Baryon Spectroscopy from Experiment and Theory

Michael Döring


supported by NSF/CAREER, NSF/PIF

HPC support by JÜLICH FORSCHUNGSZENTRUM
Elementary particles of the Standard Model

Particles like the electron (fermions, spin 1/2)

<table>
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<tr>
<th>Leptons</th>
<th>Quarks (each in 3 “colors”)</th>
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<td>$e$</td>
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$\gamma$ photon

$g$ gluon (8 “colors”)

$W^\pm$ weak interaction (Gravity is negligible.)

$Z^0$ Weak interaction

“electromagnetism”

“strong interaction”

“weak interaction”

Strong interaction
Q: How many quarks or gluons have ever been directly observed?

A: 0 (zero)

The mass of a down quark is $5$ MeV and that of an up quark is $2$ MeV.

Then, the mass of the proton ($uud$) should be $m_P \sim 9$ MeV, right?

A: $m_P = 938.272$ MeV.

It is obviously a long way from our "periodic table" of quarks and gluons to matter and its properties as we know them.

and this is the topic of this Seminar.
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It is obviously a long way from our “periodic table” of quarks and gluons to matter and its properties as we know them.

and this is the topic of this Seminar.
Even these 4% not well understood.
The full complexity: Parton shower and hadronization

Only colorless final states $\leftrightarrow$ confinement
Quark-gluon interaction: QCD

Remember from Mechanics:
\[ L = T - V \]
describes your physical system [T: Kinetic energy, V: potential energy]

\[
\mathcal{L}_{\text{QCD}} = \sum_f \bar{\psi}_f [D_\mu \gamma_\mu + m_f] \psi_f + \frac{1}{4} \sum_a G^a_{\mu\nu} G^{a\mu\nu}
\]

\( \alpha_s = \frac{g^2}{4\pi} \ll 1 \).

For small \( \alpha_s \), one can solve QCD in a controlled way (perturbation theory).
The running coupling

Deur, Burkert, Chen, Korsch, PLB665 (2008)

No easy solution of QCD at lower energies!
Lattice QCD for hadrons

- Simulate the complexity of QCD at low energies with the help of supercomputers
- Ab-initio approach: QCD $\rightarrow$ hadron masses
- Discretization in space and time, in a finite volume, to make the problem numerically treatable
The baryon spectrum: $N^*$ and $\Delta$ resonances

- Many resonances predicted in lattice calculations
  

\[
m_{\pi} = 396 \text{MeV} (!)
\]

- Search for these states in dedicated experimental programs
Photoproduction experiments: Jefferson Lab, MAMI, ELSA,...
Photoproduction cross sections

\[ \gamma + p \rightarrow X \]

\[ \gamma + p \rightarrow p + \pi^- \]

\[ \gamma + p \rightarrow p + \pi^0 \]

\[ \gamma + p \rightarrow p + \pi^0 + \Lambda \]

\[ \gamma + p \rightarrow K^+ + \Lambda \]

\[ \gamma + p \rightarrow p + \eta \]

[Data: JLab, ELSA, MAMI]
Partial wave analysis from many observables: $\gamma N \rightarrow \pi N$

**Differential cross section $\gamma p \rightarrow n\pi^+$**

- Photon: Spin 1
- Nucleon: Spin $1/2$
- Single, double, triple polarization observables
- Order principle in the chaos: Conserved quantum numbers, e.g., $J^P$: Total angular momentum $^\text{Parity}$

[CLAS measurements, PRC 79 (2009); Solid (dashed) lines: SAID (MAID) analysis; filled: CLAS, triangles: MAMI]

**Partial wave analysis**

Decompose experimental data with respect to conserved quantum numbers. Resonances have a certain, conserved $J^P$. 
Field-theoretical approach; TOPT unitarized; implemented on supercomputers. Example:

\[ \gamma N (\pi N) \rightarrow K \Sigma \]
Selected results: Two partial waves for $\pi N \rightarrow \pi N$

[Rönchen, M.D., Haberzettl, Hanhart, Huang, Krewald, Meißner, Nakayama, EPJA 49 (2013)]

[Data points: SAID (Arndt, Briscoe, Strakovsky, Workman, PRC 74 (2006))]
Fit to world data on $\pi N \rightarrow \pi N, \eta N, K \Lambda, K \Sigma \, (\sim 10^5$ exp. points)  
[Rönchen, M.D. et al., EPJA 49 (2013)]

Selected results for $\pi^- p \rightarrow K^0 \Lambda$ [almost complete experiment]
\( \pi^- p \rightarrow K^0 \Lambda \): Total cross section

Result: Partial wave content.
Resonance content: Nucleon-like resonances

- N(1440) 1/2^+
- N(1535) 1/2^-
- N(1520) 3/2^- (πΔ)
- N(1650) 1/2^-
- N(1675) 5/2^-
- N(1680) 5/2^+
- N(1710) 1/2^+
- N(1720) 3/2^+
- N(1750) 1/2^+
- N(1990) 7/2^+
- N(2190) 7/2^-
- N(2200) 9/2^+
- N(2250) 9/2^-

Physical axis: Im E=0
Future data challenges: FROST, HD-ICE at JLab
from: Eugene Pasyuk @ Meson 2012

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✓ - published  ✓ - acquired
First-ever measurement of this observable

+ global fit of world data on pion photoproduction and pion-induced reactions.

FROST $E$ in $\gamma p \rightarrow \eta p$: Excitation function

- $E$ vs. $W$ [MeV]
- $\cos \theta$ values: $-0.9 / 154^\circ$, $-0.6 / 127^\circ$, $-0.2 / 102^\circ$, $0.2 / 78^\circ$, $0.6 / 53^\circ$
Structure at $W = 1.68$ GeV: Conventional Resonances plus $K\Sigma$ Cusp

NO additional structure (non-exotic pentaquark) at $W = 1.68$ GeV $\rightarrow$ interferences & $K\Sigma$ threshold.
Photon-induced vs. Pion-induced reactions

\[ \gamma \rightarrow n \text{ Mesons} \rightarrow \text{more parameters} \rightarrow \text{Baryon} \rightarrow \text{hadronic transition} \rightarrow \pi, \pi\pi, \eta, \bar{K}, N, \Lambda, \Sigma \]

Complete Experiment

8 Observables

\[ \pi N \rightarrow \text{MB: 3 Observables} \]

Hadronic transition ALONE fix pole positions and strong branching ratios

→ Principal point of comparison with lattice QCD.

Coupled-channels: Any problematic data in MB → MB will cause problems in photoproduction analysis.
Physics opportunities with meson beams

William J. Briscoe\textsuperscript{1,a}, Michael Döring\textsuperscript{1,b}, Helmut Haberzettl\textsuperscript{1,c}, D. Mark Manley\textsuperscript{2,d}, Megumi Naruki\textsuperscript{3,e}, Igor I. Strakovsky\textsuperscript{1,f}, and Eric S. Swanson\textsuperscript{4,g}

\textsuperscript{1} The George Washington University, Washington, DC 20052, USA
\textsuperscript{2} Kent State University, Kent, OH 44242, USA
\textsuperscript{3} Kyoto University, Kyoto 606-8502, Japan
\textsuperscript{4} University of Pittsburgh, Pittsburgh, PA 15260, USA

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Abstract. Over the past two decades, meson photo- and electroproduction data of unprecedented quality and quantity have been measured at electromagnetic facilities worldwide. By contrast, the meson-beam data for the same hadronic final states are mostly outdated and largely of poor quality, or even nonexistent, and thus provide inadequate input to help interpret, analyze, and exploit the full potential of the new electromagnetic data. To reap the full benefit of the high-precision electromagnetic data, new high-statistics data from measurements with meson beams, with good angle and energy coverage for a wide range of reactions, are critically needed to advance our knowledge in baryon and meson spectroscopy and other related areas of hadron physics. To address this situation, a state-of-the-art meson-beam facility needs to be constructed. The present paper summarizes unresolved issues in hadron physics and outlines the vast opportunities and advances that only become possible with such a facility.
Applications for excited meson spectroscopy @ GlueX
Hadronic approaches to analyze resonances on the lattice
Transfer of methods from baryon to meson analysis

Ensures 3-body unitarity

+ pions from N* and Δ's
Beyond the isobar model: Three-body unitarity

- We know from baryon analysis: the importance of three-body unitarity.

- Going beyond the Breit-Wigner parameterization of the $\pi\pi$ amplitudes.

- Coupled-channels (c.f. also GlueX proposal: Kaon identification).

- Transfer of methods from baryon to meson analysis.

- Feasible for complex systems with many partial waves?

- Technical simplification of the present approach needed (tool for experimentalists).
The world as lattice sees it

\[ U_\mu(x) = \exp(i \int_x^{x+\mu} A_\mu(x') dx') \]

- Finite volume \( L^3 \)
- Finite lattice spacing \( a \)
- \( m_q(\text{Lattice}) \neq m_q \)
Resonances decaying on the lattice

Eigenvalues in the finite volume

Avoided level crossing

Energy

L (box size)

Resonance energy
**Nπ (1/2−) channel**

$m_\pi = 266$ MeV; distillation method; variational analysis using a basis of N (3 quarks) and Nπ (5 quarks) interpolators:

\[(N_\pm^{(i)})_\mu(\vec{p} = 0) = \sum_{\vec{x}} \epsilon_{abc} \left( P_\pm \Gamma_{\lambda}^{(i)} u_\lambda(\vec{x}) \right)_\mu \left( u_\lambda^T(\vec{x}) \Gamma_2^{(i)} d_\lambda(\vec{x}) \right)\]

\[\pi^+(\vec{p} = 0) = \sum_{\vec{x}} \bar{d}_a(\vec{x}) \gamma_5 u_a(\vec{x}) ,\]

\[\pi^0(\vec{p} = 0) = \sum_{\vec{x}} \frac{1}{\sqrt{2}} \left( \bar{u}_a(\vec{x}) \gamma_5 u_a(\vec{x}) - \bar{d}_a(\vec{x}) \gamma_5 d_a(\vec{x}) \right)\]

\[O_{N\pi}(I = \frac{1}{2}, I_3 = \frac{1}{2}) = p\pi^0 + \sqrt{2} n\pi^+\]

\[N\pi(\vec{p} = 0) = \gamma_5 N_+(\vec{p} = 0)\pi(\vec{p} = 0)\]

Lüscher relation ➔ phase shift:

Assuming 2 elastic resonances with identical coupling we get

\[m_1 = 1.678 \text{ GeV}\]

\[m_2 = 1.873 \text{ GeV}\]
Three unknown potentials

- \( V(\pi\pi \rightarrow \pi\pi) \)
- \( V(\pi\pi \rightarrow \bar{K}K) \)
- \( V(\bar{K}K \rightarrow \bar{K}K) \)

Expand a two-channel potential \( V \) in energy \((i, j: \pi\pi, \bar{K}K)\):

\[
V_{ij}(E) = a_{ij} + b_{ij}(E^2 - 4M_K^2)
\]

to extract phase shifts/resonances

Partial-wave mixing, coupled channels, \ldots \rightarrow in complex hadronic systems, Lüscher method needs to be complemented by techniques from experimental data analysis.
Large experimental efforts, e.g., E. Eppe et al., PRC 87 (2013).

- Coupled channels $\pi\Sigma$, $\bar{K}N$, $\eta\Lambda$, $K\Xi$
- Unitarized LO $\chi$ potential $V$ in $T = V + VGT$
- No freedom at this order $\rightarrow$ full prediction.
- UCHPT has two poles for the $\Lambda(1405)$.
- $\rightarrow$ new data not in conflict with two-pole structure of a molecular $\Lambda(1405)$ (but not yet a proof thereof).
- $\rightarrow$ needed: Statistical NLO analysis in combination with more accurate data.
Large experimental efforts, e.g., E. Epple et al., PRC 87 (2013).

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New lattice data from the GWU LQCD group (A. Alexandru, Dehua Guo)

- $N_f = 2$ calculation

- SU(3) NLO unitarized CHPT (inverse amplitude method)

- Short circuiting the Lüscher equation:
  - LQCD Eigenvalues $\rightarrow$ phases $\rightarrow$ Hadronic fit.
  - Instead: LQCD Eigenvalues $\rightarrow$ direct fit of LEC's.

- Finite volume corrections, chiral extrapolations and $N_f = 2 \rightarrow N_f = 3$ extrapolation in one step.

- Missing LEC’s for $N_f = 2 \rightarrow N_f = 3$ extrapolation determined from fit to experimental data ($\pi\pi$ in $I = 0, 2$, $\pi K$ in $I = 1/2, 3/2$, $L = 0, 1$).
  
  based on [Oller, Oset, Pelaez, PRD 1998]

- Alternatively: Resonance CHPT LO calculation in $\pi\pi, \bar{K}K$.  

The $\rho(770)$ on the lattice – NLO UCHPT results

Raquel Molina, Bin Hu, M.D., preliminary

- Separate fits to eigenlevels at different quark masses

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| Phys. Rev. D 81 | 138    | 1.22          | -2.94         |         |               |    |      |             |
| $U\chi$PT fit to exp. data | 138 | 1.29          | -3.28         |         |               |    |      |             |

Table 2: Low energy constants obtained from the fits of the lattice data that are within $m_\rho \pm 2\Gamma$. Masses are given in MeV units and the low energy constants are $\times 10^{-3}$.

- Compared to Lüscher-extracted phases

![Graphs showing the comparison of Lüscher-extracted phases with lattice data.](image)
The $\rho(770)$ on the lattice – NLO UCHPT results
Raquel Molina, Bin Hu, M.D., preliminary

Chiral extrapolation plus $N_f = 2 \rightarrow N_f = 3$ extrapolation
Resonance CHPT at LO (NLO corrections under way), in 1 channel ($\pi\pi$) to fit $N_f = 2$ LQCD data, and 2 channels ($\pi\pi, \bar{K}K$) for extrapolation to $N_f = 3$.

- $N_f = 2$: $\rho$ is now too heavy, in contrast to NLO UCHPT (!)
- $\bar{K}K$ channel again corrects qualitatively correctly—this time in opposite direction.
- Consistency check: 2-channel fit of $g, M_{\rho}, M_K$ (yes!) to exp. phases $\Rightarrow M_K = 498 \pm 185$ MeV
World data on lattice simulations of the $\rho(770)$

$N_f = 2$ simulations [Selected results]

Lattice data [Alexandru/Guo, GWU]

Data: Lang et al. (Graz)

Data: Bali et al. (Regensburg)

$N_f = 2$ extrapolation + exp. Data

$N_f = 2 \rightarrow N_f = 3$ extrapolation
The study of resonances at intermediate energies provides a key to our understanding of Quantum Chromodynamic.

JLab, MAMI, ELSA, ... provide high-precision photoproduction data. New data from FROST and HD-ICE.

The analysis allows to extract the resonance spectrum to test QCD predictions.

Baryon analysis tools can boost the analysis of lattice and meson data (upcoming GlueX experiment at JLab).

Hadronic methods to analyze Lattice QCD data and extrapolate in volume, mass, flavor: Comparison with physical world.

Next level of complexity: Complex hadronic systems (3-body) in the finite volume of a lattice simulation.
Two-body scattering
Scattering in the infinite volume limit

- Unitarity of the scattering matrix $S$: $SS^\dagger = 1$

  $$[S = 1 - i \frac{p}{4\pi E} T].$$

\[\text{Im } T^{-1}(E) = \sigma \equiv \frac{p}{8\pi E}\]

- $\rightarrow$ Generic (Lippman-Schwinger) equation for unitarizing the $T$-matrix:

  $$T = V + V G T \quad \text{Im } G = -\sigma$$

  $V$: (Pseudo)potential, $\sigma$: phase space.

- $G$: Green’s function:

  $$G = \int \frac{d^3 \vec{q}}{(2\pi)^3} \frac{f(|\vec{q}|)}{E^2 - (\omega_1 + \omega_2)^2 + i\epsilon},$$

  $\omega_{1,2} = m_{1,2}^2 + \vec{q}^2$
Discretization
Discretized momenta in the finite volume with periodic boundary conditions

\[ \Psi(x) = \psi(x + \hat{e}_i L) = \exp(i L q_i) \psi(x) \implies q_i = \frac{2\pi}{L} n_i, \quad n_i \in \mathbb{Z}, \quad i = 1, 2, 3 \]

\[ \int \frac{d^3 \vec{q}}{(2\pi)^3} g(|\vec{q}|^2) \to \frac{1}{L^3} \sum_{\vec{n}} g(|\vec{q}|^2), \quad \vec{q} = \frac{2\pi}{L} \vec{n}, \quad \vec{n} \in \mathbb{Z}^3 \]

\[ G \rightarrow \tilde{G} = \frac{1}{L^3} \sum_{\vec{q}} \frac{f(|\vec{q}|)}{E^2 - (\omega_1 + \omega_2)^2} \]

- \( E > m_1 + m_2 \): \( \tilde{G} \) has poles at \textit{free energies in the box}, \( E = \omega_1 + \omega_2 \)
- \( E < m_1 + m_2 \): \( \tilde{G} \rightarrow G \) exponentially with \( L \) (regular summation theorem).
- Formalism can be mapped to Lüscher’s \( Z_{\ell m} \).
Measured eigenvalues of the Hamiltonian (tower of lattice levels $E(L)$) → Poles of scattering equation $\tilde{T}$ in the finite volume → determines $V$:

$$\tilde{T} = (1 - V \tilde{G})^{-1} V \rightarrow V^{-1} - \tilde{G}^{-1} = 0 \rightarrow V^{-1} = \tilde{G}$$

The interaction $V$ determines the $T$-matrix in the infinite volume limit:

$$T = (V^{-1} - G)^{-1} = (\tilde{G} - G)^{-1}$$

Re-derivation of Lüscher’s equation ($T$ determines the phase shift $\delta$):

$$p \cot \delta(p) = -8\pi \sqrt{s} \left( \tilde{G}(E) - \text{Re} G(E) \right)$$

$V$ and dependence on renormalization have disappeared (!)