STUDIES OF NUCLEAR TRANSFER REACTIONS AT HIGH ENERGY

by

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To My Mother and Father,

and to Linda
ABSTRACT

Single nucleon transfer reactions resulting from the bombardment of carbon by 148 MeV nitrogen-14 ions have been experimentally investigated in an attempt to elucidate the reaction mechanisms. The energy spectra of the $^{12}\text{C}(^{14}\text{N}, ^{13}\text{N})^{13}\text{C}$, $^{12}\text{C}(^{14}\text{N}, ^{15}\text{N})^{11}\text{C}$, and $^{12}\text{C}(^{14}\text{N}, ^{15}\text{O})^{11}\text{B}$ reactions, as well as those of the deuteron transfer reaction $^{12}\text{C}(^{14}\text{N}, ^{13}\text{O})^{19}\text{B}$ and the $^{14}\text{N} - ^{12}\text{C}$ elastic and inelastic scattering have been obtained simultaneously. Angular distributions corresponding to resolvable final states are presented.

The one and two nucleon transfer reactions characterized by the highly selective population of only a few levels in the product nuclei are shown to be consistent with a reaction mechanism favoring the formation of states with dominant single particle configuration. Evidence is found for the most strongly excited levels being those for which the angular momentum transferred in the reaction most nearly matches that of the shell-model orbital into which the particle is transferred.

The smooth angular distributions observed for all the transfer reactions are in disagreement with the predictions of the Frahn-Venter diffraction model, but may be qualitatively accounted for by the more recent theories of Dar and Dodd and Greider. In consequence, it has not been possible to deduce the relative importance of nuclear and Coulomb reaction amplitudes, as inherent in the Frahn-Venter and similar diffraction reaction models.

The $^{12}\text{C}(^{14}\text{N}, ^{15}\text{N})^{11}\text{C}$ and $^{12}\text{C}(^{14}\text{N}, ^{15}\text{O})^{11}\text{B}$ mirror reactions evidence similar energy spectra and angular distributions; within the present experimental accuracy, it appears that these reactions are charge independent.

The elastic and inelastic scattering angular distributions display characteristic oscillations. The inelastic scattering mechanism is found to be highly selective, favoring collective residual excitations. Both the levels populated and the shapes of their angular distributions are in accord with previous measurements on similar systems.

The particle identification system and techniques developed for this work are described in some detail; further experimental investigations suggested by this work are discussed.
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I. INTRODUCTION

A. Orientation

The heavy ion transfer reaction, in which a nucleon, or a cluster of nucleons, is exchanged during the scattering of two complex nuclei, has been intensively studied since its initial observation over a decade ago\(^{(1)}\). The study has been both experimental and theoretical\(^{(2)}\), and has had the dual objectives of elucidating both the reaction mechanisms and the structural details of the nuclei involved.

Much of the original impetus for heavy ion studies resulted from a suggestion by Breit, Hull, and Gluckstern\(^{(3)}\) that low energy transfer reactions should provide a sensitive probe for the exploration of the nucleon density in the nuclear surface. For low-\(Q\) reactions, this is equivalent to the determination of reduced widths for nucleons in their bound states; that is, it should be possible to deduce from the transfer cross section the normalization of the tail of the initial or final state wave function of the transferred nucleon outside the nuclear potential well to which it is bound.

There are several advantages to using heavy ions. As a direct consequence of the very short mean free path of heavy ions in matter, the interactions induced by heavy projectiles are localized on the nuclear surface. Highly selective population of the simple single- or multi-particle (or hole) states is characteristically observed in heavy ion transfer reactions.

Large angular momenta are characteristic of heavy projectiles, and so a reaction mechanism favoring the population of high angular momentum states should be enhanced relative to reactions induced by lighter projectiles. There is also the possibility of transferring several nucleons, which allows the examination of multi-nucleon correlations or clusters in the nuclei involved.

There are, of course, concomitant disadvantages. Wave functions are considerably less well known than for the lighter projectiles, making the extraction of such quantities as reduced widths and spectroscopic factors dependent upon what frequently amounts to quite crude approximations. There
are also experimental difficulties: because of the characteristically large number of open reaction exit channels, cross sections for any given process are generally considerably smaller than in the corresponding light projectile case. The resultant large number of reaction products poses a formidable experimental problem in the isolation of a particular species. Similarly, the high energy loss rates of the complex nuclei in targets and windows make it difficult to achieve adequate resolution to distinguish individual residual states.

Both the theoretical and experimental problems are considerably less complex at energies below or near the Coulomb barrier, and for this reason many of the previously mentioned studies have been concerned with this energy region. Breit and his collaborators have shown that for the low energy case the reaction may be considered semi-classically, and the complex phenomena arising during the intimate contact of the collision partners may be largely neglected. Although within its range of applicability, this treatment has been remarkably successful, the somewhat overly idealized tunneling model must be modified to provide closer and more general quantitative agreement with experimental results. The approximations introduced in this process make the extraction of the reduced widths for the transferred nucleons, which constitute the primary information sought in these studies, a somewhat equivocal procedure.

In view of these difficulties, it is not surprising that alternate formulations have arisen. As is true of any semi-classical nuclear theory, the need is for a rigorous translation into a quantum mechanically valid formalism. This has been attempted in several seemingly disparate, but actually closely related, treatments, which not only provide fuller justification for the original approach, but also try to extend the range of validity to the high energy situation where the strong, short-range nuclear interactions play an important role.

When the incident energy is significantly greater than the Coulomb barrier, or for the large angle (i.e., small impact parameter) cases at intermediate energies, it is necessary to take the strong nuclear absorption into account before agreement with the experimental results is obtained.
There are three major approaches which have been applied to the high energy transfer problem: the complex potential model (i.e., distorted wave Born approximation), the finite range diffraction model of Frahn and Venter, and its extension by Dar, and the T-matrix formulation of Greider, with subsequent modifications by Dodd and Greider to include the effects of momentum recoil.

These reaction models are alike in that they carefully describe the motion of the heavy cores along their essentially Coulombic trajectories, and treat the nucleon transfer as a perturbation of this motion. However, the means by which this is accomplished and the approximations introduced are quite different.

The DWBA approach describes the reaction in terms of an optical potential. The scattering waves are distorted by the effect of the strong nuclear interactions. Heavy ion elastic and inelastic scattering and the lower energy transfer reactions have been well represented by this treatment. However, several authors have cast grave doubts upon the usage and interpretation of a local complex potential in the presence of very strong absorption; and consequently it may be that the potential model simply provides a convenient phenomenological parameterization of the problem. This will be discussed in greater detail in Section II.

Frahn and Venter have described the single nucleon transfer process with what amounts to a generalized phase analysis. The reaction model is an extension of their previous formulation to represent elastic scattering in regions where both Coulomb and nuclear absorption effects are important. Since this treatment is based upon suitable assumptions concerning the asymptotic behavior of the wave function of the stripped particle, this model must be regarded as entirely phenomenological. Specific predictions about the shape of the angular distributions and the energy dependence of the cross section are made. In simplest terms, the expressions are approximately factorable into Coulombic and nuclear terms; the former is expected to result in a smooth, essentially exponential angular distribution, whereas the latter
leads to an oscillatory diffraction pattern. In physical situations in which both the Coulomb and nuclear interactions are important, the relative magnitude of the diffraction modulation of the exponential angular dependence should provide a direct measure of the contribution of the nuclear forces to the reaction. The possibility of separating these Coulombic and nuclear contributions was one of the motivations for the present research.

Dar has extended and modified the work of Frahn and Venter in his finite range smooth cutoff diffraction model. He proposes a somewhat different mechanism for the damping of the characteristic diffraction oscillations. In Dar's treatment, the diffuseness of the nuclear surface appears as a phenomenological parameter. This diffraction model has been able to fit the data from a wide variety of scattering and transfer reactions with considerable success.

Greider has also used a finite range diffraction model to describe the high energy transfer reaction, but employs three-body T-matrix theory to derive his results. Agreement with experimental measurements is excellent for low energy transfer reactions. A later treatment includes terms of the order of the ratio of the transferred mass to the core mass (recoil effects) in the phase of the transition matrix elements. The effect of this modification is to eliminate the oscillations in the angular distributions ordinarily expected for a diffraction model, without invoking the interference effects proposed by other theories. The work of Dodd and Greider includes finite range as well as recoil effects and also predicts that high energy angular momentum states of the final nucleus are preferentially populated. This is in good agreement with previous semiclassical arguments and with experimental observations.

The present work is largely concerned with an experimental investigation of these questions, and their implications for future nuclear spectroscopic studies with the well-defined, high energy, heavy projectile beams soon to be available from the large tandem Van de Graaff accelerators.

Another subject of much recent interest is concerned with the excitation of isobaric analogue states (that is, those states of nuclei of the same mass
number which are identical in all respects save for the Z projection of the isotopic spin quantum number) in reactions where isospin would not appear to be a good quantum number. The energy differences of such analogue states are assumed calculable on the simple basis of the Coulombic energy attributable to the extra protons. Data on such systems is of interest because it adds to our knowledge of the charge independence of nuclear forces and provides a sensitive means of determining the charge radius of nuclei. The reactions under investigation here were particularly chosen for the symmetry of their reaction products and in the expectation that such analogue states would be excited. As will be seen, this permits specific predictions concerning the relative cross sections for these levels.

This research is then an experimental study of the interactions induced by a beam of 148 MeV nitrogen-14 ions impinging on a carbon target. A particle identification system has been developed for the purpose, and will be described in detail. The most significant feature of this system is that it permits the simultaneous acquisition of data on many nuclear reactions resulting from the same combination of beam and target. A complete set of test data is thus obtained which is not subject to the normal difficulties concerned with inter-system calibration and normalization. Such data should facilitate detailed theoretical studies of reaction mechanisms and nuclear structure.

B. Purpose and Scope of This Study

The experiments to be described in this report were conceived with the intention of extending the study of the theoretically important and experimentally challenging heavy ion transfer reaction to the relatively unexplored energy region far above the Coulomb barrier. As noted in the preceding section, several theoretical models for these reactions have been proposed, but little in the way of detailed experimental confirmation had been achieved. This was largely because of the experimental difficulty in the resolution of individual states in the residual nuclei. Radiochemical techniques, which
account for the bulk of this previous work, are, in general, completely incapable of performing this function.

For this reason, a particle identification system, capable of resolving individual levels in several reaction products simultaneously, was developed for this work. The system employs a 20,000 channel multiparameter analyzer and relies heavily upon digital computer techniques. It is capable of resolving isotopes at least as heavy as fluorine.

In particular, the angular dependence of the differential cross section for single nucleon transfer was of interest. Frahn and Venter\textsuperscript{(11)} had proposed a formalism in which the Coulomb and nuclear effects are essentially separable; the relative contributions can, in principle, be ascertained by the degree of oscillation of the angular distribution, which characterizes the strong nuclear absorption. The reactions studied in the present work were selected because Coulomb effects are very small, and thus the observation of the above mentioned diffraction structure would have provided at least a partial confirmation of the model, as well as information about the reaction mechanism.

While these measurements were in progress, and when it became clear that no oscillations were, in fact, present both Dar\textsuperscript{(13, 16, 17)} and Greider and Dodd\textsuperscript{(15)} proposed mechanisms, other than the Coulomb interference damping of Frahn and Venter, that could smooth the diffraction oscillations. Greider and Dodd's suggestion is that momentum recoil and finite range effects, neglected in all the other treatments, produce this damping while Dar considers both diffuse surface and angular momentum coupling effects.

As previously mentioned, the present experimental system has the ability to accumulate data simultaneously on several different nuclear reactions resulting from the same combination of beam and target. From the theoretical viewpoint, the simplest transfer reaction involves the pick up of a single neutron, and, for that reason, the reaction $^{12}\text{C}(^{14}\text{N}, ^{15}\text{N})^{11}\text{C}$ is the focal point of this work. However, in the limit of the charge independence of nuclear forces, the reaction corresponding to proton transfer is expected
to exhibit essentially identical behavior (since Coulombic differences are expected to be quite small at the high energies involved). Complementary data on the proton pickup, $^1\text{C}^{12}(^1\text{N}^{14}, ^{15}\text{O})^1\text{B}$, were therefore obtained concurrently with that of the neutron transfer in all cases.

Data was also obtained for the reactions $^1\text{C}^{12}(^1\text{N}^{14}, ^1\text{O})^1\text{B}$, $^1\text{C}^{12}(^1\text{N}^{14}, ^1\text{N})^1\text{C}^{13}$, $^1\text{C}^{12}(^1\text{N}^{14}, ^1\text{N}^*)^1\text{C}^{12}$, and $^1\text{C}^{12}(^1\text{N}^{14}, ^1\text{N}^{14*})^1\text{C}^{12*}$ corresponding to deuteron transfer, neutron transfer to the target, elastic and inelastic scattering, respectively. Discussion of these reactions will necessarily be brief, but full energy spectra and angular distributions will be presented. Since the present system eliminates the usual inter-system calibration difficulties resulting from the employment of varying analyzers, electronics, targets, and detectors, dead time and normalization are identical for all species investigated, and so accurate comparisons of relative cross sections and excitations of any levels in any reaction product are possible.

An effect commonly observed in complex ion transfer reactions is the highly selective population of energy states in the product nuclei. A further aim of this work was to investigate this phenomenon and to evaluate a previously advanced suggestion that one of the basic mechanisms underlying this selectivity is the mismatch of the angular momentum transferred to that required to put the transferred nucleon into its presumed shell-model orbit. Dodd and Greider and Durand have recently provided a sounder mathematical basis for this semi-classical argument.

It should be pointed out that, in a sense, much of this work is developmental, in that the quality of this data for nuclear spectroscopic purposes is limited by the relatively poor energy resolution of the HILAC beam. Similarly, the large loss in beam intensity resulting from the introduction of energy-degrading absorber foils and subsequent momentum analysis precluded the possibility of excitation function measurements, which would have been of considerable interest.

The quality of the data indicates, however, that this method will be a useful tool for nuclear research, particularly when the Emperor Tandem Van de Graaff accelerator under construction at Yale becomes available.
The following section of this report presents a discussion of the various theoretical treatments of high energy heavy ion transfer reactions. Differences in the approximations and techniques will be pointed out, particularly as they apply to the present work. Following that, the experimental apparatus and techniques are described, including a discussion of the data reduction procedure. The experimental results, both for the nucleon transfers and the other reactions listed above, are presented and discussed in the fourth section. Suggestions for extensions of this work, and a series of appendices describing the data analysis in greater detail are also included.
II. THEORETICAL STUDIES OF HIGH ENERGY TRANSFER REACTIONS

A. Basic Considerations

The general motivation for the study of nuclear reactions is the hope of obtaining information not only about the structure of nuclei, but also about the reaction mechanisms themselves. While the nuclear spectroscopic information is often available directly from experimentally measurable quantities, sometimes it must be inferred from the nature of the reaction mechanism. A knowledge of the reaction mechanism is almost always essential to an evaluation of the credibility of any spectroscopic quantities obtained from the study of a given reaction.

The heavy ion transfer reaction has been intensively studied with these objectives, and several theories have been proposed concerning the mechanism for the exchange of the nucleon or nucleon cluster, as well as for the extraction of spectroscopic information from the experimental measurements. These theories are necessarily only models of the reaction, as no exact mathematical formulation of the problem is currently available. There are two reasons for this: (i) the nature of the nuclear forces involved is imperfectly known, and phenomenological potentials must be employed; (ii) the transfer reaction is a rearrangement scattering in a quantum mechanical system comprising at least three members, and general techniques for the solution of three-body nuclear problems are, at present, insufficiently developed for application to such a complex situation.

1. Two- and three-body reaction models.

The heavy ion transfer reaction has been represented by two general classes of models. The first consists of three-body theories which treat the reaction participants as the two nuclear cores and the transferred particle or cluster \(^{(19)}\). That is, the reaction is viewed schematically as

\[
(a + c) + b \rightarrow a + (b + c)
\]

where \(a\) and \(b\) are the heavy cores, and \(c\) is the transferred particle or
cluster, parentheses indicating bound states. The bound states are approximated by phenomenological potentials $V_{ac}$ and $V_{bc}$ in the entrance and exit channels, respectively. Even with this assumption, the formal scattering equations can be solved only approximately. The resulting approximate formulation of the problem is still, however, potentially capable of supplying information of a spectroscopic nature\(^{(20)}\).

The second general approach involves even more drastic approximations than the first. The three-body problem is now replaced by an "equivalent" two-body reaction in which the scattering of the two nuclei involved is modified by phenomenological considerations dictated by the nature of the perturbing transfer process\(^{(21)}\). Sometimes this is done by using a zero-range approximation in the three-body matrix elements; that is, the interaction of the transferred particle and the cores is reduced to a delta-function at the nuclear surface. By eliminating all details of the interactions $V_{ac}$ and $V_{bc}$, the possibility of treating nuclear structure effects accurately is precluded, and spectroscopic information cannot be obtained quantitatively from such a model. These relatively simple parameterizations are, however, capable of correlating large bodies of experimental data, and of providing insights into the nature of the reaction mechanism.

2. Low energy transfer: the tunneling model.

Because of their great mass, heavy projectiles may be regarded as being essentially classical in nature at presently attainable laboratory energies. It is then meaningful to consider the localization in space of the primary reaction partners, and to associate an angular momentum with their relative motion. The original tunneling theory of Breit and Ebel\(^{(4)}\) was the first successful formulation of a three-body problem in nuclear physics. It is based upon a semi-classical description of the motion of the heavy cores along Rutherford trajectories, with the transfer mechanism being regarded as a quantum mechanical tunneling of the neutron from the potential well of the nucleus to which it is initially bound to that of the nucleus to which it is finally bound.
For reactions below the Coulomb barrier, the model provides good agreement with experimental data. Although the tunneling theory has been modified and extended by several authors, the central assumption (considering the transfer as a perturbation or distortion of the Rutherford trajectories) is common to all. However, the methods of treating the motion of the heavy aggregates and the transfer interaction differ considerably.

3. High energy transfer: diffraction models.

As the incident energy becomes significantly greater than the Coulomb barrier, the tunneling mechanism ceases to provide a valid description of the transfer process. At these high energies, the situation becomes complicated because the effects of the short range nuclear forces become important. This is manifested by the large number of open reaction channels, resulting in the depletion of the flux into any particular channel. Several simply-parameterized reaction models for these high energy transfer reactions have recently been developed.

These models are similar in that they represent the effects of the nuclear absorption by effectively eliminating a region of space in the integrations to evaluate the transition amplitude. This confines the effective reaction zone to a small region between the colliding nuclei. Semi-classically, this implies that only a relatively narrow band of angular momentum values is effective in producing a transfer. Particle trajectories corresponding to large angular momenta are too distant to be of consequence and those with too small angular momenta are removed by the strong nuclear absorption. This situation typifies a surface reaction, and the analogy with classical optical Fraunhofer diffraction is apparent. In consequence, these models are generally classed as nuclear diffraction models.

Historically, the diffraction concept was introduced by Bethe and Placzek\(^{(22)}\) who used the idea of a "black nucleus" to explain the appearance of the small-angle oscillations observed in the cross sections for high energy neutron scattering. The scattering of heavier charged particles has been
treated by the sharp cut off model of Blair\cite{23} in which all partial waves below a critical value of the angular momentum, \( t \), corresponding roughly to a grazing trajectory, are considered to make no contribution to the cross section. This model was subsequently modified by Greider and Glassgold\cite{24} who relaxed the sharp cut-off condition, generalized the form of the reflection coefficients in the partial wave analysis, and included the possibility of a real nuclear phase shift. McIntyre and collaborators\cite{25} made allowance for a finite surface diffuseness, so that a gradual transition from complete absorption to pure Rutherford scattering is effected. The theoretical justification for these generalizations has recently been provided\cite{96}.

It is convenient to define absorption as reflecting any process by which particles are removed from the entrance channel. In a diffraction model, this is readily incorporated into the formalism by assuming the nucleus black, or highly opaque, to certain incident partial waves. This is directly expressible in terms of the reflection coefficients. In a model employing a reaction potential, absorption is represented by the addition of an imaginary part to the average interaction potential. Both techniques have had considerable success in the description of heavy ion scattering.

The models to be discussed in the following sections employ these general ideas in somewhat different fashions, and, as a result, in some cases make divergent predictions as to nuclear behavior. The emphasis in the discussion will be on the basic physical principles and assumptions, rather than on the derivation of mathematical formulae, since it has been one of the objectives of the present work to differentiate experimentally between these model alternatives.

B. Complex Potential Model

One of the most effective theoretical approaches to the representation of nucleon-nucleus scattering has been demonstrated to involve the use of a complex potential to describe the interaction between the nucleon and nucleus.
The basic idea of the heavy ion complex potential model is that these optical nucleon-nucleus techniques can be summed effectively for the case of nucleus-nucleus scattering.

This is usually extended from elastic to inelastic interactions within the framework of the distorted-wave Born approximation (DWBA), and a high degree of success has been achieved in fitting elastic and inelastic scattering involving both light and heavy projectiles, and transfer reactions induced by light nuclei\(^{(6)}\).

Basically, the physical ideas for the transfer process are similar to those already discussed. A three-body representation of the reaction is used, elastic scattering is assumed to be the dominant process, and the particle transfer is treated in first order perturbation theory. The transfer is thus regarded as a transition between elastic scattering states (distorted waves); the distorted waves are generally obtained from a numerical solution of the Schrödinger equation, with an optical model potential which reproduces the experimentally observed elastic scattering from the nucleus at that energy.

In general, the quality of the fits to charged particle angular distributions has been excellent, and within the context of the assumptions noted above, it would seem that application of this method to high energy transfer reactions would be a reasonable procedure. The strong absorption condition is easily described by the imaginary part of the potential.

However, the physical significance of the concept of an average single-particle potential in the presence of strong absorption is subject to grave doubts. In effect, the complex potential model is a representation of a complex many-body problem, with the additional complications arising from the overlap of strongly interacting systems, by a static local potential. The fact that excellent fits are obtained does not necessarily imply the physical significance of the resultant parameters.
The great advantage of the complex potential model over diffraction treatments is that the former provides, in principle, detailed information concerning the interior of the nucleus. However, this is achieved at the cost of intuitive simplicity; the physical interpretation of the relevant parameters is no longer obvious. The extent to which this information conforms to physical reality in heavy ion interaction situations is a moot point at present.

For example, Kuehner and Almqvist found that to describe the elastic scattering of $^{16}\text{O} + ^{12}\text{C}$, a small absorptive term was necessary to fit the large diffraction oscillations observed. However, this implies that the $^{12}\text{C}$ nucleus penetrates $^{16}\text{O}$ to a depth of about 5 fermis, which seems unreasonable. Similar results are found for much of the $\alpha$-particle scattering data. Kuehner and Almqvist are led to conclude that despite the good fits, the optical model for heavy particles may simply provide a convenient parameterization with little or no physical significance.

Breit has remarked that perhaps the success of these fits is traceable to the similarities in mathematical form between a generalized nuclear reaction theory (i.e., parameters in the numerator and essentially resonance energies in the denominator) and the expansion of the optical potential results.

Frahn and Venter have compared the values of partial wave amplitudes obtained from the numerical integration of the optical model Schrödinger equation with those obtained from their phenomenological diffraction model (see Section II-D) for elastic scattering. They find

(i) the imaginary phase shifts are in agreement for all $l$ values
(ii) real phase shifts agree above the critical $l$ value but differ widely below it.

However, these lower $l$ partial waves are strongly absorbed, and so this difference is of little importance in the fitting of differential cross sections.

The present results have not been analyzed using an optical potential model. The principal reason is that insufficient elastic scattering data is
presently available to characterize the optical potential for the heavy ion reactions considered in this report.

It is apparent that a consistent set of experimental results including elastic and inelastic scattering, transfer reactions, etc., is essential for a meaningful test of the optical model for heavy ion reactions. Although experimental limitations have forced the incompleteness of the data presented in this report, it will be demonstrated that the techniques developed should be capable of providing such sets of test data when used in conjunction with the large electrostatic accelerators soon to be available.

C. The Core-Core Interaction Model (Greider).

The core-core interaction model is based upon what Greider has termed the "indirect" aspect of direct interactions; that is, the dominant interaction in the rearrangement scattering process is taken to be that between the heavy cores, and not between the transferred particle and the core. The reason for this choice is based upon the major role that the Coulomb interactions play in these reactions and the consequent desire to describe these interactions as accurately as possible. Greider represents the Coulomb potential between the cores in a higher order of approximation than do the DWBA and other theories which use the Coulomb interaction merely to distort the entrance and exit channel wave functions.

Greider's approach was the first to explicitly include the effects of strong absorption, and as such was able to describe the complete angular distribution for low energy transfer reactions.

The theory is developed within the formalism of quantum mechanical scattering theory, employing integral equation techniques and representing the core-core interaction by t-matrices, taken to be factorable into Coulomb and nuclear terms. These t-matrices are approximately the transition amplitudes for Coulomb and nuclear elastic scattering. Initial and final state scattering wave functions are considered asymptotic plane waves acted upon
by distortion operators, which distort the plane waves via the nuclear or Coulomb interaction, depending upon which is considered dominant for the process.

For the reactions considered in this report, the nuclear interaction is dominant, and the theory is formulated in terms of Coulomb distortion of the nuclear potential. If pure nuclear forces are assumed and the term arising from the Coulomb interaction is neglected, a preliminary calculation may be carried out in plane wave approximation.

The present state of the core-core theory requires making the additional assumptions that angular momentum, spin, and momentum recoil effects are negligible. Nuclear structure of the cores and transferred particle is also neglected.

Sachs\(^{(29)}\) has carried out such a calculation, using Fourier transforms of the momentum distributions for the bound state wave functions and with numerical evaluation of the resulting overlap integrals, for the reaction \(\text{C}^{12}(\text{Be}^{11},\text{Be}^{10})\text{N}^{13}\), at an energy of 115.5 MeV. The results, in view of the rather drastic assumptions, are surprisingly good. The smooth shape of the angular distribution is predicted, although the values of mean momenta required to produce a fit are not in good agreement with those found from other measurements. All angle-independent factors were neglected in the calculation, and so no prediction is made as to the absolute magnitudes. It is also important to note that, in this degree of approximation, the reaction is identified only by the interaction radius of the incident system and the reduced mass of the transferred particle.

For this last reason, such an analysis was not performed for the present data, as the results are expected to be almost identical with the Sachs calculation. No additional information about the reaction mechanism could be obtained from repeating such a calculation.

It may well be that the core-core interaction method, since it employs the powerful formalisms of scattering theory in a direct and physically reasonable fashion, will prove the most useful approach to the problem of
high energy transfer reactions. However, until a more detailed development of the theory, incorporating Coulomb, angular momentum, and spin effects is available, and until the plane wave approximation may be replaced with more suitable bound state wave functions, the more phenomenological methods to be discussed in succeeding sections are more useful for the analysis of high energy transfer reaction data.

D. Phenomenological Diffraction Model (Frahn and Venter).

1. Formulation of the strong-absorption model.

The so-called "phenomenological" model of Frahn and Venter treats the single nucleon transfer reaction at high energies as a quasi-elastic process, using a previously developed formalism for elastic nuclear scattering in the presence of strong absorption. The model involves two sets of approximations: those made in the original analysis of the elastic scattering and the further assumptions necessary to include the transfer reaction within this general framework.

It is convenient to first outline the general formalism and then specialize to the transfer case. For simplicity, consider a spinless, neutral projectile incident on a spin-zero spherical target nucleus (these restrictions may be subsequently removed). Then, in the usual partial wave notation, the scattering amplitude is represented by

$$f(\theta) = \frac{-i}{2k} \sum_{\ell \neq 0}^{\infty} (2\ell + 1) (1-\eta_{\ell}) P_{\ell} (\cos \theta)$$  \hspace{1cm} (II-1)

The infinite set of complex scattering amplitudes, $\eta_{\ell}$, is denoted the scattering function (S-function). The scattering problem is completely determined by these quantities, which represent the primary connection between physical observables and nuclear properties.

The following assumption is central to this model:
Let
\[ \text{Re } \eta_{\ell} = g_1(t) + \epsilon_1 \left( 1 - g_1(t) \right) \] (II-2a)
\[ \text{Im } \eta_{\ell} = \frac{\mu dt g_2(t)}{\Delta_2} + \epsilon_2 \left( 1 - g_1(t) \right), \] (II-2b)

with \( g_1 \) and \( g_2 \) continuous functions of \( t = \ell + 1/2 \). Parameters \( \Delta_1 \) and \( \Delta_2 \), representing the width, in \( \ell \)-space, of the region between zero and maximum absorption, characterize the \( g \)-functions. The analytical treatment to follow is allegedly independent of the specific form of these functions, which behave essentially like unit step functions at \( \ell = \ell_1 + 1/2 \), where \( \ell - 1/2 \) is approximately the angular momentum corresponding to some defined interaction radius.

Classically, this is
\[ \ell + 1/2 = kR \left( 1 - 2\eta/kR \right)^{1/2}, \] (II-3a)
where \( R \) is the interaction radius and \( \eta \) the Coulomb parameter
\[ \eta = \frac{mZ_1Z_2 e^2}{\hbar^2 k} \] (II-3b)

The parameter \( \mu \) allows for the presence of a real nuclear phase shift, while \( \epsilon_1 \) and \( \epsilon_2 \) allow for a possible small transparency for lower-\( \ell \) partial waves. The general form of \( \eta_{\ell} \) is depicted in Figure (1).

This assumption is in accord with the concept of the very limited effective reaction zone previously discussed. Note that the only significant contributions from \( \text{Im } \eta_{\ell} \) occur approximately concurrently with the rapid increase of \( \text{Re } \eta_{\ell} \) from a small value to unity. The assumption is very general, in that both the sharpness of the \( \ell \)-cutoff as well as the opacity of the nucleus are variable parameters. The Woods-Saxon shape has been found convenient for heavy ion scattering, and will be assumed throughout this discussion.

That is
\[ g(t) = \frac{1}{1 + \exp \left( \frac{T-t}{\Delta} \right)} \] (II-4)
Note that as \( \Delta \) tends to zero, \( g(t) \) goes over to the sharp cutoff case.

When assumption (II-2) is inserted in (II-1), the general properties of the \( g(t) \) functions make it possible to evaluate analytically the sum over partial
Figure 1. Qualitative form of the real and imaginary parts of the scattering function for the Frahn-Venter diffraction model. (from reference 12).
waves. This is a departure from the previously mentioned diffraction treatments, which require numerical computation. The strong absorption assumption implies \((\Delta/T) < < 1\) (that is, the width of the effective reaction zone in \(l\)-space is a small fraction of the angular momentum involved in the reaction), which in turn means that \((\ t\cdot\theta\ ) \sim \delta\ (t-T)\). If \(P_\ell (\cos \theta)\) is replaced by the leading term of its asymptotic expansion, Venter\(^\text{(12)}\) has shown that the main contribution to the sum arises in the vicinity of \(t = T\). Because of the large number of partial waves included in the sum, the Poisson sum formula may be used to convert the sum to an integral, which is evaluated in reference\(^\text{(12)}\). This procedure is then generalized to include charged particles, and projectiles with spin. Using solely analytical methods, closed form expressions are obtained for the elastic scattering amplitudes and total reaction cross sections.

An additional hypothesis must be made to determine the excitation functions (that is, \(\sigma (\theta, E)\) as a function of \(E\) for fixed \(\theta\)), since the energy dependence of the parameters \(T, \Delta,\) and \(\mu\) is unknown. Frahn and Venter use semiclassical, intuitive arguments to define the interaction radius \(R\) and the surface diffuseness, \(d\), by

\[
T = kR \left(1 - (2\eta/kR)\right)^{1/2} \quad \text{(II-5a)}
\]

\[
\Delta = kd \frac{(1-(\eta/kR))}{(1-(2\eta/kR))^{1/2}} \quad \text{(II-5b)}
\]

If now \(R\) and \(d\) are assumed to be energy-independent, these equations provide an analytical expression for the variation of the parameters \(T\) and \(\Delta\) with energy. Little is known about the parameter \(\mu\), but setting \(\mu = 0\) in the energy distributions seems consistent with presently available data.

In the process of evaluating (II-1), the Fourier transforms of the derivatives \((d\gamma_i/\partial t)\) are introduced:

\[
F_i (\Delta, \theta) = \int_{-\infty}^{\infty} \frac{d\gamma_i(t)}{dt} e^{-i (t-T)\theta} dt \quad \text{(II-6)}
\]
These quantities contain almost all the information about the nuclear surface and consequently are called "form factors" by Frahn and Venter.

In summary, the generalized phenomenological model for elastic scattering contains seven parameters, $T_1, T_2, \Delta_1, \Delta_2, M_1, \epsilon_1,$ and $\epsilon_2$, and is characterized by an (arbitrary) smoothing function, $g(t)$, consistent with the strong absorption conditions. In practice, however, $\epsilon_1 = \epsilon_2 = 0$, since low $\ell$ partial waves for heavy nuclei will be completely absorbed.

Further simplification is afforded by setting

\[
T_1 = T_2 \equiv T; \Delta_1 = \Delta_2 \equiv \Delta; \text{and } g_1(t) = g_2(t) \equiv g(t)
\]

leaving only three adjustable parameters, $T$, $\Delta$, and $\mu$.

With this model, Frahn and Venter have achieved excellent fits to both the angular distributions and excitation functions of almost all the He$^3$, He$^4$, and heavy ion elastic scattering data available.

2. Extension to transfer reactions.

It is now necessary to examine the basis for including the transfer reaction in such a formulation, and to investigate the modifications necessitated by the three-body rearrangement.

The observed angular distributions for single nucleon transfer characteristically exhibit a peak whose position, $\theta'_c$, width, and height vary systematically with energy. McIntyre et al. found empirically that the variation of $\theta'_c$ with center of mass energy corresponds approximately to Rutherford scattering along a classical trajectory with a constant distance of closest approach, $R'$. $R'$ is the value of the apsidal distance $D$, at the critical angle, where $D$ is given by

\[
D = \left( \frac{\mu}{k} \right) \left( 1 + \frac{dsc}{2} \right).
\]

A plot of $d\sigma/dD$ will peak at approximately the same value of $D = R'$ for different energies, suggesting that the primary collision partners move along classical Coulomb trajectories, little affected by the transfer, which occurs only in a narrow zone around $D = R'$.

Thus the conditions for application of the phenomenological strong absorption model for elastic scattering are well met by the heavy ion transfer reaction.
1. The reaction Q values are \( < < \) incident energy. That is, the process can be considered quasi-elastic.

2. Generally, \( 2\pi \eta > > 1 \), which satisfies an approximation made in reference (13) in the derivation of the scattering amplitude.

3. A large number of incident partial waves are involved in the process: \( 2\pi T' > > 1 \).

4. The contributing \( \ell \) values are much greater than the change in orbital angular momentum brought about by the transfer.

5. The spins of the colliding nuclei as well as that of the transferred nucleon may be neglected with respect to the relevant orbital angular momenta.

The reaction amplitude is then given by

\[
f(\theta) = \frac{i}{2k} \sum_{\ell=0}^{\infty} (2\ell + 1) \eta_{\ell} e^{2i\sigma_{\ell}} P_{\ell} (\cos \theta)
\]

where \( k \) is entrance channel wave number and \( \sigma_{\ell} \) is the Coulomb phase shift. Assumption (II-2) is taken as

\[
\eta_{\ell} = \tau \frac{dg(t)}{dt}
\]

with

\[
g(t) = \frac{1}{1 + \exp \left( T' - t/\Delta' \right)}
\]

the symbols have the same meaning as previously, and primes indicate the transfer reaction. \( \tau \) is a measure of the relative strength of the transfer interaction, and is taken to be real for simplicity. \( \eta_{\ell} \) is seen to be represented by an approximately symmetrical peak in \( \ell \)-space, centered at \( T' = L' + 1/2 \), and with half-width \( \Delta' \) and height \( \tau/4 \Delta' \).

Using the methods of reference (12) the differential cross section is expressible as

\[
\frac{d\sigma}{d\theta} = \tau^2 \frac{T'}{k^2} (F^-)^2 \left[ 1 + (F^+/F^-)^2 + 2(F^+/F^-) \sin (2T'\theta) \right]
\]
\( F_{\pm} \) is the "form factor", given for the Woods-Saxon shape by

\[
F_{\pm} = F \left[ \Delta' \left( \Theta_{\pm} \right) \right] = \frac{\pi \Delta'(\Theta_{\pm})}{\sinh[\pi \Delta'(\Theta_{\pm})]} \tag{II-13}
\]

The predicted cross section is seen to be composed of a smoothly varying part which peaks at \( \Theta_{\pm} \), and an oscillatory part, described by the \( \sin(2T')(\Theta_{\pm}) \) term, produced by diffraction at the surface of the interaction region. The condition for damping of these oscillations is

\[
(F^+/F^-) < < 1 \tag{II-14}
\]

At \( \Theta = \Theta_{\pm} \) this becomes approximately

\[
4\pi \eta (d'/R') >> 1, \tag{II-15}
\]

where primed quantities analogous to those of (II-5a) and (II-5b) have been introduced.

The total transfer cross section is given by

\[
\sigma(E) = \int \left( \frac{d\sigma}{d\Omega} \right) d\Omega = \frac{\pi}{k^2} \sum_{\ell=0}^{\infty} (2\ell+1) |\eta_\ell|^2 \tag{II-16}
\]

For large \( \ell \), this may be approximated by

\[
\sigma(E) = \frac{1}{3} \pi \left( \frac{\tau}{\Delta'} \right)^2 R'd' \left( 1 - \frac{V_c(R')}{2E} \right) \tag{II-17}
\]

where \( V_c(R')/E \) denotes the ratio of the Coulomb barrier at \( R' \) to the energy. If \( R', d' \) and \( \left( \tau/\Delta' \right) \) are assumed energy independent, then \( \sigma(E) \) increases monotonically with \( E \) and approaches the constant limit

\[
\sigma(\infty) = \frac{1}{3} \pi \left( \frac{\tau}{\Delta'} \right) R'd' \tag{II-18}
\]

3. Discussion

The success of the phenomenological model in fitting the angular and energy distributions for heavy ion reactions, without any assumptions about the details of the nuclear interaction mechanisms, indicates that to a large extent the description of strong-absorption scattering in the presence of a Coulomb field is independent of the particular model chosen. This is an
"equivalent" two-body theory, and therefore cannot be expected to provide specific information about the internal constitution of the nucleus. The parameters primarily are influenced by the grosser properties of the interaction, such as the radius and shape of the interaction region and the diffuseness of the nuclear surface. The most that can be hoped is that nuclear structure effects will be reflected in a systematic variation of these parameters.

Most of the information about the nuclear surface is contained in the form factors, $F(\Delta'\Theta)$. Since $F$ is the Fourier transform of the derivative of the scattering function, $S'(\ell)$ (see II-6), any feature of $S'(\ell)$ in $\ell$-space shows up approximately as the corresponding Fourier-transformed feature of the angular distribution. This is directly attributable to the wave mechanical complementarity between the angular momentum $\ell$ and scattering angle $\Theta$.

Thus, for example, an increase of the diffuseness, $\Delta$, in $\ell$-space will result in a sharpening of the angular distribution about $\Theta_c$ due to the reciprocity of the half-widths of the $\eta_{\ell}$ and $d\gamma/d\Omega$ distributions. This indicates the difference between a sharp cut-off model and a modified model with a finite surface diffuseness, and highlights the inadequacy of a semi-classical description, which would predict the contrary. Because of their reluctance to attach semi-classical interpretations to essentially wave mechanical quantities, Frahn and Venter prefer to regard the basic assumption on the form of $S(\ell)$, (II-2), as a basic postulate which provides a means of consistently analyzing strong-interaction processes wherein a large number of partial waves participate.

Perhaps the most salient characteristic of this diffraction model is its prediction of oscillatory structure in the angular distribution. With decreasing Coulomb parameter, this oscillatory pattern becomes more pronounced until it completely dominates the angular distribution. These oscillations are directly attributable to the nuclear components of the interaction; therefore a measure of the relative diffractive modulation of the essentially exponential angular distribution due to the Coulombic forces could potentially provide a
a means of determining the relative importance of these individual contributions.

These oscillations had never been experimentally observed when this work was initiated. That in itself did not constitute a refutation of the theory, because in many heavy ion experiments condition (II-7) for the damping of the oscillations by strong Coulomb interference was well-satisfied, and a diffraction mechanism might still have been involved. Frahn and Venter suggested (11) that the detection of oscillations of the proper frequency, by high angular resolution measurements, in a case where Coulomb effects are negligible, would be a crucial test of the correctness of the model. A specific objective of the present work was to determine whether these oscillations were indeed present in such a case.
The recently proposed finite range diffraction model of Dar (31) is capable of describing the heavy ion cluster transfer reaction both at energies below and above the Coulomb barrier. For low energy cases, a quantum mechanical derivation yields results in accord with the tunneling theory. At higher energies, the work is a natural extension of the Frahn-Venter model of the preceding section, although there are several significant differences. Principal among these is that Dar's is a three-body theory, with provision for an interaction potential between the transferred cluster and the heavy cores, in both the initial and final states. Nuclear structure effects are therefore expected to be reflected in the predictions of the model. In addition, Coulomb effects are included explicitly, and analytical forms are derived for the cross sections predicted to correspond to specific angular momentum transfers.

1. Formulation

The transfer reaction is considered to be schematically represented as

\[(b+c) + A \rightarrow b + (c+A)\]

where parentheses indicate a bound state and

\[(b+c) \equiv a; \ (c+A) \equiv B\]

Using the standard distorted-wave Born approximation formalism, the transition amplitude for the reaction \(A(a,b)B\) is computed as a matrix element between product wave functions:

\[T_{fi} = \langle \Psi_B \ | \ \Psi_b \chi_f^{(-)} (\vec{k}_f, \vec{r}_f) \rangle \langle V \ | \ \Psi_A \ | \ \Psi_a \chi_i^{(+)} (\vec{k}_i, \vec{r}_i) \rangle \]

(II:21)

where the \(\Psi\)'s are the internal wave functions for the non-interacting particles denoted by the subscripts, \(V\) is the interaction whose off diagonal matrix elements are responsible for the transfer, \(\chi_f^{(-)}\) and \(\chi_i^{(+)}\) are the distorted waves describing the relative motion in the exit and entrance channels, respectively, and \(\vec{r}\) and \(\vec{k}\) denote the relative coordinate and wave number in the corresponding channels.
The transition amplitude may be written as
\[ T_{fi} = \int d\vec{r}_f \int d\vec{r}_i \chi_f^{(-)}(\vec{k}_f, \vec{r}_f) \langle B, b | V | a, A \rangle \chi_i^{(+)}(\vec{k}_i, \vec{r}_i) \] (II-22a)

with
\[ \langle B, b | V | a, A \rangle = \int \psi_B^* \psi_b^* V \psi_a \psi_A d\xi \] (II-22b)

where \( \xi \) represents all coordinates independent of \( \vec{r}_i \) and \( \vec{r}_f \). This matrix element serves as the effective interaction, or form factor, for the transition between elastic scattering states \( \chi_i^{(+)} \) and \( \chi_f^{(-)} \). All the information on nuclear structure, angular momentum, and selection rules is contained in this matrix element. In general, this is a six-dimensional integral which must be evaluated numerically, unless, as is usually the case, a zero-range approximation is made, reducing the problem to that of evaluating a three-dimensional integral. Physically, the zero-range approximation implies that the emission of particle \( b \) and the absorption of particle \( a \) take place at the same point (i.e., point transfer).

In the strong-absorption case, finite range effects may be treated directly. This is a result of the fact that the only wave functions which appear in the overlap integrals are the Coulomb waves and the tails of the bound state wave functions, since the distorted waves are assumed to vanish in the nuclear interior and in the classical shadow regions. Outside of these regions, the Coulomb waves are assumed to be undistorted and the overlap integral is evaluated by Dar in the Coulomb wave Born approximation. A type of WKB approximation (32) is employed to replace the Coulomb waves by their classical counterparts. The dominant interaction is taken to be between the transferred cluster and the heavy core to which it is not bound; that is, between \( c \) and \( A \) in the entrance channel and between \( b \) and \( c \) in the exit channel. The physical picture is thus one of Coulomb waves diffracting around a strongly absorbing nucleus. The method of partial waves is used with a diffuse surface assumed for the nucleus. This is equivalent to a smooth cut-off of the low-\( \ell \) partial waves, and is achieved by multiplying
the amplitude of each partial wave by the square root of its reflection coefficients using a method discussed by Durand and Chiu\(^{32}\). A Woods-Saxon form is assumed for the \(t\)-dependence of the reflection coefficients:

\[
\eta_t = \left[1 + \exp \left(\frac{L_0 - t}{\delta}\right)\right]^{-1} \tag{II-23a}
\]

with

\[
L_0 = \frac{\hbar}{kR} \left[1 - 2\eta/\hbar R\right]^{1/2} \tag{II-23b}
\]

and

\[
\delta = \frac{\hbar}{kd} \left[1 - \eta/\hbar R\right] \left[1 - 2\eta/\hbar R\right]^{-1/2} \tag{II-23c}
\]

where \(L_0\) is the angular momentum corresponding to a grazing collision, \(R\) is the interaction radius, \(\delta\) and \(d\) the diffusenesses of the nuclear surface in \(t\) - and \(r\) - space, respectively. The averages are over the entrance and exit channel values.

The result, for cases of physical interest, is given by

\[
\frac{d\sigma}{d\theta} \propto \left[\frac{1}{\cosh^2 \left(\pi \delta \left(\theta - \theta_c\right)\right) - \cos^2 \left(\pi \delta \left(\Delta'/2\right)\right)}\right]^{\frac{1}{2}}
\]

\[
+ \left[\frac{1}{\cosh^2 \left(\pi \delta \left(\theta + \theta_c\right)\right) - \cos^2 \left(\pi \delta \left(\Delta'/2\right)\right)}\right]^{\frac{1}{2}}
\]

\[
+ \left[\frac{(-1)^L \left\{\cosh \left(2\pi \delta \theta_c\right) - \cos(\pi \delta \Delta) \cosh(2\pi \delta \theta) \cdot \cos((2L_0 + 1) \theta - \pi/2)\right\}}{\left(\cosh^2 \left(\pi \delta \left(\theta - \theta_c\right)\right) - \cos^2 \left(\pi \delta \left(\Delta'/2\right)\right)\right) \left(\cosh^2 \left(\pi \delta \left(\theta + \theta_c\right)\right) - \cos^2 \left(\pi \delta \left(\Delta'/2\right)\right)\right)}\right]^{\frac{1}{2}}
\]

\[
- \sin(\pi \delta \Delta) \sinh \left(2\pi \delta \theta_c\right) \sin((2L_0 + 1) \theta - \pi/2)\right]\]

\(\theta\) is the center of mass scattering angle; \(\theta_c\) represents the scattering angle for Coulombic trajectories corresponding to an apsidal distance equal to the nuclear interaction radius; \(\Delta'/2\) is the ratio of the average wave number for the binding of cluster and core to the average wave number for the relative motion of the colliding nuclei, and \(L\) is the angular momentum transferred.

The final term is an interference term, and may be neglected when the condition for Coulomb damping of the oscillations is achieved. The ratio
of the interference terms to the first two terms is \( e^{-2\pi \delta \theta c'} \), and so the damping condition is:

\[
2\pi \delta \theta c' \gg 1 \tag{II-25a}
\]

or approximately

\[
4\pi \eta (d/r) \gg 1 \tag{II-25b}
\]

When these conditions are satisfied, the cross section is closely approximated by the first two terms of (II-24). This situation pertains in reactions at energies well below the Coulomb barrier, and for reactions at energies above the barrier with high value of \( \eta \). The interpretation of the interference term for the present high energy, low-\( \eta \) case is discussed below.

2. Discussion.

It should first be emphasized that while the formulation of this model is couched in the terms of the DWBA, the techniques involved are very different. The diffraction model requires fewer parameters, and the relationship of these parameters to the physical situation is much clearer (this point will be discussed in part E of this section). Moreover, the parameters are evaluated analytically in the diffraction treatment, whereas the DWBA requires extensive numerical computations.

The finite range model is capable of making specific predictions about nuclear structure. Analytic expressions have been derived which multiply the proportionality shown in (II-24). Unfortunately these are dependent upon a knowledge of the wave functions and interaction potentials for the heavy nuclei involved, both of which are too imperfectly known at present to permit calculation. The predictions of the model will therefore be arbitrarily normalized throughout the present report.

A frequently raised objection to diffraction models in their extension of a small-angle approximation into larger angle regions which results in an apparent violation of time-reversal invariance. Dar's model bypasses this difficulty because a small angle approximation is not required in the derivation.
The conditions for Coulomb damping of the diffraction oscillations in
the Dar equations are not satisfied for the reactions under consideration in the
present work, because of the small value of \( \eta(\eta \approx 2) \). This is attributable
to the relatively low charge and high energies of the nuclei concerned. Taking
as average values \( \delta = 1 \), and \( \Theta = 8 \) degrees, condition (II-25) becomes
0.9 > 1, and the neglect of the interference terms is clearly unjustified.

A computer program was written to evaluate (II-24) explicitly; the results for
the \( ^{12}\text{C}(^{14}\text{N},^{15}\text{N})^{11}\text{C} \) ground state reaction for \( \ell = 0, 1 \) are shown in Figure 2. The
value of the surface diffuseness is taken as 0.2 fermis, in accord with fits of
this model to other reaction data \( ^{(31)} \) and with the range of values found
for heavy ion elastic scattering. Figure 2 shows the effects of using a larger
diffuseness.

A strongly oscillatory pattern is predicted for any discrete value of
\( \ell \), in agreement with other diffraction models, but in sharp contrast to the
experimental results.

Equation (II-24) is the result of a recent reformulation of this approach
by Dar and Kozlowsky \( ^{(55)} \) in which angular momentum effects have been
specifically considered throughout. In Dar's former result \( ^{(13)} \), which differed
from equation (II-24) only in that the \((-1)^\ell \) factor multiplying the interference
term was absent, an oscillatory angular distribution was predicted, unless
the interference term was neglected, on rather arbitrary grounds. \( ^{(16,17)} \)
The most recent development interprets this term as follows.

If the transferred cluster is considered to be in definite subshells
\( \ell_i, j_i, \ell_f, j_f \) in the projectile and final nucleus, respectively, the following
selection rules are obtained:

(i) \( \ell + \ell_i + \ell_f \) even, with \( \ell \) the angular momentum transferred.

(ii) \( |\ell_i - \ell_f| < \ell < \ell_i + \ell_f \)

(iii) \( |j_i - j_f| < \ell < j_i + j_f \)

In consequence of (i), the interference factor becomes \((-1)^{\ell_i + \ell_f}\).

Thus for the case of negligible Coulomb damping and low \( \eta \), Dar concludes
(a) for the transfer of a cluster from a definite subshell $l_i j_i$ into a definite subshell $l_f j_f$, the angular distribution should always exhibit an oscillatory structure. Transitions with the same parity (i.e., same values of $l = l_i + l_f$) are in phase, those of different parity are out of phase. This effect is illustrated in Figure 2 in the curves for $l = 0$ and $l = 1$.

(b) the sum over $l$ implicit in the derivation of equation (II-24) can produce a damping of the oscillations if both odd and even values of $l$ are involved. This situation can result only from a configuration mixing for the transferred particle. The effect of this mixing is shown in Figure 2 by the curves for different sums over $l$. For an even number of terms, exact cancellation of the oscillations occurs; for an odd number of terms the damping increases with the number of terms included in the sum.

(c) mixing of $l$ values with different parity may also be the result of a particle transfer accompanied by a core excitation.

As will be demonstrated later in this report, condition (c) may be excluded for some of the excitations observed. For those cases, only condition (b) is capable of explaining the observed non-oscillatory angular distributions.

Dar has also investigated the results obtained by Frahn and Venter, and arrives at a surprising conclusion. He has elsewhere applied the techniques outlined above, within the general framework of the strong-absorption model, to the problem of collective excitation via inelastic scattering, and obtains, in the case of Coulomb damping, precisely the same angular dependence for the cross section for inelastic scattering as Frahn and Venter obtain for single nucleon transfer. Dar contends, without proof, that the result of Frahn and Venter is attributable to their (arbitrary) choice of the form for the scattering function (i.e., then $\eta_l$'s as given by II-2 and II-3), and is therefore invalid.

Note that damping of the diffraction oscillations because of angular momentum mixing is not expected for inelastic scattering. The damping is a direct result of the three-body nature of the reaction and inelastic scattering.
Figure 2. Predictions of the finite range diffraction model for the 
$C^{12}(N^{14},N^{15})C^{11}$ (ground states) reaction, calculated using equation 
(II-24). The various curves show the effects of varying the number 
of terms included in the sum over angular momenta, and of varying 
the surface diffuseness, d.
FINITE RANGE DIFFRACTION MODEL
PREDICTIONS (DAR) for C\(^{12}\)(\(N^{14}\),\(N^{15}\))C\(^{11}\)

\(E_{N^{14}} = 148\) MeV  \(Q = -7.878\) MeV

\(d\sigma/d\theta,\) RELATIVE UNITS

\(\theta_{cm},\) DEGREES
is viewed as a two-body process in this formalism. Because there is no mass transferred, the oscillations are expected to persist in the absence of Coulomb damping. This prediction will be checked experimentally in a later section of this report.

F. Recoil and Finite Range Effects in Transfer Reactions (Dodd and Greider)

The discrepancy between the oscillatory angular distributions predicted by any reasonable diffraction model (in the absence of strong Coulomb effects) and the smooth distributions recently observed experimentally could indicate the inadequacy of strong absorption models to represent these reactions. However, this is quite improbable in view of the considerable success that diffraction models have enjoyed in the interpretation of the experimental results for elastic scattering, low energy transfer reactions and reactions induced by lighter projectiles. The behavior of the transfers is especially puzzling in light of the diffraction oscillations observed for elastic scattering in these same systems, since the transfer reactions are quasi-elastic in very good approximation.

A possible explanation for the smooth distributions and the absence of diffraction oscillations is that they reflect a very diffuse interaction surface; however, this is at variance with information obtained from elastic scattering and cannot be seriously considered. The suggestions of Dar for the damping mechanism have been discussed in Section III-E of this report. They are somewhat difficult to evaluate quantitatively in the present cases because the value of the angular momentum transferred is not known unequivocally.

Dodd and Greider have recently proposed an alternate mechanism to account for the absence of the predicted oscillatory structure. They point out that all previous diffraction theories have neglected consideration of recoil effects; that is terms of order 1/A, where A is the ratio of the mass of a heavy "core", to that of the transferred cluster bound to that core. Similarly, many previous treatments have used a zero-range approximation for the interaction potential. Dodd and Greider show that a smooth angular
distribution, in good accord with existing data, results even when a sharp
cutoff diffraction model is used, provided that both recoil and finite potential
range are properly accounted for. The effect is independent of the particular
form of model chosen, and is a reflection of the three-body nature of the
transfer process.

To demonstrate how this rather remarkable result comes about, consider the transfer reaction as

\[(a+c) + b \rightarrow a + (b+c).\]

Let subscripts i and f refer to the initial and final states respectively. \(k_i\)
is the relative momentum of nucleus b and bound system \((a + c)\) in the entrance
channel; in the final state, \((b+c)\) and a are moving apart with relative
momentum \(k_f\).

For convenience, consider the matrix element of the transition \(i \rightarrow f\) as given by the distorted wave Born approximation:

\[T_{if} = \int \int \chi_f^{(-)} * (k_f; \vec{r} - \vec{r}'/A_f) u_f^{*}(\vec{r}') V_{ac}(\vec{r} + \vec{r}').\]

\[\tag{11-26}\]

where \(u_i\) and \(u_f\) are the bound state wave functions for c in the initial and final
states, respectively. The mass ratios are

\[A_i = \frac{m_a + m_c}{m_c}; A_f = \frac{m_b + m_c}{m_c}\]

\[\tag{II-27}\]

\(A_f\) and \(A_i\) are assumed large with respect to unity. The vector \(\vec{r}\) relates
the centers of mass of the cores a and b, while \(\vec{r}'\) describes the position of
c c relative to b. The dominant interaction is taken to be that between core a
and transferred particle c, and is considered a finite range interaction.

The distorted waves \(\chi_i^{(+)}\) and \(\chi_f^{(-)}\) will be taken to be plane waves
modified by the effects of the strong absorption. Following the general
diffraction model prescription, this is achieved by modulating the plane
waves by functions $B_i$ or $B_f$ which become zero inside a sphere of radius $R_0$ (the "black nucleus") and in the shadow scattering region. In this approximation $B_i$ and $B_f$ are functions of $r$ only. This is just the sharp cut off diffraction model already discussed.

The central assumption made by Dodd and Greider is that although terms in $1/A$, related to the recoil momentum of the appropriate core, will be considered negligible with respect to either $k_i$, $k_f$, or $q$, the momentum transfer, in the matrix elements, these terms in $1/A$ will be retained in the phase of the distorted waves.

The transition amplitude may now be written

$$T_{if} = \int d^3r e^{iq \cdot r} B_i(r) B_f^* \left( \frac{r}{r} \right) G_{if}(r)$$

where $G_{if}(r)$ represents the transfer function and is given by

$$G_{if}(r) = \int d^3r' u_i^*(r') V_{ac} \left( \frac{r + r'}{r} \right) u_i \left( \frac{r + r'}{r} \right) e^{ip \cdot r}$$

The factor $e^{ip \cdot r}$ represents the recoil where

$$p = - \left( \frac{k_i}{A_i} + \frac{k_f}{A_f} \right)$$

For simplicity consider the following case

(i) spin effects are neglected

(ii) $u_i$ and $u_f$ are considered harmonic oscillator wave functions

(iii) $c$ is taken to have zero angular momentum relative to $a$ in the initial state, and $L$ angular momentum, with projection $M$, relative to $b$ in the final state

(iv) $V_{ac}$ is taken to be a gaussian finite range potential.

The bound-state wave functions and the interaction potential $V_{ac}$ are now expanded in spherical harmonics and the resulting integrals are evaluated by invoking the approximations of the ring-locus diffraction model which restricts the integration to an annular region.

The result is given by
The inclusion of recoil effects in the diffraction formalism also leads to the prediction that the population of selected angular momentum states will be enhanced during high energy transfer reactions. This places an effect long observed experimentally and heretofore only explained classically, on a sounder physical basis. Classically, the inhibition of the low-\(L\) states is expected because the transferred particle carries the fraction \(1/A_i\) of the incident momentum in the entrance channel. When \(c\) is captured by \(b\) at the nuclear surface \(R_b\), it imparts an angular momentum \(L_c = k_i R_b / A_i\) to the final nucleus. The implication is that states involving an angular momentum...
transfer \approx L_c will be preferentially populated, others correspondingly
inhibited, other factors being equal. This consideration of angular momentum
mismatch was first proposed, on an intuitive basis, by Pehl\(^{(4)}\) and later
demonstrated by Sachs et al.\(^{(5)}\). In the Dodd-Greider formalism, this enhance-
tment comes about when the oscillations of the recoil term \(e^{i\beta \cdot r}\) interfere
constructively with those of the bound state functions in the transfer function
\(G_{if}(r)\). In consequence the enhancement factors are energy and mass dependent
but the dependence is a slow one.

The simple model used in the Dodd-Greider calculation outlined
above represents an extreme case, in that the introduction of more realistic
assumptions will result in still further damping of the oscillations. The one-
parameter ring-locus model employed here yields the maximum diffraction
oscillations in the cross section. A more recent calculation\(^{(36)}\), employing
more realistic assumptions, indicates that the \(q\) dependence of the angular
distribution may be better represented by \(q^{-4}\) than by \(q^{-3}\).

Although the calculation above considered only an \(l=0\) to \(L=L\)
transition, the independence of the monotonic angular distributions on the
angular momentum suggests the generalization that all transitions \(l=L'\rightarrow L\),
will exhibit this same smooth nature. An experiment, in general, will
represent mixtures of several values of transferred angular momentum, each
of which should yield the same monotonic \(q\)-dependence.

Durand\(^{(95)}\) has recently provided an alternative approach to this
problem, based upon the observation that multi-particle problems can be re-
formulated by replacing composite systems by "elementary" particles with
vanishing wave function renormalization constants\(^{(97)}\). In this formalism,
the recoil effects noted above appear as simple kinematic consequences of the
model. In addition, the principal dependence of the cross section on \(q\) is obtained,
in agreement with the result of Dodd and Greider, as well as the prediction that
population of states of low angular momentum will be inhibited at high energies.
G. Summary and Discussion

Cluster transfer reactions between complex ions have been discussed within the framework of several contemporary reaction models. All these models assume that the reaction is quasi-elastic to good approximation, and that the strong nuclear absorption can be taken into account by the exclusion of a spatial region in the evaluation of transition amplitudes.

Although detailed distorted-wave Born approximation calculations, employing an optical potential, have not yet been performed for heavy ion transfer reactions at high energy, to the author's knowledge, there seems little doubt that they could yield adequate fits to the experimental angular distributions. However, there are so many free parameters involved, and the concept of a complex local potential for strongly-interacting, many-body systems is so ill-defined that the physical significance that could be attached to such a fit is unclear. If optical model techniques represent just a convenient phenomenological parameterization of the problem, as appears likely, then the techniques of the diffraction models discussed above appear both simpler and more justifiable.

The gross features of high energy transfer reactions are well-described by a simple zero-range diffraction model, based on the assumption that only encounters on a well-defined ring around the target nucleus contribute significantly to the transfer cross section. The phenomenological model of Frahn and Venter refines this simple, semi-classical approach by including smoothing of the nuclear surface diffuseness. Dar generalizes this idea, and extends the model by including Coulomb effects, cluster-core interactions, and specific angular momentum transfer. His approach amounts to a WKB approximation to the DWBA procedure.

The diffraction theories of Dar and of Dodd and Greider are both able to account for the absence of diffraction oscillations in the recently observed experimental angular distributions for the non-Coulomb damped case. Dar proposes that the smoothing is a direct consequence of the angular momentum carried by the transferred cluster, which causes an equal mixing of oscillatory distributions which are 180° out of phase with each other.
Dodd and Greider maintain that a smooth angular distribution reflects the non-negligible effects of finite range and of recoil terms, neglected in other treatments. It appears possible that both effects contribute to general low η transfer reactions, such as those investigated in the present work.

It is interesting to note that the independence of the transfer cross section upon the details of the reaction, as found by Dodd and Greider and by Durand is quite the obverse of the result of Dar, who bases his explanation of the smooth angular distributions on the specific structure of the nuclei involved, and ignores the effects of momentum recoil in his derivation.

In the Dodd-Greider approach the inclusion of the momentum recoil terms in the phase of the transition matrix elements leads to the prediction of a strong inhibition of the population of low angular momentum states by transfer reactions.

All strong absorption models are able to account adequately for the experimentally observed excitation functions for single nucleon transfer situations, characterized by an initial steep rise and a saturation as higher energies are reached. This dependence is a direct consequence of the strong absorption of the low energy (low-q) partial waves and the large number of open reaction channels deleting the flux as energy increases.

It is the contention of Dar(13,34,35) that the Frahn-Venter model(11) , despite its ability to fit a large body of experimental data, contains an erroneous assumption. The arbitrary choice of the form of the scattering function leads to a result that, according to Dar, is valid only for collective excitations via inelastic scattering.

The models discussed in this section have evolved in parallel with the work to be presented in this report. When this study was initiated, the available models, which had enjoyed striking successes in fitting heavy ion elastic scattering and neutron transfer reactions on heavy targets (i.e., large η reactions), made specific predictions for the equivalent transfer reactions.
when light targets were involved (i.e., low $\eta$ reactions). The present experiments were conceived largely with the intention of utilizing these predictions in an investigation of the relative importance of the Coulomb and nuclear reaction amplitudes, presumably separable in these low $\eta$ reactions. As noted above, the non-appearance of the predicted oscillations in the experimental angular distributions, as measured in the present studies, and in previous work at this laboratory$^{17,29}$ have provided some of the impetus for the revision of the early reaction models.

It was also the goal of this research to provide a consistent set of related data on elastic and inelastic scattering, and single and multiple nucleon transfer in the hope of making possible a significant test of the complex potential, or optical model approach. Such a set of data is presented in this report, although present experimental limitations have restricted the range of angles at which data has been obtained.

In addition to these objectives, it was hoped that it would be possible to extract significant nuclear spectroscopic information, by exploiting the demonstrated high selectivity of heavy ion reactions in the population of excited states. It was hoped that this selectivity would highlight certain components of the wave functions of the strongly populated states.

Of particular interest has been the examination of neutron and proton transfer reactions leading to analog states in the mirror pair $^{15}$N$^{15}$-O$^{15}$. Much detailed information regarding the wave functions of the states concerned is available for this system from other sources.

In succeeding sections, the results of these experiments are presented as they bear upon these objectives.
III. EXPERIMENTAL EQUIPMENT AND TECHNIQUES

The scattering chamber, particle detection apparatus, and pulse amplification electronics used for these studies have already been described in detail elsewhere.\textsuperscript{(29)} As the modifications necessary for the present work were quite minor, only a brief summary will be presented. The particle identification system and the methods of data analysis, however, represent significant departures from the aforementioned applications, and so these topics will be treated extensively in this section and in an appendix to this report.

A. Accelerator and External Beam Facilities

The $^{14}$N beam of the Yale University Heavy Ion Linear Accelerator was used throughout this work. This accelerator provides a pulsed beam of approximately 10.5 MeV/nucleon energy. An external previously calibrated system of analyzing magnets is used to provide a momentum analysis of the beam.\textsuperscript{(38)} Nominally, the beam energy is defined to within 1%; experience indicates that it is probably somewhat better than this. As will be seen, calibration of the data acquisition equipment is largely predicated upon the value of the incident beam energy, and it is this uncertainty which imposes the ultimate limitation upon the use of the Heavy Ion Accelerator for nuclear spectroscopic studies.

Variation of beam energy is achieved by the insertion of absorbing foils in the beam trajectory; subsequent momentum analysis produces a severe loss in beam intensity. This precluded the usage of any but the full energy beam for the relatively low cross section transfer processes studied in the present work. Using a relativistically correct form of the equations of motion for a charged particle moving in a uniform magnetic field, the beam energy was determined to be 148.1 MeV.
B. Scattering Chamber

The scattering chamber utilized for these experiments has been described by Sachs and Ollerhead\(^{(29, 39)}\), and is depicted schematically in Figure 3. It has three individually rotatable lids, each capable of holding detectors, and an externally adjustable target assembly which allows changing of up to four targets under vacuum. Beam collimation was provided by two brass collimators with circular apertures of 1/8 inch diameter, 18-5/32 inches apart, the closest one being 12-9/64 inches from the target center. These are followed by a brass anti-scattering snout, which minimizes the incidence of slit-scattered particles on the target. Although the calculated beam diameter at the target was 0.292", burn spots on paper calibration targets revealed that the effective beam profile is somewhat smaller. This is not surprising, as the calculations assumed a fully illuminated slit at the exit of the accelerator, and the beam is known to be well-collimated and convergent at this point.

A new rough-pumping port and associated equipment was added to the chamber for these experiments. This performed several functions. Primarily, a series of baffles and a needle valve permitted very gentle pumping down and letting up to air, thus preventing damage to the fragile foil targets and detector windows. A nitrogen cold trap lessened the deposition of pump oil residues on the targets and detectors, and an ionization gauge mounted directly on the chamber provided more meaningful information about the chamber pressure during pump down than the former one located at the accelerator exit valve.

A mercury diffusion pump maintained an operating pressure typically between $5 \times 10^{-5}$ and $1 \times 10^{-6}$ mm Hg.

In the case of the solid targets, the beam was collected in a cylindrical Faraday cage, 1-1/4" in diameter, 7-1/8" long, with its entrance 13" beyond the target in the beam line. Outscattering of electrons from the Faraday cup was magnetically suppressed, although experience indicates that this is probably superfluous. For the gaseous targets, the beam was stopped in the back wall of the target cell itself, and directly integrated.
Figure 3. Schematic diagram of scattering chamber, (from reference 29).
SCATTERING CHAMBER

- Rotating Insert
- Detector Ports
- To Detectors
- Rotating Top Plate
- Detector Port
- Vernier
- Target Assembly
- Rotating Bottom Plate
- Target Position Indicator
C. Targets

The carbon target used for the bulk of this work was a self-supporting foil of natural isotopic carbon, prepared by standard vacuum evaporation and flotation techniques. A detailed treatment of these methods and a full discussion of residual impurities has been given by Sachs, and will not be repeated here.

The average thickness was determined, by a weighing procedure and by α-particle range measurements, to be 0.21 ± 0.03 mg/cm², and represented a compromise between maximization of the reaction yield and minimization of the energy loss of the beam and reaction products in the target. Energy losses in the target have been ignored in this work; they are of the order of tenths of a percent.

It has been shown that carbon targets fabricated in this fashion contain three major types of contaminants: C¹³, heavy impurities, and hydrogen. A heavy impurity with mass about 30, probably phosphorus from the stripping agent, was observed, and causes no difficulty. Similarly the hydrogen presents no problem at the energies and angles studied here. The C¹³, however, is a potential source of confusion in these reactions, and since it is present in at least its natural isotopic abundance of 1.11%, it was felt necessary to investigate this in detail.

For this purpose gaseous targets of 58.6% C¹³-enriched methane and ordinary research grade methane were employed. A target cell designed for use with this chamber was used, although Havar foils were substituted for the previously used Mylar entrance and exit foils, and a new gas handling system was constructed. Pressure was measured with a mercury manometer, readable to within 1 mm of Hg. Provision was included for recovering the gas and for easily substituting other gases for comparison and calibration purposes. Temperature of the cell was monitored by using a thermometer in thermal contact with the cell.

Commercial gold and aluminum leaf were used for calibration purposes throughout these experiments.
D. Detectors

The dE/dx x E particle telescope utilized for these experiments has been described in detail elsewhere \(^{(29, 39)}\); it is pictured in Figure 4. The ionization chamber is a parallel-plate, gridded, high resolution detector with a rise time fast enough to allow its usage with the heavy species under investigation. Operating parameters for this work were: gas pressure = 60 mm Hg, collector and repeller voltages of ±2000 volts, respectively. The gas used was Matheson P-10(90% argon, 10% methane); it was flowed through the chamber by a mechanical pump, pressure being maintained by a regulating device \(^{(46)}\) and monitored continually on a closed-circuit television network. The ionization chamber has lucite reentrant snouts, each containing a collimator, so as not to perturb the electric field between the plates. A third collimator, 1/8" in diameter, also fabricated of lucite, was added to the rear snout for these experiments. Its size and placement were calculated to make it serve as an anti-scattering slit. It was found that outscattering events were significantly reduced by this modification. Typical resolution of the ionization chamber for 145 MeV \(^{14}\)N ions elastically scattered from gold is the order of 3.5%.

The residual energy detector was a gold surface barrier, n-type silicon semiconductor detector \(^{(47)}\), with a depletion depth of over 500 microns, sufficient to stop all isotopes of interest for this experiment. The detector resolution was typically between 0.9 - 1.1% for \(^{14}\)N ions which had passed through the ionization chamber. The energy spread of the beam contributes significantly to this resolution, as does the variation in the energy lost in the ionization chamber.

The solid angle subtended by the particle telescope is \(4.92 \times 10^{-5}\) steradians. For some of the runs, the detector was cooled by ethylene glycol recirculating through copper coils in thermal contact with the detector block. The refrigerant was itself cooled in a heat exchanger connected to the experimental vacuum system refrigerator \(^{(48)}\). Auxiliary cooling was provided by mechanical blowers, which dissipated the heat generated in the preamplifiers. In all cases, the detector leakage current was less than 1.0 \(\mu\) ampere, with usual values.
Figure 4. Ionization chamber, showing location of the semiconductor E-detector (from reference 29).
below 0.5 $\mu$ ampere. Detector noise was always less than 10 mv (at the output of the preamplifier) and was generally less than 5 mv.

Another surface barrier detector was affixed to the rotatable bottom lid of the scattering chamber; it was kept at a fixed angle and used to monitor the operation of the beam charge integrator.

E. Electronics

The output signals of all detectors were fed directly to conventional thermionic voltage preamplifiers, mounted directly onto the flanges holding the detectors. The preamplifiers used with the semiconductor detectors are modified versions of one designed by R. Beringer\(^{(29, 49)}\). For the present work the decay time was decreased from about 30 microseconds to 5 microseconds in order to permit faster counting without pulse pile-up. The ionization chamber preamplifier is Baird Atomic Model 219, slightly modified to reduce the noise. All preamplifiers make negligible contribution to the noise levels.

White cathode followers are used to drive the signal transmission cables coupling the preamplifiers to the experimental control area. These cables are approximately 250 feet long, and include delay lines to provide the proper temporal sequence at the inputs to the data sorting equipment.

The amplifiers, single channel analyzers, and external coincidence circuitry used in all this work were developed and built at Yale under the supervision of C. E. L. Gingell\(^{(50)}\). They are part of a modular, self-powered, transistorized instrumentation system of high linearity and stability.

The amplifiers were used in the double delay line clipping mode, with a clipping time of 0.75 $\mu$second. As will be discussed shortly, these amplifiers constitute all the equipment necessary for the operation of the particle identifier with the multiparameter analyzer (MPA). For monitoring purposes, the outputs of the amplifiers were scaled after passing through single channel analyzers set to match the discrimination levels of the MPA.

The monitor counter signal, after amplification, was required to pass single channel analyzer gates set on the structure of interest in the spectrum
(usually the elastic peak), and was counted in a scaler. The monitor spectrum was also collected in a RIDL 400 channel analyzer, so that the detector performance could be continuously evaluated.

Total beam charge was measured by a commercial integrating device¹. Under the present operating conditions the manufacturer claims an accuracy of 1%. Care was taken to collect data using the same range of this instrument whenever possible, so as to minimize the possibility of inter-scale calibration deviations. For all runs, the ratio of integrator counts to the counts accumulated in the monitor counter scaler (which has negligible dead time) was computed, and only those runs where this ratio remained constant were retained.

For the runs where the gaseous target was used, the target cell itself served as the beam catcher. Electric suppression of secondary electrons was provided. The absolute accuracy of this measurement is not important in the context used here, where only a comparison of relative cross sections for the C¹² and C¹³-methane targets is being sought.

F. Particle Identification System

The identification of the reaction products of heavy ion reactions at energies significantly above the Coulomb barrier is an important and challenging experimental problem. Essentially all reaction channels are open at these energies, and the investigation of a particular energy level of a given isotope will generally not be amenable to kinematic coincidence techniques in themselves, especially for low-cross section reactions.

One much used approach to this problem has involved the collection of the reaction products in stacks of thin foils; subsequent measurement of the induced radioactivity then allows the identification of a particular species by means of its characteristic half-life for radioactive decay. The activity of the individual foils provides a measure of the range of the particle, and therefore, its energy.
While much useful nuclear information has been obtained using this method \(^{(52)}\), (particularly when the intrinsically high integration capabilities of such a system have been exploited), this is more a commentary on the resourcefulness and tenacity of the experimenters than on the efficacy of the technique, whose disadvantages and limitations are all too apparent:

1. Identification and isolation of individual residual states is generally not possible.

2. Only reactions with an end product possessing a characteristic (and measurable) radioactivity may be investigated.

3. No information is available to the experimenters during the course of the experiment, making survey experiments difficult or impossible.

4. The counting procedures are both complex and tedious.

5. Limitations are placed on energy resolution because of straggling effects in the catcher foils.

6. More complicated experiments such as angular correlations, etc. are not possible.

The most widely employed procedures to surmount these difficulties are based upon the Bethe \(^{(53)}\) equation relating the rate of loss of energy of a charged particle moving through matter to its mass, charge, energy, and the properties of the absorber:

\[
-\frac{dE_1}{dx_1} = \frac{4\pi e^2 Z_1^2}{mv^2} N_2 Z_2 \left[ \ln \left( \frac{2mv^2}{I} \right) - \ln (1-\beta^2) - \beta^2 \right]
\]

(III-1)

where subscript 1 denotes the moving ion, and 2, the absorber. \(Z_1\) is the ionic charge in units of e, \(N_2 Z_2\) represents the electron density of the absorber, \(v\) is the velocity of the incident ion, \(\beta\) equals \(v/c\), \(I\) is an average ionization potential for the absorber, and \(m\) is the mass of the electron.

If the numerators and denominators are multiplied by \(M_1\), and the relativistic terms are ignored, this equation becomes

\[
\frac{dE_1}{dx_1} = k \frac{M_1 Z_1^2}{E_1} \ln \left( \frac{4(E_1 m/M_1)}{I} \right)
\]

(III-2)
where \( k \) contains only physical constants. The energy variation of the logarithmic term is slow, and, for the present situation may be considered essentially constant, so that the equation becomes

\[
\frac{dE_1}{dx_1} \cdot E_1 = K M_1 Z_1^2
\]

Thus the product of \( E \) and \( dE/dx \) provides a unique identification of the particle being measured, since no two known nuclides have the same value of \( MZ^2 \) (provided that the effective charge of the ion remains close to the nuclear charge, a situation which pertains here\(^{(29)}\)). The locus of a particular isotope in a plot of energy loss versus energy is seen to approach a rectangular hyperbola.

An alternate procedure to the one to be discussed here has been developed by Newman and his collaborators\(^{(54)}\), who use a particle telescope to identify the \( z \) of a reaction product (i.e., only the element and not the isotope) and then require a kinematic coincidence with the recoiling residual nucleus for specific identification of reaction products. The technique is limited because both product and recoil must be long-lived enough to reach the detectors, and because it is very difficult to make measurements on more than one state at a time.

The particle identification system developed by Sachs, Ollerhead\(^{(29,39)}\) and collaborators utilized an analogue pulse mutiplier to form the product of the energy loss and energy signals. With the aid of a plotting scope, multiplier parameters were adjusted so that the hyperbolic loci were linearized. By setting voltage gates on the peaks of interest in the multiplier spectrum, data on particular nuclear species was routed to individual 400 channel analyzers, or analyzer memory subgroups. In this fashion, a conventional energy spectrum is obtained directly by requiring a time coincidence between the E-detector pulse and the identifying signal from the multiplier.
For the present work, the entire multiplier system and its associated electronics, analyzers, etc. were replaced by a single instrument: a 20,000 channel multiparameter analyzer (MPA) conceived jointly by Yale University and Oak Ridge National Laboratory, and built to specification by the Tullamore Division of the Victoreen Instrument Company\(^57,58\).

Briefly, the analyzer consists of two independent, normally closed, analogue-to-digital converters (ADC’s) coupled to a 20,000 word, 24-bit random access magnetic core memory. Each memory cell thus has a capacity of \(10^6\) counts. A simplified block diagram of the analyzer logic is shown in Figure 5. Figure 6 is a photograph of the analyzer in operation in the Heavy Ion Accelerator control room.

If the \(E\) detector signal is applied to one of the ADC's and the \(\Delta E\) signal to the other, and an internal slow-coincidence (1 microsecond) is required between them, then only those events which correspond to traversal of the particle telescope will be stored. In this way the approximately hyperbolic loci predicted by Eq. (III-1) are obtained directly. The contents of the memory are dumped onto magnetic tape in a computer-compatible format, and an energy spectrum for any isotope is formed by the computer using a technique to be described below.

Since the MPA is so basic to these experiments, it is perhaps appropriate to describe it more fully before proceeding to a discussion of the data analysis. Only those features that pertain to the present work will be treated here; the above references should be consulted for a more complete description.

The analyzer memory may be used in several configurations; most of this work was done with the 100 x 200 channel format, with the \(E\)-input normally being to the 100 channel ADC. For one run, the \(E\) signal was placed on the 200 channel direction, resulting, of course, in much greater energy resolution, at the expense of isotopic separation (which, however, was still adequate for particle identification). Figure 30 illustrates the isotopic resolution of the system.
Figure 5. Block diagram of Victoreen multiparameter analyzer logic (from reference 57).
Figure 6. Multiparameter analyzer in place in the Heavy Ion Accelerator control room. The Yale modular transistorized electronics are seen at the right; the 400 channel pulse height analyzer and associated electronics for the monitor channel appear at the left. Also seen are the preset scalers and calibration pulser. Not shown is the magnetic tape unit.
One of the most useful features of this instrument is its real-time totalizing and display capability. That is, each count is added to the appropriate channel as it is received, making possible constant monitoring of the experiment in progress. Two long-persistence ten-inch oscilloscope screens provide instantaneous display of the data. One scope provides a contour map of the entire memory contents, with the intensification of a given channel proportional to its contents, on either a linear or logarithmic scale. A variable contour level and window width associated with this display enable only those channels whose contents fall within a selected range to be intensified. The sweep time for the full 20,000 channel display is less than one second.

The adjacent screen provides either a static display of any switch-selected cross section plane in either the x or y direction, or a dynamic display of successive planes through the memory.

Standard ten channel markers and a digital plane intensifier are available with both displays, making possible the precise location of any feature of interest. A third display is also available. This is an "isometric" display wherein each successive y plane is displaced slightly, creating a three-dimensional illusion. All of these displays are pictured in Figure 7 through Figure 9. Note that although these displays are identical to those observed on the MPA they were actually generated on a computer by a program described in Appendix II.

The input-output facilities of the MPA include a paper tape punch and reader, a computer typewriter, an x-y pen recorder, as well as a magnetic tape drive. The entire memory contents can be written on magnetic tape in a computer-compatible format in 13 seconds. Magnetic tape is the standard output medium and has been found economical and reliable.

The analyzer also incorporates an automatic sequence programmer which, in conjunction with three scalers which may be preset for a given number of counts, provides for added convenience and accuracy of operation. All functions of the analyzer are programmable, and the experimental operation may be advanced to its next stage using as a criterion elapsed time, accumulated counts from any source, or the application of an externally generated signal.
Figure 7. Contour map of the MPA memory contents, showing from right to left, the loci corresponding to the isotopes of oxygen, nitrogen, and part of carbon. The ordinate is E and the abscissa ΔE. Note the effect of the biased amplifiers. Thresholds were set as follows; where z represents the number of counts in a given channel: $z < 100 =$ blank; $100 \leq z < 1000 =$ low intensity; $1000 \leq z < 10000 =$ high intensity; $10000 \leq z =$ blank. Note the analogous structure in $^{15}N$ and $^{15}O$, and the high peaks corresponding to elastic and inelastic scattering. This figure was generated on an IBM 7040/7094 computer.

Figure 8. Same as Figure 7, but lower thresholds changed to highlight the structure: $z < 200 =$ blank. The peak in the upper right hand corner is due to the calibration pulser.
Figure 9. "Isometric" plot of the data shown in Figure 7 and 8.
Similarly, bias output voltages of the programmer may be used to control servo-driven external apparatus, although this was not necessary for the present work.

For these experiments the system was operated in a semi-automatic fashion. All scalers, analyzers, integrators, etc. were coupled to the MPA programmer, which was set to run for one-half hour intervals. At the end of this time, all equipment was automatically stopped and calibration pulses were sent through both the particle identifier and monitor counter electronics to check for any gain shifts. Quantities of interest on the various scalers (see Figure 10 for a schematic of the system) were recorded, and if the specified running time had elapsed the data was written on magnetic tape.

With the addition of linear amplifiers, the MPA constitutes a completely self-contained, self-powered system. Each ADC has an associated variable gain window amplifier, which enables any given portion of the input spectrum to be expanded and examined in detail. Typical procedure was to first conduct a short exploratory experiment without the window amplifiers, and then using the marker and channel identifier features and a calibrated pulser, set the window amplifiers for a detailed study of the desired region.

Set up for an experiment is a routine and extremely rapid procedure and is completely reproducible. Quite the obverse was true with the former system. The MPA can store information on many reactions simultaneously; while this was also the case for the multiplier, each new species required its own single channel analyzer as well as a multichannel analyzer memory subgroup, which places a restriction on the number of spectra that can be viewed simultaneously with good resolution because of equipment limitations. Here, too, the problem of relative dead times and inter-analyzer calibration is present. The magnetic tape output is much faster than the typewriters associated with the 400 channel analyzers (13 seconds for 20,000 channels as compared to about 12 minutes for 400 channels), which is important since this is time lost to the experimenter during the course of the experiment.

It is important to note that the information contained in the analyzer memory is not in the form of a conventional energy spectrum, but is rather
Figure 10. Block diagram of the electronic system.
Figure 11. ∆E and E detector responses. The solid lines are least squares fits to gaussians; the errors shown are statistical.
a direct representation of the approximately hyperbolic loci dictated by the Bethe energy-loss equation. Although qualitative information as to relative production of isotopes, population of states, etc. is immediately apparent from the display, the data must be processed on a computer for quantitative evaluation. This procedure is discussed in the following section.

In summary then, the particle identification system described above is characterized by its versatility, simplicity of operation, and vast data storage capabilities.

G. Data Reduction Procedures

In the above discussion of the multi-parameter analyzer particle identification system, it was pointed out that while the present technique enjoys great advantages in flexibility and ease of operation over the previous pulse multiplication methods, the data acquired must be reduced before an energy spectrum in the usual sense is available. The large amount of information obtained for any given experiment and the iterative nature of the processes necessary for its accurate interpretation strongly indicate the usage of a digital computer. In short, the usual demarcation between the acquisition of data proper and its subsequent numerical reduction and analysis has been superceded by a procedure in which the data reduction is an integral part of the experimental technique. This will be discussed more fully in Section V of this report.

It is important to emphasize at this point that the present mode of operation in no way lessens the information immediately accessible to the experimenter during the course of the experiment. Use of the variable discriminator and window width controls on the contour display of the M. P. A. graphically highlights any structure in the energy spectrum of any particular isotope, and inter-species comparisons are facilitated by this form of the display. This is demonstrated in Figure 7 where the analogous structure in the N$^{15}$ and O$^{15}$ spectra is clearly seen. The structure in other species is available simultaneously; should more quantitative information be required, it may be obtained either by typing out the channels of interest determined using the channel marker or a digital plane
identifier built into the analyzer) or by viewing a cross section display through any given plane, parallel to either the E or \( \Delta E \) axis.

When the isotopes of interest are completely resolved, the energy spectra may be simply obtained by manually adding the counts in the channels of interest from a computer printout which is a quantitative analogue of the contour display\(^{(59)}\). While this is at best a tedious procedure, it has been used successfully when light elements (and therefore large channel separations) are concerned\(^{(60)}\). However, for the heavier elements involved in the present work, where the differentiation in values of \( MZ^2 \) begins to approach the resolution of the identification system, such visual decomposition is no longer desirable. Such is also the case when the physical nature of the experiment dictates simultaneous accumulation of the spectra of several species, thereby precluding the use of the window amplifiers to expand the spectrum. It was felt that the development of a technique that could easily and accurately produce an energy spectrum from the hyperbolic locus for a given isotope was crucial to the success of this procedure; for this reason the means by which this has been accomplished are described in some detail in this section, and a description of the mathematical methods employed is appended to this report.

The problem may be summarized simply as follows: in each plane of constant energy, for each element, there is a family of gaussians, corresponding to the \( dE/dx \) detector response for each isotope of that species. If the area corresponding to each gaussian is found, then the tabulation, over all energy planes, of these areas for a given isotope is the energy distribution of that isotope as customarily obtained on a standard one-dimensional pulse height analyzer. This spectrum will exhibit the response shape of the detector used for the energy determination; for the semiconductor detector used in this work, this response is also well represented by a gaussian, as evidenced in Figure 11. The low energy tail observed is the usual one due to multiple scattering, reactions in the detector, radiation damage effects, etc.; as can be seen it makes only a small contribution to the area of the peak. The curves shown represent least
squares computer fits and the deviations at the high points are the result of the statistical weighting used.

There is nothing conceptually difficult about unfolding a sum of gaussian functions; the problem is particularly amenable to solution by the method of non-linear least squares analysis. One quite successful variant of this technique has been described by McWilliams et al\(^{61}\). In the present work, however, the problem is compounded because the large volume of data to be handled makes it impractical to prepare a set of input variables for each plane to be fitted, and also because it is desirable to leave the burden of the selection of the channels to be included in the fit to the computer. In other words, an automatic technique is sought such that for a given two-dimensional array of data, the computer will form the energy spectrum for any specified isotope.

The method adopted is outlined schematically in Figure 12. The starting point of the fitting procedure may be viewed as the selection by the experimenter of several points along the ridges of the $dE/dx \times E$ loci for all isotopes of the element in question. At present, this is done visually by inspection of a memory-image contour printout (see Appendix II). The estimates of channel location are placed on standard punched cards; more sophisticated techniques employing a light pen (a photodiode device capable of communicating the coordinates of any particular channel to the computer directly) are planned for the future. The selection of these input coordinates is not at all critical, and from six to fifteen points have been found adequate in all cases treated. A three-parameter fit to a modified form of the Bethe energy loss equation is performed for each isotope, and these parameters then enable the calculation of the approximate location of the $\Delta E$ channel corresponding to the $dE/dx \times E$ locus for a given $E$ plane (for that isotope). In this way the gaussian fitting program is supplied not only with initial estimates of each peak's height and location, but with the channels to be included in that fit in each $E$-plane. As each successive plane is fitted to a sum of gaussians, the areas of the peaks corresponding to isotopes whose energy spectra are desired are stored, and, upon completion of the fitting procedure, this information is presented in graphical and tabular form as a conventional energy spectrum.
Figure 12. Functional flow of the data reduction procedure. Dotted lines represent linkages currently manual; solid linkages are performed by the computer.
RAW DATA FROM PARTICLE TELESCOPE
To MPA

BLOCKED MAGNETIC TAPE; 20,000 CHANNELS
To IBM 1401
TEASE
TAPE EDITING
To IBM 7040/94

COMUS

ARGUS

CRT ROUTINES
1. VARIABLE INTENSITY QUALITATIVE CONTOUR MAP
2. ANY PLANE
3. 'ISOMETRIC' PROJECTION

QUANTITATIVE CONTOUR MAP (COMPUTER PRINTER)

MEDUSA

INPUT: LOCI ESTIMATES, FITTING AND OUTPUT OPTION SPECIFICATIONS

FIT EACH ISOTOPE LOCUS AND STORE PARAMETERS

CALCULATE THE LIMITS OF THE GAUSSIAN FIT AND INITIAL ESTIMATES FOR FIRST E-PLANE

TABULATE AND PLOT RESULTS
A

PERFORM NON-LINEAR LEAST SQUARES FIT TO SUM OF GAUSSIANS (ΔE RESPONSE)

TABULATE AND PLOT RESULTS

REPEAT FOR EACH SPECIFIED E-PLANE AND STORE RESULTS

CONVERT PLANE NUMBERS TO EXCITATION ENERGY IN PRODUCT NUCLEUS, USING MPA CALIBRATION AND RANGE-ENERGY PARAMETERS

FORM ENERGY SPECTRUM FOR EACH ISOTOPES SPECIFIED

TABULATE AND PLOT RESULTS

PUNCH RESULTS ON DATA CARDS

TO NEXT CASE

B
INPUT:
1. ESTIMATES OF PEAK LOCATIONS, HEIGHTS AND WIDTHS (EXCITATION OF STATE MAY BE SPECIFIED INSTEAD OF LOCATION)
2. REACTION AND CALIBRATION PARAMETERS
3. BACKGROUND AND FUNCTIONAL DEPENDENCE SPECIFICATIONS

FIT ENERGY SPECTRUM TO E-DETECTOR RESPONSE

USE RESULTS TO CALCULATE:
1. FITTED EXCITATION ENERGIES
2. CROSS SECTIONS (RELATIVE OR ABSOLUTE)
3. C.M. SCATTERING ANGLES
4. LINEAR MOMENTUM TRANSFER

TO NEXT SPECTRUM

TABULATE AND PLOT RESULTS

TABULATE
These spectra are punched on data cards by the computer and constitute the input, along with reaction parameters, to still another computer program which performs least squares fits to the response of the energy detectors. Several types of background may be specified in this gaussian fitting program; a constant background was used for the present work. From the values of peak area and locations so obtained, absolute (or relative) cross sections, reaction energies, excitations in the product nuclei, center of mass scattering angles, and linear momentum transfers are calculated. All these results are tabulated and plotted. Although not required for this work, it is a simple matter to add a routine which forms the angular distribution as well.

In order to convert the results from channel numbers in the MPA to reaction energies and excitations, the results of range-energy fits are applied to correct for the ion chamber loss, in a manner to be described. Basic to the above procedures is a highly modified version of the excellent generalized least-squares fitting package written by Moore and Ziegler. A variant of the Gauss-Newton method for solution by approximate linearization of a system of non-linear simultaneous equations is employed. As in any such iterative procedure, particularly when the parameters obtained from one fit are used as initial estimates for a subsequent problem, care must be taken to achieve convergence. Details of the mathematical means utilized to determine the statistical validity of the fits and a summary of the output obtained are presented in the above mentioned Appendix. Also appended is a description of the other computer codes used in the analysis of the energy spectra.

H. Calibration

Calibration of the MPA must be performed using the beam of the accelerator rather than standard radioactive sources, as the latter are not energetic enough to traverse the ionization chamber at operating pressure. This is unfortunate, for, as noted in Section A above, the uncertainty in beam energy is large, probably about 0.6%. It must be remembered, however, that only the
absolute calibration is affected by this; values relative to some established fiduciary should be accurate to the order of 200 keV or better. In practice, using computer programs to find the center locations of calibration peaks, the absolute calibration was generally also as good as this, although this was perhaps fortuitous.

To obtain calibration points, the full energy beam was scattered from carbon, aluminum, and gold targets. The elastic scattering peak was examined at a fairly large angle so that the mass differences of the targets resulted in separations of several channels on the MPA. Additional calibration points were obtained using well-known features of the resultant spectra, e.g. the 4.43 state of C\textsuperscript{12} in the N\textsuperscript{14} + C\textsuperscript{12} inelastic scattering. A straight line was fitted to these points using a least squares computer program.

In order to correct for the energy loss in the ionization chamber, the range-energy data of Northcliffe\textsuperscript{(63)} was used. The equivalent thickness of the ionization chamber, including its entrance and exit windows, was taken as 17.4 mg/cm\textsuperscript{2} of aluminum; this figure had been previously obtained experimentally by Sachs, and has been verified theoretically to the accuracy required by these experiments by Poth\textsuperscript{(48)}.

Northcliffe's data was fitted to a simple three parameter power series of the form

\[ \text{Range} = a_1 + a_2 \cdot \text{Energy} + a_3 \cdot (\text{Energy})^2 \]

using a generalized version of the least squares computer code mentioned in the previous section. The quality of the agreement is shown in Figure 13, and the resulting parameters are listed in Table I. Note that a simple scaling relation will produce the range of any isotope of these elements. These parameters are stored in the computer memory, and it is then a straightforward procedure for the energy spectrum fitting programs to convert channel locations to the energy at the energy detector, and then to correct for the energy loss in the ionization chamber to produce the equivalent energy at the reaction site. This result is then used to compute excitations, center of mass angles, and other quantities of interest.
Figure 13. Least squares fits to the range-energy data of Northcliffe (reference 63). The circles are data points, the curves are the computed fits to $\text{Range} = a_1 + a_2E + a_3E^2$, where $E$ is energy in MeV/amu, and the parameters are tabulated in Table I.
RANGE OF HEAVY IONS IN ALUMINUM

![Graph showing the range of heavy ions in aluminum as a function of energy. The graph includes lines for different ions, such as Be\textsuperscript{9}, B\textsuperscript{11}, C\textsuperscript{12}, N\textsuperscript{14}, and O\textsuperscript{16}. The x-axis represents energy (MeV/amu) and the y-axis represents range (mg/cm\textsuperscript{2}).]
TABLE I
Empirical Range-Energy Relation for Heavy Ions
\[ R = a_1 + a_2 E + a_3 E^2 \]

\[ R = \text{Range in mg/cm}^2 \text{ of Al} \]

\[ E = \text{Energy, MeV/amu} \]

<table>
<thead>
<tr>
<th>Isotope</th>
<th>(a_1)</th>
<th>(a_2)</th>
<th>(a_3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Be(^9)</td>
<td>(7.0087525 \times 10^{-2})</td>
<td>(2.7148686)</td>
<td>(7.0537133 \times 10^{-1})</td>
</tr>
<tr>
<td>B(^{11})</td>
<td>(3.8493499 \times 10^{-1})</td>
<td>(2.2488679)</td>
<td>(5.4410501 \times 10^{-1})</td>
</tr>
<tr>
<td>C(^{12})</td>
<td>(4.3895465 \times 10^{-1})</td>
<td>(1.8548561)</td>
<td>(4.0395963 \times 10^{-1})</td>
</tr>
<tr>
<td>N(^{14})</td>
<td>(4.6629871 \times 10^{-1})</td>
<td>(1.7368591)</td>
<td>(3.4009882 \times 10^{-1})</td>
</tr>
<tr>
<td>O(^{16})</td>
<td>(4.8240267 \times 10^{-1})</td>
<td>(1.6956507)</td>
<td>(2.9043196 \times 10^{-1})</td>
</tr>
</tbody>
</table>

Note: The number of decimal places given above is, of course, not warranted by the accuracy of the experimental results. However, the exact results of the fit are presented here to minimize the possibility of round off error in iterative calculations on a computer.
A. Presentation of Results.

1. Energy spectra.

Representative energy spectra for the reactions $^{12}_{}\text{C}(^{14}_{}\text{N},^{15}_{}\text{N})^{11}_{}\text{C}$, $^{12}_{}\text{C}(^{14}_{}\text{N},^{15}_{}\text{O})^{11}_{}\text{B}$, $^{12}_{}\text{C}(^{14}_{}\text{N},^{13}_{}\text{N})^{13}_{}\text{C}$, $^{12}_{}\text{C}(^{14}_{}\text{N},^{16}_{}\text{O})^{10}_{}\text{B}$, and $^{14}_{}\text{N} + ^{12}_{}\text{C}$ elastic and inelastic scattering are presented in Figures 14 through 19. The first four reactions are taken to represent neutron transfer to the projectile, proton transfer to the projectile, neutron transfer to the target, and deuteron transfer to the projectile, respectively. These assignments are consistent with the observation that heavy particle stripping is not expected to contribute significantly at the forward angles studied in this work\(^{(64)}\).

As mentioned previously, all these spectra were obtained simultaneously. The points are presented as obtained from the computer, as described in the appendix to this report. The lines shown simply connect the experimental points, except in Figure 15 where the detailed results of the fit are shown. The excitations indicated are obtained from a detailed computer fit (see Appendix II) and represent the average values from all cases studied. The prominent features of all spectra were completely reproducible, and variations in calculated excitations were very slight.

Also presented are two energy spectra resulting from the bombardment of methane targets with nitrogen ions. One spectrum corresponds to the use of natural methane; $^{13}_{}\text{C}$ enriched methane was used for the other.

The error bars indicated reflect not only the usual errors associated with the number of counts, but also the quality of the isotopic resolution in that energy plane.

2. Angular distributions.

Angular distributions for the resolvable peaks of the above energy spectra are presented in Figures 20 through 24. In each case, the ordinate of the curve is $(d\sigma/d\Omega)_{\text{CM}}$ in mb/steradian, and the abscissa is the center-of-mass scattering angle. The angular distributions of the transfer reactions
Figure 14. $^{13}N$ energy spectrum, $C^{12}(N^{14},N^{13})C^{13}$. The errors shown reflect statistical errors as well as the quality of the isotopic resolution in a given channel. The solid lines simply connect the experimental points.
$^{13}_N$ ENERGY SPECTRUM

$^{12}_C (^{14}_N, ^{13}_N) ^{13}_C$

$E_{INC} = 148$ MeV

$\theta_{LAB} = 9.0^\circ$
Figure 15. $N_{14}$ energy spectrum, $C^{12}(N_{14}, N_{14})C^{12}$. The significance of the error bars and the solid line is as in Figure 14. The dotted lines are the result of fitting the gaussian detector response function to this data, using a constant background and requiring fixed detector resolution.
$N^{14}$ ENERGY SPECTRUM

$C^{12}(N^{14}, N^{14})C^{12}$

$E_{\text{inc}} = 148$ MeV

$\theta_{\text{lab}} = 90^\circ$

RELATIVE YIELD

CHANNEL NUMBER
Figure 16. $^15\text{N}$ energy spectrum, $^12\text{C} (^14\text{N}, ^15\text{N}) ^{11}\text{C}$. Error bars and solid curve have same significance as in Figure 14.

Figure 17. $^15\text{O}$ energy spectrum, $^12\text{C} (^14\text{N}, ^15\text{O}) ^{11}\text{B}$. Error bars and solid curve have same significance as in Figure 14.

Figure 18. $^16\text{O}$ energy spectrum, $^12\text{C} (^14\text{N}, ^16\text{O}) ^{10}\text{B}$. Error bars and solid curve have same significance as in Figure 14.
\[ C^{12}(N^{14},N^{15})C^{11} \]

\[ E_{N^{14}} = 148 \text{ MeV} \quad \theta_{\text{LAB}} = 8.5^\circ \]
$^{12}\text{C} (^{14}\text{N}, ^{15}\text{O})^{11}\text{B}$

$E_{N^{14}} = 148 \text{ MeV} \quad \theta_{\text{LAB}} = 8.5^\circ$
$^{16}\text{O}$ ENERGY SPECTRUM

$^{12}\text{C}(^{14}\text{N},^{16}\text{O})^{10}\text{B}$

$E_{^{14}\text{N}} = 148 \text{ MeV}$

$\theta_{\text{LAB}} = 8.5^\circ$
Figure 19. $^{15}N$ energy spectra resulting from the nitrogen bombardment of natural CH$_4$ and 59% C$_{13}$-enriched CH$_4$. 
$^{15}$N ENERGY SPECTRUM $^{14}$N + CH$_4$

$E_{\text{LAB}} = 148$ MeV  \hspace{1cm} \theta_{\text{LAB}} = 11.7^\circ$
Figure 20. $N^{13}$ angular distributions, $C^{12}(N^{14}, N^{13})C^{13}$. The lines shown have no theoretical significance, and are taken to have equal slope for convenience. Absolute errors are shown.

Figure 21. $N^{15}$ angular distributions, $C^{12}(N^{14}, N^{15})C^{11}$. Lines and errors as in Figure 20.

Figure 22. $O^{15}$ angular distributions, $C^{12}(N^{14}, O^{15})B^{11}$. Lines and errors as in Figure 20.

Figure 23. $O^{16}$ angular distributions, $C^{12}(N^{14}, O^{16})B^{10}$. Lines and errors as in Figure 20.
$^{13}\text{N}$ Angular Distributions

$^{12}\text{C} (^{14}\text{N}, ^{13}\text{N}) ^{13}\text{C}$

Neutron Transfer

$E_{cm} = 68.5 \text{ MeV}$

$\left( \frac{d\sigma}{d\Omega} \times 10 \right)$

0.0 MeV State

3.8 MeV State

$(d\sigma/d\Omega)$ cm, mb/sr

$\theta_{cm}$, Degrees

16 18 20 22 24 26 28 30
N$^{15}$ ANGULAR DISTRIBUTIONS

C$^{12}$($N^{14},N^{15}$)C$^{11}$

NEUTRON TRANSFER

$E_{cm} = 68.5$ MeV

$\frac{d\sigma}{d\Omega}, \text{mb/sr}$

$\theta_{cm}, \text{DEGREES}$

$5.3 \text{ MeV STATE}$

$7.3 \text{ MeV STATE}$

$2.0 \text{ MeV STATE}$

GROUND STATES
$^{15}O$ ANGULAR DISTRIBUTIONS
$^{12}C(^{14}N,^{15}O)^{11}B$
PROTON TRANSFER
$E_{cm} = 68.5\ MeV$

$d\sigma/d\Omega \times 100$

$\theta_{cm},\ DEGREES$

2.1 MeV STATE
6.8 MeV STATE
5.2 MeV STATE
0.0 MeV STATE
$^{16}$O ANGULAR DISTRIBUTION

$^{12}$C($^{14}$N, $^{16}$O)$^{10}$B

DEUTERON TRANSFER

$E_{cm} = 68.5$ MeV

3.8 MeV STATE

$(d\sigma/d\Omega)_{cm}, \text{mb/sr}$

$\theta_{cm}, \text{DEGREES}$
Figure 24. $^{14}N$ angular distributions, $^{14}N + ^{12}C$ elastic and inelastic scattering. The curves are drawn through the experimental points; absolute errors are indicated.
Figure 25. Kinematics for the $^{13}\mathrm{C}(^{14}\mathrm{N}, ^{15}\mathrm{N})^{12}\mathrm{C}$ and $^{12}\mathrm{C}(^{14}\mathrm{N}, ^{15}\mathrm{N})^{11}\mathrm{C}$ reactions. Some representative experimental points are shown.
$N^{15}$ REACTION KINEMATICS

- $C^{13} (N^{14}, N^{15}) C^{12}$
- $C^{12} (N^{14}, N^{15}) C^{11}$

$E_{N^{14}} = 148$ MeV

$L^{15}$ ENERGY, LABORATORY, MeV.

LABORATORY SCATTERING ANGLE, DEGREES.
are plotted on a logarithmic scale to emphasize their essentially exponential character. Only those runs for which the monitor counter to beam charge integrator ratio remained constant, and for which the structures of the spectrum were clearly outside statistical variations are presented.

Absolute cross sections were calculated according to the prescription

\[
\left( \frac{d\sigma}{d\Omega} \right)_{\text{mb/ster}} = 2.66 \times 10^{-13} \frac{Y(A/Z)}{Q(T)} \left( \frac{1}{\Omega} \right) \left( \frac{1}{\eta} \right)
\]

where \( Y \) is the yield in counts, \( A \) is the atomic weight of the target and \( Z \) is the effective beam charge at the integrator, \( T \) is the areal density of the target in \( \text{mg/cm}^2 \), \( Q \) denotes the integrated beam charge in Coulombs, \( \Omega \) is the solid angle subtended by the detector, and \( \eta \) is the counter efficiency.

Following the equilibrium charge distribution data of Northcliffe, \( z \) is taken as 7 for the \( N^{14} \) beam at these energies to very good approximation. Counter efficiency is considered = 1, and all the runs were normalized to the measured target thickness of .210 \( \text{mg/cm}^2 \).

The major sources of experimental error and estimates of their magnitude are

- Counting statistics and isotopic separation \(+15\%\)
- Counting loss \(<10\%\)
- Energy resolution \(1\%\)
- Integrated beam charge \(5\%\)
- Target density \(15\%\)
- Absolute angle \(\pm0.3^\circ\)
- Relative angle \(\pm0.1^\circ\)

A relative error of 10\% is assigned to these measurements; the standard error in the absolute magnitude of the differential cross section is taken to be \(\pm30\%\).

B. Calculation of Absolute Cross Sections for Transfer Reactions

The cross sections shown in Figures 20 through 23 may be considered,
to good approximation, to be represented by pure exponentials. That is,
\[ \frac{d\sigma}{d\Omega} = \sigma_0 e^{-a\theta}, \]
where \( \alpha \) is the slope of the straight line obtained directly from the logarithmic plots shown, and \( \sigma_0 \) is the value of the cross section obtained by linear interpolation to zero degrees. Since
\[ \sigma_T = \int \frac{d\sigma}{d\Omega} d\Omega = 2\pi \sigma_0 \int_0^\pi \sin \theta \ e^{-a\theta} d\theta, \]
this may be evaluated directly, if the observed shape is assumed to hold throughout the entire angular interval. Errors due to extrapolation to small and large angle regions are expected to be small due to the small value of the \( \sin \theta \) modulation in the former case, and the rapidly decreasing exponential term in the latter.

Bock et al. have observed rather striking increases in the cross sections for heavy ion transfer reactions at more backward angles than those presented here. Consideration of the large angle case in this work was precluded by the low energy of the nuclei involved and their consequent inability to penetrate the ionization chamber. However, this backward angle rise is attributed to heavy particle stripping reactions and so makes little contribution to the cross sections presented here, which are to be interpreted strictly as those pertaining to the indicated nucleon transfer mechanism.

The results of these calculations are tabulated in Tables (II-a) and (II-b).

C. Consideration of \( ^{13}\)C Content of the Target.

As discussed elsewhere, the contaminants present in natural isotopic carbon targets prepared in the manner previously described, present no difficulties for the reactions under consideration. Figure 15 indicates the clear resolution of the elastic scattering peak with respect to the heavier contaminants. At larger angles, the contaminant peak decreases sharply, and splits into two major components, one corresponding kinematically to a mass of about 30, tentatively identified as phosphorus from the stripping
TABLE II-A

Total Reaction Cross Sections for the $^{12}(N^{14},N^{15})C^{11}$ and $^{12}(N^{14},O^{15})B^{11}$ Mirror Reactions, Assuming Only Neutron and Proton Transfer, Respectively

<table>
<thead>
<tr>
<th>Observed Nucleus</th>
<th>$N^{15}$</th>
<th>$O^{15}$</th>
<th>$N^{15}$</th>
<th>$O^{15}$</th>
<th>$N^{15}$</th>
<th>$O^{15}$</th>
<th>$N^{15}$</th>
<th>$O^{15}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Final State Excitation</td>
<td>Ground States</td>
<td>Ground States</td>
<td>2.0 MeV in $C^{11}$</td>
<td>2.1 MeV in $B^{11}$</td>
<td>5.3 MeV in $N^{15}$</td>
<td>5.2 MeV in $O^{15}$</td>
<td>7.3 MeV</td>
<td>6.8 MeV</td>
</tr>
<tr>
<td>Functional Parameters</td>
<td>$\alpha$</td>
<td>7.19</td>
<td>6.91</td>
<td>7.19</td>
<td>6.91</td>
<td>7.19</td>
<td>6.91</td>
<td>7.19</td>
</tr>
<tr>
<td></td>
<td>$\sigma_0$, mb</td>
<td>1.52</td>
<td>1.22</td>
<td>0.779</td>
<td>0.544</td>
<td>0.87</td>
<td>0.63</td>
<td>5.18</td>
</tr>
<tr>
<td></td>
<td>$\sigma_{Total}$, mb</td>
<td>0.181</td>
<td>0.158</td>
<td>0.092</td>
<td>0.071</td>
<td>0.103</td>
<td>0.082</td>
<td>0.616</td>
</tr>
<tr>
<td></td>
<td>$\sigma_{Total}$</td>
<td>$1.3 \times 10^{-4}$</td>
<td>$1.1 \times 10^{-4}$</td>
<td>$6.4 \times 10^{-5}$</td>
<td>$4.9 \times 10^{-5}$</td>
<td>$7.1 \times 10^{-5}$</td>
<td>$5.7 \times 10^{-5}$</td>
<td>$4.3 \times 10^{-4}$</td>
</tr>
<tr>
<td></td>
<td>$\sigma_{Geometric}$</td>
<td>$1.45 f$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$E_{cm} = 68.5$ MeV

$\sigma_{Geometric} = \pi r_0^2 \left( A_1^{1/3} + A_2^{1/3} \right)$

$(d\sigma/d\Omega) = \sigma_0 e^{-\alpha \theta}$, $\alpha$ for cm angles in radians

$\sigma_G = 1450$ mb
### TABLE II-B

Total Reaction Cross Sections

<table>
<thead>
<tr>
<th>Reaction</th>
<th>$^{12}$C($^{14}$N,$^{13}$N)$^{13}$C</th>
<th>$^{12}$C($^{14}$N,$^{16}$O)$^{10}$B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transferred Particle</td>
<td>Neutron</td>
<td>Neutron</td>
</tr>
<tr>
<td></td>
<td>Deuteron</td>
<td>Deuteron</td>
</tr>
<tr>
<td>Final State Excitation, MeV</td>
<td>Ground States 6.0 in N$^{13}$-$^{16}$O</td>
<td>Ground States 6.0 in N$^{13}$-$^{16}$O</td>
</tr>
<tr>
<td>Functional Parameters</td>
<td>$\alpha$ 9.07 9.07 5.90</td>
<td>$\sigma_0$, mb 5.2 10.7 2.16</td>
</tr>
<tr>
<td>$\sigma_{total}$, mb</td>
<td>0.39 0.81 &lt;.05 .24</td>
<td>$\sigma_{total}$ 1450 1450 1450</td>
</tr>
<tr>
<td>$\sigma_{geometric}$, mb</td>
<td>1450 1450 1450</td>
<td>$\sigma_{total}$ 2.69 x 10$^{-4}$ 5.57 x 10$^{-4}$ &lt;3.5 x 10$^{-5}$ 1.66 x 10$^{-4}$</td>
</tr>
</tbody>
</table>

$E_{cm} = 68.5$ MeV

$\sigma_{geometric} = \pi r_0^2 (A_1^{1/3} + A_2^{1/3})$

$\sigma_{geometric} = \pi r_0^2 (A_1^{1/3} + A_2^{1/3})$ $r_0 = 1.45f$

$\frac{d\sigma}{d\Omega} = \sigma_0 e^{-\alpha \theta}$; $\alpha$ for cm angles in radians.
agent used in the target fabrication, and the other to approximately mass 16, tentatively identified as oxygen, which has been demonstrated to be present in this type of target

Of far greater concern is the effect of C\textsuperscript{13}, present in its natural isotopic abundance of 1.1%. Since the last neutron is relatively weakly bound in C\textsuperscript{13}, it seems quite feasible that the cross section for the C\textsuperscript{13} (N\textsuperscript{14}, N\textsuperscript{15}) C\textsuperscript{12} reaction might be considerably enhanced with respect to the C\textsuperscript{12} (N\textsuperscript{14}, N\textsuperscript{15}) C\textsuperscript{11} reaction, thereby distorting the results of the N\textsuperscript{15} yield measurement. Figure 25 shows a plot of the kinematics of the two reactions, with some representative experimental points indicated. The identification seems quite unambiguous, but, as an added precaution, it was decided to compare the results of nitrogen bombardment of C\textsuperscript{12} and C\textsuperscript{13} gaseous targets. The hydrogen components of the methane targets used cannot lead to any contamination of the results, since the kinematics restricts the reaction products to more forward angles than those studied here. The normalized yields are shown in Figure 19. If C\textsuperscript{13} were responsible to any degree for the production of the N\textsuperscript{15}, it is expected that the yield for the 59% enriched C\textsuperscript{13} target would rise sharply\textsuperscript{(56)}. In fact, it is observed that the yield has fallen to about 40% of the natural methane (i.e., C\textsuperscript{12}) yield for the excited states around 7 MeV, in good agreement with expectations, and is less than 65% of the C\textsuperscript{12} yield for the ground states.

The conclusion to be drawn is that C\textsuperscript{13} contributes negligibly to the yield of the C\textsuperscript{12} (N\textsuperscript{14}, N\textsuperscript{15}) C\textsuperscript{11} reaction.

D. Discussion: Selective Population of Excited States by High Energy Transfer Reactions.

The most striking feature of the energy spectra for the neutron, proton, and deuteron transfer reactions presented in IV-A is the presence of sharp structure, indicating that isolated excited states in the product nuclei are being selectively populated. Especially notable is that in each case the population of the ground state and low-lying levels is considerably inhibited
with respect to a level or levels of greater excitation. These spectra are of interest not only because of the spectroscopic inferences that can be drawn concerning both the populated and non-populated states, but also because the body of such data yields insight into the dominant reaction mechanisms involved.

In this section, the spectra are discussed individually; the following section treats the $\text{N}^{15}$ - $\text{O}^{15}$ system in greater detail.

1. $\text{C}^{12}(\text{N}^{14}, \text{N}^{13})\text{C}^{13}$: Neutron transfer to the target.

In this reaction it is possible to assign uniquely all excitations observed, other than that corresponding to the $\text{N}^{13}$ and $\text{C}^{13}$ both in their ground states, to excited states in $\text{C}^{13}$ since only the ground state of $\text{N}^{13}$ is stable against particle emission. All excitations observed must then be in the complementary $\text{C}^{13}$ nucleus since excited $\text{N}^{13}$ configurations do not have adequate lifetime to enable them to reach the detectors.

Two strong peaks at 0.0- and 3.8-MeV are observed, and two weaker excitations are found at 7.6- and 9.5-MeV, where the numbers represent averages over all the angles considered.

Since this reaction involves the stripping of a neutron by $\text{C}^{12}$, it is expected, using the simplest model for this reaction, that the states excited in $\text{C}^{13}$ will have dominant parentage ($\text{C}^{12}$ g.s. + n). That is, the reaction is viewed as

$$\text{C}^{12} + (\text{N}^{13} + n) \rightarrow \text{N}^{13} + (\text{C}^{12} + n)$$

Evidence for at least four states of character ($\text{C}^{12}$ g.s. + n) has been found$^{(66)}$. These are

<table>
<thead>
<tr>
<th>Energy, MeV</th>
<th>$J^\pi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>$1/2^-$</td>
</tr>
<tr>
<td>3.09</td>
<td>$1/2^+$</td>
</tr>
<tr>
<td>3.85</td>
<td>$5/2^+$</td>
</tr>
<tr>
<td>7.68</td>
<td>$3/2^+$</td>
</tr>
</tbody>
</table>
Examination of the spectrum of Figure 14 suggests, on the basis of the peak halfwidths and the known purity of the ground state peak, that the peak at an indicated energy of 3.8 MeV corresponds to the excitation of a single level in $^{13}\text{C}$. This is borne out by the detailed computer fit to this spectrum.

It is tempting to identify this strong excitation as the $5/2^+$ state at 3.85 MeV in $^{13}\text{C}$, since, as calculated by Tombrello and Phillips,$^{(67)}$ this state has a parentage of 0.986 for the $(^{12}\text{C} \text{g.s.} + n)$ configuration, with the neutron in a relative $d_{5/2}$ orbit. The small high energy tail on the peak would then correspond to very weak population of the $1/2^+$ state at 3.09-MeV. The 3.68-MeV state, within the same model framework,$^{(66)}$ is predicted to have strong parentage involving $^{12}\text{C}^*(2^+, 4.43-\text{MeV})$ and a $p_{3/2}$ neutron. Excitation of this level would involve a sequential mechanism, which while not expected to be as probable as the one-stage excitation mechanism, cannot be excluded on the basis of the present results.

The observed excitation at 7.6-MeV may be tentatively identified as the $3/2^+$ state at 7.68-MeV, since this state has strong parentage for the configuration $(^{12}\text{C} \text{g.s.} + d_{3/2} \text{neutron})$. The much smaller population of this state than of that at 3.85 MeV may reflect a more complex configuration for the upper state.

The state at an indicated excitation of 9.5-MeV, may well be the $7/2^-$ state with configuration $(^{12}\text{C} \text{g.s.} + f_{7/2} \text{neutron})^{(66)}$. The energy of this state is placed at 9.51-MeV through examination of the resonant scattering of neutrons of energy 4.95-MeV. It is of interest to note that McGruer, Warburton, and Bender$^{(1)}$ found, in a study of the $^{12}\text{C}(d,p)^{12}\text{C}$ reaction at $E_d = 14.8-\text{MeV}$, that this state was relatively weakly excited, and that its angular distribution exhibited weak peaking at an angle consistent with $l = 3$ neutron transfer. This 9.51-MeV state has also been observed weakly in the $^{12}\text{C}(n, n', \gamma)^{12}\text{C}$ inelastic scattering reaction proceeding via the 4.43-\text{MeV} level of $^{12}\text{C}$.$^{(69)}$
On the basis of these identifications, the experimental spectrum of Figure 14 is consistent with a transfer mechanism which involves the transfer of a neutron to the $^{12}\text{C}$ target to form well-defined single particle states.

2. $^{12}\text{C}(^{14}\text{N},^{15}\text{N})^{11}\text{C}$: Neutron pickup by the projectile.

The two principal features exhibited are again highly selective population and an inhibition of the excitation of the lowest lying states. This reaction differs from the neutron stripping discussed above in that it is no longer possible to definitely ascribe the excitation to one product nucleus on physical grounds.

The distinguishable excitations are 0.0-, 2.0-, and 5.3-MeV. There is a broad, intense group with highest point at 7.3-MeV and a wide shoulder with center at about 9.4-MeV.

As before, the simple interpretation of the nucleon transfer reaction leads to the expectation of strong population of states with a configuration based to large degree upon ($^{14}\text{N}$ g.s. + n). The population of the 0.0-MeV level is well understood on this basis: the $^{15}\text{N}$ ground state is well represented by $^{14}\text{N}$ + a $p_{1/2}$ neutron.

The small peak on the low energy side of the ground state is identified consistently by the computer fit to be of excitation 2.0-MeV. This can only be the 1.99-MeV state in $^{11}\text{C}$, since the first excited state of $^{15}\text{N}$ is more than 3 MeV higher. The appearance of this peak may be interpreted as follows. Neutron transfers from $^{12}\text{C}$ to form the ground state of $^{15}\text{N}$ usually result in the residual nucleus, $^{11}\text{C}$, being left in its ground state; this is the structure observed at 0.0-MeV. However, in some fraction of the transfers, the $^{11}\text{C}$ nucleus is left in its first excited level, resulting in the peak observed at 2.0-MeV. The ratio of the areas of the two peaks measures the relative probability of these events.

There is no evidence for any excitation of the 4.26- and 4.75-MeV states in $^{11}\text{C}$. The next clearly observed peak is the weak one at 5.3-MeV;
since no $^{11}$C levels occur at this energy (within the system resolution), this peak is ascribed to the 5.270-MeV ($J^\pi = 5/2^+$) and the 5.299-MeV ($J^\pi = 1/2^+$) levels. The energy resolution of the system is not sufficient to distinguish these. Their weak population is consistent with the intermediate shell model calculations of Halbert and French, who find that these states are based primarily on a $T = 1, N^{14}$ core. Only states with strong $T = 0$ parentage should be significantly populated by this reaction. This assignment is borne out by $^{15}$C beta-decay studies. $^{15}$C$^\beta$ → $^{15}$N (5.3-MeV) is found to be a favored transition, indicating the $T = 1$ nature of the core for this state (since $^{15}$C has $T = 3/2$).

The absence of structure corresponding to any sizeable population of the 6.33-MeV level of $N^{15}$ is not unexpected, since the state has long been thought to be of predominantly single hole character, $(O^{16} \text{g.s.} + p_{3/2} \text{hole})$, following its strong excitation in the $O^{16}(\gamma, p)N^{15}$ reaction, for example. It was considered surprising, however, that population of this state was so strongly inhibited, in view of the expected contaminant amplitudes, some of which should be accessible to the transfer reaction.

Bock et al. in a study of the $^{12}C(O^{16}, N^{15})C^{12}$ reaction saw significant population of only this and the ground state. Figure 26 is a comparison of Bock's results with the present work. We note in passing that the Heidelberg results are contaminated by the presence of the $^{14}N$ ground state peak, since their particle identification system was capable of resolving only elements and not individual isotopes. The $O^{16}$-reaction would be expected, from the simple transfer viewpoint, to populate a $p_{3/2}$ hole state strongly. The $(2J + 1)$ statistical weighting ratio would predict that the $3/2$ level will be enhanced by a factor of two relative to the $p_{1/2}$ hole ground state. Quite the obverse is seen to be true, making the relative populations of these states differ from expectations by about a factor of four, which is difficult to explain from the single-particle model viewpoint.

Both Bock's results and the present non-observance of this level are made explicable by the recent work of Lopes et al. at Oxford. These
Figure 26. Comparison of the $N^{15}$ energy spectra from the $C^{12}(N^{14},N^{15})C^{11}$ and $B^{11}(O^{16},N^{15})C^{12}$ reactions. The latter curve is from reference 73.
workers, through a systematic study of the mixing ratios of the $3/2^-$ to $1/2^-$, $6.15$ and $6.33$ MeV-to-ground transitions in $^{15}O$ and $^{15}N$, respectively, have conclusively demonstrated that both these states are mainly collective in character with strong parentage based on the $6.92$-MeV ($J^\pi = 2^+$) state in $^{16}O$.

On the basis of the computer fits, the remaining large excitation has major components at $7.3$-MeV, and at about $8.6$-MeV, although there are certainly several other states contributing. Definite identification is not possible within the present experimental resolution, but by synthesizing shell-model calculations (70,75) beta-decay data and calculations (71,76) and gamma-ray de-excitation studies (77,78), it is possible to conjecture as to the most probable excitations.

The trio of states in $^{15}N$ at $7.16$ MeV ($5/2^+$), $7.31$ MeV ($3/2^+$), and $7.57$ MeV ($7/2^+$) are all based, in large degree, in the shell model picture (70,78), on the $^{14}N$ ground state core ($J^\pi = 1^+$, $T = 0$), with the first and last levels having a $d_{5/2}$ nucleon coupled to the core, and the $7.31$ MeV level an $s_{1/2}$ nucleon coupling. According to shell model calculations (70), all these levels exhibit large neutron reduced widths, and so the observance of all three levels is expected on this basis. Warburton et al. (77) have advanced convincing arguments for identification of the $6.79$-MeV state in $^{15}O$ as the mirror partner of the $7.31$-MeV state. Marion et al. (99) have observed a strong threshold for the analogue state in the $^{14}N(d,n)^{15}O$ reaction, confirming its dominant single-particle parentage; it also appears strongly in the $^{14}N(p,\gamma)^{15}O$ reaction (100). All these data would lead to the conclusion that the $7.31$-MeV state is strongly populated in the neutron transfer. There is one piece of contradictory evidence, however. The beta decay of $^{15}C$ is reported to feed the $7.31$-MeV state in $^{15}N$ with a log $ft$ value between 4 and 5, suggesting a significant parentage component of the state based on the $T = 1$ excited state of $^{14}N$, which is inaccessible to the transfer reaction.

The favored beta-decay of $^{15}C$ to the $8.31$ MeV level (71) ($log\; ft \sim 4 - 5$) leads to the belief that, despite its presumably dominant configuration of an $s_{1/2}$ nucleon coupled to the $^{14}N$ ground-state core and its large neutron reduced
width, no major population of this level should be observed.

Strong population of the 8.57-MeV level is indicated, although the configuration of this level is probably not so simple\(^{(78)}\). However, a large component of the configuration is probably \((N^{14}\text{g.s.} + d\text{-nucleon})\).

Little can be said about the levels above this, except that the population of the 9.05-MeV level is not expected since the beta-decay of \(C^{15}\) is super-allowed \((\log ft \sim 3)\)\(^{(76)}\), indicating that the \(T = 0\) component of the wave function must be very small. The 9.16-MeV level is probably a hole state, and so its population would, in that case, be less than that of the 9.22-MeV level, which appears the most likely candidate in this region.

An excitation of almost 9.3-MeV is also expected, corresponding to simultaneous population of the 7.3-MeV \(N^{15}\) levels and the 1.99-MeV \(C^{11}\) level, as discussed above.

The data in this shoulder region is not well enough resolved to allow any comparisons, but it seems clear that there are at least two and possibly more levels being populated between 8 and 10 MeV of excitation.

In summary, strong excitations at 0.0- and 7.3-MeV are observed, with lesser populations at 2.0- and 5.3-MeV. The strong levels correspond to the \(N^{15}\) ground state, and, probably, levels at 7.15-, 7.31-, and 7.56-MeV. The weak excitations are clearly identified as the 1.99-MeV first excited state of \(C^{11}\) and the 5.220- and 5.279-MeV doublet in \(N^{15}\). A broad low energy shoulder on the 7.3-MeV peak probably results from the population of several levels in the 8 - 10 MeV excitation range, but identification is outside the present experimental resolution. The 6.33-MeV state in \(N^{15}\) is not observed, consistent with its recently discovered predominantly hole and collective character.

Because the states of \(N^{15}\) and of \(C^{11}\) are unbound against neutron and alpha-particle emission at excitations of 10.208- and 7.544-MeV, respectively, it would not be anticipated that excitations significantly in excess of 10.208 MeV would be observed. It is clear from the spectra that \(N^{15}\) nuclei are observed at energies corresponding to this forbidden region.
These may arise from two sources: (i) mutual excitation, wherein the transfer process leaves both nuclei in excited states, (ii) population of $\text{N}^{16}$ and $\text{O}^{16}$ at excitations in excess of their neutron and proton binding energies, respectively, with the consequence that prior to reaching the detectors these states decay into $\text{N}^{15} + \text{a nucleon}$, leaving a low energy $\text{N}^{15}$ to be detected. Part of the excitation observed at 9.3-MeV very probably can be attributed to the first process, via the 7.3-MeV states in $\text{N}^{15}$ and the 2.0-MeV state in $\text{C}^{11}$.

3. $\text{C}^{12}(\text{N}^{14}, \text{O}^{15})^{11}$: Proton pickup by the projectile.

In the limit of charge independence of nuclear forces, the energy spectra of the mirror nuclei $\text{N}^{15}$ and $\text{O}^{15}$ should be identical, with all of the arguments presented above for $\text{N}^{15}$ carrying over to $\text{O}^{15}$ without change. A verification of this prediction was, in fact, one of the motivations for the present work.

Figure 31 is a level diagram\(^{(77)}\) of the $\text{N}^{15}$ and $\text{O}^{15}$ nuclei, with mirror levels in the two nuclei connected by solid lines if the correspondence is well-established, and by dashed lines if the correspondence is uncertain or speculative. The qualitative agreement with the energy spectra of Figures 15 and 16 is evident.

Comparison of the spectra of Figures 16 and 17 reveal that qualitatively the spectra appear identical from the ground level to the large excitations at 7.3 MeV and 6.8 MeV in $\text{N}^{15}$ and $\text{O}^{15}$ respectively. Comparison of the experimentally found excitations with the level schemes of these two nuclei, presented in Figure 31, reveal that quantitatively all the above statements with regard to the levels of $\text{N}^{15}$ carry over to the $\text{O}^{15}$ case. That is, the 0.0-, 2.1-, 5.2-, and 6.9-MeV peaks may be identified with the ground state, 2.14-MeV level in $\text{B}^{11}$, 5.188-, 5.240-MeV doublet, and the excitations at 6.79-, 6.85-, and 7.17-MeV respectively. The 6.16-MeV state is absent, for the same reason that the 6.33-MeV state was not observed in $\text{N}^{15}$.

The fact that the major peak at 6.9-MeV is sharper than is the case
with the corresponding $N^{15}$ peak is due to the fact that the threshold for particle emission in $O^{15}$ is at 7.291 MeV, and so any of the states formed at excitations higher than this will not live long enough to be detected as $O^{15}$, and so make no contributions to the low energy shoulder. The shoulder is observed to be considerably smaller than in the $N^{15}$ case, where the particle breakup threshold does not occur until 10.208 MeV. The excitations observed above 7.29-MeV in the $O^{15}$ are believed due to the two mechanisms already mentioned in the preceding section.

The neutron and proton transfer reactions will be further discussed in Section E.

4. $^{12}C(N^{14}, O^{16})B^{10}$: Deuteron pickup.

This reaction is not to be interpreted as representing the transfer of a physical deuteron, but simply a neutron-proton pair. The similarity of the selection mechanism for strong population indicates however that the two nucleons are transferred as a cluster.

Only two resolvable excitations at 0.0- and 6.0-MeV are observed. The dominant excitation at the latter energy corresponds to one or both of the 6.06-MeV ($J^\pi = 0^+$) and 6.13-MeV ($J^\pi = 3^-$) levels in $O^{16}$. On the simple two particle transfer model, it would be anticipated that two-particle-two-hole states in $O^{16}$ would be preferentially populated, and, at the present energies, $(p_{1/2}^{-2}(d_{5/2})^{2})^{(101)}$ configurations in particular. On the basis of the recent calculations of Kelson, for example, the $3^-$ state at 6.13-MeV in $O^{16}$ is indeed the only low lying excitation with such a dominant predicted structure. However, the 6.06-MeV level, while predominantly of four particle-four hole character, does have significant $(p_{1/2}^{-2}(d_{5/2})^{2}$ amplitude and so could also contribute to the observed spectrum.

E. Further Consideration of the $^{12}C(N^{14}, N^{15})C^{11}$ and $^{12}C(N^{14}, O^{15})B^{11}$ Mirror Reactions.

The reactions $^{12}C(N^{14}, N^{15})C^{11}$ and $^{12}C(N^{14}, O^{15})B^{11}$ exhibit a high degree of symmetry in both the entrance and exit channels, and in the limit
of the charge symmetry of nuclear forces should manifest essentially identical characteristics. The qualitative truth of this statement has been demonstrated in the preceding section.

It has not been possible to make a definite correspondence between the isobaric analogue states in the two product nuclei in the present study because of the experimental limitations imposed by the energy resolution. With an electrostatic accelerator, this limitation is removed (see Section V), and such definite correspondences should be possible in many nuclear reactions of this type.

A second and more interesting observation has to do with the relative population of the analog states in the mirror nuclei. Apart from small differences in the kinematic factors involved, negligible in the context of the present discussion, the absolute cross sections should be identical if, as is generally believed, the transition matrix elements are charge independent. Any discrepancy in the cross sections could reflect a difference in the transferred nucleon's reduced width in the narrow reaction zone at the interaction surface. This is equivalent to saying that the normalization of the tails of the bound state wave functions for the proton and neutron are different because of the effects of Coulomb forces, which is a violation of the concept of charge symmetry.

To be more specific, if the differential cross section is given by

$$\frac{d\sigma}{d\Omega} \sim \frac{k_f}{k_i} |T_{fi}|^2,$$

for example, then the statement that the proton and neutron transfer reactions are charge independent actually involves three requirements:

1. The scattering of the nuclei, as represented by $\chi$ must be charge independent.
2. The inter-nucleon interactions in each nucleus cannot depend on the type of nucleons involved. That is, the internal wave functions, $\psi$, must be charge independent.
3. The perturbation potential, $V$, whose off-diagonal elements are responsible for the transition, must be charge independent. The cross section is then proportional to

$$\frac{d\sigma}{d\Omega} \sim \frac{k_f}{k_i} \left( T_a \Delta T_c ; T_{za} T_{zb} - T_{za} T_{zb} \right)^2 \left( T_A \Delta T_c ; T_{ZA} T_{ZB} - T_{ZA} T_{ZB} \right)^2 \times |T_{fi}'|^2$$

with the reaction symbolically represented by $A(a,b)B$, with $a = (b + c)$, $B = (c + A)$, as before. $T_i$ is the isotopic spin of particle $i$, $T_{Zi}$ its $Z$-projection, $\Delta T_c$ is the isotopic spin of the transferred cluster, and $T_{fi}'$ is the transition matrix element, now assumed to contain only charge independent terms. The terms in brackets are the familiar Clebsch-Gordan vector addition coefficients.

Using these assumptions, $|T_{fi}'|^2$ is identical for reactions to analogue states, and the ratio of the reaction cross sections is simply given by the ratio of the appropriate isospin vector addition coefficients and phase space factors.

Bromley et al. \(^{(98)}\) investigated the $^{12}\text{C}(\text{He}^3,n)^{14}\text{O}$ and $^{12}\text{C}(\text{He}^3,p)^{14}\text{N}^{14*}$ (2.31-MeV) states at low $\text{He}^3$ energies and found that the ratio of these cross sections, away from the neutron reaction threshold, attained the value given by the abovementioned ratio. More recently, Cerny and Pehl \(^{(79)}\) have investigated the reactions $^{16}\text{O}(p,t)^{14}\text{O}$ and $^{16}\text{O}(p,\text{He}^3)^{14}\text{N}^{14*}$ (2.31-MeV) with 48 MeV protons, and reported a 12% difference between the cross sections, with the $(p,t)$ cross section the larger. They noted that the difference could be accounted for by a very small degree of isospin impurity.

Robson \(^{(80)}\), using the R-matrix theory of resonance reactions, has shown that the nuclear interior is expected to be a region of relatively pure isotopic spin, and that the most significant mixing of isotopic spin will occur in an external region. The present reactions should be particularly sensitive to such an effect, since the interactions are localized on the nuclear
surface. These reactions differ from those discussed by Robson in that he treats resonances in the continuum of unbound states, whereas the present work is concerned with specific bound levels. It is of interest to see how the difference in the behavior of the bound and unbound state wave functions in the external region is reflected in the mixing of the isotopic spin.

For the N\textsuperscript{15} and O\textsuperscript{15} mirror reactions, the ratio of the cross sections, following the assumptions above, may be expressed as

$$\frac{d\sigma_n}{d\Omega} = \frac{d\sigma_p}{d\Omega} = \frac{kn}{kp} (0 1/2; 0 1/2 | 1/2 1/2)^2 (0 1/2; 0 -1/2 | 1/2 -1/2)^2$$

$$= \frac{kn}{kp} \approx 1$$

where the subscripts n and p refer to the neutron and proton transfer reactions, respectively, and k is the approximate exit channel wave number. The ratio of kn/kp is considered unity to within 1/2% because the Q values for the reactions are almost the same (-7.88 MeV and -8.60 MeV for the n and p transfers, respectively).

In order to compare the N\textsuperscript{15} - O\textsuperscript{15} experimental yields, the sum of all counts below corresponding points in the energy spectra of the two nuclei was computed. This is a more reliable procedure than comparing the relative cross sections of similar structures in the spectra, for the latter procedure would then involve any errors introduced in the unfolding of each spectrum. The summation procedure is justified by the fact that all levels in one nucleus have an analogue in the spectrum of the mirror product. The non-resolution of the states around the large peaks in each spectrum prevent the inclusion in the sum of anything but the two lowest excitation peaks (i.e., the peaks at 0.0- and 2-MeV), because of the large difference in the particle emission thresholds in N\textsuperscript{15} and O\textsuperscript{15}, as noted above, and the necessity for summing only corresponding levels in the two nuclei. This method of summation, although limited, should be quite accurate, within the statistical uncertainties.
present. In the performance of such a calculation, it is imperative to subtract the background present in the energy spectra in a reasonable and consistent fashion. Some comments as to the suspected origin of this background are therefore in order.

In the present work, one of the primary objectives was the simultaneous accumulation of data on the several reactions reported on above. In order to fit the dE/dx x E loci into the available memory, the spectrum was not expanded to the point where the ΔE resolution was commensurate with the calibration (i.e., MeV/channel) of the MPA. However, the isotopes were clearly enough resolved that structure in the spectrum of one isotope was not reflected in that of its neighbor. This was very carefully checked, and in fact, some of the valleys in the $N^{15}$ spectrum, for example, correspond to peaks in the $N^{14}$ spectrum. However, a statistical analysis reveals a high degree of correlation between the parameters of adjacent loci, as it must. In the case of $N^{15}$, the adjacent hyperbola, $N^{14}$, is always between one and two orders of magnitude more populated. Since the detector response is a gaussian, some of the tail of the $N^{14}$ spectrum in each energy plane is superimposed on the $N^{15}$ gaussian. The computer program (see Appendix I) is designed to take this into account, but it must be remembered that this is a least squares fit, and so only minimizes the squared error. Thus, because of statistical variations, and the non-exactness of the gaussian response function used (see Figure 11), it is possible that part of the tail of the $N^{14}$ peak is being systematically included in the $N^{15}$ spectrum. No such situation pertains in the $O^{15}$ case, since the $O^{16}$ population is too sparse to contribute significantly.

It should be emphasized that this effect, if present, is not peculiar to the present method of particle identification, but would result with any type of identifier whenever two peaks of greatly differing magnitudes are incompletely resolved. In fact, it is an advantage of the present system that the situation may be dealt with analytically.

The low energy tails of the E detector response functions (see
Figure 11) are not expected to make a significant contribution to the N\textsuperscript{15} and O\textsuperscript{15} spectra, and so were neglected. The magnitude of the background was estimated by averaging the value of the background in the five channels immediately above (i.e., at higher energy) the ground state peak in the spectrum of interest, and applying this as a constant correction throughout the energy spectrum for a given case. In the O\textsuperscript{15} spectrum, the correction was minor, as expected. Since the limited region of interest in the N\textsuperscript{15} spectrum corresponds to a slowly varying portion of the N\textsuperscript{14} spectrum, the approximation to the background should be reasonably good.

For the cases considered, the ratio of the N\textsuperscript{15} and O\textsuperscript{15} cross sections was found to be statistically distributed about a mean value of 1.02. The deviations from this value were as large as 25% in some cases, which reflects the uncertainties in the background subtraction as well as the statistical variation in the data. Clearly more accurate measurements are needed, but within the present experimental uncertainty, the neutron and proton transfers appear charge independent.

F. Discussion: Angular Distributions and Diffraction Model Predictions

The essentially exponential angular dependence of the differential cross sections for the single nucleon transfer reactions studied in this work is illustrated in Figures 20 through 23.

In Figure 27, a representative cross section, that of the N\textsuperscript{15} ground state, has been converted to \(\frac{d\sigma}{d\theta}\) c.m. and plotted against center-of-mass scattering angle in order to facilitate comparison with the theoretical predictions, which are presented in this form.

Shown on the figure is the prediction of the Frahn-Venter model, equation (II-12) with the following parameters: \(d = 0.39f\), \(r_0 = 1.55f\), as suggested by Frahn and Venter\textsuperscript{11} for a very similar case. The curve has been normalized at \(\theta_{\text{cm}} = 19.0^0\). Also shown, normalized to the same point, is the prediction of the Dar model, equation (II-24) with the interference term neglected, as previously discussed. The parameters used for this
Figure 27. Comparison of the predictions of the Frahn-Venter and of the Dar diffraction models with the experimental angular distribution for the $^{12}$C($^{14}$N, $^{15}$N)$^{11}$C (g.s.) reaction. The curves have been arbitrarily normalized to the data. The interference term has been neglected in the Dar calculation.
COMPARISON OF THE PREDICTIONS OF TWO DIFFRACTION MODELS WITH N\textsuperscript{15} ANGULAR DISTRIBUTION (GROUND STATES)

\[ C^{12}(N^{14}, N^{15})C^{11} \]

\[ E_{cm} = 68.5 \text{ MeV} \]

NEUTRON TRANSFER

DAR: EQN. (II-24)

\[ d = 0.2f \]

(FILTERING EFFECTS NEGLECTED)

FRAHN-VENTER: EQN. (II-12)

\[ d = 0.39f \]

\[ (d\sigma/d\theta)_{cm}, \text{ RELATIVE UNITS} \]

\[ \theta_{cm}, \text{ DEGREES} \]
The conclusion seems clear: the oscillations predicted by the Frahn-Venter model (and by the Dar model when the interference term is not neglected, see Figure 2) are not present. A slight oscillation cannot be ruled out because of the experimental uncertainties associated with the measurements, but the predicted oscillation is well outside any such small variations.

On the other hand, the finite range diffraction model, when interference terms are neglected, and an arbitrary normalization is applied, is able to represent the data very well, for some representative cases. No attempt was made to fit the best line through the data (that is, as shown by Figure 28, to vary the value of the diffuseness) although this is simply done. The demonstrated small variation of $d$ in such fits is not significant in view of the errors associated with the $(d\sigma/d\theta)$ values. This is especially true in the present case, where the primary interest is in the shape of the angular distribution. The curves have been arbitrarily normalized and are displaced for clarity. A reasonable fit to the angular distributions is provided in all cases, including those not shown.

Figure 29 is a composite representation, for some of the transfer reactions for which angular distributions are available, of the variation of $(d\sigma/d\gamma)_{cm}$ as a function of linear momentum transfer, $q$. An adequate fit to the data is obtained using the Dodd-Greider prediction.

The fact that all the reactions to the various states in different nuclei can be represented on the same plot emphasizes a significant difference in the mechanisms proposed by Dar and by Dodd and Greider for the smoothing of angular distributions. The recoil–finite range effect explanation specifically builds the details of nuclear structure out of the formalism, as clearly evident in Figure 29, whereas the angular momentum mixing effect is strongly dependent upon the nature of the levels involved in the transfer.

The good fit to the $q^{-4}$ dependence, once the absence of oscillations.
Figure 28. Comparison of the Dar model predictions with the experimental angular distributions for various transfer reactions. The curves have been arbitrarily normalized and displaced for clarity.
Figure 29. Representative transfer reaction differential cross sections as a function of linear momentum transfer, $q$. The predictions of Dodd and Greider are shown. The data has been arbitrarily normalized.
DIFFERENTIAL CROSS SECTION vs. LINEAR MOMENTUM TRANSFER for $^4$He + $^{12}$C TRANSFER REACTIONS

$E_{cm} = 68.5$ MeV

- $^{12}$C($^4$He,$^1$H)$^{13}$C (3.8 MeV)
- $^{12}$C($^4$He,$^3$H)$^{11}$C (7.3 MeV)
- $^{12}$C($^4$He,$^5$He)$^{10}$B (6.8 MeV)
- $^{12}$C($^4$He,$^5$Li)$^9$B (g.s.)
- $^{12}$C($^4$He,$^6$He)$^{10}$B (6.0 MeV)
Figures 30a and 30b. Resolution of the particle identifier for the isotopes of nitrogen and oxygen, respectively. The curves shown are the computer fits to the data.
TYPICAL RESOLUTION OF PARTICLE IDENTIFIER FOR NITROGEN ISOTOPES
TYPICAL RESOLUTION OF PARTICLE IDENTIFIER FOR OXYGEN ISOTOPES
Figure 31. Energy level diagram of $\text{N}^{15}$ and $\text{O}^{15}$, showing mirror levels, (from reference 77). Correspondences thought certain are indicated by solid lines; those less certain or speculative are shown by the dotted lines.
<table>
<thead>
<tr>
<th>Energy</th>
<th>State</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.834</td>
<td>N&lt;sup&gt;14&lt;/sup&gt; + n 3/2&lt;sup&gt;+&lt;/sup&gt;</td>
</tr>
<tr>
<td>10.54</td>
<td>5/2</td>
</tr>
<tr>
<td>10.0699</td>
<td>1/2&lt;sup&gt;+&lt;/sup&gt;</td>
</tr>
<tr>
<td>9.83877</td>
<td>3/2&lt;sup&gt;+&lt;/sup&gt;</td>
</tr>
<tr>
<td>10.45</td>
<td>3/2</td>
</tr>
<tr>
<td>10.46</td>
<td>C&lt;sup&gt;11&lt;/sup&gt; + α 1/2&lt;sup&gt;+&lt;/sup&gt;</td>
</tr>
<tr>
<td>10.208</td>
<td>C&lt;sup&gt;14&lt;/sup&gt; + p 3/2</td>
</tr>
<tr>
<td>9.49</td>
<td>3/2&lt;sup&gt;-&lt;/sup&gt;</td>
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<tr>
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<tr>
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<td>3/2&lt;sup&gt;-&lt;/sup&gt;</td>
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<tr>
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<tr>
<td>6.299</td>
<td>1/2&lt;sup&gt;-&lt;/sup&gt;</td>
</tr>
<tr>
<td>5.270</td>
<td>5/2&lt;sup&gt;-&lt;/sup&gt;</td>
</tr>
<tr>
<td>5.240</td>
<td>5/2</td>
</tr>
<tr>
<td>5.188</td>
<td>1/2&lt;sup&gt;-&lt;/sup&gt;</td>
</tr>
</tbody>
</table>

**Figure 31**
has been accounted for, is to be expected, as this is essentially the Ruther­
ford elastic scattering dependence, and these reactions are known to be
quasi-elastic in good approximation. Although not so obvious, this is, in
effect, also built into Dar's formalism. Because of the low value of the
Coulomb parameter for these reactions, the critical angle, $\theta_\text{c'}$, correspon­
ding to the distance of closest approach of the nuclei on their Rutherford
trajectories, occurs at an angle smaller than experimentally attainable.
The transfer cross section, for higher $\eta$ reactions, is known to peak at
$\theta_\text{c'}$, and there is no reason to believe that this is not also true for the low
$\eta$ reactions. In that case, the angular distributions observed correspond
to the "back" side of this peak, and this is well-represented by the finite
range diffraction model, which uses the approximate elasticity of the
reaction as its starting point.

An experimental test to indicate whether one or both of these
damping mechanisms is in effect is suggested in Section V.

G. Discussion - Elastic and Inelastic Scattering in the $^\text{14}N + ^\text{12}C$ System.

While both the elastic and inelastic scattering of complex nuclei
constitute problems of considerable current interest, it is not within the
scope of the present work to present a detailed quantitative analysis of
these reactions. These experiments were conceived with the primary
intention of obtaining a finely resolved angular distribution for the rela­
tively low cross section single nucleon transfer reactions. The low duty­
cycle of the HILAC and the finite amount of experimental time available
precluded the extension of the elastic and inelastic measurements to a
larger range of angles than those presented here.

It is, however, possible to draw several conclusions on the basis of
the data presented here. Two features of the results are immediately
obvious:

(i) a highly selective inelastic scattering mechanism is populating
the excited states.
(ii) the angular distributions for both the elastic and inelastic scattering are highly oscillatory in nature, in good agreement with the results of similar experiments\(^{(81-86)}\).

The second point is an especially important one in the context of this report, because it indicates that the absence of oscillations in the transfer reaction angular distributions is not the result of an equipment malfunction or normalization error.

The first point is in agreement with the previously reported\(^{(83-86)}\) preferential excitation of collective states in the inelastic scattering of heavy ions. The principal excitations observed at 4.4, 9.6, and 14.0 MeV are identified as the 4.43-, 9.63- and 14.0 ± 0.5-states in C\(^{12}\), previously found by Garvey et al.\(^{(84)}\) be strongly excited in scattering from a C\(^{12}\) target. The angular distributions of these states are in agreement with the "phase-rule" of J. S. Blair\(^{(87)}\), which states that, on the assumption of a single stage excitation, the oscillation of the inelastic angular distribution will be in phase with the elastic scattering if the parity of the level excited in the target nucleus is negative, and out of phase if the level excited has positive parity. Thus the 2+, 4.43 MeV level is out of phase with the elastic scattering.

The 9.63-MeV \((J^\pi = 3^-)\) level would, on the basis of the phase rule, be expected to be in phase with the elastic scattering. The level, in fact, displays a rather structureless angular distribution in agreement with previous measurements\(^{(84)}\). This is probably a result of the incomplete resolution of this level from adjacent states, as evidenced by the breadth of the observed peak. The fact that the 14.0-MeV level is out of phase suggests the assignment of a positive parity to this level, also in agreement with former assignments\(^{(86)}\).

It is interesting to note the pronounced inhibition of the population of the 2.312 \((0^+, T = 1)\) first excited state of N\(^{14}\). No evidence was seen for this state at any angle studied, and we conclude that its cross section is at least two orders of magnitude less than that of the 4.43 state. The
population of this state is forbidden by conservation of isotopic spin, and is also inhibited by the single-particle nature of the level and the selection rules for this process.

The phenomenon of mutual excitation, observed in both the C\textsuperscript{12} - C\textsuperscript{12} and O\textsuperscript{16} - C\textsuperscript{12} systems\textsuperscript{(85,86)} is either not significantly present in this reaction, or is part of the small excitation observed in the 7 - 9 MeV excitation region (i.e., the mutual excitation of the 3.95-MeV level of N\textsuperscript{14} and the 4.43-MeV level in C\textsuperscript{12}). The absence of mutual excitation is a reflection of the fact that no individual states of N\textsuperscript{14} are excited strongly enough to contribute to the process to any great extent. This data would seem to indicate that these strongly excited states in carbon are much more collective in character than any states in N\textsuperscript{14}.

The small peak on the low energy side of the 4.4-MeV excitation is found by the computer fit to correspond to an average excitation of 7.66 MeV, although the statistics are quite low, and there is considerable variation about this value. The excitation may be tentatively assigned to the 7.66-MeV ($J^\pi = 0^+$) state in C\textsuperscript{12}, previously observed in C\textsuperscript{12}(C\textsuperscript{12},C\textsuperscript{12})C\textsuperscript{12}, C\textsuperscript{12}(\alpha,\alpha')C\textsuperscript{12*}, C\textsuperscript{12}(O\textsuperscript{16},O\textsuperscript{16})C\textsuperscript{12*} inelastic scattering\textsuperscript{(85)}. As in the present case, the level's population was strongly inhibited in these reactions. This is perhaps attributable to the 2J + 1 statistical factor.

The shape of the angular distribution for the elastic scattering is in good qualitative agreement with the data of Steigert et al.\textsuperscript{(81,82)} on N\textsuperscript{14} + C\textsuperscript{12} elastic scattering, although the range of angles studied do not overlap appreciably. The present measurements include center of mass scattering angles between 18.4 and 37.4 degrees, while Steigert and collaborators investigated the region of about 5 to 19 degrees. The period of the diffraction oscillations observed is in excellent agreement in these measurements. The location of the first minimum observed in the work reported herein is $\theta \text{ cm} = 20.0$ degrees, which is in agreement with the trend of the Steigert data.

No analysis of the elastic scattering data will be presented in this report, since it is clear that only very qualitative agreement can be obtained
with a plane-wave calculation, and the limited range of data available does not justify a distorted wave calculation. However, such calculations would be of great value for detailed tests of the optical model approach to the reactions of complex nuclei, as was discussed previously, and it is anticipated that measurements of the type reported can be extended in the future to cover the full angular variation required.

It is interesting to compute the slope of the envelope of the angular distributions, for, as discussed by Garvey\(^{(86)}\), the relative steepness is directly related to the number of phonons taking part in the excitation. This quantity was computed by taking the ratio of successive maxima for a given excitation. The results are compared below with the values obtained for \(^{16}O - ^{12}C\) \(^{(85)}\) and \(^{12}C - ^{12}O\) scattering \(^{(86)}\), and those constructed from a universal curve for \((\alpha, \alpha')\) scattering of Blair\(^{(86)}\). Average values are found to be in good agreement.

<table>
<thead>
<tr>
<th>Reaction Q, MeV</th>
<th>Average Steepness</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \langle N^{14}, C^{12} \rangle )</td>
</tr>
<tr>
<td>0</td>
<td>3.8</td>
</tr>
<tr>
<td>-4.43</td>
<td>2.6</td>
</tr>
<tr>
<td>-14.0</td>
<td>1.3</td>
</tr>
</tbody>
</table>

It will be recalled that in Section II-D it was reported that Dar asserts that the expression derived by Frahn and Venter for the differential cross section for single nucleon transfer, is, in fact, the correct expression for the differential cross section for inelastic scattering via collective excitation. Dar's contention is that this results from a faulty assumption on the part of Frahn and Venter regarding their choice of the form of the reflection coefficients for this problem. Although the general shape and period of the inelastic angular distribution is observed to correspond roughly to that calculated using the Frahn–Venter transfer equation, the scope of the present data is too limited to allow a judgment on the validity of Dar's contention.
V. EXTENSIONS OF THIS WORK

The experiments suggested in this section are proposed with the assumption that all future work with the system described in this report will be done in conjunction with the Yale tandem Van de Graaff accelerator. This will result in three major improvements:

1. The significantly greater beam flux will make it feasible to study very low cross section processes.
2. The excellent energy resolution of the beams will permit much more precise determinations of excitation energies.
3. The ability to precisely vary the energy of the beam with no loss in intensity will permit the accurate determination of excitation functions.

The suggestions are of two types. The first type consists of experiments to clear up specific points that arose in the current work and could not be resolved because of experimental limitations; the second type concerns modifications of the apparatus and techniques developed in the present work that would facilitate such research.

A. Further Experiments

1. The variable energy beam of a tandem accelerator should be capable of determining whether one or both of the diffraction oscillation damping mechanisms proposed by Dodd and Greider and by Dar are in effect. Recoil effects are dependent upon energy, damping due to angular momentum mixing is not. Thus, in a low \( \eta \) reaction with a smooth angular distribution at high energies, the appearance of oscillations as the energy was decreased would indicate the presence of recoil effects and the absence of angular momentum mixing. The persistence of the smooth character would signify the presence of the angular momentum effect, but
would not rule out the action of the recoil at higher energies. This last can only be done by judicious choice of a reaction so that no contribution from Dar's mechanism is expected.

2. The diffraction models of Frahn and Venter and of Dar make specific predictions concerning the excitation functions for nucleon transfer reactions. These have never been examined in detail for the low η case, and it is possible that an unexpected result would be obtained just as was the case with the angular distributions. An experimental check using an electrostatic accelerator would be an extremely simple matter.

3. High energy resolution measurements should be able to clearly distinguish the states populated in the $^{12}_C(N^{14}, N^{15})^{11}_C$ and $^{12}_C(N^{14}, O^{15})^{11}_B$ reactions. This would further establish the correspondence of analogue states in the $N^{15} - O^{15}$ system, which is of great theoretical interest because of its relation to the closed p shell.

4. The questions raised earlier concerning the relative population of isobaric analogue states in heavy ion reactions, and the important implications regarding the conservation of isotopic spin, could similarly be resolved by measurements with good statistics that were able to precisely identify the states involved.

5. It would be interesting to extend the transfer reaction angular distributions to backward angles where the mechanism of heavy particle stripping is expected to produce a backward peaking of the angular distribution. The presence or absence of diffraction oscillations in the backward angle region is an unresolved point at present, as is the question of interference between the two mechanisms at intermediate angles.

6. The simultaneous acquisition of a broad set of data including elastic and inelastic scattering and single and multi-nucleon
transfer over the full range of angles should be straightforward. As previously noted, such a well-calibrated and consistent set of data would be very important for the detailed testing of the validity of the widely-used optical potential model for heavy ion reactions.

B. Instrumentation

The uncertainty associated with the energy lost in the ionization chamber is reflected directly in the resolution of the E detector. For the reactions considered in this report, this uncertainty is typically several hundred keV, and so the resultant broadening of the E-detector response is considerably greater than that due to the beam spread of an electrostatic accelerator. The resolution can be considerably improved by electronically adding the E and AE pulses prior to analysis, so that the inputs to the MPA become the total energy and the energy loss. The same result can be achieved more easily by performing the experiment as described in this report, and performing the addition in the computer, using a calibration for the energy loss based on range-energy relations.

This addition, however, will not significantly improve the isotopic resolution, which is largely a function of the AE detector response. It would seem worthwhile to investigate the substitution of a high resolution solid state transmission counter for the ionization chamber. Such devices of suitable thickness (≤ 50 μ) have recently become available. The difficulty is that such thin detectors are generally nonuniform in thickness, and so for the case of a finite sized beam may yield results no better than those obtainable with the ionization chamber. Again, the energy resolution should be significantly improved by adding the E and AE pulses.

A minor instrumental addition is suggested by the case of two possible excitations, known to be in the conjugate nucleus, where only one is stable against particle emission. A coincidence between the output of the particle telescope and that of a third detector, positioned at the kinematic recoil angle,
will occur only for the stable level. Therefore gating the MPA with the recoil detector pulse will distinguish between the two possibilities. Provision for a third, individually rotatable detector is already present in the scattering chamber, and the fast external gate is an integral part of the MPA circuitry.

It has already been mentioned that the usage of a digital computer is, in a sense, an integral part of the experimental technique developed in this work. The physical addition of an on-line computer to the experimental apparatus that has been described is then a logical extension of the procedures already in use, and, conceptually, presents no difficulties. Plans are currently being made at the Yale Nuclear Structure Laboratory for the installation of a computer of sufficient power to perform the calculations necessary for real-time reduction of \( \frac{dE}{dx} \times E \) data. In addition to the obvious benefits regarding speed and convenience, there are several distinct advantages that accrue from such a procedure.

Since energy spectra are then available at any time during the progress of an experiment, quantitative calculations (many of which can be built into the computer and use the energy spectra directly as accumulated) and subsequent decision-making with regard to the course of the experiment are greatly facilitated. For example, it is a simple matter to form the angular distributions directly, performing all coordinate transformations, normalizations, etc. in the process.

The computer may either receive the necessary information from a coupling to the memory of the MPA, or the computer may replace, with the addition of analogue-to-digital converters, the MPA entirely. In the latter case, the memory of the computer is used on a time-shared basis for data acquisition and data analysis.

The proposed Yale installation will include a graphic terminal, comprised of a CRT display screen and a device called a light pen. The latter is capable of communicating directly to the computer the coordinates of any point on the CRT screen at which it is pointed. The memory display then becomes a two-way medium for the exchange of information between
the physicist and the computer. In addition to the usual display functions, the CRT-light pen combination may be used to provide the computer with the locations of the $dE/dx \times E$ loci, prominent features of an energy spectrum, angular distribution, etc. In each case, this is accomplished simply by sketching the curve or boundary desired on the CRT with the light pen. The operation of computer programs is greatly accelerated and simplified in this manner.

An on-line computer can also perform basic monitoring functions by means of feedback networks and appropriate servo-mechanisms. In this fashion, it is planned to continually inspect and regulate such quantities as system linearity, integrator-monitor ratio, as well as the energy, intensity, and spatial localization of the beam. The advantage to the experimenter is two-fold: the job is done more accurately than otherwise possible, and his time is freed for tasks more directly related to the experiment per se.

This brief summary is intended only to indicate the vast possibilities of such a scheme. For a recent and complete discussion of the current proposed uses of on-line digital computers in nuclear physics, see references 88 and 89.
High energy single nucleon transfer reactions in the $^\text{14}\text{C} - ^\text{12}\text{C}$ system have been the subject of an experimental study, with the investigation of the reaction mechanisms a primary objective. Information of a spectroscopic nature concerning the nuclei involved has also been obtained.

A new particle identification system, capable of resolving individual isotopes up to at least mass 17, has been developed for this research, and is described in detail. The system employs a high resolution $\text{dE/dx} \times E$ particle telescope, developed previously in this laboratory, and a 20,000 channel multi-parameter pulse height analyzer. An integral part of the experimental technique is the subsequent computer analysis of the data, and the methods employed are described in an appendix to this report.

A major advantage of this system is its ability to gather data simultaneously on several nuclear reactions resulting from the same beam-target combination; this eliminates the problems usually associated with inter-experiment normalization. Energy spectra and angular distributions are obtained in this work for the $^\text{12}\text{C}(^\text{14}\text{N},^\text{15}\text{O})^\text{11}\text{C}$, $^\text{12}\text{C}(^\text{14}\text{N},^\text{16}\text{O})^\text{10}\text{B}$, $^\text{12}\text{C}(^\text{14}\text{N},^\text{13}\text{O})^\text{13}\text{C}$, and $^\text{12}\text{C}(^\text{14}\text{N},^\text{15}\text{N})^\text{12}\text{C}$ reactions, corresponding to neutron, proton, neutron, and deuteron transfer, and elastic and inelastic scattering, respectively.

All the single nucleon transfer reactions were observed to exhibit highly selective population of individual states, with a characteristic inhibition of the low-lying levels, and major population of states in the excitation region between 6 and 9 MeV in the observed nucleus. In each case, a reaction mechanism, involving the transfer of a nucleon, is consistent with the observed spectrum. Levels of a single-particle nature with configuration $\text{(Core}^{+} \text{+ transferred nucleon)}$ are preferentially excited, with the angular momentum of the transferred nucleon a controlling factor in the relative population for a given level. States involving low angular momentum transfer are inhibited, with optimum conditions arising when the angular momentum carried by the transferred nucleon "matches"
the angular momentum of the shell-model orbit into which it becomes bound. Reactions involving an angular momentum transfer of 2 or 3 units appear to be favored by the reactions under consideration. A theoretical basis for the inhibition of reactions proceeding via low angular momentum transfer has recently been proposed by Dodd and Greider (15).

The foundations of several contemporary reaction models for cluster nucleon transfer and their predictions for the shape of the angular distributions of reaction products have been discussed. All these theories take account of the strong nuclear absorption inherent in high energy heavy ion reactions and, in the absence of Coulomb-damping, predict large oscillations in the angular distributions caused by nuclear diffraction effects at the interaction surface. The disagreement with the smooth, exponentially varying dependences observed in these experiments, and in a previous investigation at this laboratory (29), has led to a revision of two of these theories (13, 15). Mechanisms have been proposed which are able to damp the oscillations and which are in excellent qualitative agreement with the results of the present measurements. One explanation of the damping involves the mixing of out-of-phase contributions to the cross-section as a result of the angular momentum transferred in the reaction (55), the second (15) shows that the exponential dependence results if a finite range potential is used in the derivation of the transition amplitude and terms corresponding to recoil effects are retained in the phase of the matrix elements. The former suggestion is strongly dependent on nuclear structure while the latter is essentially independent of it. An experimental test to distinguish between them has been suggested.

The mirror reactions involving proton and neutron transfer have been shown to exhibit highly analogous behavior in accord with the concepts of the charge symmetry of nuclear forces.

The measured absolute cross sections for the formation of analog states are equal, as anticipated, within the accuracy of the present experiment. However the limitations imposed by energy resolution and counting statistics in the present work preclude any definite conclusions about a possible violation of iso-spin
conservation in heavy ion transfer reactions. A further experiment, using the present methods and an electrostatic accelerator, could resolve this point.

The deuteron transfer reaction is only briefly studied, due to the low statistics. It is found to have the same characteristics as the single nucleon transfer reactions, lending support to previously advanced suggestions that the deuteron is transferred as a cluster and preferentially populates states of a two-particle character.

The elastic and inelastic scattering differential cross sections have been found to exhibit the oscillations characteristic of such reactions. The elastic scattering angular distribution is in good qualitative agreement with a previous measurement in an adjacent angular region.

Strong selective population of states with collective character by the inelastic scattering has been observed, in excellent agreement with previous measurements in similar reactions.

Suggestions are advanced for the extension of this work, employing the more precise and versatile tandem van de Graaff accelerator soon to be available at this laboratory, in conjunction with the equipment and techniques developed herein. It has been demonstrated that a broad spectrum of internally consistent data can be obtained rapidly and reliably; it is expected that such data will be of increasing importance for detailed nuclear model testing and spectroscopic studies.
APPENDIX I

GENERALIZED LEAST SQUARES FITTING USING A DIGITAL COMPUTER

The purpose of this appendix is to present a brief summary of the mathematical techniques employed by the fitting programs referred to in the body of this report. The notation and treatment will follow that of Moore and Ziegler\(^{(62)}\). There exists a copious literature on the method of least squares at any level of mathematical sophistication desired, and the intent here is simply to outline the procedures that were deemed most appropriate for the present problem. It has been demonstrated, however, that the simplicity and power of this approach make it applicable to a great variety of problems\(^{(61,62)}\).

A. Mathematical Method

The method to be described was first proposed by Gauss, and although conceptually simple, it involves computation of such complexity that its use became feasible only with the advent of high speed computing machinery. It is used in two ways in this work. In the fitting of the \(\text{dE/dx} \times E\) curve for each isotope, the values of the parameters obtained possess no inherent interest and represent only a consistent way of generating the data for future calculations. However, in the fitting of the gaussian response curves of the DE and E detectors, the fitting parameters, and their standard deviations, have definite physical significance (area, peak location, etc.) and are, in fact, the reason for performing the fit.

If we assume that we are given both a function

\[
y = f(x_1, \ldots, x_m; \alpha_1, \ldots, \alpha_p)
\]

(A-1)

with independent variables \(x_i\) \((i \leq m)\) and parameters \(\alpha_k\) \((k \leq p)\) and a set of \(n\) observations \((y_i, x_{i1}, \ldots, x_{im})\) where \(i = 1, 2, \ldots, n\), and \(n > p\), then in performing a least squares fit, by definition, it is required that the sum of the squares of the deviations of the observed \(y_i\) from the function be minimized.

That is

\[
Q = \sum_{i=1}^{n} w_i \left[ y_i - f(x_{i1}, \ldots, x_{im}; \alpha_1, \ldots, \alpha_p) \right]^2
\]

(A-2)
is to be a minimum. The parameter $w_i$ appearing here is the weight associated with the $i^{th}$ observation, and is completely arbitrary, i.e., at the disposal of the experimenter. The values of the parameters $\alpha_k$ which produce the above minimization will be denoted by $\alpha_k$. We tentatively assume only one actual minimum although there may be subsidiary local extrema. The choice of the least squares criterion is by no means a unique one, nor is it necessarily the most appropriate one for a given problem. It is simple to show that the choice of the least squares criterion is precisely equivalent to the selection of the arithmetic mean of a number of observations as the best estimate of the quantity being measured. In other words, the least squares principle minimizes the average squared error, but allows large deviations from the predicted values. This is frequently the most useful choice, particularly when the data is intrinsically "noisy" and a smoothing operation is desired: this is what is generally achieved by drawing the "best" curve through a number of experimental observations.

It should be made clear that this is not always desirable; for example, it may be more appropriate, in a given physical problem, to minimize the maximum deviation, or perhaps the sum of the absolute values of the deviations. Hamming\(^{90}\) has discussed these questions at length, with particular reference to computer applications. It has been concluded that in the present problem the least squares approach provides the most tractable form of solution.

Formally, the sum of the squares, $Q$, can be minimized with respect to the parameters, $\alpha_k$, by differentiating $Q$ with respect to each $\alpha_k$ and equating the result to zero. A second differentiation demonstrates that this extremum is indeed a minimum. The ensuing set of $p$ simultaneous equations is known as the "normal equations" of the problem. These may be written

$$\frac{\partial Q}{\partial \alpha_k} = -2 \sum_{i=1}^{n} w_i \left( \frac{\partial f}{\partial \alpha_k} \right)_i \left[ y_i - f(x_{1i}, \ldots, x_{mi}, \alpha_1, \ldots, \alpha_p) \right] = 0 \quad (A-3)$$

for $k = 1, \ldots, p$, with $(\partial f/\partial \alpha_k)_i$ denoting the value of the $k^{th}$ partial derivative for the $i^{th}$ observation.
Transposing and setting \( f_i = f(x_{1i}, \ldots, x_{mi}; \alpha_1, \ldots, \alpha_p) \), the equations appear in their more usual form:

\[
\sum_{i=1}^{n} w_i f_i \left( \frac{\partial f}{\partial \alpha_1} \right)_i = \sum_{i=1}^{n} w_i y_i \left( \frac{\partial f}{\partial \alpha_1} \right)_i
\]

\[
\sum_{i=1}^{n} w_i f_i \left( \frac{\partial f}{\partial \alpha_p} \right)_i = \sum_{i=1}^{n} w_i y_i \left( \frac{\partial f}{\partial \alpha_p} \right)_i
\]  \hspace{1cm} (A-4)

In general, this is a system of \( p \) simultaneous non-linear equations, and if a solution does exist, it is not necessarily unique. If the equations are linear in the parameters, or may be transformed into a linear form, then the solution does exist, and the elegant processes of linear regression analysis may be employed.

It is instructive to write out the normal equations for the linear problem, taking the special case

\[
f(x_1, \ldots, x_m; \alpha_1, \ldots, \alpha_m) = \alpha_1 x_1 + \alpha_2 x_2 + \ldots + \alpha_p x_p \]  \hspace{1cm} (A-5)

The normal equations are simply:

\[
a_1 \sum_{i=1}^{n} w_i x_{1i}^2 + \ldots + a_p \sum_{i=1}^{n} w_i x_{1i} x_{pi} = \sum_{i=1}^{n} w_i x_{1i} y_i
\]

\[
a_1 \sum_{i=1}^{n} w_i x_{pi} x_{1i} + \ldots + a_2 \sum_{i=1}^{n} w_i x_{pi}^2 = \sum_{i=1}^{n} w_i x_{pi} y_i \]  \hspace{1cm} (A-6)

For the case of a non-linear functional relationship, such as that encountered in both aspects of the present problem, it is necessary to search for a set of parameters that produces a minimum in \( Q \). There is, however, always the danger that this will be only a relative or local minimum, or that another combination of the parameters will yield an equally good minimum.

The Gauss approach treats this situation by linearizing the desired function with respect to each of the parameters. In this way a system of linear
equations is obtained, which may be solved by a variety of techniques. The solution yields a correction to the original estimates of the parameters, enabling the formation of new, improved estimates which become the input for the next iteration. The process is repeated until the desired standard of convergence has been achieved.

The linearization is accomplished by expanding the function to be fitted in a Taylor's Series around a point in the p-dimensional parameter space, and then truncating the series so that only the linear terms are kept. Suppose that this point is given by the initial estimates of the parameters, and is denoted by \((a_{1,0}, a_{2,0}, \ldots, a_{p,0})\). Then, for each of the observation points \(i\), \((i = 1, \ldots, n)\),

\[
y_i = f(x_{i1}, x_{i2}, \ldots, x_{im}; a_{1,0}, a_{2,0}, \ldots, a_{p,0}) + \Delta y_{i,0}
\]

where \(\Delta y_{i,0}\) is, by definition, the difference between the observed \(y_i\) and the function evaluated at the estimation point.

We let

\[
\Delta y_{i,0} = \sum_{k=1}^{p} \left( \frac{\partial f}{\partial \alpha_k} \right)_{i,0} \left( \Delta a_{k,1} \right)
\]

where the partial is evaluated at \((a_{1,0}, \ldots, a_{p,0})\), \(\Delta y_{i,0}\) be the dependent variables, the \((\delta f/\delta \alpha_k)_{i,0}\) the independent variables, and the \(\Delta a_{k,1}\) the parameters to be estimated, and substitute into the normal equations:

\[
\sum_{i=1}^{n} w_i \left( \frac{\partial f}{\partial \alpha_1} \right)_{i,0}^2 (\Delta a_{1,1}) + \ldots + \sum_{i=1}^{n} w_i \left( \frac{\partial f}{\partial \alpha_1} \right)_{i,0} \left( \frac{\partial f}{\partial \alpha_p} \right)_{i,0} (\Delta a_{p,1})
\]

\[
\sum_{i=1}^{n} w_i \Delta y_{i,0} \left( \frac{\partial f}{\partial \alpha_1} \right)_{i,0}
\]

\[
\sum_{i=1}^{n} w_i \left( \frac{\partial f}{\partial \alpha_1} \right)_{i,0} (\Delta a_{1,1}) + \ldots + \sum_{i=1}^{n} w_i \left( \frac{\partial f}{\partial \alpha_p} \right)_{i,0}^2 (\Delta a_{p,1})
\]

\[
= \sum_{i=1}^{n} w_i \Delta y_{i,0} \left( \frac{\partial f}{\partial \alpha_1} \right)_{i,0}
\]
Comparison of these equations with the equations (A-6) previously written for the special case of the linear problem, with the equivalences mentioned above, reveals that this problem has been reduced to one which is mathematically identical to the linear case. By discarding all second and higher order terms, a set of \( p \) linear simultaneous equations has been produced.

If a solution can be obtained, then a set of values \( \Delta a_{k,1} \) is available to modify the original estimates: \( a_{k,1} = a_{k,0} + \Delta a_{k,1} \). The entire process is repeated with the adjusted values taking the place of the original estimates until after some number of iterations the fit is deemed sufficiently "good". This is not an obvious decision and will be discussed below.

The normal equations may be written, in the usual matrix notation, as \( AX = B \), with elements as follows (for the first iteration):

\[
A_{jk} = \sum_{i=1}^{n} w_i \left( \frac{\partial f}{\partial \alpha_j} \right)_i \left( \frac{\partial f}{\partial \alpha_k} \right)_i
\]

\[
B_k = \sum_{i=1}^{n} w_i \Delta y_{i,0} \left( \frac{\partial f}{\partial \alpha_k} \right)_i
\]

\[
X_k = (\Delta a_{k,1})
\]

The solution is simply \( X = A^{-1} B \); \( A \) has been assumed non-singular and this points up a limitation of this technique. Not only is it necessary for the function to be fitted and its derivatives to be well-behaved at all data points, but it is also required that none of the derivatives be zero for all \( n \) data points. In that case, one row and column of \( A \) are identically zero, \( A \) is therefore singular and no solution exists. Even when these problems do not occur mathematically, they may well be the practical result of calculations on a necessarily finite computer.

It can be shown that unbiased estimates of the variances and covariances of the parameters \( \alpha_k \) may be obtained from the elements of the
inverse matrix when the $y_i$ are normally distributed, and that these may be
used in a consistent way to compute confidence intervals. While these
and other considerations of fit validity are quite important, they will not be
treated here. The interested reader is referred to the work of Moore and
Ziegler previously cited for these and related matters.

It has been thought necessary to present these mathematical details
because the present problem is of a highly non-linear nature. The question of
extracting the data accurately by hand is out of the question, and while it
is true that for simple cases, a simple linearizing transformation is some-
times adequate, the nature of this data obviates that possibility. The
technique outlined above has been found rapid and reliable.

B. A System of Subroutines to Calculate the Energy Distributions of
Reaction Products

The actual fitting operation is based on the "P" and "R" subroutines
of Moore and Ziegler, which have been translated to FORTRAN IV and
considerably modified for use on the IBM 7040/7094 Directly Coupled System
at the Yale Computer Center. Although the output format of these routines
has been essentially retained, the internal workings have been altered: a
double precision matrix inversion program has been used, with provision
for handling over- and underflow problems; routines enabling functional de-
pendences between the parameters and allowing for various types of back-
grounds have been added, as well as new linkages to the various input, output,
and calculational programs.

Most significant, perhaps, has been the inclusion of a technique which
both improves the chance of convergence in difficult cases, and simultaneoulsy
provides a meaningful criterion for determining when convergence has been
achieved. The technique was first reported by Levenberg and subsequently
has been applied by Hobbie and Pinsonneault.

One of the difficulties inherent in the non-linear least squares
procedure is its instability; large matrices of experimentally determined
data are notoriously ill-conditioned and their inverses may be quite spurious\(^{(94)}\). The sum of the squares of the deviations, \(Q\), does not always decrease with each iteration and the Los Alamos procedure of stopping the process when the fractional change of each parameter becomes less than a predetermined (arbitrary) number does not guarantee that the values of the parameters corresponding to the minimum of \(Q\) will be obtained.

Levenberg showed that if each diagonal element of the matrix \(A\) is multiplied by a certain quantity before inverting it, then \(Q\), which is the direct measure of goodness-of-fit, must monotonically decrease with each iteration unless it is already at a relative minimum. This quantity is well approximated by

\[
R = 1.0 + 2 \sum (B_k^2/A_{k,k})/Q ,
\]

where the notation is as above.

This method of damping the oscillations of the least squares matrix frequently permits convergence in cases not otherwise amenable to the Gauss technique. Termination of the iterative procedure may be consistently accomplished by requiring that the decrease in \(Q\) for successive iterations be less than some specified percentage. If the value of \(Q\) increases, the results of the preceding iteration are used.

Any number of the parameters in the present program may be held fixed, a procedure which proves useful when a difficult case is encountered. Also added is the ability of specify a functional dependence between the parameters; for example it may be known from the physics of the situation that the energy corresponding to a particular peak bears a known relation to the energy of another peak included in the fit. This relation then places an additional constraint on the values of peak location.

Usually when detector resolution is essentially constant, one specifies that the half-widths of the peaks be forced equal to each other in the fit. For widely separated peaks, this can lead to error and so a more exact procedure has been adopted in this work: it is required that the ratio of the peak width
to peak center location be a constant. This is just the resolution, $\Delta E/E$, in the usual notation. This ratio is recomputed after each iteration using the improved estimates of these quantities, and correcting for the effective zero channel of the analyzer in the calculation.

Various types of background may be specified: parabolic, exponential, gaussian, straight-line, or point-to-point. All but the last are calculated by the program from two or three input points.

The signs of parameters may be held fixed throughout the fitting, allowed to vary freely, or held fixed for five iterations and then allowed to vary freely. This also helps damp the oscillations resulting from poor initial parameter estimates.
APPENDIX II

SUMMARY OF COMPUTER PROGRAMS USED IN DATA REDUCTION

A. TEASE - Tape Editing and Selection Program for the Yale Multiparameter Analyzer (IBM 1401).

This program enables magnetic tapes written on the MPA to be examined and edited on the IBM 1401 computer prior to actual data processing on the 7040/7094 system. This procedure is followed for two reasons:

1. Inherent inaccuracies in the writing circuitry of the MPA and its associated tape equipment occasionally produce errors which require editing prior to data analysis.

2. It is frequently desirable to process or retain only selected blocks of the data on a given reel of tape; a means of extraction and duplication is needed.

It is advisable to use the 1401 for these functions as machine time is both easier to obtain and considerably less expensive than for the large computers.

The program proceeds in two phases, with a programmed halt between them. Phase one scans the tape and prints the exact location and nature of any error detected; phase two performs the operations necessary to produce the desired output tape. Parameters specifying the configuration of the input tape and the operations required to produce the output tape are punched into control cards. Merging, deletion, insertion, copying, editing, skipping, and other functions are available, as well as automatic parity correction, end of file marking, etc.

B. COMUS - Contour Representation of Multiparameter Spectra

This 7040/7094 program produces a quantitative "contour map" of the analyzer memory. Several condensed formats as well as the full printout are available. If desired, the total number of counts in any block of data is found. FORTRAN IV and MAP language subroutines are employed.
C. MEDUSA - Multiparameter Energy Distributions, with Undistorted Spectral Analysis.

This is the system of subroutines referred to in Appendix I and the body of this report. The input to the program, besides the data tape of the MPA, consists of estimates of the position of hyperbolae of interest, beam energy, MPA calibration parameters, and integrated beam charge. After the appropriate fitting operations are performed, a plotted and tabulated energy spectrum, corrected for energy loss in the ion chamber or any other absorber present is printed. This may be expressed in terms of excitation energy in the observed product nucleus, if specified. This information is also punched on cards for subsequent analysis, storage, or plotting.


The set of FORTRAN IV and MAP subroutines utilizes the on-line, high resolution cathode ray tube and associated micro-film camera equipment of the 7040/7094 to produce a graphical record of data accumulated in the MPA. All the controls of the latter instrument have been digitally simulated on the computer; the following forms of display are presently available. All scaling, etc. is automatic.

1. Contour display with the following options:
   a. Two individually variable thresholds and window widths - a channel will appear either as a blank, low intensity or high intensity spot according to the levels chosen.
   b. Channel marker.
   c. Individual channel (plane) identifier.
   d. Title.
   e. Individually variable gains.
   f. Axes.

   Note that all of the above are specified digitally as control parameters on data cards.

2. Planar display - any cross section through the data, in either
direction, will be plotted, with or without axes and title, with variable
gain, in either spot intensity.

3. Isometric display - a pseudo-isometric plot, obtained by displacing
successive planes by a variable amount. All the options of the contour
plots are available here.

Output options include standard 35 mm film, (which is conveniently
made into slides or prints) and IBM aperture cards. The latter are data
processing cards with either a positive or negative image of the film on part
of the card; the remaining columns are available for punching any identifying
information. As the card may be processed by either computer or card
sorting equipment, this provides an efficient and rapid means of both infor-
mation storage and retrieval.

E. Energy Spectrum Fitting Program.

This program determines the best fit, in the least squares sense,
to the one-dimensional energy spectrum which has been formed from the
two-dimensional $\Delta E \times E$ matrix, in the manner described in Appendix I.
The function employed is again a sum of gaussians, since this well approxi-
mates the response of the energy detector (see Fig. 11). The area under
the peaks of interest is converted to cross section; absolute or relative,
and the excitation energy corresponding to the peak location is calculated
using the system calibration and the range-energy relations. Center-of-
mass angle and cross section and linear momentum transfer are also
determined for each peak.

The output cards of program MEDUSA served directly as input for the
present work. However, the program is generally written and is easily
adapted to either cards or tape from any source. The results of the program
are both tabulated and plotted. The program is written in FORTRAN IV and
MAP for the IBM 7040/7094 computer. The fitting of five peaks, with forty
observation points and straight-line background takes less than two seconds
on the 7040/7094,
The mathematical technique is identical to that described in Appendix I; however, certain additional options are available in this program.

Among these is the ability to specify a wide variety of backgrounds. This may be supplied on a point-to-point basis, or may be calculated by the program from a specification of the functional form and an estimate of three points that the background passes through. Gaussian, exponential, straight line, and parabolic backgrounds are presently included; others may easily be added.

Ordinarily, in a program of this type, estimates of peak location are provided in terms of channel number. In this work an alternate procedure has been frequently applied. The location and excitation of a reference peak serves as the basis for the conversion of initial estimates of a peak's excitation to its location with full correction for absorbers, etc. The reference peak need not be included in the channels being fitted, and the locations so determined may either be held fixed, allowed to vary freely, or forced to bear an arbitrary (linear) functional dependence to any other peaks. The same holds true for any of the other parameters employed in the procedure.

The significance of these procedures is that they allow the full utilization of physical information in the fitting process. For example, suppose that the excitation of one peak in a spectrum is clearly known, (and this is generally the case) and it is desired to identify a neighboring peak from among several possible close-lying levels. Furthermore it is known that the detector response is constant over the limited energy range involved. These facts are easily incorporated as mathematical constraints, and the fit is performed successively with the unknown peak location held fixed at each of the possible excitations. The procedure is sensitive enough to variation of the parameters involved that a unique identification can generally be obtained if the peak separation is comparable to the system resolution.

The specification of an excitation rather than a channel location is frequently useful in another way. Often data taken at a given angle permits
an unambiguous identification of an isolated peak, which because of the reaction kinematics becomes a shoulder on a neighboring peak at other angles. By requiring that the excitation (location) of the peak marked by the shoulder be fixed at the previously determined energy, and that the widths of the peaks be in the relation required by the detector resolution, it is a straightforward matter to extract the areas of the peaks concerned.

F. Utility Programs.

A generalized version of the non-linear least squares fitting program above was used to accurately find peak locations used in the calibration procedure, and to determine the effective zero channel of the MPA, range-energy calibration parameters, etc.

The program is written so that any well behaved function (see Appendix I) may be specified by changing the statements defining the function and its partial derivatives, and recompiling the subroutine in which they appear. A library of interchangeable functions is thus made available for least squares fitting.
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