Quantum Circuits

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Why do we care?

Using quantum properties, quantum computers have the potential to perform certain calculations faster than classical computers:

Superposition: allows for the simultaneous processing of lots of information

Entanglement: allows for many connected calculations to occur at once
What is a Qubit?

Classical computers store information in bits of 0 and 1

- 1 Light on
- 0 Light off

In the quantum world, bits are no longer in either the 0 state or the 1 state

- They can be in a superposition of both states

Think Schrodinger's Cat:

Like Schrodinger’s cat, qubits aren’t in either state until measured
What is a Qubit?

Qubits can be in $|0\rangle$ or $|1\rangle$ or in a superposition of the two.

$|0\rangle$ and $|1\rangle$ are both column vectors:

$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

These are the basis states of a two dimensional Hilbert space.

- A Hilbert space is a vector space used for quantum mechanics.
What is a Quantum State?

A quantum state describes the probability that a qubit will be in the $|0\rangle$ or $|1\rangle$ state when measured.

Unlike seemingly ‘probabilistic’ things in nature, the state that qubits collapse to are completely probabilistic- there are no hidden variables.
What is a Quantum State?

General state, describing the state of a qubit:

\[ |\psi\rangle = \alpha |0\rangle + \beta |1\rangle \]

Where \( |\alpha|^2 \) is the probability that \( |\psi\rangle \) is in state \( |0\rangle \), and \( |\beta|^2 \) is the probability that \( |\psi\rangle \) is in state \( |1\rangle \)

Note that because a qubit must be in \( |0\rangle \) or \( |1\rangle \), \( |\alpha|^2 + |\beta|^2 = 1 \) (meaning it is normalized)
Pauli Matrices

The Pauli Matrices are a set of matrices defined by scientist Wolfgang Pauli. They are:

\[
\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}
\]

\[
\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}
\]

\[
\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}
\]
Gates - Single Qubit

Gates are used to change the nature of the state of a qubit. There are many different kinds of gates; some act on one qubit, some act on more than one qubit. Perhaps the most basic gates are the three pauli gates, formed using the pauli matrices.

These gates act on one qubit, and their results are listed below (these can be calculated by multiplying the pauli matrices by the $|0\rangle$ and $|1\rangle$ states):

- **X gate** = swaps $|0\rangle$ to $|1\rangle$ and vica versa
- **Y gate** = $|0\rangle$ to $i|1\rangle$ and $|1\rangle$ to $-i|0\rangle$
- **Z gate** = takes $|0\rangle$ to $|0\rangle$ and $|1\rangle$ to $-|1\rangle$
Gates - Single Qubit

Hadamard Gate

\[ |0\rangle \text{ to } |+\rangle \]
\[ \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \]

\[ |1\rangle \text{ to } |-\rangle \]
\[ \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) \]
Gates - Multiple Qubits

Controlled Not (Controlled X) Gate

Applies X gate to target bit if control bit is $|1\rangle$

$|00\rangle$ to $|00\rangle$

$|01\rangle$ to $|01\rangle$

$|10\rangle$ to $|11\rangle$

$|11\rangle$ to $|10\rangle$
Gates - Multiple Qubits

Controlled Z Gate

Applies Z gate to target qubits if the control qubit is in $|1\rangle$

$|00\rangle$ to $|00\rangle$

$|01\rangle$ to $|01\rangle$

$|10\rangle$ to $|10\rangle$

$|11\rangle$ to $|11\rangle$
Gate - Multiple Qubits

Controlled Y Gate

Applies Y gate to target bit if control bit is in $|1\rangle$

$|00\rangle$ to $|00\rangle$
$|01\rangle$ to $|01\rangle$
$|10\rangle$ to $i|11\rangle$
$|11\rangle$ to $-i|10\rangle$
Bell Circuits

$q_0$ and $q_1$ both initialized to $|0\rangle$ or $|1\rangle$

Final state:

$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$

$\frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$

$\frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$

$\frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$

These circuits create entangled qubits because the final state cannot be factored - think the states cannot be separated
Unitary

Matrix that follows rule:

\[ U^\dagger U = \text{Identity} \]

Examples include all the Pauli Matrices

The Pauli Matrices are also special matrices where \( U^\dagger = U \)

Unitary Comprised of CZ, CNOT, and CY Gate
Exponentiation Gadget

If the state is $|\psi\rangle$, whereas a normal unitary operator would result in $U|\psi\rangle$, the exponentiation gadget would result in $e^{i\frac{\theta}{2}U}|\psi\rangle$

Exponentiation Gadgets are used to create more complex gates in restrictive environments
Quantum Oracles

$q_0$ and $q_1$ both initialized to $|0\rangle$ or $|1\rangle$

$q_0$ state stays the same

$q_1$ goes to $q_0 \oplus \text{CNOT}q_1$

So:

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<tr>
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<th>$q_0$</th>
<th>$q_1$ Initial</th>
<th>$q_1$ Final</th>
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Quantum Oracle Using CNOT Gate
Sources
