Spin Transport in Ultracold Fermi Gas

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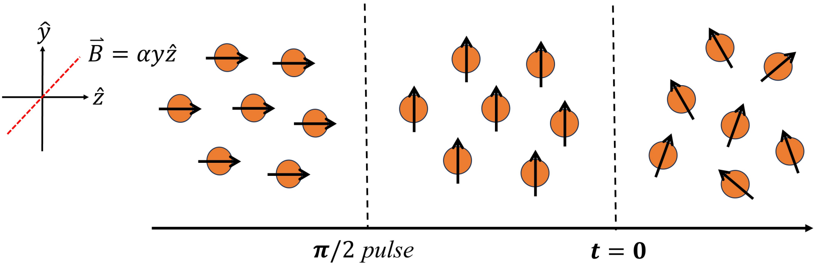
Preliminaries: Setup

Prepare fully polarized ^{6}Li at $T = 0.25\,T_F$ in a box potential, apply $\pi/2-$pulse and magnetic field gradient to release the system.

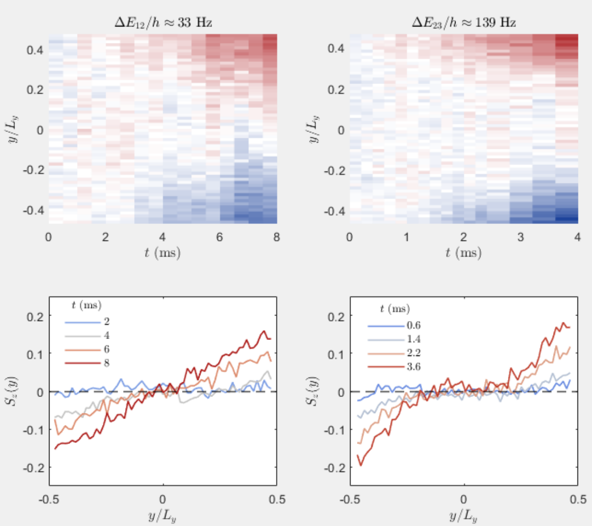


Intuitively we should observe

- Larmor precession at different rates
- Diffusion $\Rightarrow$ Collisional interaction with Pauli exclusion

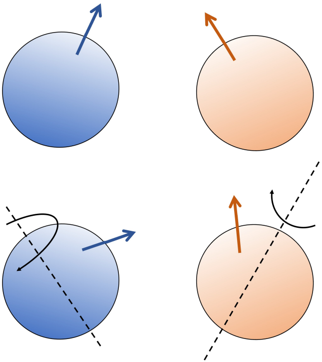
Preliminaries: Observation

Spin segregation occurring at rate much faster than Stern-Gerlach[1]



Preliminaries: Identical Spin Rotation

Semi-classical picture: exchange interaction⇒when particles come close via diffusion, their spin vectors precess around each other



This leads to **spin wave excitation**

Kinetics: Hamiltonian

$H = H_0 + U + H_{int}$, where

$$\begin{aligned} H_0 &= \sum_\sigma \int d^3x\, \psi_\sigma^\dagger \left(-\frac{\hbar^2}{2m} \nabla^2 \right) \psi_\sigma \\ U &= \text{Box potential} \\ H_{int} &= \underbrace{\sum_\sigma \int d^3x\, \psi_\sigma^\dagger \left(\frac{1}{2} \hbar \boldsymbol{\Omega}_0 \cdot \boldsymbol{\sigma} \right) \psi_\sigma}_{\text{Magnetic field gradient}} + \underbrace{g \int d^3x\, \psi_\uparrow^\dagger \psi_\downarrow^\dagger \psi_\downarrow \psi_\uparrow}_{\substack{\text{s-wave scattering} \\ \text{between opposite spin particles}}} \end{aligned} \tag{1}$$

Phase space formalism

- Wigner-transform the density matrix and solve for time evolution[2]
- Project to spin bases
- **kinetics equation** for magnetization phase space distribution $\boldsymbol{M}(\boldsymbol{r}, \boldsymbol{p}, t)$

Kinetics Equation

Define mean field angular velocity and spatial magnetization distributionn respectively as

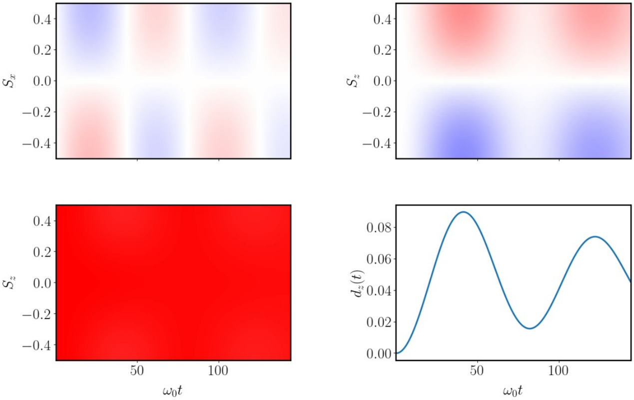
$$\boldsymbol{\Omega}_{\mathrm{MF}} = -\frac{2g\boldsymbol{S}}{\hbar}, \quad \boldsymbol{S} = \frac{1}{2}\int \frac{d^3p}{(2\pi\hbar)^3}\boldsymbol{M} \tag{2}$$

- **Kinetics Equation**[3]

$$\underbrace{\left(\partial_t + \frac{\boldsymbol{p}}{m}\cdot\nabla_r\right)\boldsymbol{M}}_{\text{Material derivative}} - \underbrace{\frac{\hbar}{2}\nabla_r\left[\boldsymbol{\Omega}_0 + \boldsymbol{\Omega}_{\mathrm{MF}}\right]\cdot\nabla_p f_0(p)}_{\text{Stern-Gerlach dipole force}} = \underbrace{(\boldsymbol{\Omega}_0 + \boldsymbol{\Omega}_{\mathrm{MF}})\times\boldsymbol{M}}_{\text{Precession}} \tag{3}$$

- Neglect the Stern-Gerlach term for now
- Box potential enters as boundary condition

Kinetics Equation: Simulation



- S_z segregation observed
- d_z dipole oscillatory
- S_y varies slowly $\Rightarrow$ steady state approximation

Leggett-Rice Equation

To obtain more analytic results, we define spin current $\boldsymbol{J}$

$$\boldsymbol{J} = \frac{1}{2}\int \frac{d^3p}{(2\pi\hbar)^3}\frac{\boldsymbol{p}}{m}\boldsymbol{M} \tag{4}$$

Neglect Stern-Gerlach term. Integrating the kinetics equation on momentum gives the Leggett Rice equation[4]

$$\text{Precession} \quad \partial_t \boldsymbol{S} + \partial_y \boldsymbol{J} = \boldsymbol{\Omega}_0 \times \boldsymbol{S} \tag{5}$$

$$\text{Diffusion} \quad v_0^2 \partial_y \boldsymbol{S} = \underbrace{-\frac{2g\boldsymbol{S}}{\hbar} \times \boldsymbol{J}}_{\text{Spin rotation}} - \frac{\boldsymbol{J}}{\tau} \tag{6}$$

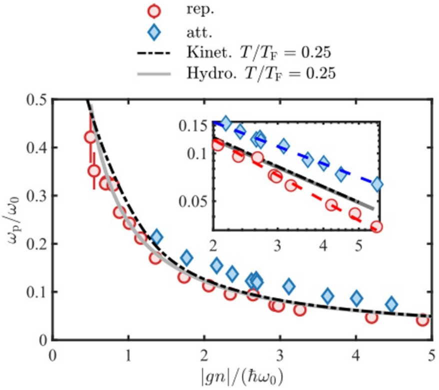
- Dispersion relation of spin wave
- Analytically predict spin segregation

Linear Response: Dispersion Relation

Leggett-Rice equation under weak magnetic field gradient $\Rightarrow$

$$\omega = -\frac{gn}{2\hbar} \pm \sqrt{\left(\frac{gn}{2\hbar}\right)^2 + v_0^2 q^2} \tag{7}$$

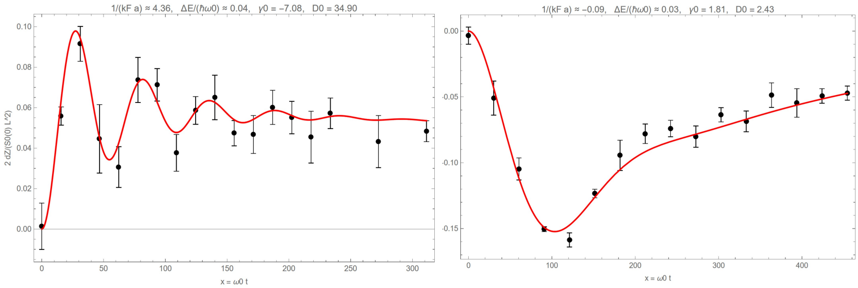
Expect high symmetry between attractive and repulsive branch.



Linear Response: Dipole Amplitude

Performing multiple scale expansion to second order in magnetic field gradient, we obtain a **quasi-steady state solution**:

$$S_z = \sum_n \frac{\alpha S_0 b_n}{D_0 k_n^2} \underbrace{\left(\gamma e^{-Dk_n^2 t}\cos\left(\gamma D k_n^2 t\right) + e^{-Dk_n^2 t}\sin(\gamma D k_n^2 t) - \gamma\right)}_{\text{Fast decay of transients controlled by diffusion, followed with steady state}} \sin\left(\frac{2n+1}{L}\pi y\right) \tag{8}$$

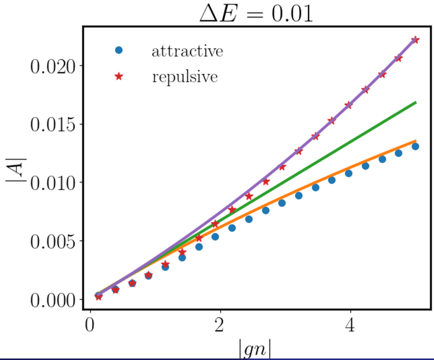


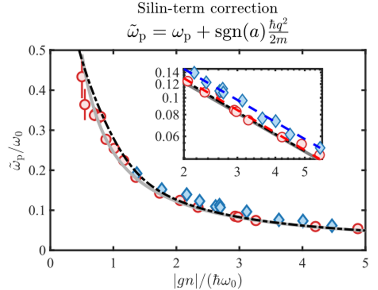
Linear Response: Resolving the Asymmetry

Retaining the dipole force term accounts for asymmetry in dispersion and amplitude, modifying the square of mean velocity

$$\left(v_0^2 - \frac{gn}{2m}\right)\partial_y \boldsymbol{S} = -\frac{2g\boldsymbol{S}}{\hbar} \times \boldsymbol{J} - \frac{\boldsymbol{J}}{\tau} - \frac{n\hbar}{4m}\partial_y \boldsymbol{\Omega}_0 \tag{9}$$

Qualitative: gradient in internal field facilitates diffusion for attractive branch $\Rightarrow$ higher frequency, suppressed dipole strength





Outlook

- Non-linear spin wave
- Beyond mean field correction
- Momentum distribution and more complex dipole structure under Landau damping

References I

[1] X Du et al. "Observation of anomalous spin segregation in a trapped Fermi gas". In: *Physical review letters* 101.15 (2008), p. 150401.

[2] Jean-Noël Fuchs, Dimitri M Gangardt, and Franck Laloë. "Large amplitude spin waves in ultra-cold gases". In: *The European Physical Journal D-Atomic, Molecular, Optical and Plasma Physics* 25.1 (2003), pp. 57–75.

[3] VP Silin. "Oscillations of a Fermi-liquid in a magnetic field". In: *Sov. Phys. JETP* 6 (1958), p. 945.

[4] Linton Reid Corruccini et al. "Spin-wave phenomena in liquid3He systems". In: *Journal of Low Temperature Physics* 8.3 (1972), pp. 229–254.