Nuclear Astrophysisc
Lecture 2: Nucleosynthesis

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Objectives of Lecture 2

- Understand types of nucleosynthesis
- Understand where various types are made
- Be able to write down equations for a reaction network
the astrophysical formation of the elements

Chart of the Nuclides

proton number $Z$

neutron number $N$

mass number $Z + N = A$

$^4\text{He}$
the astrophysical formation of the elements

solar system isotopic and elemental abundances

http://nedwww.ipac.caltech.edu/level5/Pagel/Figures/figure1_4.jpeg
some terminology

\( n_j \) number of species \( j \) per unit volume

\( W_j \) atomic weight (or molar mass) of species \( j \)

\( \rho_m \) mass density

\[
\rho_m = \frac{\sum n_j W_j}{N_A}
\]

\( \rho \) baryon mass density

\[
\rho = \frac{\sum n_j A_j}{N_A}
\]

\( X_j \) nucleon fraction, or mass fraction

\[
X_j = \frac{n_j A_j}{\rho N_A}
\]

\( Y_j \) mole fraction, or abundance

\[
Y_j = \frac{X_j}{A_j} = \frac{n_j}{\rho N_A}
\]

Note \( \sum_j X_j = \sum_j \frac{n_j A_j}{\rho N_A} = \frac{1}{\rho} \sum_j \frac{n_j A_j}{N_A} = 1 \)
some terminology

\[ y_j \] rescaled abundances

In meteoritics, abundances are normally scaled relative to silicon
(set number of silicon atoms to be \(10^6\)):
\[
\log y_j = \log f_{Si} + \log Y_i
\]
where \(f_{Si}\) is the appropriate normalizing constant. For \(Y_{Si} = 2.529 \times 10^{-5}\), \(\log f_{Si} = 10.5970\).

Astronomers sometimes use a scale relative to hydrogen
(set number of hydrogen atoms to be \(10^{12}\)):
\[
\log y_j = \log f_H + \log Y_i
\]

More commonly, astronomers express abundances as ratios relative to solar :
\[
\log [n_i / n_j]_{\text{star}} - \log [n_i / n_j]_{\text{solar}} \equiv [i / j]
\]
For example, a ratio of sodium to iron which is half solar would be \([\text{Na/Fe}] = -0.3\).
the astrophysical formation of the elements

big bang nucleosynthesis

~3/4 H

~1/4 He

some Li, Be, B
the astrophysical formation of the elements

Fusion in Stars,
Explosive burning

H → He
He → C, O
C,O → Si
Si → Fe group

neutron number $N$
the astrophysical formation of the elements
How to determine what elements an astrophysical environment produces

What information is needed?
How to determine what elements an astrophysical environment produces

My list:

- astrophysical conditions, i.e. temperature and density
- nuclear physics input (masses, reaction rates, decay rates)
How to determine what elements an astrophysical environment produces

What data can you compare with?
How to determine what elements an astrophysical environment produces

My list:

- solar system abundances (sun, earth)
- meteorites
- spectroscopic data from other stars
example abundance patterns

\[
\log(\varepsilon_i) = \log_{10}(n_i/n_\text{H}) + 12
\]
some terminology

\( y_j \) rescaled abundances

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example abundance patterns

\[ \log(\varepsilon_i) = \log_{10}(n_i/n_H) + 12 \]
the astrophysical formation of the elements

nuclear properties

nuclear reaction rates

astrophysical conditions

nuclear network code

→

evolution of nuclear abundances
building a nuclear network code: nuclear properties

atomic/nuclear masses

building a nuclear network code: nuclear properties

radioactive decay rates \( N(t) = N_0 e^{-\lambda t} \)

\( \lambda \) decay constant
\( \tau \) mean lifetime, \( \tau = 1/\lambda \)
\( T_{1/2} \) half-life, \( T_{1/2} = \ln 2/\lambda \)

How determined?

Fermi’s 'Golden Rule'

\[
rate = \frac{2\pi}{\hbar} |\langle f|H_{\text{int}}|i\rangle|^2 \rho(E)
\]

\( f, i \) final and initial state wavefunctions
\( H_{\text{int}} \) weak interaction Hamiltonian
\( \rho(E) \) density of states for the final particles
cross section for the reaction $i + j \rightarrow k + l$

$$\sigma_{ij}(v) = \frac{\text{number of reactions per nucleus } i \text{ per second}}{\text{flux of incoming projectiles } j}$$

$$\sigma_{ij}(v) = \frac{r_{ij}}{n_i} \frac{1}{n_j v_{ij}}$$

$r_{ij}$ number of interactions $i(j,k)l$ per second

$v_{ij}$ relative velocity of particles $i, j$

So the reaction rate per unit volume is just:

$$r_{ij} = n_i n_j v_{ij} \sigma_{ij}(v)$$
In astrophysical environments the relative velocity $v_{ij}$ is not constant, but instead there exists a distribution of relative velocities, which can be described by the probability function $P(v)$, where:

$$\int_{0}^{\infty} P(v)dv = 1$$

So the reaction rate can be generalized to:

$$r_{ij} = n_i n_j \int_{0}^{\infty} vP(v)\sigma_{ij}(v)dv$$

$$r_{ij} = n_i n_j \langle ov \rangle_{ij}$$
building a nuclear network code: reaction rates

If the nuclei are nonrelativistic and nondegenerate, their velocities can be described by a Maxwell-Boltzmann distribution

\[ P(v)dv = \left( \frac{m_{ij}}{2\pi kT} \right)^{3/2} e^{-m_{ij}v^2/2kT} 4\pi v^2 dv \]

where:
- \( m_{ij} \) reduced mass, \( m_{ij} = m_i m_j / (m_i + m_j) \)
- \( T \) temperature
- \( k \) Boltzmann constant, \( k = 8.6173 \times 10^{-5} \) eV/K

The velocity distribution can be written as an energy distribution, since \( E = m_{ij}v^2/2 \)

\[ P(v)dv = P(E)dE = \frac{2}{\sqrt{\pi}} \frac{1}{(kT)^{3/2}} \sqrt{E} e^{-E/kT} dE \]
Now consider the rate of change in the number density of species $j$:

$$\frac{dn_j}{dt} = n_k n_i \langle \sigma v \rangle_{kl,j} - n_j n_i \langle \sigma v \rangle_{jl,n} + n_i \lambda_{i,j} - n_j \lambda_{j,m} + K$$

Note for reactions involving identical particles, a term of the form:

$$\frac{n_i^2}{2!} \langle \sigma v \rangle_{ii,j} \text{ (two body)} \quad \text{or} \quad \frac{n_i^3}{3!} \langle \sigma v \rangle_{iii,j} \text{ (three body)}$$

is needed.

The above can be written in terms of abundances as:

$$\frac{dY_j}{dt} = Y_k Y_j \rho N_A \langle \sigma v \rangle_{kl,j} - Y_j Y_i \rho N_A \langle \sigma v \rangle_{jl,n} + Y_i \lambda_{i,j} - Y_j \lambda_{j,m} + K$$

this is often what we call the reaction rate in a network code
Conversion of $^1$H to $^4$He via:

$$^1\text{H} + ^1\text{H} \rightarrow ^2\text{H} + e^+ + \nu_e$$
$$^2\text{H} + ^1\text{H} \rightarrow ^3\text{He} + \gamma$$
$$^3\text{He} + ^3\text{He} \rightarrow ^4\text{He} + ^1\text{H} + ^1\text{H}$$
Recall the abundances evolve as

\[
\frac{dY_j}{dt} = Y_k Y_i \rho N_A \langle \sigma v \rangle_{kl,j} - Y_j Y_i \rho N_A \langle \sigma v \rangle_{jl,n} + Y_i \lambda_{i,j} - Y_j \lambda_{j,n} + K
\]

So here we have a system of four differential equations:

\[
\begin{align*}
\frac{dY_H}{dt} &= 2Y_{3_{He}}^2 \rho N_A \langle \sigma v \rangle_{3_{He}3_{He},4_{He}} - Y_{H}^2 \rho N_A \langle \sigma v \rangle_{HH,D} - Y_{D}Y_{H} \rho N_A \langle \sigma v \rangle_{DH,3_{He}} \\
\frac{dY_D}{dt} &= \frac{1}{2} Y_{H}^2 \rho N_A \langle \sigma v \rangle_{HH,D} - Y_{D}Y_{H} \rho N_A \langle \sigma v \rangle_{DH,3_{He}} \\
\frac{dY_{3_{He}}}{dt} &= Y_{D}Y_{H} \rho N_A \langle \sigma v \rangle_{DH,3_{He}} - Y_{3_{He}}^2 \rho N_A \langle \sigma v \rangle_{3_{He}3_{He},4_{He}} \\
\frac{dY_{4_{He}}}{dt} &= \frac{1}{2} Y_{3_{He}}^2 \rho N_A \langle \sigma v \rangle_{3_{He}3_{He},4_{He}}
\end{align*}
\]
PPI network equations

But note first D, then $^3$He will come into steady-state:

$$
\frac{dY_D}{dt} = \frac{1}{2} Y_H^2 \rho N_A \langle \sigma v \rangle_{HH,D} - Y_D Y_H \rho N_A \langle \sigma v \rangle_{DH,^3He} = 0, \text{ so}
$$

$$
Y_D = \frac{\langle \sigma v \rangle_{HH,D}}{2 \langle \sigma v \rangle_{DH,^3He}} \sim 10^{-17}
$$

$$
\frac{dY_{^3He}}{dt} = Y_D Y_H \rho N_A \langle \sigma v \rangle_{DH,^3He} - Y_{^3He} Y_H^2 \rho N_A \langle \sigma v \rangle_{^3He,^4He}
$$

$$
\frac{dY_{^3He}}{dt} = \frac{1}{2} Y_H^2 \rho N_A \langle \sigma v \rangle_{HH,D} - Y_{^3He}^2 \rho N_A \langle \sigma v \rangle_{^3He,^3He,^4He} = 0
$$

$$
Y_{^3He} = \sqrt{\frac{\langle \sigma v \rangle_{HH,D}}{2 \langle \sigma v \rangle_{^3He,^3He,^4He}}} \sim 10^{-5}
$$
Higher T than the sun; nuclear statistical equilibrium

When strong and electromagnetic interactions come into equilibrium at high temperatures, the nuclear abundances are no longer sensitive to individual reaction rates and only depend on the temperature $T$, density $\rho$, and the neutron-richness of the composition.

Nuclear statistical equilibrium (NSE) abundances are given by:

$$Y_i = (\rho N_A)^{A_i-1} \frac{G_i}{2^A_i A_i} A_i^{3/2} \left( \frac{2\pi \hbar^2}{m_c kT} \right)^{3(A_i-1)} \exp \left[ \frac{B_i}{kT} \right] Y_p^{Z_i} Y_n^{N_i},$$

where $G_i$ is the nuclear partition function and $B_i$ is the binding energy.

We also require mass and charge conservation:

$$\sum_i Y_i A_i = 1 \quad \sum_i Y_i Z_i = Y_e$$

electron fraction
sample NSE compositions

entropy $s/k = 10, \ Y_e = 0.5$

from Meyer (1994)

entropy $s/k = 100, \ Y_e = 0.5$
explosive burning in core-collapse supernovae

Outgoing shock wave heats inner Si and O layers – NSE is achieved and rapidly freezes out as shock passes
explosive burning in core-collapse supernovae

Outgoing shock wave heats inner Si and O layers – NSE is achieved and rapidly freezes out as shock passes

Thielemann et al (2010)
explosive burning in core-collapse supernovae

explosive Si, O, Ne, and C burning follow as the shock moves outward

Thielemann et al (2010)
$^{44}\text{Ti}$

created in alpha-rich freezeout from NSE, close to the mass cut $T_{1/2} \sim 60$ years

x-rays from the decay chain observed in SNe remnants Cas-A and RX J0852.0-4622
What about the heavier elements?

- p-process
- s-process
- r-process
SNe nucleosynthesis – proton-rich heavy elements

heavy $p$-process nuclei are made by $(\gamma,n)$ photodissociations of pre-existing $r$- and $s$- process nuclei
SNe nucleosynthesis – proton-rich heavy elements

vp-process

Thought to occur in proton-rich ejecta from the inner regions of the SNe

\[ p + \bar{\nu}_e \rightarrow n + e^+ \]

Thielemann et al (2010)
neutron capture nucleosynthesis
neutron capture nucleosynthesis
separating s- and r-process abundance patterns

Käppeler et al (2010)
the astrophysical site of the s-process

main component – low mass AGB stars
the astrophysical site of the s-process

\[ ^{12}\text{C}(p,\gamma)^{13}\text{N}(\beta^+\nu)^{13}\text{C} \rightarrow ^{13}\text{C}(\alpha,n) \]
Next lecture: r-process