Nuclear Chiral Symmetry in the Mass A ~ 130 Region

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Abstract

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In triaxial deformed nuclei, the short, intermediate, and long deformation axes may be used to define a coordinate system. In 1997, Frauendorf and Meng [Fr97] proposed that the handedness of the nuclear system, the chirality, can be defined by the orientation of the total angular momentum vector with respect to these axes. With a counterclockwise rotation from the tip of the total angular momentum vector, an axis order of short-intermediate-long defines a right-handed coordinate system. An axis order of long-intermediate-short defines a left-handed system.

In their model, for a nucleus to be chiral, not only must the nucleus be triaxial, but the total angular momentum must be aplanar. It cannot be along one of the deformation axes, nor can it be along a plane defined between the deformation axes. In odd-odd nuclei, the total angular momentum is the vector sum of a single proton angular momentum, a single neutron angular momentum, and a collective rotation. With the appropriate particle numbers, all three angular momentum vectors may be aligned orthogonally, leading to an aplanar total angular momentum vector and thus a chiral system.

In reference [Fr97], Frauendorf and Meng suggest that a symmetry between the right- and left-handed chiral systems should be manifested by an energy degenerate pair of $\Delta I = 1$ rotational bands with the same microscopic configuration. With mixing of the intrinsic chiral solutions the energy degeneracy may be broken in the laboratory frame. In this case, a pair of $\Delta I = 1$ bands with a slight degeneracy breaking may be observed. They also suggested that a pair of rotational bands observed in $^{134}$Pr have the experimental signature expected of chiral band pairs. In that mass region, calculations
predict a triaxial deformation due to the shape polarizing effects of the unpaired valence nucleons in this nucleus. In addition, the unpaired proton and neutron angular momentum vectors are predicted to have orthogonal angular momenta when the nucleus is near the ground state.

If the arguments for chiral behavior in $^{134}$Pr are valid, then neighboring odd-odd nuclei, having similar particle numbers and deformations, should also show chiral behavior. That is, if chiral behavior exists, it should be a regional effect. To test this, experiments were performed to populate high spin states in the neighboring nuclei $^{136}$Pm and $^{138}$Eu. To test the limits of the region, experiments were also performed to populate high spin states in $^{140}$Eu. The results of this work have been published as a Rapid Communication in Physical Review C [He01] as a conference proceeding in Nuclear Physics A [Be01], in the Physical Review Letters study of chiral behavior in the N = 75 isotones [St01a], as a conference proceeding [He02], and most recently as an article in Physical Review C [He03].

These results are put in a more global context by examining the recent work on candidate chiral band pairs in other nuclei in the mass A ~ 130 region. In addition, calculations were performed with models that allow a triaxial deformation and an aplanar total angular momentum, and these calculations agree favorably with experimental results.
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Chapter 1

Introduction

1.1 Chirality

"I call any geometrical figure, or group of points, chiral, and say it has chirality, if its image in a plane mirror, ideally realized, cannot be brought to coincide with itself."

- Lord Kelvin, Baltimore Lectures, 1884.

Chirality, coined from the Greek word for hand, is well known in static systems. The obvious example is seen in a comparison of the left and the right hand. Regardless of translation or rotation, the left and right hand, mirror images of each other, remain distinct. Chirality is also well known in chemistry as distinct enantiomer states. Thalidomide is one such molecule, which in its different states may be either a beneficial pain relieving drug or a teratagen. Another well known molecular example is that of DNA, which actually has a corkscrew shape, the direction of the rotation of the screw following a right hand rotation as one moves along the length of the molecule.

There are examples of chirality in kinetic systems as well. Neutrinos and anti-neutrinos have a spin vector aligned anti-parallel and parallel, respectively, to their direction of motion (in the reference frame in which they were created). These mirrored states are termed “−1 helicity” for neutrinos and “+1 helicity” for anti-neutrinos.

In 1997, Frauendorf and Meng [Fr97] proposed that a type of chirality was possible in some triaxial deformed nuclei. In their model, the handedness is a mix of the static and the kinetic: the chirality of the system is determined by the alignment of the angular momentum vectors with respect to the underlying nuclear deformation. For a
triaxial deformation, the nucleus possesses three distinct axial lengths: a short (1), intermediate (2), and long (3) axis. These orthogonal axes can define a coordinate system, and an aplanar total angular momentum vector may be oriented in a region in which the axes are ordered 1,2,3 with a counterclockwise rotation with respect to the angular momentum, or 3,2,1 with the same rotation. That is to say, the angular momentum may be in a right-handed or left-handed coordinate system, depending on its orientation with respect to the nuclear deformation axes (see figure 1.1).

Experimentally, a symmetry between the right- and left-handed chiral systems should be manifested by an energy degenerate pair of $\Delta I = 1$ rotational bands with the same microscopic configuration. With a mixing of the intrinsic chiral solutions – if there is a breaking of symmetry in the intrinsic frame which is restored in transforming to the laboratory frame – the energy degeneracy may be broken in the laboratory frame. In this case, a pair of $\Delta I = 1$ bands with a slight degeneracy breaking may be observed.

Figure 1.1: Schematic diagram showing a deformed nucleus. The total angular momentum ($I$ in this work but written as $J$ in reference [St01a] from which this diagram comes), a sum of the rotational ($R$) and single particle ($j_s, j_v$) angular momenta, may be in two different chiral orientations.

Frauendorf and Meng state that the conditions are appropriate for chiral behavior in the nucleus $^{134}$Pr, and that a pair of $\Delta I = 1$ bands in that nucleus are chiral bands. For these arguments to be valid, neighboring odd-odd nuclei with similar particle numbers
and deformations should also show chiral behavior. That is, chiral behavior should be a regional effect. To search for chiral band pairs in the neighboring odd-odd isotones, and to test the limits of the region, the high spin structure of $^{136}\text{Pm}$, $^{138}\text{Eu}$, and $^{140}\text{Eu}$ was studied in the current work. Experiments were performed at the Wright Nuclear Structure Laboratory (WNSL) Yale University, to populate high spin states in $^{136}\text{Pm}$ and $^{140}\text{Eu}$, and at Daresbury Laboratory, UK, to populate $^{138}\text{Eu}$. The results on $^{136}\text{Pm}$ and $^{138}\text{Eu}$ were summarized and published as a Rapid Communication in Physical Review C [He01] and as a conference proceeding in Nuclear Physics A [Be01]. The results discussed in these papers were also published in the seminal Physical Review Letters study of chiral behavior in the $N = 75$ isotones [St01a]. The results for the $N = 77$ nucleus $^{140}\text{Eu}$ study have been published as a conference proceeding [He02] and have recently been published as an article in Physical Review C [He03]. These experiments and results are presented in the present work.

By now, many examples of candidate chiral band pairs have been proposed in the mass $A \sim 130$ region. Following the discussion of the experiments of the current work, an inclusive review of the work known to date allowing a systematic analysis of the results and a comparison with predicted trends is presented.

1.2 Layout of the dissertation

This dissertation begins with a discussion of symmetries and their measurable experimental manifestations. This leads to a discussion of spherical, deformed, and rotating systems, which in turn leads to a discussion of angular momentum in triaxial nuclei and of chirality. Background on nuclear de-excitations, detection of these de-excitations, and methods of analysis of these data are discussed. This groundwork being laid, experiments and results in a search for chiral symmetry breaking are discussed.

Chapter 1 serves as a brief introduction to the concept of chirality and presents an overview of the structure of the dissertation. In chapter 2, the theoretical framework leading up to and including the chiral model is presented. An introduction to the
experimental techniques used in the current work follows in chapter 3. Chapters 4 and 5 present the experiments and results for the N=75 nuclei $^{136}$Pm and $^{138}$Eu and the N = 77 nucleus $^{140}$Eu. Chapter 6 places these results in a more global context by presenting details on candidate chiral band pairs in other nuclei in the A ~ 130 region. In chapter 7, alternative explanations for the nearly degenerate rotational bands are explored. The observations and models are summarized and conclusions are presented in chapter 8.
Chapter 2

Models of nuclear systems

2.1 Independent particle spherical shell model

It has long been known that a description can be made of the behavior of the nucleus using the nuclear shell model [Ge65]. The nuclear shell model was motivated by a wealth of empirical evidence such as the trend in the two neutron separation energy ($S_{2n}$) as a function of neutron number, the energy gap to the first excitation in even-even nuclei as a function of nucleon number, and the $\beta$-transformation energy (the energy released in transforming between $N_{\text{even}}Z_{\text{odd}}$ and $N_{\text{odd}}Z_{\text{even}}$ nuclei) as a function of nucleon number. For example, a plot of the two neutron separation energy over the range $N = 94$ to 132, presented in figure 2.1, shows a discontinuity in the strength with which neutrons are held, dropping dramatically just above $N = 126$. This sudden drop in separation

Figure 2.1: Two neutron separation energies for several nuclei as a function of N, from [Hf03].
energy implies a change in structure, and is similar to the discontinuities in electron binding energies that motivated the development of the atomic electron shell model. These discontinuities in the nuclear data, marking the spherical shell closures, occur for 2, 8, 20, 28, 50, 82, and 126 protons or neutrons, and these numbers are referred to as the "magic" numbers.

In its simplest form, the nuclear shell model assumes a central mean field potential \( V(r) \), with a potential minimum at \( r = 0 \), and non-interacting nucleons. Further refinements take into account residual interactions between the nucleons. For simplicity, the potential may be modeled in terms of the simple harmonic oscillator (SHO) model, such that

\[
(2.1) \quad V_{\text{SHO}}(r) = \frac{1}{2} m \omega^2 r^2,
\]

where \( \omega \) is the oscillator frequency. With the kinetic energy term,

\[
(2.2) \quad T = \frac{p^2}{2m},
\]

the simple Hamiltonian can be written

\[
(2.3) \quad H = T + V(r) = \frac{p^2}{2m} + \frac{1}{2} m \omega^2 r^2.
\]

The eigenenergies for the three dimensional SHO are simply

\[
(2.4) \quad \varepsilon_n = \hbar \omega (N + 1/2) = \hbar \omega (2n + l + 3/2)
\]

with \( n = 1, 2, 3, \ldots \), \( l = 0 \hbar, 1\hbar, 2\hbar, \ldots \) and \( N = 0, 1, 2, \ldots \). The \( n \) and \( l \) terms arise from working in a three dimensional spherical coordinate system, and represent the principal quantum number and the orbital angular momentum, respectively. \( N \) represents the oscillator shell. The orbital angular momentum values are often referred to by letters, with the values \( l = 0, 1, 2, 3, 4, 5 \ldots \) being denoted by \( s, p, d, f, g, h \ldots \)

In this simple approximation, all levels of the same \( N \) are energy degenerate. Since the relation between \( N, n, \) and \( l \) (eqn. 2.4) implies that the energy of levels with the same integer value of \( 2n + l \) are degenerate, and since \( n \) is an integer, the values of \( l \) in a single oscillator shell are either all odd or all even. This simple model reproduces the first few
major shell closures, see figure 2.2 (left). Not surprisingly, differences at higher nucleon numbers require modifications to the model.

Figure 2.2: Simple harmonic oscillator levels (left) with the addition of an angular momentum dependent $l^2$ attractive term to make the potential well more square (center) and with the further addition of the spin orbit term (right), from [Ca00]. The $j$-shells (on the right) are labeled by the quantum numbers $n,l,$ and $j$. 
The short range of the nuclear force suggests that modeling the potential more as a square well than as a harmonic oscillator may be appropriate. This is accomplished by adding an attractive term which goes as \( l^2 \) to the potential. The orbitals with higher orbital angular momenta lie farther out from the nuclear center and an attractive force acting preferentially on these orbitals has the desired effect of squaring off the potential well [Ca00]. When this term is included in the Hamiltonian, the calculated levels are as shown in figure 2.2 (center). Since the potential now depends on \( l \), the degeneracy is broken for levels which originate in the same oscillator shell, \( N \), but which have different orbital angular momenta.

In addition to an orbital angular momentum term, \( l \), nucleons have an intrinsic spin, \( s \), with values \( s = \pm 1/2 \). By including a spin-orbit coupling, which goes as \( l \cdot s \), the model reproduces the higher lying shell closures very well, as can be seen in figure 2.2 (right). Since the state with total angular momentum \( j = l + s = l + 1/2 \) is lowered in energy, the spin-orbit term is taken to be attractive. The spin-orbit potential is frequently taken to have the form [La80]

\[
V_{s-o}(r) = -\text{const} \cdot \frac{1}{r} \frac{dV_{\text{SHO}}(r)}{dr} (l \cdot s).
\]

(2.5)

For large \( j \), this coupling is strong enough to move the level by several MeV. The resulting energy levels are shown in the third column of figure 2.2 (right). The levels are labeled by the quantum numbers \( n, l, \) and \( j \). The angular momentum vector \( j \) can have different orientations, and the magnetic substate, \( m \), describes the projection of \( j \) on a symmetry axis. For a spherical potential, there is no preferred orientation for the projection axis and, by symmetry, all \( 2j + 1 \) values of the \( m \) substates are degenerate. These \( 2j + 1 \) levels with the same \( nlj \) numbers are considered part of the same minor shell, or \( j \) shell. The levels between the "magic" number energy gaps are considered part of the same major shell.
2.2 Residual interactions and quasiparticles

The independent particle spherical shell model assumes that the nucleons move in a mean field central potential without interaction with one another. There is a wealth of experimental evidence of residual interactions in addition to the mean field, such as the attractive pairing interaction between nucleons. One example of the strong pairing interaction is the ground state spin of even-even nuclei having zero spin. It is also seen in the staggering of the separation energies of neutrons as a function of neutron number, with even N nuclei having higher neutron separation energies than their odd N neighbors. The same holds true for protons when comparing even and odd Z nuclei.

Because of the pairing interaction, nucleons pair with anti-parallel angular momentum vectors, so the total angular momentum of the pair is the vector sum $j_1 + j_2 = I = 0$, where $I$ is the total angular momentum. The pairing interaction may be written as [Ca00]

$$\langle j_1, j_2 | \mathcal{V}_{p pairing} | j_3, j_4 \rangle = -G \sqrt{(j_1 + 1/2)(j_3 + 1/2)} \delta_{j_1 j_2} \delta_{j_3 j_4} \delta_{j_0 j_0}$$

(2.6)

where $G$ is the interaction strength, $G \sim 20$ MeV/A [Ca00].

The pairing interaction acts not only on the lowest $0^+$ level, but acts equally between $I = 0^+$ states that are composed of different anti-aligned $j$ values, $|j_a^1 I = 0^+\rangle$ and $|j_b^2 I = 0^+\rangle$. These diagonal matrix elements mix $0^+$ states and create partially occupied levels. Without this mixing, the level is either occupied by particles or it is not, and the level occupancy stops abruptly at the Fermi energy, denoted here as $\varepsilon_F$ (see figure 2.3). With the mixing, the levels near the Fermi surface can be occupied by fractions of particles, which are termed quasiparticles. This has the effect of spreading out the occupancy around the Fermi energy, as described schematically in figure 2.3 (right). This spreading is characterized by the gap parameter $\Delta$, where $\varepsilon_F + \Delta$ represents the energy at which the levels have a 10% occupancy and $\varepsilon_F - \Delta$ represents the energy for 90% occupancy. The pairing gap parameter can be defined in terms of a sum over the orbits i,j, as
(2.7) \[ \Delta = G \sum_{i,j} U_i V_j. \]

\( U_i^2 \) and \( V_i^2 \) describe the probability of occupancy and vacancy, respectively, of state \( i \), and are limited by the condition \( U_i^2 + V_i^2 = 1 \). \( \Delta \) is typically on the order of 700 to 1000 keV [Ca00].

**Without Pairing**

![Without Pairing Diagram]

Levels 0 1 occupancy

\( \varepsilon_F \)

**With Pairing**

![With Pairing Diagram]

Levels 0 1 occupancy

\( \varepsilon_F+\Delta \)

\( \varepsilon_F \)

\( \varepsilon_F-\Delta \)

Figure 2.3: Level occupancy with no mixing due to the pairing interaction (left) and with mixing (right). The lines on the left of each occupancy diagram represent an idealized set of levels which can have double occupancy. The Fermi surface is at the Fermi energy, \( \varepsilon_F \). Note the sharp change in occupancy at the Fermi surface without the pairing interaction (left) and the smoother change with the pairing interaction (right). The pairing gap parameter, \( \Delta \), describes the spread of the quasiparticle occupancy (right).

### 2.3 Deformation

As mentioned above, for a spherical system with a central potential, there is no preferred axial orientation, and a level having angular momentum \( j \) is \( 2j + 1 \) fold degenerate. While the two body Hamiltonian, the interaction between individual nucleons, is isotropic, for nuclei away from closed shells, the residual interactions between nucleons often lead to deformed ground states, resulting in a non-isotropic potential. The breaking of spherical symmetry means there is a definable axis for the angular momentum projection, \( m \), and breaks the \( 2j + 1 \) degeneracy of the magnetic
substates. However, the orbitals with the same $nlj$ values described by $m$ and $-m$ (which represent time reversed states of one another) are still degenerate.

### 2.3.1 Deformation parameters $\beta$ and $\gamma$

The shape of the nucleus may be easily described by modeling the nucleus as an incompressible nuclear fluid with a sharp surface, a liquid drop [Ri80]. One way to parameterize the nuclear surface is with a radial length, $R$, that is a function of angle around the nucleus [Bo75],

\[
R(\theta, \phi) = cR_0[1 + \sum_{l=2,\text{max}} \sum_{m=-l}^l \alpha_{lm} Y_{lm}(\theta, \phi)],
\]

where $c$ is determined by the volume conservation condition, $R_0 = r_0A^{1/3}$, and $l$ is the order of the spherical harmonic. $l_{\text{max}}$ is the multipole order of the nuclear deformation (for a quadrupole deformation, $l_{\text{max}} = 2$). The $l = 0$ term is absorbed into the normalization, as it describes a spherical shape centered on the origin, and the $l = 1$ term is omitted as it is equivalent to displacement of the center of the sphere. The spherical harmonics are described in more detail in appendix A. Since the $l = 0$ and 1 terms are not explicitly used, for a quadrupole deformation only the $l = 2$ terms are required and the equation becomes

\[
R(\theta, \phi) = cR_0[1 + \sum \alpha_{2m} Y_{2m}(\theta, \phi)].
\]

As the volume is conserved, to describe the three axial lengths requires two independent variables, typically chosen as $\alpha_{20}$ and $\alpha_{22} = \alpha_{2.2}$.

The quadrupole deformed shape is more usually described using the parameters $\beta_2$ and $\gamma$, which relate to the $\alpha$ parameters via

\[
\alpha_{20} = \beta_2 \cos \gamma, \quad \text{and}
\]

\[
\alpha_{2+2} = \alpha_{2.2} = \frac{1}{\sqrt{2}} \beta_2 \sin \gamma.
\]

The deformation in terms of $\alpha$ and $\beta$ can be expressed as [He94]

\[
R(\theta, \phi) = cR_0\{1 + \beta_2 \sqrt{\frac{5}{16\pi}} [\cos \gamma (3 \cos^2 \theta - 1) + \sqrt{3} \sin \gamma \sin^2 \theta \cos^2 \phi]\}.
\]
Roughly speaking, the parameter $\beta$ measures the total deformation since, in terms of $\alpha_{2m}$,

\begin{equation}
\beta_2^2 = \sum_m |\alpha_{2m}|^2,
\end{equation}

while the parameter $\gamma$ describes the triaxiality of the system. For an axially symmetric deformation, $\gamma = 0$ and equation 2.19 reduces to

\begin{equation}
R(\theta, \phi) = cR_0\left[1 + \sum_{l=2, l_{\text{max}}} \beta_l Y_{l0}(\theta, \phi)\right] = cR_0\left[1 + \beta_2 Y_{20}(\theta, \phi)\right].
\end{equation}

The higher orders of $\beta$ define octupole, etc., deformations. As only $\beta_2$ is used in describing pure quadrupole deformations, for shorthand it is referred to as simply $\beta$.

In terms of the $\beta$ and $\gamma$ parameters, the axial lengths of the nucleus may be expressed as

\begin{equation}
R_n = cR_0\left[1 + \frac{5}{4\pi} \beta \cos \left(\gamma - \frac{2\pi}{3} n\right)\right], \quad n = 1, 2, 3.
\end{equation}

A plot of radial length $R_n$ (normalized to $R_0$) as a function of $\gamma$ is presented in figure 2.4. The deformation $\beta = 0.2$ was used, a typical value for the moderately deformed chiral candidate nuclei in the $A \sim 130$ region. As $\gamma$ changes, the deformations are repeated each 120 degrees, but with an interchange of axes.

An indication of nuclear deformation can be found in the intrinsic quadrupole moment, $Q_0$, which, assuming a rigid moment of inertia, is related to the nuclear deformation parameter $\beta$ by [Ca00].

\begin{equation}
Q_0 = \frac{3}{\sqrt{5\pi} Z} R^2 \beta(1 + 0.16\beta)
\end{equation}

$Q_0$ is a valuable indication of deformation as it is an experimentally measurable quantity. More details on this are given in appendix A.
Figure 2.4: Radial length (normalized by $R_n/R_0$) as a function of $\gamma$ (in degrees), following the definitions of equation 2.15 and using $\beta = 0.2$, a typical value for many of the candidate chiral nuclei. The solid line is for the $n=1$ axis, short dash $n=2$, long dash $n=3$.

Figure 2.5: Deformation and nuclear rotation following the Lund convention. This figure relates to figure 2.4 through the choice of $n = 2$ as the rotational axis.
In this thesis, the Lund convention, as illustrated for several values of $\gamma$ in figure 2.5, is used to specify both the nuclear shape and the rotation axis. For two axes of the same length (e.g. $\beta \neq 0$ and $\gamma = 0^\circ$), the remaining axis is the symmetry axis and the nucleus has an axially symmetric shape. When the symmetry axis is the longest axis (e.g. $\beta > 0, \gamma = 0^\circ$), the deformation is called prolate. When it is shortest (e.g. $\beta > 0, \gamma = 60^\circ$ or $\beta < 0, \gamma = 0^\circ$), which is not as commonly observed in nuclei, the deformation is oblate. For intermediate values of $\gamma$, the deformation changes smoothly and the nucleus has a triaxial shape.

2.3.2 The deformed shell model

To describe the orbits in a deformed nuclear potential, Nilsson in 1955 [Ni55] first proposed the modified harmonic oscillator model, using a deformed nuclear potential. The Hamiltonian for this model is written as

$$\tag{2.17} H_{\text{Nilsson}} = -\frac{\hbar^2}{2m} \nabla^2 + \frac{m}{2} (\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2) - 2\kappa \hbar \omega_0 \mathbf{l} \cdot \mathbf{s} - \mu (l^2 - \langle l^2 \rangle_N).$$

The terms in the square brackets represent the spin orbit force and an orbital term which serves to flatten the potential well, similar to those described in section 2.1. The factors $\kappa$ and $\mu$ determine the strength of these terms and $\omega_0$ is the general oscillator frequency, with $\omega_i$ representing the oscillation frequency along the $i$ axis. For an axially symmetric deformation, the oscillator frequencies can be expressed as a function of deformation as

$$\omega_x^2 = \omega_y^2 = \omega_0^2 (1 + \frac{2}{3} \varepsilon) \text{ and } \omega_z^2 = \omega_0^2 (1 + \frac{4}{3} \varepsilon),$$

where $\varepsilon = \beta$ (exactly, $\beta = 4 (\pi/5)^{1/2} [((\varepsilon_2/3) + (\varepsilon_2/3)^2 + ...] = \varepsilon_2$ [Fi96]).

The energy variation of the orbitals as a function of deformation is summarized graphically in the Nilsson diagram, examples of which are shown in figures 2.6 and 2.7 for proton and neutron orbitals, respectively. The levels in Nilsson diagrams are typically labeled by their asymptotic quantum numbers $\Omega^\pi [N n_z \Lambda]$. Here, $n_z$ is the number of nodes of the wave function along the symmetry axis and $\Lambda$ is the projection of the orbital angular momentum, $l$, along the symmetry axis. $N$ is the principal quantum number (here
Figure 2.6: Nilsson diagram for protons, $50 \leq Z \leq 82$. ($\varepsilon_2 \sim \beta$, $\varepsilon_4 = \varepsilon_2/6$), from [Si02]. Negative parity states are represented by dashed lines, positive with solid lines. Note the $1h_{11/2}$ negative parity orbitals.
Figure 2.7: Nilsson diagram for neutrons, $50 \leq N \leq 82$ ($\varepsilon_2 - \beta$, $\varepsilon_4 = \varepsilon_2/6$), from [Si02]. Negative parity states are represented by dashed lines, positive with solid lines. Note the $1h_{11/2}$ negative parity orbitals.
capitalized) and $\Omega$ is the projection of the total angular momentum along the symmetry axis $(z)$ (see figure 2.8). $\pi$ specifies the parity (the behavior under reflection) of the orbital, and is given by $\pi = (-1)^l$.

![Diagram of orbitals around a prolate deformed nucleus](image)

Figure 2.8: Orbitals around a prolate deformed nucleus, with their respective $\Omega$ projections. The different orbitals represent different $m$ states from the same $j$-shell.

As can be seen in figures 2.6 and 2.7, for a prolate nuclear deformation ($\epsilon_2 > 0$), the low $\Omega$ orbitals from a single $j$-shell have a lower energy than the high $\Omega$ orbitals. This situation is reversed for oblate nuclei ($\epsilon_2 < 0$). A schematic diagram illustrating the orbitals and their angular momentum projections for prolate shapes is shown in figure 2.8. Due to the difference in energies, as a particular $j$-shell is filled around a prolate nucleus, the low $\Omega$ orbitals are occupied first.

Following the Pauli exclusion principle, in the ground state, the nucleons fill the levels from the bottom upward to the Fermi surface. Due to the difference in orbital energies for a prolate nucleus, the low $\Omega$ orbitals of a particular high-$j$ shell are occupied first. The orbital configuration near the ground state of a deformed nucleus can be determined by the position of the Fermi surface on the Nilsson diagram. The most easily excited states are those that lie near the Fermi surface. In this way, using the Nilsson model, the orbital configuration of the quasiparticles most likely to take part in an excitation may be modeled. The nuclei relevant to our study have $Z \sim 60$, $N \sim 75$, and a
deformation of $\beta \sim 0.2$. The Fermi surface of the protons lies near the low $\Omega$ projections of the proton $h_{11/2}$ orbital, as shown in figure 2.6. Similarly, the neutron Fermi surface lies near the high $\Omega$ projections of the neutron $h_{11/2}$ orbital, as can be seen in figure 2.7.

Levels of the same $\pi$ and $\Omega$ projection may mix and repel one another. The deformed wave functions for a level $i$, $\psi_{i,\text{Nilsson}}$, may be described in terms of the intrinsic orbital wave functions of a spherical nucleus, $\phi_j$, as

$$\psi_{i,\text{Nilsson}} = \sum_j C^i_j \phi_j,$$

where $C^i_j$ are the configuration mixing coefficients. The Nilsson labels refer to the pure states of the spherical nucleus from which the Nilsson levels may be traced. The mixture and repulsion may be seen in the curving of the levels on the Nilsson diagrams. As mentioned, the parity of the levels can be found by $\pi = (-1)^j$. By equation 2.4, the parity of a level may also be found by $\pi = (-1)^N$, where $N$ is the oscillator shell number. Thus, the parity is the same for all the orbitals in a major shell, with the exception of the high-$j$ level brought down from a higher shell by the spin-orbit interaction. Such a level is called an intruder state or a unique parity orbit. The wave functions of these unique parity orbits are very pure, as they are far away from other orbitals of the same parity. This can be seen by the straight line of the negative parity $h_{11/2}$ orbitals in the Nilsson diagrams, which are presented in figure 2.6 and 2.7 as dashed lines to distinguish them from the positive parity orbitals in the region. The chiral band structures found in the mass $A \sim 130$ region are based on the double intruder $\pi h_{11/2} \otimes \nu h_{11/2}$ configuration (here $\pi$ and $\nu$ refer to the proton and neutron configurations, respectively). This allows an easy identification of the orbital structure, and makes modeling the intrinsic structure more straightforward.

2.4 Rotation

2.4.1 Energy of a rotating nucleus

Classically, the energy of a rotating system is given by

$$E(I) = \frac{I^2}{2J},$$

(2.20)
where $S$ is the moment of inertia and $I$ is the angular momentum. In the quantum mechanical picture

$$E(I) = \frac{\hbar^2}{2S} [I(I+1)],$$

with a quantized angular momentum, $I$. In a generally accepted shorthand in nuclear physics, $I$ is sometimes referred to simply as spin when the context does not allow a confusion with the intrinsic spin, $s$. For collective rotations, this equation results in regularly spaced energy levels as a function of spin. Assuming a constant moment of inertia, the transition energy ($\gamma$-ray energy) between levels which differ by, for example, 2 units of angular momentum ($\Delta I = 2$) is

$$E(I \rightarrow I - 2) = \frac{\hbar^2}{2S} [I(I + 1) - (I - 2)(I - 1)] = \frac{\hbar^2}{2S} [4I - 2].$$

Thus, the energy spacing between adjacent transitions in the rotational band may be very regular:

$$E(Ia \rightarrow Ia - 2) - E(Ib \rightarrow Ib - 2) = \frac{\hbar^2}{2S} 4[Ia - Ib]$$

![Superdeformed Band](image)

Figure 2.9: Spectrum from a superdeformed rotational band (on right) in $^{137}\text{Ce}$ [Sa95]. Note the regular picket fence pattern.
Spectra from super-deformed nuclei (which have a major to minor elliptical deformation axis ratio of about 2:1 and a large, fairly constant moment of inertia) beautifully show the predicted regular, picket fence pattern, an example of which is shown in figure 2.9.

However, in a normal deformed rotating nucleus, the model of a rigid rotor is not always appropriate. For example, for an even-even well deformed nucleus in the $A \sim 170$ region, the energy of the $1^{\text{st}}$ excited state is $E(2^+) - E(0^+) \approx 90$ keV, giving an experimental value of $\frac{\hbar^2}{2\mathcal{I}} \approx 15$ keV [Kr88]. For a rigid rotor,

\begin{equation}
\mathcal{I}_{\text{rigid}} = \frac{2}{5}MR_{av}^2(1 + 0.31\beta),
\end{equation}

which in the mass $A \sim 170$ region, gives $\frac{\hbar^2}{2\mathcal{I}_{\text{rigid}}} = 6$ keV [Kr88]. Taking the other extreme, a fluid inside a rotating ellipsoidal vessel, the moment of inertia is given by

\begin{equation}
\mathcal{I}_{\text{fluid}} = \frac{9}{8\pi}MR_{av}^2\beta,
\end{equation}

or $\frac{\hbar^2}{2\mathcal{I}_{\text{fluid}}} = 90$ keV [Kr88]. Since the orbital frequency of the nucleons is $\omega \sim 10^{24}/\text{sec}$, and collective rotation is far slower, on the order of $\omega \sim 10^{20}/\text{sec}$, this model of irrotational flow of the nucleons is not unreasonable. Note that, from the experimental values of the moment of inertia, it appears that the true moment of inertia for many nuclei lies between the rigid and fluid extremes. While this is only a difference in magnitude in the moment of inertia of an axially symmetric rotor, for a triaxial rotor, this irrotational flow may also determine the axis of rotation.

The moment of inertia is also dependent on spin, due to effects such as centrifugal stretching, changes in the pairing strength, Coriolis mixing, and a change of orbital configuration as observed in band crossing (which will be discussed shortly). What Casten [Ca00] calls a "local" moment of inertia may be defined at each spin, via equation 2.21. To describe the changing nature of the moment of inertia, a kinetic moment of inertia, defined as

\begin{equation}
\mathcal{I}^{(1)} = I/\omega,
\end{equation}
and a dynamic moment of inertia, defined as

\[ \mathcal{J}^{(2)} = dI/d\omega, \]

are used.

2.4.2 Single particles in rotating nuclei

For deformed nuclei, the total angular momentum vector, \( \mathbf{I} \), is a combination of the collective rotation angular momentum, \( \mathbf{R} \), plus the contributions from any unpaired nucleons, \( \mathbf{j} \). The projection of the single particle angular momentum, \( \mathbf{j} \), on the symmetry axis is denoted by \( \Omega \), while the projection of the total angular momentum is denoted by \( \mathbf{K} \). For collective rotation about an axis perpendicular to the symmetry axis, \( \Omega \) and \( \mathbf{K} \) are used interchangeably [Ca00]. The level energies for a rotational band built upon an orbital with projection \( \mathbf{K} \) are given by

\[ E(I) = \frac{\hbar^2}{2I} [I(I + 1) - K(K + 1)]. \]

When dealing with several unpaired nucleons it is important to remember that their angular momentum vectors need not have the same orientation. Indeed, effects based on the coupling of angular momenta with different orientations are important for our study and will be discussed.

2.5 Principal axis cranking (PAC) – the cranked shell model

A common method of modeling rotations of deformed, axially symmetric nuclei is using the Principal Axis Cranking (PAC) model, also referred to as the cranked shell model. The nucleus in the laboratory frame can be modeled through “cranking up” the body fixed Hamiltonian \( h_0 (\beta) \) by the addition of a rotational term, with rotational frequency \( \omega \) about the rotational axis, \( x \), perpendicular to the symmetry axis, \( z \). The principal axis cranking (PAC) Hamiltonian [Be79] is

\[ h_{\text{rot}} = h_0 (\beta) - \omega \cdot \mathbf{j} = h_0 (\beta) - \omega j_z. \]

With the \( \omega \cdot \mathbf{j} \) term in the rotational Hamiltonian, distinguishing between \(+m\) and \(-m\) states, the final level degeneracy is removed.
2.5.1 Calculations based on PAC

A common calculation based on the PAC model is the total Routhian surface (TRS) calculation. The energy of the particles in an orbital configuration near the Fermi surface is calculated as a function of the deformation parameters $\beta_2$, $\beta_4$, and $\gamma$. Input parameters allow a control over the particle numbers $N$ and $Z$, the pairing strength, and the number of unpaired particles involved and their parities and signatures (which will be discussed). The TRS code calculates the potential energy of the nucleus as a function of deformation, and presents not only the calculated potential minimum, but an equipotential contour plot in polar coordinates, with the radial axis being $\beta_2$ and the angle being $\gamma$. The lowest deformation potential is taken to be the predicted deformation. The

![Figure 2.10: Total Routhian surface (TRS) calculation for $^{132}$Ce, at $\hbar\omega = 0.12$ MeV. The minimum Routhian energy is -1.32 MeV at a deformation of $\beta_2 = 0.201$, $\gamma = -14.4^\circ$, and $\beta_4 = -0.014$. Contours are about 175 keV apart.](image-url)
potential changes as a function of cranking frequency, and so the plots are frequency specific. An example of the TRS contour plot output is presented in figure 2.10 for the ground state band in $^{132}$Ce at a frequency of $\hbar \omega = 0.12$ MeV. The potential energy minimum at this frequency, with the ground state orbital configuration, is $\beta_2 = 0.201$, $\gamma = -14.4^\circ$, and $\beta_4 = -0.014$. The very gradual change in the calculated potential as a function of $\gamma$ at $\beta_2 = 0.2$ predicts a $\gamma$ softness for the nucleus. All TRS calculations presented in this dissertation were performed by the author using code supplied from the University of Liverpool.

Another very useful code based on PAC calculates the quasiparticle Routhian energies as a function of rotational frequency. Quasiparticle Routhian plots model the Routhians for the partially filled levels near the Fermi surface, over a range of frequencies. An example of this type of plot is shown in figure 2.11 for $^{132}$Ce. The input

![Figure 2.11: Quasiparticle Routhian calculation for quasi-protons in $^{132}$Ce using the deformation parameters $\beta_2 = 0.201$, $\gamma = -14.4^\circ$, and $\beta_4 = -0.014$. The parity and signature $(\pi,\alpha)$ of the orbitals is given in the key above the plot.](image-url)

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parameters to the code include the proton and neutron number, and the deformation parameters of the rotating nucleus. The deformation parameters used in the example calculation are $\beta_2 = 0.201$, $\gamma = -14.4^\circ$, and $\beta_4 = -0.014$, taken from the above TRS calculation, figure 2.10.

Several nuclear properties are reproduced in figure 2.11. At zero frequency, the large gap and the clustering of levels represents the pairing gap, $\Delta$, and the effect of the pairing interaction. The levels at $\hbar\omega = 0$ are at approximately the Nilsson predicted level energies with respect to the Fermi energy, with the addition of the pairing gap. The quasiparticle angular momentum in the intrinsic frame (experimentally estimated by subtracting a reference spin from the spin in the lab frame (see eqn. 2.36)), is called the alignment, $i_x$, and can be found from the slope of the Routhian,

$$i_x = -\frac{d\epsilon}{d\omega}.$$  

With the rotation of the nucleus, time reversal symmetry is broken, and the 2-fold level degeneracy of levels in the static deformed nucleus is broken, as can be seen in figure 2.11 in moving away from $\hbar\omega = 0$. The lower level of each pair is called the favored signature and the higher level of each pair is called the unfavored signature. The favored signature can usually be determined from

$$\alpha_{\text{favored}} = \frac{1}{2} \left[ (-1)^{j_x^{-1/2}} + (-1)^{j_x^{-1/2}} \right],$$

where $j_x$ and $j_y$ are the unpaired proton and neutron angular momenta, respectively. For singly odd nuclei, the appropriate part of equation 2.31 is used. Thus, for an even-even or a doubly odd nucleus, the signature is 0 or 1, and for a singly odd nucleus it is $+1/2$ or $-1/2$. The splitting between the favored and unfavored signatures, which can be found by $\Delta\epsilon' = \epsilon_{\text{unfavored}}' - \epsilon_{\text{favored}}'$, is called the signature splitting.

The signature of a rotational band can be found from the spins of that band through the relation $I = \alpha + 2n$, where $n$ is an integer. In nuclei in which the total angular momentum is not near a principal axis, signature does not remain a good quantum
number. This last point will be demonstrated clearly in figure 2.16 and is discussed
below.

In the PAC regime (with the angular momentum along a principal axis) the
orbitals are labeled by their parity and signature \((\pi, \alpha)\). As a shorthand, the different
parity and signature combinations are sometimes denoted by letters: \(A\) and \(C = (+, +1/2),\)
\(B\) and \(D = (+, -1/2),\) \(E\) and \(G = (-, -1/2),\) \(F\) and \(H = (-, +1/2).\) Of each pair, the left letter
denotes the lowest lying orbital or quasiparticle, the right letter the next higher. For
protons the letters are capitalized, for neutrons they are written in lower case. Several of
these are marked in figure 2.11.

In the quasiparticle Routhian diagram, the energy of the levels is shown to
decrease as a function of frequency. At a certain rotational frequency, it may be
energetically favorable to acquire angular momentum from breaking a pair of high-\(j\)
particles, called an alignment. With the alignment of the angular momentum vectors, at
least one of the pair of particles must be raised to a higher level, due to the Pauli
exclusion principal. At the frequency at which it becomes energetically favorable for
alignment, the Routhian of the aligned particles reach a minimum. This is shown in
figure 2.11 for the lowest negative parity levels, \(E\) and \(F,\) at \(\hbar \omega \sim 0.35\) MeV. This
predicted alignment frequency will be compared with experimental results shortly. The \(E\)
and \(F\) levels correspond to the \(\alpha = -1/2\) and \(+1/2\) signature partners, respectively, of the
proton \(h_{11/2}\) orbital.

In singly or doubly odd nuclei, the level above the Fermi surface \((\epsilon' = 0)\) may
already be occupied. Because of the Pauli principle, that first level is considered
"blocked." For example, in the case for the \(h_{11/2}\) quasiparticles in figure 2.11, the
alignment at \(\hbar \omega \sim 0.35\) MeV would be blocked. The alignment wouldn't become
energetically favorable until a higher frequency, corresponding to \(\hbar \omega \sim 0.48\) MeV in
figure 2.11. For negative parity bands, if the first alignment (EF) is blocked, the F and G
particles participate in the alignment. When speaking of positive parity bands, the first
crossing is the AB crossing, the second is the BC crossing.
As with the TRS calculations, all of the quasiparticle Routhian calculations in this dissertation were performed by the author using code provided by the University of Liverpool.

2.5.2 Comparison of the PAC model with experiment

A plot of the experimental spin as a function of rotational frequency for $^{132}\text{Ce}$ is presented in figure 2.12. While the excitation levels have discrete spins and energies, the rotational frequency can be estimated from the classical relation

$$\omega = \frac{dE}{dI}$$

which for $\Delta I = 2$ may be expressed $\omega \approx \frac{E_r}{2}$. As can be seen, there is a discontinuity in the plot, which corresponds with the change in orbital configuration with an alignment of a pair of $h_{11/2}$ protons. The change in the orbital configuration causes a sudden change in the moment of inertia, leading to the discontinuity of the behavior of the rotational band, termed a backbending.

![Graph](image)

Figure 2.12: Backbend in $^{132}\text{Ce}$ caused by the alignment of a pair of $h_{11/2}$ protons. Below the backbend, the angular momentum is from the core rotation. At the backbend, about 10 units of angular momentum are from the aligned particles.

The experimental Routhian may be found by subtracting a reference energy based on the cranked shell model from the experimental energy [Be79],

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\[ (2.33) \quad e'(\omega) = E'(\omega) - E'_{\text{ref}}(\omega), \]

where \( E'(\omega) = E - I_x \omega \) and the reference energy \( E'_{\text{ref}} \) is calculated as

\[ (2.34) \quad E'_{\text{ref}}(\omega) = -\frac{1}{2} \omega^2 \mathcal{S}_0 - \frac{1}{4} \omega^2 \mathcal{S}_1 + \frac{\hbar^2}{8 \mathcal{S}_0}. \]

The aligned angular momentum, \( i_x \), is defined as

\[ (2.35) \quad i_x = I_x(\omega) - I_{\text{ref}}(\omega), \]

where the calculated reference spin is

\[ (2.36) \quad I_{\text{ref}}(\omega) = \frac{1}{\hbar} \left( \mathcal{S}_0 \omega + \mathcal{S}_1 \omega^3 \right). \]

The parameters \( \mathcal{S}_0 \) and \( \mathcal{S}_1 \) are called the Harris parameters, and are chosen to subtract out the core rotation. A proper choice of these parameters gives a constant slope in the Routhian plot. A good choice of Harris parameters makes the alignment horizontal. That is, the alignment stays the same as long as the configuration remains constant. The experimental Routhian energies and alignments are presented for the yrast band in \(^{132}\)Ce in figure 2.13.

Certain features of the alignment and Routhian plots should be noted. The frequency of the pair breaking can be determined by the frequency at which the bands plotted in the Routhian diagram cross (at \( \hbar \omega \sim 0.35 \text{ MeV} \)), also called the band crossing. The alignment plot, very similar to figure 2.12 but with a modeled core rotation subtracted, shows the same backbending as seen in figure 2.12. The two horizontal lines in the alignment plot correspond to the aligned angular momentum of the nucleus in the two distinct configurations discussed: \( i_x = 0 \) when all the particles are paired, \( i_x \sim 10 \) when a pair of \( h_{1/2} \) protons align (\( i_x \text{ max} = 11/2 + 9/2 \approx 10 \)), and there are no other single particle contributions. The gain in alignment, \( \Delta i_x = 10 \), is determined by the distance between these two lines.

In the example of \(^{132}\)Ce, the band crossing observed is due to the alignment of a pair of \( h_{1/2} \) protons. The frequency of the band crossing is \( \hbar \omega \approx 0.35 \). This is where the EF crossing was predicted in the quasiparticle Routhian calculation. This lends credence
to this band crossing being identified as the EF crossing. The good match also implies a good choice of deformation parameters.

\[ 132^{\text{Ce}} \]

![Graph](image)

**Figure 2.13:** Routhian energies (top) and aligned angular momentum (bottom) for $^{132}\text{Ce}$. The horizontal scale is in MeV. Note the change in the figures as a pair of $h_{11/2}$ protons aligns. The band crossing is at $\hbar\omega \approx 0.35 \text{ MeV}$.

### 2.6 Tilted axis cranking (TAC)

A generalization of the cranked shell model allows rotation around an axis other than a principal axis. With a change in the orientation of $\omega$, equation 2.29 is rewritten:

\[
 h_{\text{rot}} = h_{\varepsilon}(\beta) - \omega \cdot j = h_{\varepsilon}(\beta) - \omega(j_x \sin \phi + j_z \cos \phi),
\]

with $\omega$ making an angle $\phi$ with the symmetry axis $z$ [Fr96]. This model is called the tilted axis cranking model (TAC). For a total angular momentum vector that has considerable contributions from single particle angular momentum vector(s), the collective rotational
vector does not dominate. Thus the total angular momentum may lie off of a principal axis. One regime in which TAC is important is in explaining shears bands.

Figure 2.14: Regularly spaced energy levels, evident in the spectrum of $^{199}$Pb, which is predicted to be nearly spherical, from [Cl98].

The shears model was motivated by the observation of regularly spaced "rotational-like" bands in nuclei that are nearly spherical, see figure 2.14 for example. Recall that regular spacing is a hallmark of collective rotation, whereas spherical nuclei typically have complicated excitation spectra arising from single particle excitations.

Figure 2.15: Coupling of the 2 proton angular momentum with the single neutron hole state, in the $\pi (h_{9/2}i_{13/2})_1 \otimes v_{i_{13/2}}$ configuration of $^{197}$Pb. Note the very slight oblate deformation of the nucleus. At low excitation energy (left) the proton angular momentum vector is oriented along the short deformation axis and the neutron angular momentum vector is oriented along the long deformation axis. At higher energy, as the angular momentum vectors align (right), the total angular momentum increases.
However, spectra with regularly spaced peaks were found in many nearly spherical nuclei. In $^{197}$Pb [Co01], for example, the nucleus is slightly oblate and the proton Fermi surface is low in the high-$j$ $h_{9/2}$ and $i_{13/2}$ orbitals. In contrast, the neutron Fermi surface is high in the $i_{13/2}$ orbital, which could also be thought of as a high-$j$ hole. In excited configurations based on unpaired $\pi$ ($h_{9/2} i_{13/2}$)$_{11}$ particles and $\nu$ $i_{13/2}$ holes, the repulsive particle/hole interaction can lead to an orthogonal coupling of the angular momenta, as represented on the left in figure 2.15.

As the nucleus is not very deformed, less energy is required for the nucleus to gain angular momentum through an alignment of these particle/hole vectors than through a collective rotation or through the breaking of additional particle/hole pairs. The coupling of these vectors to a more oriented state, resulting in a larger total angular momentum which is not in the direction of a principal axis, is represented on the right in figure 2.15. Note that in this case, the total angular momentum of the protons is greater than the angular momentum of the neutron hole. To model this, in the TAC model the total angular momentum is not restricted to a principal axis. Though shears bands were a motivation for the TAC model, TAC is not limited to such nearly spherical nuclei. It has also been successful in modeling the coupling of single particle and rotational angular momenta in deformed systems [Fr01]. This coupling of angular momentum vectors with distinct orientations is an important step which led to predictions of chiral structure in nuclei.

Figure 2.16 shows how the level energy evolves as the symmetry of the system is decreased. The spherical harmonic oscillator level is shown on the left hand side. Moving to the right, the degeneracy of the different $\Omega$ projections is broken as the nucleus is deformed. In the next panel, as the nucleus is rotated about a principal axis, the 2-fold signature degeneracy of levels in the deformed nucleus is broken. In the following panel, the axis of cranking is changed, with $\phi$ varying from 0 $^\circ$ to 90 $^\circ$. At $\phi$ = 0 $^\circ$, the cranking is PAC, and the signatures are based on rotation about the 3 axis. At $\phi$ = 90 $^\circ$, the cranking is once again PAC, and the signatures are based on rotation about the 1
axis. In between, in the tilted axis cranking regime, signature is not a good quantum number.

![Diagram of orbital energies](image)

Figure 2.16: Model of orbital energies as they transition from spherical to prolate deformed, from static prolate deformed to principal axis rotation, from principal axis rotation to tilted axis rotation to principal axis rotation about a different axis, and finally from this new principal axis rotation to static deformed, in which levels once again are the same energy as the original static deformed levels, adapted from [CI00].

2.7 Aplanar tilted axis cranking (3D-TAC) and the particle + rotor model

In 1997, Frauendorf and Meng [Fr97] presented an even more general TAC model, an aplanar TAC, also called 3D-TAC. In this model, the total angular momentum vector may be not only off of a principal axis, but also away from the principal planes. In an axially symmetric nucleus, a plane may always be defined by the angular momentum vector and the symmetry axis. To specify an aplanar angular momentum requires a breaking of axial symmetry. This is found in triaxial nuclei, with the three deformation
axes having three distinct lengths. If the total angular momentum is near any of these axes, the system can be described by principal axis cranking, and along any of the planes by tilted axis cranking. In a triaxial nucleus with an appropriate coupling of the axial angular momentum vectors, as will be described, the total nuclear rotation may be aplanar. Naming the principal axes as the short (1), intermediate (2), and long (3) axis (also labeled s,i,and l) and with the angle from the axis of cranking to the long axis (3) described by $\theta$, and to the short axis (1) by $\phi$, the aplanar cranking vector becomes:

\begin{equation}
\omega = (\omega \sin \theta \sin \phi, \omega \sin \theta \cos \phi, \omega \cos \theta).
\end{equation}

A diagram of the total angular momentum vector showing these angles from the deformation axes is presented in figure 2.17. The Hamiltonian is expressed as

\begin{equation}
h_{\text{rot}} = h_0(\beta) - \omega \cdot j = h_0(\beta) - \omega(j_x \sin \theta \sin \phi + j_y \sin \theta \cos \phi + j_z \cos \theta).
\end{equation}

3-D TAC calculations are performed for several nuclei at different frequencies using this Hamiltonian. At each frequency, the total energy is minimized by allowing the orientation of the total angular momentum vector to vary over $\theta$ and $\phi$.

The system formed by the short (s), intermediate (i), and long (l) axes in a triaxial nucleus possess a chirality, when looking from the tip of the total angular momentum vector [Fr97]. Counting the axes counterclockwise from this vector, the octant of the nucleus in which the order is s,i,l (right side of figure 2.18) is right-handed (i.e. the axes form a right-handed coordinate system). If the order is l,i,s (left side of figure 2.18) the octant is left-handed. The aplanar TAC model predicts distinguishable systems with degenerate energies for these two systems [Fr01]. With another symmetry defined, that of a handedness (chirality), another degeneracy is revealed. The chiral model predicts identical rotational bands for each chirality. If the symmetry is broken, the energy degeneracy of the bands may be broken, resulting in two experimentally discernible rotational bands. 3D-TAC does not distinguish between the chiral band pairs, but can describe the average energies and average transition strengths as a function of frequency. 3D-TAC calculations are presented for $^{136}$Pm and $^{138}$Eu following the experimental findings, in chapters 4 and 6, and are presented in a systematic comparison of Routhian energies for $^{132}$Cs, $^{134}$Pr, $^{136}$Pm, and $^{138}$Eu in chapter 7. All TAC and 3D-TAC
calculations in this dissertation were performed by Jing-ye Zhang of the University of Tennessee, Knoxville.

Figure 2.17: Schematic of the Euler angles in a triaxial nucleus from [Fr97]. The x, y, and z axes are chosen so the total angular momentum, \( \mathbf{I} \) (written as \( \mathbf{J} \) in reference [Fr97] from which this diagram comes), is along the z axis. Before acting on the system with the Euler rotations, the x, y, and z axes corresponded to the s, i, and l deformation axes of the nucleus.

Figure 2.18: Total angular momentum vector in a left-handed (left) and right-handed (right) coordinate system.
Another model which allows an aplanar total angular momentum is the particle + rotor model, which is described in reference [Bo75]. This model is based on an even-even core rotating about a principal axis. With the addition of unpaired particles, the orthogonal single particle angular momentum vectors can couple, leading to the aplanar total angular momentum. In addition, like the TAC models, a triaxial deformed rotor may be used (see reference [MV75] for example). Where the TAC calculations give the orbital energies as a function of cranking frequency, the particle + rotor model gives the energies in the laboratory frame, with spin as a good quantum number. That is, rather than through Routhians, the calculated level energies may be compared directly with experimental level energies. Also unlike the aplanar TAC model, the particle + rotor model can distinguish between the chiral band pairs, and the energy splitting between the chiral solutions in the lab frame can be calculated. These calculations follow the experimental findings in chapters 5, 6, and 7. As with the TAC and 3D-TAC calculations, unless otherwise noted, all particle + rotor calculations in this dissertation were performed by Jing-ye Zhang of the University of Tennessee, Knoxville.

2.8 Summary and comparison of the models

The three angular regimes and predictions of the PAC, TAC, and aplanar TAC, are compared in this section. These comparisons are made using transformations based on the Euler angles $\phi$, $\theta$, and $\psi$, which describe the angle of the total angular momentum vector $\mathbf{I}$ with respect to the deformation axes. The $x,y,$ and $z$ axes are chosen so the total angular momentum, $\mathbf{I}$, is along the $z$ axis. Before acting on the system with the Euler rotations, the $x,y,$ and $z$ axes correspond to the $s,i,$ and $l$ deformation axes of the nucleus. The result of rotation by the Euler angles is pictured in figure 2.17, showing the relation between the $x,y,z$ (total angular momentum aligned) and $1,2,3$ ($s,i,l$ - body fixed) coordinate systems.

For the PAC solution, the angles to which the angular momentum vector is confined are

$$\theta = 0, \frac{\pi}{2} \quad \phi = 0, \frac{\pi}{2}$$
and signature, $\alpha$, is a good quantum number. As the total spin values are constrained to $I = \alpha + 2n$, the PAC solution allows one $\Delta I = 2$ rotational band for each signature $\alpha$.

For the standard TAC solution, the angles are

$\theta \neq 0, \pi/2$, $\phi = 0, \pi/2$

or

$\theta = 0, \pi/2$, $\phi \neq 0, \pi/2$.

Signature symmetry is lost as $\theta$ or $\phi$ moves away from 0 or $\pi/2$. The solution for a distinct set of good quantum numbers represents one $\Delta I = 1$ band.

For aplanar TAC, the angles are

$\theta \neq 0, \pi/2$, $\phi \neq 0, \pi/2$.

As with standard TAC, signature symmetry is lost and all spins are possible, giving a $\Delta I = 1$ band. In the aplanar TAC model, the angular momentum can be the same angle from the three axes, but in a different octant of the triaxial nucleus, by a transformation from $|\psi, \theta, \phi\rangle$ to $|\psi, \theta, \pi/2 - \phi\rangle$. There are two degenerate solutions: the aplanar TAC model predicts two degenerate $\Delta I = 1$ bands.

2.9 Chirality
2.9.1 Mechanisms for aplanar angular momentum

The doubling of levels (as predicted in the aplanar TAC solution) was suggested by Frauendorf and Meng in 1997 [Fr97] who called the phenomenon nuclear chiral symmetry. Their original idea is that for a certain class of nuclei, namely doubly-odd, moderately deformed triaxial nuclei, the total angular momentum $I$ (the vector sum of the collective nuclear rotation, $R$, the proton angular momentum, $j_p$, and the neutron angular momentum, $j_n$) may be aplanar and, in this case, a doubling of levels should be observed. For a triaxial deformed nucleus with a suitable choice of particle numbers, the proton Fermi surface can lie low in a high-$j$ orbital (low $K$) and the neutron Fermi surface can lie high in a high-$j$ orbital (high $K$). That is to say, $j_n$ and $j_p$ lie near the short and long
deformation axes, respectively. $\boldsymbol{R}$ may lie near the intermediate axis due to irrotational flow in the nucleus. Recall that for a rigid rotor, the largest moment of inertia for a rotation is along the short axis. However, the nucleus is not a rigid rotor. For a system modeled with the motion of an irrotational and incompressible liquid with axial lengths $R_{i,j,k}$, the moments of inertia, $\mathcal{I}_{i,j,k}$, are described as [Bo69]

$$\mathcal{I}_i = \left( \frac{R_j^2 - R_k^2}{R_j^2 + R_k^2} \right)^2 \mathcal{I}_{\text{rigid}}^i,$$

with cyclic permutations, and the rigid moments of inertia are

$$(\mathcal{I}_{\text{rigid}}^i) = \rho_0 \int (x_j^2 + x_k^2) \, d\tau = \frac{1}{5} (R_j^2 + R_k^2).$$

Thus, the nuclear moment of inertia is greatest along the intermediate axis. In this way, the proton, neutron, and collective rotation angular momentum vectors may be orthogonal, and may couple to form an aplanar total angular momentum vector.

Experimental manifestations of this idea are found in several rotational bands in odd-odd nuclei with $Z \sim 60$ and $N \sim 75$. As will be shown for several such nuclei, TRS calculations predict a triaxial deformation. In addition, the Nilsson model predicts the proton and neutron Fermi surfaces to lie low and high, respectively, in the high-$j$ $h_{11/2}$ orbital. The rotational bands built on this $\pi h_{11/2} \otimes \nu h_{11/2}$ configuration show evidence of doubling. This doubling is observed with a slight energy degeneracy breaking, as will be discussed shortly.

### 2.9.2 The chirality operator

The orientation of the total angular momentum vector with respect to the short, intermediate, and long axes defines the handedness, or chirality, of the system. As mentioned, Frauendorf suggests that we may call the coordinate system right-handed if these axes are ordered counterclockwise, and left-handed if they are ordered clockwise, with respect to an aplanar total angular momentum vector [Fr01], see figure 2.18. It is not possible to change the chirality by operating on the system with only a rotation, $\boldsymbol{R}$, nor is the chirality interchanged by a mirror reflection of the angular momentum vectors.
The direction of angular momentum vectors parallel to the mirror are reversed, but angular momentum vectors orthogonal to the mirror are not changed. The net effect of a mirror reflection on a coordinate system defined by angular momentum vectors is equivalent to a rotation. Instead of a rotation or mirror reflection, an interchange of chirality can be achieved through a combination of a rotation and the time reversal operator, $T$, which anticommutes with the total angular momentum. The chirality operator is $O = T R_y (\pi)$, where $R_y (\pi)$ denotes the rotation by 180° about the y axis in the laboratory reference frame [St01b]. The wave functions of the right- and left-handed systems can be denoted by $| R \rangle = O | L \rangle$ and $| L \rangle = O | R \rangle$.

![Diagram](image)

Figure 2.19: In this diagram, with $j_{\pi}, R$, and $j_{\nu}$ along the short, intermediate, and long nuclear axes, respectively, the total angular momentum vector, written as $J$ in this diagram from reference [St01b], is in a right-handed (on left) or a left-handed (on right) coordinate system. This diagram was presented with a more general explanation in figure 1.1.

When moving from the intrinsic frame to the laboratory frame, the chiral symmetry may be broken. This symmetry may be restored by finding a set of basis states in the lab frame $|\text{IM}\rangle$ (describing the total angular momentum and the magnetic substate) which are also eigenvectors of $T R_y (\pi)$ [Bo69]. The basis may be chosen from linear combinations of the chiral states such that

$$(2.42a) \quad O |\text{IM}+\rangle = |\text{IM}+\rangle,$$

$$(2.42b) \quad O |\text{IM}−\rangle = |\text{IM}−\rangle.$$
These new basis states, denoted $|\uparrow\rangle$ and $|\downarrow\rangle$, are indeed linear combinations of the intrinsic chiral states, such that

$$(2.43a) \quad |\uparrow\rangle = \frac{1}{\sqrt{2}} (|R\rangle + |L\rangle), \quad |\uparrow\rangle = O|\uparrow\rangle,$$

and

$$(2.43b) \quad |\downarrow\rangle = \frac{i}{\sqrt{2}} (|R\rangle - |L\rangle), \quad |\downarrow\rangle = O|\downarrow\rangle.$$

The Hamiltonian in this basis [St01c] has off diagonal elements, and is given by

$$(2.44) \quad H = \begin{pmatrix} \epsilon + \delta_1 & \delta_2 \\ \delta_2 & \epsilon - \delta_1 \end{pmatrix}$$

where $\epsilon = \langle R | H | L \rangle = \langle L | H | R \rangle$, $\delta_1 = \text{Re} \langle R | H | L \rangle$, $\delta_2 = \text{Im} \langle R | H | L \rangle$.

The eigenstates have energies given by

$E_1 = \epsilon - \Delta$ and $E_2 = \epsilon + \Delta$, where $\Delta = \sqrt{\delta_1^2 + \delta_2^2} = \sqrt{\langle L | H | R \rangle \langle R | H | L \rangle^*}$.

for the case in which $\langle R | H | L \rangle = 0$, there is a doubling of levels at the same energies. It has been suggested that the chiralities mix by $\gamma$ soft vibrations, which allows the rotational angular momentum vector, $R$, to oscillate between the intrinsic solutions [St01a]. With the mixing, $\langle R | H | L \rangle \neq 0$, there is a level separation of $2\Delta$. In other words, with the mixing of the intrinsic chiralities, there is a breaking of the energy degeneracy in the lab frame. The two $\Delta I = 1$ rotational bands would lose their energy degeneracy and may be observed as distinct $\Delta I = 1$ bands.

In the paper on chiral structure in the N = 75 isotones, Starosta et al. [St01a] suggest a mechanism for mixing of the chiral states. This involves a novel collective vibration, called chiral vibration. If the triaxial deformed core for these nuclei is not stable at $\gamma = 30^\circ$, but is perhaps more $\gamma$ soft, the moment of inertia along the $i$ axis is closer to those of the $s$ and $l$ axes. The collective rotation vector $R$ is not constrained to the two orientations along the $i$ axis (defining a right- or left-handed chirality) but can oscillate between the two chiral systems, figure 2.19, over the saddle point barrier across the $s$-$l$ plane. This oscillation represents a quantum mechanical vibration, which produces the energy displacement seen between the bands, but retains the chiral structure.
2.10 Experimental work

The nuclei near $Z \sim 60$ and $N \sim 75$ are known to have prolate deformation, and
known to be soft to gamma deformation [Ch83, Ca85]. Due to the shape polarizing
effects of unpaired high-$j$ particles, singly and doubly odd nuclei may show a more stable
$\gamma$ deformation and triaxiality [Fr83, St00]. For $Z \sim 60$ and $N \sim 75$, and a reasonable $\beta$
deformation of $\beta \sim 0.2$, the Nilsson picture predicts that the proton Fermi surface is low
in the high-$j$ $h_{11/2}$ orbital (high K) while the neutron Fermi surface is high in the high-$j$
h$_{11/2}$ orbital (low K), providing a mechanism for orthogonal proton and neutron angular
momentum vectors. If in addition the collective rotation angular momentum vector is
oriented near the intermediate nuclear axis, the angular momentum vectors may be
oriented as presented in figure 2.19, with an aplanar total angular momentum.

Frauendorf and Meng [Fr97] suggested that a pair of $\Delta I = 1$ bands in $^{134}$Pr [Pe96a]
(shown in figure 2.20), which are both assigned the $\pi h_{11/2} \otimes \nu h_{11/2}$ orbital configuration,
are the predicted candidate chiral partner bands. As shown in figure 2.21, the energy
degeneracy breaking between the bands reduces as a function of spin. The bands actually
become degenerate at $I \sim 15$ and at higher spins the energy ordering of the bands is
reversed.

There is no apriori reason for the nucleus $^{134}$Pr to be an isolated example of chiral
structure. Triaxial deformation is predicted for a range of particle numbers in the $A \sim$
130 region, and high-$j$ protons with the Fermi surface low in the $h_{11/2}$ shell and high-$j$
neutrons with the Fermi surface high in the $h_{11/2}$ shell are also predicted for a range of
particle numbers. To test if this is indeed a regional effect, we were motivated to search
for chiral partner bands in the neighboring odd-odd isotone, $^{136}$Pm. When we began our
research, the band pair in $^{134}$Pr was the only known example of a candidate chiral band
pair. Success in the search in $^{136}$Pm led us to the next heavier odd-odd isotone, $^{138}$Eu.
Our research was concurrent with research on lower Z nuclei, as described below, and a
region of chiral structure was found. Finally, to test the limits of this region of chiral
behavior, we investigated the $N = 77$ nucleus, $^{140}$Eu.
Figure 2.20: The $\pi h_{1/2} \otimes v h_{1/2}$ bands in $^{134}$Pr, figure adapted from [Pe96a]. The yrast band is on the left, the candidate chiral partner band is on the right. They are connected by several $\Delta I = 1$ and $\Delta I = 2$ interband transitions. The spins presented are also from [Pe96a]. In the current work, the spins used are by 1h following systematics.
As of this writing, candidate chiral bands have been found in several other odd-odd nuclei in the mass $A \sim 130$ region. These nuclei have particle numbers ranging from $N = 71$ to 77 and from $Z = 55$ to 63, i.e. nuclei centered around $^{134}$Pr. These odd-odd nuclei are $^{126-132}$Cs [Li02, Ko03], $^{130-134}$La[Ko01, St02, Ba01], $^{132-134}$Pr [Ko01, Fr97]. The nuclei studied in this work, $^{136}$Pm [He01], $^{138}$Eu [He01], and $^{140}$Eu [He03], also display candidate chiral band pairs. The results of various experimental studies on nuclei in this region are summarized in figure 2.22.

It must be noted that chiral candidate bands are not limited to the $\pi h_{11/2} \otimes vh_{11/2}$ configuration, but the conditions for these bands in the mass $A \sim 130$ region are the same conditions required for candidate chiral bands in other regions. In $^{118}$I, the candidate band has the high-$j$ $\pi g_{9/2} \otimes vh_{11/2}$ configuration [St01b] and in $^{188}$Ir the high-$j$ $\pi h_{11/2} \otimes vh_{13/2}$ configuration [Bb01]. In an example recently published [Zh03], candidate chiral bands have also been found in a 3 particle configuration in the singly odd nucleus $^{135}$Nd.
Figure 2.22: Odd-odd nuclei in the A ~ 130 region in which candidate chiral band pairs have been identified. The band configurations are $\pi h_{1/2} \otimes v h_{1/2}$. The lead institutions for the specific works are listed.

The focus of this thesis is experimental work on chiral behavior in the odd-odd, A ~ 130 nuclei $^{136}$Pm, $^{138}$Eu, and $^{140}$Eu, and on analysis of these nuclei in light of the A ~ 130 global trends for the odd-odd candidate chiral band pairs. In these nuclei, the candidate chiral band pairs are limited to the $\pi h_{1/2} \otimes v h_{1/2}$ configuration, with the possible inclusion of the high-$j$ $\pi (g_{7/2}, d_{5/2}) \otimes v h_{1/2}$ bands in $^{140}$Eu.
Chapter 3

Experimental methods

High spin states in $^{136}$Pm, $^{138}$Eu, and $^{140}$Eu were populated following heavy-ion fusion-evaporation reactions. The deexcitation $\gamma$-rays were observed using either the YRAST Ball detector array at Yale or the Eurogam Phase 1 detector array at Daresbury. The angular dependence of both the emitted and Compton scattered $\gamma$-rays provides information on the multipolarities and electromagnetic properties of the transitions. This chapter is dedicated to the experimental methods used to populate these high spin states, to detect the emitted $\gamma$-rays, and to analyze the character of the transitions.

3.1 Fusion-evaporation reactions

It is possible to fuse nuclei using a beam of particles incident on a target if the kinetic energy of the beam is high enough to overcome the Coulomb repulsion between the nuclei. The system will equilibrate to form a compound nucleus if the energy and angular momentum are low enough for the short range strong nuclear force to hold the system together. The compound nucleus will deexcite by first evaporating particles, typically neutrons or protons, and then, when the excitation energy has fallen below the particle binding energy, by emitting a cascade of photons. The process is illustrated schematically in figure 3.1.

The energy removed by the particles is on the order of 10 MeV per particle emitted, far greater than the typical $\gamma$-ray energy observed, which is on the order of several hundred keV. On the other hand, the angular momentum lost in evaporating a particle is only on the order of 1 or 2 $h$ per particle. Thus, in a plot of excitation energy as a function of angular momentum (figure 3.1) the particle evaporation process is fairly
vertical. When the excitation energy of the nucleus is below the particle emission threshold, photon emission dominates. The level with the lowest excitation energy for a given spin is called the yrast level, and a group of such levels is called the yrast line or yrast band. The particle emission threshold lies $< 10$ MeV above the yrast line. The first photons emitted are unresolved "statistical" photons, from the energy region in which there is an extremely high density of states. As the nucleus deexcites towards the yrast line, the density of states decreases considerably and $\gamma$-rays linking discrete states become visible. Several measurable properties of the $\gamma$-rays and methods of measurement are explored in the following sections.
3.2 $\gamma$-ray interactions with matter

Energetic photons interact with matter primarily in three ways: photoelectric absorption, Compton scattering, and pair production (see figure 3.2). These are exploited to allow detection and measurement of the radiation.

In the photoelectric effect, the photon is absorbed and the energy of the photon is transferred to the kinetic energy of the absorbing electron,

$$E = h\omega - \Phi,$$

where $\Phi$ is the work function to free an electron, typically on the order of 1 eV for the detector materials used. For a sense of scale, $\gamma$-ray energies range from about 100 keV to 1 MeV. The probability of a photoelectric interaction increases with the density of electrons, roughly going as [Kn00]

$$\tau = \text{constant} \times \frac{Z^n}{E^{3.5}},$$

the exponent $n$ varying between 4 and 5 over the relevant energy range. The photoelectric effect dominates at low photon energy, $E \leq 100$ keV, as shown in figure 3.3, dropping roughly as $E^{-3.5}$.

In Compton scattering, the photon may scatter off an electron, transferring some of the energy of the incident photon, $E_\gamma$, to the electron. As the photon energies involved are much greater than the typical binding energies of the electrons, the bound electron is modeled as a free electron. By conservation of energy and momentum, the energy of the scattered photon, $E'_\gamma$, is [Kn00]

$$E'_\gamma = \frac{E_\gamma}{1 + \alpha(1 - \cos \theta)},$$

where $\theta$ is the angle of scattering of the photon, $\alpha = E_\gamma/m_e c^2$, and $m_e$ is the mass of the electron. The loss of energy of the photon, and the gain in energy of the electron, is

$$\Delta E_\gamma = E_\gamma - E'_\gamma = \frac{E_\gamma \alpha(1 - \cos \theta)}{1 + \alpha(1 - \cos \theta)},$$

45
and ranges from 0 for θ = 0° to roughly E_γ at θ = 180° for E_γ large. As shown in figure 3.3, this type of scattering dominates over much of the relevant γ-ray energy range.

For pair production, the photon creates a particle-antiparticle pair. The minimum energy required for this is 2mc^2, where 2m is the mass of the particle-antiparticle pair. The lowest mass pair, the electron and positron, are created at energies at or above 511 keV × 2 = 1022 keV, with any additional energy going into the kinetic energies of the electron – positron pair. Pair production is not significant over most of the relevant energy range.

![Diagram of γ-ray interactions with matter](image)

*Figure 3.2: Schematic of γ-ray interactions with matter, from [Kn00].*

These three processes, and combinations of them, are displayed schematically in figure 3.2. The probability of a photon interacting in any of these processes is described by the fractional loss of the number of photons, I, in traversing a material of thickness dx,

\[
\frac{dI}{I} = -\mu \, dx, \text{ or } I = I_0 e^{-\mu x}. 
\]

The total attenuation coefficient, \( \mu \), is a sum of the individual attenuation coefficients for photoelectric absorption, Compton scattering, and pair production, which are all functions of the photon energy and the scattering material. The attenuation coefficients are shown in figure 3.3 for germanium, the material used in the high resolution detectors to be discussed. From about 100 keV through several MeV, Compton scattering strongly dominates.
Figure 3.3: Attenuation coefficients for the photoelectric effect (PE) (dominating at low energy), Compton scattering (CS) (at moderate energy), and pair production (PP) (at higher energy) in germanium. Coefficient data are from [NI03].

3.3 Detectors

Two types of detectors were used in the experiments in this thesis, high resolution germanium (Ge) detectors and lower resolution bismuth germanate (BGO) scintillators. Both detector types are discussed below.

3.3.1 Semiconductor detectors

The energy of ionizing radiation can be detected by measuring the number of ions created when the radiation interacts with matter. The precision is greatly enhanced with
the use of semiconductor materials, in which the energy required for ionization is modest. The average energy required to free an electron in germanium (at 80K) is 3 eV [Or03]. (The band gap is 0.7 eV, but energy is lost to phonon excitations of the lattice.) As the energy to free an electron is so small, at room temperature a flood of thermal electrons can easily be excited to the conduction band. For normal operation, germanium detectors are cooled to near liquid nitrogen temperatures.

Semiconductor detectors, such as Ge or SiLi, are operated as reverse biased p-n diodes. The reverse bias creates a depletion region, reducing the number of free charge carriers in the undisturbed crystal. As the depletion region is the active region in the detector, the bias is set high enough so the entire crystal is depleted, with an operational gradient of typically a few kilovolts over a few centimeters. The depth of the depletion region goes as

\[
d \approx \left( \frac{2eV}{eN} \right)^\frac{1}{2}
\]

where \( N \) is the impurity or dopant concentration and \( V \) the bias [Kn00]. To achieve a depletion region as large as a few centimeters without dielectric breakdown, high purity germanium detectors (HPGe), with impurities on the order of \( 10^9/cm^3 \), are typically used.

High energy photons incident on the detector crystal may free several electrons through Compton scattering, imparting enough kinetic energy to these electrons that a cascade of secondary electrons are freed. For a 1 MeV incident photon, for example, the photon may Compton scatter several times in the detector before finally being absorbed via the photoelectric effect. The primary electrons may have several hundred keV of energy, and can each free several hundred thousand secondary electrons. The immense number of charge carriers freed from an incident \( \gamma \)-ray gives semiconductor detectors very high energy resolution.

If, on the other hand, the photon scatters out of the detector, the full photopeak energy is not deposited in the detector. To improve efficiency, especially for the more penetrating higher energy \( \gamma \)-rays, detectors may be made larger and/or with denser
material. Several types of detectors make use of this geometrical dependence, both to improve and decrease efficiency at higher energies. Coaxial, clover, and LEPS detectors, all Ge detectors with different geometries, are discussed below. A discussion of higher density scintillator detectors follows in the next section.

Coaxial detectors

Germanium detectors commonly have a coaxial geometry. For this geometry, the detector crystal has a central anode, with a cathode surrounding the crystal. This type of detector is not a true coaxial shape, having a front crystal area that lacks the central anode and a slightly tapered bulletized face (see figure 3.4), which allows a slightly more uniform electric field than a flat face. The bias is applied via n- and p-type donor regions on the outer skin and the inner core of the coaxial detector. The n-type donor contacts on the detector are thicker than the p-type contacts. Hence, n-type detectors (having a slight impurity of electron donors) are constructed with the thinner p-type contact on the face of the detector, to not attenuate the incoming radiation. The electric field is fairly uniform in the coaxial section of the detector, but it is less so near the front where, for the n-type detectors commonly in use, the field lines run from the central anode to the irregular geometry of the bulletized detector front. This is good for the highly penetrating high energy photons (from several hundred keV to several MeV) but is not ideal for low energy photons. For low energy photons, most of the energy is deposited in the first few centimeters of the detector in this irregular field, and both energy and time resolution suffer. In addition, the metal vacuum housing around the detector crystal has a noticeable attenuation for low energy photons.

Ge detector efficiency is typically described relative to a standard 3” diameter × 3” long NaI crystal (with 25 cm from the source to the face of the crystal), which is defined as having a 100% relative efficiency. Using this metric, the coaxial detectors used in the current work have a relative efficiency of 20 - 25% [Be00].
Figure 3.4: Cross section schematic of a coaxial detector showing the bulletized front. The detector is positioned so the photons are incident from the left of the diagram. The example is an n-type detector, having the thinner p-type contact on the outer surface.

Clover Detectors

Four coaxial detectors are combined in a single vacuum housing in a clover detector. Thus, the clover detector provides a larger total detector volume, and a higher photopeak efficiency, by placing several crystals side by side, as seen in figure 3.5. This is very useful for several reasons.

First, since the signals from the individual crystals may be read out independently, the direction of photon scattering between the clover leaves can be determined. The total energy of the photon can still be determined by summing the energies deposited in the individual clover leaves, in a process that is called add-back. The direction of Compton scattering between pairs of the crystals in a clover detector can assist in determining the electromagnetic character of the transition, discussed later in this chapter. In this way the clover detector can act as a Compton polarimeter.

Second, the leaves may act together as a single large detector. The large size means a high efficiency. For a sense of the numbers involved [Du99], if a photon is incident on a leaf of a clover detector, there is a $\sim$25% chance of that leaf recording the total photopeak energy. The add-back increases the efficiency to $\sim$1.5 times the efficiency of four clover leaves acting independently. Compared to the standardized 3”×3” NaI crystal efficiency, clover detectors operating with add-back have a 150%
relative efficiency, about 6 times that of a coaxial detector. For a size comparison, each
clover crystal is about 2" diameter \times 4" long.

Third, Doppler broadening effects on the energy resolution increase with an
increase in the solid angle of the detector, as will be discussed. By effectively making
one large detector by combing several smaller detectors, the Doppler broadening is
reduced.

![Clover crystals](image)

Figure 3.5: Clover crystals in their relative operational positions [Du99]. The size of
each leaf is approximately 50 mm wide by 80 mm long, and leaves are separated by 2
mm. Crystals are set back from the detector face by 10 mm (1.5 mm of which is the
aluminum end cap).

**LEPS Detectors**

The poor coaxial detector efficiency at low energies motivated the development
of Ge detectors sensitive to this energy range. The Low Energy Photon Spectrometer
(LEPS) detectors function very much like the standard coaxial detectors, with modifications to the geometry to increase the low energy sensitivity. The LEPS crystal is a disc, and is considerably thinner than the standard coaxial detectors. The small geometry decreases the sensitivity of this detector to the more penetrating, high energy \(\gamma\)-rays, reducing background. In addition, the effects of the irregular electric field lines found in the front of the coaxial detectors are avoided in the LEPS detectors since the cathode and anode are coplanar, normal to the incoming photons. The thicker metal canister of the coaxial and clover detectors attenuates low energy photons. This is addressed in LEPS detectors by using a thin beryllium window on the front detector face.

3.3.2 Scintillator detectors

In scintillator detectors, the energy from the incoming \(\gamma\)-rays is absorbed in the detector material which, as it deexcites, produces scintillation light. This scintillation light is converted to an electric current by a photomultiplier, with the current proportional to the energy of the incident radiation.

The energy resolution of scintillator detectors is typically poor compared with solid state detectors. This difference can be seen a comparison of a bismuth germanate (BGO) scintillator spectrum and a clover spectrum, figure 3.6. On the other hand, the efficiency of the BGO material is quite high, about 2 1/2 times that of NaI. Recall that the clover detector has a relative efficiency of 150\%, as compared with a 3"\(\times\)3" NaI crystal. Using the rough estimate that the \(\gamma\)-ray detection takes place homogenously over the full 630 cm\(^3\) volume of the Clover, and over the 350 cm\(^3\) volume of the NaI standard, a BGO the size of a Clover detector would have > 450\% relative efficiency, three times that of a clover operated with add-back.
Figure 3.6: Comparison of the resolution of a BGO detector and a clover detector, using $^{153}$Eu as a source. Note the better energy resolution and higher peak-to-background ratio in the clover spectrum.

Figure 3.7 Cross section schematic of a clover detector surrounded by BGO Compton suppression scintillators [Be00]. The target position/γ-ray source is where the dashed lines cross.
The high efficiency of scintillator detectors makes them very useful for Compton suppression. As mentioned, when photons scatter out of a Ge detector, the full photopeak energy is not deposited in the detector. Recording this energy would add to the background noise in the spectrum. To suppress these events, many of the Ge detectors are surrounded with BGO scintillator crystals, as in figure 3.7. If there is a scintillation signal from one of these surrounding scintillators, the energy read in the Ge detector is not added to the data. This BGO Compton suppression was utilized in all the experiments discussed.

![Figure 3.8: Comparison of $^{152}$Eu spectrum taken at the same time with a clover detector without Compton suppression (upper spectrum) and with (lower spectrum). The energy of the unshielded spectrum is moved up by 10 keV to allow a comparison of the peak heights. The peak height above the background is about the same for both spectra, but the background is half as great for the shielded clover, doubling the peak-to-background ratio for that spectrum as compared with the unsuppressed spectrum.](image-url)
An example of the difference between a Compton suppressed spectrum and an unsuppressed spectrum is shown in figure 3.8. Both spectra were taken for the same amount of time with both Clover detectors the same distance from the source. The peaks are about the same height above the background, but in the unsuppressed spectrum, the background is twice as high as in the suppressed spectrum.

3.4 Detector arrays

The combination of several detectors in an array increases the photopeak efficiency, as well as allowing coincidence measurements and giving information on the angular distributions of the emitted γ-rays. YRAST Ball, at Yale University, was used for the high spin experiments on $^{136}\text{Pm}$ and $^{140}\text{Eu}$. Eurogam Phase 1, at Daresbury, was used for the high spin experiment on $^{140}\text{Eu}$. Both these detector arrays are discussed below.

3.4.1 YRAST Ball

Data on $^{136}\text{Pm}$ and $^{140}\text{Eu}$ were taken using the Yale Rochester Array for SpecTrosopy (YRAST Ball) detector array at the Wright Nuclear Structure Laboratory, Yale University [Be00]. The array is shown in figure 3.9. YRAST Ball is a powerful device for gamma ray spectroscopy with between ~ 2.5 and 3% array efficiency for the detector set-ups used in the $^{136}\text{Pm}$ and $^{140}\text{Eu}$ experiments. That is, for an emitted photon at 1.3 MeV, the chance of the full energy peak of the photon being recorded is 2.5 - 3%. This takes into account the finite solid angle of the array, as opposed to measurements of the intrinsic detector efficiencies. The array has space for ~30 Ge detectors, including 9 segmented clover Ge detectors.
Figure 3.9: YRAST Ball. Closed view from downstream, open view from upstream. The target chamber can be seen at the center of the array (center of lower picture).
Detector positions are available in 4 rings concentric to the beam line. A schematic diagram of the array with the detector angles is shown in figure 3.10. Measuring the angles with respect to the beam axis, with 0° downstream, the ring at 160° has space for 3 coaxial (or LEPS) detectors, 126.5° has space for 8 coaxial detectors, 90° has space for 9 clover detectors, and 50° has space for 8 coaxial detectors. The target to detector distance is ~21cm. At that distance, the clover detectors each contribute ~ 0.3% to the absolute array efficiency, the coaxials about 1/6th of that each.

For the $^{136}$Pm experiment [He01], YRAST Ball consisted of 4 Compton suppressed clover detectors mounted at 90° with respect to the beam axis, and 18 Compton suppressed coaxial Ge detectors, in the other 3 rings, with 7 at 50°, 8 at 126.5°,
and 3 at 160°. One of the coaxial detectors at 50° was a large volume coaxial detector with ~70% efficiency. For sensitivity to x-rays and low energy γ-rays, three LEPS detectors were mounted in the array, 1 at 50° and 2 at 90°. The total photopeak efficiency for YRAS T Ball in this configuration is ~2.5%.

For the 140Eu experiment [He03], YRAS T Ball consisted of 7 clover detectors mounted at 90° with respect to the beam axis, and 15 Compton suppressed coaxial Ge detectors mounted in the other rings, with 6 at 50°, 8 at 126.5°, and 2 at 160°. 3 LEPS detectors were also mounted in the array, 2 at 50° and 1 at 90°. The total photopeak efficiency for YRAS T Ball in this configuration is ~3%.

3.4.2 Eurogam Phase 1

The general structure of the Eurogam Phase 1 array [Bk92, Be92] is a sphere made of 12 pentagonal structures, with each pentagon having space for 5 detectors. The total array has space for a total of 60 detectors in the pentagons.

The data for 138Eu were taken for one of the commissioning runs for the Eurogam Phase 1 array and were originally used to establish the high spin structure of that nucleus [Pa94a]. At the time of the experiment, the array consisted of 45 Compton suppressed large volume coaxial Ge detectors, grouped around the central and backward array angles. The detectors used in the array had a ~70% relative efficiency at Eγ = 1.33 MeV [Bk92] and each contributes ~0.126% absolute efficiency to the array [Be92]. Compton suppressed Ge data were recorded for events in which six or more unsuppressed detector events occurred within a 50 ns coincidence window. The γ-γ coincidence matrix from this experiment was made available to us for analysis.

3.5 Electronics and data acquisition at YRAS T Ball

In this section, an overview of the associated electronics used with the Ge detectors of YRAS T Ball is given. The key tasks performed by the electronics are:
- to set a minimum signal threshold in the Ge detectors above noise levels,
- to veto the signals from Compton suppressed detectors, and
- to identify and trigger on events of at least a given multiplicity.

A simplified diagram of the electronics is presented in figure 3.11. In that figure, shapes of the signal pulses at several steps in the processing are shown beside the signal lines. Several modules, including the TFA, CFD, and the amplifiers are NIM, while the ADC and TDC modules are VME.

When a γ-ray is incident on a detector crystal, the detector outputs a current proportional to the γ-ray energy. This fast pulse of current is converted in the detector preamplifier to a sharp voltage pulse. The signal is split and processed, one part to be recorded as the photon energy (left column of figure 3.11), and the other part to give a relative time within the total event (center column of figure 3.11). The energy signal is amplified and sent to the CAMAC ADC module, which delays the signal and, given the appropriate master gate signal (discussed below), converts the analog energy signal to a digital signal to be recorded.

The time signal takes a more complicated route. The signal is first amplified in a fast timing amplifier, then sent to a constant fraction discriminator (CFD), which produces a logic signal whenever the voltage exceeds a threshold level.

If the BGO Compton shield outputs a signal at about the same time as its Ge detector (Δt depending on the electronics settings), the time signal is vetoed. Following the right hand column of figure 3.11, the shield output is amplified by a fast amplifier and sent to a discriminator. If the signal is larger than the discriminator threshold, a long (~500 ns) logic pulse is sent to the veto input of a coincidence logic unit, suppressing the time signal.

If there is no BGO output, the time logic pulse continues through that coincidence module. The time signal is split, with part going to a multiplicity gate. If the number of
coincidences in a set amount of time is at least a chosen number, N in figure 3.11, this module sends a "master gate" start signal to the CAMAC. The other part of the time

![Diagram](image)

Figure 3.11: Diagram of the electronics used for YRAST Ball data. The detector signal is split into an energy (left column) and time (center column) signal. The Compton suppression signal (right column) acts as a veto to the time signal. Pulse shapes at several points in the processing are shown beside the signal lines.

signal is delayed, to give time for the master gate, then sent to the CAMAC electronics. It must be recalled that there are many time signals arriving at the CAMAC. Once the

60
master gate fires, the first time signal to the CAMAC starts an event clock. The time elapsed from the start of the event clock to the arrival of a time signal from any of the detectors is converted to a digital signal in the CAMAC and sent on to be recorded. After a chosen time, on the order of several hundred ns, the master gate signal ends and no more data is converted from that event.

The energy gate is open longer, on the order of 15 μs. This longer gate is required because the analog energy signals have a longer rise and decay time (with about 10 μs width).

The events are sent to a VME High-Speed Memory module (CES HSM 8170), then sent without further processing to the SPARC workstation computers where they are recorded to 8mm exabyte data tapes for later analysis. The energy is recorded over 4096 channels, so the gain on the amplifiers determines the maximum energy resolution of the data. Since the energy signals do not have the antiCompton veto, and are admitted to the CAMAC as long as the master gate fires, some spurious energy signals are recorded. The channel for each energy signal and for each time signal is recorded, so those energy signals without a good associated time can be vetoed in subsequent analysis.

3.6 General techniques
3.6.1 Gain matching and efficiency calibration

The energy signals from the Ge detectors in an array are typically gain matched so that known spectral lines from a γ-ray source placed at the center of the array are read at the same channel by all the detectors. In addition, the efficiency curves are different for different detectors because of different detector geometries and/or different absorbers used. To measure the efficiency of the array, the measured intensities of known spectral lines from a calibrated γ-ray source are compared with their calculated intensities. An example calibration of the 90 ° detector ring in YRAST Ball using a^{152}Eu source is presented in figure 3.12. In the bottom of that figure, the recorded spectrum of ^{152}Eu is presented. This is compared with the known photon intensities of the transitions and
relative efficiencies for those energies are determined. A calibration curve is fit to these points, as presented in the top of figure 3.12. An absolute efficiency calibration can be determined knowing the strength of the source and the data recording time. A relative efficiency calibration, though, is enough for comparing the \( \gamma \)-ray intensities at different angles, which will be discussed.

![Graph showing relative efficiency vs. energy](image)

**Figure 3.12**: Calibration of the 90° detector ring in YR\(\text{AST} \) Ball with a \(^{152}\text{Eu}\) source. The measured spectrum (bottom) is compared with the known photointensities for each measured point and the relative intensities give a relative efficiency for each point (top). An efficiency curve is fit to these points (also top).

### 3.6.2 Doppler shift and Doppler broadening

In thin target fusion-evaporation reactions, the compound nucleus recoils through the back of the target. In this case, the \( \gamma \)-rays emitted have a Doppler shift, with energies

\[
E'(\theta) = E(1 - v/c \cos \theta),
\]

where \( \theta \) is the angle from the recoil path to the detector and \( E \) is the unshifted energy.

For the \(^{136}\text{Pm}\) and \(^{140}\text{Eu}\) experiments on YR\(\text{AST} \) Ball, the \( v/c \) was 2 - 3\%, which is about
\( v = 10 \text{ mm/ns}. \) For a 1 MeV \( \gamma \)-ray, the Doppler shift at 160° is 28 keV, a significant shift. Since the angles of the detector and velocity of the recoiling nucleus are well known, the Doppler shift is not a problem and is compensated for when analyzing the data.

However, the fastest change in the Doppler shift as a function of angle,

\[
\frac{dE}{d\theta} \propto \sin \theta,
\]

occurs around \( \theta = 90^\circ \). Due to the finite solid angle of real detectors, this leads to a Doppler broadening that is maximum at this angle. The Doppler broadening cannot be corrected for like the Doppler shift, but small solid angle detectors may be used at the angles where this broadening is greatest. Clover detectors at 90° in YRAS T Ball, composed of several small detectors, minimize the effects of Doppler broadening.

### 3.6.3 Coincidences

In typical fusion-evaporation reactions, only 1 out of every 10,000 or so beam particles actually reacts. With a beam current of a few particle nA, there are perhaps 1,000,000 reactions per second. That is, the mean time between reactions is about 1 \( \mu \)s. By setting a time gate of \(~ 100 \) ns when gathering the data, it is possible to record the individual reactions as distinct events. By gathering data on many such events, a \( \gamma-\gamma \) coincidence matrix (or \( \gamma-\gamma-\gamma \) coincidence "cube") may be assembled. Such a matrix allows an examination of the spectrum that is in coincidence with a given transition energy. By a coincidence analysis, the nuclear excitation levels and band structure are determined.

### 3.6.4 Isotope identification

There are several techniques for identifying a nucleus in the experimental data. By far, the easiest is by identifying transitions that are in coincidence with already known transitions. However, in the case of the \(^{140}\text{Eu}\) experiment discussed later, the only previous information available was on low lying transitions below a 125 ms isomer. For
this thin target experiment in YRAST Ball, where the nucleus is recoiling with \( v/c = 2.7\% \), the nucleus moves out of the sensitive area of the detector array in a time that is on the order of 100 ns. The high spin data yields only information about states above the isomer, and prompt coincidences with the known sub-isomeric levels are not observed. To assign \( \gamma \)-rays to \(^{140}\text{Eu} \), a coincidence with europium K\( \alpha \) x-rays and an experimental excitation function were examined. The energies of the K\( \alpha \) x-rays depend on the Z of the nucleus and the structure of the inner shells of electrons, so they are the same for different isotopes. For \(^{63}\text{Eu} \), the K\( \alpha_{1} \) x-ray (the most intense K\( \alpha \) line) is 41.5 keV. For the nearest Z neighbors, \(^{62}\text{Sm} \) and \(^{64}\text{Gd} \), it is 40.1 and 43.0 keV, respectively, easily resolvable with LEPS detectors using their typical < 1 keV energy resolution. The excitation function works because in a fusion-evaporation reaction, the more thermal energy that is put into the system, the more particles that are evaporated (typically about \( \sim 10 \) MeV per extra particle). By measuring \( \gamma \)-ray intensities as a function of beam energy, transitions can be assigned to nuclei produced with different mass, \( A \). Combining the information from these two techniques, giving \( A \) and \( Z \), identifies the nucleus from which a \( \gamma \)-ray was emitted.

### 3.7 Analysis of transitions

#### 3.7.1 Properties of electromagnetic radiation

Nuclei contain complicated charge and current distributions which create electric and magnetic fields. The fields vary with distance; those that vary as \( 1/r^2 \) are termed monopole, \( 1/r^3 \) dipole, \( 1/r^4 \) quadrupole, and so on to different multipolarities. The transitions between excited states, which emit \( \gamma \)-rays, are described by the electromagnetic character and multipolarity of the fields, denoted \( \sigma (= M \) or \( E \)) and \( L (= \) integer), respectively. This is discussed in further detail in appendix A. The emission of this energy takes the nucleus from an initial state, \( |I_i M_i\rangle \), to a final state, \( |I_f M_f\rangle \). This transition can be modeled by operating on the initial and final states with a multipole operator \( m (\sigma L) \).
The total transition probability can be described by

\[ T_\beta = \frac{8\pi(L+1)c}{\hbar L((2L+1)!!)^2} \left( \frac{E}{\hbar c} \right)^{2L+1} B(\sigma L; I_i \rightarrow I_f). \]

(3.9)

The reduced transition probabilities, \( B(\sigma L) \), are expressed in terms of the multipole operator as

\[ B(EL; I_i \rightarrow I_f) = \frac{1}{2I_i + 1} \left| \langle I_f \parallel m(EL) \parallel I_i \rangle \right|^2 \]

for electric transitions and

\[ B(ML; I_i \rightarrow I_f) = \frac{1}{2I_i + 1} \left| \langle I_f \parallel m(ML) \parallel I_i \rangle \right|^2 \]

for magnetic transitions. An estimate of transition probabilities as a function of transition energy and atomic mass number, \( A \), are presented in reference [Kr88]. These single particle estimates were modeled assuming the transition is due to a single particle moving between an initial and final state. For the first few multipoles they are:

\[ T(E1) = 1.0 \times 10^{14} A^{2/3} E^3 \]

(3.10)

\[ T(E2) = 7.3 \times 10^7 A^{4/3} E^5 \]

\[ T(E3) = 34 A^2 E^7 \]

(3.11)

\[ T(M1) = 5.6 \times 10^{13} E^3 \]

\[ T(M2) = 3.5 \times 10^7 A^{2/3} E^5 \]

\[ T(M3) = 16 A^{4/3} E^7 \]

where the transition rates are in decays per second. Further details are presented in appendix A. In terms of equation 3.12, the reduced transition probabilities, called transition strengths, can also be determined by comparing the experimental transition rates with the calculated single particle rates:

\[ B(\sigma L) = \frac{T(\sigma L)_{\text{experiment}}}{T(\sigma L)_{\text{single particle}}} \]

(3.13)

The transition strengths give insight into the nuclear structure. The magnetic moments are sensitive to the magnetic properties of the nucleus, and hence to single particle aspects. The electric moments are sensitive to the nuclear charge distribution, and hence to collective effects such as deformation.

Selection rules limit the transition based on the parity of the initial and final states. The parity of a transition is
\begin{align*}
(3.13) \quad \pi (ML) &= (-1)^{L+1} \\
(3.14) \quad \pi (EL) &= (-1)^{L}
\end{align*}

for magnetic transitions and

for electric transitions. The parity of the transition depends on the parity of the initial and final states, following \( \pi = \pi_{\text{final}} \times \pi_{\text{initial}} \). Selection rules also set a lower limit on the multipolarity, \( L \), of a transition, based on the initial and final spins, \( I_i \) and \( I_f \), of the system, such that

\begin{equation}
| I_i - I_f | \leq L \leq I_i + I_f.
\end{equation}

A transition is called a "stretched" transition if the change in angular momentum is the largest that is allowed for that transition multipolarity.

Selection rules may allow transitions between the same two levels to have different multipolarities. This happens commonly in parity conserving \( \Delta l = 1 \) transitions, which can have admixtures of \( M1 \) and \( E2 \) character. The degree of this mixing is typically quantified in the \( E2/M1 \) mixing ratio, \( \delta_{E2/M1}^2 \). The mixing ratio is defined by the transition width, \( \Gamma \) (which is itself related to the transition probability: \( \Gamma = \Gamma/h \)), by

\[ \delta_{E2/M1}^2 = \Gamma (E2) / \Gamma (M1) = T (E2) / T (M1) \]

Since transition rates based on these \( M1 \) and \( E2 \) multipoles go as \( E^3 \) and \( E^5 \), respectively, the admixture varies as a function of transition energy. The mixing ratio is typically small, but may slightly complicate transition analysis.

Several experimental techniques are used to determine the multipolarity and electromagnetic character of transitions in oriented nuclei. The methods include angular distribution measurements, directional correlation of oriented states measurements, and Compton asymmetry measurements. These techniques are discussed below.

\subsection*{3.7.2 Angular distribution measurements}

The angle that a \( \gamma \)-ray is emitted from a nucleus depends on the order and the orientation of the multipole field from which it is emitted. If the nucleus is not oriented,
the direction of the multipole can be derived by correlating several transitions from the same nucleus.

A simpler approach can be taken when dealing with oriented nuclei. In heavy ion fusion-evaporation reactions, the spins of the nuclei (and hence the multipoles) are typically aligned in a plane perpendicular to the beam direction. That is, the orientation of the nuclear angular momentum vector with respect to the beam axis is peaked at $\theta = 90^\circ$, but the variation as a function of $\phi$, the angle symmetric about the beam axis, is averaged out.

The orientation of the nucleus is slightly attenuated by the emission of the evaporated particles. This provides a substate alignment, an $m$-state alignment which is peaked symmetrically about $m = 0$. The angular distribution of the emitted photons is found from a weighted sum of the different $m$-state angular distributions. This can be expressed in terms of Legendre polynomials as

$$ W(\theta) = \sum_v A_v P_v(\cos \theta), $$

where $W(\theta)$ is the intensity of the gamma-rays measured at angle $\theta$, $v$ are even numbers less than or equal to $2L$, and $A_v$ are the angular distribution coefficients.

By measuring the intensity of the gamma ray distribution as a function of angle and fitting $W(\theta)$, the expansion coefficients $A_v$ can be found. For a pure “stretched” dipole transition ($L = 1$ and $\Delta I = 1$), the angular distribution is a second order polynomial in terms of cosine $\theta$, and can be expressed as

$$ W(\theta) = A_0 + A_2 P_2(\cos \theta). $$

For a pure “stretched” quadrupole transition ($L = 2$ and $\Delta I = 2$), the angular distribution is a fourth order polynomial in terms of $\cos \theta$ and can be expressed as

$$ W(\theta) = A_0 + A_2 P_2(\cos \theta) + A_4 P_4(\cos \theta). $$

The peaking of the $m$ state distribution about $m = 0$ results in a pure dipole transition having the normalized coefficient $a_2 = A_2/A_0 < 0$ and a pure quadrupole transition having $a_2 > 0$. A simple discussion based on classical multipoles is included in appendix A.
An example of an angular distribution fit is presented in figure 3.13 for the 640.6 keV transition in $^{140}$Eu. The data was taken on YRAST Ball, with detectors arranged in rings concentric to the beam line with angles of $\theta = 50^\circ$, $90^\circ$, $126.5^\circ$, and $160^\circ$. Due to the symmetry of the $50^\circ$ and $126.5^\circ$ angles with respect to the angular distribution function $W(\theta)$, the system is treated as having 3 independent angles and the fit is performed for $A_0$ and $A_2$ only. The 640.6 keV transition is a strong $E2$ transition, with a normalized coefficient of $a_2 > 0$ (i.e., $W(\theta) \propto A_2 \cos \theta$ decreases from $0^\circ$ to $90^\circ$), consistent with a quadrupole assignment.

Figure 3.13: Efficiency corrected intensities of the 640.6 keV ($E2$) peak for each of the four detector rings (at $50^\circ$, $90^\circ$, $126.5^\circ$, and $160^\circ$). The curve, $W(\theta)$, is fit by varying the parameters $A_0$ and $A_2$. 
3.7.3 Directional correlation of oriented states

The method of directional correlation of oriented states (DCO) analysis is also sensitive to transition multipolarities. The DCO ratio is defined as the intensity, I, of a measured transition at angle 1 when gated on a reference gamma ray at angle 2, divided by the intensity of the measured transition at angle 2 when gated on a reference gamma ray at angle 1 (where the reference gamma ray is of known multipolarity). That is:

\[
R_{DCO} = \frac{I_m(\theta_1, \text{gate on } \gamma_r(\theta_2))}{I_m(\theta_2, \text{gate on } \gamma_r(\theta_1))},
\]

efficiency corrected for the gating and measuring angles. If the reference gamma and the measured gamma ray have the same multipolarity, their angular distribution is similar, and the DCO ratio is expected to be 1. For different multipolarities, the angular distributions are different and the DCO ratio is either greater or less than 1, depending on the gate and measured γ-ray multipoarities.

In terms of the normalized angular distribution functions, through the second order term:

\[
R_{DCO} = \frac{W_m(\theta_1)W_r(\theta_2)}{W_m(\theta_2)W_r(\theta_1)} = \frac{(1+a_m\cos^2\theta_2)(1+a_r\cos^2\theta_2)}{(1+a_m\cos^2\theta_2)(1+a_r\cos^2\theta_2)}
\]

Taking, for simplicity, \(\theta_1 = 180^\circ\) and \(\theta_2 = 90^\circ\), the ratio reduces to

\[
R_{DCO} = \frac{I_m(180^\circ, \text{gate on } \gamma_r(90^\circ))}{I_m(90^\circ, \text{gate on } \gamma_r(180^\circ))} = \frac{1+a_m}{1+a_r}.
\]

For a sense of scale, using these detector angles and the typical sized \(a_2\) values of −0.15 for dipole (the weighted average of \(a_2\) values from used for the intense 446 keV transition in \(^{140}\)Eu) and +0.15 for quadrupole transitions, the ratio is \(R_{DCO} = 0.85/1.15 = 0.74\) when gating with a dipole and measuring a quadrupole, and \(R_{DCO} = 1.15/0.85 = 1.35\) when gating with a quadrupole and measuring a dipole. A mixing of multipoarities in either the reference or measured transition may make a determination difficult, and is why the reference transition must be well chosen.
An example of measured intensities following this scheme can be seen in figure 3.14. The 170.6 and 191.1 keV dipole transitions in $^{140}$Eu are presented at 90°, gated at 160° (left side of figure 3.14) and at 160°, gated at 90° (right side of figure 3.14). The gate used is the strong 537.4 keV quadrupole transition. The intensities of the measured transitions are markedly lower in the right figure. With the detector angles at 90° and nearly 180°, the ratio of the intensities in the left figure divided by the intensities in the right figure approximates equation 3.21, above. This ratio is greater than 1, as expected when measuring a dipole transition using a quadrupole gate using these angles.

![Graph showing energy distribution of 170.6 and 191.1 keV transitions]

**Figure 3.14:** The 170.6 and 191.1 keV dipole transitions, gated with the 537.4 keV quadrupole transition, using a 90°–160° coincidence matrix. The gating and measuring angles are represented above the figures. Note the difference in intensities for the different angles.

### 3.7.4 Polarization

With an oriented source, γ-rays have different linear polarization in the lab frame for radiation from electric and magnetic transitions. The polarization is reflected in and
can be measured by the direction of Compton scattering in the clover Ge detectors. The difference in scattering direction is most marked at 90° from the beam axis, the angle at which the clovers, by no accident, are positioned in YRAST Ball. The measurable quantity in Compton polarimetry is the number of scatters parallel (N∥) and perpendicular (N⊥) with the beam axis. The difference between the two can be striking, as demonstrated in the spectra for the 537 keV E2 transition from 140Eu in figure 3.15, taken using a clover detector mounted in YRAST Ball at θ = 90°. The full energy peak was determined using the Clover leaves in add-back only mode. For electric transitions, the Compton scattering is preferentially orthogonal to the beam direction, as can be seen experimentally in the comparison of intensities for orthogonal and parallel scattering, shown in figure 3.15.

![Graph showing Compton scattering](image)

**Figure 3.15:** Compton scattering of the 537.4 keV γ-ray from 140Eu. The data were taken using the Clover detectors in YRAST Ball. The 537.4 keV γ-ray is emitted in an E2 transition. Note the higher intensity of the parallel scattering.
To compare such differences in intensity, a normalized azimuthal asymmetry ratio is used. In this work it is referred to as the Compton asymmetry ratio, defined as

$$\varepsilon = \frac{\alpha N_\perp - N_\parallel}{\alpha N_\perp + N_\parallel},$$

where $\alpha$ is a correction factor for different geometries and efficiencies between parallel and orthogonal oriented detectors. The count rates for $N_\perp$ and $N_\parallel$ depend on the probabilities that the electric field vector has a certain direction, and following the Klein-Nishina formula (see appendix A), the count rates differ most at $\theta = 90^\circ$ from the beam axis. That is, the asymmetry ratio is most sensitive at $\theta = 90^\circ$. The asymmetry $\varepsilon$ is related to the degree of polarization $P$ by $\varepsilon = QP$, with $Q$ being the polarization sensitivity of the polarimeter [Sc94].

### 3.7.5 Measured transition strengths

Figure 3.16: Schematic of dipole ($I \to I-1$) and quadrupole ($I \to I-2$) deexcitation paths from a single level, with spin $I$.

A common way of describing the transition strengths $B(\sigma L)$ is through the ratio $B(M1; I \to I-1)/B(E2; I \to I-2)$, for transitions deexciting from a single level (figure 3.16). This ratio gives structural information, without the difficulty of measuring the individual transition strength values. Indeed, in the experimental chapters, the measured ratios from the chiral candidate bands are compared with calculations from both the TAC and particle + rotor models. The experimental ratio is defined as [Re00]

$$\frac{B(M1; I \to I-1)}{B(E2; I \to I-2)} = 0.697 \frac{1}{1+\delta_{E2/M1}^2} \frac{I_y(\Delta I = 1)}{I_y(\Delta I = 2)} \frac{E_0^2(\Delta I = 2)}{E_0^2(\Delta I = 1)} \left(\mu_N/e_b\right)^2,$$

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where $I_\gamma$ is the intensity of the transition, the transition energy $E_\gamma$ is in MeV, and $\delta^2_{E2/M1}$ is the $E2/M1$ mixing ratio for the $\Delta I = 1$ transition.

The $B(E2)$ values may be found experimentally, through lifetime measurements. Following equations 3.12 and 3.13,

\begin{equation}
\frac{1}{\tau} = B(E2) \left( 7.3 \times 10^7 A^{4/3} E^5 \right),
\end{equation}

where $\tau$ is the mean lifetime. The quadrupole moment and the $B(E2)$ values are related by

\begin{equation}
B(E2; I \rightarrow I - 2) = \frac{5}{16\pi} \langle I K 2 0 | I - 2 K \rangle^2 Q^2.
\end{equation}

Thus, the $B(E2)$ values give information on the nuclear deformation, since for axially symmetric nuclei

\begin{equation}
Q = \frac{3}{\sqrt{5\pi}} Z R_0^2 \beta(1+0.16\beta),
\end{equation}

where $R_0 \propto A^{1/3}$ and $\beta$ is the deformation parameter. For a triaxial nucleus, in the cranking approximation the deformation parameters $\beta$ and $\gamma$ are related to the quadrupole moment through [Ri82]

\begin{equation}
Q(\beta, \gamma) = Q(\beta, \gamma = 0) 2 \sin (30^\circ - \gamma).
\end{equation}
Chapter 4

Experimental results on the $N=75$ nuclei
$^{136}\text{Pm}$ and $^{138}\text{Eu}$

4.1 Introduction

As discussed in chapter 1, candidate chiral partner bands were found in the $N=75$, $Z=59$ nucleus $^{134}\text{Pr}$, with the $\pi h_{11/2} \otimes v h_{11/2}$ orbital configuration. TRS calculations for the $\pi h_{11/2} \otimes v h_{11/2}$ configuration in $^{134}\text{Pr}$ predict a deformation triaxial deformation with $\beta \sim 0.2$ and $\gamma \sim \pm 25^\circ$ at moderate spins. The $h_{11/2}$ neutron level near the Fermi surface is $9/2[514]$ in the Nilsson asymptotic quantum numbers, an orbital high in the high-$j$ $h_{11/2}$ shell ($K = 9/2$), and the relevant $h_{11/2}$ proton level is $3/2[541]$, an orbital low in the high-$j$ $h_{11/2}$ shell ($K = 3/2$). The angular momenta of these particles are oriented toward the long and short nuclear axes, respectively, providing nearly orthogonal single particle angular momentum vectors. These vectors, when coupled with a collective rotational angular momentum vector along the intermediate axis, provide for an aplanar angular momentum and chiral behavior.

The presence of such band pairs is expected to occur for nuclei with triaxial deformation for a suitable range of proton and neutron numbers. The odd-odd isotones with the next higher proton numbers, $^{136}\text{Pm}$ ($Z = 61$) and $^{138}\text{Eu}$ ($Z = 63$), are predicted to have a triaxial deformation similar to that of $^{134}\text{Pr}$. Indeed, TRS calculations for the low-lying $\pi h_{11/2} \otimes v h_{11/2}$ configuration in $^{138}\text{Eu}$ predict a considerable triaxiality over a range of rotational frequencies, from $\hbar \omega = 0$ to $\sim 0.2$ MeV, with $\beta \sim 0.2$ and $\gamma \sim \pm 25^\circ$ (see figure 4.1, bottom). For $^{136}\text{Pm}$, calculations predict similar deformations.
Figure 4.1: Polar coordinate plots of Total Routhian surface calculations for $^{136}$Pm at $h\omega = 0.2$ MeV (top) and $^{138}$Eu at $h\omega = 0$ MeV (bottom). TRS calculations plot the Routhian energies for a given quasiparticle configuration and for a PAC frequency as a function of $\beta$ and $\gamma$. The energy minima are interpreted as the probable deformations. The figures are for proton parity and signature $(\pi,\alpha) = \text{neutron} (\pi,\alpha) = (-, -1/2)$, which describes the $\pi h_{11/2} \otimes \nu h_{11/2}$ configuration. There are potential energy minima in both plots for $\beta \sim 0.2$, $\gamma \sim +25^\circ$ and $-25^\circ$. Contours are 175 keV apart.

$\beta \sim 0.2$ and $\gamma \sim \pm 25^\circ$ (see figure 4.1, top). For $Z = 61$ (Pm) to 63 (Eu) the proton Fermi surface is around the $3/2[541]$ and $5/2[532]$ levels of the $1h_{11/2}$ shell (see Nilsson diagrams, figures 2.7 and 2.8. Thus, the orientation of the unpaired proton and neutron
angular momentum vectors in $^{136}$Pm and $^{138}$Eu is predicted to be similar to those of $^{134}$Pr. That is, the conditions are favorable for candidate chiral band pairs to exist in these nuclei. If the arguments for chiral behavior are valid, the neighboring nuclei with similar particle number and deformation should show similar behavior. Indeed, we have published experimental results describing candidate chiral bands [He01] based on this work.

4.2 Previously known band structure

Some information on high spin states in $^{136}$Pm and $^{138}$Eu was previously known from several different studies. In the work by Beausang et al. on $^{136}$Pm [Be87], two bands are presented, the positive parity yrast band based on the $\pi h_{11/2} \otimes v h_{11/2}$ configuration, and a negative parity band based on the $\pi (g_{7/2}, d_{5/2}) \otimes v h_{11/2}$ configuration, see figure 4.2. For both bands, the neutron occupies the 9/2[514] Nilsson level. The $\pi h_{11/2} \otimes v h_{11/2}$ band was known from a level of spin 7$^+$ to a tentative level of spin 17$^+$, with a proton configuration based on the 3/2[541] Nilsson level.

![Diagram of the $^{136}$Pm level scheme from [Be87]. The left band is the $\pi h_{11/2} \otimes v h_{11/2}$ band.](image)

Figure 4.2: $^{136}$Pm level scheme from [Be87]. The left band is the $\pi h_{11/2} \otimes v h_{11/2}$ band.

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In the work by Paul et al. on $^{138}$Eu [Pa94a], three bands are described, as shown in figure 4.3. As expected, the yrast band has the even parity $\pi h_{11/2} \otimes v h_{11/2}$ configuration, and the two side bands have odd parity and are assigned the $\pi d_{5/2} (h_{11/2})^2 \otimes v h_{11/2}$ and $\pi g_{7/2} (h_{11/2})^2 \otimes v h_{11/2}$ configurations. The $\pi h_{11/2} \otimes v h_{11/2}$ yrast band was known from a level of spin $7^+$ to a level of spin $24^+$.

![Figure 4.3: $^{138}$Eu level scheme from [Pa94a]. The center band is the $\pi h_{11/2} \otimes v h_{11/2}$ band.](image)

4.3 Populating high spin states in $^{136}$Pm and $^{138}$Eu

High spin states in $^{136}$Pm were populated in the $^{116}$Sn($^{24}$Mg, p3n) reaction at beam energies of 130 and 135 MeV [He01]. The $^{24}$Mg beam was delivered by the ESTU tandem Van de Graaff accelerator at the Wright Nuclear Structure Laboratory (WNSL),
Yale University. The target consisted of two stacked foils of $^{116}$Sn, each of 0.8 mg/cm$^2$ thickness. $\gamma$-rays were detected using the YRAST Ball detector array [Be00] which at the time of the experiment consisted of 18 coaxial Ge detectors, each with ~25% relative efficiency, three low energy LEPS detectors, and four clover detectors of ~150% relative efficiency each. The total absolute efficiency of the array, the percentage of full photopeaks measured at $E_\gamma \sim 1.3$ MeV from a source at the center of the array, was ~2.5%. In the five day experimental run, a total of $6.7 \times 10^8$ unfolded $\gamma-\gamma$ coincidences were accumulated. The current level scheme (figure 4.4, left) was constructed with the Radware analysis software [Ra95].

High spin states in $^{138}$Eu were populated following the $^{106}$Cd($^{35}$Cl, 2pn) reaction at a beam energy of 150 MeV [Pa94a]. The beam was delivered by the Daresbury Laboratory tandem Van de Graaff accelerator. The decay $\gamma$-rays were measured using the Eurogam Phase 1 detector array which consisted of 45 large volume Ge detectors [Bk92, Be92]. A total of $5.7 \times 10^8$ unfolded $\gamma-\gamma$ coincidences were recorded. This data set had been utilized previously to establish the high spin structure of $^{138}$Eu presented in figure 4.3 [Pa94a]. The total $\gamma-\gamma$ coincidence matrix from this data set was made available to us for analysis. As with $^{136}$Pm, Radware analysis software [Ra95] was used to construct the current level scheme.

4.4 Results

A doubles and triples coincidence analysis of the $^{136}$Pm data set resulted in the extension of the previously known positive parity $\pi h_{11/2} \otimes \nu h_{11/2}$ band and the negative parity $\pi g_{7/2} \otimes \nu h_{11/2}$ side band. In addition, a new band consisting of strong $\Delta I = 1 M1/E2$ transitions with $\Delta I = 2 E2$ crossover transitions was observed. This new band is linked to the yrast band by a series of $\Delta I = 1 M1/E2$ and $\Delta I = 2 E2$ transitions. A partial level scheme of $^{136}$Pm showing the new transitions alongside the yrast band is presented in figure 4.4. The width of the transition arrows represents the relative intensity of the
Figure 4.4: Partial level scheme of $^{136}\text{Pm}$ and $^{138}\text{Eu}$ from the present work. Arrow widths describe intensity. Absolute intensities are not compared between the nuclei.
transitions within the nucleus. The more strongly populated band, shown on the right for each nucleus in figure 4.4, is the $\pi h_{11/2} \otimes \nu h_{11/2}$ yrast band and the less strongly populated band, on the left for each nucleus, is referred to as the yrare band and is our candidate chiral partner band partner.

Figure 4.5: Gate on the 850 keV interband (top) and 590 keV yrare band (bottom) transitions in $^{136}$Pm. Transition energies are labeled for the corresponding peaks. Plain numbers are for the yrast band. Symbols are given in the legend.
Figure 4.6: Gate on the 99, 168, 285 keV transition in the yrast band of $^{136}$Pm. Transition energies are labeled for the corresponding peaks. Plain numbers are from the yrast band, symbols are given in the legend.

Typical gated spectra of transitions in the yrare and yrast bands of $^{136}$Pm are presented in figures 4.5 and 4.6. The peaks are labeled with their transition energies in the figures. Transitions in the yrare band are also marked with a +, and transitions connecting between the yrast and yrare band are also marked with a *, while yrast transitions are marked only with their transition energies. The top part of figure 4.5 shows a coincidence gate on the 850 keV transition in $^{136}$Pm, the $15^+ \rightarrow 13^+$ transition in the yrare band. The 364 and 289 keV yrare band transitions are visible, as are the 590 and 595 keV interband transitions. The low lying 99, 168 and 285 keV transitions in the yrast band are also apparent. In the bottom of figure 4.6 a gate on the 590 keV interband transition is presented. Several yrare band transitions (289, 364, 424, 426, and 850 keV) are visible, as are the yrast band transitions below the 590 keV interband transition (99 and 168 keV). Neither the 850 nor the 590 keV interband transitions are in coincidence with high lying members of the yrast band. Figure 4.6 shows a gate on the low lying 99,
168, and 272 keV transitions in the yrast band. In contrast to figure 4.5 which used yrare and interband transitions as gates, the gating transitions in figure 4.6 are in coincidence with both the yrare band and high lying members of the yrast band. These and other coincidence gates were used to verify the previously known \( \pi h_{11/2} \otimes \nu h_{11/2} \) band and place the new yrare band in \(^{136}\text{Pm}\), as presented in the partial level scheme of figure 4.4.

A doubles coincidence analysis of the \(^{138}\text{Eu}\) data set resulted in the confirmation of the previously observed \( \pi h_{11/2} \otimes \nu h_{11/2} \) band, consisting of strong \( \Delta I = 1 \) \( M1/E2 \) transitions with \( \Delta I = 2 \) \( E2 \) crossover transitions. In addition, a new \( \Delta I = 1 \) band with \( \Delta I = 2 \) crossover transitions was observed to feed into the previously known \( \pi h_{11/2} \otimes \nu h_{11/2} \) band via several \( \Delta I = 1 \) and \( \Delta I = 2 \) transitions. A partial level scheme is presented on the right in figure 4.4, showing the yrast \( \pi h_{11/2} \otimes \nu h_{11/2} \) band and the new \( \Delta I = 1 \) yrare band. A typical coincidence spectrum, gated on the 351 keV transition in the yrare band, is

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**Figure 4.7:** Gate on the 351 keV yrare band transition in \(^{138}\text{Eu}\). Transition energies are labeled for the corresponding peaks. Plain numbers are for the yrast band. Symbols are given in the legend.
presented in figure 4.7. As in the \( ^{136}\text{Pm} \) spectra, the spectrum peaks are labeled with their transition energies. Transitions in the yrare band are also marked with a +, interband transitions are marked with a *, and yrast band transitions are marked only with their transition energies. Not only are the low lying members of the yrast band visible in the figure (104, 167, and 273 keV) but transitions in the yrare band (164, 299, 383, 405, and 478 keV) are visible, as well as the interband transitions (522, 524, 547, 630, 688, and 788 keV). In contrast, the low lying 104, 167, and 273 keV yrast band transitions are in coincidence with the higher lying members of the yrast band as well as several yrare band and interband transitions, as can be seen in figure 4.8. These and other coincidence gates were used to verify the previously identified \( \pi h_{11/2} \otimes \nu h_{11/2} \) band and confirm the placement of new transitions as an yrare band in the new level scheme, as presented in figure 4.4.

![Figure 4.8: Gate on the 104, 166, and 272 keV transitions in \( ^{138}\text{Eu} \). Transition energies are labeled for the corresponding peaks. Symbols are given in the legend. Plain numbers are for the yrast band.](image-url)
DCO ratios and Compton asymmetry ratios were utilized to assign transition
multipolarities and electromagnetic character for the transitions in $^{136}$Pm and $^{138}$Eu. The
polarization sensitivity of the YRAST Ball clover detectors was used to determine the
electromagnetic character of several of the new transitions in $^{136}$Pm, and confirm several
of the known ones. Compton asymmetry ratio values [Sc94, Pi01] were also determined
for known transitions in the neighboring nucleus, $^{135}$Nd. The weighted average of the
Compton asymmetry ratios for known $M1$ values, $\varepsilon = 0.074(25)$, was determined from
measured values for the 199, 362, 233, 477, and 250 keV low lying $M1$ transitions in
$^{135}$Nd. The weighted average for known $E2$ values, $\varepsilon = -0.076(34)$, was determined from
measured values for the 595, 728, and 835 keV transitions in $^{135}$Nd. The data are

![Compton asymmetry ratio graph]

Figure 4.9: Compton asymmetry measurements for transitions in $^{136}$Pm. Lines and
dashes are error limits for known $M1$ and $E2$ transitions, respectively. Interband and
yrare transitions are marked with their energies in keV. See text for details.
presented in appendix B and are summarized in figure 4.9, presented with the average asymmetry ratios from the known transitions for comparison. The error limits for the known $M1$ and $E2$ transitions are demarcated by dashed and solid lines, respectively.

For the measured interband transitions between the new yrare band and the yrast band, with energies of 590, 595, 666, 684, and 751 keV, the polarization asymmetry values found were 0.03(38), 0.07(3), 0.07(12), 0.26(7), and -0.09(4), respectively, see figure 4.9. While the errors are very large, reflecting the weak intensity of those transitions, one can say that these values are in general consistent with an $M1$ assignment. The outlying value of 0.26(7) falls about two standard deviations from the limits of the known $M1$ transitions, but about four standard deviations from the limits of the known $E2$ transitions. The 751 keV transition lies in the limits of the known $E2$ transitions, at -0.09(4), but by the spin and parity assignments suggested by the other transitions placed in the level scheme, it is assigned as magnetic.

The angular dependence of the $\gamma$-ray intensities was investigated through DCO analysis. The data for $^{136}$Pm were collected on YRAST Ball, with detector angles at 50°, 90°, 126.5°, and 160°. A matrix was constructed for coincidence data between the detector ring at 90° and the one at 160°, for maximum contrast in the DCO ratios between dipoles and quadrupoles. A DCO analysis was possible for several of the transitions in both the yrast band and the new yrare band, as well as on several of the transitions connecting the two bands. The DCO ratios were established by measuring the intensity of $\gamma_1$ at 90° when gated by $\gamma_2$ at 160°, divided by the intensity of $\gamma_1$ at 160° gating on $\gamma_2$ at 90°. Ratios of

\[
R_{DCO} = \frac{I_{\gamma_1}(90°), \text{ gate on } \gamma_2(160°)}{I_{\gamma_1}(160°), \text{ gate on } \gamma_2(90°)}
\]

are presented in appendix B and are summarized here in figure 4.10. The DCO ratios in the figure were taken by gating on any of several strong low lying dipole transitions. As with the Compton asymmetry ratios, the DCO ratios of known $M1$ and $E2$ transitions from the neighboring nucleus $^{135}$Nd are plotted for comparison. The weighted average for known $M1$ transitions, $R_{DCO} = 1.01(7)$, was determined from measured values for the
233, 250, 362, and 477 keV dipole transitions in $^{135}$Nd, gating on the 199 keV dipole transition. The weighted average for known $E2$ transitions, $R_{DCO} = 0.45(2)$, was determined from measured values for the 595, 728, and 836 keV quadrupole transitions in $^{135}$Nd, with the same gate. The known transitions are summarized in figure 4.10 in which the error limits for the known $M1$ and $E2$ transitions are demarcated by dashed and solid lines, respectively. This is a typical figure, and there are many combinations of gates that can be used to establish the character of a particular transition.

![Graph showing DCO ratio vs transition energy for $^{136}$Pm transitions](image)

Figure 4.10: DCO ratios for transitions in $^{136}$Pm using a dipole gate. Lines and dashes are error limits for known $M1$ and $E2$ transitions, respectively. Interband and yrare transitions are marked with their energies in keV. See text for details.

Nearly all the DCO ratios for the known dipole transitions in $^{136}$Pm are greater than one. This could easily be due to the impurity of the gating dipole transition. In
addition, the admixture of $M1/E2$ character in the gating transition can cause the DCO ratio for an observed dipole transition to be high. The DCO values for the 289 and 365 keV transitions in the yrare band, 2.5(1.8) and 2.7(1.4) respectively, with error bars just above the limit of the known dipoles, are more consistent with those transitions being assigned as dipole than as quadrupole. The 424 keV yrare band transition has a DCO ratio of 0.87(39), close to the limits of the known dipoles, but with significant error bars as well. The multipolarity of the interband transitions connecting the new yrare band and the yrast band are important for identification of the yrare band. The 595 and 684 keV interband transitions have DCO ratios of 2.60(58) and 1.20(33), respectively. The 684 keV interband transition can be clearly labeled as a dipole. The 595 keV transition lies two and a half standard deviations above the limit of the known dipoles, but four standard deviations from the limit of the known quadrupoles, and is more consistent with being assigned as a dipole transition. A dipole assignment for the 595 keV transition is also consistent with the Compton polarimetry analysis.

4.5 Discussion

The yrast bands in figure 4.4 have been previously assigned the $\pi h_{11/2} \otimes v h_{11/2}$ configuration [Be87, Pa94a]. Now, in the present work, we also assign the newly observed yrare bands the same $\pi h_{11/2} \otimes v h_{11/2}$ configuration through their relation with the yrast bands. Both the proton and neutron $h_{11/2}$ orbitals are negative parity intruder orbitals, together forming a positive total parity band. If the new yrare bands have negative parity, there may only be parity non-conserving interband transitions between the yrare and yrast bands. The interband transitions in $^{136}$Pm have DCO and Compton asymmetry ratios consistent with their being parity conserving $M1/E2$ and $E2$ transitions, requiring that the yrare band has positive parity. It is very unlikely that the yrare bands are composed of two positive parity orbitals. In a single transition mediated by the magnetic dipole operator, the probability of a change in the parity of both the proton and neutron orbital configurations is vanishingly small. Thus, the most likely configuration for the yrare bands is the positive parity $\pi h_{11/2} \otimes v h_{11/2}$ configuration.
$^{136}$Pm Alignment and Routhian

Figure 4.11: Alignment plot (top) and Routhian plot (bottom) for $^{136}$Pm. Squares and triangles are the yrast and yrare $\pi h_{11/2} \otimes \nu h_{11/2}$ bands. Filled (open) figures are for transitions between odd (even) spin levels. Additional levels for $^{136}$Pm have been included from reference [Ha01].
Figure 4.12: Alignment plot (top) and Routhian plot (bottom) for $^{138}$Eu. Squares and triangles are the yrast and yrare $\pi_{h_{1/2}} \otimes \nu_{h_{1/2}}$ bands. Filled (open) figures are for transitions between odd (even) spin levels.
Further support for the $\pi h_{11/2} \otimes \nu h_{11/2}$ configuration assignment for the new bands can be found in the alignment plots for $^{136}$Pm and $^{138}$Eu, see figures 4.11 and 4.12. As can be seen, the yrare and yrast bands have the same large initial intrinsic angular momentum, $i_\kappa = 7 - 8$ h, consistent with a $\pi h_{11/2} \otimes \nu h_{11/2}$ assignment. The Harris parameters used, $\mathcal{Z}_0 = 5$ h$^2$/MeV and $\mathcal{Z}_0 = 5$ h$^4$/MeV$^2$, were chosen to subtract out the angular momentum of the core for $^{134}$Pr, to be consistent with a systematics plot to follow. The slight upward slope in figure 4.11 is due to fitting to that neighboring nucleus instead of individually fitting to the even-even cores for $^{136}$Pm and $^{138}$Eu.

The behavior of the yrare and yrast bands as a function of frequency is very similar in both plots, and in marked contrast to that of the negative parity side bands. The bands that are observed to high enough frequency show a sharp backbend at between $\hbar \omega = 0.40$ and $0.45$ MeV. The large increase in alignment of $\Delta i_\kappa = 5$ h observed in is consistent with the angular momentum gain from the alignment of a pair of $h_{11/2}$ particles. The energy difference between the bands, determined by the difference in the Routhian energies, is $\sim 300$ keV for both nuclei.

The behavior of the other bands in these nuclei stands in contrast. For the $\pi h_{11/2} \otimes \nu h_{11/2}$ band, the first proton $h_{11/2}$ band crossing (EF) is blocked. The first crossing observed is the FG crossing, at a higher frequency. On the other hand, for the $\pi (d_{5/2} \otimes g_{7/2}) \otimes \nu h_{11/2}$ band in $^{136}$Pm the EF proton $h_{11/2}$ crossing is not blocked, and it is observed at a much lower crossing frequency than the crossing in the $\pi h_{11/2} \otimes \nu h_{11/2}$ bands. This is clearly seen in the alignment plot in the top of figure 4.11, and the crossing frequency can be noted directly from the Routhian plot in the bottom of figure 4.11. In addition, the negative parity band in $^{136}$Pm starts with a lower alignment, consistent with a lower angular momentum contribution from the unpaired proton than that found in the $\pi h_{11/2} \otimes \nu h_{11/2}$ band. In $^{138}$Eu, the crossing frequency for the negative parity bands is also lower than that of the $\pi h_{11/2} \otimes \nu h_{11/2}$ bands, see figure 4.12. In fact, the $\pi d_{5/2} \otimes \nu h_{11/2}$ and $\pi g_{7/2} \otimes \nu h_{11/2}$ have so far been observed only above their crossing frequency, as $\pi (h_{11/2})^2 d_{5/2} \otimes \nu h_{11/2}$ and $\pi (h_{11/2})^2 g_{7/2} \otimes \nu h_{11/2}$ configuration bands [Pa94a].
The parity conservation of the interband transitions can be further argued by contrasting them with known parity changing transitions in $^{136}\text{Pm}$ and $^{138}\text{Eu}$. These parity changing transitions have been found feeding into the previously known $\pi \, h_{11/2} \otimes \nu \, h_{11/2}$ bands from negative parity bands, and presented in [Be87, Pa94a]. Several parity changing $E1$ transitions between the $\pi (d_{5/2}, g_{7/2}) \otimes \nu \, h_{11/2}$ band and the yrast band in $^{136}\text{Pm}$ can be seen in figure 4.2. In $^{138}\text{Eu}$, the parity changing interband transitions observed are also of $E1$ character (fig 4.3). In both of these nuclei, the interband transitions appear as $E1 \, \Delta I = 1$ transitions, while $E1 \, \Delta I = 0$ or $M2 \, \Delta I = 2$ interband transitions have not been seen between such bands. The observation of only $\Delta I = 1$ transitions between bands of opposite parity in these nuclei stands in contrast with the observation of both $\Delta I = 1$ and $\Delta I = 2$ transitions between the yrare and yrast $\pi h_{11/2} \otimes \nu h_{11/2}$ bands.

The transition strength ratio for $\gamma$-rays depopulating a level are a sensitive experimental observable which may be compared with theory. Ratios such as $B(M1; I \rightarrow I - 1)/B(E2; I \rightarrow I - 2)$ for the in-band transitions were extracted from the data for the yrast and yrare $\pi h_{11/2} \otimes \nu h_{11/2}$ bands in $^{136}\text{Pm}$ and $^{138}\text{Eu}$. These ratios are presented in figure 4.13 for both nuclei.

In $^{138}\text{Eu}$, the ratios for the $\pi h_{11/2} \otimes \nu h_{11/2}$ yrast band first decrease and then increase (after the band crossing) as a function of spin. Values derived from the yrare band have a more limited spin range, but have a similar value to those of the yrast band. In addition, both bands show the same staggering of values as a function of spin, with the even spin values being slightly lower, though the error bars and the measurement of only two yrare values make this somewhat uncertain.

The values for the yrast band in $^{136}\text{Pm}$ also decrease as a function of spin, and show a similar staggering. The yrare band for $^{136}\text{Pm}$ has similar values for the range of spins over which it has been measured. With the caveat that the error bars and limited
Figure 4.13: Measured $B(M1; I \rightarrow I-1)/B(E2; I \rightarrow I-2)$ values for the yrast (closed symbols) and yrare (open symbols) $\pi h_{11/2} \otimes v h_{11/2}$ bands. The solid lines are the results of 3D-TAC calculations.
range of observation make the staggering somewhat uncertain, the yrare band shows a similar staggering as a function of spin. The clear similarity of the \(B(M1)/B(E2)\) in-band values of the yrast and yrare bands provides further support for the yrare band being assigned the same \(\pi h_{11/2} \otimes v h_{11/2}\) orbital configuration as the yrast band.

The similarity of \(B(M1; I \rightarrow I-1)/B(E2; I \rightarrow I-2)\) values for the yrast and yrare bands can also be seen in their contrast with values from bands of different orbital configurations. One such contrasting example is presented in figure 4.14, in which the \(B(M1; I \rightarrow I-1)/B(E2; I \rightarrow I-2)\) in-band values for the yrast \(\pi h_{11/2} \otimes v h_{11/2}\) band (labeled "band 2") and the odd parity \(\pi d_{5/2} (h_{11/2})^2 \otimes v h_{11/2}\) band (labeled "band 3"), derived in reference [Pa94a], are presented. The bands have a notably different values over most of the spin range, and especially in the low spin regime where both yrare and yrast \(\pi h_{11/2} \otimes v h_{11/2}\) values were presented in the figure 4.13.

\[B(M1)/B(E2) \times (\mu_N/e\hbar)^2\]

Figure 4.14: Measured \(B(M1; I \rightarrow I-1)/B(E2; I \rightarrow I-2)\) for inband transitions in the yrast \(\pi h_{11/2} \otimes v h_{11/2}\) band (labeled "band 2") and the \(\pi d_{5/2} (h_{11/2})^2 \otimes v h_{11/2}\) band (labeled "band 3") from [Pa94a]. Level spins are \(1\hbar\) lower than presented in the current work. Note the contrast in the values of these bands.
The solid lines in figure 4.13 are the results of 3D-TAC calculations for the $\pi h_{11/2} \otimes v h_{11/2}$ configuration in $^{136}$Pm and $^{138}$Eu. These calculations were performed for triaxial deformed nuclei using the deformation parameters $\varepsilon_2 = 0.194$, $\varepsilon_4 = 0.028$, and $\gamma = -25^\circ$ for $^{136}$Pm and $\varepsilon_2 = 0.202$, $\varepsilon_4 = 0.032$, and $\gamma = -24^\circ$ for $^{138}$Eu. The deformations were obtained from TRS calculations at a frequency of $\hbar \omega \sim 0.25$ MeV for the $\pi h_{11/2} \otimes v h_{11/2}$ configuration for these nuclei. Calculated $B(M1)/B(E2)$ values along with the calculated tilt angle of the total angular momentum are presented in appendix B. The tilt angle is considerably away from any of the principal axes for most of the calculated spin range, which means the angular momentum is aplanar and provides for the system to be chiral. The calculations reproduce both the magnitude and the general trend of the experimental data. The agreement of these aplanar angular momentum calculations points towards the yrast and yrare $\pi h_{11/2} \otimes v h_{11/2}$ bands as being chiral partner bands.

4.6 Additional work on $^{136}$Pm

Following our analysis, additional experimental work was performed on $^{136}$Pm by D. Hartley et al. [Ha01]. The level scheme from that work is presented in figure 4.15. Using the high efficiency Gammasphere array [Le92], the level scheme presented in the current work was confirmed, and the yrare band was observed to higher spins. High spin data from this work was also used to extend the alignment plot presented in the previous section (figure 4.11).

The $B(M1; I \rightarrow I-1)/B(E2; I \rightarrow I-2)$ ratios extracted from the data in [Ha01] are presented in figure 4.16. These values are similar to the values extracted from the current work, also presented in figure 4.16. The ratios for the yrast band decrease with spin and display an odd/even staggering, with higher values for odd spins. It appears that there is a similar reduction in value of the yrare band as a function of spin. With the larger error bars, it is difficult to determine if there is an odd/even staggering in the yrare band. The similarity of the measured $B(M1)/B(E2)$ values of the yrare band and the yrast $\pi h_{11/2} \otimes v h_{11/2}$ band and the excellent agreement with the TAC calculation serves to further confirm the $\pi h_{11/2} \otimes v h_{11/2}$ assignment of the yrare band.
Figure 4.15: Partial level scheme for $^{136}$Pm, from [Ha01].

Results of particle + rotor calculations have been included in figure 4.16.

Deformation parameters for the rotor were taken from TRS calculations, $\varepsilon_2 = 0.194$, $\varepsilon_4 = 0.028$, and $\gamma = -25^\circ$, and were kept constant. This calculation is in good agreement with the experimental ratios for the $\pi h_{1/2} \otimes v h_{1/2}$ bands, and also reproduces the trend of the data, with higher values at low spin. The calculations even reproduce the odd-even staggering that is visible at higher spins.
**Figure 4.16:** Measured $B(M1; I \rightarrow I-1)/B(E2; I \rightarrow I-2)$ values for the yrast and yrare $\pi h_{11/2} \otimes \nu h_{11/2}$ bands from the present work, open symbols. Data from reference [Ha01] is included, closed symbols. The solid and dashed lines are the results of particle + rotor calculations with a triaxial deformed rotor for the yrast and yrare $\pi h_{11/2} \otimes \nu h_{11/2}$ bands, respectively. All blue symbols and lines are yrast, red are yrare.

### 4.7 Summary

Based on the similarities in the alignments, Routhians, level energies, and $B(M1)/B(E2)$ values, and based on the parity conserving properties of the interband transitions, the new yrare bands in $^{136}$Pm and $^{138}$Eu are assigned the same $\pi h_{11/2} \otimes \nu h_{11/2}$ configuration as the yrast $\pi h_{11/2} \otimes \nu h_{11/2}$ bands. Candidate chiral band pairs in $^{134}$Pr have the same $\pi h_{11/2} \otimes \nu h_{11/2}$ orbital configuration for both band pairs, which suggests the band pairs in $^{136}$Pm and $^{138}$Eu as potential chiral bands. The predicted degeneracy breaking between the candidate chiral band pairs in this region is about 300 keV [Zh02a], which is what has been found experimentally for both nuclei. (see figures 4.11 and 4.12). In addition, TRS calculations predict similar triaxial deformations for all three nuclei, a requirement for this type of chiral behavior. Calculations based on 3D-TAC and on the particle + rotor model, both modeling triaxial nuclei with aplanar total angular
momentum, are in good agreement with the experimental data. Following this, both of
the ΔI = 1 band pairs in $^{136}$Pm and $^{138}$Eu are offered as candidate chiral band partners.
Further calculations and comparisons with data are presented in chapters 6 and 7.
Chapter 5

Experimental results on the \( N = 77 \) nucleus \( ^{140}\text{Eu} \)

5.1 Introduction

The doubly-odd \( ^{140}\text{Eu} \) (\( N = 77, Z = 63 \)) nucleus lies at the extremes of both the \( N \) and \( Z \) range in which chiral structures have been found in the \( A \sim 130 \) mass region. It is at the limit where the proton and neutron Fermi surfaces are optimal for the required perpendicular coupling of the single particle angular momenta (for \( Z = 63 \) and \( N = 77 \) the \( k = 5/2 \) proton and \( 11/2 \) neutron projections of the \( h_{11/2} \) orbitals are lowest). In addition, for essentially any low lying configuration, TRS calculations for \( ^{140}\text{Eu} \) show well developed potential energy minima that are close in energy near both \( \gamma = +25^\circ \) and \( -25^\circ \) which persist up to high frequencies.

Until recently, excited states in \( ^{140}\text{Eu} \) were known only from \( \beta \)-decay studies, up to a 125 ms, \( 5' \) isomeric level at an excitation energy given as < 234 keV in reference [Fi91] (reproduced in figure 5.1, left). Recently, two independent studies of \( ^{140}\text{Eu} \) were carried out [Ta02, Cu02], in addition to our own [He02, He03], and several rotational bands have been reported based on prompt coincidence data [He02]. The isomer decay tagging method was used to identify several transitions which feed the \( 5' \) level as well as a new, higher lying, \( \sim 300 \) ns isomer [Ta02] (see figure 5.1, right). Two rotational bands were reported, assigned as the \( \pi h_{11/2} \otimes \nu h_{11/2} \) and \( \pi h_{11/2} \otimes \nu g_{7/2} \) configurations, feeding into the \( \sim 300 \) ns and into the 125 ms, \( 5' \), isomeric levels, respectively [Cu02] (see figure 5.2).
Figure 5.1: $^{140}\text{Eu}$ level scheme from [Fi96] (left) and [Ta02] (right). Note the 125 ms ($S'$) and ~300 ns isomers. The 170.7, 191, 361.7, and 285.3 keV transitions above the $S'$ isomer (right figure) are identified with the 170.6, 191.1, 362, and 285.4 keV transitions in bands 1 and 2 of the current work (see figure 5.3).

In the current work, high statistics prompt $\gamma-\gamma$ and $\gamma-\gamma-\gamma$ coincidence data from YRASST Ball were used to develop and greatly expand the level scheme for $^{140}\text{Eu}$. Transitions were assigned to $^{140}\text{Eu}$ based on $K_\alpha$ x-ray coincidences and following an excitation function. Other transitions were assigned through coincidences with these $\gamma$-rays. Several of the results reported in references [Ta02, Cu02] were confirmed and many new levels and several new bands were identified. Several of the transitions earlier reported in [He02, Cu02] were found to form another new band, which is presented as a candidate chiral partner band. These new results are published in reference [He03].
5.2 Populating high spin states in $^{140}$Eu

High spin states in $^{140}$Eu were populated following the reaction $^{92}$Mo($^{51}$V, 2pn) at a beam energy of 205 MeV, the target consisting of 2 stacked 700 µg/cm$^2$ foils. The beam was provided by the ESTU tandem Van de Graaff accelerator at the Wright Nuclear
Structure Laboratory at Yale University. Prompt decay γ-rays were measured using the YRAST Ball detector array [Be00]. For this experiment, YRAST Ball consisted of 7 Compton suppressed segmented clover Ge detectors [Du99] mounted at 90° with respect to the beam axis, each with ~ 150% efficiency relative to a standard 3" × 3" NaI detector. In addition, 16 Compton suppressed coaxial Ge detectors, each with ~ 25% relative efficiency, were mounted in three rings at 50° (6 detectors), 126° (8 detectors), and 160° (2 detectors) with respect to the beam axis. For additional sensitivity to low energy γ-rays and x-rays, three LEPS detectors were also mounted in the array, two at 50° and one at 90°. The total photopeak efficiency of YRAST Ball in this configuration is about 2.5%.

The bombardment energy of 205 MeV was chosen following an excitation function measurement where the beam energy was varied from 190 - 220 MeV. The yield of several high intensity transitions from 140Gd, another 3-particle (p2n) evaporation channel whose level scheme is well known, was found to be optimal at 205 MeV. At this energy, about 15% of the reaction intensity went into 140Eu, with the strongest population going into the nucleus 140Gd. In a five-day experiment, a total of 1.0×10^9 unfolded double and 4.7×10^8 unfolded triple coincidences were measured using YRAST Ball.

The level scheme of 140Eu deduced from this data set is presented in figure 5.3. The level scheme was constructed using both triples and doubles gamma coincidence data with the Radware analysis software [Ra95]. The use of triples coincidences gives a decided advantage over doubles, as many of the transitions in 140Eu are similar in energy to γ-rays that occur with significant intensity in neighboring nuclei, and single gates placed on these transitions are inevitably strongly polluted. To illustrate the quality of the triples data, some typical double-gated γ–γ–γ spectra are presented in figures 5.7, 5.8, and 5.10. Details of these spectra will be discussed below. The measured γ-ray transition energies and total intensities, together with the extracted DCO ratios, angular distributions, and Compton asymmetries for transitions in 140Eu are presented in table
140Eu

Figure 5.3: 140Eu level scheme from the current work, see reference [Hed03].
format in appendix B. In general, the centroid values for the transition energies were
determined using the doubles coincidence data and only when uncontaminated gates were
not available, using the triples coincidences. The intensities in the table are presented as
relative intensities, with the intensity of the 640.6 keV transition defined as 100.

![Graph](image)

**Figure 5.4:** Angular distribution $a_2$ values for transitions in $^{140}$Eu. Dashed (solid)
bounded regions are error limits of known quadrupole (dipole) transitions from
neighboring nuclei. Open (closed) symbols are assigned dipole (quadrupole) transitions.
The $\gamma$-ray efficiencies are not well established for transitions below $\sim 200$ keV, which
probably accounts for the upward trend in the $a_2$ values for the low energy transitions.

Transition multipolarities were determined following angular distribution, angular
correlation, and Compton polarimetry analyses. For the angular distribution analysis, the
four detector angles of YRAST Ball, 160°, 125°, 90° and 50° with respect to the beam
axis, were used. Due to the frequent occurrence of overlapping peaks and the weak
intensity of others, a *gated* angular distribution analysis was utilized. The angular
distribution function $W(\theta) = A_0 + A_2 P_2(\cos \theta)$ was fit to the data, as discussed in chapter
3. The results, given as the ratio \( a_2 = A_2/A_0 \), are presented in appendix B. The \( a_2 \) values are also presented in figure 5.4 together with a weighted average of \( a_2 \) values from known transitions in \(^{137}\text{Sm}\), 0.38(7) for known quadrupole and -0.15(6) for known dipole transitions. An \( M1/E2 \) admixture could account for the lower magnitude values for some of the known dipole transitions.

![Figure 5.5: DCO ratios for transitions in \(^{140}\text{Eu}\). Symbols are the same as in figure 5.4.](image)

A directional correlation of oriented states (DCO) analysis was also used to aid in the assignment of transition multipolarities. For this analysis the 90° and 160° detector rings were utilized, the data being sorted into a E(90°) versus E(160°) coincidence matrix. Results of the DCO analysis are presented in appendix B and are summarized in figure 5.5. For comparison, weighted averages from several measured DCO ratios for known dipole and quadrupole transitions from \(^{137}\text{Sm}, \,^{138}\text{Sm}, \,^{138}\text{Eu}, \,^{141}\text{Gd}\) are also included in figure 5.5. When corrected for the efficiency of both the gating and
coincident transitions, ratios of $R_{\text{DCO}} \sim 0.90(4)$ and $\sim 0.62(2)$ ($\sim 1.71(8)$ and $\sim 0.85(2)$) are found for the known $\Delta I = 1$ and $\Delta I = 2$ transitions, respectively, when gating on a $\Delta I = 1$ ($\Delta I = 2$) transition.

![Graph showing asymmetry ratio, ε, for transitions in $^{140}$Eu. Symbols are the same as in figures 5.4 and 5.5.](image)

The YRAST Ball clover detectors were used as Compton polarimeters [Du99, Sc94] to help assign the electromagnetic character of the transitions. Extracted values for the Compton asymmetry ratios, $\varepsilon$, are summarized in figure 5.6 and appendix B for many of the transitions assigned to $^{140}$Eu. Compton asymmetry ratios were also extracted for several known $M1$ and $E2$ transitions from the neighboring nuclei $^{137,138}$Sm. The weighted averages for these transitions, 0.09(2) and -0.06(1) respectively, are included in figure 5.6 for comparison.
5.3 The level scheme

As mentioned above, the low lying structure of $^{140}\text{Eu}$ is characterized by two isomeric states, a long lived (125 ms) $5^-$ isomer and a recently identified, higher-lying, $\sim 300$ ns isomer [Ta02], linked to the $5^-$ isomer via several intermediate levels and transitions. In the present experiment, the recoil velocity of the $^{140}\text{Eu}$ nuclei was $v/c \sim 3\%$ or $\sim 10$ mm per ns. Thus the experiment is not sensitive to the $\gamma$-rays directly depopulating the isomeric levels known in this nucleus, but prompt feeding, which bypasses the isomeric states, may be observed. The 170, 191, 361, and 285 keV transitions identified in the isomer decay tagging experiment [Ta02], and connecting levels intermediate between the two isomers, were seen both in the present work and the excitation function measurement, implying prompt feeding which bypasses the $\sim 300$ ns isomer, and confirming their assignment to $^{140}\text{Eu}$.

Band 1, shown on the left hand side of figure 5.3, was placed directly feeding the $5^-$ isomer [Cu02]. The lower part of band 1 consists of a series of four $\Delta I = 1 M1/E2$ transitions with crossover $\Delta I = 2 E2$ transitions. The intense 170.6 and 191.1 keV transitions (visible in the total projection) and the weaker 362.0 keV crossover transition are identified with the 170, 191, and 361 keV transitions that directly populate the $5^-$ isomeric level in reference [Ta02]. At higher spins, the decay intensity in-band 1 is carried by $E2$ transitions with weak or unobserved dipole transitions. The $\sim 170$ keV transition is a doublet, the second member of which, 170.0 keV, is placed in band 2. A double gate placed on the 191.1 and $\sim 170$ keV transitions, figure 5.7 (a), shows the doublet character of the 170 keV transition, the expected strong coincidences with other transitions in band 1, and the higher lying transitions in band 2. The 362.0, 537.4, 715.6, and 830.0 keV transitions were also reported in reference [Cu02]. However, the higher lying $15^- \rightarrow 13^- 843$ keV transition in reference [Cu02] could not be confirmed in the present work.
Figure 5.7: (a) Double gate on the 170 and 191 keV transitions. (b) Double gate on the 345 keV transition and any of the 170, 191, 362, 537, 716 keV transitions in band 1.

The probable signature partner band, consisting of the 484.5 and 722.0 keV $E2$ transitions together with the connecting $\Delta I = 1$ transitions (244.0 and 292.9 keV), is reported here for the first time. The 925.0 keV transition is seen in both the present work and also in reference [Cu02]. Figure 5.7 (b), a double gate on the 345.4 keV transition.
and low lying members of band 1 shows that the 345.4 and 925.0 keV transitions are in coincidence with one another and members of band 1. We have placed these transitions with the 345.4 keV transition directly feeding the 111’ level of band 1, though the ordering of these two transitions is uncertain. The expected 122’ → 101’ E2 transition is not observed.

A short band consisting of a series of five ΔI = 1 transitions is shown on the left of band 1 in figure 5.3. These transitions are mutually coincident and are in coincidence with low lying transitions in band 1, though direct linking transitions have not been identified. The dashed lines in figure 5.3 indicate possible decay paths. For this short band, spin assignments are tentative and are given relative to the lowest level in the band. The transitions have been ordered such that the transition energies typically increase, and the total transition intensities typically decrease, with increasing spin.

Band 2, shown near the left in figure 5.3, was observed for the first time in our analysis. Band 2 is dominated by ΔI = 2 E2 transitions at lower spins. However, at spin I ~ 12 h, a band crossing occurs (see below) after which the nature of the band changes and dipole transitions dominate the decay with weak or unobserved ΔI = 2 E2 crossover transitions. A typical double gated spectrum for transitions in band 2 are presented in figure 5.8. Several weak branches, including the 612.9, 396.8 keV sequence and the tentative 366.3 keV transition, feed into band 2 at higher spins.

Two ΔI = 2 E2 transitions (821.0 and 709.4 keV) and two ΔI = 1 M1/E2 transitions (401.4 and 285.4 keV) connect levels in band 2 and band 1 allowing firm spin, parity, and excitation energy assignments for band 2 relative to band 1. The 285.4 keV transition, assigned as a ΔI = 1 M1/E2 transition, feeds directly from the lowest observed level of band 2 to the 51’ isomeric level. Thus band 2 is assigned negative parity with the lowest observed level having a spin I = 6. The 366.3 and 379.6 keV transitions are tentative, and are shown in figure 5.3 with dashed arrows.

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Band 3, shown near the middle of figure 5.3, at lower spins consists of a sequence of six $\Delta I = 1 M1/E2$ transitions with crossover $\Delta I = 2 E2$ transitions. A band crossing occurs around spin 15, after which the in-band decay is dominated by $\Delta I = 2 E2$ transitions. The observed transitions in band 3, up to spin $25^+$, confirm the band reported as $\pi h_{11/2} \otimes \nu h_{11/2}$ in reference [Cu02].

Relative spin assignments were made following DCO, angular distribution, and Compton asymmetry measurements. Note that the spins reported here are two units higher than in reference [Cu02]. Our spin assignments are based on a systematic study of level energies in the $\pi h_{11/2} \otimes \nu h_{11/2}$ bands in neighboring $N = 73, 75,$ and 77 nuclei [Li96]. The energies of the known $\pi h_{11/2} \otimes \nu h_{11/2}$ bands, relative to the $10^+$ level, are plotted for odd-odd nuclei from $^{55}$Cs through $^{63}$Eu and for $N = 73$ through 77, see figure 5.9. As can be seen for nuclei of the same $Z$, the level spacing increases as $N$ increases from 73 to 77, approaching the closed neutron shell at $N = 82$. In contrast, for the same
Figure 5.9: Systematics of level energies in $\pi h_{11/2} \otimes v h_{11/2}$ bands in the mass 130 region as a function of proton number, plotted for several different neutron numbers. The level energies are plotted relative to the $10^+$ level, following the level assignments of Liu et al. [Li96]. $^{140}$Eu, with $N = 77, Z = 63$, lies at the far right. Level assignments of Cullen et al. [Cu02] are connected with dashed lines. Level assignments from our work [He03] are connected with solid lines.

$N$, the level spacing decreases as $Z$ increases from 55 to 63, towards the proton mid-shell. The excitation energies for the $\pi h_{11/2} \otimes v h_{11/2}$ band in $^{140}$Eu are plotted on the far right in figure 5.9 using the spin assignments from the present work (symbols connected with solid lines) and from [Cu02] (symbols connected with dashed lines). Using the assignments from [Cu02], the excitation energies show a sudden upturn for the levels above the $10^+$ level, and sudden downturn for the $9^+$ level, contrary to the trends described. Since the energy difference between neighboring levels generally increases smoothly with spin, the relatively large difference in energy between neighboring levels
for $^{140}$Eu supports a revision of the spin assignments. The $^{140}$Eu data, plotted with our spin assignments shown on the far right in figure 5.9, connected with solid lines, continues the trend of smoothly decreasing inter-level spacing with increasing $Z$, and follows the trend of larger inter-level spacing with increasing $N$. In addition, a 71.0 keV transition not reported in reference [Cu02] was placed at the bottom of band 3 as a new $10^+ \rightarrow 9^+$ transition. The 71.0 keV transition is strong in figure 5.10 (c), a double gate on the 640.6 and 446.0 keV transitions in band 3. It is similar in energy to the low energy $10^+ \rightarrow 9^+$ transitions seen in other nuclei in the systematics plot, further supporting its placement and spin assignment.

Curiously, the low-lying, even-spin levels in band 3 are populated more strongly than the odd spin levels. The 720.6 and 831.9 keV $E2$ transitions connecting the odd-spin low-lying levels are less intense than the 640.6, 807.3, and 920.0 keV $E2$ transitions. Nevertheless, it is the odd-spin levels that are observed to higher spins (25 h compared to 16 h), indicating that the even-spin levels must lie higher in energy following the band crossing. The $16^+ \rightarrow 14^+ 920.0$ keV transition confirms the 920 keV transition reported in [Cu02]. The odd spin sequence reported here agrees with the results of reference [Cu02] up to the 991.7 keV transition. We were unable to confirm the 962, 890, and 1074 keV transitions seen in that reference.

Band 3 has been identified as the $\pi h_{11/2} \otimes v h_{11/2}$ yrast band. The energies of the levels in band 4 are similar, but the levels lie slightly higher than those of the yrast $\pi h_{11/2} \otimes v h_{11/2}$ band. In the context of a search for chiral band pairs, this near degeneracy makes band 4 very interesting. The 94.9, 406.6, 646.8, and 787.3 keV transitions were previously reported in reference [Cu02], where they were placed as a side feeding cascade, above and in coincidence with the 754.2 keV transition of band 3. While their coincidence with the 754.2 keV transition is confirmed in our experiment, a triples coincidence analysis indicates that the transitions are considerably lower in the level scheme and form a short rotational band, as presented in figure 5.3. In particular, the 787.3 keV transition is not in coincidence with any of the transitions lying between the $15^+$ and $12^+$ levels in band 3, placing it parallel to these levels. This can be seen in figure
5.10 (a), a double gate on the 787.3 and 646.8 keV transitions. Furthermore, the 406.6 and 94.9 keV transitions are not in coincidence with any band 3 member below the 15\textsuperscript{+} level, see figure 5.10 (b). Finally, figure 5.10 (c) shows a double gate on the 446.0 and 640.6 keV band 3 transitions. No evidence is seen in this spectrum for the low lying and most intense band 4 members. The observation of the 502.1, 490.4, and 483.5 keV linking transitions confirms our placement. Most of the intensity of band 4 comes from band 3 via the 787.3 keV linking transition. Within band 4, the dashed 817.5 keV transition weakly links the 616.0 keV transition as part of the same band. The 502.1 keV transition feeds from directly below the 616.0 keV transition to band 3 and takes some of the intensity away from band 4. The \( \sim 647 \) keV transition is a doublet, occurring as 646.8 keV in band 4, with the stronger partner occurring as 647.7 keV in band 3. Thus, coincidences with this transition bring back too much background to clearly see the 817.5 keV transition. However, the 817.5 keV transition is seen in double gated coincidence spectra gated by the 616.0 and 406.6 keV transitions.

DCO analysis, Compton asymmetries, and angular distribution measurements show that band 4 is comprised of four \( \Delta I = 2 \ E2 \) transitions and one \( \Delta I = 1 \) mixed \( M1/E2 \) transition, and is connected to band 3 by \( E2 \) (787.3 and 511.5 keV) and mixed \( M1/E2 \) (502.1, 490.4, and 483.5 keV) transitions. Following the arguments of \(^{136}\text{Pm}\) and \(^{138}\text{Eu}\), the parity conserving character of both the \( M1/E2 \) and \( E2 \) inter-band transitions requires a total positive parity configuration for band 4.

Finally, another new band, band 5 (shown on the right side of figure 5.3) consists of a series of three \( \Delta I = 2 \) transitions, 609.8, 619.3 and 619.3 keV, that are strongly in coincidence with both the 94.9 keV transition of band 4 and the 386.1 keV, probably \( \Delta I = 2 \) transition. The 386.1 keV transition is easily seen in double coincidence gates placed on the 94.9 and 646.8 keV transitions, and is placed at the bottom of the band. The 609.8 keV and 619.3 keV doublet are strongly in coincidence with the 386.1 and 94.9 keV transitions, but are not in coincidence with any other transitions identified in \(^{140}\text{Eu}\). A very low energy 20.5 keV transition connecting the 646.8 and 386.1 keV transitions is implied by the coincidence of the 646.8 and 386.1 keV transitions, although the 20.5 keV
Figure 5.10:  (a) Double gate on the 647 keV doublet and 787 keV transition. Low-lying transitions in band 4 and high-lying transitions in band 3 are clearly visible. The energies of low-lying transitions in band 3 that are not in coincidence are indicated with circles.  
(b) Double gate on the 647 keV doublet and either the 407 or 95 keV band 4 transitions.  
The 787 keV transition linking bands 3 and 4 is very intense and the low-lying band 4 transitions and high-lying band 3 transitions are visible. Once again, circles indicate low-lying band 3 transitions that are not seen in coincidence.  
(c) Double gate on the 641 and 446 keV transitions in band 3. High-lying members of band 3 are present. The energies of the low-lying members of band 4 that are not in coincidence are indicated with circles.
transition has not been observed. The 20.5 keV transition must be fast enough for the 646.8 and 386.1 keV transitions to both occur in the sensitive region of the array, a concern since transition half lives are inversely dependent on transition energy. Calculated Weisskopf single particle estimates [We51] for the 20.5 keV transition half-life are 30 ps and 3 ns sec for E1 and M1 transitions, respectively, fast enough to be observed in the present experiment. The large electron conversion coefficient for this low energy transition, (α ~ 100 for M1 and ~ 10 for E1 transitions [Fi96]), and the low sensitivity of the detector array at this low energy, leaves the 20.5 keV transition as only conjectural. The ordering of the other transitions in band 5 is based on intensity arguments.

5.4 Discussion

Total Routhian surface calculations for $^{140}$Eu for essentially any low-lying proton and neutron orbital configurations predict a γ–soft potential energy surface with two potential energy minima, at a deformation of β ~ 0.2 and γ ~ ± 25°, see figure 5.11. These minima persist over a wide frequency range from $\hbar \omega = 0$ to ~ 0.450 MeV, corresponding to a spin of I ~ 24 h. These minima are close in energy up to frequencies of at least $\hbar \omega ~ 0.450$ MeV and spin ~ 24 h for both low lying positive and negative parity configurations. For these deformations, proton orbitals close to the Fermi surface include the negative parity $h_{11/2}$ orbitals, originating from the low - $\Omega$ [541]3/2 Nilsson configuration, and the (mixed) positive parity $d_{5/2}$ [411]3/2 and $g_{7/2}$ [413]5/2 orbitals. For neutrons, the orbitals close to the Fermi surface include the negative parity $h_{11/2}$ orbital from the high - $\Omega$ [505]11/2 orbital and the positive parity $d_{3/2}$ [400]1/2 orbital.

5.4.1 The negative parity bands

We consider first the negative parity bands, bands 1 and 2. The aligned angular momenta for bands 1 and 2 are plotted in figure 5.12 (a) as a function of rotational frequency. The Harris parameters used, $\Sigma_0 = 5 \hbar^2/\text{MeV}$ and $\Sigma_1 = 45 \hbar^4/\text{MeV}^2$, were
Figure 5.11: Polar coordinate plots of TRS calculations at $\hbar \omega = 0.2$ MeV. The top figure is for proton $(\pi, \alpha) =$ neutron $(\pi, \alpha) = (-, -1/2)$, which describes the $\pi \, h_{11/2} \otimes \nu \, h_{11/2}$ configuration. The bottom figure is for proton $(\pi, \alpha) = (+, +1/2)$, neutron $(\pi, \alpha) = (-, -1/2)$, which describes the $\pi \, (d_{5/2}, g_{7/2}) \otimes \nu \, h_{11/2}$ configuration. There are potential energy minima in both plots for $\beta \sim 0.2$, $\gamma \sim 25^\circ$ and $-25^\circ$. Contours are 175 keV.
chosen to subtract the angular momentum of the core for \(^{134}\)Pr. The same parameters are used for consistency with the general systematics plot to follow in chapter 6. For band 1, no backbend is observed through the highest transitions identified, while band 2 shows a sharp backbend centered at \(\hbar \omega \sim 0.28\) MeV, with \(\Delta \delta \sim 6\) h. As particles in the intruder \(h_{11/2}\) orbitals have the largest angular momentum in this mass region, they are the most probable particles participating in the pair alignment. The large increase in alignment of \(\Delta \delta \sim 6\) h observed in figure 5.12 (a) is consistent with the angular momentum gain from the alignment of a pair of \(h_{11/2}\) particles.

Quasiparticle Routhian calculations were performed for the deformation \(\beta_2 = 0.19\), \(\beta_4 = -0.021\), and \(\gamma = -26^\circ\), and are presented in figure 5.13. As can be seen, the lowest frequency band crossing is calculated to occur at \(\hbar \omega \sim 0.32\) MeV, corresponding to the alignment of a pair of \(h_{11/2}\) protons, in good agreement with the experimental alignment plot for band 2. The observation of the EF \(\pi h_{11/2}\) crossing suggests that the proton configuration for band 2 below the backbend does not involve the \(\pi h_{11/2}\) orbital. The probable orbital configurations then are either proton \(d_{5/2}\) or \(g_{7/2}\) coupled to an \(h_{11/2}\) neutron. Following the Gallagher-Moszkowski rules for coupling of single particle angular momenta, the likely \(K = \Omega_p \pm \Omega_n\) for the band is 6 for the \(\pi d_{5/2} \otimes \nu h_{11/2}\) band and 2 for the \(\pi g_{7/2} \otimes \nu h_{11/2}\) band [Ga58]. It is misleading though to describe band 2 as solely \(\pi d_{5/2} \otimes \nu h_{11/2}\) or \(\pi g_{7/2} \otimes \nu h_{11/2}\) due to the mixing which is likely to occur between the two proton orbitals.

The proton \(h_{11/2}\) orbital coupled to the low lying neutron \(d_{3/2}\) orbital is the most probable configuration for the negative parity states at low spin, below the 5' isomer [Ta02]. That said, the negative parity configurations above the 5' isomer probably do not include the \(d_{3/2}\) neutron orbital. The lowest lying total negative parity configuration at higher spins is calculated to be mixed \(\pi (g_{7/2}, d_{5/2}) \otimes \nu h_{11/2}\). Thus, one possibility is that both band 1 and band 2 involve the \(\pi (g_{7/2}, d_{5/2}) \otimes \nu h_{11/2}\) configuration. We briefly note that, in reference [Cu02], the band that we have labeled as band 1 is assigned the
\( \pi h_{11/2} \otimes \nu g_{7/2} \) configuration in figure 5 of that reference and the \( \pi h_{11/2} \otimes \nu d_{5/2} \) configuration in the manuscript, both of which we consider to be less likely.

Figure 5.12: (a) Alignment plot of bands 1 and 2. Squares denote transitions between odd spin levels, circles between even spin levels. The open symbols are for band 1 while the closed symbols are for band 2. (b) Alignment plot of bands 3 and 4. Squares denote transitions between odd spin levels, circles between even spin levels. Open symbols are for band 3, closed symbols are for band 4.
Figure 5.13: Calculated Woods-Saxon quasiparticle Routhian plot for N=77, Z=63. The parity and signature (\(\pi, \alpha\)) of the orbitals are represented by solid lines, (+,+1/2); dotted lines, (+,-1/2); dot-dash lines, (-,+1/2); dashed, (-,-1/2). The deformation chosen for the calculations is \(\gamma \sim -25^\circ\), from TRS calculations. (a) Plot for neutron configurations. (b) Plot for proton configurations.
The $\Delta I = 1$ side band shown to the left of band 1 is intriguing. Though the specific linking transitions could not be identified, the band is in coincidence with band 1. Furthermore, it is tantalizingly similar to the high spin portion of band 2. The similarity leads one to speculate that these transitions form a continuation of band 1 after a band crossing, similar to that observed in band 2. Further experiments are required to clarify the structure of the bands and determine if there is a sharp backbend in band 1. These bands and their possible interpretations will be discussed further below.

5.4.2 The positive parity bands

Band 3 is the yrast band based on the $\pi h_{1/2} \otimes v h_{11/2}$ orbital configuration [He02, Cu02]. As discussed in chapter 4, strongly populated yrast bands based on a $\pi h_{11/2} \otimes v h_{11/2}$ configurations are well known in neighboring odd-odd nuclei, see figure 5.9. In addition, quasiparticle Routhian calculations suggest that both the proton and neutron $h_{11/2}$ orbitals lie near the Fermi surface for $N = 77$ and $Z = 63$ at the calculated deformation. As previously mentioned, a systematic comparison of the energy levels in band 3 with those of $\pi h_{11/2} \otimes v h_{11/2}$ double intruder bands from neighboring $N = 73, 75, 77$ nuclei strongly suggest a $\pi h_{11/2} \otimes v h_{11/2}$ positive parity configuration for band 3, confirming the assignment of [Cu02].

The orbital configuration of band 4 can be deduced by its relation to band 3. Band 4 is linked to band 3 by a series of parity conserving $M1/E2$ and $E2$ transitions, establishing band 4 as positive parity. Moreover, for both the proton and neutrons, the $h_{11/2}$ orbitals are intruder orbitals and are the only negative parity orbitals near the Fermi surface. The possible configurations for band 4 then are either the total positive parity $\pi h_{11/2} \otimes v h_{11/2}$ orbital configuration or a combination of proton and neutron orbitals that are individually positive parity. As mentioned in the previous chapter, the selection rules for $M1$ and $E2$ transitions make it exceedingly unlikely that both proton and neutron configurations are changed in a single transition, and so band 4 is assigned the same $\pi h_{11/2} \otimes v h_{11/2}$ orbital configuration as band 3.
Further evidence for this configuration assignment is provided in the alignment plot of bands 3 and 4, presented in figure 5.12 (b), where one observes the same large initial aligned angular momentum for the bands (iₚ ~ 7 h), consistent with π h₁₁/₂ ⊗ υ h₁₁/₂ assignment. Band 3 (solid points in figure 5.12 (b)) backbends at a frequency of hω ~ 0.31 MeV. Band 4 (open points) is not observed to high enough energies to complete its backbend, but both bands 3 and 4 begin to align at the same frequency, hω ~ 0.37 MeV. Cranked shell model calculations for β ~ 0.2, γ ~ +25° and β ~ 0.2, γ ~ -25°, (see figure 5.13), predict the second h₁₁/₂ band crossing (FG) at hω ~ 0.45 MeV for negative γ deformation, but at only hω ~ 0.35 MeV for positive γ deformation. The latter value of hω ~ 0.35 MeV is in fairly good agreement with experimental results, hω ~ 0.3 MeV.

Calculations of the in-band transition strengths B(M1: I → I - 1) and B(E2: I → I - 2) were performed by Jing-ye Zhang for the π h₁₁/₂ ⊗ υ h₁₁/₂ and the π g₇/₂ ⊗ υ h₁₁/₂ orbital configurations in 140Eu using the particle + rotor model with triaxial deformation, γ ~ 25°. Measured values for the reduced transition probability ratio B(M1)/B(E2) are presented along with the calculated values in figure 5.14 and appendix B. For band 3, there is reasonable agreement between our measured B(M1)/B(E2) values and the calculated values except for the first point. For band 1 on the other hand, experimental B(M1)/B(E2) values are far off those of the particle + rotor model calculations. Unlike the h₁₁/₂ negative parity "intruder orbitals", which have relatively little mixing with other orbitals, the positive parity orbitals in this mass region may be strongly mixed. Perhaps the discrepancy between experimental and modeled B(M1)/B(E2) values for band 1 is due to the difficulty of modeling the highly mixed positive parity proton orbitals of this π (g₇/₂, d₅/₂) ⊗ υ h₁₁/₂ configuration band.

Unfortunately, we do not have measurements of the individual B(M1) or B(E2) values. Lifetime measurements are needed to compare individual B(M1) and B(E2) values with calculations to clarify the discrepancy. In addition to in-band branching ratios, in-band to out-of-band transition branching ratios, B(M1: I → I -1)_{in} / B(M1: I → I -1)_{out} and B(E2: I → I -2)_{in} / B(E2: I → I -2)_{out} were measured for several transitions in bands 2 and 3. These values are also presented in appendix B.
Figure 5.14: Reduced transition strength ratio $B(M1)/B(E2)$ for in-band transitions. Open symbols are data while the filled symbols with solid lines are from particle + rotor calculations. Diamonds are for band 1, squares for band 3. See text for details.

5.4.3 Possible chiral structures in $^{140}$Eu

In the odd-odd mass $A \sim 130$ nuclei all chiral band pairs have been found to have the $\pi h_{11/2} \otimes \nu h_{11/2}$ orbital configuration. In this respect, it is important that both bands 3 and 4 have a $\pi h_{11/2} \otimes \nu h_{11/2}$ configuration. Furthermore, the bands exhibit a near degeneracy of levels of the same spin, typical of chiral partner bands, with a difference of only $\sim 30$ keV at spin 13 h and 15 h to $\sim 140$ keV at spins 11 h and 17 h, as summarized in figure 5.15 (top). Therefore, band 4 may be the chiral partner band to band 3.

However, there is no apriori reason that chiral structures may not be found in bands based on configurations different from $\pi h_{11/2} \otimes \nu h_{11/2}$. In fact, as was mentioned, candidate chiral bands with different configurations have been identified in other mass
Figure 5.15: Level energies as a function of spin in $^{140}$Eu for the $\pi h_{11/2} \otimes v h_{11/2}$ configuration bands 3 and 4 (top) and the $\pi (g_{7/2}, d_{5/2}) \otimes v h_{11/2}$ configuration bands 1 and 2 (bottom)

regions. In this regard, it is interesting that bands 1 and 2 are both assigned the same $\pi (g_{7/2}, d_{5/2}) \otimes v h_{11/2}$ orbital configuration. Similar to the relation between bands 3 and 4, the levels of the same spin in bands 1 and 2 are also nearly degenerate, with a separation
of ~ 110 keV for the 6’ and 8’ levels, and only ~ 10 keV for the 10’ level, see figure 5.15 (bottom). Added to that, the $\Delta I = 1$ band segment feeding into band 1 is similar to the $\Delta I = 1$ high spin structure of band 2. This leads to the interesting possibility that perhaps bands 1 and 2 are also chiral partners. The Fermi surface is about mid-shell for the $g_{7/2}$ and $d_{5/2}$ orbitals and their angular momenta are less than the $h_{11/2}$ orbital. Thus the proton angular momentum vector is not predicted to be as strongly oriented toward a nuclear axis as for the $h_{11/2}$ particle and the total angular momentum vector would not be as strongly aplanar. However, TRS calculations predict similar triaxial minima at $\gamma = 25^\circ$ for the $g_{7/2}$ or $d_{5/2}$ proton coupled with a $h_{11/2}$ neutron as for the $\pi h_{11/2} \otimes \nu h_{11/2}$ configuration, so the total deformation is predicted to be triaxial, and the total angular momentum still may be aplanar. Thus we have the interesting possibility that bands 1 and 2 may also be chiral partners. Of course, this is only speculative since, in contrast to the double-intruder $\pi h_{11/2} \otimes \nu h_{11/2}$ configuration, there are several natural parity proton orbitals near the Fermi surface, allowing many possibilities for a total negative parity band structure. It is always possible for an accidental degeneracy of levels in such structures to occur. To clarify the situation, further measurements are required, such as are discussed later.

However, there is a problem with the chiral interpretation. In the chiral scenario, the signature-like splitting within the yrast and yrare chiral bands should be small, leading to $\Delta I = 1$ bands. In this context, the lack of evident partners for bands 4 and 2 is troublesome. It is possible that the intensity of the partner bands may be below the sensitivity of this experiment. A rough estimate of the population intensity of the weaker partner for a variety of energy splittings can be obtained from an examination of several of the nuclei populated in the current experiment. The odd neutron neighbor $^{137}$Sm, has a signature splitting of 100-150 keV in the $\nu h_{11/2}$ yrast band [Ma 1989]. The less favored signature partner of this band is clearly identifiable in our data, with an intensity about 70% that of the favored signature. For the $\pi h_{11/2}$ yrast band of the odd proton neighbor $^{141}$Eu, the splitting is 350-400 keV and the intensity of the signature partner drops to about 10% of the favored signature [Xu91].
The typical observed splitting within a $\pi h_{11/2} \otimes \nu h_{11/2}$ chiral band in the region is about 50 - 100 keV, and the splitting within band $3 \sim 100$ keV. The weaker partner of band 3, consisting of the levels with odd spin, is populated with about 15-20% the intensity of the favored signature. Assuming, as a rough estimate, that the unobserved partners of band 4 and band 2 were populated with an intensity ratio of $\sim 15-20\%$, we would expect a relative intensity of transitions in bands 2 and 4 of $\sim 5\%$. If this rough estimate is generous, then the signature partners of bands 4 and 2 could be easily missed. Therefore the "missing partners," although troublesome, do not necessarily rule out the chiral interpretation for band 2 and band 4.

5.5 Summary

The high spin level scheme for $^{140}$Eu was developed following the reaction $^{92}$Mo($^{51}$V, 2pn) at a beam energy of 205 MeV. A total of 5 bands and 69 transitions were placed in the new level scheme. The doubly intruder $\pi h_{11/2} \otimes \nu h_{11/2}$ configuration was assigned to bands 3 and 4, while the $\pi (g_{7/2}, d_{5/2}) \otimes \nu h_{11/2}$ configuration was assigned to bands 1 and 2. Both bands 1 and 2 as well as bands 3 and 4 show some of the features expected from chiral twin bands, namely the same orbital configuration and a near degeneracy of levels of the same spin and parity. In addition, candidate chiral bands in neighboring nuclei have the same $\pi h_{11/2} \otimes \nu h_{11/2}$ orbital configuration found in bands 3 and 4. TRS calculations predict a triaxial deformation over a large range of spins for all reasonable orbital configurations. In addition, calculated values of the B(M1)/B(E2) ratio based on the particle + rotor model for a triaxial deformation of $\gamma \sim 25^\circ$ and an aplanar total angular momentum agree reasonably well with measured values from band 1 for spin 13 and above. Additional particle + rotor calculations which also show agreement with data will be presented in chapter 6. For the chiral interpretation we expect the level degeneracy between the two chiral partners to increase up to I $\sim 15$ h as indeed is observed for the odd spin sequence in bands 3 and 4 and for the even spin sequence in bands 1 and 2. The interpretation for band 5 is unclear.
Chapter 6

Analysis of the experiments in light of calculations and global trends

In chapters 4 and 5, the experiments which populated high spin states in $^{136}\text{Pm}$, $^{138}\text{Eu}$, and $^{140}\text{Eu}$ were presented, and the results of those experiments were discussed in the context of the proposed chiral band structures. In the present chapter, these results will be put into a larger context: the results are compared and contrasted with the systematic properties of candidate chiral bands observed in the mass A ~ 130 region.

These properties include the alignment properties, the energy degeneracy breaking between $\pi h_{11/2} \otimes \nu h_{11/2}$ band pairs, the staggering of the level energies as a function of spin, and/or staggering of the $B(M1; I \rightarrow I - 1)/B(E2; I \rightarrow I - 2)$ values as a function of spin. Data used in this chapter were taken from the following references: $^{128,130}\text{Cs}$, $^{130}\text{La}$, and $^{132}\text{Pr}$ [Ra03]; $^{132}\text{La}$ and $^{134}\text{Pr}$ [St01a]; $^{134}\text{La}$ [Ba01]; $^{136}\text{Pr}$ [Pr95b]; $^{136}\text{Pm}$ [He01]; $^{138}\text{Pm}$ [He01]; $^{138}\text{Eu}$ [He01]; and $^{140}\text{Eu}$ [He03].

Following a discussion of the systematic trends in the data, calculations based on the 3D-Dirac model and the particle-vortex model are presented. These calculations are compared with the data for various values of the chiral band pairs.
spin assignment for the odd-odd nuclei in the $A \sim 130$ study. A later study by Starosta et al. [St01a] confirmed the relative spin assignments of these bands in the $N = 75$ Cs, La, Pr, and Pm nuclei. Another study, by Koike et al. [Ko01], involving isomer delayed tagging analysis for $^{128}$Cs, allowed a confirmation of the spin values for that nucleus and, through a systematic analysis, for the other nuclei presented in that work. Following these studies, the spin values presented in the current work for $^{136}$Pm and $^{138}$Eu nuclei are 1h higher than those presented in the literature by Beausang et al. [Be87], and by Paul et al. [Pa94a], respectively.

Excitation Energy Systematics

![Excitation Energy Systematics Diagram]

Figure 6.1: Excitation energy systematics for the yrast $\pi h_{11/2} \otimes \nu h_{11/2}$ bands in Cs, La, Pr, Pm and Eu plotted as a function of $Z$, for the $N = 73, 75$, and $77$ isotones, all normalized to the $10^+$ level. The $\pi h_{11/2} \otimes \nu h_{11/2}$ band has not been observed in $^{136}$Eu.

To paraphrase Liu et al. [Li96], for the same band configuration in neighboring odd-odd nuclei, if the relative spin assignments are correct, a plot of the level energies as
a function of proton (neutron) number falls on a slowly varying smooth curve. For the
spin assignments used in this work, the smoothly varying curves for the $\pi h_{11/2} \otimes \nu h_{11/2}$
bands are seen in figure 6.1 (reproduced from figure 5.9).

6.2 Alignment systematics

Experimental alignment plots for the $\pi h_{11/2} \otimes \nu h_{11/2}$ yrast and yrare chiral
candidate partner bands in the doubly odd nuclei for $57 \geq Z \geq 63$ and $N = 73$, $75$, and $77$
are presented in figure 6.2. The same Harris parameters, $Z_0 = 5 \ h^2/\text{MeV}$ and $Z_1 = 45$
$h^4/\text{MeV}^3$, which were chosen to subtract out the average core rotation for $^{134}\text{Pr}$, are used
in all the plots (this explains the slow increase in the alignment in the $N = 73$ isotopes, for
example). Several of the nuclei show a band crossing, with a gain in single particle
angular momentum of $\Delta i_k \sim 6 \ h$, caused by the alignment of a pair of $h_{11/2}$ protons.

The frequency of the alignment in $^{140}\text{Eu}$ is lower than what may typically be
expected with the first $\pi h_{11/2}$ band crossing blocked, but it is consistent in fact with the
systematic variation of the alignment frequency in these nuclei. The crossing is
particularly evident in the $N = 75$ isotones, where there is a clear decrease in the crossing
frequency as proton number increases, from $\hbar \omega \sim 0.47 \ \text{MeV}$ for Pr to 0.39 MeV for Eu.
This can be seen in figure 6.2, where the open data points represent the yrast bands. For
the $N = 73$ isotones the full backbends are not observed. However, the same trend of
decreasing frequency for the start of the backbend is still present for these isotones, from
$> 0.5 \ \text{MeV}$ in La to $\sim 0.4 \ \text{MeV}$ in Pm. No information is available for the $N = 73$ nucleus
$^{136}\text{Eu}$. The alignment frequency for the $N = 77$ nucleus $^{134}\text{La}$ already occurs at $\hbar \omega \sim 0.4$
$\text{MeV}$. No alignments are observed for the $N = 77$ Pr and Pm. In light of the systematics,
it is expected that the $N = 77 \ ^{140}\text{Eu}$ backbend would occur at a frequency lower than $\hbar \omega \sim$
$0.4 \ \text{MeV}$. The low alignment frequency is indeed consistent with a $\pi h_{11/2} \otimes \nu h_{11/2}$ band
assignment. Instead, it is the lack of an observed backbend for $^{136}\text{Pr}$ and $^{138}\text{Pm}$ that is
anomalous.
Figure 6.2: Systematics of alignments of known $\pi h_{11/2} \otimes \nu h_{11/2}$ bands in doubly odd nuclei from $N = 73$ to 77 and $Z = 57$ to 63. Squares denote even spin levels, circles odd spin levels. Open symbols are for the more strongly populated yrast band, closed symbols denote the yrare chiral partner band. The Harris parameters $\mathcal{Z}_0 = 5 \hbar^2/\text{MeV}$ and $\mathcal{Z}_1 = 45 \hbar^4/\text{MeV}^3$, and $K = 6$, were used for all plots for convenience.
The alignment systematics in figure 6.2 shows the behavior of both the yrast and yrare $\pi h_{11/2} \otimes v h_{11/2}$ candidate chiral band pairs. Where observed, the yrare bands exhibit a very similar behavior to the yrast bands. They have the same initial single particle angular momentum, and backbend at a similar frequency as their respective yrast bands. The exception is $^{134}$Pr in which the alignment of the yrare band at low frequencies increases until it is $\Delta I_x \approx 2 - 3$ h higher than the yrast band. This is a larger difference than seen between the other band pairs, the next largest difference is $^{134}$La, with a difference of just $I_x \approx 1.5$ h. Above $\hbar \omega \sim 0.35$ for $^{134}$Pr, the alignment curves are parallel. In the next heavier odd-odd isotone, $^{136}$Pm, the alignment is the same for the yrast and yrare bands. At the backbend, the behavior is similar, though the yrare band backbends at a slightly lower frequency than the yrast band, the yrare and yrast band alignments centered on $\hbar \omega \sim 0.38$ and $\hbar \omega \sim 0.43$, respectively.

6.3 Degeneracy breaking of $\pi h_{11/2} \otimes v h_{11/2}$ candidate chiral band pairs

The experimental energy splitting between the yrast and the yrare candidate chiral bands may be compared by plots of experimental Routhian values or more directly by simply plotting level energies as a function of spin. Such a plot for the yrast and yrare bands in the N = 73, 75, and 77 isotones of Cs, La, Pr, Pm, and Eu is presented in figure 6.3 and, in order to amplify the small differences, the energy splitting between the yrare and yrast bands in these nuclei, $\Delta E = E(I)_{yrare} - E(I)_{yrast}$, is presented in figure 6.4.

The yrare chiral candidate bands behave similarly to the yrast bands, increasing with a smooth upward curve as a function of spin, as is expected of rotational bands. As can be seen in figure 6.3 and in more detail in figure 6.4, over the observed spin range the energy spacing between the chiral band pairs either remains fairly constant or drops with an increase in spin. The average energy splitting between the bands is low, about 250 keV. The Cs isotopes have a splitting that is even lower than that, with $^{128,130}$Cs having $\Delta E \sim 200$ keV. At the lowest observed spin for the yrare band in $^{132}$Cs, the splitting is about 450 keV, but drops rapidly with increasing spin. In fact, in $^{132}$Cs, and another N = 77 isotope $^{134}$La, the splitting drops to near zero at I = 14. The N = 75 isotones $^{136}$Pm and
$^{134}$Pr also show decreased splitting with an increase of spin. $^{136}$Pm has a stable splitting from $I \sim 11 - 16$, which then reduces gradually to almost zero at $I \sim 21$. The most dramatic decrease is found in $^{134}$Pr in which the chiral band pairs actually cross, and the splitting drops rapidly through zero to negative values at $I \sim 15$. It would be interesting to observe the chiral band pairs in $^{132}$Cs and $^{134}$La to higher spins to see if the splittings would continue their downward trend and cross through zero.

![Energy vs Spin Plot](image)

Figure 6.3: Plots of energy vs. spin for the $\pi h_{11/2} \otimes \nu h_{11/2}$ yrast (solid symbol and line) and yrare (open symbol and dash). The level energies are relative to the lowest level.
It is very interesting to note that an yrare band has been identified in $^{136}\text{Pr}$ which feeds into the yrast $\pi h_{11/2} \otimes \nu h_{11/2}$ band through several transitions, at least one of which has been identified as a $\Delta I = 1$ $M1/E2$ transition, and another as a quadrupole $\Delta I = 2$ transition [Pe96b]. This is very suggestive of a $\pi h_{11/2} \otimes \nu h_{11/2}$ configuration for the yrare band, and the yrast and yrare bands being candidate chiral pairs. These pairs haven't been interpreted in the context of chiral symmetry breaking and so they are not included in figures 6.3 and 6.4. Instead, a plot of the level energies as a function of spin and the energy difference between the bands is presented separately, in figure 6.5. Further study
and calculations are needed for this nucleus. In the context of the global trends, it is of
great interest that the bands cross and the splitting between the bands crosses through
zero at I ~ 14, similar to the two lighter N = 77 isotones discussed.

\[
\text{Level energies and energy splitting in } ^{136}\text{Pr}
\]

![Graph showing level energies and energy splitting in $^{136}\text{Pr}$](image)

Figure 6.5: (top) plots of energy vs. spin for the $\pi h_{1/2} \otimes v h_{1/2}$ yrast band and possible
$\pi h_{1/2} \otimes v h_{1/2}$ yrare band (symbols are as in figure 6.3) and (bottom) plot of energy
difference between these bands (symbols are as in figure 6.4). All data for figure 6.5 is
from $^{136}\text{Pr}$, taken from [Pe96b].

In terms of what is expected for chiral band pairs, the calculated energy
degeneracy breaking for this region using the particle + rotor model is $\Delta E \sim 300$ keV
[Zh02a], in good agreement with what has been observed regionally. The results of
calculations performed for the $\pi h_{1/2} \otimes v h_{1/2}$ band configuration in $^{136}\text{Pm}$ and $^{140}\text{Eu}$
using the particle + rotor model with a triaxial core will be presented.

A comparison of the observed level energies and the particle + rotor calculated
values is presented in figure 6.6 for the $\pi h_{1/2} \otimes v h_{1/2}$ configuration yrast and yrare bands
in $^{136}\text{Pm}$. The data is from the present work [He01] with several higher lying yrare levels
taken from reference [Ha01]. The deformation parameters used for the rotor for $^{136}\text{Pm}$
were $\varepsilon_2 = 0.194$, $\varepsilon_4 = 0.028$, and $\gamma = -25$ $^\circ$, taken from TRS calculations. As can be seen,
there is a good fit between the experimental data and calculations. The general trend of
the data is reproduced very well by the calculations, and the slight energy difference
between the bands of about 300 keV before the backbend and change in orbital
configuration (beginning at $I \sim 17$) is also reproduced. The calculations, based on aplanar total angular momentum, support the chiral solution.

![136Pm Level Energies](image)

$^{136}$Pm Level Energies

Figure 6.6: A comparison of experimental level energies of the $\pi h_{11/2} \otimes \nu h_{11/2}$ configuration yrast and yrare bands in $^{136}$Pm with calculated energies using the particle + rotor model with aplanar total angular momentum. At spin $I \sim 17 \hbar$ the bands begin their backbends and their configuration changes.

A similar comparison of the experimental level energies as a function of spin and the results of particle + rotor calculations with a triaxial deformed core ($\gamma \sim 25^\circ$) for $^{140}$Eu is presented in figure 6.7. The levels of the positive parity $\pi h_{11/2} \otimes \nu h_{11/2}$ bands are presented up to the backbend, when the orbital configuration and number of unpaired particles changes. As with $^{136}$Pm, the general trend of the experimental data is reproduced for the positive parity bands. The splitting between the yrast and yrare bands remains fairly constant for both the experimental data and calculations. The agreement
with the model using aplanar angular momentum again supports the chiral solution.

Figure 6.7: Comparison of experimental level energies and those from particle + rotor calculations (from [Zh02a]). The zero energy is arbitrary in this plot. The relative zeroes between the odd and even parity band calculations is also arbitrary.

The level energies of the negative parity bands in $^{140}$Eu, which are both mixtures of the $\pi (g_{7/2}, d_{5/2}) \otimes \nu h_{11/2}$ configuration, are also shown in figure 6.7, with filled symbols denoting the lower, more strongly populated band (band 1 in figure 5.3) and open symbols for the less strongly populated band (band 2 in figure 5.3). The energy spacing between the levels remains relatively constant for the band pair. Calculations were performed for this configuration using the particle + rotor model, with the same deformation parameters as for the even parity bands. The calculations have a reasonable fit to the data, but the upward curvature is a bit greater than the trend of the data. Perhaps the positive parity proton has a high degree of mixing with configurations that were not accounted for in the calculations.
The calculated trend between the positive and negative parity bands roughly reproduces the diverging trend between these bands in the data, though it does overstate the difference at higher energies. Overall, the good agreement of the aplanar model with the positive parity $\pi h_{11/2} \otimes \nu h_{11/2}$ bands, which are composed of fairly pure intruder orbitals, supports the chiral solution for these bands.

### 6.4 Energy staggering $S(I) = [E(I) - E(I-1)]/2I$ in the $\pi h_{11/2} \otimes \nu h_{11/2}$ bands

The energy splitting between even and odd spin levels in a rotational band can be characterized by the energy staggering parameter $S(I)$, where

$$S(I) = \frac{[E(I) - E(I-1)]}{2I}.$$  \hspace{1cm} (6.1)

Experimental values of $S(I)$ for odd-odd $A \sim 130$ nuclei have been compiled and presented in figure 6.8. As can be seen in that figure, at low spins, the even spin states of the yrast bands have lower values of this staggering parameter than the odd states. Only after reaching a critical “inversion spin” ($I_{\text{inv}}$) is the odd/even spin dependence reversed. For $^{130}\text{Cs}$, $^{132,134}\text{Pr}$, $^{134,136}\text{Pm}$, and $^{138,140}\text{Eu}$, this inversion spin is $I_{\text{inv}} \sim 17$ to 20, with a low of $I_{\text{inv}} \sim 15$ for $^{132}\text{Cs}$.

The value of the staggering parameter for the yrast bands is universally low at spin 10, but at higher spins the values tend to stabilize to a value of about 10 to 15 keV/h, as can be seen in figure 6.8. While the staggering between even and odd spins tends to reduce in several of the nuclei, the staggering becomes more clear at high spins in $^{134}\text{La}$ and appears to start this high spin behavior in $^{132}\text{La}$. In $^{130}\text{La}$, the staggering remains large through the entire observed spin range. In addition, the odd/even dependence does not reverse through the observed spin range for these isotopes, in contrast to the inversion at a critical spin seen in nearly all (8 of the 11) Cs, Pr, Pm, and Eu isotopes presented.
Figure 6.8: Plot of the staggering parameter $S(I) = [E(I) - E(I-1)]/2I$ vs. $I$ with experimental values from $\pi h_{1/2} \otimes \nu h_{1/2}$ bands in several mass $A \sim 130$ nuclei. Values from the yrast band are represented by the solid symbols and lines, yrare by the open symbol and dashed line.

Turning to the yrare candidate chiral band pairs, several of the trends seen in the yrast bands are reproduced in the yrare bands. The magnitude of the staggering parameter is typically the same as seen for the yrast bands, as is the general trend of the $S(I)$ line as a function of spin, even rising from low values near $I \sim 10$. For some of the nuclei, the odd/even dependence of the yrast staggering is reproduced, though the staggering is generally less dramatic. In the yrare bands the odd/even spin dependence in several nuclei is too vague to determine, or is actually reversed compared to that of the yrast band. On a whole, the patterns in the yrare bands are less clear.
Recall from chapter 2 that in the PAC regime the favored signature partner may typically be determined by equation 2.31:

\[
\alpha_{\text{favored}} = \frac{1}{2} [(-1)^{j_{\gamma} - 1/2} + (-1)^{j_{\gamma} + 1/2}].
\]  

(6.2)

Though signature is not a good quantum number in the TAC regime, it is still of interest to compare predictions with experimental results. The even spin levels in the \( \pi \ h_{11/2} \otimes \nu \ h_{11/2} \) bands have energies lower than the general trend, opposite from what is predicted by considering signature. It has been suggested by Bengtsson et al. [Be84] that a similar signature inversion in nuclei in the \( A \sim 160 \) region is indicative of triaxial deformation with \( \gamma > 0 \)°. The same may be true in the \( A \sim 130 \) region but, as Hartley et al. point out [Ha01], while the degree of signature inversion follows the degree of triaxial deformation that is expected (based on TRS calculations) for several of the nuclei, the signature inversion is also large for Cs and La nuclei near the \( N = 82 \) spherical shell closure (where the deformation may be significantly smaller), suggesting that it may not always be appropriate to use Bengtsson's interpretation for the \( A \sim 130 \) nuclei.

Nevertheless, the inversion of the odd/even spin dependence may imply an aplanar total angular momentum, a requirement for chiral bands. An analysis of the "signature" inversion in the \( \pi \ h_{11/2} \otimes \nu \ h_{11/2} \) bands in this region was performed in 1994 by Tajima et al. [Ta94]. In that work, calculations were carried out to model the experimental data using a particle + rotor model with a triaxial deformation. An irrotational flow moment of inertia and a proton-neutron (pn) residual interaction were used, and the model reproduced the low spin "signature inversion" found in this region, see reference [Ta94] for details. In that paper, Tajima et al. recognized that, as with the TAC model, signature is not a good quantum number, and his calculations allowed not only for rotational aligned orbitals, but also for orbitals aligned along the deformation axes of the nucleus. In principle, this allows an aplanar total angular momentum.
6.5 B(M1)/B(E2) values as a function of spin

The B(M1; I → I - 1)/B(E2; I → I - 2) values presented in chapters 4 and 5 for the π\textsubscript{h}_{11/2} ⊗ ν\textsubscript{h}_{11/2} yrast bands in \textsuperscript{136}Pm, \textsuperscript{138}Eu, and \textsuperscript{140}Eu show a staggering as a function of spin, with the values for odd spin levels generally raised and the values for even spin levels generally lowered from the general trend (for convenience, the B(M1)/B(E2) figures 4.13, 4.16, and 5.14 are all reproduced below as figures 6.10, 6.11, and 6.12).

Where there is sufficient data to observe this ratio for the yrare π\textsubscript{h}11/2 ⊗ ν\textsubscript{h}11/2 band—for two points in \textsuperscript{136}Pm and two points in \textsuperscript{138}Eu in figure 6.10, and for several measured points in the extended data of \textsuperscript{136}Pm [Ha01] in figure 6.11—a similar staggering is apparent which is in phase with the staggering observed in the yrast band.

This staggering in the B(M1; I → I - 1)/B(E2; I → I - 2) values in the π\textsubscript{h}11/2 ⊗ ν\textsubscript{h}11/2 bands is a general characteristic of the odd-odd nuclei in this mass region, as can be seen in figures 6.9(a) to (d). In the core-quasiparticle coupling model used in reference [Ko03], the restrictions imposed by the chiral geometry leads to such a staggering. The observation of such a staggering in bands other than the π\textsubscript{h}11/2 ⊗ ν\textsubscript{h}11/2 bands has not been addressed.

B(M1)/B(E2) values were calculated for the π\textsubscript{h}11/2 ⊗ ν\textsubscript{h}11/2 candidate chiral band pairs in several of the nuclei in this region using both the 3D-TAC model and the particle + rotor model. 3D-TAC calculations were presented with the data for \textsuperscript{136}Pm and \textsuperscript{138}Eu, and are presented again here in figure 6.10. As mentioned in chapter 4, the nuclei are predicted to have a triaxial deformation, with deformation parameters \( \varepsilon_2 = 0.194, \varepsilon_4 = 0.028, \) and \( \gamma = -25^\circ \) for \textsuperscript{136}Pm and \( \varepsilon_2 = 0.202, \varepsilon_4 = 0.032, \) and \( \gamma = -24^\circ \) for \textsuperscript{138}Eu, and a total angular momentum that is strongly aplanar. Since the 3D-TAC model cannot distinguish between the yrast and yrare bands and computes them as degenerate, the calculations should be compared with the average of the experimental values for each spin. The calculations generally match both the magnitude of the B(M1)/B(E2) values for each nucleus, and the trend of the measured values as a function of spin.
Figure 6.9: (a) (on left) In band $B(M1)/B(E2)$ values for yrast (solid line) and yrare (dashed line) $\pi h_{11/2} \otimes v h_{11/2}$ bands in several Cs isotopes, using $I_0 = 9$ h. Open (closed) figures are gated from above (below), from [Ko03]. (b) (on right) Values for $^{134}$Pr. The top is for the yrast $\pi h_{11/2} \otimes v h_{11/2}$ band (open squares) and $\pi g_{7/2} \otimes v g_{7/2} (h_{11/2})^2$ band (closed circles). The bottom is for the $\pi h_{11/2} \otimes v g_{7/2} (h_{11/2})^2$ band (open squares) and an unassigned band (closed circles), from [Ba01].

Figure 6.9: (c) (on left) In band $B(M1)/B(E2)$ values for yrast (closed circles) and yrare (open circles) $\pi h_{11/2} \otimes v h_{11/2}$ bands and $\pi g_{7/2} \otimes v h_{11/2}$ band (closed squares) in $^{134}$Pr, from [Pe96a]. Spins for the $\pi h_{11/2} \otimes v h_{11/2}$ bands, at least, must be raised by 1 h to correspond with spins used in the current work. (b) (on right) Values for $^{136}$Pr. Band 1 and 2 are the yrast and yrare $\pi h_{11/2} \otimes v h_{11/2}$ bands, respectively. Bands 3 and 4 are the $\pi h_{11/2} \otimes v s_{11/2} (h_{11/2})^2$ and $\pi g_{7/2} (h_{11/2})^2 \otimes v h_{11/2}$ bands, respectively, from [Pe96b]. Spins for the $\pi h_{11/2} \otimes v h_{11/2}$ bands, at least, must be raised by 1 h to correspond with spins used in the current work.
Figure 6.10: Measured $B(M1; I \rightarrow I - 1)/B(E2; I \rightarrow I - 2)$ values for the yrast (closed symbols) and yrare (open symbols) $\pi h_{11/2} \otimes \nu h_{11/2}$ bands. The solid lines are the results of 3D-TAC calculations.
Figure 6.11: Reduced transition strength ratio $B(M1)/B(E2)$ for in-band transitions in $^{140}$Eu. Open symbols are data while the filled symbols with solid lines are from particle + rotor calculations. Diamonds are for the main $\pi(g_{7/2},d_{5/2}) \otimes v_{h11/2}$ band, squares for the main $\pi h_{11/2} \otimes v_{h11/2}$ band.

Calculated $B(M1)/B(E2)$ values from particle + rotor calculations are shown in figure 6.11 for the main $\pi(g_{7/2},d_{5/2}) \otimes v_{h11/2}$ band and the yrast $\pi h_{11/2} \otimes v_{h11/2}$ band in $^{140}$Eu, and in figure 6.12 for both the yrast and yrare $\pi h_{11/2} \otimes v_{h11/2}$ bands in $^{136}$Pm. The calculations were performed using a triaxial deformed rotor, $\gamma \sim 25^\circ$ in the case of $^{140}$Eu [Zh02a] and $\varepsilon_2 = 0.194$, $\varepsilon_4 = 0.028$, and $\gamma = -25^\circ$ for $^{136}$Pm [Ha01]. Results of calculations were presented with measured values in chapters 5 and 6, and are presented again here in figures 6.11 and 6.12. For $^{140}$Eu, the calculations match the data for the $\pi h_{11/2} \otimes v_{h11/2}$ band reasonably well for $I \geq 13$. The calculations for bands 1 and 2 have less of an agreement with data. Unlike the negative parity intruder orbitals making up the $\pi h_{11/2} \otimes v_{h11/2}$ bands, which are fairly unmixed, the positive parity proton configurations in the $\pi(g_{7/2},d_{5/2}) \otimes v_{h11/2}$ bands are an admixture of several orbitals, and may be more difficult to model.
The $^{136}\text{Pm}$ calculations are presented with data from the current work supplemented with the extended data from [Ha01]. As can be seen in figure 6.12, the calculations provide a very good fit to the data. The magnitude of the data is reproduced, as well as the general trend of the data, including the initial large values for low spins. The odd/even staggering of the data is also reproduced in the calculations, for both the yrast and yrare chiral bands. The particle + rotor model distinguishes between the yrast and yrare chiral bands and predicts slightly lower values for odd spin yrare levels as opposed to the yrast levels, resulting in a weaker staggering as a function of spin for the yrare band, as seen in figure 6.12. The difference is slight and on the order of the error limits of the measured values.

![Figure 6.12: Measured B(M1; I → I−1)/B(E2; I → I−2) values for the yrast and yrare $\pi h_{11/2} \otimes v h_{11/2}$ bands from the present work (open figures). Data from reference [Ha01] is included (open symbols). The solid and dashed lines are the results of particle + rotor calculations with a triaxial deformed rotor for the yrast and yrare $\pi h_{11/2} \otimes v h_{11/2}$ bands, respectively. This plot was also presented in figure 4.16.](image)

The observation of yrast and yrare band pairs with similar B(M1)/B(E2) values in magnitude, general trend, and staggering, is consistent with the bands having the same orbital configuration assignment, $\pi h_{11/2} \otimes v h_{11/2}$. The general agreement of the model
calculations with the data is consistent with the nuclei having both a triaxial deformation and an aplanar total angular momentum. That is, the calculations support the chiral interpretation of the data.
Chapter 7

Alternate interpretations and "the smoking gun" for chirality

7.1 PAC rotation and proton $h_{11/2}$ signature splitting

Clearly before claiming a new and "exotic" interpretation for the observed $\Delta I = 1$ band pairs with the same microscopic configuration, more traditional explanations need to be considered and ruled out. A very traditional interpretation would be based on PAC. In PAC, for a single unpaired particle, the splitting between the odd and even spin levels is a manifestation of signature splitting, the time reversal symmetry of the two signatures being broken in the rotating nucleus. In the odd-odd nuclei in the current study, the traditional PAC interpretation would be that the yrare band is based on the unfavored $\pi h_{11/2}$ signature and the yrast band on the favored $\pi h_{11/2}$ signature, both coupled to both $\nu h_{11/2}$ signatures, as schematically shown in figure 7.1. Note that the proton signature splitting is expected to be considerably larger than that of the neutron.

Observing bands built on the coupling of the unfavored proton and neutron signatures is rare, but they have been seen in, for example in $^{164}$Tm [Re99]. For the chiral candidate bands in the $A \sim 130$ region, the energy difference between the yrare and yrast bands expected following the PAC interpretation is not in agreement with what is observed experimentally. This may be seen by a comparison of data with PAC calculations for several of the chiral candidate nuclei, and by comparison with experimentally observed signature splitting from neighboring odd proton nuclei. Such a
comparison is given in the following section.

Figure 7.1: Schematic of levels from PAC solution for unperturbed levels (left), favored and unfavored $\pi h_{11/2}$ signatures (center), which are coupled to the favored and unfavored $\nu h_{11/2}$ signatures (right). Symbols in parentheses represent proton and neutron signatures, ($\alpha$ proton, $\alpha$ neutron), respectively.

7.1.1 Comparison with calculations

To model the signature splitting for the odd-odd $N = 75$ isotones Cs, La, Pr, Pm, and Eu as a function of frequency, quasiparticle Routhian calculations, based on principal axis cranking, were carried out. The calculations were performed for triaxial deformed nuclei, using the deformation parameters $\varepsilon_2 = 0.2, \gamma = 25^\circ, \text{and } \varepsilon_4 = 0$, typical values expected for these nuclei. Experimental Routhian energies for the yrast and yrare $\pi h_{11/2}$ $\otimes \nu h_{11/2}$ chiral candidate bands, also plotted as a function of frequency, are presented beside these calculations in figure 7.2. For consistency, all Routhians in figure 7.2 were plotted using the same Harris parameters, the same parameters as were used in the alignment systematics plots presented in previously.

As can be seen, the experimental Routhians of the yrast bands show a fairly constant downward slope, and that slope is consistent for the different isotones. The yrare bands remain at a relatively constant level of about 300 keV above the yrast bands, with the exception of Pr, in which the lines cross. To magnify the splitting between the bands, the energy differences between the yrast and yrare bands are presented, with the same vertical scale for both experiment and calculations.
In contrast to the experimental Routhians, the calculated Routhians diverge, since the signature splitting increases at higher frequencies. The calculations presented in figure 7.2 are valid below the lowest (EF) calculated backbend frequency, after which the configuration changes. Experimentally though, this first crossing is blocked, so the configuration remains the same to a higher frequency.

The most dramatic feature of the calculations as presented in figure 7.3 is the strong increase of signature splitting as a function of spin, a trend not followed by the data, and opposite that seen in $^{134}$Pr. The highest frequency at which the experimental splitting was observed for is about 0.4 MeV for Eu and Pm. Extrapolating the calculated values predicts a splitting of 500 to 600 keV for these nuclei, considerably greater than that which is observed.

The calculated values also change dramatically as a function of proton number, though the curves make this difficult to see. Comparing the calculated values at a single frequency, say $\hbar \omega = 0.3$ MeV, the calculated splitting goes from 0.4 MeV for Eu to 0.2 MeV for Cs. Again, this is in contrast to the almost constant experimental values.

This same comparison was performed for the N = 77 nucleus $^{140}$Eu. The calculated Routhian energies are presented in figure 7.4 beside the experimental Routhians for the candidate chiral band pairs. A magnification of the experimental and calculated splittings is presented in the bottom of figure 7.4. The difference between the bands of the experimental Routhian decreases over the frequency range over which it is observed, from ~ 150 to 50 keV. In contrast, the calculated signature splitting follows the trend of the N = 75 calculations, it increases with increasing frequency over the frequency range observed, to over 400 keV at $\hbar \omega = 0.3$ MeV.
Figure 7.2: Experimental Routhians (left) and PAC quasiparticle Routhian calculations for $h_{\frac{11}{2}}$ protons (right) as a function of frequency, calculated in the present work. Triangles (squares) are transitions in the yrare (main) bands. Filled (open) symbols are transitions between even (odd) levels. Solid (dashed) lines represent the favored (unfavored) proton $h_{\frac{11}{2}}$ signature partner in the calculations and the yrast (side) band in the data.
Figure 7.3: Difference between the experimentally derived Routhians for the yrast and yrare $\pi h_{11/2} \otimes \nu h_{11/2}$ bands for each labeled nucleus (left) and calculated signature splitting between the favored and unfavored $h_{11/2}$ proton signature from PAC quasiparticle Routhian calculations (right), from the present work. Vertical scales are the same for the left and right side.
Figure 7.4: Experiment and calculations for $^{140}$Eu.

Top: Experimental Routhians (left) and PAC quasiparticle Routhian calculations (right) as a function of frequency, calculated in the present work. Solid (dashed) lines represent the favored (unfavored) proton $h_{11/2}$ signature partner in the calculations and the yrast (side) band in the data. Squares (circles) represent transition between odd (even) levels in the yrast band, while triangles represent transitions between odd levels in the yrare band.

Bottom: Difference between the experimentally derived Routhians for the yrast and yrare $\pi h_{11/2} \otimes \nu h_{11/2}$ bands (left) and difference between the favored and unfavored $h_{11/2}$ proton signature from PAC quasiparticle Routhian calculations (right).
Figure 7.5: Routhian plots of $\pi h_{1/2} \otimes \nu h_{1/2}$ band pairs (left) and TAC calculations for those bands from [Ha01] (right). The TAC calculations presented are limited to principal axes, making them effectively PAC calculations. Triangles (squares) are transitions in the yrare (main) $\pi h_{1/2} \otimes \nu h_{1/2}$ bands. Filled (open) symbols represent transitions between even (odd) levels. Solid (dashed) lines represent the favored (unfavored) proton $h_{1/2}$ signature partner in the calculations and the yrast (side) band in the data.
Calculations were also performed using the tilted axis cranking model but with the angular momentum limited to a principal axis. This limitation effectively makes it principal axis cranking, appropriate for signature splitting calculations. The calculations were performed for the N = 75 isotones Cs, La, Pr, and Pm [Ha01]. Experimental Routhian energies for the yrast and yrare $\pi h_{11/2} \otimes \nu h_{11/2}$ chiral candidate bands are presented beside the calculations in figure 7.5.

The calculated signature splittings plotted in figure 7.5 decrease from well over 1 MeV for Cs to 500 keV for Pm. Interestingly, this decrease with increasing Z is opposite the trend seen in the calculations plotted in figure 7.4. In addition, the calculated Routhians plotted in figure 7.5 not only are curved upward but the calculated signature splitting increases with rotational frequency. This is unlike the steady splitting seen in the data, and opposite the decreasing splitting seen in $^{134}\text{Pr}$.

Neither the PAC limited TAC calculation signature splitting nor the PAC based quasiparticle Routhian calculated signature splitting explains the degeneracy breaking observed between the chiral candidate band pairs in this region. A comparison with measured $h_{11/2}$ proton signature splittings in the odd Z neighboring nuclei is presented in the following section.

### 7.1.2 Comparison with measured signature splitting

A survey of signature splitting in several neighboring odd A nuclei has been performed for comparison with the observed chiral splitting. The size of the splitting in the odd neutron nuclei is comparable to the splitting between the odd and even spin levels in a single $\Delta I = 1$ band, on the order of 10s of keV.

To study the splitting between the $h_{11/2}$ proton signatures in the odd Z neighboring nuclei, Routhian plots of these nuclei are presented in figure 7.6. Since the orbital configuration changes at the backbend, the Routhians of these odd Z neighbors are plotted only up to the level at which the backbend begins. The energy difference between
The favored and unfavored bands is presented in figure 7.7. The data for the odd Z nuclei were taken from the following references: $^{131}$La [Hi89], $^{133}$La [Mo82], $^{135}$La [Se98]; $^{133}$Pr [Hi88], $^{135}$Pr [Se86]; $^{137}$Pr [Xu89]; $^{137}$Pm [Be87b]; $^{139}$Pm [Xu87]; $^{139}$Eu [Va95]; and $^{141}$Eu [Xu91].

Figure 7.6: Experimental Routhian plots of the favored (solid circle with line) and unfavored (open circle with dash) $\pi h_{1/2}$ bands observed in the odd Z nuclei from N = 74 to 78, Z = 61 to 65. The assignment of the unfavored $\pi h_{1/2}$ signature band in $^{135}$Pr (open diamonds and triangles) is offered tentatively for two separate bands [Se86].
Figure 7.7: The difference between the Routhian energies of the favored and unfavored \( \pi h_{11/2} \) signatures, as presented in figure 7.6. The symbols for \(^{135}\text{Pr}\) relate to figure 7.6.

The splitting between the favored and unfavored signature of the \( h_{11/2} \) proton orbital in these odd Z nuclei shown in figure 7.7 appears rather flat over the observed frequency range. The signature splitting in the N = 74 isotones runs from over 500 keV to about 650 keV, while the signature splitting in the N = 76 isotones is around 400 keV. A direct comparison cannot be made in the N = 76 isotope \(^{137}\text{Pm}\), as the Routhians do not overlap in frequency.

The chiral candidate bands in the N = 75 nuclei have an energy difference several hundred keV lower than the signature splitting of their odd Z neighbors. In addition, the splitting has been seen in several cases to decrease considerably as a function of frequency, almost crossing in several nuclei and actually crossing in \(^{134}\text{Pr}\). This behavior
is not seen in signature partners in neighboring odd Z nuclei.

As mentioned, the signature splitting observed in the $N = 78$ isotones is typically lower than that seen in the $N = 74$ and 76 isotones, as shown in figure 7.7. The signature splitting for the $N = 78$ isotones ranges from about 300 to 400 keV. It is more appropriate to compare these not with the $N = 75$ chiral band pairs, but with chiral band pairs from the neighboring $N = 77$ isotones. These have not been presented in a systematic fashion as there are fewer of them known. The difference in the Routhian values for the chiral band pairs in $^{146}$Eu runs from only 150 keV to about 50 keV, as shown in figure 7.4, decreasing in energy with an increase in frequency. The next lighter isotope in which candidate chiral band pairs have been found, $^{134}$La, has a splitting between the band pairs that also decreases as a function of frequency, from 317 keV to only 14 keV [Ba01]. This trend of decreasing chiral band splitting as a function of frequency is similar to that seen in chiral bands in $N = 73$ and 75 nuclei, and not seen in the signature splitting in the region presented in figure 7.7. Briefly, signature splitting does not describe the degeneracy breaking observed in the $\pi h_{11/2} \otimes \nu h_{11/2}$ bands.

7.2 Shape coexistence

Shape coexistence is well known in several nuclei in the region [Pa94b, Ca91, Pa89, Mu84], typically occurring for bands based on different orbital configurations, which polarize the core to different shapes. However, in the even-even core of $^{140}$Eu ($^{138}$Sm), two bands having the same orbital configuration but different deformations are suggested [Pa94b]. In that example, the band built on the $\pi (h_{11/2}, g_{7/2}) \otimes \nu (h_{11/2})^2$ configuration shows a negligible signature splitting (band 12 in figure 7.8) and has a calculated deformation of $\beta_2 = 0.17, \beta_4 = -0.02, \gamma = -91^\circ$. For the other band built on the same orbital configuration, with the calculated deformation $\beta_2 = 0.17, \beta_4 = -0.02, \gamma = -30^\circ$ ($\gamma$ is different), only one signature is observed (band 2 in figure 7.8).
For several of the odd-odd nuclei in the region, TRS calculations for any reasonable proton and neutron configurations predict well defined coexisting minima at $\beta \sim 0.2$ and $\gamma \sim \pm 25^\circ$, as can be seen for $^{136}$Pm, $^{138}$Eu, and $^{140}$Eu in figures 5.1 and 6.11. These
minima coexist, and persist over a wide frequency range. Following the Lund
convention, in the PAC regime, \( \gamma = +30^\circ \) and \( \gamma = -30^\circ \) means an interchange of rotational
axis and thus a different moment of inertia. In addition, the change in axis changes the
microscopic structure involved in rotational effects, and the signature splitting is
different. For example, for a positive \( \gamma \) deformation, the proton splitting may be large
and the neutron splitting small. For a negative \( \gamma \) deformation, the situation may be
reversed. In the PAC regime, the two \( \gamma \) deformations predicted for many of the odd-odd
nuclei, \( \gamma \sim \pm 25^\circ \), may produce different rotational bands, consistent with shape
coexistence. Thus, if PAC is applicable - if the proton, neutron, and rotational angular
momenta are near the same axis - it is reasonable to suggest that the observed band pairs,
while both being based on the \( \pi h_{1/2} \otimes \nu h_{1/2} \) configuration, may be based on different \( \gamma \)
deformations.

However, it must be noted that in the TAC regime, positive and negative \( \gamma \)-
deformations are treated the same, and that the sign is a relic of the PAC model in which
the Lund convention was defined. That is, for non-axial total angular momentum, a
different sign in the \( \gamma \)-deformation does not mean shape coexistence. In addition, for the
candidate chiral bands that have been observed, including those in \(^{140}\)Eu, the transition
energies are very similar as a function of spin and the splitting between the bands is
small. This implies a similar moment of inertia and microscopic structure for the band
pairs, consistent with the observed band pairs and with the TAC interpretation.

7.3 Rotational bands built on \( \gamma \)-vibrations

Besides rotational excitations, nuclei are well known to exhibit other forms of
collective motion, vibrations for example. When a quadrupole deformed nucleus vibrates
along its deformation axis, this is termed a \( \beta \)-vibration, see figure 7.9 (left). For nuclei
that are reasonably soft to \( \gamma \)-deformation, \( \gamma \)-vibration is a possible mode of excitation.
For a \( \gamma \)-vibration, the long deformation axis, denoted by \( z \), remains relatively constant
while the lateral axes, denoted \( x \) and \( y \), vibrate, see figure 7.9 (right). In \( \beta \)-vibrations,
which conserve axial symmetry, the band head is a $K = 0$ state. In $\gamma$-vibrations, the
axially symmetry is broken and the band head is a $K = 2$ state. The angular momentum
gain in such a state is via a rotation along the $z$ axis which, due to the rms of the $\gamma$
deformation fluctuations, is no longer a symmetry axis [Ca00]. In addition, as only a
handful of orbits are involved in the vibration, the transitions out of vibrational states are
typically less collective than rotational transitions, as measured through $E2$ strengths. As
Casten comments [Ca00] in even-even deformed nuclei, rotational $B(E2; 2^+ \rightarrow 0^+)$ values
can easily reach several hundred single particle units while vibrational transitions, as
from a $\gamma$-vibration band to the ground state band, are more typically 10 - 30 single
particle units.

![Diagram](image)

Figure 7.9: Schematic of a nucleus undergoing $\beta$-vibration (left) and $\gamma$-vibration (right)
with lateral view (top) and end view (bottom) with respect to the $z$ axis. Dark and light
lines correspond between the top and bottom schematics. The $z$ axis is the long axis in a
prolate deformed nucleus. In a $\beta$ vibration, the lateral circumference changes, as does the
length. In gamma vibration, the lateral cross section deforms.

Deformed nuclei may have rotational bands built on both the non-vibrating
ground state and the vibrating excited state. Another alternate explanation proposed for
the splitting between the yrast and yrare $\pi h_{11/2} \otimes \nu h_{11/2}$ band pairs has been for the yrare
rotational band to be built on a $\gamma$-vibration while the yrast rotation band is built on the
non-vibrating nucleus. Indeed, \(\gamma\)-vibration bands are well known in even-even and singly-odd nuclei in the mass \(A \sim 130\) region (see [Va95], for example).

Experimentally, however, the band heads for these \(\gamma\)-bands have been found at >500 keV above the yrast bands in even-even and singly odd nuclei in the region, at and 546 for \(^{132}\)Ce [Pa97] and at 543 keV for \(^{139}\)Eu [Va95] for example, about twice the energy difference observed between the yrare and yrast \(\pi \ h_{11/2} \otimes \nu \ h_{11/2}\) bands, perhaps making an interpretation based on \(\gamma\)-phonon excitation unlikely. In addition, the energy difference between the yrast and yrare \(\pi \ h_{11/2} \otimes \nu \ h_{11/2}\) bands in \(^{132}\)Cs, \(^{134}\)La, and \(^{134}\)Pr decreases, to nearly zero for the first two nuclei (see figure 6.4), while in \(^{134}\)Pr the bands even cross. Hartley et al. consider this a highly unlikely event for a \(\gamma\)-vibration band [Ha01].

Very recently, new calculations have been performed by S. Brant et al. for odd-odd nuclei in the framework of a modified interacting boson fermion-fermion model (IBFFM) to describe the splitting between the \(\pi \ h_{11/2} \otimes \nu \ h_{11/2}\) bands in \(^{134}\)Pr [Br03]. In this model, the odd-odd quality of the nucleus is accounted for by two fermions, and there is an additional term involved that parameterizes the \(\gamma\)-softness. The IBFFM parameters for \(^{134}\)Pr were found by modeling neighboring singly odd nuclei with the IBFM. To describe \(^{134}\)Pr, in principle, only the residual interaction between the odd proton and odd neutron has to be adjusted to the experimental data.

The spectrum of the neighboring even-even nucleus \(^{134}\)Ce is described by the Hamiltonian

\[
H_{IBM} = \varepsilon_0 n_d + \pi P \cdot P + k' L \cdot L + k' Q \cdot Q + \Theta_3 ([d^+ d^+]_2 d^+]_3 + [d^- d^-]_2 d^-]_3
\]

The first four terms represent the standard Hamiltonian from the IBM-1. The fifth term, with the strength parameter \(\Theta_3\) and with the cubic interaction, is included to add triaxiality. Though the \(\gamma\) vibration band is > 550 keV above the ground state band in \(^{134}\)Ce [Se94], the calculated values for the band splitting in odd-odd \(^{134}\)Pr are much smaller. The results of the calculation are presented in figure 7.10 (left) without a stable
triaxiality ($\Theta = 0$), and in figure 7.10 (right) with a more stable triaxial deformation ($\Theta = 0.03$ MeV). For the left panel, the core is $\gamma$–soft and the predicted energy splitting remains fairly constant at ~ 400 keV. For the right panel, the energy difference between the bands decreases and by $I \sim 17$ the bands cross. A comparison of experimental and calculated transition intensities in that work, figure 7.11, is also in relatively good agreement.

![Figure 7.10](image)

**Figure 7.10**: Calculated energy as a function of spin for the yrast and yrare $\pi^* h_{11/2} \otimes v h_{11/2}$ bands in $^{134}$Pr using the IBFFM without triaxiality but with a $\gamma$ soft core ($\Theta_3 = 0$, left panel) and with stable triaxial deformed core ($\Theta_3 = 0.03$ MeV, right panel), from [Br03].
Figure 7.11: Transition intensities from experiment (left) and from IBFFM calculations (right), from [Br03]. The intensities are generally similar, the main exception being the interband transitions.

The authors of reference [Br03] suggest that the $\gamma$-soft solutions are similar to the chiral vibrations proposed by Starosta et al. [St01a] (discussed in section 2.9) for the $N = 75$ nuclei other than $^{134}\text{Pr}$. Calculations specifically for the other $N = 75$ nuclei have not been presented. For $^{134}\text{Pr}$ specifically, a more stable triaxial deformation is suggested,
with the calculated yrast band built on the ground state of the triaxial core and the calculated yrare band predominately based on the γ band of the core. With an increase of spin, there is an increase of mixing between these bands, and at the band crossing calculations predict large admixtures of higher lying collective structures [Br03].

Transition strengths from IBFFM calculations are not presented explicitly in reference [Br03], nor are the transition intensities given numerically, although the intensities are presented graphically in figure 7.11. As transition strengths vary strongly with energy (e.g. \(B(M1) \propto E^3\) and \(B(E2) \propto E^5\)), and exact energies are not given in reference [Br03], a definite comparison of transition strengths is not currently possible. By figure 7.11, it appears that the calculated and experimental in-band \(B(M1)/B(E2)\) values may be similar, but the interband values would be very different. If the exact calculated energies and intensities were made available, it would be worthwhile comparing calculated and experimental transition strengths. It would also be interesting to compare calculated and experimental level energies and transition strengths in neighboring odd-odd nuclei in this region.

7.4 Chiral calculations and “the smoking gun” for chirality

Several explanations have been suggested to explain the observed band pairs with a small degeneracy breaking. Some of these are consistent with the chiral explanation: the mixing of the intrinsic states has been suggested to follow from γ-oscillations, reorienting the \(R\) vector and hence the intrinsic coordinate system [St01a, Br03]. Other explanations stand against the chiral interpretation, but generally do not fit the data. While in general, chiral structure is strongly suggested by the experimental data, a clear experimental signature of chiral structure would still be welcome. Calculations with the TAC and particle + rotor model suggest a possible "smoking gun" for chiral behavior.

That "smoking gun" is the predicted discontinuity in the values of the \(E2\) transition strength, \(B(E2)\) [Zh02b]. In the mass \(A \sim 130\) region, where the chiral bands are based on configurations involving proton and neutron \(h_{11/2}\) orbitals, the unpaired
proton and neutron both have angular momentum $j = 11/2$. The total angular momentum is most aplanar when the rotational angular momentum is near this value as well, giving a total angular momentum of $I \sim 15$.

Recall that the quadrupole moment is related to the triaxial deformation parameters $\beta$ and $\gamma$ through equation 3.28,

\begin{equation}
Q(\beta, \gamma) = Q(\beta, \gamma = 0) 2 \sin (30^\circ - \gamma),
\end{equation}

and that the transition strength goes as the square of the quadrupole moment, $B(E2) \propto Q^2$. Thus for a given $\beta$, when the quadrupole moment is low at $\gamma = 30^\circ$, so is the $E2$ transition strength. As the rotational vector, $R$, increases, the total angular momentum moves away from the $(J_n, J_\nu)$ plane, and the system moves into the 3D-TAC regime, where equation 7.2 does not apply. Consequently, the $B(E2)$ values are not limited to near zero and may rise sharply.

For calculations performed using "frozen alignments", where the direction of the single particle angular momentum vectors are fixed, it is at spin $I \sim 15$ that the inband $B(E2)$ values for the $\pi h_{11/2} \otimes \nu h_{11/2}$ bands are predicted to show a sharp increase [Zh02b]. Using more flexible vector alignments to better approximate what is believed to happen experimentally, the discontinuity is smoothed somewhat, but it is still discernible. Results of these calculations for $^{134}$Pr and $^{136}$Pm are presented in figures 7.13 and 7.14, respectively. The $B(M1)/B(E2)$ values from these calculations for $^{136}$Pm have already been compared with data in figure 6.12 and shown to have a good fit. The calculated values follow the same pattern in both nuclei, with a sudden increase in $B(E2)$ at around $I \sim 15$. The discontinuity is very clear for $^{134}$Pr and, although less extreme for $^{136}$Pm, a change of slope is evident around this spin range, see figures 7.13 and 7.14. It is reasonable to assume that this discontinuity may be a global trend for chiral bands.

The discontinuities are not the effects of particle alignment. To begin with, the $B(E2)$ calculations were performed for the $\pi h_{11/2} \otimes \nu h_{11/2}$ configuration bands, and did not take into account a band crossing. The discontinuities in these values are due solely to the aplanar quality of the total angular momentum. Experimentally, these
\textbf{\textsuperscript{134}Pr: calculated }B(M1)/B(E2), B(M1), B(E2)

Figure 7.13: Predicted \(B(M1)\) and \(B(E2)\) values and their ratio for the \(\pi h_{11/2} \otimes \nu h_{11/2}\) bands in \textsuperscript{134}Pr using the particle + rotor model [Zh02b]. For this nucleus, a \(B(E2)\) value of 0.5 (\(eb\))^2 \(\sim 122\) W.u., so the discontinuity is considerable.
Figure 7.14: Predicted $B(M1)$ and $B(E2)$ values and their ratio for the $\pi h_{11/2} \otimes \nu h_{11/2}$ bands in $^{136}$Pm using the particle + rotor model [Zh02a]. For this nucleus, a $B(E2)$ value of $0.5 \text{ (eb)}^2 \sim 120 \text{ W.u.}$, so the discontinuity is considerable.
discontinuities at I \sim 15 are not expected to be seen due to a band crossing. This is demonstrated in both the observed B(E2) values across a band crossing in neighboring nuclei, and in the spin in which the band crossing has been observed in $^{134}$Pr and $^{136}$Pm, above the predicted discontinuity.

Experimental B(E2) values for $^{128}$Ce which cover the spin range of an alignment in that nucleus are presented in figure 7.15. The first band crossing occurs around I \sim 10 to 12. The behavior of the B(E2) values is consistent above and below the band crossing. Unlike the predicted values for aplanar angular momentum, the values do not show a sudden and sustained increase.

![128 Ce measured B(E2) values](image)

Figure 7.15: Measured B(E2) values for $^{128}$Ce, from [Li96b]. The behavior as a function of spin is fairly constant, there is no sudden jump as expected with chiral nuclei. In addition, the observed band crossings in $^{134}$Pr and $^{136}$Pm occur at higher spins.
In addition, the observed band crossing in $^{134}$Pr and $^{136}$Pm occur at higher spins than the predicted discontinuity in the B(E2) values for these nuclei. To illustrate this, the level energies are plotted in figure 7.16 as a function of spin. To observe the change in slope due to the onset of a change in orbital configuration more easily, the level energy is subtracted from a rotational energy, modeled as a 2nd order polynomial in I. These are presented on the left side of figure 7.16 for $^{134}$Pr and the right side for $^{136}$Pm. The band crossings occur at I ~ 20 for both of these nuclei. The calculated B(E2) values for these nuclei, with their sharp discontinuities, are also included in this figure. The sudden increase in the B(E2) values, starting around I ~ 14 for both of these nuclei, represents the predicted onset of chiral behavior in these nuclei, several \hbar lower than the observed alignments.

![Figure 7.16: Predicted B(E2) values (top) for $^{134}$Pr (left) and $^{136}$Pm (right), from figures 7.13 and 7.12, respectively. The bottom figures are the level energies for each nucleus, with a rotational energy subtracted. The rotational energy is modeled as a 2nd order polynomial in I. The change in slope of these lower plots show the approximate spin at the experimental band crossing.](image-url)
Chapter 8

Summary and conclusions

The odd-odd nuclei $^{136}\text{Pm}$, $^{138}\text{Eu}$, and $^{140}\text{Eu}$ were studied in a search for candidate chiral band pairs. Details of the experimental work and results were presented. Previous high spin structure was known in $^{136}\text{Pm}$ and $^{138}\text{Eu}$, and in the current work, new bands were added to both of these nuclei. No previous high spin structure was known for $^{140}\text{Eu}$, and in the current work, 5 bands and a total of 69 transitions were identified.

Transitions were assigned a multipolarity and electromagnetic character following angular distribution, DCO, and Compton polarimetry measurements, and via their relation to the previously known band structure. Following this, the new yrare bands in $^{136}\text{Pm}$ and $^{138}\text{Eu}$ were assigned the same $\pi h_{11/2} \otimes v h_{11/2}$ band configuration as the yrast bands. In $^{140}\text{Eu}$, two bands were assigned the $\pi h_{11/2} \otimes v h_{11/2}$ configuration and two bands were assigned the mixed proton $\pi (g_{7/2}, h d_{5/2}) \otimes v h_{11/2}$ band configuration. The $\pi h_{11/2} \otimes v h_{11/2}$ band pairs exhibit the slight degeneracy breaking predicted for chiral structure, and are presented as candidate chiral band pairs. The $\pi (g_{7/2}, h d_{5/2}) \otimes v h_{11/2}$ bands in $^{140}\text{Eu}$ may also be candidate chiral band pairs.

Several candidate chiral band pairs have been identified in neighboring odd-odd nuclei. An examination of systematic trends in these bands, including those from our data in Pm and Eu, shows:
- a smooth variation in level energy, confirming the spin assignments;
- a similar energy degeneracy breaking between the yrast and yrare bands, typically about 300 keV but in some cases even smaller;
- a similar odd/even spin dependence of the staggering parameter, $S(I)$;
- and a similar odd/even spin dependence of the $B(M1)/B(E2)$ values.
The near degeneracy of the bands is in line with predictions for chiral structure. The odd/even spin dependent staggering of $S(I)$ and the $B(M1)/B(E2)$ values may be indicative of triaxial deformation, also in line with predictions for chiral structure.

Calculations were performed using the 3D-TAC model and the particle + rotor model, both for aplanar angular momentum in triaxial deformed nuclei. The calculations have a reasonable fit with data for level energies and $B(M1)/B(E2)$ values. For the particle + rotor model specifically, the fit to the $B(M1)/B(E2)$ values is very good. These calculations support the assignment of the band pairs as chiral.

Several other possible explanations were presented and experimental examples and calculations for these explanations were examined. PAC based signature splitting does not match the behavior of the candidate chiral band pairs. Shape coexistence was also presented, though the argument in favor of it is not as convincing as the chiral argument. Experimental data from neighboring even and singly-odd nuclei show $\gamma$-vibrations to be too energetic to explain the degeneracy breaking. Very recent calculations with odd-odd nuclei (IBFFM) show a smaller degeneracy breaking [Br03]. Vibrations are actually complementary to the chiral explanation as they offer an explanation for the mixing of the intrinsic chiral states. IBFFM calculations with both the yrast and yrare bands built on $\gamma$-vibrations can model the 300 keV degeneracy breaking and can maintain this value as nearly constant over a range of spins. The band crossing in $^{134}$Pr on the other hand is modeled with a rigid core for one band and a $\gamma$-vibration for the other. Both the experimental behavior of this specific nucleus and the parameters chosen for modeling it are thus somewhat different from the other candidate chiral band pairs. Transition strengths are not presented in reference [Br03].

For future work, it would be useful to compare IBFFM calculated transition strength values in $^{134}$Pr with experimental values. In addition, calculations can be performed for the other odd-odd nuclei in the mass region to compare with candidate chiral band pairs. It would also be interesting to compare calculations with experiment in the other mass regions in which candidate chiral band pairs have been identified.
The chiral model predicts a unique signature for chiral bands, a discontinuity in the $E2$ transition strengths, $B(E2)$, as a function of spin. Calculations were performed for both $^{134}$Pr and $^{136}$Pm, and the discontinuity is expected to be more clear in $^{134}$Pr. This discontinuity is based on the aplanar quality of the total angular momentum, and is not expected in regular rotational bands. Further investigation in this vein is recommended.

To this end, we performed lifetime measurements for $^{134}$Pr in a Doppler shift attenuation method (DSAM) experiment. The DSAM uses the Doppler shift of the photons a nucleus emits to determine the velocity of the nucleus as it decelerates in a thick target backing. The velocity at the time of emission gives information on the ordering of the emitting transitions and on the level lifetimes. The data were collected using the high efficiency Gammasphere detector array. The high efficiency of this array may also allow analysis of basic band structure over a larger spin range than that currently known in $^{134}$Pr. Analysis of the lifetimes and the extraction of the $B(E2)$ values is currently under way.
Appendix A

A.1 Spherical harmonics

The spherical harmonics \( Y_{lm}(\theta, \phi) \) are defined as

\[
Y_{lm}(\theta, \phi) = \frac{\sqrt{\frac{2l+1}{4\pi} \frac{(l-m)!}{(l+m)!}}} \left[ \begin{array}{c}
5 \frac{(l-m)!}{(l+m)!} \end{array} \right] P_l^m(\cos \theta) e^{im\phi}
\]

Figure A.1: The real parts of the labeled spherical harmonics.

for positive \( m \), while values for negative \( m \) may be found through the relation

\[
Y_{lm}(\theta, \phi) = (-1)^m Y^{*}_{l-m}(\theta, \phi).
\]
The real values of the spherical harmonics are plotted in figure A.1. \( P_l^m(x) \) is the associated Legendre function, and is related to the Legendre polynomial,

\[
P_l^m(x), \text{ by } P_l^m(x) = (-1)^m (1-x^2)^{m/2} \frac{d^m}{dx^m} P_l(x).
\]

A.2 Multipole fields

The true angular distribution of electromagnetic radiation from a nucleus requires a quantum mechanical treatment. That treatment will follow. A classical treatment is presented first, leading to the angular distributions of the radiated power of a multipole field, \( I_{l,m}(\theta,\phi) \), which gives a reasonable description of the direction of radiation.

The angular distribution of an electromagnetic field of multipolarity \( l \) may be described by the spherical harmonic of order \( l \), \( Y_{lm} \). The spherical harmonics are the solutions to the angular part of the wave equation:

\[
-\left( (1/\sin \theta) \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial}{\partial \theta} \right) + (1/\sin^2 \theta) \frac{\partial^2}{\partial \phi^2} \right) Y_{lm} = l(l+1) Y_{lm},
\]

which can be written as

\[
LY_{lm} = l(l+1) Y_{lm}
\]

using the differential operator

\[
L = \frac{1}{l} (r \times \nabla).
\]

\( X_{lm} \) is the normalized form of the vector spherical harmonic, \( LY_{lm} \), defined as

\[
X_{lm}(\theta,\phi) = \frac{1}{\sqrt{l(l+1)}} L Y_{lm}(\theta,\phi).
\]

In the far field approximation (\( kr >> 1 \)), the electric and magnetic fields are given by [Ja75]

\[
\text{(A.8a)} \quad B \rightarrow e^{ikr-\text{int}} \sum_{l,m} (-1)^{l+1} [a_E(l,m)X_{lm} + a_M(l,m) \mathbf{n} \times X_{lm}],
\]

\[
\text{(A.8b)} \quad E \rightarrow B \times \mathbf{n},
\]

where \( \mathbf{n} = r/r \), the unit vector in the radial direction.
The coefficients $a_E(l,m)$ and $a_M(l,m)$ specify the amounts of electric and magnetic multipole fields. The power radiated by a multipole field varies with angle according to the Poynting vector

(A.9) \[ \mathbf{S} = \mathbf{E} \times \mathbf{B}. \]

Following equations A.8a and A.8b, the time averaged power radiated per unit solid angle is

(A.10) \[ \frac{dP(l,m)}{d\Omega} = \frac{c}{8\pi k^2} |a_E(l,m)|^2 \mathbf{X}_{lm} \times \mathbf{n} + |a_M(l,m)|^2 \mathbf{X}_{lm}^2 \]

which, in the case of a pure multipole of order $(l,m)$, reduces to

(A.11) \[ \frac{dP(l,m)}{d\Omega} = \frac{c}{8\pi k^2} |a(l,m)|^2 |\mathbf{X}_{lm}(\theta,\phi)|^2. \]

The first few orders of the angular distributions $|\mathbf{X}_{lm}(\theta,\phi)|^2$ are:

- dipole $l = 1, m = 0 \ 3/8\pi \ (1 - \cos^2 \theta)$
- $l = 1, m = \pm 1 \ 3/16\pi \ (1 + \cos^2 \theta)$
- quadrupole $l = 2, m = 0 \ 15/8\pi \ (\cos^2 \theta - \cos^4 \theta)$
  - $l = 2, m = \pm 1 \ 5/16\pi \ (1 - 3\cos^2 \theta + 4\cos^4 \theta)$
  - $l = 2, m = \pm 2 \ 5/16\pi \ (1 - \cos^4 \theta)$.

As the vector sum of the spherical harmonics obeys the sum rule, giving a total power distribution

(A.12) \[ \sum_{m=-l}^{l} |\mathbf{X}_{lm}|^2 = \frac{2l+1}{4\pi}, \]

the power emitted is independent of angle for a uniform $m$ state distribution. The expression for the total angular distribution of the emitted photons is instead a weighted sum of the different $m$ state angular distributions, weighted around $m = 0$, as mentioned above. This can be expressed in terms of Legendre polynomials, as

(A.13) \[ W(\theta) = \sum_{\nu} A_{\nu} P_{\nu}(\cos \theta), \]

where $W(\theta)$ is the intensity of the $\gamma$-rays measured at angle $\theta$, $\nu$ are even numbers less than or equal to $2l$, and $A_{\nu}$ are the angular distribution coefficients. For reference, the first few even polynomials are:
\( P_0 (\cos \theta) = 1, \)
\( P_2 (\cos \theta) = \frac{1}{2} (3\cos^2 \theta - 1), \) and
\( P_4 (\cos \theta) = \frac{1}{8} (35\cos^4 \theta - 30\cos^2 \theta + 3). \)

By measuring the intensity of the \( \gamma \)-ray distribution as a function of angle and fitting \( W(\theta) \), the expansion coefficients \( A_\nu \) can be found. For nuclear reactions, the multipolarity of the transitions is usually limited to \( \ell = 2 \) \((\Delta l_{\text{max}} = 2)\), although occasionally \( E3, M3 \) or higher order transitions are encountered. The transitions are usually \( E1, M1, \) or \( E2 \), which is understandable following the selection rules and trends in transition strengths. For a pure "stretched" dipole transition \((\ell = 1 \text{ and } \Delta l = 1)\), the angular distribution is a second order polynomial in terms of cosine \( \theta \) and can be expressed as
\[
W(\theta) = A_0 + A_2 P_2 (\cos \theta).
\]

For a pure "stretched" quadrupole transition \((\ell = 2 \text{ and } \Delta l = 2)\), the angular distribution is a fourth order polynomial in terms of \( \cos \theta \) and can be expressed as
\[
W(\theta) = A_0 + A_2 P_2 (\cos \theta) + A_4 P_4 (\cos \theta).
\]

The peaking of the \( m \) state distribution about \( m = 0 \) provides for the radiation from a pure dipole transition having the normalized coefficient \( a_2 (= A_2/A_0) < 0 \), and from a pure quadrupole transition having \( a_2 > 0 \).

Quantum mechanically, the angular distribution function is similar [Ya67],
\[
W(\theta) = \sum A_k P_k (\cos \theta),
\]
with \( \kappa \) even. The coefficients are expressed
\[
A_k(J_f L_f J_l L_l J_i) = \alpha_k(J_i) A_k^{\text{max}}(J_i L_i L_j J_i),
\]
where \( \alpha_k \) is the attenuation coefficient of the alignment. The variables are defined in reference [Ya67]:
\[
A_k^{\text{max}}(J_1 L_1 L_2 J_f) = p_k(J_i) \frac{1}{1 + \delta^2} \left\{ f_k(J_f L_1 L_1 J_i) + 2 \delta f_k(J_f L_1 L_2 J_i) + \delta^2 f_k(J_f L_2 L_2 J_i) \right\},
\]
where
\[
f_k(J_f L_1 L_2 J_i) = B_k(J_i) F_k(J_f L_1 L_2 J_i),
\]

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\( F_k \) and \( f_k (\equiv B_k F_k) \) are given in tables in the same reference. The statistical tensor \( \rho_k(J_1) \) is defined in terms of the population parameters \( P_m(J) \) which specify the degree of alignment of a state with spin \( J \):

\[
\rho_k(J_1) = (2J+1)^{1/2} \sum_m (-1)^k \langle J \; m \; J - m \; k | 0 \rangle P_m(J).
\]

Finally, \( \delta \) is given by

\[
\delta = \frac{\langle J_1 \; L_2 \; J_1 \rangle}{\langle J_1 \; L_1 \; J_1 \rangle}.
\]

### A.3 Compton scattering as a probe of photon polarization

In radiating photons from a nucleus, the energy emitted in the form of a photon follows the Poynting vector, which is defined as the vector product \( \mathbf{S} = \mathbf{E} \times \mathbf{B} \). As an example we look to the simple dipole field. As a simple example with a low multipole, for an electric dipole \( (E1) \) with \( \theta = 0 \) defined along the direction of the dipole moment, the components of the electric field vector \( \mathbf{E} \) are

\[
E_r = \frac{p}{4\pi \varepsilon_0} \frac{2 \cos \theta}{r^3},
\]

and

\[
E_\theta = \frac{p}{4\pi \varepsilon_0} \frac{\sin \theta}{r^3},
\]

where \( p \) is the electric dipole moment. There is no \( \phi \) dependence in the electric field, \( E_\phi = 0 \). At \( \theta = 90^\circ \), both \( E_r \) and \( E_\phi = 0 \), and \( E_\theta \neq 0 \), thus the electric field vector is parallel to the symmetry axis of the dipole field, see figure A.2. The appearance of this dipole produces a magnetic field, which is perpendicular to the symmetry axis.

In the case of a magnetic dipole transition \( (M1) \), the magnetic field components are

\[
B_r = \frac{m\mu_0}{4\pi} \frac{2 \cos \theta}{r^3},
\]

and
\[ B_\theta = \frac{m \mu_0 \sin \theta}{4\pi r^3}, \]

where \( m \) is the magnetic dipole moment. There is no \( \phi \) dependence in the magnetic field, \( B_\phi = 0 \). At \( \theta = 90^\circ \), both \( B_r \) and \( B_\phi = 0 \), and \( B_\theta \neq 0 \), thus the magnetic field vector is parallel to the symmetry axis of the dipole field, see figure A.2. In this case, the electric field is perpendicular to the symmetry axis of the magnetic dipole.

Figure A.2: Schematic of electric dipole (top) and magnetic dipole (bottom). Note the direction of electric and magnetic fields for photons emitted at various angles. The arrows to the right represent photons emitted at \( 90^\circ \) to the dipole axis.
Figure A.3: Schematic of Compton scattering defining the scattering plane, polarization plane, the angle between the planes (ψ), and the Compton scattering angle (θ), adapted from [Sc94].

This difference in polarization for the magnetic and electric dipoles, and for higher multipoles, is discernible in the direction of scattering in a Compton scattering event. Define the plane containing the electric field of the incident photon as the polarization plane and the plane containing both the incident and scattered photons as the Compton scattering plane. As shown schematically in figure A.3, the angle of scattering between the incident and scattered photon is defined as θ (not the angle from the dipole axis from figure A.2) and the angle between the planes is defined as ψ.

The differential cross section for Compton scattering is given by the Klein-Nishina formula:

\[
\frac{d\sigma}{d\Omega} = \frac{r_0^2}{2} \left[ \left( \frac{E'}{E} \right)^2 + \left( \frac{E'}{E} \right)^2 \cos^2 \theta - 2 \sin^2 \theta \cos \theta \right],
\]

where \( r_0 = \frac{e^2}{m_0 c^2} \) is the classical electron radius [Sc94]. The differential cross section is maximum for \( \psi = 90^\circ \). That is, Compton scattering is predominately perpendicular to the electric field vector of the incident photon, see figure A.3. At the clover detector ring, \( \sim 90^\circ \) from the beam axis, which is about \( 90^\circ \) from the multipole axis due to the orientation of the nuclei in fusion-evaporation reactions, the beam direction typically lies near the Compton scattering plane for electric transitions, and is typically perpendicular.
to the Compton scattering plane for magnetic transitions. Thus, by considering scattering
between clover leaves orthogonal and parallel to the beam direction, transition
polarizations can be determined.

A.4 The transition matrix element - from classical to quantum

The average power radiated from an electric dipole, d, is

\[
P = \frac{1}{12\pi\varepsilon_0} \frac{\omega^4}{c^2} d^2,
\]

and from a magnetic dipole, μ, is

\[
P = \frac{1}{12\pi\varepsilon_0} \frac{\omega^4}{c^2} \mu^2,
\]

[Kr88]. In more general terms for higher multipoles, the radiated power is,

\[
P(\sigma L) = \frac{2(L+1)c}{\varepsilon_0 L[(2L+1)!]^2} \left( \frac{\omega}{c} \right)^{2L+2} \text{Im}(\sigma L)^2,
\]

where σ is M for a magnetic field or E for an electric field, and L is the multipolarity. In classical electrodynamics, m is the amplitude of the time varying field. In quantum
mechanics, this is treated as the multipole operator, connecting the initial and final states
of the wave function, \(\Psi\), with matrix element

\[
m_{fi}(\sigma L) = \int \Psi^*_f m(\sigma L) \Psi_i dV.
\]

The transition matrix element is of course very useful in predicting observables of
nuclear processes. The probability of transition per unit time, \(\lambda\) (also denoted \(T_\gamma\), the rate
of the transition), is the inverse of the mean lifetime of the state, \(\tau\), and proportional to the
linewidth of the state, \(\Gamma\), such that

\[
T = \lambda = 1/\tau = \Gamma/h.
\]

Following Fermi’s Golden Rule, the decay probability can be expressed as

\[
\lambda = \frac{2\pi}{h} |m_{fi}|^2 \rho(E_\gamma),
\]

where \(\rho(E_\gamma)\) is the density of final states.
Approached another way, for a transition that emits a photon of energy $\hbar\omega$ with a probability of transition per unit time $\lambda(\sigma L)$, the power can be expressed

\begin{equation}
\lambda(\sigma L) = \frac{P(\sigma L)}{\hbar\omega} = \frac{2(L + 1)}{\varepsilon_0 \hbar \mu [(2L + 1)]} \left( \frac{\omega}{c} \right)^{2L+1} \text{Im}(\sigma L)^2.
\end{equation}

The decay probability can be expressed as

\begin{equation}
\lambda(\sigma L) = \frac{P(\sigma L)}{\hbar\omega} = \frac{2(L + 1)}{\varepsilon_0 \hbar \mu [(2L + 1)]} \left( \frac{\omega}{c} \right)^{2L+1} \text{Im}(\sigma L)^2.
\end{equation}

Evaluating the multipole operator for a square well potential in $r$, and estimating the angular integrals as 1, the transition probabilities become [We51],

\begin{equation}
\lambda(EL) = \frac{4.4(L + 1)}{L[(2L + 1)]^{3/2}} \left( \frac{3}{L + 3} \right)^2 \left( \frac{E}{197\text{MeV}} \right)^{2L+1} \text{R}^{2L-2} 10^{21}/\text{sec},
\end{equation}

$\text{R}$ in fm, and for magnetic multipoles,

\begin{equation}
\lambda(ML) = \frac{1.9(L + 1)}{L[(2L + 1)]^{3/2}} \left( \frac{3}{L + 3} \right)^2 \left( \frac{E}{197\text{MeV}} \right)^{2L+1} \text{R}^{2L-2} 10^{21}/\text{sec},
\end{equation}

with $\text{R}$ in fm. For the first few multipole orders, with the units as $\lambda$ [s$^{-1}$] and $E$ [MeV], and $R = R_0 A^{1/3}$, the estimated decay constants for this single particle estimate are [Kr88]:

\begin{align*}
\lambda(E1) &= 1.0 \times 10^{14} A^{2/3} E^3 \\
\lambda(E2) &= 7.3 \times 10^{7} A^{4/3} E^5 \\
\lambda(E3) &= 34 A^2 E^2 \\
\lambda(M1) &= 5.6 \times 10^{13} E^3 \\
\lambda(M2) &= 3.5 \times 10^{7} A^{2/3} E^5 \\
\lambda(M3) &= 16 A^{4/3} E^7
\end{align*}

It is interesting to note that, following selection rules and the decay constants, for $\Delta I=1$ parity conserving transitions, magnetic dipoles ($M1$) and electric quadrupoles ($E2$) may compete favorably.

### A.5 Transition strengths: $B(\sigma L)$

There is a collectivity usually involved in nuclear transitions, which can be seen in increased transition rates compared with those estimated from only single particle transitions. The strength of a transition is determined by comparing experimental transition widths, $\Gamma_{\text{expt}}$, with those from single particle estimates, $\Gamma_{\text{sp}}$, and is defined as

\begin{equation}
B(\sigma L) = \frac{\Gamma_{\text{expt}}}{\Gamma_{\text{sp}}} = \frac{T_{\text{expt}}}{T_{\text{sp}}}.
\end{equation}
In terms of the reduced transition probabilities, and writing T instead of \( \lambda \), equation A.32 becomes

\[
T_{fi} = \frac{2(L + 1)}{\varepsilon_0 h L L_1(2L + 1)!!} \left( \frac{E}{\hbar \omega} \right)^{2L+1} B(\sigma L; I_i \rightarrow I_f).
\]  

(A.37)

The reduced transition probabilities, \( B(\sigma L) \), are expressed in terms of the reduced matrix element \( \langle I_f \parallel m(\sigma L) \parallel I_i \rangle \) as

\[
B(EL; I_i \rightarrow I_f) = \frac{1}{2I_i + 1} \left| \langle I_f \parallel m(EL) \parallel I_i \rangle \right|^2
\]

for electric transitions and

\[
B(ML; I_i \rightarrow I_f) = \frac{1}{2I_i + 1} \left| \langle I_f \parallel m(ML) \parallel I_i \rangle \right|^2
\]

for magnetic transitions. The reduced matrix element \( \langle I_f \parallel m(\sigma L) \parallel I_i \rangle \) is found through the 3j symbol from the full matrix element:

\[
\langle I_i M_i \parallel m(\sigma L, \mu) \parallel I_f M_f \rangle = (-1)^{L_i - M_i - M_f} \left( \begin{array}{ccc} I_i & L & I_f \\ -M_i & \mu & M_f \end{array} \right) \langle I_i \parallel m(\sigma L) \parallel I_f \rangle,
\]

(A.39)

where \( \mu \) is the negative value of the z component of the angular momentum, \( M_f - M_i = -\mu \). Common transition strengths measured, \( B(M1) \) and \( B(E2) \), go as

\[
B(M1; I \rightarrow I - 1) \propto (g_k - g_R)^2
\]

(A.40a)

and

\[
B(E2; I \rightarrow I - 2) \propto Q_0^2,
\]

(A.40b)

where \( g_k \) and \( g_R \), are the single particle and core rotational g factors, respectively, and \( Q_0 \) is the intrinsic quadrupole moment of the nucleus. \( g_R \sim Z/A \) while \( g_k \) depends on single particle properties such as the orbital. The \( B(E2) \) values, based on the quadrupole moment, can be very strong for deformed nuclei.

The observed quadrupole moment, \( Q \), is related to the intrinsic quadrupole moment, \( Q_0 \), via [Ca00]

\[
Q = Q_0 \left( \frac{2K^2 - I(I + 1)}{(I + 1)(2I + 3)} \right).
\]

(A.41)
For inband transitions, the $B(E2)$ and quadrupole moments relate via Clebsch-Gordon coefficients, and equation A.40b can be expressed more explicitly as

\[(A.42) \quad B(E2; I_i \rightarrow I_f) = \frac{5}{16\pi} e^2 Q_0^2 \left< I_i \right| K20 \left| I_f \right| K \right>^2.

Through the intrinsic quadrupole moment, the $B(E2)$ values give information on the nuclear deformation, since

\[(A.43) \quad Q_0 = \frac{3}{\sqrt{5\pi}} Z R_0^2 \beta (1 + 0.16\beta),
\]

where $R_0 \propto A^{1/3}$ and $\beta$ is the deformation parameter, see chapter 2 for more details. The $B(E2)$ values can be derived from lifetime measurements. Following equations 3.12 and 3.13,

\[(A.44) \quad \frac{1}{\tau} = B(E2) (7.3 \times 10^7 A^{4/3} E^5),
\]

where $\tau$ is the mean lifetime.
## Appendix B

### Tables

**Table 1: $^{136}$Pm Eγ, Intensity, Compton asymmetry ratio, DCO, and transition assignment**

<table>
<thead>
<tr>
<th>Eγ (keV)</th>
<th>intensity</th>
<th>Compton asym. ratio</th>
<th>DCO ratio</th>
<th>GateDCO</th>
<th>assignment</th>
</tr>
</thead>
<tbody>
<tr>
<td>99</td>
<td>75 (1)</td>
<td></td>
<td></td>
<td></td>
<td>$M1/E2$</td>
</tr>
<tr>
<td>168</td>
<td>$\geq 145$ (3)</td>
<td>0.006 (4)</td>
<td>1.69 (7)</td>
<td>d</td>
<td>$M1/E2$</td>
</tr>
<tr>
<td>285</td>
<td>$\equiv 100$</td>
<td>0.03 (1)</td>
<td></td>
<td></td>
<td>$M1/E2$</td>
</tr>
<tr>
<td>289</td>
<td>16 (4)</td>
<td>2.5 (1.2)</td>
<td></td>
<td>d</td>
<td>$M1/E2$</td>
</tr>
<tr>
<td>292</td>
<td>71 (3)</td>
<td>0.04 (1)</td>
<td>1.48 (13)</td>
<td>d</td>
<td>$M1/E2$</td>
</tr>
<tr>
<td>357</td>
<td>22 (1)</td>
<td>0.06 (2)</td>
<td>1.74 (24)</td>
<td>d</td>
<td>$M1/E2$</td>
</tr>
<tr>
<td>364</td>
<td>6 (1)</td>
<td>0.07 (5)</td>
<td>2.7 (1.4)</td>
<td>d</td>
<td>$M1/E2$</td>
</tr>
<tr>
<td>409</td>
<td>46 (2)</td>
<td>0.06 (2)</td>
<td>1.79 (15)</td>
<td>d</td>
<td>$M1/E2$</td>
</tr>
<tr>
<td>418</td>
<td>6 (1)</td>
<td>1.34 (1.35)</td>
<td></td>
<td>d</td>
<td>$M1/E2$</td>
</tr>
<tr>
<td>424</td>
<td>4 (2)</td>
<td>-0.08 (9)</td>
<td>0.87 (39)</td>
<td>d</td>
<td>$M1/E2$</td>
</tr>
<tr>
<td>426</td>
<td>5 (2)</td>
<td></td>
<td></td>
<td>(M1/E2)</td>
<td></td>
</tr>
<tr>
<td>453</td>
<td>9 (1)</td>
<td></td>
<td></td>
<td></td>
<td>$E2$</td>
</tr>
<tr>
<td>457</td>
<td>9 (1)</td>
<td></td>
<td></td>
<td></td>
<td>$M1/E2$</td>
</tr>
<tr>
<td>468</td>
<td>12 (1)</td>
<td>1.44 (39)</td>
<td></td>
<td>d</td>
<td>$M1/E2$</td>
</tr>
<tr>
<td>479</td>
<td>19 (2)</td>
<td>0.002 (26)</td>
<td>0.28 (35)</td>
<td>d</td>
<td>$M1/E2$</td>
</tr>
<tr>
<td>506</td>
<td>11 (1)</td>
<td></td>
<td></td>
<td></td>
<td>$M1/E2$</td>
</tr>
<tr>
<td>577</td>
<td>24 (1)</td>
<td>-0.07 (2)</td>
<td>0.55 (6)</td>
<td>d</td>
<td>$E2$</td>
</tr>
<tr>
<td>590</td>
<td>18 (4)</td>
<td>0.03 (38)</td>
<td></td>
<td></td>
<td>$M1/E2$</td>
</tr>
<tr>
<td>595</td>
<td>22 (2)</td>
<td>0.07 (3)</td>
<td>2.60 (58)</td>
<td>d</td>
<td>$M1/E2$</td>
</tr>
<tr>
<td>638$^e$</td>
<td>$\approx$ 1 (1)</td>
<td></td>
<td></td>
<td></td>
<td>(M1/E2)</td>
</tr>
<tr>
<td>653</td>
<td>2 (1)</td>
<td></td>
<td></td>
<td></td>
<td>(E2)</td>
</tr>
<tr>
<td>666</td>
<td>5 (1)</td>
<td>0.07 (12)</td>
<td></td>
<td></td>
<td>$M1/E2$</td>
</tr>
<tr>
<td>684</td>
<td>6 (1)</td>
<td>0.26 (7)</td>
<td>1.20 (33)</td>
<td>d</td>
<td>$M1/E2$</td>
</tr>
<tr>
<td>700</td>
<td>16 (1)</td>
<td>-0.15 (3)</td>
<td>0.56 (9)</td>
<td>d</td>
<td>$E2$</td>
</tr>
<tr>
<td>751</td>
<td>6 (1)</td>
<td>-0.09 (4)</td>
<td></td>
<td></td>
<td>$M1/E2$</td>
</tr>
<tr>
<td>758</td>
<td>5 (1)</td>
<td></td>
<td></td>
<td></td>
<td>(E2)</td>
</tr>
<tr>
<td>765</td>
<td>28 (1)</td>
<td>-0.1 (2)</td>
<td>0.64 (8)</td>
<td>d</td>
<td>$E2$</td>
</tr>
<tr>
<td>790$^e$</td>
<td>&lt; 2</td>
<td></td>
<td></td>
<td></td>
<td>(E2)</td>
</tr>
<tr>
<td>835</td>
<td>20 (1)</td>
<td>-0.04 (2)</td>
<td>0.48 (8)</td>
<td>d</td>
<td>$E2$</td>
</tr>
<tr>
<td>850</td>
<td>6 (1)</td>
<td>0.3 (56)</td>
<td></td>
<td></td>
<td>$E2$</td>
</tr>
<tr>
<td>879$^e$</td>
<td>&lt; 1</td>
<td></td>
<td></td>
<td></td>
<td>(E2)</td>
</tr>
<tr>
<td>897</td>
<td>27 (1)</td>
<td>-0.04 (2)</td>
<td>0.64 (9)</td>
<td>d</td>
<td>$E2$</td>
</tr>
<tr>
<td>925$^f$ lower</td>
<td>12 (1)</td>
<td>0.16 (13)</td>
<td>0.11 (10)</td>
<td>d</td>
<td>$E2$</td>
</tr>
<tr>
<td>925$^f$ upper</td>
<td>4 (1)</td>
<td></td>
<td></td>
<td></td>
<td>(E2)</td>
</tr>
<tr>
<td>975</td>
<td>12 (1)</td>
<td>-0.07 (6)</td>
<td></td>
<td></td>
<td>$E2$</td>
</tr>
</tbody>
</table>
Table 1 continued

<table>
<thead>
<tr>
<th>$E_\gamma$ (keV)</th>
<th>intensity $^b$</th>
<th>Compton asym. ratio</th>
<th>DCO ratio</th>
<th>GateDCO</th>
<th>assignment</th>
</tr>
</thead>
<tbody>
<tr>
<td>1009$^e$</td>
<td>&lt; 1</td>
<td></td>
<td></td>
<td></td>
<td>(E2)</td>
</tr>
<tr>
<td>1092$^e$</td>
<td>&lt; 1</td>
<td></td>
<td></td>
<td></td>
<td>(E2)</td>
</tr>
<tr>
<td>1095$^e$</td>
<td>&lt; 1</td>
<td></td>
<td></td>
<td></td>
<td>(E2)</td>
</tr>
<tr>
<td>1108</td>
<td>2 (1)</td>
<td></td>
<td></td>
<td></td>
<td>(E2)</td>
</tr>
</tbody>
</table>

a: Error on $E_\gamma$ is $\leq 0.5$ keV, below the given significant figures
b: Intensities given are total intensities and are relative to the intensity of the 285 keV transition.

Some intensities were found by from the total transition intensity populating (depopulating) a level that the transition in question depopulates (populates). This typically only gives a maximum or minimum possible transition intensity value.
c: Efficiencies are not well defined below $E_\gamma \approx 200$ keV
d: Intensity of the $10^\pi \rightarrow 9^+ 168$ keV transition found indirectly by adding intensities feeding into the $10^\pi$ level.
e: Tentative transitions
f: The 925 keV transition is a doublet

Transition assignments are from measurements and/or from prior assignments. Assignments implied in the current work solely by the similar structure of the yrast and yrare band are presented in parentheses.

Table 2a: $^{136}$Pm TAC calculated $B(M1; I\rightarrow I-1)/B(E2; I\rightarrow I-2)$

<table>
<thead>
<tr>
<th>Yrast Band</th>
<th>$h\omega$ (MeV)</th>
<th>$\theta$ (deg)</th>
<th>$\phi$ (deg)</th>
<th>$I$ (h)</th>
<th>$B(M1)/B(E2)$</th>
<th>$[\mu_N/eb]^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.200</td>
<td>53.43</td>
<td>71.50</td>
<td>9.86</td>
<td>15.0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.250</td>
<td>59.04</td>
<td>50.00</td>
<td>10.75</td>
<td>6.0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.300</td>
<td>62.79</td>
<td>37.00</td>
<td>11.75</td>
<td>3.3</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.350</td>
<td>68.40</td>
<td>25.00</td>
<td>12.03</td>
<td>2.2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.375</td>
<td>70.28</td>
<td>21.50</td>
<td>13.84</td>
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<td>0.00</td>
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Table 2b: $^{136}$Pm particle + rotor calculated and experimental $B(M1; I\rightarrow I-1)/B(E2; I\rightarrow I-2)$

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<tr>
<th>Yrast Band</th>
<th>experiment $B(M1)/B(E2)$</th>
<th>$[\mu_N/eb]^2$</th>
<th>particle + rotor calculations $B(M1)/B(E2)$</th>
<th>$[\mu_N/eb]^2$</th>
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<td>$[\mu_N/eb]^2$</td>
</tr>
<tr>
<td>h</td>
<td></td>
<td></td>
<td>B(E2)</td>
<td>$[\mu_N/eb]^2$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>B(M1)/B(E2)</td>
<td>$[\mu_N/eb]^2$</td>
</tr>
<tr>
<td>10</td>
<td>9.84(4.3)</td>
<td>1.764</td>
<td>0.112</td>
<td>15.75</td>
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<tr>
<td>11</td>
<td>9.84(4.3)</td>
<td>1.764</td>
<td>0.112</td>
<td>15.75</td>
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<tr>
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<td>1.504</td>
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Table 2b continued

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<th>Spin (h)</th>
<th>B(M1)/B(E2)</th>
<th>B(M1)</th>
<th>B(E2)</th>
<th>B(M1)/B(E2)</th>
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<tbody>
<tr>
<td></td>
<td>(µN/eb)^2</td>
<td>(µN)^2</td>
<td>(eb)^2</td>
<td>(µN/eb)^2</td>
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<tr>
<td>16</td>
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<td>0.154</td>
<td>0.47</td>
<td>0.33</td>
</tr>
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<td>17</td>
<td>1.8(6)</td>
<td>1.293</td>
<td>0.515</td>
<td>2.51</td>
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<tr>
<td>18</td>
<td>2.1(3.6)</td>
<td>0.103</td>
<td>0.543</td>
<td>0.19</td>
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<td>19</td>
<td></td>
<td>1.249</td>
<td>0.581</td>
<td>2.15</td>
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Yrare Band

<table>
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<tr>
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<th>B(M1)/B(E2)</th>
<th>B(M1)</th>
<th>B(E2)</th>
<th>B(M1)/B(E2)</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>(µN/eb)^2</td>
<td>(µN)^2</td>
<td>(eb)^2</td>
<td>(µN/eb)^2</td>
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<tr>
<td>10</td>
<td>1.76</td>
<td>0.116</td>
<td>15.17</td>
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<tr>
<td>11</td>
<td>1.698</td>
<td>0.154</td>
<td>11.03</td>
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<td>1.358</td>
<td>0.188</td>
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<tr>
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<td>1.219</td>
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<td>14</td>
<td>5.7(1.5)</td>
<td>0.15</td>
<td>0.233</td>
<td>0.64</td>
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<td>16</td>
<td></td>
<td>0.177</td>
<td>0.347</td>
<td>0.51</td>
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<td>0.712</td>
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<td>1.51</td>
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<td>18</td>
<td></td>
<td>0.164</td>
<td>0.47</td>
<td>0.35</td>
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<tr>
<td>19</td>
<td></td>
<td>0.192</td>
<td>0.534</td>
<td>0.36</td>
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Table 3a: \(^{138}\text{Eu}\) \(E\gamma\), Intensity, and transition assignment

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<th>(E\gamma) (keV)</th>
<th>intensity</th>
<th>assignment</th>
</tr>
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<td>104</td>
<td>78(3)</td>
<td>M1 / E2</td>
</tr>
<tr>
<td>164</td>
<td>2(2)</td>
<td>(M1 / E2)</td>
</tr>
<tr>
<td>167</td>
<td>=100</td>
<td>E2</td>
</tr>
<tr>
<td>262</td>
<td>43(2)</td>
<td>M1 / E2</td>
</tr>
<tr>
<td>273</td>
<td>64(2)</td>
<td>M1 / E2</td>
</tr>
<tr>
<td>299</td>
<td>7(1)</td>
<td>(M1 / E2)</td>
</tr>
<tr>
<td>320</td>
<td>21(1)</td>
<td>M1 / E2</td>
</tr>
<tr>
<td>351</td>
<td>9(2)</td>
<td>(M1 / E2)</td>
</tr>
<tr>
<td>362</td>
<td>29(1)</td>
<td>M1 / E2</td>
</tr>
<tr>
<td>382</td>
<td>6(1)</td>
<td>M1 / E2</td>
</tr>
<tr>
<td>383</td>
<td>4(1)</td>
<td>(M1 / E2)</td>
</tr>
<tr>
<td>405</td>
<td>5(1)</td>
<td>(M1 / E2)</td>
</tr>
<tr>
<td>412</td>
<td>7(1)</td>
<td>(M1 / E2)</td>
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<tr>
<td>416</td>
<td>3(1)</td>
<td>(M1 / E2)</td>
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<tr>
<td>420</td>
<td>&lt; 1</td>
<td>(M1 / E2)</td>
</tr>
<tr>
<td>426</td>
<td>3(1)</td>
<td>M1 / E2</td>
</tr>
<tr>
<td>428</td>
<td>15(1)</td>
<td>M1 / E2</td>
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<tr>
<td>$E_γ$ (keV)$^a$</td>
<td>intensity$^b$</td>
<td>assignment$^c$</td>
</tr>
<tr>
<td>-----------------</td>
<td>----------------</td>
<td>----------------</td>
</tr>
<tr>
<td>441$^d$ lower</td>
<td>2 (1)</td>
<td>$M_1 / E_2$</td>
</tr>
<tr>
<td>441$^d$ upper</td>
<td>8 (1)</td>
<td>$M_1 / E_2$</td>
</tr>
<tr>
<td>463$^d$ yrast</td>
<td>6 (1)</td>
<td>$M_1 / E_2$</td>
</tr>
<tr>
<td>463$^d$ yrare</td>
<td>1 (1)</td>
<td>(E2)</td>
</tr>
<tr>
<td>478</td>
<td>5 (1)</td>
<td>($M_1 / E_2$)</td>
</tr>
<tr>
<td>504</td>
<td>1 (1)</td>
<td>($M_1 / E_2$)</td>
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<tr>
<td>522</td>
<td>5 (1)</td>
<td>($M_1 / E_2$)</td>
</tr>
<tr>
<td>524</td>
<td>2 (1)</td>
<td>($M_1 / E_2$)</td>
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<tr>
<td>526</td>
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<td>($M_1 / E_2$)</td>
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<tr>
<td>535</td>
<td>23 (1)</td>
<td>E2</td>
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<td>547</td>
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<tr>
<td>625</td>
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<td>(E2)</td>
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<td>630</td>
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<td>(E2)</td>
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<td>650</td>
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<td>(E2)</td>
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<tr>
<td>680</td>
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<tr>
<td>683</td>
<td>27 (2)</td>
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<td>688</td>
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<td>($M_1 / E_2$)</td>
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<td>743</td>
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<td>($M_1 / E_2$)</td>
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<td>748</td>
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<td>(E2)</td>
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<tr>
<td>756</td>
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<td>(E2)</td>
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<td>(E2)</td>
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<td>828</td>
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<td>(E2)</td>
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<td>(E2)</td>
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<td>879</td>
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<td>E2</td>
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<td>897</td>
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<td>(E2)</td>
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<td>1043</td>
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<tr>
<td>1146</td>
<td>1 (1)</td>
<td>(E2)</td>
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a: Error on $E_γ$ is $\leq 0.5$ keV, below the given significant figures
b: $^{156}$Pm efficiency values used for analysis. $^{158}$Eu efficiency curve was not made available to us. Efficiencies are not well defined below $E_γ \approx 200$ keV. Intensities are given relative to the 167 keV transition.
c: Assignments from reference [Pa94] are presented without parentheses. Assignments implied in the current work by assuming a similar structure for all of the yrast band, and for the yrare band structure similar to the yrast band, are presented in parentheses.
d: These transitions are doublets.
### Table 3b: $^{138}$Eu experimental $B(M1; I\rightarrow I-1)/B(E2; I\rightarrow I-2) [(\mu_N/\text{eb})^2]$

<table>
<thead>
<tr>
<th>Yrast Band</th>
<th>Spin (h)</th>
<th>$B(M1)/B(E2) [(\mu_N/\text{eb})^2]$</th>
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<td></td>
<td>11</td>
<td>3.1(4)</td>
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<td>1.6(2)</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>2.1(6)</td>
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<tr>
<td></td>
<td>16</td>
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<th>$B(M1)/B(E2) [(\mu_N/\text{eb})^2]$</th>
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<td>4.0(1.0)</td>
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### Table 3c: $^{138}$Eu TAC calculated $B(M1; I\rightarrow I-1)/B(E2; I\rightarrow I-2) [(\mu_N/\text{eb})^2]$

<table>
<thead>
<tr>
<th>Yrast Band</th>
<th>$h\omega$ (MeV)</th>
<th>$\theta$ (deg)</th>
<th>$\phi$ (deg)</th>
<th>$I(h)$</th>
<th>$B(M1)/B(E2) [(\mu_N/\text{eb})^2]$</th>
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### Table 4: $^{140}$Eu $E_\gamma$ intensity, Compton asymmetry ratio, angular distribution, DCO ratio, DCO gate, and transition assignments

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<tr>
<th>$E_\gamma$ (keV)</th>
<th>intensity$^a$</th>
<th>Compton asymmetry ratio</th>
<th>angular distribution ratio</th>
<th>DCO ratio</th>
<th>gate$^b$</th>
<th>assignment$^b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>71.0 (2)</td>
<td>$\geq 130 (2)^{c,d}$</td>
<td>0.55 (55)$^c$</td>
<td>2.83 (98)$^c$</td>
<td>q</td>
<td>M1/E2</td>
<td></td>
</tr>
<tr>
<td>94.9 (2)</td>
<td>$\geq 65 (2)^{c,d}$</td>
<td>0.57 (23)$^c$</td>
<td>1.25 (42)$^c$</td>
<td>q</td>
<td>M1/E2</td>
<td></td>
</tr>
<tr>
<td>153.7 (2)</td>
<td>33 (1)$^c$</td>
<td>0.36 (24)</td>
<td>0.88 (29)$^e$</td>
<td>d</td>
<td>M1/E2</td>
<td></td>
</tr>
<tr>
<td>170.0 (2)</td>
<td>39 (7)$^c$</td>
<td>0.16 (21)$^c$</td>
<td>1.33 (9)$^c$</td>
<td>q</td>
<td>M1/E2</td>
<td></td>
</tr>
<tr>
<td>170.6 (1)</td>
<td>72 (1)$^c$</td>
<td>0.16 (21)$^c$</td>
<td>1.33 (9)$^c$</td>
<td>q</td>
<td>M1/E2</td>
<td></td>
</tr>
<tr>
<td>191.1 (2)</td>
<td>35 (1)$^c$</td>
<td>0.16 (8)$^c$</td>
<td>1.23 (7)$^c$</td>
<td>q</td>
<td>M1/E2</td>
<td></td>
</tr>
<tr>
<td>209.9 (2)</td>
<td>26 (1)</td>
<td>0.00 (19)</td>
<td>0.96 (33)</td>
<td>d</td>
<td>M1/E2</td>
<td></td>
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</table>

186
Table 4 continued

<table>
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<tr>
<th>$E_Y$ (keV)</th>
<th>intensity*</th>
<th>Compton asymmetry ratio</th>
<th>angular distribution ratio $a_2$</th>
<th>DCO ratio</th>
<th>gate&lt;sub&gt;DCO&lt;/sub&gt;</th>
<th>assignment&lt;sup&gt;b&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>230.4 (1)</td>
<td>34 (1)</td>
<td>0.16 (5)</td>
<td></td>
<td>0.73 (4)</td>
<td>d</td>
<td>M1 / E2</td>
</tr>
<tr>
<td>244.0 (2)</td>
<td>≥ 8&lt;sup&gt;d&lt;/sup&gt;</td>
<td>1.59 (75)</td>
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<td></td>
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<td>258.0 (2)</td>
<td>12 (1)</td>
<td>0.16 (7)</td>
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<td>1.00 (12)</td>
<td>d</td>
<td>M1 / E2</td>
</tr>
<tr>
<td>274.7 (3)</td>
<td>16 (1)</td>
<td>0.11 (12)</td>
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<td>1.49 (82)</td>
<td>q</td>
<td>M1 / E2</td>
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<td>275.6 (2)</td>
<td>31 (1)</td>
<td>0.29 (13)</td>
<td>-0.21 (26)</td>
<td>0.96 (21)</td>
<td>d</td>
<td>M1 / E2</td>
</tr>
<tr>
<td>285.4 (3)</td>
<td>&gt; 3&lt;sup&gt;d&lt;/sup&gt;</td>
<td>0.13 (9)</td>
<td>0.09 (12)</td>
<td>0.78 (10)</td>
<td>d</td>
<td>M1 / E2</td>
</tr>
<tr>
<td>292.9 (4)</td>
<td>&gt; 15&lt;sup&gt;d&lt;/sup&gt;</td>
<td>0.37 (11)</td>
<td>0.08 (21)</td>
<td>1.02 (20)</td>
<td>d</td>
<td>M1 / E2</td>
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<td>345.4 (3)</td>
<td>4 (1)</td>
<td>1.37 (41)</td>
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</tr>
<tr>
<td>361.3 (2)</td>
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<td>0.09 (9)</td>
<td>-0.23 (23)</td>
<td>0.63 (10)</td>
<td>d</td>
<td>M1 / E2</td>
</tr>
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<td>E2</td>
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<td>E2</td>
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<td></td>
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Table 4 continued

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<th>intensitya</th>
<th>Compton asymmetry ratio</th>
<th>angular distribution a2</th>
<th>DCO ratio</th>
<th>gateDCO</th>
<th>assignmentb</th>
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a: Intensities given are total intensities and are relative to the intensity of the 640.6 keV transition.
b: Assignments in parentheses are implied by the surrounding band structure.
c: Efficiencies are not well defined below Eγ ≈ 200 keV.
d: Some intensities were found by from the total transition intensity populating (depopulating) a level that the transition in question depopulates (populates). This typically only gives a maximum or minimum possible transition intensity value.

Table 5: 140Eu Transition strength ratios

<table>
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<tr>
<th>Band</th>
<th>Spin (h)</th>
<th>experiment B(M1)/B(E2) [(μN/eb)²]</th>
<th>particle + rotor calculations B(M1)/B(E2) [(μN/eb)²]</th>
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<td>8</td>
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<td>6.25</td>
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<td>9</td>
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<tr>
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<td>10</td>
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<td>0.57</td>
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<td>1.97(39)</td>
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<th>Band</th>
<th>Spin (h)</th>
<th>B(E2; I→I-2) out/B(E2; I→I-2) in [(μN/eb)²]</th>
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