204}$Pb, $^{205}$Pb, $^{206}$Pb. It has been remarked earlier that the use of the Kuo-Brown series is questionable on a convergence problem.

The third approach to the residual interaction, the quasi-particle plus realistic interaction used by Harvey and Clement, does better in predicting the location of the $2f_{7/2}$ spectroscopic strength. The reference cited only treated $^{205}$Pb briefly, however, with only summed spectroscopic factors quoted, without centroid energies or individual strengths. The authors do however state that the lowest $5/2^-$, $1/2^-$, $3/2^-$, and $13/2^+$ states in $^{205}$Pb should be very nearly pure one quasi-particle configurations, that is, states created by the quasi-particle creation operator acting upon the vacuum (here the $^{204}$Pb ground state). Therefore, the quoted sums from this work in Table V-2 are placed at the corresponding experimental levels. On the other hand, the $7/2^-$ and $9/2^-$ are calculated to be very admixed one quasi-particle three-quasi-particle state with greater fragmentation of the levels. A $7/2^-$ state at 1.82 MeV of excitation is predicted to have the most one quasi-particle amplitude, presumably corresponding to the observed 1.764 MeV level. The $9/2^-$ one quasi-particle amplitude is particularly diluted and thus might not be seen at all in the $(d,p)$ experiment. One of the more impressive results of the Harvey calculation is the predicted splitting of the $1i_{13/2}$ level for which the higher fragment was unknown at the time of publication. While almost all the expected spectroscopic strength is figured to go to the main fragment, a few per cent should still remain for the weaker fragment at 1.85 MeV. This is indeed what the experimental results show. These
encouraging trends would make a more detailed set of predictions based on
the BCS model a desirable effort. Also needed, instead of the usual sep-
aration-energy form factors in the reaction calculation are realistic
form factors calculated in the quasi-particle basis. This would re-
move the disconcerting normalization error which must be attached to the
quoted experimental spectroscopic factors.

In summary, it can be said that while none of the predicted sets of
theoretical spectroscopic factors agree very well with experiment, the
distribution of excitation suggested by the Harvey and Clement paper
appears impressive. More detailed results are needed, however, for a
careful comparison. At this time it can be concluded that these authors
are correct in calculating nearly all the $2f_{5/2}$, $3p_{1/2}$, $3p_{3/2}$, and $1i_{13/2}$
excitation amplitude to be undivided while the $2f_{7/2}$ hole state is much
more fragmented. The Miranda calculation seems to do very poorly in fore-
casting the experimental spectrum except insofar as the collective
$.703 \text{ MeV } 7/2^-$ level is concerned. The success of the McGrory calculation
is limited by the erroneous distribution of the $2f_{7/2}$ spectroscopic
strength which is definitely not in the $.703 \text{ MeV state}$ as he predicted.

A final note on the hole-state spectrum has direct relevance to the
forthcoming particle-state spectrum. Of the fifteen proton groups observed
with 20 MeV incident energy deuterons, only four (including two doublets)
had peak cross sections exceeding 100 $\mu$b/sr. The remaining eleven weak
fragments had maxima below 25 $\mu$b/sr. Since in addition nearly all the
hole-state spectroscopic strength is believed accounted for, there
is no expectation that there will be any but minute population of neg-
ative parity ($l_n = 1$ or 3) states above 2.57 MeV of excitation. All the
states to be reported in sections C and D have peak angular distributions in excess of 50 μb/sr. and the majority are over 100 μb/sr. These magnitudes make it extremely unlikely that the proton groups represent hole state fragments, for if they did there would be a huge shift in the centroid energies of these orbits, completely out of line with the observed low-lying spectrum of $^{205}$Pb. Therefore, only positive parity fragmentation (and the $1j_{15/2}$) were considered in the high-excitation analysis. That this very necessary simplification is possible, is a pleasing example of the underlying physical link between the hole and the particle states' excitation spectra.
C. THE PARTICLE STATE ANALYSIS: EX. > 2.57 MeV

The (d,p) spectrum of $^{205}$Pb above 2.57 MeV is found to be much more complicated than the hole-state spectrum examined in the preceding two sections. Samples of this spectrum as excited by 13.0 and 20.0 MeV incident energy deuterons are illustrated in Figures V-5 and V-6. Also represented in these figures are the decompositions accomplished by the fitting program AUTOFIT. It will be recalled from the last chapter that the view adopted was that the fit obtained to the data at any one observation angle was not by itself convincing. When however, the data analyses from all observation angles at both incident energies were closely scrutinized, it was found that the positions and number of the deduced levels were constant to a remarkable degree. The credence of the data analysis method thus confirmed, the next step is to assign angular momentum values to the extracted angular distributions. It is realized that an unpolarized (d,p) experiment cannot distinguish the total transferred spin, $j = I_n \pm 1/2$. That is, the present study cannot distinguish between a $2g_{9/2}$ and a $2g_{7/2}$ fragment at the same excitation energy, or similarly between a $3d_{5/2}$ and a $3d_{3/2}$ fragment. Such a discrimination can only be based on the assumption that the shell-model ordering given in Figure 1-1 is maintained in $^{205}$Pb.

Although the energies of the levels in $^{205}$Pb are believed to be quite well determined in this study, a larger error must be attached to the angular distribution yields of all but the few well-isolated levels. At both energies, separate but partially overlapping exposures were made. The magnetic field settings were different thereby placing the proton groups along different regions of the focal plane thus giving rise to dissimilar group shapes in the overlapping runs. Upon proper normalization of
A complete AUTOFIT of one of the 13.0 MeV particle-state spectra. The continuous line here is that predicted by the program and is not a guide to the eye. As in Figure IV-2, the overlapping lines near 4.5 MeV of excitation are samples of those predicted by the program.
$^{204}\text{Pb}(d,p)^{205}\text{Pb}$

$E_d = 13.0\text{ MeV}$

$\theta_{lab} = 118.75^\circ$

4600 $\mu\text{C}$
A complete AUTOFIT of one of the 20.0 MeV particle state spectra. Here, all the overlapping peaks are drawn in with the resultant sum omitted for clarity.
$^{204}\text{Pb}(d,p)^{205}\text{Pb}$

$E_d = 20.0$ MeV

$\theta_{\text{lab}} = 50^\circ$
the exposures, the yields of partially resolved levels of moderate strength (200 \(\mu b/sr\)) were compared and found to be constant only to about 15%, an uncertainty much larger than that attributable to scanner errors (5%) and hence a measure of the uncertainty of the deduced fits. This error increase to about 25% for the more weakly excited partially resolved peaks.

The effect of such errors is most serious in the region of excitation beginning around 4.75 MeV and continuing up to 5.72 MeV, the range in which most of the \(2g_{7/2}\) and \(3d_{3/2}\) strength is observed. In similar experiments on \(^{206}\text{Pb}\) (MC 70) and \(^{202,204}\text{Hg}\) (MO 72), Moyer et al. were greatly hampered in assigning the \(g_{7/2}\) and \(d_{3/2}\) fragments because the associated angular-distribution shapes are not appreciably distinct. Were the present more dense spectrum to have been studied at only one incident energy, it would have been very difficult to assign much \(g_{7/2}\) and \(d_{3/2}\) spectroscopic strength in \(^{205}\text{Pb}\). The data-reduction errors mentioned above further confuse the interpretation of the angular distributions. However, with a complete analysis of the spectrum at two incident energies, 13.0 and 20.0 MeV, a comparison of the relative increase in yield of corresponding proton groups enables the \(2g_{7/2}\) and the \(3d_{3/2}\) fragments to be successfully distinguished. The differences in the relative yield factors for the single-particle states in \(^{209}\text{Pb}\) are listed in Table V-3.

The differential cross sections are for the maxima in the angular distributions as experimentally obtained in the 13.0 and 20.0 MeV \(^{208}\text{Pb}(d,p)\) exposures. The ratio of these maxima defines the relative increase in yield. A comparison of these relative yield numbers shows that there is an increased intrinsic cross section for the higher \(l_n\) transfers while the intrinsic cross section for the \(l_n = 0\) transfer actually decreases. The cross section for the \(3d_{5/2}\) and the \(3d_{3/2}\) is seen to remain about the
Table V-3. The Maxima of the \((d,p)\) Differential Cross-Sections at 13.0 and 20.0 MeV Incident Energy

<table>
<thead>
<tr>
<th>(nlj)</th>
<th>(E_B^{209}) (MeV) (^a)</th>
<th>((d\sigma/d\Omega)_{\text{max}}) (^b)</th>
<th>(mb/sr)</th>
<th>Ratio</th>
<th>(E_B^{205}) (MeV) (^a)</th>
<th>(Ratio) (^c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(2g_9/2)</td>
<td>(-3.945)</td>
<td>2.03</td>
<td>4.40</td>
<td>2.17</td>
<td>(-4.038)</td>
<td>2.14</td>
</tr>
<tr>
<td>(1i_{11/2})</td>
<td>(-3.146)</td>
<td>.19</td>
<td>.50</td>
<td>2.63</td>
<td>(-3.242)</td>
<td>2.58</td>
</tr>
<tr>
<td>(1j_{15/2})</td>
<td>(-2.511)</td>
<td>.13(^d)</td>
<td>.45</td>
<td>3.46</td>
<td>(-2.733)</td>
<td>3.23</td>
</tr>
<tr>
<td>(3d_{5/2})</td>
<td>(-2.380)</td>
<td>7.90</td>
<td>8.40</td>
<td>1.06</td>
<td>(-2.571)</td>
<td>1.11</td>
</tr>
<tr>
<td>(4s_{1/2})</td>
<td>(-1.912)</td>
<td>5.45</td>
<td>2.60</td>
<td>.48</td>
<td>(-2.193)</td>
<td>.50</td>
</tr>
<tr>
<td>(2g_{7/2})</td>
<td>(-1.450)</td>
<td>2.87</td>
<td>6.45</td>
<td>2.25</td>
<td>(-1.443)</td>
<td>2.25</td>
</tr>
<tr>
<td>(3d_{3/2})</td>
<td>(-1.405)</td>
<td>6.09</td>
<td>6.10</td>
<td>1.00</td>
<td>(-1.675)</td>
<td>1.07</td>
</tr>
</tbody>
</table>

\(^a\) Binding energy of the single particle state in \(^{209}\)Pb, or the fragments' centroid in \(^{205}\)Pb.

\(^b\) The maximum of the differential cross section (\(^{208}\)Pb\((d,p)\)^{209}\)Pb) as experimentally observed at these incident energies (see Figs.

\(^c\) The experimental ratios for \(^{209}\)Pb multiplied by the Q-value corrections calculated from Eq. (3-1). The optical parameters of Table V-1 were used to compute this correction.

\(^d\) The \(1j_{15/2}\) transfer is predicted to peak at far back angles at which plates were not exposed. The value quoted here is that based on the shape of the DWBA curve as normalized to the forward angle data points.
same at 13.0 and 20.0 MeV. This behavior is a simple consequence of the fact that as the incident energy increases, the matching value of the angular momentum transfer also increases to higher values. The overall effect on the approximately equal Q-value transfers to the 3d_{3/2} and 2g_{7/2} states is for the 2g_{7/2} peak cross section to increase 2.25 times as fast relative to the 3d_{3/2} transfer. Hence a 2g_{7/2} proton group misassigned in the 13.0 MeV experiment analysis as being a 3d_{3/2} fragment would have its 20.0 MeV spectroscopic factor 2.25 times as large as the 13.0 MeV value. Such a discrepancy is well beyond the data reduction errors attached to the cross sections and it is this fact which allows 2g_{7/2} and the 3d_{3/2} fragments to be distinguished with confidence. In fact all the deduced levels were compared in this fashion. The remaining columns of Table V-3 show, in anticipation of the results to be described in Section D, that binding energy differences of a few hundred kev do not have a great effect on these relative yield numbers.

Without further ado, Table V-4 presents the energies of the 110 levels now conclusively determined in the (d,p) spectrum of \(^{205}\text{Pb}\) between 2.57 and 5.62 MeV of excitation. Between 5.62 and 5.75 MeV a few additional states are weakly populated but their density is too high to resolve even at 7 kev resolution. Beyond 5.75 and until the neutron threshold of 6.73 MeV, no states could be discerned above the background. (Above 5.75 MeV, the background level in the 13.0 MeV experiments was typically 10 \(\mu\text{b/sr per 3 kev channel}\)) Of the 110 states established, 87 can be assigned reliable angular momentum values, another 18 more weakly excited states are tentatively assigned angular momentum values, while the remaining 5 groups with very weak cross sections cannot be classified. Figures V-7 and V-8 show DW fits to selected angular distributions for
Table V-4. The Single Particle Fragments in $^{205}$Pb

<table>
<thead>
<tr>
<th>#</th>
<th>Ex. Energy</th>
<th>$\Delta$Ex</th>
<th>ntj</th>
<th>Stj</th>
<th>$\Delta$Stj</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.565</td>
<td>---</td>
<td>$2g_9/2$</td>
<td>.16</td>
<td>± .01</td>
</tr>
<tr>
<td>2</td>
<td>2.607</td>
<td>± .001</td>
<td>$2g_9/2$</td>
<td>.18</td>
<td>± .01</td>
</tr>
<tr>
<td>3</td>
<td>2.634</td>
<td>± .003</td>
<td>$(2g_9/2)_1$</td>
<td>(.017)</td>
<td>± (.002)</td>
</tr>
<tr>
<td>4</td>
<td>2.657</td>
<td>± .003</td>
<td>$(2g_9/2)_2$</td>
<td>(.013)</td>
<td>± (.002)</td>
</tr>
<tr>
<td>5</td>
<td>2.708</td>
<td>± .001</td>
<td>$2g_9/2$</td>
<td>.38</td>
<td>± .02</td>
</tr>
<tr>
<td>6</td>
<td>2.798</td>
<td>± .002</td>
<td>$2g_9/2$</td>
<td>.05</td>
<td>± .003</td>
</tr>
<tr>
<td>7</td>
<td>2.862</td>
<td>± .002</td>
<td>$(2g_9/2)_3$</td>
<td>(.009)</td>
<td>± (.002)</td>
</tr>
<tr>
<td>8</td>
<td>2.931</td>
<td>± .002</td>
<td>$2g_9/2$</td>
<td>.005</td>
<td>± .001</td>
</tr>
<tr>
<td>9</td>
<td>3.010</td>
<td>± .002</td>
<td>$2g_9/2$</td>
<td>.030</td>
<td>± .002</td>
</tr>
<tr>
<td>10</td>
<td>3.043</td>
<td>± .002</td>
<td>$2g_9/2$</td>
<td>.015</td>
<td>± .002</td>
</tr>
<tr>
<td>11</td>
<td>3.119</td>
<td>± .002</td>
<td>$2g_9/2$</td>
<td>.016</td>
<td>± .002</td>
</tr>
<tr>
<td>12</td>
<td>3.165</td>
<td>± .003</td>
<td>$(2g_9/2)_4$</td>
<td>(.005)</td>
<td>± (.001)</td>
</tr>
<tr>
<td>13</td>
<td>3.249</td>
<td>± .003</td>
<td>$(3d_5/2)_5$</td>
<td>(.006)</td>
<td>± (.001)</td>
</tr>
<tr>
<td>14</td>
<td>3.306</td>
<td>± .003</td>
<td>$(2g_9/2)_6$</td>
<td>(.008)</td>
<td>± (.002)</td>
</tr>
<tr>
<td>15</td>
<td>3.334</td>
<td>± .002</td>
<td>$(3d_5/2)_7$</td>
<td>(.006)</td>
<td>± (.001)</td>
</tr>
<tr>
<td>16</td>
<td>3.393</td>
<td>± .003</td>
<td>$1i_{11/2}$</td>
<td>.09</td>
<td>± .02</td>
</tr>
<tr>
<td>17</td>
<td>3.422</td>
<td>± .004</td>
<td>$1i_{11/2}$</td>
<td>.03</td>
<td>± .006</td>
</tr>
<tr>
<td>18</td>
<td>3.435</td>
<td>± .002</td>
<td>$(3d_5/2)_8$</td>
<td>(.006)</td>
<td>± (.001)</td>
</tr>
<tr>
<td>19</td>
<td>3.483</td>
<td>± .002</td>
<td>$3d_5/2$</td>
<td>.045</td>
<td>± .002</td>
</tr>
<tr>
<td>20</td>
<td>3.511</td>
<td>± .002</td>
<td>$1i_{11/2}$</td>
<td>.20</td>
<td>± .02</td>
</tr>
<tr>
<td>21</td>
<td>3.533</td>
<td>± .002</td>
<td>$3d_5/2$</td>
<td>.022</td>
<td>± .001</td>
</tr>
<tr>
<td>22</td>
<td>3.566</td>
<td>± .002</td>
<td>$3d_5/2$</td>
<td>.023</td>
<td>± .001</td>
</tr>
<tr>
<td>23</td>
<td>3.592</td>
<td>± .004</td>
<td>$tn \geq 4$</td>
<td>.06</td>
<td>± .01</td>
</tr>
<tr>
<td>24</td>
<td>3.613</td>
<td>± .003</td>
<td>$1i_{11/2}$</td>
<td>.09</td>
<td>± .001</td>
</tr>
<tr>
<td>25</td>
<td>3.659</td>
<td>± .002</td>
<td>$3d_5/2$</td>
<td>.009</td>
<td>± .002</td>
</tr>
<tr>
<td>26</td>
<td>3.764</td>
<td>± .002</td>
<td>$3d_5/2$</td>
<td>.009</td>
<td>± .002</td>
</tr>
<tr>
<td>27</td>
<td>3.834</td>
<td>± .004</td>
<td>$\sigma_{max} = .020 \mu b/sr^2$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>28</td>
<td>3.857</td>
<td>± .002</td>
<td>$3d_5/2$</td>
<td>.007</td>
<td>± .002</td>
</tr>
<tr>
<td>29</td>
<td>3.889</td>
<td>± .004</td>
<td>$\sigma_{max} = .035 \mu b/sr$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>3.950</td>
<td>± .003</td>
<td>$(3d_5/2)$</td>
<td>(.005)</td>
<td>(± .001)</td>
</tr>
<tr>
<td>#</td>
<td>Ex. Energy</td>
<td>$\Delta$Ex</td>
<td>$n\ell j$</td>
<td>Stj</td>
<td>$\Delta$Stj</td>
</tr>
<tr>
<td>----</td>
<td>------------</td>
<td>------------</td>
<td>--------</td>
<td>------</td>
<td>------------</td>
</tr>
<tr>
<td>31</td>
<td>3.961</td>
<td>± .002</td>
<td>3d$_{5/2}$</td>
<td>.024</td>
<td>± .003</td>
</tr>
<tr>
<td>32</td>
<td>3.988</td>
<td>± .002</td>
<td>3d$_{5/2}$</td>
<td>.055</td>
<td>± .005</td>
</tr>
<tr>
<td>33</td>
<td>4.002</td>
<td>± .002</td>
<td>1$\ell_{15/2}$</td>
<td>.38</td>
<td>± .06</td>
</tr>
<tr>
<td>34</td>
<td>4.016</td>
<td>± .003</td>
<td>(3d$_{5/2}$)</td>
<td>(.001)</td>
<td>(± .0003)</td>
</tr>
<tr>
<td>35</td>
<td>4.083</td>
<td>± .003</td>
<td>3d$_{5/2}$</td>
<td>.028</td>
<td>± .003</td>
</tr>
<tr>
<td>36</td>
<td>4.106</td>
<td>± .003</td>
<td>1n ≥ 4</td>
<td>$\sigma_{\text{max}} = .035 \mu$b/sr</td>
<td></td>
</tr>
<tr>
<td>37</td>
<td>4.125</td>
<td>± .002</td>
<td>3d$_{5/2}$</td>
<td>.009</td>
<td>± .002</td>
</tr>
<tr>
<td>38</td>
<td>4.127</td>
<td>± .002</td>
<td>3d$_{5/2}$</td>
<td>.050</td>
<td>± .005</td>
</tr>
<tr>
<td>39</td>
<td>4.156</td>
<td>± .002</td>
<td>3d$_{5/2}$</td>
<td>.014</td>
<td>± .002</td>
</tr>
<tr>
<td>40</td>
<td>4.187</td>
<td>± .004</td>
<td>(4s$_{1/2}$)</td>
<td>(.002)</td>
<td>(± .0004)</td>
</tr>
<tr>
<td>41</td>
<td>4.199</td>
<td>± .003</td>
<td>(4s$_{1/2}$)</td>
<td>(.004)</td>
<td>(± .0008)</td>
</tr>
<tr>
<td>42</td>
<td>4.214</td>
<td>± .002</td>
<td>3d$_{5/2}$</td>
<td>.019</td>
<td>± .002</td>
</tr>
<tr>
<td>43</td>
<td>4.239</td>
<td>± .003</td>
<td>(3d$_{5/2}$)</td>
<td>(.005)</td>
<td>(± .001)</td>
</tr>
<tr>
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<td>4.254</td>
<td>± .003</td>
<td>3d$_{5/2}$</td>
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<td>± .002</td>
</tr>
<tr>
<td>45</td>
<td>4.299</td>
<td>± .002</td>
<td>3d$_{5/2}$</td>
<td>.015</td>
<td>± .002</td>
</tr>
<tr>
<td>46</td>
<td>4.326</td>
<td>± .002</td>
<td>3d$_{5/2}$</td>
<td>.049</td>
<td>± .007</td>
</tr>
<tr>
<td>47</td>
<td>4.342</td>
<td>± .002</td>
<td>3d$_{5/2}$</td>
<td>.145</td>
<td>± .022</td>
</tr>
<tr>
<td>48</td>
<td>4.361</td>
<td>± .002</td>
<td>3d$_{5/2}$</td>
<td>.035</td>
<td>± .005</td>
</tr>
<tr>
<td>49</td>
<td>4.372</td>
<td>± .002</td>
<td>3d$_{5/2}$</td>
<td>.035</td>
<td>± .005</td>
</tr>
<tr>
<td>50</td>
<td>4.389</td>
<td>± .002</td>
<td>3d$_{5/2}$</td>
<td>.042</td>
<td>± .006</td>
</tr>
<tr>
<td>51</td>
<td>4.412</td>
<td>± .002</td>
<td>3d$_{5/2}$</td>
<td>.025</td>
<td>± .004</td>
</tr>
<tr>
<td>52</td>
<td>4.428</td>
<td>± .004</td>
<td>(3d$_{5/2}$)</td>
<td>(.006)</td>
<td>(± .001)</td>
</tr>
<tr>
<td>53</td>
<td>4.443</td>
<td>± .003</td>
<td>3d$_{5/2}$</td>
<td>.020</td>
<td>± .003</td>
</tr>
<tr>
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<td>4.452</td>
<td>± .002</td>
<td>3d$_{5/2}$</td>
<td>.050</td>
<td>± .008</td>
</tr>
<tr>
<td>55</td>
<td>4.497</td>
<td>± .002</td>
<td>4s$_{1/2}$</td>
<td>.340</td>
<td>± .030</td>
</tr>
<tr>
<td>56</td>
<td>4.539</td>
<td>± .002</td>
<td>4s$_{1/2}$</td>
<td>.185</td>
<td>± .020</td>
</tr>
<tr>
<td>57</td>
<td>4.558</td>
<td>± .002</td>
<td>4s$_{1/2}$</td>
<td>.100</td>
<td>± .015</td>
</tr>
<tr>
<td>58</td>
<td>4.590</td>
<td>± .002</td>
<td>4s$_{1/2}$</td>
<td>.150</td>
<td>± .020</td>
</tr>
<tr>
<td>59</td>
<td>4.624</td>
<td>± .002</td>
<td>(3d$<em>{5/2}$, 3d$</em>{3/2}$)</td>
<td>(.16, .024)</td>
<td>(± .003, .005)</td>
</tr>
<tr>
<td>60</td>
<td>4.642</td>
<td>± .004</td>
<td>(4s$_{1/2}$)</td>
<td>(.021)</td>
<td>(± .004)</td>
</tr>
<tr>
<td>#</td>
<td>Ex. Energy</td>
<td>$\Delta$Ex</td>
<td>nτj</td>
<td>Stj</td>
<td>$\Delta$Stj</td>
</tr>
<tr>
<td>----</td>
<td>------------</td>
<td>------------</td>
<td>-----</td>
<td>-----</td>
<td>-------------</td>
</tr>
<tr>
<td>61</td>
<td>4.656</td>
<td>± .005</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>62</td>
<td>4.693</td>
<td>± .003</td>
<td>(2g_{7/2})</td>
<td>.025</td>
<td>(± .005)</td>
</tr>
<tr>
<td>63</td>
<td>4.709</td>
<td>± .003</td>
<td>(3d_{5/2}, 3d_{3/2})</td>
<td>.013, .020</td>
<td>(± .003, .004)</td>
</tr>
<tr>
<td>64</td>
<td>4.722</td>
<td>± .002</td>
<td>4s_{1/2}</td>
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<td>± .010</td>
</tr>
<tr>
<td>65</td>
<td>4.745</td>
<td>± .004</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>66</td>
<td>4.760</td>
<td>± .002</td>
<td>3d_{3/2}</td>
<td>.014</td>
<td>± .003</td>
</tr>
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<td>± .002</td>
<td>3d_{5/2}</td>
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<td>± .003</td>
</tr>
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<td>3d_{3/2}</td>
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</tr>
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<td>± .002</td>
<td>3d_{3/2}</td>
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<td>± .003</td>
</tr>
<tr>
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<td>2g_{7/2}</td>
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<td>2g_{7/2}</td>
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<td>73</td>
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<td>(± .002, .003)</td>
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<td>76</td>
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<td>± .005</td>
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<td>± .006</td>
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<td>± .020</td>
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<tr>
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<td>± .012</td>
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<tr>
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<td>5.285</td>
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<td></td>
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<tr>
<td>#</td>
<td>Ex. Energy</td>
<td>ΔEx</td>
<td>nlj</td>
<td>Stj</td>
<td>ΔStj</td>
</tr>
<tr>
<td>----</td>
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<td>------</td>
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<td>± .006</td>
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<td>± .004</td>
<td>2g_{7/2}</td>
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<td>± .005</td>
</tr>
<tr>
<td>99</td>
<td>5.378</td>
<td>± .004</td>
<td>(3d_{3/2})</td>
<td>(.017)</td>
<td>(± .004)</td>
</tr>
<tr>
<td>100</td>
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<td>± .003</td>
<td>(2g_{7/2})</td>
<td>(.013)</td>
<td>(± .003)</td>
</tr>
<tr>
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<td>± .008</td>
</tr>
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<td>± .005</td>
</tr>
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<td>2g_{7/2}</td>
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<td>± .005</td>
</tr>
<tr>
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<td>5.473</td>
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<td>2g_{7/2}</td>
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<td>± .008</td>
</tr>
<tr>
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<td>2g_{7/2}</td>
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<td>± .005</td>
</tr>
<tr>
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<td>5.515</td>
<td>± .004</td>
<td>2g_{7/2}</td>
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<td>± .005</td>
</tr>
<tr>
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<td>2g_{7/2}</td>
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<td>± .005</td>
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<tr>
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<td>± .003</td>
<td>2g_{7/2}</td>
<td>.020</td>
<td>± .004</td>
</tr>
<tr>
<td>109</td>
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<td>± .002</td>
<td>(2g_{7/2})</td>
<td>(.010)</td>
<td>(± .002)</td>
</tr>
<tr>
<td>110</td>
<td>5.623</td>
<td>± .003</td>
<td>2g_{7/2}</td>
<td>.020</td>
<td>(± .004)</td>
</tr>
</tbody>
</table>

a) Maximum cross sections are for 13.0 MeV incident energy.
Fits to the strongest different particle-state fragments of the $^{204}$Pb(d,p)$^{205}$Pb reaction at $E_d = 13.0$ MeV.
Fits to the strongest different particle-state fragments of the $^{204}\text{Pb}(d,p)^{205}\text{Pb}$ reaction at $E_d = 20.0\text{ MeV}$. 
$^{204}\text{Pb (d, p)}^{205}\text{Pb}$

$E_d = 20.0 \text{ MeV}$

- $E_x = 4.497$
  - $4s_{1/2}$
  - $S = 0.33$

- $E_x = 4.342$
  - $3d_{5/2}$
  - $S = 0.152$

- $E_x = 2.708$
  - $2g_{9/2}$
  - $S = 0.38$

- $E_x = 5.418$
  - $2g_{7/2}$
  - $S = 0.046$

- $E_x = 3.511$
  - $1\ell_{11/2}$
  - $S = 0.28$

- $E_x = 5.083$
  - $3d_{3/2}$
  - $S = 0.12$

- $E_x = 4.002$
  - $1\ell_{15/2}$
  - $S = 0.45$

$\frac{d\sigma}{d\Omega}$ (mb/sr)

$\theta_{c.m.}$
Fits to the $^{208}\text{Pb}(d,p)^{209}\text{Pb}$ single-particle angular distributions at $E_d = 13.0$ MeV. A combination of contaminant interferences and intrinsic weakness (even with 600 µC) rendered the $1i_{11/2}$ and $1j_{15/2}$ angular distributions non-significant in the angles exposed ($5^\circ \rightarrow 80^\circ$).
\[ ^{208}\text{Pb} (d, p) ^{209}\text{Pb} \]

\[ E_d = 13.0 \text{ MeV} \]

\[
\begin{align*}
\text{d} \sigma / \text{d} \Omega \ (\text{mb/sr}) \\
0^\circ & \quad 50^\circ & \quad 100^\circ \\
0.01 & \quad 1.0 & \quad 10.0 \\
0.1 & \quad 1.0 & \quad 10.0
\end{align*}
\]

\( E_x = 2.033 \quad 4s_{1/2} \quad S=0.93 \)

\( E_x = 2.937 \quad 2g_{7/2} \quad S=0.97 \)

\( E_x = 1.565 \quad 3d_{5/2} \quad S=0.93 \)

\( E_x = 2.442 \quad 3d_{3/2} \quad S=0.90 \)
some of the more strongly excited fragments. It should be emphasized that the $2g_{7/2}$ and the $3d_{3/2}$ fits are not by themselves conclusive, but are correlated with the relative yield comparisons. These fits, generated by the optical parameter in Table V-1, are included as supplementary guides, since the template procedure described in Chapter Four was the primary interpretive vehicle. For comparison, the $^{209}$Pb single-particle angular distributions have the DW fits shown in Figure V-9 and III-9.

In order to facilitate the discussion of the spectrum, Figure V-10 presents the strongest values of the definitely assigned shell-model fragments as a function of the excitation energy. Also included in this figure are arrows denoting the corresponding energies of the strong states of $^{209}$Pb as well as the spectroscopically weighted centroids of the fragments in $^{205}$Pb. The spectrum will be discussed more fully in the next section, but for now attention is drawn to the apparent gap in the distribution of the $2g_{7/2}$ strength around 5.1 MeV of excitation. At that position occur very strongly populated $3d_{3/2}$ fragments whose assignments are unquestionable. However, within the data-reduction error, the spectroscopic strength of each of these $3d_{3/2}$ states could be decreased by about 10% if it were assumed that under each group a $2g_{7/2}$ state were being concealed. From Table V-3 it can be seen that at both 13.0 and 20.0 MeV, the intrinsic $3d_{3/2}$ cross section is greater than that of the $2g_{7/2}$. Such an assumption of continuous doublets would increase the total sum $2g_{7/2}$ spectroscopic strength by $\Delta S_{g_{7/2}}$, but would still leave an unmistakable dip in the $2g_{7/2}$ spectral distribution.

A final word about the particle-state results has to do with the assigned errors, $\Delta E_x$ and $\Delta S_{ij}$. Between the first and the last of the excitation energies listed in Table V-3, there could be as much as a 10
The spectroscopic strength distribution for the particle states in $^{205}$Pb. The arrows denote the positions of the $^{205}$Pb centroids and the corresponding single-particle states in $^{205}$Pb.
kev systematic error. The local error, $\Delta Ex.$, refers to the position uncertainty of a level with respect to its neighbors within about a 200 kev window. The error in the spectroscopic factor derives from the uncertainty in the deduced yields. For the well-isolated levels, the error was simply a combination of statistical and assumed scanning errors. For the overlapping levels, the assigned error was somewhat more subjective. A minimum of 15% was assigned to the moderately strong groups (200 $\mu$b/sr.) and this included the fitting error, the background error, and the assumed scanning error. For the more weakly excited levels a larger uncertainty was placed on the quoted spectroscopic factor. Such an appraisal was the result of computing a "spectroscopic factor" at each angle of a level's angular distribution at both 13.0 and 20.0 MeV incident energy. This "spectroscopic factor" was simply the ratio of the differential cross section in $^{205}$Pb to the corresponding differential cross section in $^{209}$Pb. In general it was found in this procedure that the more weakly excited of the partially resolved groups showed a larger scatter in these individual "spectroscopic factors". The assigned spectroscopic factor for such a group was a weighted average of these individual "spectroscopic factors" with the error $\Delta S_{ij}$ based on the scatter of the individual values. It should be pointed out that these comments do not apply with equal force to all the sub-orbitals. For example, the $2g_{9/2}$ and $4s_{1/2}$ strength is practically all concentrated in a few well resolved states, whereas, the $2g_{7/2}$ strength is distributed almost evenly among closely lying levels. This can also be seen from the values listed in Table V-5 which gives the centroid energies, spectroscopic sums (only the definitely assigned states are included), and the total error in the spectroscopic factor, assuming the individual errors $\Delta S_{ij}$ to be uncorrelated.
Table V-5. Summary of the Particle States' Distribution in $^{205}$Pb

<table>
<thead>
<tr>
<th>nτj</th>
<th>$E_x^{\text{cent.}}$ (MeV)</th>
<th>$\Sigma S_{\tau j}^a$</th>
<th>$\Delta S_{\tau j}^b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2g_9/2$</td>
<td>2.697</td>
<td>.84</td>
<td>± .03</td>
</tr>
<tr>
<td>$1i_{11/2}$</td>
<td>3.493</td>
<td>.38</td>
<td>± .03</td>
</tr>
<tr>
<td>$1i_{15/2}$</td>
<td>4.002</td>
<td>.38</td>
<td>± .05</td>
</tr>
<tr>
<td>$3d_{5/2}$</td>
<td>4.163</td>
<td>.74</td>
<td>± .03</td>
</tr>
<tr>
<td>$4s_{1/2}$</td>
<td>4.542</td>
<td>.83</td>
<td>± .05</td>
</tr>
<tr>
<td>$2g_{7/2}$</td>
<td>5.292</td>
<td>.66</td>
<td>± .03</td>
</tr>
<tr>
<td>$3d_{3/2}$</td>
<td>5.060</td>
<td>.88</td>
<td>± .04</td>
</tr>
</tbody>
</table>

\[ a\) Assuming the single particle strengths in $^{209}$Pb are 1.0, except for the $1i_{15/2}$ for which the value .77 was taken.\]

\[ b\) Assuming the errors in the fragments are uncorrelated.\]
The discussion of this table and also of Figure V-10 form the concluding section of this Chapter.
D. THE INTERPRETATION OF THE PARTICLE STATE SPECTRUM

In the last section it has been stated that a (d,p) experiment by itself cannot distinguish the total spin transferred, but can only determine the orbital angular momentum of the bound state. This is not true for the $1/2^+$ transfer, of course, in which there is no ambiguity. Similarly, there is almost no question that the observed $l_n = 6$ and the one $l_n = 7$ transfer correspond to $11/2^+$ and $15/2^-$ states rather than $13/2^+$ and $17/2^-$ states. Finally the work of Casten et al. (CC 73) and the $^{205}\text{Bi}$ decay studies (HA 72, RV 72) have established a $9/2^+$ spin for the strong triplet of states beginning at 2.565 MeV of excitation. In essence then there is really only one $j$-value ambiguity, that between $3d_{5/2}$ and $3d_{3/2}$ fragments. Table V-4 partially alludes to this ambiguity by failing to distinguish definitely a few $3d_{5/2}$ or $3d_{3/2}$ states around 4.7 MeV of excitation. Nonetheless the overall trend of Figure V-10 is undeniable. Unless one is willing to argue for a strange inversion of the $3d_{5/2}$ and $3d_{3/2}$ centroids, or less drastically, for an odd intermixture of these states as they appear in the spectrum, then otherwise it must be concluded that there is a concentration of spectroscopic strength in $^{205}\text{Pb}$ more or less narrowly about the single-particle centroids. This is especially true for the $2g_{9/2}$ and $4s_{1/2}$ orbitals, although less dramatically but still recognizably for the $2g_{7/2}$ orbital. In this respect, the simple shell model potential, central field plus spin-orbit term, clearly predominates over the smearing effect of the residual interaction.

At this point though, it is not obvious what form the residual interaction should take. The weak-coupling prediction of Rao (RA 71), already illustrated in Figure II-6, does not at all resemble the actual spectrum shown in Figure V-10: The success that this simple model (MR 69)
has enjoyed in $^{207}\text{Pb}$ predicting the spectrum is not repeated in $^{205}\text{Pb}$.

For example, in the particle-vibration picture, the multiplets are always displaced upward in energy from the single-particle energy which is being coupled. Accordingly, the lowest-lying particle state (here the $2g_{9/2}$) would only be weakly fragmented.

This is what experiments show for the $2g_{9/2}$ states in $^{207}\text{Pb}$ and $^{205}\text{Pb}$. In contradiction, the experimental spectrum of $^{205}\text{Pb}$ definitely has the $2g_{9/2}$ strength split among three closely-lying levels. In one sense, the model can be recovered by allowing neutrons in the $N = 5$ major shell to couple as particle (holes) with the excited core states of $^{204}\text{Pb}$($^{206}\text{Pb}$). In such a manner it would be possible to split the lowest-lying particle state. For example, by coupling the ground state $2f_{5/2}$ neutron to the excited $3^-$ core at 2.62 MeV, a multiplet is generated with a zero order energy very near the observed triplet. Within that multiplet there is a $9/2^+$ member which by admixing with the single-particle $2g_{9/2}$ state, creates a $9/2^+$ doublet. This interpretation receives strong support from the $^{205}\text{Bi}$ decay results. There the 2.607 MeV state (#2 of Table V-4) is seen to decay via an E1 transition with a .68 branching ratio to the .703 MeV $7/2^-$ level, which as section B has indicated is believed to be based mainly on the $2f_{5/2}$ neutron coupled to the first $2^+$ state of the core. The appealing view is that the $3^-$ core part of the 2.607 level is simply undergoing an E1 transition of the $2^+$ core and then that core as was seen before overwhelmingly decays to the ground state via an E2 transition. During these decays the $2f_{5/2}$ neutron plays the role of a bystander. It is noted that this scheme will only induce a doublet of $9/2^+$ states. A third $9/2^+$ state can, with more imagination, be engendered by coupling the $1i_{13/2}$ particle at 1.014 MeV to the first $4^+$ excited core at 1.0 MeV.
of excitation. An objection to this idea is that this particular core is only weakly collective (ST 69) and that the multiplet comes at the wrong zero order energy. The reason the scheme is proposed here is that the $^{205}\text{Bi}$ decay work shows very enhanced E2 rates from the 2.565 and 2.607 MeV states to the 1.014 MeV $13/2^+$ state. In fact the branching ratios, shown in Figure V-11 far exceed the single particle estimate relative to the E1 ratios ($\lambda(E2)_{s.p.} \approx 10^{-3} \lambda(E1)_{s.p.}$ at these energies). Thus the schematic diagram of Figure V-11 shows a very admixed triplet for the $2g_9/2$ splitting for which the spectroscopic factors deduced here are just the squares of the non-excited core amplitudes. No decays are shown out of the 2.70 MeV $9/2^+$ state since that energy cannot be reached from $^{205}\text{Bi}$. At this stage, it would be very interesting to calculate the splitting and decay rates of these states based on such a description. This would be the first step in trying to understand the fragmentation of the single particle states in $^{205}\text{Pb}$.

Even without a detailed knowledge of the origins of the splittings, very interesting trends emerge when the fragmentations in $^{205}\text{Pb}$ and $^{207}\text{Pb}$ are compared. In $^{207}\text{Pb}$ at least 50% of the expected shell-model strength is to be found in a single state for the $2g_9/2$, $3d_{5/2}$, $4s_{1/2}$, and the $3d_{3/2}$ shell model orbits. No such concentration of strength exists in $^{205}\text{Pb}$ but the least fragmented orbitals in $^{207}\text{Pb}$ (the $2g_9/2$ and the $4s_{1/2}$) are the same as those in $^{205}\text{Pb}$. Likewise, the most fragmented orbit in $^{207}\text{Pb}$, the $2g_{7/2}$, also displays the most pronounced smearing in $^{205}\text{Pb}$.

In what may be the most significant trend of all, the centroid binding energies for $^{209}\text{Pb}$, $^{207}\text{Pb}$, and $^{205}\text{Pb}$ appear to display consistent if puzzling systematics. The $4s_{1/2}$ and $3d_{3/2}$ orbitals show a marked increase in binding energy in going from $^{209}\text{Pb}$ to $^{205}\text{Pb}$. On the other hand, the
Evidence for an extended particle-vibration model to describe the splitting of the $2g_{9/2}$ orbit in $^{205}$Pb. The remaining de-excitation out of the 2.565 and 2.607 MeV states goes to a number of omitted levels. Moreover, there are weak E3 branches directly to the ground state whose mere existence argues for collectivity in the $9/2^+$ states.
PROPOSED CONFIGURATIONS

\[ E_x \]

\[ J^\pi \]

\[ (\sqrt{0.36} \left| ^{204}\text{Pb}_{g.s.} \otimes 2g_{9/2} \right\rangle_{9/2^+} + \alpha_1 \left| ^{204}\text{Pb}_{2.62} \otimes 2f_{5/2} \right\rangle_{9/2^+} \]

\[ + \alpha_2 \left| ^{204}\text{Pb}_{1.27} \otimes 1l_{13/2} \right\rangle_{9/2^+} \]

\[ (\sqrt{0.18} \left| ^{204}\text{Pb}_{g.s.} \otimes 2g_{9/2} \right\rangle_{9/2^+} + \beta_1 \left| ^{204}\text{Pb}_{2.62} \otimes 2f_{5/2} \right\rangle_{9/2^+} \]

\[ + \beta_2 \left| ^{204}\text{Pb}_{1.27} \otimes 1l_{13/2} \right\rangle_{9/2^+} \]

\[ (\sqrt{0.16} \left| ^{204}\text{Pb}_{g.s.} \otimes 2g_{9/2} \right\rangle_{9/2^+} + \gamma_1 \left| ^{204}\text{Pb}_{2.62} \otimes 2f_{5/2} \right\rangle_{9/2^+} \]

\[ + \gamma_2 \left| ^{204}\text{Pb}_{1.27} \otimes 1l_{13/2} \right\rangle_{9/2^+} \]

\[ (\sqrt{0.03} \left| ^{204}\text{Pb}_{g.s.} \otimes 1l_{13/2} \right\rangle_{13/2^+} + \text{other terms} ) \]

\[ (\left| ^{204}\text{Pb}_{0.899} \otimes 2f_{5/2} \right\rangle_{7/2^+} ) \]

\[ (\sqrt{0.26} \left| ^{204}\text{Pb}_{g.s.} \otimes 2f_{5/2} \right\rangle_{5/2^-} + \text{other terms} ) \]
### Table V-6. Binding Energies of the Single Particle States in the Odd Pb Nuclei

<table>
<thead>
<tr>
<th>nt,j</th>
<th>$^{209}\text{Pb}$</th>
<th>$^{207}\text{Pb}$</th>
<th>$^{205}\text{Pb}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2g_{9/2}$</td>
<td>-3.944 MeV</td>
<td>-3.978 MeV</td>
<td>-4.038 MeV</td>
</tr>
<tr>
<td>$1i_{11/2}$</td>
<td>-3.165</td>
<td>-3.223$^a$</td>
<td>-3.242$^a$</td>
</tr>
<tr>
<td>$1j_{15/2}$</td>
<td>-2.502</td>
<td>-2.618</td>
<td>-2.733$^a$</td>
</tr>
<tr>
<td>$3d_{5/2}$</td>
<td>-2.379</td>
<td>-2.431</td>
<td>-2.572</td>
</tr>
<tr>
<td>$4s_{1/2}$</td>
<td>-1.911</td>
<td>-2.085</td>
<td>-2.193</td>
</tr>
<tr>
<td>$2g_{7/2}$</td>
<td>-1.452</td>
<td>-1.466$^{a,b}$</td>
<td>-1.443</td>
</tr>
<tr>
<td>$3d_{3/2}$</td>
<td>-1.407</td>
<td>-1.526</td>
<td>-1.675</td>
</tr>
</tbody>
</table>

$^a$) More than half the expected single particle strength is missing.

$^b$) The difficulties experienced by Moyer et al. (MC70) in distinguishing $g_{7/2}$ and $d_{3/2}$ transfers make this value questionable. Since there is more unassigned $2g_{7/2}$ strength than missing $3d_{3/2}$ strength, the centroid energy quoted here assumes two other states in $^{207}\text{Pb}$ are actually $2g_{7/2}$ in addition to the only definitely assigned $7/2$ state at $E_B = 1.611$ MeV.
$2g_{7/2}$ orbital remains at about the same binding energy in these nuclei and may even become less bound in $^{205}$Pb since there is probably some unreported excitation above 5.62 MeV of excitation energy. The residual interaction seems to be much more effective in binding the $4s_{1/2}$ and $3d_{3/2}$ fragments while at the same time most severely splitting the $2g_{7/2}$ states, repelling them to slightly higher excitation energies. Naively, one might think that as particles are removed from the $^{208}$Pb core, the single particle states would become less bound, there being fewer nearby nucleons on the Fermi surface. This expectation is not borne out by the experimental values as given in Table V-6. In fact when viewed as a function of the orbital angular momentum quantum number, the binding energy trends listed in the table go counter to what should be the case, assuming that $^{204}$Pb was becoming deformed. It is well known that in the deformed region a term $Dl^2$ is introduced into the Nilsson potential in order to bind the higher-angular-momentum states at the observed energies. Such a term might explain the increased binding of the $1j_{15/2}$ and the $1i_{11/2}$ levels and the lack of a great change in the $2g_{7/2}$ and $2g_{9/2}$ suborbitals. Such an explanation cannot be a complete answer however, since it is the $4s_{1/2}$ centroid, that with zero angular momentum, which displays the greatest shift in binding energy. In fact, the binding energy question is an enigma when only the $^{208}$Pb core by itself is considered. Batty (BA 70) has made a survey of the parameters used in a Woods-Saxon well in order to reproduce the observed energies of the shell-model states in the odd nuclei immediately adjacent to $^{208}$Pb. While both the proton-hole and particle-state energies may be generated by a single set of parameters, there is as yet no single potential which will simultaneously predict the binding energies of both the particle ($N > 126$) and the hole ($N < 126$)
states. Zaidi et al. (ZD 67) have devised a parameter set which gives good agreement with the observed positions of the single particle states in $^{209}$Pb (with some modification for the non-pure $1j_{15/2}$ fragment). When used in a standard DWUCK calculation for the $^{208}$Pb(t,d) reaction (analogous to the (d,p) reaction), consistent spectroscopic factors can be extracted from the experimental angular distributions. On the other hand if the potential is used to calculate the neutron hole energies, it fails badly in that it gives too low a $l_{13/2}$ position. Moreover, when this set is used to calculate cross sections in the $^{208}$Pb(d,p) experiment an 20.0 MeV, the resulting spectroscopic factors for the single-particle states of $^{209}$Pb are at least 20% too high. This disagreement with the (t,d) analysis, wherein the derived spectroscopic factors are more reasonable, is not inexplicable. As a test of the form factor, the (d,p) reaction is superior because the proton distorted wave is much less absorbed at the nuclear surface than is the triton distorted wave. Hence the (d,p) reaction is a much more sensitive test of the nuclear interior than is the (t,d) reaction.*

A final comment on the particle-state spectrum has to do with the total spectroscopic sums given in Table V-5. In Chapter Two it was stated that in the BCS calculations for $^{204}$Pb it became necessary to introduce population of the particle-state orbits via 2p-6h correlations in order to obtain the correct ground state energy for $^{204}$Pb. If such correlations do exist (more correctly if there is more correlation in the $^{204}$Pb ground state than there is in $^{208}$Pb) then the total spectroscopic strength for the single particle states will be less than unity. No claim is made here that all the fragments in $^{205}$Pb have been identified. There could well be states

* For the same reason, the calculated (t,d) angular distributions are not especially sensitive to the use of the deuteron break-up or the more conventional deuteron optical potential.
between 5.7 and 6.7 MeV which do not rise significantly above the background level. In this respect, the background has about 3.0 mb/sr. of differential cross section in this energy region at 13.0 MeV incident energy at an observation angle of 65°. At the same angle there is about 4.0 mb/sr. of cross section missing for the single particle orbits of 205Pb. While some particle states could be buried under the background, it seems very unlikely that the outstanding spectroscopic strength can be accounted for in this excitation region. If it were, that would mean that for each particle orbit about 15% of the spectroscopic strength was displaced quite discontinuously away from the observed centroid. More likely, only a very few per cent of the unaccounted strength exists in the region above 5.75 MeV of excitation.* Roughly 10% of the total spectroscopic strength for all the single particle orbits. Whether such a number is indicative of the increased correlation in the 204Pb ground state is an open question. For example, there might be subtle differences of this order of magnitude in the 205Pb bound-state form factors as compared to those in 209Pb. At best the figure 10% probably represents a rough estimate of the amount of increased correlation to be expected in 204Pb.

This concludes the discussion of the results of the particle states' analysis. One should be careful not to lose sight of the unity displayed by the entire (d,p) spectrum of 205Pb. Both the hole and the particle regions are manifestations of the same residual interaction. To this end, Figure V-12 attempts to measure the dispersion of the observed spectroscopic strength in the complete spectrum. In a coarse fashion, this figure

* Strictly speaking, one should talk about differences in the background levels of the 205Pb and the 209Pb runs. That is the subtracted background in the 205Pb analysis should not be viewed as a source of missing strength since the analysis of the 209Pb groups (at the same plate distances) also had a background.
Two parameter condensation of the \((d,p)\) excitation spectrum of \(^{205}\text{Pb}\).
TWO PARAMETER DESCRIPTION OF
THE (d,p) SPECTRUM OF $^{205}$Pb

EXCITATION ENERGY (MeV)

$2g_{7/2}$ \[ \Gamma_{2/3}^{(2g_{7/2})} = 0.26 \text{ MeV} \]

$3d_{3/2}$ \[ \Gamma_{2/3}^{(3d_{3/2})} = 0.30 \text{ MeV} \]

$4s_{1/2}$ \[ \Gamma_{2/3}^{(4s_{1/2})} = 0.08 \text{ MeV} \]

$3d_{5/2}$ \[ \Gamma_{2/3}^{(3d_{5/2})} = 0.53 \text{ MeV} \]

$2g_{9/2}$ \[ \Gamma_{2/3}^{(2g_{9/2})} = 0.10 \text{ MeV} \]

$2f_{7/2}$ \[ \Gamma_{2/3}^{(2f_{7/2})} = 0.0 \text{ MeV} \]

$1i_{13/2}$ \[ \Gamma_{2/3}^{(1i_{13/2})} = 0.0 \text{ MeV} \]

$3p_{3/2}$ \[ \Gamma_{2/3}^{(3p_{3/2})} = 0.0 \text{ MeV} \]

$3p_{1/2}$ \[ \Gamma_{2/3}^{(3p_{1/2})} = 0.0 \text{ MeV} \]

$2f_{5/2}$ \[ \Gamma_{2/3}^{(2f_{5/2})} = 0.0 \text{ MeV} \]
describes the information contained in Tables V-2 and V-4 and in Figure V-10 according to two parameters for each subshell: \( N_{2/3} \) and \( \Gamma_{2/3} \). The number \( N_{2/3} \) is the minimum number of states which take up \( 2/3 \) of the observed spectroscopic strength and \( \Gamma_{2/3} \) is the width in MeV which is spanned by the states making up \( N_{2/3} \). As before, the arrows denote the weighted centroids of the different orbitals. This figure makes clear that the relative simplicity of the particle region should be no surprise in view of uncomplicated fragmentation of the hole states. Accordingly, the apparent shell-model ancestry of the entire spectrum should be a guide to its understanding.
VI. FINAL SUMMARY

The present study has analyzed the (d,p) spectrum of \(^{205}\text{Pb}\) from 0.0 to 5.62 MeV of excitation. The analysis confirms that in describing the gross features of the spectrum, the simple shell model works extremely well. The region up to the first expected particle state (the \(2g_9/2\)) is only sparsely populated with the major cross section strength residing as expected in the first three levels. Above the particle state threshold energy, the spectrum becomes much more dense and complicated. Nonetheless, the distribution of spectroscopic strength for the particle orbits clearly confirms that the bare shell potential, central field plus spin-orbit term, has not been overwhelmed by the residual interaction.

When the spectrum is viewed in finer detail, however, the presence of the residual interaction is manifest. Much theoretical work has been performed for the low-lying spectrum of \(^{205}\text{Pb}\). Having provided an extremely sensitive gauge of the spectroscopic strength distribution, the present work largely contradicts one (Miranda) use of the phenomenological interaction for the hole-state spectrum. Another approach, the pairing-plus-realistic interaction of Harvey and Clement seems to do a much better job of predicting the observed distribution although more detailed results would be desired. The situation for the positive parity spectrum is somewhat worse. Insofar as the fragmentation of the \(2g_9/2\) orbit is concerned, the usual intermediate coupling model as applied by Rao fails to account for the observed splitting. We propose an extension of the model to include the coupling of neutrons in the hole region to excited cores as for example

\[ |^{204}\text{Pb}^{3-}_{2.62 \text{ MeV}} \otimes 2f_{5/2} > g/2^+ \]
and

\[ ^{204}_{1.27}p_1^{4+} \times ^{11}_{13/2}i > 9/2^+ \]

Such an extension is interesting in the light of the decay modes of the two smaller \(2g_{9/2}\) fragments. For the remainder of the positive parity orbits, however, it is more doubtful that the simple model will be viable. In any event, the exacting comparison of the \((d,p)\) particle state spectrum at two incident energies has yielded the great majority of expected spectroscopic amplitude for the five lowest positive parity states \((2g_{9/2}, 3d_{5/2}, 4s_{1/2}, 2g_{7/2}, \text{and } 3d_{3/2})\) as well as traces of the \(1i_{15/2}\) and \(1i_{11/2}\) orbits. This has enabled centroid energies to be determined with added confidence in \(205\)Pb. When these are compared to those of \(207\)Pb and \(209\)Pb, consistent but inexplicable binding energy trends are observed as a function of angular momenta in these odd nuclei. That is, we are unable to account for the 300 kev increase in binding energy of the \(4s_{1/2}\) and \(3d_{3/2}\) centroids. This could well be connected with the general binding energy problem in the lead region.

Although primarily intended as a study of nuclear structure, this work has examined and contributed to a further understanding of the \((d,p)\) reaction especially as it has been utilized in the lead nuclear structure investigations. After choosing more physically correct parameters for the participating distorted waves, very uniform results appear from the analysis of a series of \(208\)Pb\((d,p)\) between 17.0 and 24.8 MeV deuteron energies. It is also suggested here that, should the outgoing proton energy correspond to an analog resonance of the elastic channel, the simple reaction amplitude generally used may give misleading results.
Finally, very peculiar stripping patterns are seen in the population of the fragmented 2f⁷/₂ orbit, whereas the more concentrated 2f⁵/₂ orbit (both \( l_n = 3 \)) shows no such difficulty. The idea is advanced that multi-step reaction processes may be involved in the excitation of 2f⁷/₂ states in the lead nuclei by (d,p) stripping.

There are several courses open for future study of this nucleus. Most interesting would be a \(^{204}\text{Pb}(d,p\gamma)\) correlation experiment, the \( \gamma \) ray being in coincidence with the excitation of the three strongest 2g⁹/₂ fragments. The gamma decay of the 2.708 9/2⁺ fragment to the .701 MeV 7/2⁻ collective level would give added confirmation to the particle-core scheme of Figure V-11. Also under discussion is the \(^{204}\text{Pb}(d^+,p)\) experiment for which the polarized beam can make a better discrimination between 3d 3/2 and 3d⁵/₂ states as well as distinguish different angular momentum transfers in the 2g⁷/₂ and 3d³/₂ region of excitation of \(^{205}\text{Pb}\). The \((\alpha, ^{3}\text{He})\) reaction which preferentially populates higher-spin transfers would have a similar advantage for those states, as well as possibly locating the missing \( 1j_{11/2} \) and \( 1j_{15/2} \) spectroscopic amplitude. Of course, the main theoretical task now would be to come up with a residual interaction which is strong enough to split the shell-model positive-parity orbits in place but not so strong as to smear the distribution over a wide range of excitation energy. Whether that can be done and still move the centroid energies to where they are experimentally observed remains to be seen.
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