ABSTRACT

The Observation of the 3/2+ State of $^{19}$F at $E_x=7101$ keV and its Relation to Explosive Stellar Hydrogen Burning

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In extremely hot and dense astrophysical environments, such as in novae events, the energy generation mechanism may shift from cyclic processes involving hydrogen fusion reactions on only low mass ($A<19$) nuclei [the HotCNO cycles], to a violent thermonuclear runaway producing heavier mass nuclei up to $^{68}$Se and higher [the rp-process]. This shift will determine the power output and the timescale of the explosive event and is, thus, clearly important to understand. One of the pathways for this transition to occur is via the $^{18}$F(p,$\gamma$)$^{19}$Ne reaction which channels material away from the HotCNO cycles to the rp-process. This reaction competes with the $^{18}$F(p,α)$^{15}$O reaction which returns material back to the HotCNO cycles. To determine the temperatures and densities under which such a transition may occur one must know the locations, widths, and spin-parities of resonant states in the compound nucleus $^{19}$Ne just above the $^{18}$F + α threshold. The purpose of this work was to check on the existence of the astrophysically important 3/2+ resonant state recently reported in $^{19}$Ne at $E_x\sim 7.07$ MeV, by searching for its isospin mirror state in $^{19}$F.

This mirror state was, indeed, observed via the $^{15}$N(α,γ)$^{19}$F reaction to lie at $E_x=7101 \pm 1$ keV, with alpha width, $\Gamma_{\alpha} = 28 \pm 1.1$ keV, γ-width, $\Gamma_{\gamma} = 0.38 \pm 0.06$ eV, and resonance strength, $\omega\gamma = 0.77 \pm 0.11$ eV. Under the assumption of equivalent alpha structure of the mirror states, the alpha width of the $^{19}$Ne state was determined to be $\sim 30$ keV. This is in resonable agreement with two previous studies, but in disagreement, by a factor of $\sim 3$, with one.

The latest information indicates that the $^{18}$F(p,γ)$^{19}$Ne reaction rate under astrophysical conditions is too low for it to be an important pathway for breakout from the HotCNO cycles to the rp-process.
The Observation of the $3/2^+$ State of $^{19}$F at $E_x=7101$ keV and its Relation to Explosive Stellar Hydrogen Burning

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Chapter 1

Introduction

1.1 General Introduction

The urge to make sense of our physical surroundings is deep-rooted in the human psyche and has motivated generations of scientists and philosophers. Among the most fundamental questions raised is that of the origin of the elements: where did all rich variety of nuclei come from? Only in this century, beginning with the landmark paper of Eddington [Edd20] in which he proposed that nuclear reactions powered the stars, has real headway been made in understanding the answers to this question of nucleosynthesis.
As suggested by Burbidge, Burbidge, Fowler and Hoyle and, independently, by Cameron in 1957 [B2FH57, Cam58], these same nuclear reactions are solely responsible for synthesizing all the elements heavier than hydrogen and helium in the cores of stars. The lightest elements, hydrogen, helium and trace quantities of lithium condensed out immediately following the Big-Bang [Hoy64, Pee66, Wag67].

The astrophysical sites where nucleosynthesis occurs may be divided into two categories: quiescent and explosive. An obvious example of a quiescent site is the steady-state burning of our sun where it fuses hydrogen to helium; a nova or supernova being typical explosive events. While both types of astrophysical sites are important for manufacturing the elements, the explosive events are especially significant since they also spew the newly created elements back into the interstellar medium from where this material will be included in future generations of stars and planets, as described below. These ejected elements are crucially important from a human perspective as well, since they end up in such places as the earth and in our bones and blood. It is important to note that a quiescently burning star may evolve over the course of a few million or billion years to become the site of an explosive event. I discuss both types of astrophysical sites below.

1.2 Quiescent Hydrogen Burning

Quiescent hydrogen burning occurs when a star produces energy by hydrogen fusion reactions in a quasi-static equilibrium. This comes about when a protostellar nebula which typically consists mostly of hydrogen gas contracts converting its gravitational energy to thermal energy. When the temperature of the core reaches $T_6 \sim 10$ (where $T_6$ is the temperature in $10^6 K$) thermonuclear reactions between the hydrogen nuclei (protons), which are the most abundant and have the lowest Coulomb barriers, can begin. These reactions produce a prodigious amount of energy which halts the gravitational infall and allows the star to shine in equilibrium for many millions or billions of years depending on the mass of the star.
At solar temperatures, $T_6 \approx 16$, and densities, $\rho \sim 150$ g/cm$^3$ the dominant energy production mechanism is the p-p chain [Wei37, Bet38, Wei38, Bet39] shown in Fig 1.1, where for any branch the net result is $4p \rightarrow ^4\text{He} + 2e^+ + 2\nu + 26.731$ MeV:

Fig 1.1: The p-p chain. These series of reactions are responsible for the bulk of energy generation in stars with temperatures $T_6 < -18$. Note that all three chains end at $^4\text{He}$. 

\[ 
\begin{align*}
\text{p}(p,e^+\nu)d(p,\gamma)^3\text{He} & \xrightarrow{86\%} ^3\text{He}(^3\text{He,2p})^4\text{He} \\
^3\text{He}(\alpha,\gamma)^7\text{Be} & \xrightarrow{14\%} ^7\text{Be}(e^-,\nu)^7\text{Li}(p,\alpha)^4\text{He} \\
\text{p}(p,\gamma)^7\text{Be} & \xrightarrow{0.02\%} ^7\text{Be}(p,\gamma)^8\text{Be}(e^-\nu)^8\text{Be}^*(\alpha)^4\text{He} \\
\end{align*} \]
At slightly higher temperatures, $20 < T_6 < 150$, which could occur in the cores of somewhat more massive stars, $M > 2 M_{\text{Sun}}$, the dominant energy production occurs via the CNO cycle [Wei37, Wei38, Bet39, Ibe65] as shown in Fig 1.2:

$$^{12}\text{C}(p, \gamma)^{13}\text{N}(e^+ + \nu)^{13}\text{C}(p, \gamma)^{14}\text{N}(p, \gamma)^{15}\text{O}(e^+ + \nu)^{15}\text{N}(p, \alpha)^{12}\text{C}$$

The overall result is still: $4p \rightarrow ^4\text{He} + 2e^+ + 2\nu + 26.731 \text{ MeV}$, the heavier elements acting only as catalysts.

Although some unsolved issues, such as the solar neutrino problem [Bah89] persist, the quasi-static process by which such stars fuse hydrogen to helium is thought to be well understood [Par86]. Note that at least one species of the seed nuclei ($^{12}\text{C}, ^{13}\text{C}, ^{14}\text{N}$ or $^{15}\text{N}$) must have been present, even in a small concentration, in the proto-stellar material for the CNO cycle to operate. This seed material must have been injected into the interstellar medium from some previous explosive event.
1.3 Explosive Hydrogen Burning

In very hot and dense astrophysical environments ($T_9=0.1-1.0$ and $\rho > 10^4$ g/cm$^3$) such as those in novae, x-ray bursts and supermassive stars, hydrogen burning may occur explosively. This means that the enormous power output from the relevant nuclear reactions disrupts the quasi-static equilibrium and material may be ejected into interstellar space in a violent explosion. (In the case of supermassive stars the prodigious power output from the central region of the star causes the weakly bound outer layers to be ejected.) For nuclei in the mass range $12<A<19$, the dominant reaction pathways are the HotCNO-I & -II cycles described below; while for $A>19$, a thermonuclear runaway known as the rapid proton capture (rp-) process [Fig 1.3] occurs, synthesizing elements up to the Fe region and producing energy at a rate $\sim 100$ times greater than the HotCNO cycles [Wal81]. The rp-process is an irreversible process in that progressively higher and higher mass nuclei are synthesized with no recycling back to the low mass ($A<19$) region. At the interface between these two mass regions lies the “gateway” nucleus $^{19}\text{Ne}$ ($t_{1/2}=17.4$ sec), which is the starting point of the rp-process via the proton-capture reaction $^{19}\text{Ne}(p,\gamma)^{20}\text{Na}$. We discuss first the HotCNO cycles followed by the “breakout” process via $^{19}\text{Ne}$ into the rp-process:

At temperatures $T_9=0.1-0.2$ and $\rho > 10^4$ g/cm$^3$ the proton capture rate on the unstable $^{13}\text{N}$ will outpace its positron decay rate ($t_{1/2}=9.96$ min.), and the CNO cycle will be diverted into the dominant HotCNO-I cycle [Cau62, Hoy65, Cau77, Wie82] as shown in Fig 1.3:

$$^{12}\text{C}(p,\gamma)^{13}\text{N}(p,\gamma)^{14}\text{O}(e^+\nu)^{14}\text{N}(p,\gamma)^{15}\text{O}(e^+\nu)^{15}\text{N}(p,\alpha)^{12}\text{C}$$
At somewhat higher temperatures, $T_9=0.2-1.0$, the $^{14}\text{O}(\alpha,p)^{17}\text{F}$ reaction occurs more rapidly than the $^{14}\text{O}(e^+\nu)^{14}\text{N}$ decay ($t_{1/2}=70.5$ s), and the dominant reaction cycle shifts to the HotCNO-II cycle, which further increases the power output and creates a path to higher mass nuclei as shown in Fig 1.3:

$$^{12}\text{C}(p,\gamma)^{13}\text{N}(p,\gamma)^{14}\text{O}(\alpha,p)^{17}\text{F}(p,\gamma)^{18}\text{Ne}(e^+\nu)^{18}\text{F}(p,\alpha)^{15}\text{O}(e^+\nu)^{15}\text{N}(p,\alpha)^{12}\text{C}$$

Fig 1.3: The HotCNO-I & -II cycles dominate the energy production in astrophysical sites with low mass nuclei, $A<19$, and temperatures $0.1<T_9<1.0$. Also shown is the rp-process which is an irreversible thermonuclear runaway process which operates in the same temperature range for nuclei $A>19$ and produces elements up to the Fe peak. The power output from the rp-process is about 100 times that of the HotCNO-II cycle [Wal81]. The nucleus $^{19}\text{Ne}$ is critically important since it lies at the "gateway" between the HotCNO cycles and the rp-process. A possible breakout reaction $^{18}\text{F}(p,\gamma)^{19}\text{Ne}$ is shown. It is important to know the branching ratio $^{18}\text{F}(p,\gamma)^{19}\text{Ne}$ vs. $^{18}\text{F}(p,\alpha)^{15}\text{O}$ to understand the probability of escape to the rp-process vs. re-cycling in the HotCNO cycles.
To close this cycle it is required that the rate of $^{18}\text{F}(p,\alpha)^{15}\text{O}$ be greater than the competing $^{18}\text{F}(p,\gamma)^{19}\text{Ne}$ reaction since the latter reaction will produce the "gateway" nucleus $^{19}\text{Ne}$ causing an irreversible "breakout" from the HotCNO-II cycle into the rp-process [Wal81]. (An alternate breakout reaction is $^{15}\text{O}(\alpha,\gamma)^{19}\text{Ne}$; its rate is currently uncertain by an order of magnitude [Oli97]).

Since the HotCNO-II cycle and the rp-process produce very different mass nuclei and since their energy production rates are so vastly different, it is clearly important to understand the relative rates of the breakout versus re-cycling, $^{18}\text{F}(p,\gamma)^{19}\text{Ne}$ vs. $^{18}\text{F}(p,\alpha)^{15}\text{O}$, reactions. The aim of this thesis is to clarify the spin-parity assignment of what is thought to be the dominant $^{18}\text{F} + p$ resonant state [Utk98] which is crucial for determining this reaction branching. However, before discussing the work done on determining these relative rates I introduce some general formalism particular to stellar reaction rate theory.

1.4 Stellar Reaction Rate Formalism

The ultimate aim of nearly all nuclear astrophysics experiments is the determination of stellar reaction rates. Since stellar conditions differ markedly from the neat and controlled conditions in the nuclear laboratory, we describe the framework for translating the laboratory results into astrophysically meaningful quantities.

Consider two different species of nuclei, a and b with number densities $N_a$ and $N_b$ in a stellar interior; assuming their relative velocity is v, the rate at which the nuclear reactions can proceed between them is given by [eg: Rol88]:

$$r_{ab} = N_a N_b v \sigma(v)$$  \hspace{1cm} (1.1)

where $\sigma(v)$ is the velocity dependent cross section for the reaction of interest to occur. However, the particles in a stellar environment do not have a single velocity but a velocity distribution $\phi(v)$ characteristic of the temperature, T, of the plasma.
We thus modify eq 1.1 to reflect this:

\[
\rho_{ab} = N_a N_b \int_0^\infty \phi(v) v \sigma(v) \, dv = N_a N_b \langle \sigma \, v \rangle .
\]  
(1.2)

where the quantity \( \langle \sigma \, v \rangle \) is the average reaction rate per particle pair. The velocity distribution for either species can be well described by a Maxwell-Boltzmann function:

\[
\phi(v_i) = 4\pi v_i^2 \left( \frac{m_i}{2\pi kT} \right)^{3/2} e^{-\frac{m_i v_i^2}{2kT}} \quad i = a \text{ or } b
\]  
(1.3)

where \( k \) is the Boltzmann constant and \( m_i \) the nuclear mass of either species. It can be shown [Cla83] that the relative velocities also follow a Maxwell-Boltzmann distribution, with the individual masses replaced by the “reduced-mass” \( \mu = m_a m_b / (m_a + m_b) \):

\[
\phi(v) = 4\pi v^2 \left( \frac{\mu}{2\pi kT} \right)^{3/2} e^{-\frac{\mu v^2}{2kT}}
\]  
(1.4)

the reaction rate per particle pair is then given by:

\[
\langle \sigma \, v \rangle = 4\pi \left( \frac{\mu}{2\pi kT} \right)^{3/2} \int_0^\infty v^3 \sigma(v) e^{-\frac{\mu v^2}{2kT}} \, dv
\]  
(1.5)

or in terms of the energy in the center of mass frame, \( E = \mu v^2 / 2 \):

\[
\langle \sigma \, v \rangle = \left( \frac{8}{\pi\mu} \right)^{1/2} \frac{1}{(kT)^{3/2}} \int_0^\infty E \sigma(E) e^{-\frac{E}{kT}} \, dE
\]  
(1.6)

Thus finding the stellar reaction rates boils down to determining the relevant energy dependent reaction cross sections.
1.5 The Gamow Window

As mentioned in section 1.2, when a proto-stellar cloud of (mostly) hydrogen collapses, a temperature of \(10^7\)K is required before nuclear reactions can begin. This high temperature is needed since the protons have positive charge and repel each other through the Coulomb force. This temperature is, however, much lower than the \(6.4 \times 10^9\)K one would \textit{classically} require for two protons to overcome their 550 keV Coulomb barrier and fuse. That nuclear reactions can happen at all at this relatively “cool” temperature is due to the fact that in the Maxwell Boltzmann velocity distribution a small number of particles are present at very high velocities. It is these few particles in the high-velocity tail of the Maxwell-Boltzmann distribution that have sufficient probability of quantum-mechanically tunnelling through the Coulomb barrier.

From eq 1.6 we can see explicitly how the contributions to the integral from the high energies is quenched due to the presence of the exponential term from the Maxwell-Boltzmann distribution. On the other hand, at low energies the tunnelling cross sections are exceedingly small. It is in the middle ground where the product of these two terms contributes the most to the integral and results in a peak in the stellar reaction rate [Fig 1.4]. This energy region is called the Gamow window and is centered on energy \(E_o\) with 1/e width \(\Delta E_o\) [Rol88]:

\[
E_o = 1.22 \left( Z_e Z_b \mu T_b \right)^{1/3} \text{keV} \tag{1.7}
\]

\[
\Delta E_o = 0.749 \left( Z_e Z_b \mu T_b \right)^{1/6} \text{keV} \tag{1.8}
\]

Nuclear fusion reactions in stars take place predominantly in this region, \(E_o \pm \Delta E_o/2\). Thus this is the most important range of energies where nuclear physics information must be obtained. For our reaction of interest \(^{18}\text{F} + p\ (Q=+6411\ \text{keV})\) at nova conditions of \(T_9=0.1-1.0\), the Gamow window lies from \(\sim 100-800\ \text{keV}\) above the threshold, depending on the temperature considered. So we are interested in \(^{19}\text{Ne}\) resonances with excitation energies from about 6500 keV to 7300 keV. Although this energy region in \(^{19}\text{Ne}\) has been studied, there are still a number of states that have yet to be located [Utk98]. (These states
are known to be missing by a simple comparison with the known states in the mirror nucleus $^{19}\text{F}$, as explained in Section 2.3.) The aim of this thesis was to clarify the spin-parity assignment of the $E_x \sim 7.070$ MeV state of $^{19}\text{Ne}$ by locating its mirror state in $^{19}\text{F}$.

![Gamow peak illustration](image)

Fig 1.4: A qualitative illustration of the Gamow peak. This occurs where the product of the Maxwell-Boltzmann distribution and the Coulomb barrier tunneling probability is maximum. Most nuclear fusion reactions between species $a$ and $b$ in a star of central temperature, $T_6$, occur in the Gamow window: $E_o \pm \Delta E_o/2$ where:

$$E_o = 1.22 \left(\frac{Z_a^2 Z_b^2}{\mu} T_6^{1/3}\right) \text{ keV}$$

$$\Delta E_o = 0.749 \left(\frac{Z_a^2 Z_b^2}{\mu} T_6^{1/6}\right) \text{ keV}$$

and the Gamow energy, $E_o = 978.1 Z_a^2 Z_b^2 \mu$ keV.

The Gamow window also defines the range of energies for which nuclear physics information must be obtained in order to determine the stellar reaction rates. Figure taken from [Rol88].
1.6 Resonant Reaction Rates

In the previous section, we have seen how the concept of the Gamow window allows one to narrow down the energy region of interest for a particular fusion reaction in a given astrophysical setting.

In many instances the reaction cross section in this energy window will be dominated by resonances. A resonant fusion reaction is a two-step process: the two initial particles are captured to an excited state of the compound nucleus which then decays by emitting a particle or a gamma ray to the final state. In order to populate the resonance the sum of the Q-value for the reaction and the center of mass energy, E, must equal the excitation energy of the resonant state, \( E_R = E + Q \). The excited state thus formed has a lifetime \( \tau \), before decaying to the final state. Therefore, it must also have some conjugate energy "width" given by the uncertainty principle \( \Gamma = \hbar / \tau \). This total width is the sum of widths, \( \Gamma_i \), to each allowed decay channel. The cross section for a resonance reaction is given by a Breit-Wigner formula [eg: Bla52]:

\[
\sigma(E) = \pi \frac{\hbar^2}{(2J_a + 1)(2J_b + 1)} \frac{\Gamma_x(E) \Gamma_y(E)}{(E - E_R)^2 + (\Gamma / 2)^2}
\]

where:
- \( \hbar \) is the de Broglie wavelength of the incoming particle
- \( J_R, J_a, J_b \) = spins of the resonant state and the entrance channel particles
- \( \Gamma_x, \Gamma_y \) = partial widths of the formation and decay channels respectively

The partial widths in eq 1.9 are, in general, energy dependent. For narrow resonances \( (\Gamma << E_R) \) where the partial widths do not vary significantly over the resonance the
Breit-Wigner cross section may be inserted in eq. 1.6 and integrated to yield:

\[
<\sigma \nu> = \left( \frac{2\pi}{\mu kT} \right)^{3/2} \hbar^2 (\omega \gamma) e^{-\frac{E_\gamma}{kT}}
\]  

(1.10)

where: \( \omega = \frac{(2J_\gamma + 1)}{(2J_\gamma + 1)(2J_\gamma + 1)} \) and \( \gamma = \frac{\Gamma_r \Gamma_\gamma}{\Gamma} \)

and the product \( (\omega \gamma) \) is referred to as the resonance strength. If several narrow resonances exist, their total contribution to the reaction rate will be the sum of the individual rates:

\[
<\sigma \nu>_{\text{tot}} = \sum_i <\sigma \nu>_i
\]  

(1.11)

For broad resonances eq 1.10 is not valid and the energy dependence of the partial widths must be known in order to integrate eq 1.6 directly.

1.7 Calculation of Partial Widths

In case an experimental determination of partial widths is not possible or available, it may be necessary to estimate these quantities analytically.

For gamma decays a very crude estimate of the partial width can be made using the Weisskopf model [Bla52] which yields:

\[
\Gamma_L (E_\gamma) = \alpha_L E_\gamma^{2L+1}
\]  

(1.12)

where \( E_\gamma \) is the gamma-ray energy, \( L \) is the multipolarity of the electromagnetic transition, and \( \alpha_L \) is a constant dependent on \( L \), the nuclear mass \( A \), and the type of transition, electric or magnetic.
For example the lowest multipolarities' partial $\gamma$-widths in units of eV are [Bla52]:

$$\Gamma^{	ext{E1}}(E_{\gamma}) = 6.8 \times 10^{-2} \, A^{2/3} \, E_{\gamma}^{3}$$

$$\Gamma^{	ext{M1}}(E_{\gamma}) = 2.1 \times 10^{-2} \, E_{\gamma}^{3}$$

$$\Gamma^{	ext{E2}}(E_{\gamma}) = 4.9 \times 10^{-8} \, A^{4/3} \, E_{\gamma}^{5}$$

where $E_{\gamma}$ is in units of MeV and $A$ is the atomic number of the compound nucleus. It should be emphasized that the $\gamma$-widths thus obtained are very crude estimates, and may well disagree with the data by factors of $\sim 10^{3}$. The calculation of $\gamma$-widths is discussed in further detail in Section 2.5.

For charged particle decay, the partial width is dependent on the particle's ability to penetrate the Coulomb and angular momentum barriers. If we assume the excited state to consist of the decay particle plus a residual spherically symmetric nucleus, then we can write the wavefunction for the system in spherical coordinates as:

$$\Psi(r, \theta, \phi) = \frac{\chi_{l}(r)}{r} \, Y_{lm}(\theta, \phi)$$

(1.13)

where $\chi_{l}(r)$ is the radial part of the wavefunction and $Y_{lm}$ is the appropriate spherical harmonic. The penetrability of the decay particle is given simply by the ratio of the probability of finding it at infinity to that of finding it at the nuclear surface [eg: Rol88]:

$$P_{l}(\eta, \rho) \equiv \left| \frac{\chi_{l}(r = \infty)}{\chi_{l}(r = R_{n})} \right|^{2}$$

(1.14)

where $\eta = e^{2} z_{1} z_{2}$ and $\rho = \mu v r / \hbar$, with $v$ defined as the relative velocity in the CM system, $v = \sqrt{2E / \mu}$. The rate of decay, $\lambda$, can be written as the probability flux at infinity:

$$\lambda = \lim_{r \to \infty} v \int_{\Omega} |\Psi|^{2} \, r^{2} \, d\Omega = \sqrt{2E / \mu} \left| \chi_{l}(\infty) \right|^{2}$$

(1.15)
Using eq. 1.14 we can write this as:

$$\lambda = \sqrt{\frac{2E}{\mu}} P_i(\eta, \rho) \left| \chi_i(R_n) \right|^2 .$$  \hspace{1cm} (1.16)$$

Assuming a uniform probability density, the probability of finding the decay particle at the nuclear surface is given by the ratio of the volume of a spherical shell at the surface to the total nuclear volume:

$$\left| \chi_i(R_n) \right|^2 dr \equiv \frac{4 \pi R_n^2 \, dr}{\frac{4}{3} \pi R_n^3} \Rightarrow \left| \chi_i(R_n) \right|^2 \equiv \frac{3}{R_n} .$$  \hspace{1cm} (1.17)$$

We use the approximate sign in eq 1.17, since our derivation assumes that the excited state is made up only of the decay particle plus the residual nucleus. Of course, other nuclear configurations are also possible: to take these into account we define the dimensionless reduced width, \( \theta_i^2 \), which provides a measure of the similarity of the real excited compound nucleus to the decay channel:

$$\left| \chi_i(R_n) \right|^2 = \frac{3}{R_n} \theta_i^2 \quad \text{with} \quad \theta_i^2 \leq 1 .$$  \hspace{1cm} (1.18)$$

Finally, since \( \Gamma = \hbar \lambda \), inserting eq 1.18 in eq 1.16 we have:

$$\Gamma_i(E) = \frac{3 \hbar}{R_n} \sqrt{\frac{2E}{\mu}} P_i(\eta, \rho) \theta_i^2 .$$  \hspace{1cm} (1.19)$$

A more rigorous derivation involving the nuclear harmonic oscillator model as opposed to our "uniform" nucleus model, yields nearly the same result [Bla52]:

$$\Gamma_i(E) = \frac{2 \hbar}{R_n} \sqrt{\frac{2E}{\mu}} P_i(\eta, \rho) \theta_i^2 .$$  \hspace{1cm} (1.20)$$
Thus, physically, the charged particle partial width is proportional the product of two probabilities: one is the probability of finding the decay channel configuration in the excited compound nucleus, $\theta^2$, and the other is the probability that this decay will actually occur, given by $P_i(\eta, \rho)$. To calculate the partial widths one must know the penetrabilities as defined in eq 1.14. Since the radial solutions to the Schroedinger equation are the well known regular and irregular Coulomb wavefunctions, $F(\eta, \rho)$ and $G(\eta, \rho)$ [see Appendix], the penetrabilities can be evaluated:

$$P_i(\eta, \rho) = \frac{1}{F_i^2(\eta, \rho) + G_i^2(\eta, \rho)}$$

(1.21)

As the penetrability decreases very rapidly with increasing orbital angular momentum, $l$, only the minimum $l$ partial wave allowed by considerations of angular momentum and parity is used in calculating the reaction rates. Lastly, we have mentioned only the decay channel in this derivation; the results, however, apply exactly to the formation channel as well since the physics is unchanged under time reversal.

Since we now have energy dependent expressions for the partial widths we are able to write the stellar rates for reactions through broad resonances. First, using eq. 1.9, we can parametrize the energy dependence of the cross section as [Rol88]:

$$\sigma(E) = \sigma_R \frac{E_R}{E} \frac{\Gamma_x(E)}{\Gamma_x(E_R)} \frac{\Gamma_y(E)}{\Gamma_y(E_R)} \frac{(\Gamma_R/2)^2}{(E-E_R)^2 + (\Gamma(E)/2)^2}$$

(1.22)

where the resonant cross section, $\sigma_R = \sigma(E=E_R)$, and the resonant full-width, $\Gamma_R = \Gamma(E=E_R)$, are typically known from experiments. We can thus insert eq. 1.22 into eq. 1.6 and evaluate the integral to arrive at the broad resonance contribution to the stellar reaction rate.
1.8 Importance of Spin-Parity in Stellar Reaction Rates

As one of the aims of this dissertation was to clarify the spin-parity assignment of a state in $^{19}$Ne at 7.070 MeV, we now discuss why the spin and parity of resonant states are important in calculating reaction rates. From eq. 1.9 we can see that the resonant reaction cross section is proportional to the product of the formation and decay partial widths. Substituting eq. 1.21 in eq. 1.20, we can write for either partial width:

$$\Gamma_i(E) = \frac{2h}{R_n} \frac{\sqrt{2E/\mu}}{F_i^2(\eta, \rho) + G_i^2(\eta, \rho)}$$  \hspace{1cm} (1.23)$$

Since the sum, $F_i^2(\eta, \rho) + G_i^2(\eta, \rho)$, is strongly $l$-dependent (increasing sharply with increasing $l$'s), and because the $l$ used in the calculation is determined as the minimum allowable for correctly coupling to the resonant state spin-parity, this means that knowledge of the spin-parity of the resonance is critical in calculating the partial widths and thus the stellar reaction rates. For example, comparing the cross section of $^{18}$F(p,α)$^{15}$O occurring through a 3/2$^+$ versus a 7/2$^+$ state in the compound nucleus $^{19}$Ne$^*$:

\[ \begin{array}{cccc}
\text{I.} & J^\pi: & 1^+ & 1/2^+ & l_{in}=0 & 3/2^+ & l_{out}=1 & 1/2^- & 0 \\
\text{II.} & J^\pi: & 1^+ & 1/2^+ & l_{in}=2 & 7/2^+ & l_{out}=3 & 1/2^- & 0 \\
\end{array} \]

Considering only the difference in penetrabilities for the two cases we find that [see Appendix]:

$$\frac{\sigma_{3/2^-}}{\sigma_{7/2^-}} = \frac{F_2^2(\eta_{in}, \rho_{in}) + G_2^2(\eta_{in}, \rho_{in})}{F_0^2(\eta_{in}, \rho_{in}) + G_0^2(\eta_{in}, \rho_{in})} \times \frac{F_3^2(\eta_{out}, \rho_{out}) + G_3^2(\eta_{out}, \rho_{out})}{F_1^2(\eta_{out}, \rho_{out}) + G_1^2(\eta_{out}, \rho_{out})} = 8.2 \times 10^6$$ \hspace{1cm} (1.24)$$

where the subscripts “in” and “out” refer to the incoming and outgoing channels. (Here, $\eta_{in}=1.7$, $\eta_{out}=2.38$ and $\rho_{in}=0.9$, $\rho_{out}=3.85$.) Thus, clearly, it is important to know the spin-parity of the resonant states in the relevant compound nucleus at excitation energies corresponding to the Gamow window.
Chapter 2

Framework of the Experimental Goals

2.1 Introduction

To determine the conditions under which explosive hydrogen burning will shift from the HotCNO-II cycle to thermonuclear runaway in the rp-process, it is important to know the relative rates of the competing reactions, $^{18}\text{F}(p,\alpha)^{15}\text{O}$, and $^{18}\text{F}(p,\gamma)^{19}\text{Ne}$ [see Sect. 1.3]. To determine these rates it is imperative to know the locations, spin-parities, and strengths of the resonances in the compound nucleus $^{19}\text{Ne}$, at excitation energies corresponding to the Gamow window. Recently, two separate direct measurements of $^{19}\text{Ne}$ states via the radioactive ion beam reaction $p(^{18}\text{F},^{15}\text{O})\alpha$ suggested the presence of a
broad $3/2^+$ state at $E_x \sim 7.070$ MeV [Cos95, Reh95]. However, mainly due to the current state of radioactive ion beam technology, both these experiments suffered from low beam currents and, thus, low yields, as well as poor energy resolution due to the necessarily thick targets used. Since a $3/2^+$ state at this energy would be an $l=0$ resonance in $^{18}\text{F} + \text{p}$, and would dominate the explosive stellar reaction rates above $T_\theta \sim 0.4$ [Utk98], it was important to be absolutely certain of its existence. If indeed this state did exist in $^{19}\text{Ne}$, then it must also exist in its isospin-mirror nucleus, $^{19}\text{F}$ [see Sect. 2.3]. The subject of this thesis is a study of the properties of this $3/2^+$ state in $^{19}\text{F}$ via the reaction $^{15}\text{N}(\alpha,\gamma)^{19}\text{F}$.

In this chapter we give a synopsis of the previous work done on the 7.070 MeV state of $^{19}\text{Ne}$, followed by a discussion of isospin-mirror nuclei, a review of earlier work on $^{19}\text{F}$ in the same energy regime, and, lastly, a discussion of electromagnetic decay theory relevant to our study of $^{15}\text{N}(\alpha,\gamma)^{19}\text{F}$.

2.2 Previous Results on the $\sim 7.070$ MeV State of $^{19}\text{Ne}$

The 7.070 MeV state of $^{19}\text{Ne}$ was first seen using the $^{20}\text{Ne}(^3\text{He},\alpha)^{19}\text{Ne}$ reaction in a systematic study of the $^{19}\text{Ne}$ nuclear structure [Gar70]. The $^{19}\text{F}(^3\text{He},t)^{19}\text{Ne}$ reaction was then used to specifically explore the astrophysically interesting region of $^{19}\text{Ne}$, just above the $^{18}\text{F} + \text{p}$ threshold at 6.411 MeV [Gos73,Kou74]. In order to determine the proton and alpha partial widths of the relevant $^{19}\text{Ne}$ resonances, a more recent experiment used $t-\alpha$ and $t-\text{p}$ coincidences to measure the branching between $^{19}\text{F}(^3\text{He},t\alpha)^{15}\text{O}$ and $^{19}\text{F}(^3\text{He},tp)^{18}\text{F}$ reactions at resonances in this energy region [Utk98]. Because of improved statistics, this experiment was also able to extract the total width of the 7.070 MeV state, $\Gamma_{\text{tot}} = 39 \pm 10$ keV, as well as the partial widths, $\Gamma_\alpha = 25 \pm 6$ keV and $\Gamma_\text{p} = 14.5 \pm 4$ keV.

Two later experiments were carried out to directly probe the resonances in $^{18}\text{F} + \text{p}$ using radioactive ion beams of $^{18}\text{F}$ on polyethelene targets, $(\text{CH}_2)_n$. In one of these experiments the yield of $^{15}\text{O}$ ions was measured [Reh95] and in the other the yields of alphas and protons was determined [Cos95]. Despite the fact that both these experiments suffered
from low yields and large target thicknesses (80-190 keV) as well as poor beam purity, they agreed on a $3/2^+$ assignment of the resonance at $E_x \sim 7.07$ MeV in $^{19}$Ne. However, there was significant discrepancy on the location, and on the total and partial widths of this state as summarised in Table 2.1

<table>
<thead>
<tr>
<th>Study</th>
<th>$E_x$ (keV)</th>
<th>$\Gamma_{tot}$ (keV)</th>
<th>$\Gamma_\alpha$ (keV)</th>
<th>$\Gamma_p$ (keV)</th>
<th>$J^\pi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{19}$F($^3$He,t)$^{19}$Ne [Utk98]</td>
<td>7070 ± 7</td>
<td>39 ± 10</td>
<td>25 ± 7</td>
<td>14 ± 4</td>
<td>$(3/2^+)$</td>
</tr>
<tr>
<td>p($^{18}$F,$^{15}$O)α [Reh95,96,97]</td>
<td>7063 ± 4</td>
<td>13.6 ± 4.6</td>
<td>8.6 ± 2.5</td>
<td>5 ± 1.6</td>
<td>$3/2^+$</td>
</tr>
<tr>
<td>p($^{18}$F,$^{15}$O)α [Cos95]</td>
<td>7049 ± 15</td>
<td>37 ± 5</td>
<td>-18.5</td>
<td>-18.5</td>
<td>$3/2^+$</td>
</tr>
</tbody>
</table>

Table 2.1: The properties of the ~7.07 MeV state of $^{19}$Ne from various studies.

A spin-parity assignment of $3/2^+$ for this state, as suggested by the radioactive beam studies, would be very important since it would make this state an $I=0$ resonance for $^{18}$F + p. That together with its width and location would make it the dominant resonance for the $^{18}$F(p,α)$^{15}$O reaction at temperatures of $T_9 = 0.4$-1.0.

Since the data supporting the conclusions of both [Cos95] and [Reh95] were less than ideal, and because of the overriding importance of this $^{19}$Ne state for understanding HotCNO breakout to the rp-process [see Section 1.3], it was decided to check on its $3/2^+$ assignment by looking for its mirror state in $^{19}$F.
2.3 Isospin Mirror Nuclei

Nuclei that can be formed from each other by completely exchanging protons for neutrons - and vice versa - are called isospin mirrors. Since the Strong force between nucleons is almost charge independent, the energy spectra of isospin mirror nuclei will be essentially the same except for small shifts due to the different Coulomb interactions of protons and neutrons. An exact treatment of the shifts is given in [Ehr51, Tho52]. Typically, these shifts are less than \(-200\) keV for low mass nuclei. The concept of isospin mirrors is, then, a powerful spectroscopic tool since it allows one to deduce spin-parity assignments and other spectroscopic properties for excited states of a given nucleus based on knowledge of its mirror. For example, the astrophysically relevant energy states of the mirror nuclei \(^{19}\)Ne and \(^{19}\)F, as determined prior to this study, are shown in Fig 2.1. (By comparison with \(^{19}\)F, on the basis of isospin symmetry, at least seven states in \(^{19}\)Ne in the energy region shown in Fig 2.1 are still undiscovered.)

The isospin mirror argument can, and often is, pushed beyond simply identifying mirror states. For example, because of missing experimental information in \(^{19}\)Ne it has been customary to adopt the reduced alpha widths of mirror states in \(^{19}\)F, \(\theta_\alpha^2(19\text{Ne}^*) = \theta_\alpha^2(19\text{F}^*)\), as well as their partial \(\gamma\)-widths, \(\Gamma_\gamma(19\text{Ne}^*) = \Gamma_\gamma(19\text{F}^*)\). However, as pointed out by [Oli97], the hypothesis of equal reduced alpha widths presupposes an equivalent alpha structure of the relevant states in both nuclei, which may or may not be the case. The wavefunctions for a pair mirror states of \(^{19}\)F and \(^{19}\)Ne may be written:

\[
\begin{align*}
|^{19}\text{F}^* &> = a_1 |^{18}\text{F} \otimes n > + a_2 |^{15}\text{N} \otimes \alpha > + a_3 |^{12}\text{C} \otimes ^7\text{Li} > + a_4 |^{11}\text{B} \otimes ^8\text{Be} > + \ldots \\
|^{19}\text{Ne}^* &> = b_1 |^{18}\text{F} \otimes p > + b_2 |^{15}\text{O} \otimes \alpha > + b_3 |^{12}\text{C} \otimes ^7\text{Be} > + b_4 |^{11}\text{C} \otimes ^8\text{Be} > + \ldots 
\end{align*}
\]

where the abbreviations \(a_i = \theta_i(19\text{F}^*)\) and \(b_i = \theta_i(19\text{Ne}^*)\) have been used. Thus, only if the alpha structure of the mirror states is about the same, will the assumption of equal reduced alpha widths of the two mirror states be borne out. However, since it is believed that at least for some states in \(^{19}\)F*, the \(^{15}\text{N} \otimes \alpha\) component is only a small admixture to the global wavefunction [Des87, Wie80, Oli97], the equivalence of the reduced alpha widths has to be questioned.
Fig 2.1: The energy spectra of the mirror nuclei $^{19}$F and $^{19}$Ne above the $^{18}$F+p threshold as determined prior to this study. As can be seen, the situation is far from ideal: many mirror identifications have not been made, and many states in $^{19}$Ne have simply not been located. Indicated mirror states are based on the relative populations and angular distributions of excited states in three sets of mirror transfer reactions: $^{20}$Ne(d,$^3$He)$^{19}$F and $^{20}$Ne(d,t)$^{19}$Ne, and $^{16}$O($^6$Li,$^3$He)$^{19}$F and $^{16}$O($^6$Li,t)$^{19}$Ne [solid arrows, Utk98], and $^{12}$C($^6$B,$^3$He)$^{19}$F and $^{12}$C($^6$B,t)$^{19}$Ne [dotted arrow, Che97]. Vertical arrows show the locations of the Gamow window, $E_0 \pm \Delta E_g/2$, for temperatures $T_\psi=0.2$, 0.5, 1.0.

Figure adapted from [Utk98].
For mirrors states, the accuracy of the assumption of equivalence of the partial $\gamma$-widths is dependent on the multipolarity considered: mirror $E1$ transitions will have approximately the same $\Gamma_\gamma$, whereas the $\Gamma_\gamma$ of mirror $M1$ transitions may differ by a factor of $\sim 2$ [War69].

The aim of this study was to check on the existence of the $3/2^+$ state found in $^{19}\text{Ne}$ at $E_x \sim 7.07$ MeV [Cos95, Reh95] by searching for its mirror analog state in $^{19}\text{F}$. Furthermore, under the assumption of equal reduced widths for the mirror states, we hoped to examine the discrepancy in the experimentally determined widths for the $^{19}\text{Ne}$ state.

2.4 Previous Results on the $\sim 7.1$ MeV State of $^{19}\text{F}$

The first suggestion of a $3/2^+$ state in $^{19}\text{F}$ at $E_x \sim 7.1$ MeV came from an $^{15}\text{N}(\alpha,\alpha)^{15}\text{N}$ elastic scattering experiment [Smo61] in which the scattered alpha-particles were detected at six center-of-mass angles that corresponded to zero-crossings of the Legendre polynomials of order 0 through 4. It was argued on the basis of a partial wave analysis that a $7/2^+ - 3/2^+$ doublet at $E_x \sim 7.1$ MeV would give the best agreement to the measured yield of elastically scattered alphas at all angles. Later, Mo and Weller [Mo72] re-analysed this data between $E_x = 7-8$ MeV in $^{19}\text{F}$ and concluded that it was “unnecessary” to invoke the $3/2^+$ state in order to get a reasonable agreement with the measured yields. However, in their analysis they took a broad, $\Gamma_{cm}=65$ keV, state at $E_x=7.35$ MeV to be a $7/2^+$ state, when, in fact, it was later found to be a $1/2^+$ state. Since the the $7/2^+$ state at 7.1 MeV would no longer be in the same partial wave as the nearby broad 7.35 MeV state, this leaves in serious doubt any conclusions from their re-analysis.

Other evidence conflicting with the suggestion that only a $7/2^+$ state exits at $E_x \sim 7.1$ MeV in $^{19}\text{F}$ came from an $^{19}\text{F}(e,e')^{19}\text{F}$, inelastic electron scattering experiment [Bro85] in which it was found that the data at 7.11 MeV was in “disagreement” with the assumption of a lone $7/2^+$ state.
Because of this uncertainty about the existence of the 7/2\textsuperscript{+}-3/2\textsuperscript{+} doublet at \( E_x \approx 7.1 \text{ MeV} \) in \( ^{19}\text{F} \), I re-analysed the original elastic scattering data in a narrow energy segment, \( E_x = 7.01-7.25 \text{ MeV} \), in the framework of a partial wave analysis. The differential cross section in the center-of-mass (CM) frame for the scattering of a spin 0 alpha-particle from a spin 1/2 \( ^{15}\text{N} \) nucleus may be expressed in terms of a non-spin-flip amplitude, \( g \), and spin-flip amplitude, \( h \) [Cri49]:

\[
\frac{d\sigma}{d\omega} = \lambda^2 (|g|^2 + |h|^2) \quad (2.1)
\]

where \( \lambda = \frac{h}{\mu \nu} \), with \( \nu \) defined as the relative velocity of the particles in the CM frame, and \( \mu \) as the reduced mass, \( \frac{m_\text{alpha} m_\text{N}}{m_\text{alpha} + m_\text{N}} \). The amplitudes \( g \) and \( h \) are complex functions expressed in terms of partial waves of angular momentum \( J = l \pm \frac{1}{2} \), where \( l=0,1,2,\ldots,\infty \):

\[
g = -\frac{1}{2} \eta \csc^2(\theta/2) e^{i\eta \ln(\csc^2\theta)} + \frac{1}{2i} \sum_{l=0}^{\infty} P_l(\theta) e^{2i\alpha_l} [(l+1) f_l^+ + l f_l^-] \quad (2.2)
\]

\[
h = -\sin\theta \sum_{l=1}^{\infty} \frac{dP_l(\theta)}{d\cos\theta} e^{2i\alpha_l} [f_l^+ - f_l^-]
\]

where \( \theta \) is the CM scattering angle, \( \eta \equiv \frac{e^2 Z_{\text{alpha}} Z_{\text{N}} Z_{\text{N}}}{\hbar v} \), the \( P_l(\theta) \)'s are the Legendre polynomials of order \( l \), and the \( \alpha_l \) and \( f_l^\pm \) are the Coulomb scattering phase shifts, and the pure nuclear scattering amplitudes, respectively, given by:

\[
\alpha_0 = 0, \quad \alpha_l = 2 \sum_{j=1}^{l} \tan^{-1}(\eta/s) \quad (2.3)
\]

\[
f_l^\pm = e^{2i\delta_l^\pm} - 1 \quad (2.4)
\]

where the \( \delta_l^\pm \) are the pure nuclear scattering phase shifts, extracted from the fit to the data.
In the vicinity of a narrow resonance of energy, $E_r$, total width, $\Gamma$, and elastic alpha partial width, $\Gamma_\alpha$, the $f_{i\pm}^\pm$ of the resonant partial wave may be written:

$$f_{\text{res}} = e^{2i\delta_{\text{res}}} \left[ 1 + \frac{\Gamma_\alpha}{\Gamma} (e^{2i\beta} - 1) \right]$$

(2.5)

where $\delta_{\text{bgd}}$ is the background phase shift in the resonant partial wave and the angle, $\beta \equiv \tan^{-1}\left(\frac{\Gamma_\alpha}{E - E_r}\right)$. In the case of several nearby resonances of different $J^\pi$, the quantities $f_{\text{res}}$ may be calculated independently for each resonant channel. However, in the case of $N$ resonances of the same $J^\pi$, a modification to eq. 2.5 is necessary:

$$f_{\text{res}} = e^{2i\delta_{\text{res}}} \left[ 1 + \sum_{k=1}^{N} \frac{\Gamma_\alpha}{\Gamma} (e^{2i\beta_k} - 1) \right]$$

(2.6)

The $^{15}\text{N}(\alpha,\alpha)^{15}\text{N}$ elastic scattering data of [Smo61] in the energy range $E_\alpha = 3.8-4.1$ MeV ($E_x = 7.01-7.25$ MeV) were re-analysed based on the formulas given above using a modified version of a computer code [Kal96] that considered all partial waves $l \leq 3$.

The fitting routine varied the overall normalization and the resonant and background phase-shifts for fixed input values of $E_{\text{res}}$, $J^\pi$, and, $\Gamma = \Gamma_\alpha$, for each resonance. Three possibilities of the spin of the questionable state at $E_x \approx 7.1$ MeV were explored; in separate fits it was first input as a lone $7/2^+$ state, then as a lone $3/2^+$ state, and finally as a $7/2^+ - 3/2^+$ doublet. In each fit two known broad resonances at, $E_x = 6.989$ MeV ($1/2^-$), and, at $E_x = 7.364$ MeV ($1/2^+$) were also included in the fit with their parameters fixed. The results of the fits are shown in Fig 2.2. As can be seen, the data are consistent with a $7/2^+ - 3/2^+$ doublet at $E_x \approx 7.1$ MeV, as suggested by [Smo61]; although it is not obvious that the doublet fit is significantly better than the $7/2^+$ singlet, especially when one considers the additional free parameters that have been included in the fit by the introduction of the $3/2^+$ state.
Fig 2.2: Fits to the $^{15}$N(α,α)$^{15}$N data of [Smo61] based on a partial wave analysis code that considered all $I \leq 3$. In each case shown, the characteristics of the unknown resonance(s) at $E_\alpha = 7.1$ MeV ($E_\alpha \sim 3.91$ MeV) were input, and the best fit based on these values generated. The input values for $E^*$, $J^*$, and $\Gamma$, and $\Gamma_\alpha$, for each case are:

(I) Single state: $E^* = 7.11$ MeV, $J^* = 7/2^+$, $\Gamma = \Gamma_\alpha = 32$ keV.

(II) Single state: $E^* = 7.11$ MeV, $J^* = 3/2^+$, $\Gamma = \Gamma_\alpha = 32$ keV.

(III) Doublet: $E^* = 7.11$ MeV, $J^* = 7/2^+$, $\Gamma = \Gamma_\alpha = 32$ keV, $E^* = 7.10$ MeV, $J^* = 3/2^+$, $\Gamma = \Gamma_\alpha = 8$ keV.
The $^{19}$F excited states in this region have also been explored by many transfer reactions [eg: $^{18}$O(³He,d)$^{19}$F, Gre70; $^{16}$O(⁷Li,α)$^{19}$F, Tse74; $^{12}$C(⁹Be,d)$^{19}$F, Jar79], and although some of their results were consistent with a 7/2⁺ state at 7.1 MeV [eg: $^{15}$N(⁷Li,t)$^{19}$F, Mid69], none of them could conclusively rule out the existence of a 7/2⁺ - 3/2⁺ doublet since such reactions are known to be selective in their population of the compound nuclear states. The only previous radiative capture reaction carried out in this excitation energy region, $^{15}$N(α,γ)$^{19}$F, did not “clearly identify” any resonances at $E_x$~7.1 MeV [Dix77]. This experiment, however, was carried out using solid Ti$^{15}$N targets with a ~30% Ge(Li) detector. (The percentage is relative to a standard 3x3 inch NaI scintillation detector). We hoped that by studying the same reaction using a windowless gas target and two ~100% High Purity (HP)Ge-detectors in a close geometry, that the extra sensitivity would allow us to conclusively determine whether the 3/2⁺ state exists in this energy region. We will identify and differentiate between the possible 3/2⁺ and 7/2⁺ states by examining their different γ-decay schemes.

2.5 Electromagnetic Transitions in Nuclei

An excited state of spin-parity, $\tilde{J}_a^{*a}$, may decay to a lower energy state of spin-parity, $\tilde{J}_b^{*b}$, by emitting a γ-ray of angular momentum, $I = \tilde{J}_a - \tilde{J}_b$. In analogy with classical physics, this electromagnetic radiation may arise from oscillating electric charges which set up oscillating electric fields, or it may be due to variations of the nuclear electric currents which induce oscillating magnetic fields. Thus we have,

$$I = \tilde{J}_a - \tilde{J}_b \Rightarrow |J_a - J_b| \leq l \leq J_a + J_b, \quad l \neq 0,$$

and $m_I = m_a - m_b$.

where m is the z-projection of the γ-ray angular momentum vector, and $m_a$ and $m_b$ are the z-components of the respective nuclear states' angular momenta. Since no radiation exists with $l$=0 this means that γ-transitions between two states of spin $J_a = J_b = 0$, are strictly forbidden. In addition to these selection rules arising from the conservation of angular
momentum of the system, there are additional selection rules imposed by the conservation of parity of the nucleus and radiation field system. If the initial state is described by the wavefunction, \( \psi_a \), the final state by, \( \psi_b \), and the interaction operator is written as \( H_{lm}(\vec{r}) \), for the parity of the system to be conserved it is required that:

\[
P[<\psi_b | H_{lm}(\vec{r}) | \psi_a>] = +1 <\psi_b | H_{lm}(\vec{r}) | \psi_a> \tag{2.8}
\]

where \( P \) denotes the parity inversion operator (\( \vec{r} \to -\vec{r} \)). Thus, only states with either the same or opposite parity will be connected via \( H_{lm}(\vec{r}) \), depending on the parity of the operator itself. Before giving the mathematical formulas for these electromagnetic transition operators, we use physical arguments to deduce their parities.

Since the electric dipole (E1) transition operator must be proportional to \( q_i \vec{r}_i \) (where \( q_i \) and \( \vec{r}_i \) are the charge and position of the \( i \)th proton), and thus have odd parity, this means that the parity of the states \( |\psi_a> \) and \( |\psi_b> \) must be opposite to satisfy eq 2.8 above. The electric quadrupole (E2) transition operator has an extra \( \vec{r}_i \) term, and thus being even under parity, it will connect states of the same parity. Each additional electric multipole operator will add a factor proportional to \( \vec{r}_i \) and, thus, successive electric operators will connect states of alternating parity: E1 – opposite parity states, E2 – same parity states, E3 – opposite parity states, etc. Similarly, for the magnetic transition operators, the magnetic dipole (M1) transition ensues from an oscillation in the nuclear magnetic moments, each of which are proportional to the axial vector, \( \vec{r}_i \times \vec{j}_i \), where \( \vec{j}_i \) are the individual proton currents. Since the axial vector term, \( \vec{r}_i \times \vec{j}_i \), is unchanged under parity inversion, this means that the magnetic dipole (M1) operator must connect nuclear states of the same parity. As in the case of the electric transition operators, each successive magnetic multipole operator will add a factor proportional to \( \vec{r}_i \). Thus, successive magnetic transition operators will also connect states of alternating parity: M1 - same parity states, M2 - opposite parity states, M3 – same parity states, etc. These parity selection rules for electromagnetic transitions in nuclei are summarized in Table 2.2.
<table>
<thead>
<tr>
<th>$l$</th>
<th>Polarity: $2^l$</th>
<th>Name</th>
<th>Parity Change of Nuclear States a and b?</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Electric</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>Dipole</td>
<td>yes</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>Quadrupole</td>
<td>no</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
<td>Octupole</td>
<td>yes</td>
</tr>
<tr>
<td></td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>$l$</td>
<td>$2^l$</td>
<td>$2^l$-pole</td>
<td>$E_l$ has parity $(\cdot)$</td>
</tr>
</tbody>
</table>
The operators, $H_{lm}(\mathbf{r})$, in eq. 2.8 are the quantum mechanical electromagnetic interaction Hamiltonians. Since the classical interaction energy of particles in an electromagnetic system is given by, $H_{\text{class}} = \frac{1}{c} \int j \cdot \mathbf{A} \, dV$, where $\mathbf{A}$ is the vector potential of the classical electromagnetic field, the quantum mechanical analogues can be derived in terms of a nuclear current density, $\mathbf{j}_N$, and the vector potential of the quantized electromagnetic field, $\mathbf{A}_\text{lm}^\dagger$:

$$H_{lm}(\mathbf{r}) = \frac{1}{c} \int \mathbf{j}_N(\mathbf{r}) \cdot \mathbf{A}_\text{lm}^\dagger(k\mathbf{r}) \, dV$$  \hspace{1cm} (2.9)

it being understood that for electric transitions $\mathbf{A}_\text{lm}^{(E)r}$ is used, and for magnetic transitions $\mathbf{A}_\text{lm}^{(M)r}$ is used [War69]. (The complex conjugate form of the vector potential arises from the convention for emission processes; for absorption of radiation we would write the normal forms.) Mathematically, the vector potential must satisfy the wave equation:

$$\left( \nabla^2 + k^2 \right) \mathbf{A}_\text{lm}^{(E \text{ or } M)r} = 0,$$

and the classical electrodynamics equation, $\mathbf{E} = \frac{1}{\mu} \nabla \times \mathbf{B}$. It can be shown that the following expressions [War69] satisfy these conditions:

$$\mathbf{A}_\text{lm}^{(E)r}(k\mathbf{r}) = (-i)^{|l+1|} \frac{\mathbf{\nabla} \times (\mathbf{\nabla} \times \mathbf{\nabla}) \mathbf{f}_l(kr) \mathbf{Y}_{lm}(\mathbf{r})}{k \sqrt{l(l+1)}}$$  \hspace{1cm} (2.10)

$$\mathbf{A}_\text{lm}^{(M)r}(k\mathbf{r}) = (-i)^{|l+1|} \frac{\mathbf{\hat{r}} \times \mathbf{\nabla} \mathbf{f}_l(kr) \mathbf{Y}_{lm}^*(\mathbf{r})}{\sqrt{l(l+1)}}$$  \hspace{1cm} (2.11)

where $\mathbf{f}_l(kr)$ is the regular spherical Bessel function of order $l$, $\mathbf{\hat{r}}$ is the radial unit vector in the $\mathbf{r}$ direction, and $k=2\pi/\lambda$, is the wavenumber of the radiation. The total interaction Hamiltonian will, of course, be a sum over the individual Hamiltonians of all multipoles.
The approximations for sources of small extent, kr<<1, are interesting since they give a qualitative understanding of the transition probabilities of the various multipolarities. For kr<<1, we can write:

\[ j_l(kr) \sim (kr)^l/(2l+1)! \]

where \((2l+1)!! = 1 \times 3 \times 5 \times \ldots \times (2l-3) \times (2l-1) \times (2l+1)\).

Apart from factors of the order of unity we can then approximate:

\[ |A_{lm}^{(E)}(kr)|^2 \sim (kr)^{2l-2} \quad \text{and} \quad |A_{lm}^{(M)}(kr)|^2 \sim (kr)^{2l} \] (2.13, 14)

We, thus, find that the probability for an electric multipole transition of given \(l,m\) is enhanced by at least two orders of magnitude (since \(kr \sim 0.1\)) over the corresponding magnetic multipole transition, just from considering the structure of the vector potentials.

Also, from the approximations in eq. 2.13 and eq. 2.14, successive multipoles of the same type (E or M) will supressed by at least two orders of magnitude, as well.

Assuming point nucleons, the nuclear current density may be split into a convective term, arising from the motion of the protons, and a spin term, arising from a variation in the alignment of the magnetic moments of all nucleons [War69],

\[ \vec{J}_N(\vec{r}) = \sum_{i=\text{protons}} \vec{J}_{i}^{\text{conv}}(\vec{r}) + \sum_{i=\text{all nucleons}} \vec{J}_{i}^{\text{spin}}(\vec{r}) \]

\[ \Rightarrow \]

\[ \vec{J}_N(\vec{r}) = \sum_{i=\text{protons}} e_p \left[ \frac{\vec{p}_i}{2M_p} \delta(\vec{r} - \vec{r}_i) + \delta(\vec{r} - \vec{r}_i) \frac{\vec{p}_i}{2M_p} \right] + c\mu_0 \sum_{i=\text{all nucleons}} \mu_i \vec{\sigma}_i \times \vec{\sigma}_i \delta(\vec{r} - \vec{r}_i) \] (2.15)

where the nucleon magnetic moments, \(\mu_i\), are in units of nuclear magnetons, \(\mu_0 = \frac{e\hbar}{2M_pc}\), \(M_p\) is the proton mass, \(e_p\) is the proton charge, \(\vec{\sigma}_i\) is the Pauli spin vector, \(\vec{p}_i\) is the quantum-mechanical momentum operator, \(-i\hbar\vec{\nabla}\), and, \(\delta(\vec{r} - \vec{r}_i)\) is the delta function.
A full derivation of the matrix elements representing the radiative transition probability between two nuclear states $a$ and $b$ is too lengthy to reproduce here and may be found in [Bla52]. It can be shown that both the electric and magnetic matrix elements may be written as a sum of convective and spin terms:

\[ <\psi_b | H^{(E)}_{lm}(r) | \psi_a> = Q^{(E)}_{lm}^{\text{conv}} + Q^{(E)}_{lm}^{\text{spin}} \]  
\[ <\psi_b | H^{(M)}_{lm}(r) | \psi_a> = Q^{(M)}_{lm}^{\text{conv}} + Q^{(M)}_{lm}^{\text{spin}} \]

where,

\[ Q^{(E)}_{lm}^{\text{conv}} = e_p \sum_{i=\text{all protons}} \int r_i \ Y^*_{lm}(\vec{r}) \ \psi^*_b \ \psi_a \ dV \]  
\[ Q^{(E)}_{lm}^{\text{spin}} = -\frac{i k \mu_e}{l+1} \sum_{i=\text{all nucleons}} \int r_i \ Y^*_{lm}(\vec{r}) \ \vec{\nabla} \cdot [\psi^*_b (\vec{r}_i \times \vec{\sigma}_i) \psi_a] \ dV \]  
\[ Q^{(M)}_{lm}^{\text{conv}} = -\frac{2 \mu_o}{l+1} \sum_{i=\text{all protons}} \int r_i \ Y^*_{lm}(\vec{r}) \ \vec{\nabla} \cdot (\psi^*_b \ \vec{L}_i \ \psi_a) \ dV \]  
\[ Q^{(M)}_{lm}^{\text{spin}} = -\mu_o \sum_{i=\text{all nucleons}} \mu_i \int r_i \ Y^*_{lm}(\vec{r}) \ \vec{\nabla} \cdot (\psi^*_b \ \vec{\sigma}_i \ \psi_a) \ dV \]

where, in eq. 2.17c, we have used the fact that the quantum-mechanical angular momentum operator may be expressed, $\vec{L} = -i \ \vec{r} \times \vec{\nabla}$. All the integrations are carried out over the entire nuclear volume. It can be seen by simply counting the polar and/or axial vectors, as well as the powers of $r$, in each matrix elements that the equations 2.17 do, indeed, impose the parity selection rules on the nuclear wavefunctions, $\psi_a$ and $\psi_b$, shown in Table 2.2.
Although equations 2.17 give the analytical expression for the radiative transition probabilities between nuclear states, in practice, it is exceedingly difficult to carry out these calculations since the true nuclear wavefunctions, $\psi_a$ and $\psi_b$, are unknown (however, they may be very well approximated using, for instance, the nuclear shell model). Despite this problem, it is possible to arrive at a rough estimate of the rates by making some simplifying assumptions. We have already assumed that $kr \ll 1$. If we take the radial nuclear wavefunction to be a constant (i.e. constant probability density inside the nuclear volume) and assume that the radiative transition of multipolarity $l$ is due only to one proton changing its angular momentum from $l$ to 0, we arrive at the single-particle or independent-particle model of [Bla52]. The transition rate estimates thus derived are called the Weisskopf single-particle rates (in 1/sec), and are given by:

$$\omega(El) = \frac{2(l+1)}{l[(2l+1)!!]} \left( \frac{3}{l+3} \right)^2 \left( \frac{E_\gamma}{\hbar c} \right)^{2l+1} \alpha c R^{2l}$$  \hspace{1cm} (2.18a)$$

$$\omega(Ml) = \frac{20(l+1)}{l[(2l+1)!!]} \left( \frac{3}{l+3} \right)^2 \left( \frac{\hbar}{M_p c} \right)^2 \left( \frac{E_\gamma}{\hbar c} \right)^{2l+1} \alpha c R^{2l-2}$$  \hspace{1cm} (2.18b)$$

where, $R =$ radius of the emitting system in fm = $1.2A^{1/3}$ fm for nuclei,

- $M_p =$ proton rest mass = 938 MeV/c$^2$,
- $\alpha =$ the electromagnetic fine structure constant $= \frac{e^2}{\hbar c} = 1/137$,
- $\hbar c =$ 197 MeV fm,
- $c =$ speed of light $= 3 \times 10^{23}$ fm/sec,
- $E_\gamma =$ energy of the emitted $\gamma$-ray in MeV.

The approximate $\gamma$-widths are then given by (in eV) : $\Gamma_\gamma = \hbar \omega = 6.57 \times 10^{-16} \omega$, as mentioned in Section 1.7. The estimated transition rates, calculated for $A=19$, are plotted in Fig 2.3. It should be noted that if, according to the angular-momentum selection rules, a given transition can proceed via several $l = |J_a - J_b| ... J_a + J_b$, the following multipolarities will compete:

If there is parity change between states $a$ and $b$: E1, M2, E3, M4...until $l_{max}$.

If there is NO parity change between states $a$ and $b$: M1, E2, M3, E4...until $l_{max}$.
Fig 2.3: A log-log plot of the Weisskopf Single-Particle decay rates calculated for $A=19$ using equations 2.18a,b. These are very crude estimates, and may typically overestimate the decay rates by factors of $-10^3$. If $l=1,2$ are both allowed, the $E1$ and $M2$ multipolarities will compete if there is a change of parity between the initial and final nuclear states. In case there is no change of the parity, the $E2$ and $M1$ multipolarities will compete. Note that the $E2$ multipole is suppressed by a factor $-10^3$ compared with $M1$, whereas the $M2$ multipole is suppressed by a factor $-10^4$ compared with $E1$. 

Weisskopf Single-Particle Transition Rate $1/sec$

$Ey (MeV)$

$1.0 \times 10^8$

$1.0 \times 10^9$

$1.0 \times 10^{10}$

$1.0 \times 10^{11}$

$1.0 \times 10^{12}$

$1.0 \times 10^{13}$

$1.0 \times 10^{14}$

$1.0 \times 10^{15}$

$1.0 \times 10^{16}$

$1.0 \times 10^{17}$

$1.0 \times 10^{18}$

$E\gamma (MeV)$

$1$

$2$

$3$

$4$

$5$

$6$

$7$

$8$

$9$

$10$
Apart from those cases where large collective effects are observed, the Weisskopf rates typically overestimate the transition rates by factors $\sim 10^3$. Comparisons between the experimental data and the calculated rates have been compiled by [End79] and the results for E1 transitions are shown in Fig 2.4. Clearly, the Weisskopf rates are not an acceptable substitute for experimental measurement of $\Gamma_\gamma$'s. However, the roughness of the theoretical approximation needs to be appreciated in the context of the enormous total range of radiative transition rates which can span about 23 orders of magnitude.

![Histogram illustrating the relation between observed and calculated transition rates for nuclei A=6-44 (white) and for a subset of nuclei A=6-20 (black). The transitions are binned according to the log of the ratio of their experimentally measured $\gamma$-widths, $\Gamma_m$, to their estimated Weisskopf $\gamma$-widths, $\Gamma_w$. As can be seen, Weisskopf estimates typically overestimate the true rates by factors $\sim 10^3$. Figure taken from [End79]](image)
2.6 Strategy for Identifying the 3/2$^+$ State of $^{19}$F at $E_x \approx 7.1$ MeV

Our strategy in searching for the 3/2$^+$ state of $^{19}$F assumed that only E1, M1 and E2 transitions would produce a detectable γ-yield via the $^{15}$N(α,γ)$^{19}$F reaction. This assumption is valid, not only from considering the Weisskopf rates, as shown in Fig 2.3, but also in view of the extensive compiled experimental data in light nuclei [End79]. This, then, gave us a method to identify and separate at least some transitions coming from the putative 3/2$^+$ state at $E_x \approx 7.1$ MeV versus those that may be coming from the nearby 7/2$^+$ state at $E_x = 7.114$ MeV. Since the selection rules for E1, M1 and E2 operators allow decays from a 3/2$^+$ state to one set of states (7/2$^+$, 5/2$^+$, 5/2$^-$, 3/2$^+$, 3/2$^-$, 1/2$^+$, 1/2$^-$), and from a 7/2$^+$ to another set of states (11/2$^+$, 9/2$^-$, 9/2$^+$, 7/2$^+$, 7/2$^-$, 5/2$^-$, 5/2$^+$, 3/2$^+$), one can see that any decays observed occurring to 3/2$^-$, 1/2$^+$, or 1/2$^-$ states, must have originated from a 3/2$^+$ state and not from the 7/2$^+$. Of course, simply because a decay is allowed by angular momentum and parity considerations does not mean that it will necessarily occur. Such a situation may occur when the overlap of the nuclear parts of the initial and final states' wavefunctions is zero or very small (e.g., the E1 operator will not connect states with total isotopic spin difference $\Delta T \geq 2$; see [War69]). Thus, there was no a priori guarantee that even if the 3/2$^+$ state did exist where we presumed it should, that it would decay via any of the "signature" transitions to 3/2$^-$, 1/2$^+$, or 1/2$^-$ states.
Chapter 3

THE $^{15}$N($\alpha,\gamma$)$^{19}$F EXPERIMENT

3.1 Introduction

The $^{15}$N($\alpha,\gamma$)$^{19}$F reaction was chosen to search for a $3/2^+$ state in $^{19}$F at an excitation energy of about 7.1 MeV. Since it was suggested that this $3/2^+$ state would overlap the known $7/2^+$ state at approximately the same energy, an analysis of the very different gamma-decay schemes for the two resonances would provide the clearest separation of the states. It should be noted that since $^{15}$N has a $1/2^-$ ground state, the $^{15}$N($\alpha,\gamma$)$^{19}$F reaction will preferentially populate the $3/2^+$ state (since this would be an $l=1$ capture) rather than the $7/2^+$ state (which would require an $l=3$ capture). $^{15}$N + $\alpha$ is the only open (energetically allowed) channel for studying these resonances. A previous $^{15}$N($\alpha,\gamma$)$^{19}$F experiment [Dix77], did not observe the $3/2^+$ state at $E_\alpha \sim 7.1$ MeV.
3.2 Experimental Facility

The $^{15}$N($\alpha$,γ)$^{19}$F experiment was carried out on the 90° Beam-line at the University of Stuttgart 4MV Dynamitron facility in Germany [Fig. 3.1]. This facility is ideally suited to study low energy alpha-particle radiative capture reactions since high-intensity beam currents of $^4$He$^+$ are readily available [Ham79]. In addition, since the RHINOCEROS recirculating windowless gas target system can be used, this means that there are no problems associated with backgrounds due to gas-cell entrance and exit windows [Kne87, Ham98]. Since during our run in August and September 1997 the Dynamitron was pushed to its design limit voltage of 4 MV, the currents ranged between only 50-80 μA.

The laboratory beam energy, $E_\alpha$, was determined online by using Ge detectors to measure the energy of the entrance channel to ground state γ-transition of the $^{19}$F* nucleus, $E_\gamma$. The beam energy is then simply the difference between $E_\gamma$ and the Q-value (4013.8 keV) converted to the lab system: $E_\alpha = \frac{19.007}{15.0001} (E_\gamma - Q)$.

3.3 The Gas Target System

For the $^{15}$N($\alpha$,γ)$^{19}$F experiment, commercially available $^{15}$N$_2$ enriched to 99% was used in conjunction with the RHINOCEROS recirculating windowless gas target system [Fig. 3.2]. This system afforded many important advantages over solid targets or gas cells with entrance and exit windows [Ham97]:

- Windowless gas targets can withstand a high beam power for an extended period without degradation of the target material.
- No reaction backgrounds from backings, oxides or entrance / exit windows is seen.
- The beam was stopped about 1.5m from the target area, minimizing the beam-stop background.
- No carbon buildup was witnessed. This is especially important since the $^{13}\text{C}(\alpha,n)^{16}\text{O}$ can lead to aggravating neutron backgrounds.

- Target thickness could be controlled simply by varying the gas pressure.

- The target gas was continuously cleaned using an adjustable temperature cryotrap and zeolite adsorption trap.

Fig 3.1: The University of Stuttgart 4MV Dynamitron facility. The $^{12}\text{N}(\alpha,\gamma)^{19}\text{F}$ experiment was carried out on the 90° beam-line using the RHINOCEROS recirculating windowless gas target. Figure taken from [Kne97].
Fig. 3.2: The RHINOCEROS windowless, recirculating gas-target system. The beam enters from the left and proceeds to the target chamber (not shown). All apertures are made from Tungsten whose high-Z suppresses background nuclear reactions. The system can maintain a pressure differential of about 8 orders of magnitude between the beam-line and the target chamber. For pumping details see Fig 3.3.
Figure taken from [Mor 97].
The RHINOCEROS system maintained a pressure differential of about 8 orders of magnitude between the target chamber and the upstream beam tube by using a series of water-cooled differential pumping apertures [Fig. 3.3]. All apertures have tungsten inserts to minimize beam-induced background reactions; in addition, the target chamber and the beam-tube following the target chamber are plated with about 20μm of gold. The target chamber pressure was maintained to better than 1% by using an electronic needle valve in a feedback system linked to a BARATRON capacitance manometer on the target chamber. Two settings of the $^{15}$N$_2$ were used: 3.0 mbar and 1.51 mbar corresponding to 26 and 13 keV energy loss for the ~3.9 MeV incident alpha particles.

3.4 The Target Chamber

The gas target chamber has a flat disk shape with a 6cm diameter by 2.6cm wide active volume [Fig. 3.4] which allows the Ge detectors to be used in a close geometry, maximizing the solid angle subtended. It is constructed of high-purity stainless steel and the inside is coated with 20μm of gold to suppress background nuclear reactions and prevent any chemical reactions. The upstream and downstream beam apertures are 5mm in diameter and are water cooled and coated with about 40μm of gold. Since the beam spot is known to be 1mm at the beam energies of interest, the beam interaction with the apertures should be minimal in any case.

There are six modular inserts that can be mounted radially on the chamber at various lab angles: 30°, 60°, 45°, 90°, 135°, 155°. Two of these inserts were used to hold Si particle detectors at 60 degrees and 135 degrees in the laboratory. These were used to monitor the yield of the elastically scattered alpha particles. The third insert was used for a high-precision BARATRON capacitance manometer and the last was used for the gas supply to the chamber.
Fig. 3.3: A schematic of the RHINOCEROS recirculating windowless gas target. (The darker shadings indicate higher gas pressure.) The pressure in the target chamber is reduced by a factor of $10^{-5}$ in the five differential pumping stages before the connection to the DYNAMITRON beamline. The gas in the first two stages is recirculated back to the target chamber by a combination of a high-load Turbo pump and six Roots Blowers. Before re-entering the target chamber the gas is cleaned using a Zeolite Trap which is always kept below $0^\circ C$, and a Cryo Trap held at $-65^\circ C$. The pressure in the Target chamber is controlled by an electronic flow control valve in a feedback loop with a high-precision BARATRON capacitance manometer.
Fig 3.4: A vertical section through the target chamber. The beam enters from the left, reaches the central high pressure area of 6cm diameter, and exits to be dumped in the Faraday cup ~1.5m away. The inlet and outlet differential pumping apertures are water cooled due to the high beam currents of ~80μA. The radial attachments are used to mount the monitor Si detectors, the gas inlet and the BARATRON capacitance manometer which is used to regulate the target cell pressure. The Ge detectors are aligned perpendicular to the plane of the page, on either side to the target chamber.

Figure taken from [Ham97].
3.5 The Particle Detectors

Since during data taking, the beam-stop Faraday cup has about 0.05 mbar of gas present which is ionized by beam, one cannot directly measure the beam current. Instead one relies on the count rate of the elastically scattered alphas to indicate the beam current. This is measured using two Passivated Implanted Planar Silicon (PIPS) detectors with a 50mm$^2$ active surface and a 100μm active depth each. The alpha particle energy resolution for both was approximately 20 keV when biased to +25V.

These detectors were mounted at 60 and 135 degrees in the laboratory since the cross-section for $^{15}$N(α,α), which is needed to correctly interpret the count rate, had previously been measured at these angles [Smo61].

The geometry relevant to the particle detectors is shown in Fig. 3.5. The beam traversing the extended gas target sets up a cylindrical target volume; of this volume there is an effective central target length, $l_a$, which is defined by a combination of a slit of width, $s$, and an aperture of radius, $r$, separated by a distance, $f$. If we call the distance from the aperture to the target volume, $d$, and assuming $d >> l_a$, then we have [Rol88]:

$$l_a d\Omega = \frac{\pi}{\sin \psi} \frac{sr^2}{fd}$$  \hspace{1cm} (3.1)

Where $d\Omega$ is solid angle and $\psi$ is the angle of the detector relative to the beam, both in the laboratory system. The number of elastically scattered alpha particles can then be related to the number of beam particles, and thus to the beam current, by:

$$N_{a, beam} = \frac{N_{a, Detected}}{\frac{d\sigma}{d\Omega}_{LAB} \rho_{gas} l_a d\Omega}$$  \hspace{1cm} (3.2)
Fig 3.5: The geometry used to calculate the effective target lengths and solid angles relevant to the Si monitor detectors. The diameter of the beam is \(-1\) mm. The elastically scattered alphas from the effective target length, \(l_\alpha\), are detected in the Si detectors with unit efficiency after passing through the slit-collimator combination. Figure taken from [Mor97].

The target atom density, \(\rho_{\text{gas}}\) was known and stable to better than 1% during all runs.

Table 3.1 Lists the relevant parameters for the August/September 1997 \(^{15}\text{N}(\alpha,\gamma)^{19}\text{F}\) run:

<table>
<thead>
<tr>
<th>Lab Angle</th>
<th>Dimension</th>
<th>60 deg</th>
<th>135deg</th>
</tr>
</thead>
<tbody>
<tr>
<td>60 deg</td>
<td>(s)</td>
<td>0.15 mm</td>
<td>0.3 mm</td>
</tr>
<tr>
<td></td>
<td>(r)</td>
<td>1.5 mm</td>
<td>1.5 mm</td>
</tr>
<tr>
<td></td>
<td>(d)</td>
<td>326.4 mm</td>
<td>325.1 mm</td>
</tr>
<tr>
<td></td>
<td>(f)</td>
<td>301.0 mm</td>
<td>303.3 mm</td>
</tr>
<tr>
<td></td>
<td>(l_\alpha)</td>
<td>0.19 mm</td>
<td>0.30 mm</td>
</tr>
<tr>
<td></td>
<td>(d\Omega)</td>
<td>66.3 (\mu)Sr</td>
<td>66.8 (\mu)Sr</td>
</tr>
</tbody>
</table>

Table 3.1 Geometrical Attributes of the Particle Detectors. See Fig 3.5 for definitions.
This method allows one to determine the beam current throughout the run when the Faraday cup cannot be used due to the ionized gas present. Thus, one can determine the true collected charge during a run even if the beam current is not stable. During our $^{15}\text{N}(\alpha,\gamma)^{19}\text{F}$ beam time we used this method to monitor the current continuously; however, since we found that the current was steady enough (±3%) throughout the approximately hour-long runs we did not need to use the yields of the elastically scattered alphas to correct for the current variations. A simple average of the start-of-run and end-of-run current was sufficient to determine the true collected charge.

The Si detectors have the incidental benefit of indicating any impurities that may be contaminating the gas in the target chamber. Any such impurities result in additional peaks in the energy spectra of the elastically scattered alpha particles, as shown in Fig 3.6. The most prevalent contaminant was $^{14}\text{N}$, which was typically present at less than ~1 % level. Other contaminants were present at less than ~0.1 % level.

Fig. 3.6: Sample energy spectrum of the elastically scattered $\alpha$'s detected at 90 degrees. This spectrum is from a particularly contaminated run to illustrate the additional contaminant peak due to $^{14}\text{N}$. 
3.6 The Gamma Detectors

Two n-type High Purity Germanium (HPGe) detectors were used in a close geometry to the target chamber to measure the gamma rays from the $^{15}\text{N}(\alpha,\gamma)^{19}\text{F}$ reaction as shown in Fig. 3.7. The 90-degree geometry was chosen to minimize the distance of the Ge crystal from the target chamber and thus maximize the geometrical efficiency. Both detectors were actively shielded against Compton-scattered gammas using a veto signal from the Bi$_4$Ge$_3$O$_{12}$ (BGO) shields surrounding the central Ge crystal. Detector 1 was used with a commercially manufactured BGO array with 8 segments, whereas Detector 2 had a 4-segment BGO array that was made locally at the University of Stuttgart. Detector 1 was not moved throughout the entire experiment, so that there would be one cohesive data-set while the location of Detector 2 was changed several times in various attempts to optimize peak-shape and dead-time versus efficiency. The data from Detector 2, although useful during the run, was not used in the analysis since every change in position would require running a separate lengthy GEANT simulation as well as a $^{60}\text{Co}$ calibration, to determine the true efficiency (see section 3.12). The characteristics of both detectors are summarised in Table 3.2. Both detectors, together with their BGO arrays, were placed in passive shields made from lead and iron bricks to minimize background counts.

<table>
<thead>
<tr>
<th>Det</th>
<th>Ge Diameter (mm)</th>
<th>Ge Length (mm)</th>
<th>FWHM 1.33 MeV Line from $^{60}\text{Co}$</th>
<th>Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>75.6</td>
<td>92.7</td>
<td>2.18 keV</td>
<td>95.3%</td>
</tr>
<tr>
<td>2</td>
<td>75.2</td>
<td>96.8</td>
<td>2.39 keV</td>
<td>101.5%</td>
</tr>
</tbody>
</table>

Table 3.2: The Characteristics of the Ge Detectors. The Percentage Efficiency is with respect to a 3 x 3 inch NaI Detector Efficiency.
Fig. 3.7: A horizontal section through the experimental set-up. The beam enters from the top and the γ-transitions that occur in the region of the target chamber are picked-up with the highest efficiency. The detectors are placed at 90 degrees to the beam as this allows them to be as close as possible to the target chamber. This geometry also ensures that the Doppler shifts of the γ-rays are symmetric [see Sect. 3.10]. Both detectors are actively shielded against Compton scattering by using BGO anticoincidence scintillation crystals: Detector 1 has a commercially manufactured array whereas Detector 2’s BGO shield was made in-house at the University of Stuttgart. The position of Detector 1 was not changed throughout the experiment, whereas Detector 2 was moved in an effort to reduce the doppler broadening in the extended gas target (see Section 3.10). The data from Detector 2 was not used in the analysis. Here the detectors are shown as close as possible to the target cell. The actual target-detector separation in our run is shown in Fig 3.13. Figure taken from [Ham97].
3.7 Electronics

For each run, the following quantities were acquired, histogrammed, and stored on a PC using the data acquisition program TMCA from Köln University [Tar96]:

- Raw energy spectrum from Ge Detector 1 (80 – 9200 keV).
- Raw energy spectrum from Ge Detector 2 (80 – 9200 keV).
- BGO-suppressed energy spectrum from Ge Detector 1 (80 – 9200 keV).
- BGO-suppressed energy spectrum from Ge Detector 2 (80 – 9200 keV).
- TAC spectrum from Detector 1 relative to BGO 1 veto signal.
- TAC spectrum from Detector 2 relative to BGO 2 veto signal.
- Energy spectrum from 60 degree Si-detector.
- Energy spectrum from 135 degree Si-detector.

The electronic set-up for the Ge detectors is shown in Fig. 3.8 and Fig. 3.9 below. For detector 1, one of the pre-amplified energy signal from the Ge detector is sent to a Silena 7618 amplifier and then digitized via an 8K Silena 7423 ADC and histogrammed as the raw energy spectrum. The timing output from the pre-amplifier is used as the Time to Amplitude Converter (TAC) Start signal, after passing through a Timing Filtering Amplifier (TFA) and Constant Fraction Discriminator (CFD). The Stop for the TAC is provided by an 80ns cable delayed logic signal from the BGO array. (The two CFD signals from the BGO crystals are logic summed prior to this delay.) If the Start-Stop time difference in the TAC is less than 50ns then a 5V logic veto signal is sent to an ADC whose output forms the BGO-suppressed energy spectrum. The TAC output is sent to a third ADC and histogrammed.

The electronics setup for detector 2 is almost identical except that the BGO array for detector 2 has only 4 crystals (as compared to 8 for BGO 1), and so only one CFD is used and no logic sum is needed.
The typical count rate for the Ge-detectors is about 1000 Hz. The dead-time of the detectors was assessed on-line by feeding a pulser signal to the detector pre-amplifiers and measuring what fraction of the original pulser "counts" were "detected".

The Si-detectors' outputs are simply amplified, digitized and histogrammed and their electronics are completely independent of the Ge-BGO electronics. The typical count rate for the Si-detectors is about 20 Hz.

Fig. 3.8: The electronics setup for Detector 1. All ADC's are read into the PC.
Fig. 3.9: The electronics setup for Detector 2. All ADC's are read into the PC.
Since there is no trigger structure to the acquisition of the $\gamma$-spectra, one needs to understand various aspects of the spectra in order to meaningfully extract the required physics offline. For example, background lines may overlap with regions of interest, and doppler broadening due to the extended target and the close detector geometry may spread the $\gamma$-ray lines of interest. These and other effects are discussed below.

3.8 The $\gamma$-Background

To correctly interpret the alpha capture $\gamma$-spectra measured by the Ge detectors, one needs to understand the contributions from the background. There are two components to this $\gamma$-background; there is a beam induced background and a room background which is present even without beam.

The room background was measured at the start and at the end of the 3 week run, as well as periodically during the run when time allowed. A typical spectrum is shown in Fig 3.10, and the identified lines are listed in Table 3.3.

With the beam on there is also a significant neutron-induced background resulting from neutron inelastic scattering, $(n,n'\gamma)$, and from neutron capture reactions, $(n,\gamma)$, on the material of the target chamber and on the Ge detectors themselves. In addition $(\alpha,\gamma)$ and $(\alpha,X\gamma)$ reactions may take place with contaminants present in the gas. Since the target chamber is made from high-purity Cr-Ni stainless steel, and is plated with about 20$\mu$m of gold the $(\alpha,\gamma)$ and $(\alpha,X\gamma)$ reactions on the material of the target chamber itself are suppressed. A typical $\gamma$-spectrum including beam-induced background reactions is shown in Fig 3.11, and the resulting identified background lines are listed in Table 3.4.
Fig 3.10: The room γ-background. This spectrum was collected with no beam. The natural radioactivity present in the atmosphere and room materials gives rise to these lines.
<table>
<thead>
<tr>
<th>Gamma Energy (keV)</th>
<th>From Background Source:</th>
</tr>
</thead>
<tbody>
<tr>
<td>77.1</td>
<td>Bi atomic transition</td>
</tr>
<tr>
<td>87.2</td>
<td>Bi atomic transition</td>
</tr>
<tr>
<td>93.0</td>
<td>$^{234}$Th</td>
</tr>
<tr>
<td>238.6</td>
<td>$^{212}$Pb</td>
</tr>
<tr>
<td>351.9</td>
<td>$^{214}$Pb</td>
</tr>
<tr>
<td>511.2</td>
<td>$\beta^+$ annihilation</td>
</tr>
<tr>
<td>569.7</td>
<td>$^{207}$Bi</td>
</tr>
<tr>
<td>583.1</td>
<td>$^{208}$Tl</td>
</tr>
<tr>
<td>609.3</td>
<td>$^{214}$Bi</td>
</tr>
<tr>
<td>661.6</td>
<td>$^{137}$Cs</td>
</tr>
<tr>
<td>727.3</td>
<td>$^{212}$Bi</td>
</tr>
<tr>
<td>766.36</td>
<td>$^{234}$Pa$^m$</td>
</tr>
<tr>
<td>896.6</td>
<td>$^{209}$Po</td>
</tr>
<tr>
<td>911.0</td>
<td>$^{228}$Ac</td>
</tr>
<tr>
<td>969.0</td>
<td>$^{226}$Ac</td>
</tr>
<tr>
<td>1001.03</td>
<td>$^{234}$Pa$^m$</td>
</tr>
<tr>
<td>1063.7</td>
<td>$^{207}$Bi</td>
</tr>
<tr>
<td>1120.3</td>
<td>$^{214}$Bi</td>
</tr>
<tr>
<td>1238.1</td>
<td>$^{214}$Bi</td>
</tr>
<tr>
<td>1377.7</td>
<td>$^{214}$Bi</td>
</tr>
<tr>
<td>1460.8</td>
<td>$^{40}$K</td>
</tr>
<tr>
<td>1509.2</td>
<td>$^{214}$Bi</td>
</tr>
<tr>
<td>1588.2</td>
<td>$^{228}$Ac</td>
</tr>
<tr>
<td>1620.5</td>
<td>$^{212}$Bi</td>
</tr>
<tr>
<td>1633.4</td>
<td>$^{207}$Bi sum line</td>
</tr>
<tr>
<td>1661.3</td>
<td>$^{214}$Bi</td>
</tr>
<tr>
<td>1729.6</td>
<td>$^{214}$Bi</td>
</tr>
<tr>
<td>1764.5</td>
<td>$^{214}$Bi</td>
</tr>
<tr>
<td>1770.2</td>
<td>$^{207}$Bi</td>
</tr>
<tr>
<td>1847.4</td>
<td>$^{214}$Bi</td>
</tr>
<tr>
<td>2105.4</td>
<td>$^{208}$Tl single escape (2614 keV)</td>
</tr>
<tr>
<td>2118.6</td>
<td>$^{214}$Bi</td>
</tr>
<tr>
<td>2204.2</td>
<td>$^{214}$Bi</td>
</tr>
<tr>
<td>2447.9</td>
<td>$^{214}$Bi</td>
</tr>
<tr>
<td>2614.5</td>
<td>$^{208}$Tl</td>
</tr>
</tbody>
</table>

Table 3.3: The room background lines and their sources
Fig 3.11: Sample spectrum showing the gamma-ray lines due to background nuclear reactions. Numbered lines refer to Table 3.4. Room background lines are marked "B" and \(^{19}\text{F}\) primary, secondary and tertiary decays are marked "F".
<table>
<thead>
<tr>
<th>Energy (keV)</th>
<th>Type</th>
<th>Background Reaction(s)</th>
<th>Number on Fig. 3.10</th>
</tr>
</thead>
<tbody>
<tr>
<td>73.77</td>
<td>narrow</td>
<td>Pb atomic transitions</td>
<td>1</td>
</tr>
<tr>
<td>139.4</td>
<td>narrow</td>
<td>$^{74}\text{Ge}(n,\gamma)^{75}\text{Mn}\text{Ge}$</td>
<td>2</td>
</tr>
<tr>
<td>277.4</td>
<td>narrow</td>
<td>$^{207}\text{Pb}(n,\gamma)^{208}\text{Pb}$</td>
<td>3</td>
</tr>
<tr>
<td>351.9</td>
<td>narrow</td>
<td>$^{214}\text{Pb}$ decay or $^{17}\text{O}(\alpha,\gamma)^{21}\text{Ne}$</td>
<td>4</td>
</tr>
<tr>
<td>352.3</td>
<td>narrow</td>
<td>$^{57}\text{Fe}(n,n'\gamma)^{57}\text{Fe}$</td>
<td>5</td>
</tr>
<tr>
<td>440</td>
<td>narrow</td>
<td>$^{19}\text{F}(\alpha,\gamma)^{23}\text{Na}$</td>
<td>6</td>
</tr>
<tr>
<td>563.4</td>
<td>broad</td>
<td>$^{76}\text{Ge}(n,n'\gamma)^{76}\text{Ge}$</td>
<td>7</td>
</tr>
<tr>
<td>595</td>
<td>broad</td>
<td>$^{74}\text{Ge}(n,n'\gamma)^{74}\text{Ge}$</td>
<td>8</td>
</tr>
<tr>
<td>669</td>
<td>narrow</td>
<td>$^{63}\text{Cu}(n,n'\gamma)^{63}\text{Cu}$</td>
<td>9</td>
</tr>
<tr>
<td>693</td>
<td>broad</td>
<td>$^{72}\text{Ge}(n,n'\gamma)^{72}\text{Ge}$</td>
<td>10</td>
</tr>
<tr>
<td>718</td>
<td>narrow</td>
<td>$^{10}\text{B}(\alpha,\alpha'\gamma)^{10}\text{B}$</td>
<td>11</td>
</tr>
<tr>
<td>834</td>
<td>broad</td>
<td>$^{72}\text{Ge}(n,n'\gamma)^{72}\text{Ge}$</td>
<td>12</td>
</tr>
<tr>
<td>843</td>
<td>narrow</td>
<td>$^{23}\text{Na}(\alpha,\gamma)^{23}\text{Al}$</td>
<td>13</td>
</tr>
<tr>
<td>847</td>
<td>narrow</td>
<td>$^{56}\text{Fe}(n,n'\gamma)^{56}\text{Fe}$ or $^{55}\text{Mn}(n,\gamma)^{56}\text{Mn}$</td>
<td>14</td>
</tr>
<tr>
<td>962</td>
<td>narrow</td>
<td>$^{63}\text{Cu}(n,n'\gamma)^{63}\text{Cu}$</td>
<td>15</td>
</tr>
<tr>
<td>1014</td>
<td>narrow</td>
<td>$^{27}\text{Al}(n,n'\gamma)^{27}\text{Al}$</td>
<td>16</td>
</tr>
<tr>
<td>1037</td>
<td>narrow</td>
<td>$^{56}\text{Fe}(n,n'\gamma)^{56}\text{Fe}$</td>
<td>17</td>
</tr>
<tr>
<td>1039.5</td>
<td>broad</td>
<td>$^{70}\text{Ge}(n,n'\gamma)^{70}\text{Ge}$</td>
<td>18</td>
</tr>
<tr>
<td>1115</td>
<td>narrow</td>
<td>$^{65}\text{Cu}(n,n'\gamma)^{65}\text{Cu}$</td>
<td>19</td>
</tr>
<tr>
<td>1129.7</td>
<td>narrow</td>
<td>$^{25}\text{Mg}(n,\gamma)^{25}\text{Mg}$</td>
<td>20</td>
</tr>
<tr>
<td>1273</td>
<td>narrow</td>
<td>$^{28}\text{Si}(n,\gamma)^{28}\text{Si}$</td>
<td>21</td>
</tr>
<tr>
<td>1327</td>
<td>narrow</td>
<td>$^{63}\text{Cu}(n,n'\gamma)^{63}\text{Cu}$</td>
<td>22</td>
</tr>
<tr>
<td>1368</td>
<td>narrow</td>
<td>$^{24}\text{Mg}(n,n'\gamma)^{24}\text{Mg}$</td>
<td>23</td>
</tr>
<tr>
<td>1435.8</td>
<td>narrow</td>
<td>$^{10}\text{B}(\alpha,\alpha'\gamma)^{10}\text{B}$</td>
<td>24</td>
</tr>
<tr>
<td>1481</td>
<td>narrow</td>
<td>$^{65}\text{Cu}(n,n'\gamma)^{65}\text{Cu}$</td>
<td>25</td>
</tr>
<tr>
<td>1547</td>
<td>narrow</td>
<td>$^{63}\text{Cu}(n,n'\gamma)^{63}\text{Cu}$</td>
<td>26</td>
</tr>
<tr>
<td>1633</td>
<td>narrow</td>
<td>$^{16}\text{O}(\alpha,\gamma)^{20}\text{Ne}$ or $^{17}\text{O}(\alpha,\gamma)^{21}\text{Ne}$</td>
<td>27</td>
</tr>
<tr>
<td>1725</td>
<td>narrow</td>
<td>$^{56}\text{Fe}(n,\gamma)^{56}\text{Fe}$</td>
<td>28</td>
</tr>
<tr>
<td>1779</td>
<td>narrow</td>
<td>$^{24}\text{Mg}(\alpha,\gamma)^{26}\text{Si}$</td>
<td>29</td>
</tr>
<tr>
<td>1808</td>
<td>narrow</td>
<td>$^{25}\text{Mg}(n,\gamma)^{25}\text{Mg}$</td>
<td>30</td>
</tr>
<tr>
<td>2614</td>
<td>narrow</td>
<td>$^{208}\text{Tl}$ decay or $^{207}\text{Pb}(n,\gamma)^{208}\text{Pb}$</td>
<td>31</td>
</tr>
<tr>
<td>3491</td>
<td>narrow</td>
<td>$^{18}\text{O}(n,\gamma)^{22}\text{Ne}$ or $^{17}\text{O}(\alpha,\gamma)^{21}\text{Ne}$</td>
<td>32</td>
</tr>
<tr>
<td>3680</td>
<td>narrow</td>
<td>$^{10}\text{B}(\alpha,\gamma)^{14}\text{N}$</td>
<td>33</td>
</tr>
<tr>
<td>3683.9</td>
<td>narrow</td>
<td>$^{10}\text{B}(\alpha,\gamma)^{15}\text{C}$</td>
<td>34</td>
</tr>
<tr>
<td>3853.2</td>
<td>narrow</td>
<td>$^{10}\text{B}(\alpha,\gamma)^{15}\text{C}$</td>
<td>35</td>
</tr>
<tr>
<td>7415</td>
<td>narrow</td>
<td>$^{24}\text{Mg}(\alpha,\gamma)^{28}\text{Si}$</td>
<td>Not shown</td>
</tr>
<tr>
<td>7645</td>
<td>narrow</td>
<td>$^{56}\text{Fe}(n,\gamma)^{57}\text{Fe}$</td>
<td>Not shown</td>
</tr>
</tbody>
</table>

Table 3.4: The Beam-induced Background $\gamma$-lines
3.9 Energy Calibration of the Ge Detectors

To obtain an accurate energy calibration, a $^{226}$Ra gamma source which provided 12 prominent gamma lines in the region 300-2500 keV was used [Fig. 3.12]. Since we were interested in transitions that could be as high as 7200 keV however, and no gamma sources exist with lines in that region, we used a high intensity $^{241}$Am-Be neutron source to induce the $^{57}$Fe(n,$\gamma$) reaction with lines at 7631.7 and 7645.5 keV. Thus we were able to get an accurate energy calibration from about 300-7650 keV. All 14 energy lines used for the calibration are listed in Table 3.5 and the energy-channel calibration plot is shown in Fig 3.13.

<table>
<thead>
<tr>
<th>Energy (keV)</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>351.9</td>
<td>$^{226}$Ra</td>
</tr>
<tr>
<td>609.3</td>
<td></td>
</tr>
<tr>
<td>768.35</td>
<td></td>
</tr>
<tr>
<td>934.06</td>
<td></td>
</tr>
<tr>
<td>1120.29</td>
<td></td>
</tr>
<tr>
<td>1238.11</td>
<td></td>
</tr>
<tr>
<td>1509.228</td>
<td></td>
</tr>
<tr>
<td>1764.49</td>
<td></td>
</tr>
<tr>
<td>2118.55</td>
<td></td>
</tr>
<tr>
<td>2204.22</td>
<td></td>
</tr>
<tr>
<td>2293.36</td>
<td></td>
</tr>
<tr>
<td>2447.86</td>
<td></td>
</tr>
<tr>
<td>7631.7</td>
<td>$^{57}$Fe(n,$\gamma$)</td>
</tr>
<tr>
<td>7645.5</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.5: The gamma lines that were used for the Ge Energy Calibration
Fig 3.12: A sample $^{226}$Ra spectrum used for the energy calibration. The $\gamma$-lines used for the calibration are numbered with their energies in keV. As can be seen, there are many more lines from $^{226}$Ra and its eight decay daughters, but the 12 shown are more than sufficient to obtain an accurate energy calibration.
Fig 3.13: The energy-channel calibration for Ge detector 1. The calibration for both detectors is extremely linear and is accurate to \(-1\) keV over an approximately 7500 keV range. The algebraic expression for the best fit lines are:

Detector 1: \( E_y = (1.1234 \times \text{Channel #}) + 25.5237 \) (keV)
3.10 Doppler Broadening in the Extended Gas Target

The main contribution to the peak widths seen in the gamma spectra of the $^{15}$N(α,γ)$^{19}$F reaction (eg: Fig. 3.11) is from the Doppler effect which arises from the combination of the close geometry of the Ge detectors and the extension of the gas target. Since the $^{19}$F nucleus is moving at a velocity $v$ (in the lab. system) when it emits the gamma ray of energy $E_γ$, this means that the energy of the detected gamma ray, $E_{Det}$, will be Doppler-shifted up or down according to the projection of the velocity along the path of the gamma-ray [Fig 3.14]:

$$E_{Det} = E_γ(1 + \frac{v}{c}\cosθ) \tag{3.3}$$

The angle between the gamma ray direction and the velocity vector is defined as $θ$. Typically, the Doppler broadening of the gamma peaks is about 25 keV FWHM, compared to the intrinsic resolution of the Ge of about 2 keV and the center-of-mass target energy loss of 10 keV.

Fig 3.14: The energy of the detected gamma-ray, $E_{Det}$ will be shifted up or down relative to the original $E_γ$ due to the motion of the emitting nucleus, in our case $^{19}$F. The amount of shift is proportional to the projection of the velocity along the gamma-ray direction, $\cosθ$. Two extreme cases are shown in the figure. The construction of the target cell is shown schematically here; a detailed drawing of the target chamber is shown in Fig 3.4.
3.11 Angular Distributions of the $\gamma$-Radiation

In order to determine the true resonance strength of the $3/2^+$ state of interest it is important to understand and correct for the effects of coupling the finite solid angles of the Ge detectors with the angular distributions of the various transitions from that state.

A state with spin $J_1$ and z-projection $m_{J_1}$ can decay to another with spin $J_2$ and z-projection $m_{J_2}$ by emitting an electromagnetic radiation of multipolarity $l$. This radiation will have an angular distribution defined by $l$ and $m = m_{J_2} - m_{J_1}$ [Bla52]:

$$Z_{lm}(\theta) = \frac{1}{2} \left[ 1 - \frac{m(m+1)}{l(l+1)} \right] |Y_{l,m+1}|^2 + \frac{1}{2} \left[ 1 - \frac{m(m-1)}{l(l+1)} \right] |Y_{l,m-1}|^2 + \frac{m^2}{l(l+1)} |Y_{l,m}|^2$$  \hspace{1cm} (3.4)

Where $|m| \leq l$ and the $Y_{l,m}$'s are the spherical harmonics. For each initial sub-state $m_{J_1}$, we must sum over all possible $m$'s linking the two states to arrive at the overall angular distribution of the $J_1 \rightarrow J_2$ transition via multipolarity $l$:

$$\omega_l(\theta) = \sum_{all \ m_{J_1}} \sum_{all \ possible \ m's \ from \ m_{J_1}} Z_{lm}(\theta)$$  \hspace{1cm} (3.5)

If, from each initial $m_{J_1}$, all $m = -l, \ldots, 0, \ldots, +l$ decays are possible then it can be shown that the $\omega_l(\theta)$ is isotropic. However, often not all $m$'s that are allowed are actually possible simply because the final state may have too low a spin, $J_2$. Two cases are illustrated in Fig. 3.15 below.
Fig 3.15: Two E1 decays from an initial 1/2⁺ state are shown above. In (a) all allowed m values are possible and the decay is isotropic. In (b) the dotted decays are not possible (even though they are "allowed" for l=1) since the mJ₂=+3/2 and the mJ₂=-3/2 are not present for the final state J=1/2⁻. Thus in (b) there is an "extra" m=0 term which leads to an overall sin²θ term and makes the decay non-isotropic.
Of course, when considering nuclear reactions it may be that only a few of the possible initial \( m \) sub-states are populated. In our case of the \( ^{15}\text{N}(\alpha,\gamma)^{19}\text{F} \) experiment, if we take the direction of the \( \alpha \)-beam to be the \( z \)-axis, then the projection of the orbital angular momentum, \( l_o \), along the \( z \)-axis must be 0 (i.e. \( m_l = 0 \)). Since the intrinsic spin of \( ^{15}\text{N} \) is \( \frac{1}{2} \) and that of the \( \alpha \)-particle is 0, this means that only the \( m_f = \pm \frac{1}{2} \) sub-states can be populated in \( ^{19}\text{F} \). Thus for our \( 3/2^+ \) state of interest we need consider only decays from the \( m = \pm \frac{1}{2} \) sub-states.

Of all the decays that are seen from this state (see Section 4.4) we see that the only ones that may have a small non-isotropic component are the relatively weak decays to the \( 3/2^+ \) states at 1554.0 and 3908.2 keV, \( \text{assuming} \) they have an E2 contribution and are not pure M1. It is possible to calculate the maximum error incurred in \( \omega \gamma \) in assuming that these decays are isotropic. By using eq. 3.4 we can calculate the angular distribution for these transition in the worst case scenario of pure E2 decay:

\[
\omega(\theta) = \frac{15}{8\pi} \left( 1 + \frac{1}{3} \cos^4 \theta \right) \tag{3.6}
\]

However since the detectors are in a close geometry to the target chamber we essentially integrate over a range of \( \theta \)'s \( (-40^\circ \leq \theta \leq -140^\circ) \). We can compare the global average value of this function with the average value for our detector solid angle by integrating \( \omega(\theta) \) [Fig 3.16]. Doing this, we see that the maximum percentage error in assuming these decays are isotropic is \( \sim 8 \% \). Again, any M1 admixture in the decays will reduce this error. Furthermore, since these two decays together constitute less than \( \sim 10\% \) of the total resonance strength, the overall maximum error incurred in \( \omega \gamma \) in assuming that they are isotropic is less than 1\%.
Fig 3.16: Plotted is the angular distribution function for the two weak $3/2^+ \rightarrow 3/2^+$ transitions in the "worst-case" scenario of pure E2 multipolarity:

$$\omega(\theta) = \frac{15}{8\pi} (1 + \frac{1}{3}\cos^4 \theta)$$

Any M1 admixture will produce a more isotropic function.

The approximate angular range of the detector solid angle is $-40^\circ \leq \theta \leq -140^\circ$. If the yield in this region is extrapolated isotropically, as is done in the analysis, we underestimate the total yield by for these two decay branches by $\sim 8\%$. (We cannot make an *ad hoc* 8% correction since we do not know the M1 admixture in the transition).
3.12 The Efficiency of Ge detectors and the GEANT Simulation

The efficiencies of the Ge detectors was needed to determine the resonance strength, $\omega \gamma$, of the $3/2^+$ state of interest. Since the efficiency of the Ge detectors is a function of gamma-ray energy and position along the extended gas target it was necessary to carry out a Monte-Carlo simulation to determine all the efficiency contours. The GEANT code, version 3.21 [Bru93] was chosen since it is the most comprehensive simulation package available and considerable experience in its use has been gained at the University of Stuttgart. This simulation took into account the following electromagnetic processes between 10 keV and 7200 keV:

- Photoelectric effect: $X(\gamma,e^-)X$
- Pair-production: $X(\gamma,e^-)Xe^+$
- Compton Scattering: $e'(\gamma,\gamma)e^-$ (inelastic)
- Rayleigh effect: $e'(\gamma,\gamma)e^-$ (elastic)
- Positron Annihilation $e^-(e^+,\gamma)\gamma$
- Bremsstrahlung
- Photofission of heavy elements $X(\gamma,Y)Z$

The GEANT input geometry and ten sample rays are shown in Fig 3.17 below. To save CPU time only positive values of x (from 0 to +200mm) were chosen as ray sources since the geometry is symmetric about the y-z plane containing the center of the chamber. For each position and energy 250,000 rays were calculated.

To normalize the results of the GEANT simulation an absolute efficiency curve as a function of position for $E_\gamma = 1332.5$ keV was generated using a $^{60}$Co source ($t_{1/2}=1925.5$ days) with a known decay rate of 393 kBq in 1/1/84 [Fig 3.18]. The sharp cut-off in the position dependence is due to the lead collimator in front of the Ge detector.

The “true” calculated efficiency contours are shown in Fig 3.19 and the efficiency at x=0 as function of $\gamma$-ray energy is shown in Fig 3.20.
Fig 3.17: An example of the geometry used by the GEANT Monte-Carlo simulation package to calculate the $\gamma$-detection efficiency as a function of the $\gamma$-ray energy $E_\gamma$ and position along the beamline, $x$. Ten sample rays are shown emanating from $x=0$. Here the closest possible target-detector geometry is shown. The actual GEANT simulation for detector 1 for our run considered a beam-detector separation of 84 mm as shown in Fig 3.14.
Fig 3.18: The absolute efficiency calibration measurement for Detector 1. This curve was produced by measuring the yield of the 1332.5 keV line of $^{60}$Co as a function of position. The $^{60}$Co source had a known decay rate of 393 kBq in 1/1/1984 ($t_{1/2}$=1925.5 days). The GEANT simulation output was normalized using this data.
Fig 3.19: The Efficiency contours as calculated by the Monte-Carlo simulation GEANT. Note the y-axis is logarithmic scale. The output was normalized to the 1332.5 keV curve (see Fig. 3.18). The widths of the higher energy contours are slightly larger since these gamma-rays can penetrate more of the lead collimators.
Fig 3.20: The efficiency as a function of the $\gamma$-ray transition energy as calculated by GEANT for the center of the target chamber, $x=0$. The results have been normalized to the absolute efficiency measurement for the 1332.5 keV line of $^{60}$Co [see Fig 3.18]. The straight line through the data points is a simple interpolation intended to guide the eye. (The error in the measured efficiency has been propagated to the calculated points).
Chapter 4

ANALYSIS AND DISCUSSION

4.1 Introduction

The $^{15}$N(α,γ)$^{19}$F experiment was carried out to determine the resonance energy, $E_{\text{res}}$, total width, $\Gamma_{\text{tot}}$, and resonance strength, $\omega \gamma$, of the "missing" 3/2$^+$ state in $^{19}$F. Since it had been reported [Smo61] (but never subsequently confirmed) that this state would lie at approximately the same excitation energy as a known 7/2$^+$ state at $E_x \sim 7.1$ MeV [Ajz87], a primary goal was identifying those γ-decays coming from the 3/2$^+$ state versus those from the 7/2$^+$. Once this identification was made, the yield of this transition as a function of excitation energy gave the resonance energy, $E_{\text{res}}$, and the total width, $\Gamma_{\text{tot}}$, of the resonance. To obtain the resonance strength, $\omega \gamma$, the absolute efficiency of the Ge Detector 1 was determined and yields to all allowed final states, from the initial 3/2$^+$, summed. These procedures are described below in more detail.
4.2 Identification of the $3/2^+$ vs. $7/2^+$ State at $E_x \sim 7.1$ MeV

To identify those decays coming from the $3/2^+$ state as opposed to those coming from the $7/2^+$ state we relied on the fact that only E1, M1 and E2 $\gamma$-decay multipolarities have high transition probabilities [see Section 2.5]. As shown in Fig 2.3, M2 and higher multipolarities are greatly suppressed, and one would not thus expect a large yield from such transitions in a capture experiment. Thus, for both members of the supposed doublet we can write the possible final states that we would expect:

$$3/2^+ \rightarrow 7/2^+, 5/2^+, 5/2^*, 3/2^+, 3/2^*, 1/2^+, 1/2^*$$

*suppressed if from $7/2^*$

$$7/2^+ \rightarrow 11/2^+, 9/2^+, 9/2^*, 7/2^+, 7/2^*, 5/2^+, 5/2^*, 3/2^*$$

*suppressed if from $3/2^*$

It should be noted that there is no a priori guarantee that any transitions that uniquely determine the initial state must occur as it is entirely possible that decays would occur only to the common final states of $7/2^+, 5/2^+, 5/2^*, or 3/2^+$. However, since $^{15}$N has a ground state spin of $1/2^-$, this means that the $3/2^+$ state will be preferentially populated over the $7/2^+$ state since they are $l=1$ and $l=3$ captures respectively. This fact was obviously helpful in our search for the $3/2^+$ state.

4.3 Location and Width of the $3/2^+$ State of Interest

The existence of the $3/2^+$ state of interest was inferred from the enhanced yield of the Resonance (R) $\rightarrow$ 1459 keV ($3/2^-$) state transition at $E_x \sim 7.1$ MeV. Two $\gamma$-spectra, one off-resonance ($E_x = 3817$ keV) and another on-resonance ($E_x = 3910$ keV), are compared in Fig 4.1. To determine accurately the excitation energy corresponding to each run, we used the Ge-Detector $\gamma$-spectra to find the centroids of the R $\rightarrow$ 0, R $\rightarrow$ 197.1 and R $\rightarrow$ 1459 keV transition peaks, and averaged these to arrive at $E_x$ for each run:

$$E_x = \frac{E_{R \rightarrow 0} + (E_{R \rightarrow 197} + 197.14) + (E_{R \rightarrow 1458.7} + 1458.7)}{3}$$

(4.1)
These transitions were selected because, firstly, they were intense enough so that the statistical error in determining the centroid was low, and, secondly, they were at high enough energy that the $\gamma$-background was small. As a check, this method was used to find the location of a known resonance, the $7/2^-$ state at $E_x = 6926.5 \pm 1.7$ keV [Ajz87]. From our data we determined the location of this resonance to be $6924.9 \pm 0.9$ keV, in good agreement with the standard compilations. (The resonance strength of this state was also determined as described in Section 4.4).

In this fashion the excitation energies were found for the runs that scanned the $3/2^+$ state of interest. The measured yield per incident alpha particle of the $R \rightarrow 1459$ keV transition at each energy are plotted in Fig 4.2, together with the background yields of that transition at a few energies below the resonance. To determine the centroid and width of the $3/2^+$ state the background subtracted yields at each of the 10 energies that scanned the state were determined. In order to find the best-fit parameters of the Breit-Wigner function that described the resonance a computer program was written which varied the overall normalization, $N$, the total width, $\Gamma$, and the resonance energy, $E_{\text{res}}$. The calculated yields at each energy, $Y_i$, were then determined as coming from the intrinsic Breit-Wigner averaged over a 10 keV CM energy loss bite:

$$Y_{\text{calc},i} = \frac{1}{10.0} \int_{E_i-10}^{E_i+10} \frac{N}{(E - E_{\text{res}})^2 + (\Gamma/2)^2} dE$$

(4.2)

For each parameter set $(N, \Gamma, E_{\text{res}}, k)$, the total deviation from the 10 experimental values was determined as the square root of the sum of the squares of the fractional deviations for each energy:

$$\sigma_k = \sqrt{\sum_{i=1}^{10} \left( \frac{Y_{\exp} - Y_{\text{calc}}}{Y_{\exp}} \right)_i^2}$$

(4.3)

The parameter set that gave the least deviation, $\sigma_k$, then gave our best-fit values for the intrinsic Breit-Wigner curve: $E_{\text{res}} = 7101 \pm 1$ keV, and $\Gamma = 28 \pm 1.1$ keV.
Fig 4.1: Comparison of the $\gamma$-spectra (a) off resonance at $E_{\gamma}=3817$ keV and (b) on resonance at $E_{\gamma}=3910$ keV. Even off resonance radiative captures may occur through the tails of broad resonances (BT). Note the enhanced yield of the $R \rightarrow 1459$ keV transition, which is the signature of the $3/2^+$ state, on resonance. In both (a) and (b) most lines below ~1700 keV are secondary and tertiary decays.
Fig 4.2: The excitation function of the $R \rightarrow 1459$ keV transition. The solid line is a least-squares best fit to the data using a Breit-Wigner function folded with a 10 keV center-of-mass target energy loss (Section 4.3). The parameters of the intrinsic Breit-Wigner are: $E_{\text{res}} = 7101 \pm 1$ keV, and $\Gamma = 28 \pm 1.1$ keV. The corresponding solid curve above has width $\sim 30$ keV. The dotted line is the assumed uniform background extrapolated from lower energies. Higher energies were not possible due to the voltage limit of the accelerator. The x-axis energy uncertainty in the data is approximately the size of the dots.
4.4 The Resonance Strength of the 3/2\(^+\) State

To determine the resonance strength, \(\omega\gamma\), of the 3/2\(^+\) state we examined all possible E1, E2 and M1 decays from that state and assumed that they would not be fed by the nearby 7/2\(^+\) state. This is a valid assumption since although the 7/2\(^+\) state is broad, its peak lies about 13 keV higher in energy. Furthermore, because the 7/2\(^+\) state is an \(l=3\) resonance its population is suppressed in the alpha capture process in the first place compared to the 3/2\(^+\) state which is an \(l=1\) capture.

The following transitions were found to have a significantly enhanced yield at \(E_\alpha = 3910\) keV, the peak of the 3/2\(^+\) resonance:

\[
\begin{align*}
7101 \text{ keV (3/2\(^+\))} & \rightarrow 197.1 \text{ keV (5/2\(^+\)) M1/E2} \\
& \rightarrow 1458.7 \text{ keV (3/2\(^+)\) E1 (signature of the 3/2\(^+\) state)} \\
& \rightarrow 1554.0 \text{ keV (3/2\(^+\)) M1/E2} \\
& \rightarrow 3908.2 \text{ keV (3/2\(^+\)) M1/E2} \\
& \rightarrow 4377.7 \text{ keV (7/2\(^+\)) E2} \\
& \rightarrow 5463.5 \text{ keV (7/2\(^+\)) E2}
\end{align*}
\]

The yield curves of these transitions are shown in Fig 4.3. Since the peaks of the yield curves all coincide, this further bolsters our assumption that the 7/2\(^+\) state is not significantly contributing to the yields. The maximum yields of these transitions after subtracting the background are listed in Table 4.1 below, and the experimentally determined \(\gamma\)-decay scheme from the 3/2\(^+\) level is shown in Fig. 4.4

<table>
<thead>
<tr>
<th>Transition</th>
<th>(\gamma)-Energy (keV)</th>
<th>Detected Yield per (\alpha)</th>
<th>Efficiency Compensated Yield per (\alpha)</th>
<th>% Branch</th>
</tr>
</thead>
<tbody>
<tr>
<td>R \rightarrow 197.1</td>
<td>6904</td>
<td>(1.4 \pm 0.07)E-15</td>
<td>(1.24 \pm 0.06)E-12</td>
<td>28 \pm 2</td>
</tr>
<tr>
<td>R \rightarrow 1458.7</td>
<td>5642</td>
<td>(2.9 \pm 0.1)E-15</td>
<td>(1.85 \pm 0.07)E-12</td>
<td>41 \pm 2</td>
</tr>
<tr>
<td>R \rightarrow 1554.0</td>
<td>5547</td>
<td>(4.9 \pm 1.7)E-16</td>
<td>(2.9 \pm 1)E-13</td>
<td>6 \pm 3</td>
</tr>
<tr>
<td>R \rightarrow 3908.2</td>
<td>3193</td>
<td>(4.2 \pm 2.1)E-16</td>
<td>(1.4 \pm 0.7)E-13</td>
<td>3 \pm 2</td>
</tr>
<tr>
<td>R \rightarrow 4377.7</td>
<td>2724</td>
<td>(2.5 \pm 0.3)E-15</td>
<td>(7.9 \pm 1)E-13</td>
<td>18 \pm 3</td>
</tr>
<tr>
<td>R \rightarrow 5463.5</td>
<td>1638</td>
<td>(9.3 \pm 4)E-16</td>
<td>(1.8 \pm 0.8)E-13</td>
<td>4 \pm 2</td>
</tr>
</tbody>
</table>

Table 4.1: Peak \(\gamma\)-Yields from the 3/2\(^+\) Resonance at 7101 keV
Fig 4.3: The efficiency compensated yields of all resonant transitions from the 3/2$^+$ state of $^{19}$F at $E_x=7101$ keV. The yields of the $R \rightarrow 3908$ keV and $R \rightarrow 5464$ keV transitions have been multiplied by 0.1 and 0.5, respectively, in the plot to separate the data points. The "signature" transition of the 3/2$^+$ state, $R \rightarrow 1459$ is also the most intense. The x-axis energy uncertainty is smaller than the size of the markers.
Fig 4.4: The decay scheme from the 3/2+ state of $^{19}$F. The γ-ray energies in MeV together with the % branching ratios are shown for each transition. The darker 5.642 MeV transition is the "signature" of the 3/2+ state.
For the $^{15}\text{N}(\alpha,\gamma)^{19}\text{F}$ reaction, we can write the infinitesimal $\gamma$-yield per incident alpha as:

$$dY = \sigma n \, dx$$

where $\sigma$ is the cross section for the process, $n$ is the number of active target nuclei per unit volume, and $dx$ the infinitesimal distance travelled by the incident alpha particle through the target. Since the stopping cross section, $S$, is defined:

$$S \equiv -\frac{1}{n} \frac{dE_{\text{lab}}}{dx} \quad (4.4)$$

we can change variables and write the integrated yield as:

$$Y = -\frac{1}{S} \int_{E_i}^{E_f} \sigma(E_{\text{lab}}) \, dE_{\text{lab}} = \frac{1}{S} \frac{M_{^{15}\text{N}} + M_\alpha}{M_{^{15}\text{N}}} \int_{E_i}^{E_f} \sigma(E_{\text{CM}}) \, dE_{\text{CM}} \quad (4.5)$$

Furthermore, because the energy dependence of the cross section is well described by a Breit-Wigner function, we can write:

$$Y = \frac{1}{S} \frac{M_{^{15}\text{N}} + M_\alpha}{M_{^{15}\text{N}}} \frac{\lambda^2}{4\pi} (\omega_\gamma) \Gamma_{\text{tot}} \int_{E_i}^{E_f} \frac{dE_{\text{CM}}}{(E_{\text{CM}} - E_R)^2 + \left(\Gamma_{\text{tot}}/2\right)^2} \quad (4.6)$$

where $\lambda = h/p_\alpha$, the de Broglie wavelength of the incident alpha particle.
A complication having to do with the energy and position dependence of the Ge-detector efficiencies arises when relating the measured $\gamma$-yield of a given transition, $Y_M$, to the yield calculated in eq. 4.6. Rigorously, one must also include the efficiency, $e(E, x)$, in the integral in eq. 4.6 to arrive at the measured yield. In practice, this is not only complicated but unnecessary. The energy dependence of the efficiency for a given transition is very weak over the approximately 13 keV energy loss of the alpha particle and can simply be taken outside the integral. However, the position dependence of the efficiency must be examined more closely.

It is important to know the range of positions from which the $\gamma$-decays from $^{19}F^*$ contribute to the yield since this determines the effective target length and thus the limits of the integration in eq 4.6 above. Two factors define the effective target length: one is the density of target atoms (the pressure profile) shown in Fig 4.5 and the other is the position dependence of the efficiency. Both these factors have a nearly step-function position dependence and were approximated as such for the analysis. (The position dependence of the efficiency was determined in Section 3.12.) A sample efficiency step-function approximation is shown in Fig. 4.6, and the energy dependence of the effective step-function efficiencies are plotted in Fig 4.7. As a summary, some sample approximated efficiency curves, along with the gas pressure profile and the physical target chamber are shown together in Fig. 4.8. As can be seen, in our energy regime the effective target length is always defined by the efficiency step functions since these have smaller widths than the gas pressure profile width. The energy dependence of the effective target length is shown in Fig 4.9.

Since both the energy- and position-dependence of the efficiency can be removed from the integral by an appropriate choice of the limits of integration, the measured yield for a
Fig 4.5: The pressure profile in the target chamber [Kne87]: the steep drop in pressure is due to the differential pumping. Even though the physical extension of the target chamber is 60 mm, the pressure profile was approximated by a step of width 83 mm as shown. This width was chosen to equalize the added area at the top of the step with that subtracted at the bottom.
Measured Efficiency (Detector 1 @ 1332.5 keV)

0.008
0.007
0.006
0.005
0.004
0.003
0.002
0.001
0
-100 -80 -60 -40

Position x (mm)

Effective Extension of 1.5 mbar $^{15}$N Target Chamber Pressure

Fig 4.6: A sample of the step-function approximations to the efficiency curves. This data are for the 1332.5 keV transition of $^{60}$Co. The width of the step is determined from the width of the curve at half the peak (x=0) efficiency. The height of the approximated step function is then chosen to equalize the areas lost and gained compared to the real curve. The areas outside the effective extension of the chamber pressure are multiplied by 1/20 when comparing the lost and gained areas since the pressure profile is diminished by approximately this factor in this region. The efficiency falls off around $x = \pm 35$ mm due to the attenuation of the lead collimators.
Fig. 4.7: The energy dependence of the effective efficiencies, as determined by the heights of the step function efficiency approximations (e.g., Fig. 4.6). Also shown for reference are the actual normalized data points calculated by GEANT for $x=0$ (see Section 3.12). The effective efficiencies for the lower energy γ-ray transitions are more highly suppressed compared to their $x=0$ values, since the lead collimators have greater attenuation at lower energies. The solid line through the data points is a simple interpolation intended to guide the eye.
Fig 4.8: The effective target length is given by folding the pressure profile with the efficiency curves. The approximated forms of the pressure profile (from Fig 4.5) and 3 sample approximated efficiency curves (eg: Fig 4.6) are shown here together with the top half of the physical target chamber (the entire target chamber is shown in Fig 3.4). Since the higher energy $\gamma$-rays are more penetrating, the widths of their efficiency curves are larger. Note that the efficiency step functions always determine the effective target lengths since their widths are always less than the pressure profile's.
Fig 4.9: The energy dependence of the effective target length. The effective target length is defined by the widths of the efficiency step functions for different energies as illustrated in Fig 4.6.
given transition, \( Y_M \), may be related to the calculated total yield, \( Y \):

\[
Y_M = \varepsilon_{E\gamma} B Y = \frac{\varepsilon_{E\gamma}}{S} \frac{B M_{15N} + M_a}{M_{15N}} \frac{\lambda^2}{4\pi} (\omega\gamma) \Gamma_{\text{tot}} \int_{E_f}^{E_i} \frac{dE_{CM}}{(E_{CM} - E_R)^2 + (\Gamma_{\text{tot}}/2)^2}
\]

(4.7)

where \( B \) is the branching ratio of the given transition and \( \varepsilon_{E\gamma} \) is the effective detection efficiency of the gamma ray. Since all factors on the right except for \( \omega\gamma \) are determined, one can solve for it's value. We find \( \omega\gamma = 0.77 \pm 0.11 \text{ eV} \), and, thus, \( \Gamma_\gamma = 0.38 \pm 0.06 \text{ eV} \).

To check our method of analysis we also determined the resonance strength of the \( 7/2^- \) state of \( ^{19}\text{F} \) at \( E_\pi = 6.925 \) MeV, which also has an approximately isotropic \( \gamma \)-decay angular distribution and which had been previously measured by [Dix77]. Our results are consistent with [Dix77] within the quoted uncertainties, as shown in Table 4.2.

<table>
<thead>
<tr>
<th>( E_\pi ) (kev)</th>
<th>( \omega\gamma ) (eV)</th>
<th>( ^{15}\text{N}(\alpha,\gamma)^{19}\text{F} )</th>
<th>( ^{15}\text{N}(\alpha,\gamma)^{19}\text{F} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>6924.9 ± 0.9</td>
<td>8.4 ± 1.8</td>
<td>6925 ± 2</td>
<td>9.7 ± 1.4</td>
</tr>
</tbody>
</table>

Table 4.2: Comparison of results with measurement of [Dix77] for the \( 7/2^- \) state of \( ^{19}\text{F} \) at \( E_\pi = 6925 \) keV.

Having thus checked the basic accuracy of our method of analysis against a known resonance with a similar \( \gamma \)-decay angular distribution in the same energy region, we did not attempt to further check our method against all the known resonance strengths in \(^{19}\text{F} \), for two main reasons. Firstly, the degree of approximation in assuming that the \( \gamma \)-decays are isotropic varies from resonance to resonance; secondly, the intrinsic width of the resonance can affect the accuracy of the method: for an excited state with a narrow width, the resonance is located entirely within the target and centered at a precise point. Thus one needs to make no effective efficiency approximations that account for the spatial extent of the resonant yield (as we did for the \( 3/2^+ \) state, eg: Fig 4.6). For these reasons, one does not necessarily learn more about the accuracy of this method of obtaining the resonance strength simply by comparing against a larger sample of known resonances.
4.5 Results

A 3/2\(^+\) state in \(^{19}\)F at \(E_x=7101\) keV was observed by detecting its unique \(\gamma\)-signature (the \(R \rightarrow 1459\) keV transition) using the \(^{15}\)N(\(\alpha,\gamma\))\(^{19}\)F reaction. This state is the isospin mirror of the astrophysically important 3/2\(^+\) state recently observed in \(^{19}\)Ne at \(E_x\approx 7070\) keV [Cos95, Reh95, Utk98].

The characteristics of the 3/2\(^+\) state of \(^{19}\)F are summarised in Table 4.3 below. The measured value of the alpha width, \(\Gamma_\alpha = 28 \pm 1.1\) keV implies a reduced alpha width, \(\theta_\alpha^2 = 0.05\).

<table>
<thead>
<tr>
<th>(E_{\text{Res}})</th>
<th>(7101 \pm 1) keV</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Gamma_{\text{tot}})</td>
<td>(28 \pm 1.1) keV</td>
</tr>
<tr>
<td>(\Gamma_\alpha)</td>
<td>(28 \pm 1.1) keV</td>
</tr>
<tr>
<td>(\Gamma_\gamma)</td>
<td>(0.38 \pm 0.06) eV</td>
</tr>
<tr>
<td>(\omega\gamma)</td>
<td>(0.77 \pm 0.11) eV</td>
</tr>
</tbody>
</table>

Table 4.3: The characteristics of the 3/2\(^+\) State of \(^{19}\)F at 7.101 MeV as determined by this study.

The main sources of errors in determining the location and width of the 3/2\(^+\) resonant state were the accuracy of the channel-energy calibration and the statistical uncertainty in the individual peak counts plotted in Fig. 4.2. The largest error in determining the resonance strength, \(\omega\gamma\), was the uncertainty in the absolute efficiency, which, in turn, was due mainly to the low number of counts “detected” at the higher energies in the GEANT Monte-Carlo simulation. (The uncertainty in the stopping power which also enters into the yield calculation, eq. 4.7, was much smaller than this.)
Lastly, it should be mentioned that no statistically significant resonant $\gamma$-yield was observed to $11/2^+$, $9/2^-$, $9/2^+$, or $7/2^-$ states which would be populated in decays from the $7/2^+$ state at $E_x=7.114$ MeV but not in decays from the $3/2^+$ state at 7.101 MeV. Although resonant transitions were observed to some states ($3/2^+$, $5/2^+$, $7/2^+$) that may have been fed from both a $3/2^+$ and a $7/2^+$, these transitions were thought to have originated from the $3/2^+$ state based on the location of their maxima. The reason that we did not observe the $7/2^+$ state using the $^{15}\text{N}(\alpha,\gamma)^{19}\text{F}$ reaction, may be because this is an $l=3$ resonance for $^{15}\text{N} + \alpha$, and is thus suppressed by the angular-momentum barrier in the capture process.
Chapter 5

Conclusions

5.1 Introduction

The shift from explosive hydrogen burning in the HotCNO cycles to thermonuclear runaway in the rp-process controls the production of low mass (A<19) versus higher mass (A=19–68) nuclei in explosive astrophysical environments such as novae. This transition also determines the power output and the timescale of the explosive event and is, thus, clearly important to understand. (The power output of the rp-process is ~100 times that of the HotCNO cycles [Wal81]). One of the pathways for this transition to occur is via the $^{18}\text{F}(p,\gamma)^{19}\text{Ne}$ reaction which channels material away from the HotCNO-II cycle to the rp-process. This reaction competes with the $^{18}\text{F}(p,\alpha)^{15}\text{O}$ reaction which returns material back to the HotCNO cycles. To determine the temperatures and densities
under which such a transition may occur one must know the locations, widths and spin-parities of resonant states in the compound nucleus $^{19}$Ne just above the $^{18}$F + p threshold. The purpose of this work was to check on the existence of an astrophysically important $3/2^+$ state recently reported in $^{19}$Ne at $E_x \approx 7.07$ MeV [Cos95, Reh95] by searching for its isospin mirror state in $^{19}$F. We have observed this state via the $^{15}$N($\alpha$,\gamma)$^{19}$F reaction at $E_\alpha = 7101 \pm 1$ keV, with alpha width, $\Gamma_\alpha \equiv \Gamma_{\text{tot}} = 28 \pm 1.1$ keV and ($\alpha$,\gamma) resonance strength, $\omega\gamma = 0.77 \pm 0.11$ eV.

The observed parameters of this resonance are consistent with the earlier $^{15}$N($\alpha$,\alpha)$^{15}$N elastic scattering data of [Smo61] as shown in Fig. 5.1, which was generated using the method of Section 2.4. Comparison with Fig. 2.2 shows that the $7/2^+$ resonance dominates the resonant elastic scattering cross-section - the input width of the $3/2^+$ state seems to have only a small effect on the fitted cross-section. Within the range of parameters we consider here, the fit to the elastic scattering data is essentially insensitive to the input width of the $3/2^+$ state.

![Fig 5.1: Fit to the $^{15}$N($\alpha$,\alpha)$^{15}$N data of [Smo61] based on the parameters of the $3/2^+$ state as determined from this study. The input values for $E_{\text{res}}$, $J^*$, and, $\Gamma_\alpha$, for the two resonances ($7/2^+$-$3/2^+$ doublet) at $E_x = 7.1$ MeV ($E_\alpha = 3.91$ MeV) are:

Doublet: $E_{\text{res}} = 7.11$ MeV, $J^* = 7/2^+$, $\Gamma = \Gamma_\alpha = 32$ keV,
$E_{\text{res}} = 7.10$ MeV, $J^* = 3/2^+$, $\Gamma = \Gamma_\alpha = 28$ keV.

This figure should be compared to Fig. 2.2 to appreciate the dominant effect of the $7/2^+$ state.

\[ \frac{d\sigma}{d\omega} \text{ (mb/str)} \]

\[ \text{E}_\alpha \text{ (MeV)} \]

<table>
<thead>
<tr>
<th>$\theta_{\text{cm}}$</th>
<th>$\frac{d\sigma}{d\omega}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$90^\circ$</td>
<td></td>
</tr>
<tr>
<td>$125.3^\circ$</td>
<td></td>
</tr>
<tr>
<td>$169.1^\circ$</td>
<td></td>
</tr>
</tbody>
</table>

\[ \text{E}_\alpha \text{ (MeV)} \]

\[ \frac{d\sigma}{d\omega} \text{ (mb/str)} \]
5.2 Translation of the $^{19}$F results to the $^{19}$Ne System

Since there was a significant discrepancy in the reported alpha widths of the mirror $^{19}$Ne state [see Section 2.3], we have examined this inconsistency under the assumption of equal reduced alpha widths for the isospin mirror states. If the underlying nuclear structure of mirror states is similar then the alpha-particle widths should scale with the Coulomb barrier penetrabilities, as described in Section 1.7. Using eq 1.23, we can explicitly "correct" for these different penetrabilities to translate results between the mirror states:

$$ (\Gamma_\alpha)_{^{19}Ne} = \left[ \frac{\rho}{F_i^2(\eta, \rho) + G_i^2(\eta, \rho)} \right] \left[ \frac{F_i^2(\eta, \rho) + G_i^2(\eta, \rho)}{\rho} \right] (\Gamma_\alpha)_{^{19}F} \quad (5.1) $$

where $\eta = \frac{e^2 z_i z_j}{\hbar}$ and $\rho = \mu \nu v / \hbar$. Our measurement of $(\Gamma_\alpha)_{^{19}F} = 28 \pm 1.1 \text{ keV}$ translates to $(\Gamma_\alpha)_{^{19}Ne} \approx 30 \text{ keV}$ for the $^{19}$Ne state at $-7.07 \text{ MeV}$, in agreement with the results of [Cos95] and [Utk98]. The results for this important state are listed in Table 5.1. We also include the new information in Fig 5.2.

<table>
<thead>
<tr>
<th>Study</th>
<th>$E_x$ (keV)</th>
<th>$\Gamma_{tot}$ (keV)</th>
<th>$\Gamma_\alpha$ (keV)</th>
<th>$\Gamma_p$ (keV)</th>
<th>$\Gamma_\gamma$ (eV)</th>
<th>$J^\pi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{19}$F($^{3}$He,\alpha)$^{19}$Ne [Utk98]</td>
<td>7070 $\pm$ 7</td>
<td>39 $\pm$ 10</td>
<td>25 $\pm$ 7</td>
<td>14.5 $\pm$ 4</td>
<td>$-$</td>
<td>(3/2$^+$)</td>
</tr>
<tr>
<td>$p(^{19}$F,$^{15}$O)$\alpha$ [Reh95,96]</td>
<td>7063 $\pm$ 4</td>
<td>13.6 $\pm$ 4.6</td>
<td>8.6 $\pm$ 2.5</td>
<td>5 $\pm$ 1.6</td>
<td>$-$</td>
<td>(3/2$^+$)</td>
</tr>
<tr>
<td>$p(^{19}$F,$^{15}$O)$\alpha$ [Cos95]</td>
<td>7049 $\pm$ 15</td>
<td>37 $\pm$ 5</td>
<td>$-18.5$</td>
<td>$-18.5$</td>
<td>$-$</td>
<td>(3/2$^+$)</td>
</tr>
<tr>
<td>$p(^{18}$F,$^{19}$Ne)$\gamma$ [Reh97]</td>
<td>$-$</td>
<td>$-$</td>
<td>$-$</td>
<td>$-$</td>
<td>$\leq 3$</td>
<td>(3/2$^+$)</td>
</tr>
<tr>
<td>This study: $^{15}$N($\alpha,\gamma)^{19}$F</td>
<td>$-$</td>
<td>$\geq 30$</td>
<td>$=30$</td>
<td>$-$</td>
<td>$=0.38$</td>
<td>(3/2$^+$)</td>
</tr>
</tbody>
</table>

Table 5.1: The properties of the $-7.07 \text{ MeV}$ state of $^{19}$Ne from various studies. The value of $\Gamma_\alpha$ calculated for $^{19}$Ne from the $^{15}$N($\alpha,\gamma)^{19}$F study is based on the assumption that the mirror states have equal alpha structure (see Section 2.3). The value of $\Gamma_\gamma$ for the $^{19}$Ne state obtained from the $^{15}$N($\alpha,\gamma)^{19}$F study assumes that mirror states will have the same $\gamma$-width.
Fig 5.2: The energy spectra of the mirror nuclei $^{19}$F and $^{19}$Ne above the $^{18}$F+p threshold including results as determined by this study (bold solid arrow) and others [thin solid arrows-Utk98; dotted arrow-Che97]. Vertical arrows show the locations of the Gamow window, $E_0 \pm \Delta E_0/2$, for temperatures $T_\gamma=0.2, 0.5, 1.0$. Figure adapted from [Utk98].
Based on the assumption of the same $\Gamma_\gamma$ for the mirror states we find that $\Gamma_\gamma = 0.38$ eV for the 7.070 MeV state of $^{19}$Ne which is consistent with the limit of [Reh97] of $\Gamma_\gamma \leq 3$ eV.

The latest calculation of the competing, $^{18}$F(p,γ)$^{19}$Ne, and, $^{18}$F(p,α)$^{15}$O, astrophysical reaction rates [Utk98], indicates that the breakout reaction, $^{18}$F(p,γ)$^{19}$Ne, is a factor of $10^3$–$10^4$ slower than the recycling reaction, $^{18}$F(p,α)$^{15}$O at $0.1 < T_9 < 1.0$. This calculation assumed that the radiative partial width of the then unconfirmed $3/2^+$ state of $^{19}$Ne at 7.07 MeV was, $\Gamma_\gamma = 1$ eV. Thus, the present work, which confirms the existence of this state and finds that $\Gamma_\gamma = 0.38$ eV for its radiative width, further downgrades the importance of the $^{18}$F(p,γ)$^{19}$Ne reaction as a pathway for HotCNO breakout to the rp-process. However, even with the slightly modified parameters of the 7.07 MeV state of $^{19}$Ne as determined from this study, the basic conclusions of [Utk98] remain unchanged: the breakout reaction, $^{18}$F(p,γ)$^{19}$Ne, will be $10^3$–$10^4$ times slower than the recycling reaction, $^{18}$F(p,α)$^{15}$O at $0.1 < T_9 < 1.0$. In this temperature range, the $^{15}$O(α,γ)$^{19}$Ne reaction is a better candidate for HotCNO breakout to the rp-process since it is more effective in producing $^{19}$Ne. However, this reaction rate is still uncertain by about an order of magnitude [Oli97].

Further direct experiments using radioactive $^{18}$F beams on hydrogenous targets may resolve the discrepancy in the alpha width of this astrophysically important $^{19}$Ne state. Lastly, from a comparison of $^{19}$F states in this region, it should be noted that at least seven states in the astrophysically important region ($6.4$ MeV $\leq E_x \leq 7.4$ MeV) are still missing in the $^{19}$Ne spectrum. However, by comparison with states in the $^{19}$F spectrum [Utk98], these states are thought to have small reduced charged particle widths and are not believed to make a large contribution to the astrophysical reaction rates.
Appendix: The Coulomb Wavefunctions

The regular and irregular Coulomb wavefunctions, \( F_\ell(\eta, \rho) \) and \( G_\ell(\eta, \rho) \) respectively, are the solutions to the Schrödinger equation in a spherically symmetric Coulomb potential. There is no analytical form for these solutions and tabulations of them can be found in [eg: Blo51]. The substitutions, \( \eta \equiv \frac{e^1 z_1 z_2}{\hbar v} \) and \( \rho \equiv \mu vr / \hbar = kr \), (with \( v \) defined as the relative velocity in the CM system, \( v = \sqrt{2E/\mu} \)) are made to put the radial Schrödinger equation in a simpler form:

\[
\left[ \frac{d^2}{d\rho^2} + 1 - \left( \frac{2\eta}{\rho} \right) - \left( \frac{l(l+1)}{\rho^2} \right) \right] \chi_\ell = 0, \quad \text{where } \chi_\ell \text{ is a linear combination of } F_\ell \text{ and } G_\ell
\]

For an outgoing particle the asymptotic form of the solution (\( r \) or \( \rho \to \infty \)) must be that of the free particle, \( \chi_\ell(\eta, \rho \to \infty) = e^{ikr} = e^{i\rho} \). The asymptotic forms of the Coulomb wavefunctions are, ignoring the phase shifts [Blo51]:

\[
\lim_{r \to \infty} F_\ell \to \sin(\rho) \quad \text{and},
\]

\[
\lim_{r \to \infty} G_\ell \to \cos(\rho)
\]

The general outgoing solution to the Schrödinger equation is a linear combination of the regular and irregular solutions, \( \chi_\ell(\eta, \rho) = A F_\ell(\eta, \rho) + B G_\ell(\eta, \rho) \), where \( A \) and \( B \) are constants. Since this must match the asymptotic form, \( e^{i\rho} \), this means that, \( A = i \) and \( B = 1 \).
Thus, since the penetration factor is defined as [see Section 1.7]:

\[ P_i(\eta, \rho) \equiv \left| \frac{\chi_i(r = \infty)}{\chi_i(r = R_n)} \right|^2 \]

we can write (eq 1.21):

\[ P_i(\eta, \rho) = \frac{1}{F_i^2(\eta, \rho) + G_i^2(\eta, \rho)} \]

For a given system of an outgoing particle and a residual spherically symmetric core, the ratio of the penetration factor for relative angular momentum, \( l \), to that for relative angular momentum 0, has been approximated analytically in [Rol88]:

\[ \frac{P_i(\eta, \rho)}{P_0(\eta, \rho)} = e^{-\frac{7.61 l(l+1)}{2\mu z_1 z_2 R_n}} \]

where \( \mu \) is the reduced mass in amu and \( R_n \) is the nuclear radius in Fermis. This formula shows explicitly the extreme dependence on the relative angular momentum in calculating penetration factors and thus charged particle partial widths.

The above formula was used in obtaining eq. 1.24.
References


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[Tho52] R. G. Thomas, Phys Rev. 88(1952)1109


