MULTIPLE COULOMB EXCITATION STUDIES
IN EVEN-EVEN SAMARIUM NUCLEI

by

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The spectroscopy of the low-lying collective levels of the even-even samarium nuclei $^{148}$, $^{150}$, $^{152}$, and $^{154}$ have been studied by detecting the de-excitation gamma radiation following coulomb excitation with oxygen ions up to 65 Mev in energy. Information is presented both on the energy level structure of these nuclei and the reduced transition probabilities for excitation of some of the levels, and these are discussed within the framework of the existing collective models. The most striking deviations from the simple models occur in $^{152}$Sm and an attempt has been made to explain these deviations by first order interband mixing between the ground state band and the beta and gamma vibrational bands. The analysis is performed from measurements of branching ratios of interband transitions. Apparent success in explaining the deviations from the simple axial rotor model is obtained by invoking the hydrodynamic model to calculate the absolute interband transition probability. However, in this dissertation a measurement of the absolute transition probability negates this and other seemingly successful attempts based on the hydrodynamic model.

Reduced transition probabilities for vibrational states are obtained for $^{148}$Sm and $^{150}$Sm, and these new reduced transition probabilities for excitation of even parity states are in fair agreement with the relative magnitudes predicted for pure harmonic oscillations of the nuclear surface. Evidence for collective $3^-$ excitations in the four nuclei is presented, and the measured excitation probabilities are given.
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I. Introduction

A. Introduction

The collective model of Bohr and Mottelson\textsuperscript{1,2} has had far reaching success in describing the static and dynamic properties of the low-lying nuclear states. Even in its simplest form the model provides bases for a qualitative understanding of the relative spacings, spins, parities, moments and de-excitation modes of levels up to one or two Mev. Considerable success has been achieved in understanding gross characteristics of collective nuclear phenomena from a consideration of the effects of long and short range components of the nuclear forces. In the vicinity of major closed shells the short range components dominate and lead to spherical equilibrium nuclear shapes. As extra-core nucleons are added the long range components of the force act to distort the nucleon orbitals and produce a stable deformation. The collective models discussed here are based on the simplifying assumption that one may treat low-lying nuclear levels in terms of the motion of the nuclear surface, omitting consideration of the origin of the deformation.

For spherical nuclei the excitations of the nuclear surface are oscillations about the equilibrium shape. If the nuclear surface is non-spherical, then rotations of the deformed surface are also possible excitations. In the simplest form of the collective model the rotations and vibrations are considered to
be independent of one another, and rotational levels are expected to have the energy-angular momentum dependence of a symmetric top. However, in many nuclei this energy dependence is not followed exactly.

In recent years, at this laboratory and others experiment,\textsuperscript{3,4} experimental studies have been carried out on the deviations from the simple rotational model, and an effort has been made to correlate these deviations with perturbations in the collective Hamiltonian which had heretofore been neglected. This dissertation concerns itself with a study of the low-lying levels in the even-even samarium nuclei, performed in order to investigate selected perturbations by means of their effects on gamma ray transition probabilities. In this study coulomb excitation with oxygen ions has been employed to investigate the energy levels and gamma ray transitions in the samarium isotopes 148, 150, 152, and 154.

The samarium isotopes are of particular interest because they extend across the transition region at 88 to 90 neutrons at which nuclear characteristics change rapidly with neutron number. The Sm\textsuperscript{148} and Sm\textsuperscript{150} nuclei, with 86 and 88 neutrons, have level structures that may be considered vibrational in character, while Sm\textsuperscript{152} and Sm\textsuperscript{154}, with 90 and 92 neutrons, have well-developed rotational bands. The change in the shape of the nuclear surface with neutron number from spherical to strongly-deformed is reflected in the energies of the first excited states, as shown in Fig. 1.

From the point of view of the simplest shell models it is difficult to understand this sudden onset of nuclear deformation.
Closed shell nuclei are spherical, and one would expect a gradual increase in core distortion as nucleons are added to the unfilled shell. Yet this transition region is only a few neutrons wide in a shell containing 78 neutrons. An adequate explanation has come only through the superconducting model of the nucleus which considers pairing and quadrupole interactions of extra-core nucleons and their effect on the core. The deformation parameter $\beta$ is calculated by assuming a quadrupole moment and solving for the stable deformation that would produce that quadrupole moment. In the notation of Belyaev,\textsuperscript{5} for $\theta_N$ near $\theta_{N_o}$, the deformation is proportional to

$$\beta \sim \frac{\theta_N}{\theta_{N_o}} \sqrt{1 - \frac{\theta_{N_o}}{\theta_N}}$$

(1)

where the occupation number $\theta_N$ is

$$\theta_N = \frac{2N}{\Omega} \left(1 - \frac{N}{2\Omega}\right)$$

(2)

and $\theta_{N_o}$ is a parameter that is a constant for each nucleon shell. The number of nucleons in a shell is $N$, where $2\Omega$ is the maximum number in that shell. The spherical shape is unstable and deformation occurs for $\theta_N$ greater than $\theta_{N_o}$, so that the square root becomes real. The rapid change in $\beta$ for $\theta_N$ near $\theta_{N_o}$ is evident from Eq. 1 and the graph of $\beta/\beta_{\text{max}}$ as a function of $\theta_N/\theta_{N_o}$, given in Fig. 2.
Lengths of principal axes of an ellipsoid plotted as differences from a sphere.

\[ \frac{\delta R_k}{\sqrt{\frac{5}{4\pi}} R_0 \beta} \text{ vs } \gamma \]

\[ \delta R_3 \quad \delta R_1 \quad \delta R_2 \]

0°  60°  120°  180°  240°  300°  360°

\[ \theta_k/\theta_{no} \]

\[ \beta/\beta_{max} \]

FIG. 2

FIG. 3
The simple collective model prediction for the energy dependence of the rotational levels of an axially symmetric nucleus is given by

\[ E_J = \frac{\hbar^2}{2I} J(J+1) \]  

(3)

where \( I \) is the moment of inertia and \( J \) is the angular momentum of the level. The first excited state in Sm\(^{154}\) lies at 82 kev\(^6\) and the second excited state, which was unknown until this study, lies at 267 kev. The ratio \( E_4/E_2 \) given by Eq. 3 is 3.33, and the experimental ratio is 3.26. These are in fair agreement. However, in Sm\(^{152}\) the first and second excited states are at 121.8 and 366.6 kev\(^7\) and the ratio is \( E_4/E_2 = 3.010 \). This is a significant deviation from the rotational model value, and is investigated in detail in this dissertation.

The Sm\(^{148}\) and Sm\(^{150}\) nuclei are important for study because level structures resembling vibrational modes of oscillation are not well understood at present. The transition probabilities between first and second excited states have not been measured previously, and data on these transition strengths are necessary in order to make detailed comparisons with any collective model.

Coulomb excitation has been chosen as a means for studying the collective states of samarium nuclei for the following reasons. The coulomb excitation cross sections for exciting collective states are sufficiently large that adequate data may be obtained in reasonable amounts of time with existing accelerators and beam intensities. Most important is the fact...
that the excitation mechanism is well known. The energy transfer from the long range electromagnetic field of the bombarding particle is well understood, and nuclear structure information may be extracted from the data without complications from the nuclear force problem. The coulomb excitation calculations are discussed in more detail below.
B. Collective Models

1. The Hamiltonian for Collective Nuclear Motion

The nucleus has been represented with varying success by models which range from a treatment of the nucleons as strongly interacting, as in a liquid drop, to the other extreme of complete independence, as in the extreme independent particle or spherical shell model. The latter approach has been carried out with a high degree of success in the prediction of ground state properties and low energy excitations by Mayer and Haxel, Jensen, and Suesc. The simplest independent particle model considers nucleons moving in a self-consistent spherically symmetric potential. It gives a satisfactory explanation for the magic number effects observed as discontinuities in binding energy, capture cross sections, isotopic abundances, and beta and alpha decay systematics, for the so-called islands of isomerism, and for general trends in nuclear ground state spins and parities. A major failure of the simple shell model is reflected in the fact that the magnitude of the experimental quadrupole moments is much larger than the single particle predictions, suggesting the existence of collective nuclear motion involving many nucleons. In order to explain this discrepancy various collective models have been proposed that consider the effects of coupled particle motion.

An early explanation of the large quadrupole moments was proposed by Rainwater, who calculated the polarization of the nuclear core by a particle outside the core. In this case the
nucleons move in a deformed potential and the large quadrupole moment may be accounted for by the involvement of many particles. S. G. Nilsson, T. D. Newton, and others have performed calculations, to include nuclear distortion effects in the single particle orbits of the shell model, by adding terms involving the deviations from symmetry in the Hamiltonian used in the shell model. In these collective models the deformation of the nucleus from non-spherical shapes is postulated as one of the basic parameters. Early attempts by Brueckner considered a combination of all of the single particle orbits to obtain a deformation in a self-consistent manner, but calculations are very complex and obtaining useful results is difficult. More recently there has been marked success with this approach in calculations by Kelson and Levinson. The collective models of Bohr and Mottelson and of Davydov and Fillipov will be considered here as representative. In these models the equilibrium deformation is again assumed to be one of the given parameters.

Before discussing the particular assumptions and aspects of the collective models, it is of interest to consider an analogy, discussed by A. Bohr, between the motion of nucleons in a nucleus and electrons in a diatomic or more complex molecule. In the molecule electrons move in a non-spherical potential, with their motion perturbed by the much slower rotations and vibrations of the heavy nuclei. In the nucleus, each nucleon moves in an average force field that is roughly constant during the time of one orbit. The rotations and vibrations
of the nucleus as a whole occur more slowly because of the large effective mass. It is this mass difference which allows the approximate separation of collective coordinates specifying the orientation and magnitude of the nuclear deformation from the intrinsic coordinates specifying the particle orbits.

In the simple hydrodynamic collective model the nuclear surface is expressed as

\[ R = R_0 \left[ 1 + \sum_{\lambda u} \alpha_{\lambda u} Y_{\lambda u}(\theta, \phi) \right] \]  

in the space-fixed frame of reference, where \( R_0 \) is the radius of the equilibrium sphere and \( \lambda \) is the multipole order of the deformation. Deformations for \( \lambda = 1 \) are equivalent to a translation of the entire nucleus and may be neglected. Quadrupole deformations \((\lambda = 2)\) will be discussed in detail since it was the existence of large quadrupole moments that led to the introduction of collective models.

In this case there are just five \( \alpha_{\lambda u} \) quantities which are then the coordinates of the nuclear surface ellipsoid. The collective motion is described in terms of oscillations of the \( \alpha_{2u} \). For small changes in \( \alpha_{2u} \) the oscillations may be considered harmonic so that the Hamiltonian is written as

\[ H = \sum_{u} \left[ \frac{1}{2} B |\dot{\alpha}_{2u}|^2 + \frac{1}{2} C |\alpha_{2u}|^2 \right] \]  

The coefficients \( B \) and \( C \) are the mass parameter and the effective surface tension, in analogy with the mass and force constant of
a simple harmonic oscillator. Similarly, the frequency of oscillation is

$$\omega = \sqrt{\frac{C}{D}}$$  \hspace{1cm} (6)

It is customary to transform from the space-fixed to the body frame of reference that coincides with the principal axes of the ellipsoid by a rotation through the Euler angles $\theta_i$:

$$a_{\nu} = \sum_{\mu} a_{2\mu} D_{\mu\nu}^2$$  \hspace{1cm} (7)

where $D_{\mu\nu}^2$ is the rotation matrix for transforming the spherical harmonics from axes with $\mu$ quantum numbers to $\nu$ quantum numbers. In this coordinate system $a_2 = a_{-2}$ and $a_1 = a_{-1} = 0$, so that the five coordinates are $a_0$, $a_2$, and the three Euler angles of the rotation. A further transformation allows a more physical interpretation of these parameters:

$$a_0 = \beta \cos \gamma$$  \hspace{1cm} (8)

$$a_2 = \frac{1}{\sqrt{2}} \beta \sin \gamma$$

Since

$$\sum_{\mu} |a_{2\mu}|^2 = \sum_{\nu} |a_{\nu}|^2 = a_0^2 + 2a_2^2 = \beta^2$$  \hspace{1cm} (9)

the parameter $\beta$ is directly related to the total nuclear deformation. The significance of $\gamma$ is obtained by writing the
lengths of the principal axes as differences from the spherical values.

\[ \delta R_\kappa = \frac{5}{\sqrt{4\pi}} R_0 \beta \cos(\gamma - \frac{2\pi \kappa}{3}) \]  

where \( \kappa \) designates the 1, 2, or 3 axis. It is seen that the \( \gamma \) parameter determines the type of ellipsoid. For \( \gamma = 0 \) or an integral multiple of \( \pi/3 \) there is symmetry about one of the axes. With positive \( \beta \), the shape for \( \gamma = 0 \) is a prolate spheroid, while for \( \gamma = \pi/3 \) the shape is that of an oblate spheroid. It is convenient to limit the range of \( \gamma \) to \( \pi/6 \) and always choose the symmetry axis as the 3-axis. The range from \( \gamma = \pi/6 \) to \( \pi/3 \) is just the negative of the above range if the 2 and 3 axes are interchanged. (See Fig. 3.) For intermediate values of \( \gamma \) the nucleus is an ellipsoid.

The potential energy when written in terms of \( \beta \) becomes

\[ V = \frac{1}{2} C \sum_u |\alpha_{2u}|^2 = \frac{1}{2} C \beta^2 \]  

When expressed in terms of \( \beta \) and \( \gamma \) the kinetic energy term in Eq. 5 separates naturally into rotational and vibrational parts, representing rotations of the equilibrium shape and vibration about the equilibrium with fixed spatial orientation.

\[ T = \frac{1}{2} B (\beta^2 + \beta^2 \gamma^2) + \frac{1}{2} \sum_\kappa \frac{L_\kappa^2}{I_\kappa} \]
where $I_\kappa$ is the angular momentum component along the $\kappa$ axis and $J_\kappa$ is the moment of inertia about that axis,

$$I_\kappa = 4B^2 \sin^2(\gamma - \frac{2\pi \kappa}{3})$$

(13)

The lowest vibrational modes of excitation of a deformed nucleus are called $\beta$ and $\gamma$ vibrations for the following obvious reasons. For the case of axial symmetry ($\gamma = 0$), $\beta$ vibrations occur with $\gamma$ constant, and axial symmetry is maintained. Gamma vibrations take place with $\beta$ fixed, and axial symmetry is destroyed. The nucleus may rotate while undergoing shape oscillations, so that there are rotational bands based on the beta and gamma oscillations. These vibrational bands are discussed further below.

The complete nuclear Hamiltonian for collective motion is the sum of Eq. 11 and 12 and may be written in the form given by Preston and Kiang:

$$H = T_{vib} + T_{rot} + U_3 + V(\beta, \gamma)$$

$$= -\frac{\hbar^2}{2B} \left\{ \frac{1}{\beta^2} \frac{\partial}{\partial \beta} \left( \beta \frac{\partial}{\partial \beta} \right) + \frac{1}{\beta^2 \sin 3\gamma} \frac{\partial}{\partial \gamma} (\sin 3\gamma \frac{\partial}{\partial \gamma}) \right\}$$

$$+ \left[ \frac{\hbar^2}{4I_1} + \frac{\hbar^2}{4I_2} \right] [J(J+1) - K^2] + \frac{\hbar^2}{2I_3} K^2$$

$$+ \left[ \frac{\hbar^2}{4I_1} - \frac{\hbar^2}{4I_2} \right] [J_1^2 + J_2^2] + V(\beta, \gamma)$$

(14)
In this equation $\mathcal{J}$ is the kinetic energy, for $\beta$ and $\gamma$ vibrations, while the second and third terms are the rotational kinetic energy. The term in $J_1^2 - J_2^2$ is small and can be treated as a perturbation. It is non-zero for nuclei that are not axially symmetric. If the potential $V$ is a function of $\beta$ only, as in Eq. 11, the wave function is separable and may be written as a product of functions of $\beta$ and $\gamma$ and Euler angles:

$$\Psi = f_{J}(\beta) \sum_{K=-J}^{J} g_{K}(\gamma) D_{MK}^{J}(\theta_i)$$ (15)

For the special case of axial symmetry, for which $\gamma = 0$ and $U_3 = 0$, $K$ is a good quantum number and the sum over $K$ is omitted.

Antisymmetrization of the wave function imposes restrictions on the range of $J$ and $K$ values. The function $g_{K}$ is zero unless $K$ is even. Further, the wave function must be written as a sum of $D$ functions for positive and negative $K$.

$$\Psi = f_{J}(\beta) \sum_{K=0}^{J} g_{K}(\gamma) \Psi_{\text{rot}}$$ (16)

$$\Psi_{\text{rot}} = \frac{1}{\sqrt{16\pi^2 (1+\delta_{KJ})}} \left( D_{MK}^{J} + (-)^{J} D_{MK,-K}^{J} \right)$$ (17)

Thus far the discussion has been limited to the oscillations of the nuclear surface. For odd-mass nuclei it is necessary to consider the coupling of the angular momentum of individual nucleons to the angular momentum of the nuclear surface. For
even-even nuclei the angular momenta of pairs of nucleons add to zero, and one need only consider the surface oscillations. The following discussion is restricted to even-even nuclei.

2. Collective Behavior in Nuclei With Static Deformation

The rotational energy for the axially symmetric even-even nucleus, in which $J_1 = J_2$ and $U_3 = 0$, is given by

$$\langle \psi_{\text{rot}} | H_{\text{rot}} | \psi_{\text{rot}} \rangle = \frac{\hbar^2}{2I_1} [J(J+1) - K^2] - \frac{\hbar^2}{2I_3} K^2$$

$$= E_J$$ (18)

Since the $3$-axis is an axis of symmetry, then rotations about that axis are indistinguishable quantum mechanically and there is no energy associated with the motion. Therefore the second term in Eq. 18 must be zero, which requires $K = 0$. Another approach is to note that if $I_3$ is very small, the rotational energy is very large, and for a low-lying rotational band $K$ must be zero. In either case, for $K = 0$

$$E_J = \frac{\hbar^2}{2I} J(J+1)$$ (19)

Only even values of $J$ occur since the wave function $\psi_{\text{rot}}$ vanishes for $K = 0$ and $J$ odd.

The ground state rotational band discussed above is based on the equilibrium shape of the nucleus, for fixed beta and gamma values. It is also possible for the nucleus to undergo shape oscillations at the same time as rotations, and this
leads to rotational bands based on these vibrations. With beta
and gamma vibrations these are called beta and gamma vibrational
bands. Since beta vibrations occur with axial symmetry maintained,
\( K = 0 \) and possible \( J \) values are 0, 2, 4, etc. For the gamma
band axial symmetry is destroyed during the vibration and \( K \) is
non-zero. Since odd \( K \) is excluded by the symmetry properties
of the wave functions, the lowest gamma band has \( K = 2 \). Odd-spin
states may occur in this band with spin values of \( K, K+1, 
K+2, \) etc.

Collective rotations in which there is no restriction to
\( \gamma = 0 \) have been considered in greater generality by Davydov
and Fillipov.\(^{19}\) The nucleus is treated as an ellipsoid, and
rotations with fixed \( \beta \) and \( \gamma \) are considered. The range of gamma
is restricted to 0 to \( \pi/6 \) in their analysis (see above discussion)
and the wave function is

\[
\Psi = \sum K g_K^{J}(\gamma) \Psi_{\text{rot}}
\] (20)

where \( \Psi_{\text{rot}} \) is given in Eq. 17. The rotational Hamiltonian is

\[
\frac{3}{2} \sum \frac{L_x^2}{2I_x}
\]

\( x = 1 \) (21)

and \( I_x \) is given in Eq. 13. Again, \( K \) is even and may have several
values, up to and including \( J \), for each \( J \). If \( J \) is odd, \( K 
\) cannot be zero since \( \Psi_{\text{rot}} \) vanishes. There is then only one 0+
state, two 2+ states, one 3+ state, three 4+ states, etc. The
energy levels are given as a function of
\[ E/(\hbar^2/4B\beta^2) \], where
and are shown in Fig. 4. It is seen that a group of states above the ground state rotational band become much lower in energy as $\gamma$ is increased. These have the same spins as the gamma vibrational band in the symmetric nucleus. The ratio of the energies of the first two $2^+$ levels in a nucleus may be used to determine the value of $\gamma$, and then predict the positions of the other energy levels.

This asymmetric rotor model has been successful in accounting for energy level spacings and reduced transition probabilities for a number of nuclei. For examples one may refer to data on osmium nuclei,\textsuperscript{23} which lie in a transition region, and data on ruthenium and cadmium nuclei,\textsuperscript{24} which are classed as vibrational. However, as will be shown further in this thesis, the model does not fare so well for nuclei at the lower mass region of the rare earths.

The simplest asymmetric rotor model does not consider the effect of collective vibrations. For example, this model ignores the existence of the 0+ state usually identified with the beta-vibrational band in the even-even nucleus. A later paper by Davydov and Chaban\textsuperscript{25} considers beta vibrations by writing the collective Hamiltonian as

$$H = -\hbar^2 \left\{ \frac{1}{\beta^3} \frac{\partial}{\partial \beta} \left( \beta^3 \frac{\partial}{\partial \beta} \right) \right\} - \sum_{\kappa} \frac{L_\kappa^2}{2 r_\kappa}$$  \hspace{1cm} (22)

in the same notation as above. Solutions for the energy are found in terms of $\gamma$ and a new parameter $\mu$, which is related to
After G.R. DeMille et al., Can. J. Phys. 37, 1036 (1959)

FIG. 4
the strength of the beta vibration by

\[ \mu^2 = \frac{\hbar \omega_0}{C \beta_0^2} \]  

(23)

The equilibrium value of \( \beta \) is \( \beta_0 \), and \( C \) is the elasticity constant mentioned above. The energies are presented as ratios to the first excited state. (See Fig. 5) For small \( \mu \) values (\( \mu < 0.3 \)) nuclei are heavily rotational, while for large values (\( \mu > 0.5 \)) the spectra are vibrational in character. Similar calculations have been carried out for odd-mass nuclei by J. R. Roesser.\(^{26}\)

Vibrations of the gamma variety are treated in a paper by Davydov\(^{27}\) by including a term

\[-\frac{1}{\sin 3\gamma} \frac{\partial}{\partial \gamma} (\sin 3\gamma \frac{\partial}{\partial \gamma}) \]  

(24)

in the Hamiltonian of Eq. 22 and writing the potential energy as

\[ V(\beta, \gamma) = \frac{1}{2} C (\beta - \beta_0)^2 + \frac{1}{2} C \gamma \beta_0^2 (\gamma - \gamma_0)^2 \]  

(25)

where the subscript indicates equilibrium values. An additional group of levels is obtained from gamma vibrations, one of which is a 0+ state. Thus either beta or gamma vibrations give rise to 0+ states. As pointed out by Davydov, for the non-zero gamma band states, if the equilibrium value of \( \gamma \) is zero, then there is no way to distinguish between levels of an asymmetric rotor and levels from gamma vibrations. Bohr has discussed a related
\[ \frac{E(4^+)}{E(2^+)} \]

\[ \frac{E(2')}{E(2^+)} \]

FIG. 5
point,\textsuperscript{28} that the existence of an equilibrium value of $\gamma$ is not distinguishable from an average or rms value of $\gamma$ oscillations.

It should be noted that the relative spacings of the ground state rotational band became much less than the predictions of Eq. 19, for finite $\gamma$ values. Thus an alternative explanation to the mixing analysis presented in section C can be made with the asymmetric rotor model. The two approaches will be discussed with the data presented herein.

3. Collective Behavior in Nuclei With Non-Static Deformation

For a spherical even-even nucleus the lowest modes of excitation are collective oscillations. For quadrupole excitations the levels expected for the harmonic oscillations are a $2^+$ state and a degenerate triplet of $0^+$, $2^+$, and $4^+$ states at twice the energy of the first state. The triplet of levels would not be expected to remain degenerate in any real nucleus due to perturbations. For example in Sm$^{148}$, in which only one member of the triplet is known, the ratio of its energy to the energy of the first $2^+$ level is 2.14.\textsuperscript{29} In Sm$^{150}$ the second $2^+$ level that has been identified is far from the $0^+$ and $4^+$ levels, and the energy ratios to the first $2^+$ level are 2.2, 3.1, and 2.3.\textsuperscript{30,31}

These ratios indicate that there is not strict adherence to the harmonic oscillator. However, the reduced transition probabilities for the first excited states in these nuclei are sufficiently large to suggest validity for a collective model description. If the experimental data of Elbek, Oleson, and
Skilbreid\textsuperscript{32} for the reduced transition probabilities (See Table I) are expressed in terms of the single-particle values used by Alder, Bohr, Huus, Mottelson, and Winther\textsuperscript{33} we obtain 39 and 56 single particle units for Sm\textsuperscript{148} and Sm\textsuperscript{150}. This definition of the single particle unit (abbreviated as s.p. unit) is

\[ B(E\lambda)_{\text{s.p.}} = (2\lambda+1) \frac{e^2}{4\pi} \left( \frac{3}{3+\lambda} \right)^2 R_o^{2\lambda} \]  \hspace{1cm} (26)

where \( \lambda \) is the multipolarity of the transition and

\[ R_o = 1.2 A^{1/3} \times 10^{-13} \text{ cm} \]  \hspace{1cm} (27)

The single particle values are listed in Table II. The \( B(E\lambda) \) value is related to the absolute transition probability by

\[ T = \frac{8\pi(\lambda+1)}{\lambda[(2\lambda+1)!!]^2} \left( \frac{\mu}{\mu_0} \right)^{2\lambda+1} \frac{1}{\hbar} B(E\lambda, J_f \rightarrow J_i) \]  \hspace{1cm} (28)

The ratio of reduced matrix elements for the second phonon levels has been calculated by Choudhury\textsuperscript{34} in the simple vibrational model, obtaining

\[ \frac{B(E2, J \rightarrow 2)}{B(E2, 2 \rightarrow 0)} = 2 \]  \hspace{1cm} (29)

where \( J = 0, 2, \) or 4. Thus the validity of harmonic oscillation can be tested by determining the relative transition probabilities in these vibrational nuclei. Further predictions of the model are that the crossover radiations from the second 2+ state to the ground state are much less probable than the cascade E2...
Table I

B(E2) Values for Excitation of First Excited States in Even-Even Samarium Nuclei

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>B(E2,0 → 2) $[e^2 \times 10^{-48} \text{cm}^4]$</th>
<th>$\frac{B(E2)<em>{exp}}{B(E2)</em>{s.p.}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>148</td>
<td>$0.89 \pm 0.10$</td>
<td>38</td>
</tr>
<tr>
<td>150</td>
<td>$1.32 \pm 0.06$</td>
<td>56</td>
</tr>
<tr>
<td>152</td>
<td>$3.40 \pm 0.15$</td>
<td>142</td>
</tr>
<tr>
<td>154</td>
<td>$4.61 \pm 0.20$</td>
<td>190</td>
</tr>
</tbody>
</table>

Table II

Single Particle B(E1) Values for Even-Even Samarium Nuclei

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>B(E1) $[e^2 \times 10^{-26} \text{cm}^2]$</th>
<th>B(E2) $[e^2 \times 10^{-48} \text{cm}^4]$</th>
<th>B(E3) $[e^2 \times 10^{-74} \text{cm}^6]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>148</td>
<td>5.4</td>
<td>0.0232</td>
<td>0.91</td>
</tr>
<tr>
<td>150</td>
<td>5.5</td>
<td>0.0236</td>
<td>0.94</td>
</tr>
<tr>
<td>152</td>
<td>5.5</td>
<td>0.0239</td>
<td>0.96</td>
</tr>
<tr>
<td>154</td>
<td>5.6</td>
<td>0.0243</td>
<td>0.98</td>
</tr>
</tbody>
</table>
radiations and that the cascade $M_1$ radiations through the first $2^+$ state are forbidden. These latter characteristics are also predicted in near-harmonic models by Sharff-Goldhaber and Weneser,\textsuperscript{35} Kraushaar and Goldhaber,\textsuperscript{36} and Wilets and Jean.\textsuperscript{37}

It is possible to account for the positions of energy levels in many nuclei classified as vibrational by determining a suitable $\gamma$ parameter in the asymmetric rotor model. The simplest form of the model contains no $0^+$ excited state, but the later modification\textsuperscript{25} to include beta vibrations brings in a $0^+$ state and a new parameter (see discussion above), so that it may be possible to fit the observed level energies in terms of the asymmetric rotor model. This model has had some degree of success in fitting energy levels in palladium and cadmium isotopes.\textsuperscript{38}

Higher multipole collective oscillations than quadrupole undoubtedly exist, and many nuclei have $3^-$ states\textsuperscript{39} with associated large transition probabilities to the ground state. These states may be interpreted as the $\lambda = 3$ harmonic oscillation. In deformed even-even nuclei a rotational band may be based on the octupole vibration, giving rise to odd-spin, odd-parity states $1^-, 3^-, 5^-$, etc. The $1^-$ level in such an octupole band is interesting in that it is excited to a very low degree.

Calculations by Lipas\textsuperscript{40} have considered this $1^-$ state as a quadrupole-octupole interaction to obtain reduced transition probabilities. These estimates will be discussed in connection with the data.
C. Rotation - Vibration Interactions

The energy dependence of the levels of the ground state band of an even-even nucleus given in Eq. 19 was derived under the conditions of axial symmetry and fixed nuclear shape during rotation. This is an adiabatic approximation of no interaction between collective rotations and vibrations. This approximation is valid only to the extent that Eq. 19 is satisfied experimentally. The deviation from the adiabatic approximation may be expressed in terms of a slowly convergent power series in \( J(J+1) \) in much the same fashion as in molecular spectroscopy. It is shown herein that centrifugal distortion of the nucleus and deviations from axial symmetry lead to terms in \( J^2(J+1)^2 \) and \( J^n(J+1)^n \) generally.

The method to be followed is to write the deviations from axial symmetry and the centrifugal distortion as perturbing terms in the collective Hamiltonian, and thereby obtain the change in energy of the ground state rotational band. These perturbing terms bring in admixtures of components of vibrational band wave functions to the ground state band wave functions, and vice versa. These admixtures may have strong effects on the transition rates between the ground state and vibrational bands, as is shown below. Sheline, Nielsen, Lipas, and Preston and Kiang have carried out such calculations for admixtures of wave functions and examined the consequent changes in gamma ray transition probabilities between the bands. These transitions are shown schematically in Fig. 6. The notation followed here
Schematic of Gamma Ray Inter-Band Transitions

Fig. 6
Formation of a wave function by the admixture of wave functions of different $K$ bands is a means of expressing the fact that $K$ is not a good quantum number. The relative amount of admixture is determined by the matrix element of the perturbing Hamiltonian. Even if the amount of admixture is small, the effects on transition rates may be quite large, as is discussed below.

The $U_3$ term neglected for axial symmetry in the collective Hamiltonian (Eq. 14) gives rise to the gamma band mixing. As noted earlier, the rms value of a gamma vibration is equivalent to a small gamma value. In $U_3$ the operator $J_1^2 - J_2^2$ may be written as

$$J_1^2 - J_2^2 = \frac{1}{2} (J_1 + iJ_2)(J_1 + iJ_2) + \frac{1}{2}(J_1 - iJ_2)(J_1 - iJ_2)$$

which is the sum of raising and lowering operators, giving a double raise or lowering. This operator connects wave functions of the same spin and parity, but with $K$ differing by $\pm 2$. It therefore mixes gamma and ground state bands.

Writing the moments of inertia as in Eq. 13, and expanding for small $\gamma$, $U_3$ becomes

$$U_3 = \left[ \frac{\hbar^2}{4I_1} - \frac{\hbar^2}{4I_2} \right] (J_1^2 - J_2^2)$$

$$\approx - \frac{\hbar^2}{3B\beta_0} \frac{\gamma}{3} (J_1^2 - J_2^2)$$
neglecting higher order terms in γ. The matrix element of $U_3$ is

$$
\langle \Psi_f | U_3 | \Psi_i \rangle = -\frac{\hbar^2}{3B^2_0} \langle g(\gamma) | Y_j g(\gamma) \rangle \sqrt{(J-1)J(J+1)(J+2)}
$$

$$
= -C_\gamma \sqrt{(J-1)J(J+1)(J+2)}
$$

The wave functions are written as

$$
\Psi_{gs} = \Psi_{gs} - \frac{C_\gamma}{\hbar \omega_\gamma} \sqrt{(J-1)J(J+1)(J+2)} \Psi_{\gamma}
$$

$$
\Psi_{\gamma} = \Psi_{\gamma} + \frac{C_\gamma}{\hbar \omega_\gamma} \sqrt{(J-1)J(J+1)(J+2)} \Psi_{gs}
$$

where $\hbar \omega_\gamma$ is the energy of the gamma vibration. The energy perturbation is

$$
\Delta E = -\frac{\langle |U_3| \rangle^2}{\hbar \omega_\gamma}
$$

$$
= \frac{C_\gamma^2}{(\hbar \omega_\gamma)^2} (J-1)(J+1)J(J+2) \hbar \omega_\gamma
$$

$$
= -\varepsilon_\gamma^2 (\hbar \omega_\gamma)[J^2(J+1)^2-J(J+1)]
$$

$$
= A_\gamma J(J+1) + B_\gamma J^2(J+1)^2
$$
where \( B = -\varepsilon \gamma^2 \hbar \omega \gamma \) and \( \varepsilon \gamma = C \gamma / \hbar \omega \gamma \). The term in \( A \gamma \) represents a renormalization of the moment of inertia, while the term in \( B \gamma \) is the rotation-vibration interaction.

As a possible mechanism to account for centrifugal distortion of the nucleus during rotation one considers beta band mixing. In analogy with gamma vibrations, the rms value of a beta vibration is equivalent to a larger beta value. The addition of a small component of beta vibration effectively adds a component of a larger beta value and increases the moment of inertia. This effect is included by expanding the moments of inertia in terms of small oscillations about an equilibrium value \( \beta_o \).

\[
I_1 = I_2 = 3B\beta^2
\]

\[
= 3B[\beta_o + (\beta - \beta_o)]^2
\]

\[
= 3B\beta_o^2 [1 + \frac{\beta - \beta_o}{\beta_o}]^2 \tag{36}
\]

The perturbing part of the Hamiltonian is obtained from

\[
\left[ \frac{\hbar^2}{4I_1} + \frac{\hbar^2}{4I_2} \right] J^2 = \frac{\hbar^2}{6B\beta_o^2} [1 - 2(\frac{\beta - \beta_o}{\beta_o}) + \ldots] \tag{37}
\]

keeping only the first term in \((\beta - \beta_o)/\beta_o\). This operator connects states of the same spin and parity but differing in the value of \( \beta \). The eigenvalues of this operator, denoted as \( H_1 \), are
\[ \langle \psi_f | H_i | \psi_i \rangle \]
\[ = \frac{\hbar^2}{3B_0^2} \langle f(\beta) | \frac{\beta-\beta_0}{\beta_0} | f(\beta) \rangle \ J(J+1) \]
\[ = C_\beta \ J(J+1) \quad (38) \]

The wave functions are
\[ \psi_{gs} = \psi_{gs} - \frac{C_\beta}{\hbar \omega_\beta} \ J(J+1) \ \psi_\beta \]
\[ \psi_\beta = \psi_\beta + \frac{C_\beta}{\hbar \omega_\beta} \ J(J+1) \ \psi_{gs} \quad (39) \]

and the energy perturbation is
\[ \Delta E_\beta = - \left( \frac{\langle | H_i | \rangle^2}{\hbar \omega_\beta} \right) \]
\[ = - \frac{C_\beta^2}{(\hbar \omega_\beta)^2} (\hbar \omega_\beta) \ J^2(J+1)^2 \]
\[ = - \epsilon_\beta^2 (\hbar \omega_\beta) \ J^2(J+1)^2 \quad (40) \]

where
\[ \epsilon_\beta = \frac{C_\beta}{\hbar \omega_\beta} \quad (41) \]

Thus it is seen that terms in \( J^2(J+1)^2 \) can be accounted for on the basis of perturbations to the Hamiltonian. Higher
order terms in $J(J+1)$ are not obtained because this analysis is only carried out to first-order in the perturbation. We now calculate the reduced transition probabilities for E2 transitions within and between bands.

The reduced transition probability is defined by

$$B(E2, i \rightarrow f) = \Sigma_{\mu M_f} |\langle \psi_f | \mathcal{M}(E2, \mu) | \psi_i \rangle|^2$$

(42)

where $\mathcal{M}(E2, \mu)$ is the quadrupole moment, proportional to $r^2 Y_{2M}$. To refer this operator to the body axis

$$Y_{2M} = \Sigma_m D_{M_m}^2 Y_{2m}(\theta', \phi')$$

(43)

where $\theta', \phi'$ are relative to the body axes. Writing the total wave function as

$$\psi_{JK} = f(\gamma, \beta)[D_{MK}^J + (-)^J D_{M,-K}^J]$$

(44)

we see that integrals will occur in the product of three D-functions, which are evaluated by Nilsson

$$\int D_{M_f M_o}^{J_f} L_{M_m}^{J} \frac{d\Omega}{2J_f + 1} (J_o L M_o M | J_r M_r) (J_o L K_o m | J_f K_f)$$

(45)

where $M = M_f - M_o$ and $m = K_f - K_o$. The expressions in parentheses are Clebsch-Gordan coefficients, with the phase and normalization of Condon and Shortley.

Within a constant factor the matrix element is
\[
\begin{align*}
\langle \Psi | m(E_L, u) | \Psi_{J_f K_f} \rangle &= \sum_m \langle \Psi | D_m^L | \Psi_L^* | \Psi \rangle \\
&= \sum_m \int r^2 Y_{Lm}^* f_o r^2 dr \\
&= \sum \left[ D_{M_f K_f}^{J_f} + (-1)^{J_f} D_{M_f K_f}^{J_f} \right] D_{M_o K_o}^L \left[ D_{M_o K_o}^{J_o} + (-1)^{J_o} D_{M_o K_o}^{J_o} \right] d\Omega \\
&= \frac{8\pi^2}{2J_f+1} (J_o L M_o M | J_f M_f) \sum_m \int r^2 Y_{Lm}^* f_o r^2 dr \\
&= \left( J_o K_o M | J_f K_f \right) + (-1)^{J_f} \left( J_o L K_o m | J_f - K_f \right)
\end{align*}
\]

For \( L = 2 \) transitions, and for particular \( K_o \) and \( K_f \),
\[ \underline{K_o = 2, K_f = 2}: \]
\[
\langle | m | \rangle = Q_{22} (J_o 220 | J_f 2) (J_o 2M_o M | J_f M_f) \\
Q_{22} = \frac{16\pi^2}{2J_f+1} \int r^2 Y_{22}^* f_o (K=2) r^2 dr 
\]

\[ \underline{K_o = 2, K_f = 0}: \]
\[
|m| = Q_{20} (J_o 222 | J_f 0) (J_o 2M_o M | J_f M_f) \\
Q_{20} = \frac{16\pi^2}{2J_f+1} \int r^2 Y_{22}^* f_o (K=0) r^2 dr 
\]
\( K_o = 0, K_f = 0: \)

\[
\langle |m| > = Q_{00}(J_o200|J_f0)(J_o2M_oM|J_fM_f)
\]

\[
Q_{00} = \frac{16\pi^2}{2J_f+1} \int f_f(K=0)r^2Y_{20}^* f_0(K=0)r^2dr
\] (49)

In obtaining these expressions one must note that the term in 
\( D_{M,-K}^J \) does not occur in the wave function when \( K = 0 \), and that

\[
(J_oL-K_o-m|J_f-K_f) = (-)^{J_o+L+J_f+2K_f} (J_oLK_o-m|J_fK_f)
\] (50)

With these expressions and the admixed wave functions, one may obtain for the gamma band de-excitations to the ground state band,

\[
B(J_o \rightarrow J_f) = \sum_{MM_f} |(J_o2M_oM|J_fM_f)|^2
\]

\[
[Q_{20}(J_o222|J_f0) + \epsilon_{\gamma}(J_{-1}J_o(J_{+1})(J_{-1}))]
\]

\[
Q_{00}(J_o200|J_f0) - \epsilon_{\gamma}(J_{+1}J_o(J_{-1})(J_{+1}))Q_{22}(J_o220|J_f2)\]

(51)

The sum \( \sum_{MM_f} |(J_o2M_oM|J_fM_f)|^2 = 1 \) since this is just the orthogonality condition. If the moments of inertia in the two bands are assumed to be equal,

\[
Q_{00} = Q_{22} = Q
\] (52)
Let $Q_{20} = P$

Then

\[ B(J_0 \rightarrow J_f) = \left[ P(J_0 202 | J_f 2) - \varepsilon \sqrt{(J_f-1)_{J_f} (J_f+1)(J_f+2)} P(J_0 202 | J_f 2) \right] + \varepsilon \sqrt{(J_f-1)_{J_f} (J_f+1)(J_f+2)} P(J_0 202 | J_f 2) \] \] (53)

Similarly, for the beta band de-excitations to the ground state band,

\[ B(J_0 \rightarrow J_f) = |(J_0 200 | J_f 0)|^2 \]

\[ \left[ P - \varepsilon J_f (J_f+1) Q + \varepsilon J_0 (J_0+1) Q \right]^2 \] (54)

For transitions with the ground state band, with gamma band mixing,

\[ B(J_0 \rightarrow J_f) = \left[ Q(J_0 200 | J_f 0) - \varepsilon_{\gamma} \sqrt{(J_f-1)_{J_f} (J_f+1)(J_f+2)} P(J_0 202 | J_f 2) \right] + \varepsilon_{\gamma} \sqrt{(J_f-1)_{J_f} (J_f+1)(J_f+2)} P(J_0 220 | J_f 2) \] (55)

and with beta band mixing,

\[ B(J_0 \rightarrow J_f) = |(J_0 200 | J_f 0)|^2 \left[ Q - \varepsilon_{\beta} J_f (J_f+1) P + \varepsilon_{\beta} J_0 (J_0+1) P \right]^2 \] (56)
These B(E2) values have been listed in Table III for various possible spins, and are given in terms of a parameter \( z \), defined by

\[
\begin{align*}
  z_\gamma &= \epsilon_\gamma \sqrt{\frac{24}{Q}} = \epsilon_\gamma \alpha_\gamma \sqrt{\frac{24}{Q}} \\
  z_\beta &= 2\epsilon_\beta \frac{Q}{P_\beta} = 2\epsilon_\beta \alpha_\beta
\end{align*}
\]

(57)

The prime in Table III and Fig. 7 and 8 indicates the vibrational state.

When \( z \) is zero, the B(E2) ratios are equal to the Alaga predictions. \(^{45}\) Alaga pointed out that in the collective model with pure rotational bands, the branching ratio for transitions from a level in one band to different levels in another band may be expressed just as ratios of Clebsch-Gordan coefficients. These values are noted in the graphs of B(E2) ratios in Fig. 7 and 8.

The ratio \( \alpha_\beta \) or \( \alpha_\gamma \) is much greater than one, and it is seen that the effects of any admixture are strongly enhanced for transitions between bands, while for transitions within one band the mixing effects are depressed. In the ratios of B(E2) values for transitions between bands, the \( P^2 \) cancels out and one may determine \( z \) by obtaining the experimental gamma ray branching ratio. Extraction of \( \epsilon \) requires a knowledge of \( \alpha \), which may be obtained from the absolute transition probability, or from a ratio of the B(E2) value within the band to that between the bands. For example,
Table III

\( B(E2) \) Values for Even-Even Nuclei With Wave Function Admixtures

**Gamma Band De-Excitation**

\[
B(2' \rightarrow 0) = \frac{1}{5} P^2 (1 + z_\gamma)^2
\]

\[
B(2' \rightarrow 2) = \frac{2}{7} P^2 (1 - 2z_\gamma)^2
\]

\[
B(2' \rightarrow 4) = \frac{1}{7} P^2 \left[ \frac{1}{\sqrt{10}} - z_\gamma \sqrt{\frac{45}{2}} + 3z_\gamma \sqrt{\frac{2}{5}} \right]^2
\]

\[
B(3' \rightarrow 2) = \frac{5}{14} P^2 (1 - z_\gamma)^2
\]

\[
B(3' \rightarrow 4) = \frac{1}{7} P^2 (1 - 6z_\gamma)^2
\]

\[
B(4' \rightarrow 2) = \frac{5}{42} P^2 (1 + 5z_\gamma)^2
\]

**Beta Band De-Excitation**

\[
B(0' \rightarrow 2) = \frac{1}{5} P^2 (1 - 3z_\gamma)^2
\]

\[
B(2' \rightarrow 0) = \frac{1}{5} P^2 (1 + 3z_\gamma)^2
\]

\[
B(2' \rightarrow 2) = \frac{2}{7} P^2
\]

\[
B(2' \rightarrow 4) = \frac{18}{35} P^2 (1 - 7z_\gamma)^2
\]

\[
B(4' \rightarrow 2) = \frac{2}{7} P^2 (1 + 7z_\gamma)^2
\]

\[
B(4' \rightarrow 4) = \frac{20}{77} P^2
\]

\[
B(4' \rightarrow 6) = \frac{5}{11} P^2 (1 - 11z_\gamma)^2
\]

**Ground State Band De-Excitation**

\[
B(0 \rightarrow 2) = Q^2 (1 - z_\gamma/a_\gamma^2) = Q^2 (1 - 3z_\beta/a_\beta^2)
\]

\[
B(2 \rightarrow 4) = Q^2 \left[ \sqrt{\frac{18}{35}} - 5z_\gamma/a_\gamma^2 \sqrt{\frac{3}{42}} + z_\gamma/a_\gamma^2 \sqrt{\frac{5}{42}} \right]^2
\]

\[
= \frac{18}{35} Q^2 (1 - 7z_\beta/a_\beta^2)^2
\]
B(E2) BRANCHING RATIOS FOR DE-EXCITATION OF GAMMA BAND LEVELS AS A FUNCTION OF $Z_\gamma$.

\[
\frac{B(2'\rightarrow 2)}{B(2'\rightarrow 0)}
\]

\[
\frac{B(3'\rightarrow 4)}{B(3'\rightarrow 2)}
\]

ALAGA RATIOS

-4 -3 -2 -1 0 .1 .2 .3 .4 .5

FIG. 7

B(E2) BRANCHING RATIOS FOR DE-EXCITATION OF 2+ LEVEL IN BETA BAND AS A FUNCTION OF $Z_\beta$

\[
\frac{B(2'\rightarrow 4)}{B(2'\rightarrow 0)}
\]

\[
\frac{B(2'\rightarrow 4)}{B(2'\rightarrow 2)}
\]

ALAGA RATIOS

-2 -1 0 .1 .2 .3 4 .5

FIG. 8
\[
\frac{B(E2, 0^+ \rightarrow 2^+)}{B(E2, 0^- \rightarrow 2^-)} = \alpha^2 (1 + 3z\beta)^2
\] (58)

In the analysis considered here the form of the perturbing Hamiltonian has been used to obtain the spin dependence of the mixing. The strength of the interaction has been lumped into the parameter \( \epsilon \), which is determined from \( z \) and experimental branching ratios. However, the hydrodynamic formulation may be used to calculate the interaction strength, and also the form of \( \alpha \). In the notation of Preston and Kiang, \( ^{22} \)

\[
\epsilon_\beta \alpha_\beta = \frac{1}{3} \rho = \frac{1}{3} \frac{\hbar \omega_o}{\hbar \omega_\beta}
\] (59)

\[
\epsilon_\gamma \alpha_\gamma = \frac{1}{3} \gamma^{2} \frac{\hbar \omega_o}{\hbar \omega_\gamma}
\]

and

\[
\alpha_\beta^2 = \frac{2}{\rho} = 2 \frac{\hbar \omega_\beta}{\hbar \omega_o}
\] (60)

\[
\alpha_\gamma^2 = \frac{1}{\gamma^{2} 10} = \frac{\hbar \omega_\gamma}{\hbar \omega_o}
\]

where \( \hbar \omega_o \) is the energy of the 2+ state in the ground state band, and \( \hbar \omega_\beta \) and \( \hbar \omega_\gamma \) are the energies of the first levels in the beta and gamma bands. It will thus be of interest to compare experimental values of \( \alpha \) to the hydrodynamic values.

The procedure for examining the effects of rotation-vibration interaction is first to excite levels in the beta
and gamma vibrational bands and to measure the gamma ray branching ratios. These branching ratios determine a value of $z$. The $\epsilon$ value is extracted from $z$ by determining $\alpha$ from absolute transition probabilities, therefore requiring a knowledge of the excitation mechanism. Then with this $\alpha$ one may calculate the coefficient of the $J^2(J+1)^2$ term in the ground state band energy equation, and compare to the experimental coefficient.
D. Coulomb Excitation

1. Introduction

The coulomb excitation process may be described as the inelastic scattering of a charged particle from a nucleus, in which energy is transferred from the incident particle to the nucleus, leaving the nucleus in one of its excited states. The interaction takes place through the time dependent electromagnetic field of the projectile. When bombarding energies are well below the coulomb barrier the projectile does not come into contact with the nucleus, and all problems relating to specific details of nuclear forces are avoided. In addition, the long-range electromagnetic interaction is well understood. These conditions make it possible to calculate accurately the excitation probability, and therefore to provide a powerful tool in obtaining nuclear structure information.

Experimental measurements may be made of the inelastically scattered particles, the de-excitation gamma rays, and de-excitation conversion electrons. Coincidence measurements may also be performed with combinations of these measurements. In the work reported in this dissertation pure metal targets of samarium were bombarded with oxygen ions up to 64 Mev in energy. The data consist of direct gamma ray spectra, gamma ray spectra detected in coincidence with other gamma rays, and gamma ray spectra in coincidence with oxygen ions scattered into backward directions.
Nuclear excitation by long range electric interactions was proposed early in nuclear reaction studies by Rutherford, Chadwick, and Ellis,\(^{46}\) and was suggested as the mechanism by which an isomeric activity observed in indium\(^ {47}\) was produced. In subsequent years the theoretical calculations were carried out in enough detail to allow experimental determinations of transition matrix elements. Large numbers of nuclei were studied by bombarding targets with protons or alpha particles at energies up to the coulomb barrier and observing the de-excitation gamma radiation. Mention should be made of the intensive data survey by Heydenburg and Temmer.\(^ {48}\) An extensive tabulation of the early data from coulomb excitation, as well as the theory involved, appears in an excellent review article by Alder, Bohr, Huus, Mottelson, and Winther\(^ {33}\) (hereafter designated as ABHMW). An outline of the theory is presented here in order to introduce the notation and general concepts, which will be referred to when discussing the experimental data.

2. First-Order Perturbation Calculation

The theory was first approached from a semi-classical viewpoint for the case of electric quadrupole excitations\(^ {49}\) and gave satisfactory agreement with the experimental data. The semi-classical calculation neglects the energy loss of the scattered projectile due to nuclear excitation so that hyperbolic orbits are involved. This method is valid if the Sommerfeld parameter \(\eta\), given by

\[
\eta = \frac{Z_1 Z_2 e^2}{\hbar v} \tag{61}
\]
is much greater than unity.

The method employed is to write the differential cross section for excitation of a state of spin $J_f$ and magnetic quantum number $M_f$ as a transition probability times the Rutherford scattering cross section.

$$d\sigma = P \, d\sigma_R = P \, \frac{a^2}{4} \sin^{-4}(\frac{\theta}{2}) d\Omega$$

(62)

where $P$ may be written as

$$P = \frac{1}{2J_o + 1} \sum_{M_i M_f} |b_{if}|^2$$

(63)

Eq. 63 holds for transitions from an un-oriented initial state of spin $J_i$ and magnetic quantum number $M_i$ to a final state $J_f, M_f$. The quantity $b_{if}$ is the transition amplitude between initial and final states and is expressed as

$$b_{if} = \frac{1}{i\hbar} \int_{-\infty}^{\infty} \langle f | H(t) | i \rangle e^{i\omega t} dt$$

(64)

in time dependent perturbation theory, where

$$\omega = \frac{E_f - E_i}{\hbar} = \frac{\Delta E}{\hbar}$$

(65)

The $b_{if}$ are evaluated by making the usual multipole expansion and factoring the resulting expression, for each angular momentum value, into the reduced matrix element of the nuclear multipole moment, a factor involving energy, charge, and mass,
and an orbital integral. For electric multipole excitation the Hamiltonian $H(t)$ involves the scalar potential (see Fig. 9 for the coordinate system)

$$\phi = \frac{Z_1 e}{|\mathbf{r} - \mathbf{r}_p(t)|} - \frac{Z_1 e}{r_p}$$  \hspace{1cm} (66)

while for magnetic excitation one has the vector potential

$$\mathbf{A}(\mathbf{r},t) = \frac{Z_1 e}{c} \frac{\mathbf{v}_p(t)}{|\mathbf{r} - \mathbf{r}_p(t)|}$$  \hspace{1cm} (67)

which is of the order of $v/c$ times $\phi$ in absolute magnitude.

Since the cross sections are proportional to the square of the matrix element, the magnetic excitations are reduced by an amount $(v/c)^2$ relative to the electric excitations. For the maximum energies used in this experiment $v/c = 0.1$ and at most 1\% as much magnetic excitation would be expected as electric of the same multipolarity. Magnetic excitation will not be considered further.

The orbital integral mentioned above is a function only of the scattering angle $\theta$ and a parameter $\xi$,

$$\xi = \frac{\alpha A}{\Delta E} = \frac{Z_1 Z_2 e^2}{\Delta E} = \eta \frac{\Delta E}{2E}$$  \hspace{1cm} (68)

where $\alpha$ is

$$\alpha = \frac{Z_1 Z_2 e^2}{m_0 v^2}$$  \hspace{1cm} (69)
CENTER-OF-MASS COORDINATE SYSTEM FOR COULOMB EXCITATION PERTURBATION CALCULATION.

**FIG. 9**
and is equal to half the distance of closest approach of nuclear centers in a head-on collision, with \( m_0 \) being the reduced mass. The characteristic parameter \( \xi \) for the coulomb excitation process is known as the adiabatic parameter. Eq. 68 may be re-written as

\[
\xi = \left( \frac{a}{v} \right) \left( \frac{\Delta E}{\hbar} \right) = \frac{a/v}{\hbar/\Delta E}
\]  

(70)

which represents the ratio of collision time to nuclear time. For a large value of \( \xi \) the collision is adiabatic and excitation is less probable. This is somewhat analogous to forced oscillations of a harmonic system, in which energy is transferred only when the applied and characteristic frequencies are comparable.

The differential cross section is usually written as

\[
d\sigma_{E\lambda} = \left( \frac{Z_1 e^2}{\hbar v} \right) a^{-2\lambda+2} B(E\lambda) \, d\xi_{E\lambda}(\theta, \xi)
\]

(71)

where \( d\xi_{E\lambda} \) is a function of the orbital integrals and \( \sin^\frac{\theta}{2} \). For the total cross section, the integration over \( \theta \) leads to a function \( f_{E\lambda}(\xi) \). Both functions are tabulated in ABHW for \( 0 \leq \xi \leq 4.0 \) and \( 0 < \lambda < 4 \). Fig. 10 gives a graph of the functions \( f_{E\lambda} \).

Corrections may be made for the energy loss of the incident projectile by calculating \( \xi \) and \( a \) from an average velocity.

\[
\xi = \frac{Z_1 Z_2 e^2}{\hbar} \left( \frac{1}{v_f} - \frac{1}{v_i} \right) = \eta_f - \eta_i
\]

(72)
where η is the Sommerfeld parameter. In Eq. 71, v is replaced by \( v_i \). An analysis by Brussaard and Biedenharn\(^{50}\) from the correspondence principle shows that the correct value of v for orbit modification is given by

\[
v \Rightarrow v_i v_f \left( \frac{v_i + v_f}{3} \right)^{1/3}
\]

(74)

In this experiment \( \Delta E/E_{in} \) is only a few per cent for the higher bombarding energies, at which cross sections are largest. Corrections to the average v used in Eq. 72 and Eq. 73 will be small and will only differ slightly from Eq. 74.

The coulomb excitation cross sections have also been evaluated quantum mechanically. The quantum calculation\(^{51}\) treats the coulomb excitation process as the transfer of a photon from the projectile to the nucleus. The result for \( f_{E\lambda} \) in the total cross section may be expressed in terms of a factor R dependent upon \( \Theta, \zeta, \) and η, which multiplies the classical \( f_{E\lambda} \) expression to give the quantum mechanical value. As is seen in Fig. 11, R is nearly equal to unity for η greater than 3 or 4. For a 60 Mev oxygen ion η = 40.5 and use of the semi-classical expression for that energy and lower is well justified. Although greater discrepancies would be expected between quantum mechanical and semi-classical differential cross sections than for total cross sections, calculations by J. Bang\(^{52}\) for the differential
QUANTUM MECHANICAL CORRECTIONS TO CLASSICAL CROSS SECTION FUNCTION \( f_{E\lambda} \)

FIG. 11

Differential cross sections for double Coulomb excitation of an even-even nucleus.

\[
\frac{d\sigma_j(\theta)}{d\sigma_j(\pi)} \text{ vs. } \theta
\]

FIG. 12
cross section function for \( \gamma = 0 \) and for E2 excitation found
that for \( \eta = 4 \) the difference is only about 1% at angles greater
than 40°, and somewhat more for smaller angles. Again, the use
of semi-classical expressions in this experiment is valid since
the minimum involved is about 40. A review paper by Breit and
Glückstern\(^5\) considers the assumptions and calculations for the
complete quantum mechanical approach.

The angular distribution of gamma rays following coulomb
excitation has been calculated for El and E2 excitation and is
given in ABHMW. The correlation is expressed as a modification
of the usual gamma-gamma cascade, in which

\[
W(\theta_\gamma, \phi_\gamma) = \sum_{k} a_{k\lambda}(\theta_\gamma) A_k Y_{kk}(\theta_\gamma, \phi_\gamma)
\]

The \( A_k \) is the normal gamma-gamma angular correlation coefficient,
while \( a_{k\lambda} \) is a factor, dependent upon the scattering angles \( \theta \)
and \( \phi \), that arises from the calculation of the coulomb excitation
process in terms of the equivalent gamma ray. The angles \( \theta_\gamma \) and
\( \phi_\gamma \) specify the direction of the gamma ray.

If particles are detected symmetrically about the beam
direction the \( \phi_\gamma \) and \( k \) dependence is removed. For no particle
detection the relevant particle factor is \( a_{k\lambda} \), in which the
differential cross section has been integrated over the scattering
angle. The angular correlation is then

\[
W(\theta_\gamma) = \sum_k a_{k\lambda}(\gamma) A_k P_k(\cos \theta_\gamma)
\]
For a gamma-gamma cascade following coulomb excitation a triple correlation is involved between the particle and two gamma ray directions. Again the correlation is expressed in terms of a particle factor times the first gamma ray transition, the factor used being appropriate to the experimental conditions for particle detection.

3. Second-Order Perturbation Calculation

In early coulomb excitation experiments only one level of a nucleus was populated, and the probabilities for excitation were small enough to be treated adequately by first-order perturbation calculations. However, at higher bombarding energies it becomes possible to observe gamma radiation from energy levels not directly excited from the ground state, but rather through an intermediate level in a double process. Newton and Stephens observed double coulomb excitation in tungsten by bombarding with oxygen ions and observing gamma rays from 2+ and 4+ states. They found that the energy dependence of the yield was in good agreement with the energy dependence of the product of two first-order E2 cross sections.

Excitation cross sections have been calculated with second-order perturbation theory for population of a final level through an intermediate level. The calculation is semi-classical, as in the first-order calculation. The result may be expressed as the sum of three terms, a first-order excitation cross section \(\sigma^{(1)}\) (same as Eq. 42), a second-order cross section \(\sigma^{(2)}\), and the interference term \(\sigma^{(1,2)}\) between the two. A. C.
Douglas has calculated and prepared tables of $\sigma^{(2)}$ and $\sigma^{(2)}$ for excitation of 0+, 2+ and 4+ levels in an even-even nucleus through a lower-energy 2+ level over the range of parameters $\xi_1$ and $\xi_2$ from 0 to 0.5 in 0.1 intervals. No inclusion of the interference term need be made for 0+ and 4+ levels since the direct transitions are negligible. The sign of the interference term depends upon the relative sign of nuclear matrix elements. For 2+ excitation, calculations in this dissertation will be given with both signs of the interference term. Fig. 12 shows angular dependences of the differential cross sections for $\xi_1 = \xi_2 = 0.2$, given as a function of scattering angle $\Theta$. The reduced matrix element dependence and relative magnitudes are removed by normalizing the curves at $\Theta = \pi$.

4. Multiple Coulomb Excitation

When heavy ions are used in bombarding particles, the coulomb excitation cross sections may become very large. The dependence of the cross section (Eq. 71 with corrections for energy loss of scattered projectile) upon the charge and mass of the incident beam is

$$Z_1^2 A_1 \left[ (1 + \frac{A_1}{A_2}) Z_1 \right]^{-2\lambda+2} \approx A_1 Z_1^{4-2\lambda}$$

which depends upon the multipolarity of the transition. For projectiles of different charge, the cross section comparison should be made at incident energies bearing the same relationship to the coulomb barrier. Since experiments are usually performed
at energies at or below the coulomb barrier so that gamma rays from nuclear reactions will not complicate the analysis, the coulomb barrier energy should be included as a measure of the largest experimentally useful cross section. The energy dependence of the cross section is

\[ E^{\lambda - 2} [E - (1 + \frac{A_1}{A_2}) \Delta E]^{\lambda - 1} \sim E^{2\lambda - 3} \]  

(78)

Replacing \( E \) by \( E_B \) where

\[ E_B = \frac{Z_1Z_2e^2}{r} \sim \frac{Z_1Z_2}{\frac{1}{A_1^{1/3}} + \frac{1}{A_2^{1/3}}} \sim Z_1 \]  

(79)

and multiplying by the \( A_1Z_1 \) dependence,

\[ A_1Z_1^{4-2\lambda} [Z_1]^{2\lambda-3} = A_1Z_1 \]  

(80)

and it is evident that higher mass and higher charge projectiles will lead to larger cross sections. Thus far \( \tilde{z} \) dependence has been neglected, but since

\[ \tilde{z} \sim \frac{Z_1A_1^{1/2}}{E^{3/2}} \sim \frac{Z_1A_1^{1/2}}{Z_1^{3/2}} = \frac{A_1}{Z_1^{1/2}} \]  

(81)

the effects dependent upon \( \tilde{z} \) are of higher order.

It is therefore seen that heavy ions are very useful in obtaining large cross sections for coulomb excitation. However, it is possible for the cross sections to become so large that
the first-order perturbation theory is no longer applicable, and even in second-order may not be very accurate. With heavy ions many levels of a nucleus may be populated, and this "multiple coulomb excitation" process has been treated by Robinson and Alder and Winther by a direct solution of the Schrodinger equation. Since Robinson's method involves numerical calculations for all cases, the work of Alder and Winther will be discussed here.

The starting point for Alder and Winther's method is the "sudden approximation," in which the limit of $\gamma = 0$ is used. Classical orbits are again used for the incident heavy ions. A new parameter is introduced as the square root of the probability $P$ for a transition from state 1 to 2 calculated by first-order perturbation theory (Eq. 71)

$$\chi_{1\rightarrow 2}(\theta, \gamma) = \pm \sqrt{P_{1\rightarrow 2}(\theta, \gamma)} \quad (82)$$

For $\gamma = 0$ the parameter is $\chi(\theta)$, while $\chi$ is used for $\theta = \pi$. If $\chi(\theta, \gamma)$ is much smaller than one, first-order perturbation theory may still be applied, while for values comparable with unity the perturbation approach is no longer valid. The probability for excitation is again expressed in terms of a transition amplitude squared (see Eq. 63), where in this case the amplitude is a function of $\chi(\theta)$.

For the case of a rotational band based on the ground state of a nucleus, it is convenient to introduce the parameter
where \(Q_0\) is the intrinsic quadrupole moment. This is related to the \(\chi\) parameter, and in an even-even nucleus

\[
q = \frac{\sqrt{\frac{16\pi}{5}} \frac{Z}{A}}{\hbar/\sqrt{2}} \sqrt{\mathcal{B}(E2, 0^{-})}
\]

(85)

One may employ rotational wave functions

\[
\psi = \sqrt{\frac{2J+1}{4\pi}} D^{J}_{MK}(\alpha, \beta, 0)
\]

(86)

to write the transition amplitudes for excitations of a state \(J_f, M_f\) in the ground state rotational band as

\[
a_{J_f M_f} = \sum_{J} (2J_f + 1)^{1/2}(2J_i + 1)^{1/2}(-)^{M_i + M_f}
\]

\[
\times \langle J_f J_i M_i M_i | J M_i - M_i \rangle (J_f J_i - KK | J0) A_{J, M_f - M_i}(\theta, q)
\]

(87)

In an even-even nucleus this reduces to

\[
a_{J_f M_f} = (2J_f + 1)^{1/2} A_{J_f M_f}(\theta, q)
\]

(88)
The form of $A_{JM_{\alpha}}^{M_{\alpha}}$ is such that a convenient approximation is to neglect terms for which $M_{\alpha}$ is non-zero, and then to express all angle dependence through $q(\theta)$, where $q(\theta)/q$ is a function of $\theta$ only through ratios of orbital integrals. Thus

$$A_{JM}(\theta,q) \approx A_{J0}(\pi,q(\theta)) \quad (89)$$

The excitation probabilities are given in Fig. 13 for the ground state band of an even-even nucleus, where $d\sigma = P \, d\sigma_R$.

Corrections may be made for finite $\xi$ values by adding a term to $P$, so that

$$P_J(\theta,q) = P_J(\pi,q(\theta)) + \xi \sum_j J(q(\theta)) \quad (90)$$

However, the second order terms in $\xi$ are not negligible, and more recent calculations by Alder$^{59}$ should be used for finite $\xi$ corrections. These are performed by diagonalizing a matrix that includes five rotational states. In this case the excitation probability is given as a function of $\chi$ and $\xi$. These calculations are discussed in detail in Appendix A.

In deformed nuclei it is possible to excite rotational bands based on vibrational excitations. The probability for such an excitation may be calculated by treating the most probable direct excitation by first-order perturbation theory, and then considering rotational band excitation from that level. Lutken and Winther$^{60}$ perform such calculations, writing the total transition amplitude for a particular multipolarity as
FIG. 13

$P_j(q)$ vs $q$

$P_0$

$P_2$

$P_4$

$P_6$

$P_8$

$P_{10}$
$b_{if} = -i\chi(\theta, \xi) B_{J_fM_f}^\lambda (q(\theta)) \delta_{M_f0}$  \hspace{1cm} (91)

where $\chi(\theta, \xi)$ is the amplitude for excitation to the directly attainable member of the band, and $B_{J_fM_f}^\lambda$ is the same as $a_{J_fM_f}$ in Eq. 87. The differential cross section is

$$d\sigma = P d\sigma_R$$

$$= \frac{1}{2J_i+1} \sum_{M_iM_f} |b_{if}|^2 d\sigma_R$$

$$= \frac{1}{2J_i+1} \sum_{M_iM_f} [\chi(\theta, \xi)]^2 |B_{J_fM_f}^\lambda|^2 d\sigma_R$$

$$= |B_{J_fM_f}^\lambda|^2 d\sigma_{E\lambda}$$  \hspace{1cm} (92)

where $d\sigma_{E\lambda}$ is the direct excitation cross section given in Eq. 71.

The equivalence of $B_{J_fM_f}^\lambda$ and $a_{J_fM_f}$ suggests that the mechanism may be viewed as a direct excitation to one level in the vibrational band, followed by a rotation to another level if there is sufficient energy. The directly attainable member of the band is in effect a new ground state. However, this picture is not strictly correct because the calculation also includes excitations which are a rotation in the ground state band followed by a transition to the vibrational band. These multiple processes are most probable at high incident ion energy and large scattering angles for which the energy transfer is largest. At
lower energies or forward scattering the energy transfer to the nucleus is small and the first-order or direct processes will be dominant.

As in the case of $a_{JM}$, the terms in $B_{JM}^{1}(q)$ for which $M$ is non-zero are small and may be neglected to a good approximation. Again, all angle dependence is expressed through $q(\theta)$. Fig. 14 and 15 show graphs of $B_{JM}^{1}$ as a function of $q$ for $M = 0$ and $\lambda = 2, 3$. These were evaluated from Eq. 87 in the sudden approximation ($\xi = 0$).

It should be pointed out that this calculation of excitation probabilities for levels in a vibrational band is dependent upon a particular nuclear model, in this case the collective model for an axially symmetric nucleus with pure $K$ bands. As discussed above there can be deviations from this simple case, which will affect the relative and total excitation probabilities of the band. Such effects have been considered by Lutken and Winther and will be discussed in connection with the data.

The sudden approximation may also be applied to the excitation of higher vibrational states, but the problem can be solved for arbitrary values of $\xi$. The resultant probability is a function of $\chi(\theta, \xi)$:

$$P_N = \frac{1}{N!} e^{-[\chi(\theta, \xi)]^2} [\chi(\theta, \xi)]^{2N}$$

(93)

where $N$ is the total quantum number determined by $E = \hbar \omega (N + \frac{1}{2})$. This result is derived on the assumption that the nucleus is a harmonic oscillator and is therefore model dependent. If $P_N$
Multiple Coulomb excitation of the $K=0$ beta vibrational band in an even-even nucleus.

$|b^2_j|_0^2 \ vs \ q$

Multiple Coulomb excitation of the $K=0$ octupole vibrational band in an even-even nucleus.

$|b^3_j|_0^2 \ vs \ q$
is small, cross sections can be calculated in the second order perturbation theory. This avoids questions of the validity of harmonic oscillations and is therefore more reliable.
II. Apparatus

A. Beam Considerations

The coulomb excitation experiments reported herein were first performed using the oxygen beam of the Yale University heavy ion accelerator. This accelerator produces beams of constant velocity, corresponding to 10.5 Mev/nucleon, which are reduced to lower energies by passing them through aluminum foils. The reduced-energy beam is momentum analyzed with a magnet that bends the beam trajectory through 45°. The oxygen beam is the highest-Z beam that is accelerated with good intensity and was used for that reason. In this experiment oxygen beams with 6+ charge state and up to 65 Mev energy were employed.

The heavy ion accelerator is a pulsed machine with macroscopic duty cycle of 2%. Coincidence measurements are extremely important in coulomb excitation studies of complex nuclei in order to determine nuclear level schemes and to ascertain excitation mechanisms, yet it is difficult to obtain adequate statistics in reasonable amounts of time with this low duty cycle. In order to make coincidence measurements in greater detail, the model EN tandem Van de Graaff accelerator at the Oak Ridge National Laboratory* was utilized in measurements carried out for the maximum energy available of approximately 49 Mev.

*The assistance of Drs. F. K. McGowan and C. D. Moak is gratefully acknowledged.
This maximum energy was less than was possible with the heavy ion accelerator, and coulomb excitation yields were correspondingly lower. However, background radiations were of sufficiently low intensity that measurements with the lower-energy incident beams were entirely possible. All of the coincidence data presented here was taken with the tandem Van de Graaff accelerator.
B. Gamma Ray Detection and Target Chambers

Of the types of measurements possible in coulomb excitation, detection of the de-excitation gamma rays was chosen as the most convenient and as yielding the greatest amount of information about the nucleus. Measurements of relative populations of nuclear levels by detecting the inelastically scattered particles was not feasible with heavy ions under the present experimental conditions because targets could not be made thin enough for individual level yields to be resolved. Such measurements would not give the desired information on branching ratios. Measurements of conversion electrons from the gamma rays give branching ratio data and are useful in determining multipolarities and obtaining high resolution; however, until the recent advent of lithium-drifted detectors the conversion electron measurements could have been made only with a single-channel spectrometer and with great expenditures of time. Also, the conversion coefficient is strongly dependent upon transition energy (for example, the K shell coefficient in $^{152}$Sm at 100 kev $\alpha_K = 1.1$ and at 800 kev $\alpha_K = .004$) and the higher energy transitions would not be detected efficiently. Three types of gamma ray measurements have been made in order to unravel the complex gamma ray spectra that occur and to answer the question of whether the observed gamma rays were from coulomb excitation of the target or were from background sources. Measurements have been made of direct gamma ray spectra, of gamma rays in coincidence with incident ions scattered into backward directions, and of gamma rays in coincidence with other
gamma rays. Gamma-gamma coincidence spectra are especially important in determining the assignment of gamma rays in the level scheme.

The target chamber used at Yale for coincidences between gamma rays and coincidences with ions scattered into backward angles (hereafter called gamma-particle coincidences) is shown in Fig. 16. The counters are shown in the position used for gamma-gamma coincidences. The collimation limited the beam to a 1/4" diameter spot on the target. At Oak Ridge direct and gamma-gamma coincidence spectra were taken with a target chamber (Fig. 17) that insured more reliable beam integration. In this chamber the beam was collimated to a spot about 1 mm x 6 mm. At both accelerators the beam consisted of 0^{16} ions with 6+ charge state. The gamma ray counters in each case were shielded from each other with a lead block to prevent gamma rays scattering from one crystal to the other and producing false coincidences. The lead block also served to position the counters accurately.

The gamma ray counters used at Yale consisted of 1 1/2" diameter by 2" long sodium iodide (thallium activated) crystals optically coupled to RCA 6810 A photomultiplier tubes with Dow-Corning stopcock grease. A fast signal for timing purposes was taken from the anode and a linear signal from the eleventh dynode. This dynode was maintained at ground potential to reduce gain dependence upon counting rate by allowing d.c. coupling to the linear system. Counting rate dependence was further minimized by using a low resistance voltage-divider chain with 3 ma current. To absorb the high power dissipation and avoid temperature
FIG. 17

COULOMB EXCITATION TARGET CHAMBER

LEAD SHIELD

CRYSTAL

TARGET
shifts the resistors were immersed in a water-cooled transformer oil bath. The crystal and photomultiplier were housed in a lead cylinder lined with a graded shield of tin and copper to absorb lead X-rays.

A block diagram of the electronics appears in Fig. 18. It is basically a standard fast-slow coincidence unit, but with the additional provision that accidental coincidences were measured simultaneously with true coincidences by means of a second identical coincidence circuit with additional delay at one input. The r.f. frequency of the heavy-ion accelerator is 70.2 megacycles, so that particles occur in bunches 14 nanoseconds apart. Therefore, a correct measurement of accidental coincidences can be made only if the resolving time for the accidentals coincidence circuit overlapped the same number of beam pulses and was in the same relative phase to the beam as the true coincidence resolving time. This was accomplished by making the additional delay an integral number of r.f. cycles, the number chosen being 9. The resolving time used for the data was \( 2\tau = 30 \times 10^{-9} \) seconds.

The inclusion of a linear gate in both of the gamma ray channels greatly reduced the counting rates at the inputs to the single channel analyzers and multichannel analyzer (hereafter called MCA). Since the zero level of most multichannel analyzers is sensitive to counting rate, a linear gate is very necessary with pulsed beams where the counting rate varies from zero to maximum and back again with each pulse. The linear gate used at Yale is essentially a d.c. clamp circuit which is inhibited by
BLOCK DIAGRAM for FAST - SLOW COINCIDENCE SYSTEM
COULOMB EXCITATION EXPERIMENT

FIG. 18
a square pulse to open the gate (see Appendix B for a circuit diagram). The linear gate for 0-10 v. signals used at Oak Ridge is also in Appendix B.

The recording of accidental coincidences should be pointed out. The linear gate may be opened by either an accidental or a true coincidence pulse. The pulses going to the MCA were then routed into separate halves of the memory for true or accidental coincidences.

The slow coincidence condition that a pulse from the gating counter was within the single channel analyzer window was imposed by requiring that a pulse from the single channel analyzer open a gate in the MCA. The total number of true and accidental coincidence pulses, the total number of pulses from each counter, and the number of true and accidental coincidence pulses within each single channel analyzer range were all recorded on scaling circuits. The degree to which the ratio of true to accidental coincidences remained constant served as an additional check on the system stability during extended periods of data collection.

The gamma ray counters used at Oak Ridge were 3" x 3" sodium iodide (thallium activated) crystals, commercially available in a unit mounted on the 3-inch DuMont 6363 photomultiplier tube. The cross-over coincidence method was used for obtaining a timing pulse. This was performed with transistorized electronics in which the fast coincidence pulse was derived from the zero-point of the cross-over in a double differentiated amplifier output pulse. From an analysis by Fairstein of the statistics involved in detection processes, for the large dynamic range of
gamma ray energies desired in this experiment, about 80-1400 kev, the cross-over coincidence method is expected to require a minimum of about $200 \times 10^{-9}$ seconds for 90% or better efficiency. It was found experimentally that a resolving time of $2\tau = 200 \times 10^{-9}$ seconds was required to obtain a flat top on the resolving time curve. A block diagram of the electronics is shown in Fig. 19. Accidental coincidences were measured by inserting 0.4 microseconds delay between amplifier 1 and the single channel analyzer.
BLOCK DIAGRAM for FAST-SLOW COINCIDENCE SYSTEM

FIG. 19
C. Detection of Inelastically Scattered Particles

The gamma-particle coincidence spectra are especially useful because, as will be seen below, gamma rays from elements with lower Z than the target nuclei are not detected. Lower elements are found on the target surface from oxidation of the rare earth target and from condensation of pump oil vapors. The bombardment of target surface impurities with oxygen ions produces nuclear reactions and background gamma radiation. However, in such reactions there are no oxygen ions scattered into angles greater than 90° in the laboratory. By requiring coincidences between gamma rays and oxygen ions scattered into backward angles, only coulomb excitation gamma rays are recorded.

Protons and alpha particles are emitted in backward directions from the compound nucleus reactions, but are of lower energy than most ions scattered from the target. By placing a lower limit on the voltage pulse accepted from the particle detector, it was possible to discriminate against the contamination particles. The gamma rays were then measured in coincidence with pulses above the discriminator level and could only result from coulomb excitation. Comparing spectra taken in this way with the direct spectrum, gamma rays appearing in direct spectra and not in gamma-particle spectra are probably from target contaminants, while those that appear in gamma-particle spectra alone are from coulomb excitation of the target.

Inelastically back-scattered ions were detected by an array of 1 cm² surface-barrier gold-silicon junction detectors.
(Fig. 20) with the sensitive surfaces located in a plane about one inch from the target surface. The detectors were connected in pairs and could all be switched in parallel to obtain a large solid angle. As many as 32 detectors could be used, although in practice the maximum number used was 20. The advantage of this switching arrangement was that if one of the detectors began to draw excessive current, resulting in a higher electronic noise level during a long accumulation of data, it could be switched out and the run continued without loss in resolution, although with slightly reduced effective solid angle.

The junction detectors were used at a bias such that the depletion region was adequate for stopping 65 Mev oxygen ions, but only about 3 Mev protons or 10 Mev alpha particles. An alpha particle spectrum from thorium B and C' is given in Fig. 21, showing that the resolution of the detector array is about 0.34 Mev. The individual detector resolution is much better, but the signal in each detector is divided across the entire capacitance of the array and the resultant signal-to-noise ratio is lower.

The location of the discriminator level in the thick target particle spectrum for 49 Mev oxygen ions on Sm$^{152}$ is shown in Fig. 22. The particle spectrum from an aluminum target is given for comparison, showing that the discriminator level is well above the particles from low-Z materials. This particular discriminator level does not accept oxygen ions scattered from nuclei as high as 37 in Z. A further aspect of the discriminator level is the determination of an effective target thickness. Some oxygen
THORIUM B AND C' ALPHA PARTICLE SPECTRUM IN JUNCTION DETECTOR ARRAY.

8.78 MeV

6.04

COUNTS PER CHANNEL

CHANNEL NUMBER

FIG. 21

JUNCTION DETECTOR SPECTRA
UNNORMALIZED
BACK-SCATTERED O16 IONS
SPECTRUM FROM ALUMINUM TARGET
PULSES ACCEPTED BY DISCRIMINATOR

COUNTS PER CHANNEL

CHANNEL NUMBER

FIG. 22
ions penetrating into the target and losing energy before and after scattering inelastically from a target nucleus will have less than the minimum energy accepted. The energy of the discriminator cut-off and the associated target thickness have been calculated in Appendix A.
D. Target Preparation

One is confronted with two problems when considering coulomb excitation of even-even rare earth nuclides. First, one must work with separated isotopes. Natural samarium has four stable isotopes that are 10% or more of the total by weight. This is in sharp contrast to odd-A rare earths which are very nearly monoisotopic in nature. Second, all rare earths oxidize readily, and separated isotopes are available in the oxide form. If targets were made of the rare earth oxide, nuclear reactions with the oxygen would yield large numbers of gamma rays that would completely mask all but the most prominent gamma radiations from the rare earth under investigation. Therefore pure metal targets are of great importance.

The reduction and purification of many rare earths has been accomplished by F. Spedding and A. Daane. In particular, they reduced samarium oxide with La in an inert atmosphere, dealing with 20-200 gram samples. Since separated isotopes cost on the order of dollars/mg and about 100 mg. were needed in a single target, it was necessary to effect an extensive miniaturization of the process. It has been possible at this laboratory to obtain a yield of at least 50% of the enriched samarium on a target area 7/8" in diameter. About 25 mg/cm² of Sm are needed to stop a 65 Mev O¹⁶ beam. This amount was obtained as an average when a charge of 250 mg Sm was reacted with La filings.

The evaporation was performed in a standard bell jar at a pressure of about 5 x 10⁻⁵ mm Hg. The crucible used for the
evaporation (Fig. 23) was made from 1/4" tantalum tubing with .010" walls. The strips at the top were formed by slitting the tube and bending the strips back, then filing them down to about 1/16" width. At the bottom the tubing was pinched shut and two .010" strips were spot-welded onto the base. These two operations resulted in completely uniform heating of the entire surface of the cylinder. Omission of either technique leads to a cooler area at the end of the cylinder and allows condensation of samarium at that point.

The samarium was evaporated onto a tantalum foil backing clamped to a water-cooled copper plate at about 1/2" from the mouth of the cylinder. The target area was defined by a .010" tantalum mask to facilitate estimation of target thickness. Also, samarium evaporated onto the mask could be reclaimed. Water-cooling is necessary in order to prevent re-evaporation of samarium from the target surface since its proximity to the cylinder would lead to extensive heating by radiation. Since samarium does not adhere well to cold tantalum, the water was not turned on until the target was completely covered with a small amount of samarium. Water cooling also has the effect of yielding an optically reflective samarium surface, while without cooling the samarium may appear grey or black, as might be expected for a granular surface. Optically reflective targets are more resistant to oxidation than grey or black targets, presumably because of the lower surface area.
1/4" dia. tantalum tubing
.010" wall

.010" tantalum strips

copper current leads

FIG. 23
The basic materials needed for the evaporation process are a bell jar and vacuum system capable of reaching a pressure of about $10^{-6}$ mm Hg. The current requirements are about 120 amps a.c., continuously variable from 0 to maximum. A Variac was used in the primary of a 2 kilowatt transformer with a current amplification factor equal to about 25. As stated above, water cooling is needed for the target backing. The target holder must also be movable since lanthanum is often stored in oil and this oil is driven off in the initial heating of the cylinder. The evaporation process is given in Appendix C.
III. Data Presentation

A. Samarium - 152

The data on Sm$^{152}$ are considered first since this nucleus showed the most complex and interesting spectrum of the four nuclei studied. It is a deformed nucleus and has well-developed ground state, beta, and gamma vibrational bands as described earlier. Evidence for an octupole vibrational band has also been obtained. The level scheme (Fig. 24) known from beta decay studies included at least one member of each rotational band, but in this experiment more levels have been populated and a more complete picture is available.

Direct gamma ray spectra are shown in Fig. 25 and Fig. 26. Gamma ray transitions have been observed in previous work at 122, 245, 563, 841, 963, 965, 1087, and 1114 kev, with possible evidence for 689 and 811 kev transitions from a 2+ level at 811 kev. These have been confirmed and have been more definitely assigned in the decay scheme. A low intensity peak occurs at 82 kev, reflecting a small target impurity of Sm$^{154}$. The isotopic content of the targets used is given in Table IV. Small peaks from isotopic impurities can be noticed below the first excited state de-excitation radiation in many of the targets, and will not be mentioned specifically. A new gamma ray is found at 340 kev, and this will be shown to be from the third member of the ground state band, a 6+ to 4+ transition.
\[ 62 \text{Sm}^{152} \]

FIG. 24
GAMMA RAY SINGLES SPECTRUM

49.00 MeV $^0$He IONS ON Sm$^{152}$ THICK TARGET

ABSORBER: 0.031" Pb, 0.040" Sn, 0.060" Cu

-x--x--x-- 511 keV RADIATION SUBTRACTED

CHANNEL NUMBER

COUNTS PER CHANNEL

FIG. 25
SINGLES SPECTRA

$^{16}$ O IONS ON Sm$^{52}$ THICK TARGET
ISOTOPIC ENRICHMENT 99.2% 

ABSORBERS: 0.027" Pb
0.040" Sn
0.060" Cu

YIELDS UNNORMALIZED
### Table IV

Isotopic Content of Targets by Per Cent

<table>
<thead>
<tr>
<th>Isotope</th>
<th>144</th>
<th>147</th>
<th>148</th>
<th>149</th>
<th>150</th>
<th>152</th>
<th>154</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sm$^{148}$</td>
<td>.36</td>
<td>.81</td>
<td>96.26</td>
<td>1.41</td>
<td>.34</td>
<td>.55</td>
<td>.28</td>
</tr>
<tr>
<td>Sm$^{150}$</td>
<td>.05</td>
<td>.34</td>
<td>.36</td>
<td>.87</td>
<td>96.37</td>
<td>1.51</td>
<td>.5</td>
</tr>
<tr>
<td>Sm$^{152}$</td>
<td>&lt;.02</td>
<td>.19</td>
<td>.07</td>
<td>.11</td>
<td>.09</td>
<td>99.06</td>
<td>.48</td>
</tr>
<tr>
<td>Sm$^{154}$</td>
<td>&lt;.01</td>
<td>.08</td>
<td>.07</td>
<td>.21</td>
<td>.05</td>
<td>.38</td>
<td>99.21</td>
</tr>
</tbody>
</table>

Accuracy of each per cent to ±.05 or better
The gamma rays at 511 and above about 1100 kev are from target surface contaminants (see Section E). The different incident energies used in Fig. 26 serve chiefly to identify the 563 and 689 kev transitions as coming from levels of the beta band. The 563 kev line falls off quickly with energy, while the 689 kev line remains at the same relative intensity. These energy dependences will be shown to be characteristic of transitions from the 0+ and 2+ members of the beta band.

A gamma-particle coincidence spectrum appears in Fig. 27. This spectrum, and all other coincidence spectra presented here, have been corrected for accidental coincidences in the manner described in Chapter IV. The corrections are of the order of 10% for most coincidence spectra. The absence of 511 kev annihilation radiation is evidence of the effectiveness of this type of coincidence spectrum in discriminating against background radiation. Most of the transitions prominent in the direct spectrum appear again except for energies greater than 1260 kev. These are possibly from contaminants. Since the particle detectors are placed at large scattering angles (about 138 to 162 degrees) the ions detected are those for which the nucleus experienced the highest electric field gradient. Accordingly, transitions to higher energy states should be more dominant here than in the direct spectrum. This is observed to be the case in the ground state band when comparing the relative intensities of 122 and 245 kev gamma rays, the 245 coming from a higher level. The comparison for higher energy gamma rays is complicated by the great reduction in background and the fact that the relative
GAMMA-PARTICLE COINCIDENCE SPECTRUM

49.00 MeV O\textsuperscript{16} IONS ON Sm\textsuperscript{152} THICK TARGET

ABSORBER: 0.031" Pb, 0.040" Sn, 0.060" Cu

Counts Per Channel vs. Channel Number

Fig. 27
excitation of vibrational band members is a strong function of scattering angle.

Gamma-gamma coincidence spectra are shown in Fig. 28, Fig. 29, and Fig. 30. The settings of the single channel analyzers on the gating counter spectrum are shown in Fig. 31. The most efficient use of single channel analyzers (gates) is to set them on the more intense low energy transitions. Since most high energy transitions are in coincidence with the lower energy transitions from the first few excited states, a spectrum taken in coincidence with a low-energy transition yields several high energy transitions. A spectrum taken in coincidence with a high energy gamma ray yields only one or two low energy transitions. The shorthand notation of T + 122 will be used to designate gamma ray spectra taken in coincidence with the 122 kev transition, T + 245 for coincidences with 245 kev, and so on. Again, these spectra have been corrected for accidental coincidences.

The ground state band transitions appear strongly in T + 122, T + 245, and T + 340. In T + 340 the cascade following the 340 kev transition should show equal intensities of 245 and 122 kev transitions, but the heavy absorber removes much of the 122 kev radiation. The 444 kev transition appears in T + 122 and T + 245, but not in T + 340, and is therefore involved in a cascade through the 4+ level. This is further confirmed by the T + 444 coincidence spectrum (Fig. 32), which shows only the 245 kev gamma ray. The 630 kev transition is possibly from a target surface contaminant. The 689 kev transition is a double line since a 670 kev gamma ray is found in T + 245. The only
COINCIDENCES WITH 122 keV TRANSITION
49.00 MeV 0\(^{16}\) IONS ON Sm\(^{152}\) THICK TARGET
ABSORBER: 0.031° Pb, 0.040° Sn, 0.060° Cu

COUNTS PER CHANNEL

CHANNEL NUMBER

FIG. 28
COINCIDENCES WITH 245 keV TRANSITION
49.00 MeV O^{16} IONS ON Sm^{152} THICK TARGET

ABSORBER: 0.031" Pb, 0.040" Sn, 0.060" Cu

--x--x-- 511 keV RADIATION SUBTRACTED
COINCIDENCES WITH 340 keV TRANSITION

49.00 MeV $^{16}O$ IONS ON $^{152}Sm$ THICK TARGET

ABSORBER: 0.031" Pb, 0.040" Sn, 0.060" Cu

--x--x-- 511 keV RADIATION SUBTRACTED

COUNTS PER CHANNEL

CHANNEL NUMBER

FIG. 30
DIRECT SPECTRUM IN GATING COUNTER SHOWING LOCATION OF GATES.
49.00 MeV O$^{16}$ IONS ON Sm$^{152}$ THICK TARGET

ABSORBER: 0.040" Sn, 0.060" Cu

COUNTS PER CHANNEL

10^{5}

10^{4}

10^{3}

CHANNEL NUMBER

FIG. 31
COINCIDENCES WITH 444 keV TRANSITION
49.00 MeV O^{16} IONS ON Sm^{152} THICK TARGET

ABSORBER: 0.031" Pb, 0.040" Sn, 0.060" Cu
gamma rays absent from these coincidence spectra that appeared in the direct spectra are at 811 and 1087 kev, indicating that these are direct transitions to the ground state, as expected. Intensities and decay scheme assignments are discussed further in Chapter V.
B. Samarium - 154

The only known levels in this deformed nucleus at the beginning of this experiment was the 2+ member of the ground state band at 82 kev. Since then additional members of the ground state band have been reported, first by J. S. Greenberg et al. and later by J. de Boer et al. Direct spectra (Fig. 33) showed no strong gamma rays other than the ground state band transitions and it was assumed that the vibrational bands lie moderately high in energy and are not easily excited in coulomb excitation.

However, coincidence spectra reveal several new gamma rays. The best gamma-particle spectrum (Fig. 34), taken at 49 Mev, shows some of these transitions. The 474 kev peak is probably a sum coincidence since its intensity is very close to the 3% of the 278 kev peak intensity predicted by relative detection efficiencies. Low intensity peaks occur at 553, 630, and 740 kev, and slightly stronger ones at 840, 920, 1030, and 1440 kev. The 920 kev transition also appears in the direct spectrum, though weakly. The gamma-gamma coincidence measurements again show some of these weaker transitions. The settings of gates are shown in Fig. 38. The T + 185 spectrum (Fig. 35) gives the most information, with the 630, 740, and 920 kev transitions standing out. The T + 82 spectrum (Fig. 36) has only the 630 kev transition. The T + 278 spectrum is shown in Fig. 37. Most of these transitions are assigned to an octupole band on the basis of intensities and branching ratios. The 1030 and 1440 kev transitions are possibly from the beta and gamma bands.
GAMMA RAY SINGLES SPECTRUM
49.00 MeV O^{16} IONS ON Sm^{154} THICK TARGET

ABSORBER: 0.030° Cu

CHANNEL NUMBER

COUNTS PER CHANNEL

0 20 40 60 80 100 120 140 160 180 200

FIG. 33
GAMMA-PARTICLE COINCIDENCE SPECTRUM

49.00 MeV O\(^{16}\) IONS ON Sm\(^{154}\) THICK TARGET

ABSORBER: 0.030° Cu

COUNTS PER CHANNEL

CHANNEL NUMBER

FIG. 34
COINCIDENCES WITH 185 keV TRANSITION
49.00 MeV O\textsuperscript{16} IONS ON Sm\textsuperscript{154} THICK TARGET

ABSORBER: 0.030" Cu

--x--x--x-- 511 keV RADIATION SUBTRACTED
COINCIDENCES WITH 82 keV TRANSITION
49.00 MeV O\textsuperscript{16} IONS ON Sm\textsuperscript{154} THICK TARGET

ABSORBER: 0.030" Cu
COINCIDENCES WITH 278 keV TRANSITION
49.00 MeV $^{16}$O IONS ON $^{154}$Sm THICK TARGET

ABSORBER: 0.030" Cu

511 keV RADIATION SUBTRACTED

CHANNEL NUMBER FIG. 37
DIRECT SPECTRUM IN GATING COUNTER
SHOWING LOCATION OF GATES.
49.00 MeV O^{16} IONS ON Sm^{154} THICK TARGET

ABSORBER 0.030" Cu
C. Samarium - 150

The first excited state in the Sm\(^{150}\) nucleus is a 2+ level at 334 kev. Higher energy levels are reported by Groshev et al.\(^{36}\) based on thermal neutron capture gamma ray studies (Fig. 39). There are 0+ and 4+ levels at 740 and 773 kev, a 2+ level at 1047 kev, and a 3- level at 1071 kev. The direct gamma ray spectrum (Fig. 40) shows the strong 334 kev transition from the first excited state, the 4+ to 2+ 439 kev transition, as well as the 2+ to 2+ 712 kev transition unresolved from the 3- level at 1071 kev. Two other peaks occur with low intensity at 859 and 1026 kev.

Several more gamma rays are found in the coincidence spectra. In gamma-particle coincidences (Fig. 41) peaks occur at 583 and 1193 kev. The region around 773 kev is filled in by a sum coincidence of 334 and 439 kev gamma rays, while sum coincidences of 334 with 712 and 736 kev gamma rays appear at 1046 and 1071 kev. The T + 334 (Fig. 42) spectrum again has the 583 kev gamma ray, but the 1193 kev transition is very weak or absent, suggesting that it is a direct transition to the ground state. Further coincidence spectra were not very useful since they yielded only 334 kev radiation and low intensity lines from extremely weakly populated higher levels.

More recent thermal neutron capture gamma studies by R. K. Smither\(^{31}\) include coincidence measurements and angular correlation studies. Smither's decay scheme has significant differences
GAMMA RAY TRANSITIONS in $^{150}$Sm

L.V. Groshev et al, Nucl. Phys. 43, 669 (1963)

FIG. 39
GAMMA RAY SINGLES SPECTRUM
49.00 MeV O\textsuperscript{16} IONS ON Sm\textsuperscript{150} THICK TARGET

ABSORBER: 0.030° Cu

---X---X--- 511 keV RADIATION SUBTRACTED

COUNTS PER CHANNEL

1,000,000

100,000

10,000

CHANNEL NUMBER

0 20 40 60 80 100 120 140 160 180 200

FIG. 40
GAMMA-PARTICLE COINCIDENCE SPECTRUM

49.00 MeV O^{16} IONS ON Sm^{150} THICK TARGET

ABSORBER: 0.030" Cu

COUNTS PER CHANNEL

CHANNEL NUMBER

FIG. 41
COINCIDENCES WITH 334 keV TRANSITION
49.00 MeV $^{16}\text{O}$ IONS ON Sm$^{150}$ THICK TARGET

ABSORBER: 0.030" Cu

- - - - - - 511 keV RADIATION SUBTRACTED

COUNTS PER CHANNEL

CHANNEL NUMBER

FIG. 42
(see Fig. 43) from Groshev's decay scheme. These differences will be discussed in Chapter V, together with the assignment of the stronger observed transitions.
GAMMA RAY TRANSITIONS in Sm$^{150}$

R.K. Smither

FIG. 43
D. Samarium - 148

The direct gamma ray spectrum from Sm$^{148}$ (Fig. 44) shows only one strong transition, and it is not completely resolved from the 511 annihilation radiation. There are lines at 420, 705, and 1020 kev, but these are of low intensity and can be attributed to expected lines from surface contaminants. Coincidences with the 551 kev first excited state (Fig. 45) show a definite peak at about 620 kev and a series of very weak transitions above that. From the decay scheme known by beta decay studies (Fig. 46) the 620 kev line is probably approximately equal amounts of 611 and 630 kev radiation from 3- and 4+ levels. The 914 kev de-excitation from the 1- level is clearly detected in the gamma-particle spectrum (Fig. 47). The absence of 0+ and 2+ levels of a second-phonon triplet in the known level scheme suggests that these were not populated in beta decay and that the weaker lines in T + 551 might be from these levels. This will be discussed further in Chapter V.
GAMMA RAY SINGLES SPECTRUM

49.00 MeV $^0$H IONS ON Sm$^{148}$ THICK TARGET

ABSORBER: 0.030" Cu

--X--X--X-- 511 keV RADIATION SUBTRACTED

COUNTS PER CHANNEL

10,000

100,000

CHANNEL NUMBER

0 20 40 60 80 100 120 140 160 180 200

FIG. 44
COINCIDENCES WITH 551 keV TRANSITION

49.00 MeV O

IONS ON Sm\textsuperscript{148} THICK TARGET

ABSORBER: 0.030\textsuperscript{o} Cu

\textbf{---x---x---} 511 keV RADIATION SUBTRACTED

COUNTS PER CHANNEL

CHANNEL NUMBER

FIG. 45
GAMMA-PARTICLE
COINCIDENCE SPECTRUM

49.00 MeV \(^{16}\text{O}\) IONS ON \(^{148}\text{Sm}\) THICK TARGET

ABSORBER: 0.030" Cu

COUNTS PER CHANNEL

CHANNEL NUMBER

FIG. 47
E. Contamination Gamma Rays

The two principal sources of background gamma radiation, other than ambient background, are target surface contaminants from oxidation of the target material and from condensation of pump oil vapors. The coulomb barriers for oxygen on oxygen or oxygen on carbon are about 12 and 15 Mev, and so an incident beam of 49 Mev easily induces compound nucleus reactions. The gamma rays resulting from cascades from highly excited states can, in some cases, be identified with particular daughter nuclei resulting from the decay of the compound nucleus. Fig. 48 shows some of the higher energy gamma rays from 49 Mev O\textsuperscript{16} on a Sm\textsuperscript{152} target. The gamma rays from 1.27 Mev and higher were found in every target.

Considering the case of 49-Mev oxygen ions incident upon an oxygen nucleus, the excitation of the compound nucleus is

\[
E_{\text{exc}} = Q + \frac{E_0 M_2}{M_1 + M_2}
\]

\[
= (16.000 + 16.000 - 31.982)931 + \frac{49(16)}{32}
\]

\[
= 41.3 \text{ Mev}
\]

(94)

The S\textsuperscript{32} nucleus can emit more than one nucleon in order to lose this energy. For emission of an alpha particle, the excitation energy in the final nucleus 14S\textsuperscript{28} is
GAMMA RAY SINGLES SPECTRUM

49.00 MeV O$^{16}$ IONS ON Sm$^{152}$ THICK TARGET

FIG. 48
E = (31.982 - 4.004 - 27.986)931 - E_{exc}(S^{32})

= 23.9 Mev (95)

We shall consider a few level schemes of daughter nuclei from $^{16}_{8}O + ^{16}_{8}O \rightarrow ^{32}_{16}S^{*}$. For the emission of one neutron or one proton, we have levels in $^{16}_{5}S^{31}$ and $^{15}_{1}P^{31}$, as shown in Fig. 49. The gamma rays at 1.27, and 2.23 Mev are found in Fig. 48, and the .960 is found with low intensity in several coincidence spectra. These are found in gamma-gamma coincidence spectra since they are coincident with some high energy transition whose compton pulses lie under any gate set at low energy. We note also that position annihilation radiation will occur in coincidence spectra from Compton scattered radiation corresponding to the 1.27 Mev gamma rays.

Emission of two nucleons from the compound system gives $^{16}_{5}S^{30}, ^{15}_{1}P^{30}$, and $^{14}_{1}Si^{30}$. The level schemes are shown in Fig. 50. The gamma rays at 1.98, 1.26 or 1.27, 2.54, 2.72, and 2.24 are certainly evident in Fig. 4%. The .686 and .705 Mev transitions occur weakly in several spectra (see Fig. 43 and 44). These would be expected to be less prominent since the higher energy transitions are more probable in cascades from high-lying states. The existence of $^{30}_{1}P$ as one of the sources of background radiation was verified by finding a 2.7 minute activity for 511 radiation after bombarding the target for a few minutes. The possible nuclear levels for mass 29 and 28 systems are shown in Fig. 51 and 52. None of these additional
FIG. 49

\[ ^{16}\text{Si}^{31} \]

2.6 sec.

\[ \frac{1}{2}^+ \]

\[ ^{31}\text{P}^{15} \]

FIG. 50

\[ ^{30}\text{Si}^{14} \]

2.6 min.

\[ ^{30}\text{P}^{15} \]

\[ \beta^+ \]

1.27 2.23

0.96

1.27 2.23
gamma rays was prominent.

Targets bombarded by an amount of beam in excess of 200 microcoulombs were yellowed, presumably indicating a dissociation of pump oil molecules. Emission of one nucleon from the $\text{Si}^{28}$ compound system leads to $\text{Si}^{27}_{14}$ and $\text{Al}^{27}_{13}$ with low level structures shown in Fig. 53, and for two nucleons out, $\text{Si}^{26}$, $\text{Al}^{26}$, and $\text{Mg}^{26}$ (Fig. 54).

Since the gamma radiations from coulomb excitation of the target nuclei are less than $1.2 \text{ Mev}$ energy, only the lower-energy target contaminant radiations need be considered. Fortunately, only the $511 \text{ kev}$ radiation is very intense, but the other transitions must be noted to avoid erroneous assignments to weakly populated levels. From the above analysis gamma rays are expected at $418, 686$ and $705, 824, 842$, and $1015 \text{ kev}$. The $418, 686$ and $705$, and $1015 \text{ kev}$ radiations occur in all spectra, although they are often hidden by coulomb excitation gamma rays. No evidence was found for $824$ or $842 \text{ kev}$ contaminant radiation.

Another source of gamma radiation other than the target nuclei is from inelastic scattering of neutrons from sodium and iodine in the crystal. The levels of Na$^{23}$ and I$^{127}$ excited by inelastic neutron scattering are given in Fig. 55 and 56. Again there are possible sources for the contamination radiation at about $420 \text{ kev}$. The gamma rays at $43, 59, 144, 203, 359$ and $375 \text{ kev}$ are not prominent in any of the spectra. The $203 \text{ kev}$ transition is strong in the data of Lind and Day. Since the transition is not observed in direct spectra to any extent, we may conclude that inelastic scattering of neutrons from the
\( \text{Na}^{23} \)

\( 3/2^+ \)

\( 5/2^+ \)

\( 7/2^+ \)

\( 1/2^+ \)

\( \text{I}^{127} \)

\( \text{FIG. 55} \)

\( \text{FIG. 56} \)

TO EITHER 0 OR .059 LEVEL
sodium iodide crystal does not contribute appreciably to the background. In gamma-gamma spectra some of these transitions may appear, but only the 438 kev gamma ray in Na$^{23}$ is known to be in a cascade from higher states.
IV. Data Processing

A. Treatment of Data

The gamma ray spectra obtained in the direct and coincidence measurements were analyzed to determine the transition energies and absolute gamma ray intensities. Below are discussed a number of considerations that were involved in reducing the raw data.

The energies of gamma rays were determined by calibrations with standard gamma ray sources carried out at the same counting rate as that used in the experiment. Internal calibrations were also made by the use of known gamma ray transitions in the nucleus studied. Monoenergetic gamma ray sources prepared on 0.00025" mylar foil were used to measure the response of the sodium iodide crystals to gamma rays as function of gamma ray energy. These standard spectral shapes, which were obtained in counter geometries exactly simulating those used in the experiments, were employed in determining the relative fraction of the detector pulses lying within the photopeak.

The gamma ray spectra measured in coincidence with other gamma rays or with back-scattered particles were corrected for accidental coincidences by subtracting from the data a direct spectrum normalized by the ratio of the total number of accidental to true coincidence pulses.

From the actual number of counts in the photopeak the absolute intensity of the gamma ray transitions was calculated
by applying several correction factors. These included the efficiency of the detector, the fraction of gamma rays removed by the absorber, and the multichannel analyzer counting losses. Also, to convert from the total number of gamma ray transitions from a given level to the total number of transitions from that level, a correction was made for the number of conversion electrons emitted. The total efficiency of 1 1/2" x 2" and 3" x 3" sodium iodide crystals used here was that calculated by Vegors, Marsden, and Heath\textsuperscript{71} for a crystal at 10 cm from the source. This calculation involves integrating the absorption cross section over the entire solid angle subtended by the crystal.

To obtain the ratio of probabilities for a count to fall in the photopeak rather than the Compton distribution of a spectrum, measurements were made with no absorber on the counter other than the 0.030" of aluminum used in mounting the crystal on the photomultiplier. These agreed well with measurements by Heath.\textsuperscript{72}

The conversion coefficients used are those of Sliv and Band,\textsuperscript{73} which are corrected for finite nuclear size effects. The photopeak attenuation by the various absorbers used were calculated from the measured thickness and cross sections given by Wapstra.\textsuperscript{74} These were checked experimentally with monoenergetic sources. The photopeak intensity of the 122 kev gamma ray in Sm\textsuperscript{152} is reduced by a factor of 0.0070 by the 0.031" lead absorber and is thus very dependent upon the thickness measurement. However, the absorber correction was checked from the relative attenuations of 122 and 245 kev gamma rays.
from Sm$^{152}$ direct spectra taken with and without the lead. The error from this determination is discussed in Section 3.

Measurements were made of the ratio of the efficiency of the gamma ray counters at the positions used in the experiment to the efficiency at 10 cm (See Fig. 57 and 58) and these were used in determining the total efficiency of the detector. During the experiment the counters were placed at 9.2 cm from the target for Sm$^{152}$ and Sm$^{154}$ spectra, but were moved to 6.2 cm for Sm$^{148}$ and Sm$^{150}$ spectra to improve the true to accidental coincidence ratio. The normalization of efficiencies between gamma-gamma and gamma-particle spectra was checked experimentally by comparing the intensities of the transitions from the first and second excited states in Sm$^{152}$, Sm$^{154}$, and Sm$^{148}$ using the direct spectra taken at two positions. The need for a re-normalization of the gamma-particle spectra arose from the poor integration of the beam by the chamber used in the gamma-particle experiments. The geometry used in these experiments did not allow efficient trapping of secondary electrons so that the integrator read too high a value. Since the geometry used in the gamma-particle coincidence experiments was the same for all four isotopes, an internal check was then obtained on efficiencies in all the experiments. In this way all efficiencies, including the beam integration, could be referred to the gamma-gamma geometry used for Sm$^{152}$ and Sm$^{154}$. The error used in making these comparison was 5%.

The total number of transitions was determined from the efficiencies and the number of gamma ray counts in the photopeak
PHOTOPEAK EFFICIENCY FOR COUNTER GEOMETRY USED RELATIVE TO CALCULATED GEOMETRY.

FIG. 57

GAMMA - PARTICLE SPECTRA

Cd$^{109}$ Hg$^{203}$ Sn$^{113}$ Sr$^{85}$ Cs$^{137}$ Mn$^{54}$ Zn$^{65}$

GAMMA-GAMMA SPECTRA

E (keV) FIG. 57

PHOTOPEAK EFFICIENCY FOR GEOMETRY USED IN Sm$^{148}$ AND Sm$^{150}$ SPECTRA RELATIVE TO Sm$^{152}$ GEOMETRY.

COUNTER AT 90°

Ce$^{144}$ Sn$^{113}$ Sr$^{85}$ Cs$^{137}$ Mn$^{54}$ Zn$^{65}$

COUNTER AT 0°

E (keV) FIG. 58
of a spectrum; these are related by

$$T = \frac{S(1+\alpha)}{\eta A \lambda}$$

(96)

$T$ = total number of transitions

$S$ = number of counts in photopeak

$\alpha$ = conversion coefficient

$\eta$ = efficiency for a gamma ray to give a pulse in the photopeak for a counter at the distance used

$A$ = fraction of gamma rays remaining in photopeak after passing through absorber

$\lambda$ = live time of multi-channel analyzer

For the gamma-gamma coincidences the efficiency and absorber of the second counter must be included, as well as the width of the single channel analyzer window compared to the width of the photopeak on which it was set. The total number of transitions determined by the coincidence spectra is then given by

$$T = \frac{C(1+\alpha_1)(1+\alpha_2)}{\eta_1 A_1 \eta_2 A_2 G}$$

(97)

$C$ = number of counts in photopeak

$G$ = fraction of photopeak lying within the single channel analyzer window

where 1 and 2 refer to counters 1 and 2 and all other symbols are the same as above.

For the gamma-particle spectra the number of counts in the photopeak was corrected in the same manner as were the direct spectra. The yield obtained in this manner was compared with
the yield calculated from the differential cross section and the junction detector solid angle. The particle detector arrangement used for quantitative work is shown schematically in Fig. 59. The sensitive area of each detector was divided into 2 mm by 2 mm squares, and the expression

\[ d\Omega = \frac{dA \cos \theta}{r^2} \]  \hspace{1cm} (98)

was used to determine the total solid angle as a function of scattering angle. The differential cross sections were then integrated over the angle and solid angle obtained in this manner. Details of the calculation are given in Appendix A.

As was mentioned above, the pulse height selection that was performed on the scattered heavy ion pulses determined the effective target thickness. The highest energy in the particle spectrum corresponds to the energy of the particles elastically scattered from the target surface. Heavy ions penetrating into the target to such depth that they emerge with an energy below that selected by the discriminator level do not contribute to gamma-particle coincidences. The energy loss before scattering from a nucleus and after emerging from the target has been calculated as a function of target thickness and is shown in Fig. 60. The position of the discriminator level is indicated.

The incident energy for which thick target integrations must be terminated is not the energy of the incident ion corresponding to the discriminator level, but the energy of the ion before it scatters from a nucleus in the target. From the curve in Fig. 60
SCHEMATIC DIAGRAM OF JUNCTION DETECTOR POSITIONS.

FIG. 59

EFFECTIVE TARGET THICKNESS $d$ (mg/cm$^2$)

FIG. 60
and the settings used in this experiment, the cut-off energy was found to be 45.0 Mev. Unfortunately, due to uncertainties in multi-channel analyzer calibration and system linearity, this method is not the best way of determining the cut-off energy. The number actually used was obtained by requiring agreement between the \( B(E2) \) values determined from the direct and gamma-particle spectra for the first excited state in \( \text{Sm}^{148} \). This excitation is expected to be well-described by first-order perturbation theory, so that the energy cut-off is only subject to uncertainties in determining gamma ray intensities. The energy cut-off obtained for the 49 Mev incident beam was \( 45.5 \pm 0.25 \) Mev. Further justification for this energy cut-off was obtained from the agreement of experimental and calculated coulomb excitation gamma radiation yields in \( \text{Sm}^{152} \) and \( \text{Sm}^{154} \) for the ground state bands.

The effect of the correlations, between the directions of the gamma rays and the beam direction, on yield calculations has only been considered in an approximate way. The effects are generally small, as will be indicated in the following, and in most cases can be neglected compared to other errors. If the gamma ray polarization is not measured, and if scattered ions are detected symmetrically about the beam direction, then the angular correlation may be written as in Eq. 76,

\[
W(\theta_{\gamma}) = \sum_{\text{even } k} a_k A_k P_k(\cos \theta_{\gamma})
\]

(99)

where \( \theta_{\gamma} \) is measured relative to the beam direction.
For measurements in which the inelastically scattered particles are not detected, the particle coefficient $a_k$ for $k = 4$ is much smaller than for $k = 2$, so that $k = 4$ terms may be neglected. The particle coefficients have been evaluated for direct E2 excitation (see ABHMW) as a function of $\xi$. These must be integrated over energy for the thick target and normalized by

$$a^t_k = \frac{\int a_k \frac{\sigma(E)dE}{E/ds}}{\int \frac{\sigma(E)dE}{E/ds}}$$

where the superscript $t$ indicates the thick target coefficient.

For the case of 49 Mev $0^{16}$ ions on a Sm$^{152}$ thick target and direct E2 excitation of the 0.811 Mev level,

$$a^t_0 = 1$$

$$a^t_2 = 0.711$$

$$a^t_4 = -0.028$$

The correction for gamma rays recorded at $90^\circ$, as in the direct spectra in this experiment, is then

$$W(90^\circ) = 1 - 0.127 - 0.012$$

$$= 0.861$$

for the case of pure E2 de-excitation to the ground state. It is
seen that neglect of the $k = 4$ term is only a 1% error. These coefficients are also attenuated by the finite size of the crystal. The calculations of Yates\textsuperscript{75} have been used for the attenuation factors.

The gamma rays in gamma-particle coincidence spectra were measured at 55° to the beam direction. At this angle $P_2(\cos \theta \gamma)$ is zero, and there is no contribution from the $k = 2$ term in $W(\theta \gamma)$. For $\theta \gamma = 55^\circ$, $P_4(\cos 55^\circ) = -0.23$, and the angular correlation is

$$W(55^\circ) = 1 - 0.23 a_4 A_4$$ \hspace{1cm} (103)

Since the particle coefficient $a_4$ is much smaller than unity, and since the correlation is attenuated by the finite solid angle of the crystal, the $k = 4$ term may be neglected. Thus there are no angular correlation corrections to be made for gamma ray intensities obtained from gamma-particle spectra.

The particle coefficients $a_k$ depend upon the excitation mechanism. For double or multiple excitation the evaluation involves lengthy numerical integrations and has not been carried out. In the case given above, $A_2$ is one of the larger coefficients possible, for a 0+ to 2+ to 0+ analogous gamma-gamma cascade. For 0+ to 2+ to 2+ and 0+ to 2+ to 4+ the coefficients are -.077 and 0.102. It is estimated that neglect of angular correlations in the direct spectra will lead to errors of about 10%. The $\text{Sm}^{148}$ and $\text{Sm}^{150}$ spectra were recorded with the gamma ray counters at about 6 cm from the target to obtain higher absolute
efficiencies, and this proximity reduces the angular correlation so that it may be neglected with only small error.

One case has been calculated in detail for a triple correlation. The 2+ state in the beta band of Sm\textsuperscript{152} has been assumed to be populated entirely by direct E2 excitation, for which the direct E2 particle coefficients may be used with the triple correlation of three gamma rays. The tables of Sharp et al.\textsuperscript{76} and of Biedenharn and Rose\textsuperscript{77} were used to calculate the angular correlation coefficients for de-excitations from the 2+ level to 2+ and 4+ levels in the ground state band. Details of the calculation are given in Appendix D.
B. Cross Section Calculations

The reduced matrix elements were obtained from the experimental yields and the energy dependence of the cross section in the following manner:

\[ Y = \left( \frac{I}{q} \right) (q \Delta x \rho \frac{N_{Av}}{A_2}) \sigma(E) \]

\[ = \frac{I \rho N_{Av}}{A_2} \sigma(E) \Delta x \]

(109)

\( \Delta x = \) target thickness
\( \rho = \) density of target material
\( A_2 = \) atomic number of target
\( I = \) number of incident ions
\( N_{Av} = \) Avogadro's number
\( q = \) cross-sectional area of the beam

For a thick target an integration must be carried out over \( x \), yielding

\[ Y = \frac{I \rho N_{Av}}{A_2} \int_0^x \sigma(E) dx = \frac{I \rho N_{Av}}{A_2} \int_0^E \frac{\sigma(E) dE}{dE/dx} \]

\[ = \frac{I \rho N_{Av}}{A_2} \int_0^E \frac{\sigma(E) dE}{|dE/dx|} \]

(105)
It is more convenient to express the yield in terms of transitions per microcoulomb of 6+ charge state $^{16}$ projectiles by setting

$$I = 1 \ \mu \text{Coulomb} = \frac{10^{-6}}{6 \times 1.67 \times 10^{-14}} = 1.04 \times 10^{12} \ \text{ions} \ (106)$$

Also, to convert $dE/dx$ from Mev/cm to Mev/mg/cm$^2$

$$\frac{dE}{dx} = \rho \times 10^3 \frac{dE}{ds} \ (107)$$

The yield per microcoulomb becomes

$$Y = \frac{6.26 \times 10^8}{A_2} \int \frac{\sigma(E)dE}{dE/ds} \ (108)$$

where $\sigma(E)$ is given in barns.

Differential cross section yields were obtained from the expression

$$Y(\theta) = \frac{6.26 \times 10^8}{A_2} \int d\sigma(\theta) \frac{dE}{d\Omega} \frac{dE}{ds} \Delta\Omega(\theta) \ (109)$$

and summed over $\Delta\Omega$ for the range of $\theta$ involved. The junction detector solid angle was divided into four annular rings, each being six degrees wide in $\theta$, and the calculations were performed at the center angle of each ring. The discrepancy between this and a calculation for two-degree wide rings was less than 0.5% for the 8+ level excitation in the $^{154}$Sm ground state band, which has the strongest angular dependence of the cross sections occurring here.
Calculations for direct excitations are simplified by the use of expressions and tables given in ABHMW.

\[ \sigma_{E_A} = C_{E_A} E^{\lambda-2} (E - \Delta E')^{\lambda-1} B(E_A) f_{E_A}(\xi) \]  
\text{(110)}

where

\[ \xi = \frac{Z_1 Z_2 A_1^{1/2} \Delta E'}{12.65 (E - \frac{1}{2} \Delta E')^{3/2}} \left[ 1 + \frac{5 (\Delta E')^2}{2 \lambda E} \right] + \ldots \]

\[ \Delta E' = (1 + \frac{A_1}{A_2}) \Delta E \]

\[ C_{E_1} = 0.02498 Z_1^2 A_1 \]  
\text{(111)}

\[ C_{E_2} = 4.82 \left(1 + \frac{A_1}{A_2}\right)^{-2} \frac{A_1}{Z_2^2} \]

\[ C_{E_3} = 9.298 \times 10^2 \left(1 + \frac{A_1}{A_2}\right)^{-4} \frac{A_1}{Z_1^2 Z_2^2} \]

The dE/ds values used in this experiment were obtained from the semi-empirical range curves of Hubbard\textsuperscript{78} and Northcliffe\textsuperscript{79} for oxygen ions in silver and gold. A linear extrapolation in nuclear charge was made between silver (Z = 47) and gold (Z = 79) to obtain the range of oxygen ions in samarium (Z = 62). This curve is reproduced to better than 3\% for 8-80 Mev oxygen ions by the second order polynomial.
\[ S = 0.000976 \ E^2 + 0.297 \ E + 1.5 \ \text{mg/cm}^2 \quad (112) \]

where \( E \) is in Mev, and is differentiated to obtain

\[ \frac{dE}{ds} = \frac{1}{0.002E + 0.297} \quad (113) \]

It is seen that \( dE/ds \) is not strongly dependent upon the energy.

As an example of the energy dependence of the yield calculation, the yield for excitation of the 0.811 Mev level in \( \text{Sm}^{152} \) for a loss of 1 Mev in the incident beam has been plotted as a function of beam energy in Fig. 61. The excitation mechanism is direct \( E2 \), as given by Eq. 110. The yield is observed to diminish much more rapidly than the \( f_{E2} \) function above (cf. Fig. 10) due to the energy factor. However, this is not true for \( E1 \) excitation since the dependence is of the order \( E^{2\lambda - 3} \). The principal contribution is in the first 10 or 15 Mev lost by the incident ion. The thick target integrations were carried out by means of a program written for an IBM 709 computer, which is listed in Appendix E.

The cross section calculations for double \( E2 \) excitation were made with the aid of tables computed by A. C. Douglas for excitation of an \( I = 0^+, 2^+, \) or \( 4^+ \) level via a lower-energy \( 2^+ \) level. The total cross section is

\[ \sigma^{(2)}(J) = \left( \frac{Z}{A} \right)^4 a^{-6} B(0\rightarrow 2) B(2\rightarrow J) F(\xi_1, \xi_2, J) \quad (114) \]
CALCULATED YIELD FOR E2 COULOMB EXCITATION OF THE 0.811 MeV LEVEL IN Sm$^{152}$ BY O$^{16}$ IONS.

\[
\frac{\text{YIELD/}\mu\text{C}}{B(E2, 0 \to 2)} \text{ FOR } \Delta E = 1\text{ MeV}
\]

\[\xi = 0.47\]

\[\xi = 0.69\]

\[\xi = 1.16\]
where the reduced matrix elements \( B(0\rightarrow 2) \) and \( B(2\rightarrow J) \) are in units of \( e^2 \, \text{cm}^4 \) and \( F(\xi_1, \xi_2, J) \) is tabulated by Douglas. In terms of cross section parameters

\[
\sigma^{(2)} = 4.52 \left(\frac{A_1}{Z_1}\right)^2 \frac{E(E - \Delta E')}{A_1} \frac{B(0\rightarrow 2) \, B(2\rightarrow J) \, F(\xi_1, \xi_2, J)}{[Z_1(1 + \frac{A_1}{A_2})]^{6.3}} \] (115)

in the same notation as Eq. 111. For the differential cross section \( dF(\xi_1, \xi_2, J_\theta) \) is substituted for \( F \).
C. Discussion of Errors

The errors involved in determining gamma ray intensities are largely due to the uncertainty of determining the area under the photopeak in a given spectrum. These errors depend upon the complexity of the spectrum and are included in the table of gamma ray intensities as standard deviations.

The absorber corrections are expected to be known to better than 3% for all but the first excited state de-excitation in Sm$^{152}$ and Sm$^{154}$ at 122 and 82 kev. These were checked from spectra taken with and without part of the absorber for the 122 kev gamma ray. For the 82 kev gamma ray these were checked from the relative intensities of 82 and 185 kev gamma rays in cascade in the T + 278 spectrum. The errors are estimated as 5% and 8% for the 122 and 82 kev gamma rays.

Uncertainties in the efficiency for detecting a gamma ray with the pulse in the photopeak reflect uncertainties in detector position as well as in ratios of pulses in the photopeak to total number of pulses and in total detection efficiencies. The positioning uncertainty leads to about a 7% uncertainty. Each total detection efficiency was normalized to the 9.2 cm position used for Sm$^{152}$ and Sm$^{154}$ spectra since this position was well known, thus giving internal consistency. The total photopeak efficiencies are believed to be known to an accuracy of ± 5%, based on measurements of Bell et al. and of Stelson and McGowan.
Uncertainties in accidental coincidence rates, analyzer counting losses, and beam integration have also been considered. In coincidence spectra the uncertainty in the fraction of the photopeak within the gate must be included. These additional uncertainties have been estimated for each spectrum and included in the total standard deviation given with the gamma ray intensities.

Gamma ray transition energies in these nuclei have been established to ±1 kev in other work, but new transitions found here are accurate to only ±1% from system non-linearity and gain shifts.

Errors in determining nuclear matrix elements are from uncertainties in gamma ray intensities and in the calculation of excitation yields. If the mechanism of excitation is known the calculation errors are from errors in dE/dx and in interpolation between tabulated values of cross section functions. Errors in dE/dx are estimated to be about ±3%, based on a comparison of semi-empirical range curves with range measurements for oxygen in nickel and the validity of the linear extrapolation in Z of material. Interpolation errors are estimated at about 0.5% for direct excitation calculations, 1% for multiple coulomb excitations, and 2% for double coulomb excitations.

Additional uncertainties are due to errors in parameters used in the theory, such as the q in multiple excitation theory. These are included in the tabulated calculations as standard deviations.
V. Discussion of Data

A. Samarium - 152

1. Decay Scheme

As will be evident from the discussion, this nucleus is the more interesting of the four studied and is discussed first. From earlier work levels have been assigned to the ground state rotational band, and to beta, gamma, and octupole vibrational bands (see Fig. 24). At least two levels in each band are populated in the coulomb excitation studies reported here. New gamma ray transitions have been observed in this work and have been assigned to higher members of the various bands. Of special interest are the transitions from the beta and gamma bands to the ground state band because they give information on wave function admixtures, as outlined in Chapter I.C. We proceed with the determination of the level scheme.

The ground state rotational band has well-known 2+ and 4+ states at 121.8 and 366.6 kev. The 2+ to 0+ and 4+ to 2+ E2 radiations are observed as intense transitions in the T + 122, T + 245, T + 340, and gamma-particle spectra. The 340 kev gamma ray appears in the T + 245 spectrum and has been assigned as the 6+ to 4+ transition from a third level in the ground state band at 707 kev. This assignment is further justified by the comparison of gamma ray intensities, taken from the gamma-particle spectrum, with the intensities predicted in the multiple
coulomb excitation theory (see Chap. I.D. and Table V).

A fit to the first two levels of the ground state band with

$$E = AJ(J + 1) + BJ^2(J + 1)^2$$

(116)

yields $A = 21.15$ kev and $B = -0.141$ kev, but these in turn predict an excitation of 639 kev for the 6+ level. Inclusion of a term $CJ^3(J+1)^3$ leads to

$$A = 21.39 \text{ kev}$$

$$B = -0.193 \text{ kev}$$

$$C = 0.0020 \text{ kev}$$

These parameters result in a prediction of 1288 kev for the 8+ level. However, this cannot be verified because the weak corresponding gamma ray which would be at 581 kev is masked by other transitions. There is evidence for a gamma transition at about 480 kev in gamma-particle spectra taken at higher bombarding energies, which might be assigned to the 8+ level. A term in $J^4(J+1)^4$ with a negative coefficient would be necessary to account for this energy.

The change in magnitude of the calculated coefficient $B$ with the inclusion of the 6+ level should be noted, being $-0.141$ kev when only two levels are known, and $-0.193$ kev with the third level included. The $B$ value is very sensitive to the energies of the 2+ and 4+ levels, changing by about 5% for a 1 kev energy difference. A 1 kev discrepancy in the 6+ level energy has only a 0.5% effect on $B$. Therefore it is important to have
an extensive knowledge of ground state band levels before comparing coefficients to predictions from the band mixing theory.

A comparison can be made between the ground state band energies to the predictions of the simple asymmetric rotor model. Adopting the usual convention, the ratio of energies of the 2+ levels in the ground state and gamma bands is used to determine the values of the asymmetry parameter $\gamma$. This ratio is 8.91 and from Fig. 4 yields $\gamma = 13.2^\circ$. With this $\gamma$ value one obtains 403 and 827 kev for the energies of 4+ and 6+ levels, instead of the measured values of 367 and 707 kev, respectively. This procedure may be reversed and a $\gamma$ value obtained from the ground state band 4+ and 2+ levels. The result is $\gamma = 22^\circ$, which does not agree very well with $13.2^\circ$. This simple version of the asymmetric rotor model does not give a satisfactory fit to the experimental energy levels. The more recent modifications of the model will be discussed later.

Four gamma ray transitions are observed which are assigned to the gamma band. These are the 1087 and 965 kev de-excitation from the 2+ level, and the 869 and 1114 de-excitation from the 3+ level. As is shown in Fig. 62, the multiple coulomb excitation process is not expected to populate the 3+ level strongly. The case exhibited there is for a larger $q$ value than obtained in these experiments. The parameter $q$ is related to the reduced transition probability for excitation of the first rotational level by (see Eq. 84 and 85):
MULTIPLE COULOMB EXCITATION OF THE GAMMA VIBRATIONAL BAND IN AN EVEN-EVEN NUCLEUS.

From Lütken and Winther, Kgl. Danske Videnskab. Selskab Mat.-Fys. Skr. 2, No.6 (1964)

$q = 3 \quad \chi^{(2)} = 0.4$

FIG. 62
The amount of 3+ level excitation relative to 2+ is about 10% in Fig. 62. A lower initial q value, and an integration over energy that would bring in contributions from q values that were lower yet, would reduce the relative amount of multiple processes that lead to excitation of the 3+ level to less than 10%.

The relative amount of 3+ state excitation obtained experimentally (see Table VI) is about 9%, which suggests that either or both of the 859 and 1114 kev lines are composite. The 869 kev line is possibly composite since the energy of the peak in T + 245 corresponds more closely to 850 kev.

There is some evidence for population of a 4+ level in the gamma band, based on a weak transition at about 1260 kev in the gamma-particle spectrum and a weak transition at about 1015 kev in the T + 245 spectrum. These gamma rays suggest that a 4+ level at about 1382 kev in the gamma band is populated to a small degree.

The 0+ and 2+ beta band levels at 685 and 811 kev are strongly populated in coulomb excitation, and the gamma rays at 563, 444, 689, and 811 kev are evident in direct and coincidence spectra. Determinations of the intensities of the 444 and 689 transitions must take into account gamma rays at about the same energy from other states. A correction for the weak contaminant at about 420 kev is made from the intensity obtained in the T + 340 spectrum, where there is no 444 kev gamma ray. A 670 kev
transition, or composite transition, appears in T + 245 and contributes to the intensity of 689 in T + 122.

Two possible interpretations for the 670 transition are the E2 de-excitation from a 4+ beta band level, and the E1 de-excitation from a 3- level, both of which would lie at 1037 kev. A branch to the 2+ level should occur at 915 kev in either case, and one does indeed see that the 965 kev gamma ray in T + 122 is unduly wide on the low energy side. Hansen and Nathan report a 3- level at 1070 kev from inelastic alpha particle scattering, but believe that the transition they observe is composite. It is possible that they excite the gamma band 2+ level to some degree.

The relative differential cross sections for excitation of 0+, 2+, and 4+ levels in the beta band are shown in Fig. 63 for 49 Mev incident ions. As discussed in Chapter I.D.3, the cross section is expressed as the product of $d\sigma_{E2}$, the direct excitation cross section for population of the 2+ level, and $|B_{J0}^2|^2$, the function that expresses the probability for redistribution of the amplitudes among the various levels. The differential cross section for the J level in the beta band is equal to (see Eq. 92 and 110)

$$|B_{J0}^2|^2 d\sigma_{E2} = |B_{J0}(q(\theta))|^2 C_{E2}(E-\Delta E')B(E2) d\sigma_{E2}(\theta,\xi)$$

(118)

Only the angle-dependent part $|B_{J0}(q(\theta))|^2 d\sigma_{E2}(\theta,\xi)$ has been plotted in Fig. 63.
MULTIPLE COULOMB EXCITATION of Sm$^{152}$ BETA BAND.
RELATIVE DIFFERENTIAL CROSS SECTIONS at 49 MeV.

$df_{E2} / |B_{j0}|^2$ vs $\theta$

$\xi = 0.416$
$q = 1.545$

$J = 2$

$df_{E2}$

$J = 0$

$J = 4$

FIG. 63

MULTIPLE COULOMB EXCITATION of Sm$^{152}$ OCTUPOLE BAND.
RELATIVE DIFFERENTIAL CROSS SECTIONS at 49 MeV.

$df_{E3} / |B_{j0}|^2$ vs $\theta$

$q = 1.545$
$\xi = 0.495$

$J = 3$

$J = 5$

$J = 1$

FIG. 64
The angular distributions of differential cross sections are useful for identifying the beta band transitions. It is seen that the 0+ and 4+ level excitations are much higher at large scattering angles, and one would expect that in the gamma-particle spectrum the gamma ray transitions from the 0+ level would be dominant. This is evident in Fig. 27, where the 563 kev gamma ray is much more prominent than the 2+ level de-excitations at 444, 689, and 811 kev. In the total excitation cross section the sum over all scattering angles yields a relatively higher amount of 2+ level excitation. Further, at lower oxygen ion energies, such as occur deeper in the target, the direct excitation is more probable than the multiple processes which populate the 0+ and 4+ levels. This is seen from the dependence of $|B_{J0}^2|^2$ upon $q$ (Fig. 13), noting from Eq. 83 that $q$ is proportional to the energy to the three-halves power. The direct excitation cross section function $df_{E2}$ at 49 Mev for the 2+ level alone has been drawn in Fig. 63 as a dotted line, showing the relative importance of higher order effects.

The 963 kev transition from the 1- level is not separable from the gamma band 965 kev gamma ray, but the 841 kev branch is in T + 122. The branching ratio has been measured by Grodzins as 0.72 for 963 to 841 and may be used to correctly determine the 965 kev gamma band intensity. As mentioned above, a 3- level may exist at 1037 kev. We refer to differential cross section dependences upon angle and energy to estimate the likelihood of its existence.
The relative differential cross sections for multiple coulomb excitation of the octupole band are shown in Fig. 64 for 49 Mev incident ions. Again the cross section is written as the product of the direct excitation cross section and a redistribution function. In this case the $3^-$ level is excited directly. The differential cross section is equal to

$$|B_{J0}^3|^2 \sigma_{E3} = |B_{J0}^3(q(\theta))|^2 C_{E3} E(E-\Delta E')^2 B(E3) \, df_{E3}(\theta, \xi)$$

(119)

Only the angle-dependent part, $|B_{J0}^3(q(\theta))|^2 \, df_{E3}(\theta, \xi)$, is plotted in Fig. 64. The $df_{E3}$ alone is shown as a dotted line. The parameter $q$ has been chosen the same as in the beta band for the sake of comparison. The actual value depends upon the position of the $3^-$ level, which is not certain.

It is seen in Fig. 64 that the multiple processes leading to $1^-$ and $5^-$ level excitation are more probable at large scattering angles, and would be more evident in gamma-particle spectra. In gamma-gamma spectra, where all scattering angles are included, the $3^-$ excitation will dominate. Also, the integration over energy for a thick target yields relatively more $3^-$ excitation since multiple processes are dependent upon $q$ (see Fig. 15) and $q$ is proportional to the three-halves power of the energy.

From the relative intensities of $3^-$ and $1^-$ excitation in Fig. 64 and from the above discussion, it seems possible that part of the 670 kev radiation is from a $3^-$ level at 1037 kev. The relative amount due to beta band excitation could theoreti-
cally be separated from the octupole band excitation by comparing gamma-gamma and gamma-particle spectra, but the intensities in the gamma-particle spectrum obtained here are very low and are not well resolved from adjacent lines.

From gamma-particle spectra taken at higher energies there is evidence for gamma rays at about 410 and 750 kev. Since the multiple processes are strongly energy dependent (see above discussion), these have been tentatively assigned as de-excitations from a 5- level at 1117 kev in the octupole band.

The rotational parameters for the vibrational bands may be determined from the level spacings so that moments of inertia might be compared. Calculating the A and B parameters from

\[ E_J - E_{J_0} = A[J(J+1) - J_0(J_0+1)] \]

\[ + B[J^2(J+1)^2 - J_0^2(J_0+1)^2] \]  \hspace{1cm} (120)

where \( J_0 \) is the spin of the first member of the band, we obtain for the gamma band

- \( A = 32.01 \) kev
- \( B = -0.399 \) kev

The beta band moment of inertia is closer to that of the ground state band, with

- \( A = 22.46 \) kev
- \( B = -0.243 \) kev
The octupole band has a very large moment of inertia with $A = 7.4$ kev, but this depends upon the 1037 kev level position, which is not well known.

2. Reduced Transition Probabilities

In order to extract reduced transition probabilities from the data it has been necessary to rely upon the calculations of the multiple coulomb excitation theory. The excitation probabilities for states within the rotational bands are so large that first and second-order processes are insufficient to account for the yields obtained. However, transitions between rotational bands are, in general, much less probable and are treated in first order perturbation theory. A specific model of an axially symmetric nuclei with pure K bands is assumed initially in these calculations. Deviations from the model in terms of mixing of wave functions from different K bands have been discussed in Chapter I, and their effect upon excitation probabilities is discussed below with the data.

Excitations of the ground state band are calculated for rotations of the equilibrium shape of the nucleus and expressed in terms of a parameter $q$ (see Chapter I.4). These cross sections are evaluated in the $\mu = 0$ approximation, where contributions from non-zero quantum states are neglected. This approximation is well-satisfied at backward angles. The cross sections are evaluated for $\xi = 0$, but corrections for finite values of $\xi$ have been carried out in two ways. A first-order correction was calculated in the original paper by Alder and
Winther,⁵⁸ but the series expansion in 5 diverges and more direct inclusion of 5 is necessary. Finite 5 calculations have been made by Alder⁵⁹ by a matrix diagonalization procedure to determine excitation probabilities. Both calculations appropriate to Sm¹⁵² were made and compared to the data (see below).

The excitation of rotational bands based on vibrational modes of excitation is calculated in the multiple coulomb excitation theory by treating transitions within the band as excitations between rotational states and using the sudden approximation, and by treating the transitions between bands as perturbations and using the first-order direct excitation theory. The resulting cross section factors into two terms. One is the direct excitation cross section dσEₐ, and the other is the redistribution function |Bₐ(q(θ))|². These have been discussed above and are given in Fig. 62, 63, and 64 for the different bands. The sum of the redistribution function over all levels in the band is normalized to unity⁵⁸

\[ \sum J |Bₐ(J0(q(θ))|² = 1 \] (121)

and it is seen that the total excitation cross section for the band is equal to the direct excitation cross section. Thus one may extract the reduced transition probability for the direct excitation by adding the intensity of each level population.

The finite 5 corrections have not been made for transitions within rotational bands. The cross sections will thus over-
estimate multiple processes. Further, the \( \mu = 0 \) approximation is used for these intra-band transitions. In the beta and octupole vibrational bands the errors are small, but in the gamma band the 3+ level excitation occurs only for non-zero values. Individual level populations have been calculated only for the beta band since the data is more precise for beta band transitions. We proceed with the ground state band calculations.

In the multiple coulomb excitation theory for levels of a ground state rotational band the relative excitation probabilities are calculated from functions of the parameter \( q \). Since \( q \) does not factor out of the cross section functions and cannot be determined directly from the experimental yield, calculations have been made with \( B(E2) \) values from the inelastic proton scattering data of Elbek et al.\(^{32}\)

The comparison of experimental gamma ray intensities from gamma-particle spectra to calculations in the multiple coulomb excitation theory is made in Table V. The number of gamma rays from a level is corrected for efficiencies and given in Column 5 as transitions per microcoulomb. To obtain the total population of a particular level by the excitation mechanism alone, the number of transitions is corrected for cascades from de-excitation of higher states, and for sum coincidences. These are listed in Column 6. Columns 7, 9, and 11 list the calculated yield \( Y \) for three different cases and columns 8, 10, and 12 give the ratio \( R \) of calculated to experimental yield. Case number one is a calculation made with the first-order correction in \( \xi \) given in
### TABLE V  EXCITATION of GROUND STATE ROTATIONAL BANDS

<table>
<thead>
<tr>
<th>J</th>
<th>$E_Y$ [keV]</th>
<th>Total Counts</th>
<th>$\alpha$</th>
<th>$\gamma A$</th>
<th>Transitions per $\mu$C</th>
<th>Corrected $^\dagger$</th>
<th>$Y_1$</th>
<th>$R_1$</th>
<th>$Y_2$</th>
<th>$R_2$</th>
<th>$Y_3$</th>
<th>$R_3$</th>
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<tbody>
<tr>
<td><strong>Sm$^{152}$</strong></td>
<td></td>
<td></td>
<td></td>
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<td></td>
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<tr>
<td>2</td>
<td>122</td>
<td>4.64±0.91 (3)</td>
<td>1.14</td>
<td>0.000253±0.000025</td>
<td>3.83±0.89 (5)</td>
<td>3.37±0.78 (5)</td>
<td>3.01±0.21 (5)</td>
<td>0.89±0.19</td>
<td>0.90±0.19</td>
<td>2.86±0.20 (5)</td>
<td>0.85±0.18</td>
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<td>4</td>
<td>245</td>
<td>3.66±0.18 (4)</td>
<td>0.11</td>
<td>0.00959±0.00087</td>
<td>3.94±0.45 (4)</td>
<td>3.74±0.43 (4)</td>
<td>4.81±0.30 (4)</td>
<td>1.28±0.15</td>
<td>1.05±0.13</td>
<td>3.32±0.21 (4)</td>
<td>0.89±0.11</td>
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<td>6</td>
<td>340</td>
<td>2.45±0.25 (3)</td>
<td>0.039</td>
<td>0.0120±0.0011</td>
<td>1.96±0.28 (3)</td>
<td>1.95±0.28 (3)</td>
<td>3.23±0.22 (3)</td>
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<td>0.93±0.14</td>
<td>1.38±0.08</td>
<td>0.71±0.11</td>
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</tr>
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<td>6.10±0.18 (4)</td>
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<td>3.71±0.48 (5)</td>
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<td>3.32±0.24 (5)</td>
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<td>4</td>
<td>185</td>
<td>8.35±0.25 (4)</td>
<td>0.261</td>
<td>0.0255±0.0023</td>
<td>6.30±0.70 (4)</td>
<td>6.30±0.70 (4)</td>
<td>7.98±0.52 (4)</td>
<td>1.36±0.17</td>
<td>1.24±0.16</td>
<td>6.20±0.40 (4)</td>
<td>1.05±0.13</td>
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<tr>
<td>6</td>
<td>278</td>
<td>5.59±0.22 (3)</td>
<td>0.073</td>
<td>0.0227±0.0021</td>
<td>4.02±0.44 (3)</td>
<td>3.64±0.40 (3)</td>
<td>7.59±0.43 (3)</td>
<td>2.08±0.26</td>
<td>1.47±0.18</td>
<td>4.11±0.22 (3)</td>
<td>1.13±0.14</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>360</td>
<td>2.01±0.40 (2)</td>
<td>0.033</td>
<td>0.0193±0.0018</td>
<td>1.63±0.37 (2)</td>
<td>1.47±0.33 (2)</td>
<td>3.94±0.17</td>
<td>2.68±0.61</td>
<td>1.33±0.30</td>
<td>1.41±0.06</td>
<td>0.96±0.22</td>
<td></td>
</tr>
</tbody>
</table>

Number in parentheses is power of 10 for multiplying factor.

$^\dagger$ Corrected for cascades from higher states and for sum coincidences.
<table>
<thead>
<tr>
<th>$E_y$ [keV]</th>
<th>Spectrum</th>
<th>Counts</th>
<th>$\alpha_1$</th>
<th>$\eta_1 \alpha_1$</th>
<th>$\alpha_2$</th>
<th>$\eta_2 \alpha_2$</th>
<th>$\mu C$</th>
<th>Transitions per $\mu C$</th>
<th>Corrected for angular correlation</th>
</tr>
</thead>
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<tr>
<td><strong>Gamma Band Transitions</strong></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>1087</td>
<td>Direct</td>
<td>9.3±2.3(3)</td>
<td>0.002</td>
<td>0.0060 ± 0.0005</td>
<td></td>
<td></td>
<td>19.5</td>
<td></td>
<td>8.0±2.5(4)</td>
</tr>
<tr>
<td>965</td>
<td>Direct</td>
<td>1.71±0.43(4)</td>
<td>0.003</td>
<td>0.0068 ± 0.0006</td>
<td></td>
<td></td>
<td>19.5</td>
<td></td>
<td>1.29±0.40(5)*</td>
</tr>
<tr>
<td>965</td>
<td>T+122</td>
<td>2.20±0.33(3)</td>
<td>0.003</td>
<td>0.0068 ± 0.0006</td>
<td>1.14</td>
<td>0.0087 ± 0.0008</td>
<td>714</td>
<td></td>
<td>1.05±0.25(5)*</td>
</tr>
<tr>
<td>1114</td>
<td>T+122</td>
<td>1.5±0.6 (2)</td>
<td>0.002</td>
<td>0.0059 ± 0.0005</td>
<td>1.14</td>
<td>0.0087 ± 0.0008</td>
<td>714</td>
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<td>8.77±4.16(3)</td>
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<tr>
<td>869</td>
<td>T+245</td>
<td>6.8±1.0 (2)</td>
<td>0.003</td>
<td>0.0074 ± 0.0007</td>
<td>0.11</td>
<td>0.0166 ± 0.0016</td>
<td>714</td>
<td></td>
<td>8.58±1.72(3)</td>
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<tr>
<td><strong>Beta Band Transitions</strong></td>
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<tr>
<td>444</td>
<td>Direct</td>
<td>8.1±3.3 (3)</td>
<td>0.017</td>
<td>0.0114 ± 0.0010</td>
<td></td>
<td></td>
<td>19.5</td>
<td></td>
<td>3.70±1.51(4)</td>
</tr>
<tr>
<td>T+122</td>
<td>7.1±2.1 (3)</td>
<td>0.017</td>
<td>0.0114 ± 0.0010</td>
<td>1.14</td>
<td>0.0087 ± 0.0008</td>
<td>714</td>
<td></td>
<td>2.18±0.75(4)</td>
<td></td>
</tr>
<tr>
<td>T+245</td>
<td>2.18±0.22(3)</td>
<td>0.017</td>
<td>0.0114 ± 0.0010</td>
<td>0.11</td>
<td>0.0166 ± 0.0016</td>
<td>714</td>
<td></td>
<td>1.82±0.30(4)</td>
<td></td>
</tr>
<tr>
<td>T+444</td>
<td>1.41±0.42(3)</td>
<td>0.11</td>
<td>0.0096 ± 0.0009</td>
<td>0.017</td>
<td>0.0114 ± 0.0011</td>
<td>714</td>
<td></td>
<td>1.98±0.65(4)</td>
<td></td>
</tr>
<tr>
<td>$\gamma-p$</td>
<td>1.4 ± 0.6 (2)</td>
<td>0.11</td>
<td>0.0114 ± 0.0010</td>
<td></td>
<td></td>
<td>107.7</td>
<td>2.0 ± 0.8 (2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>563</td>
<td>T+122</td>
<td>1.77±0.35(3)</td>
<td>0.008</td>
<td>0.0103 ± 0.0009</td>
<td>1.14</td>
<td>0.0087 ± 0.0008</td>
<td>714</td>
<td></td>
<td>5.96±1.51(4)</td>
</tr>
<tr>
<td>$\gamma-p$</td>
<td>1.61±0.16(3)</td>
<td>0.008</td>
<td>0.0103 ± 0.0009</td>
<td></td>
<td></td>
<td>107.7</td>
<td>1.46±0.21(3)</td>
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<tr>
<td>670</td>
<td>T+245</td>
<td>1.96±0.20(3)</td>
<td>0.006</td>
<td>0.0093 ± 0.0008</td>
<td>0.11</td>
<td>0.0166 ± 0.0006</td>
<td>714</td>
<td></td>
<td>1.99±0.33(4)</td>
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<tr>
<td>689</td>
<td>Direct</td>
<td>7.43±2.1 (3)</td>
<td>0.006</td>
<td>0.0090 ± 0.0009</td>
<td></td>
<td></td>
<td>19.5</td>
<td></td>
<td>5.76±1.79(4)</td>
</tr>
<tr>
<td>T+122</td>
<td>1.25±0.15(3)</td>
<td>0.006</td>
<td>0.0090 ± 0.0009</td>
<td>1.14</td>
<td>0.0087 ± 0.0008</td>
<td>714</td>
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<td>4.80±0.92(4)</td>
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<tr>
<td>811</td>
<td>Direct</td>
<td>5.7 ± 1.5 (3)</td>
<td>0.004</td>
<td>0.0079 ± 0.0079</td>
<td></td>
<td></td>
<td>19.5</td>
<td></td>
<td>3.72±1.15(4)</td>
</tr>
<tr>
<td>915</td>
<td>T+122</td>
<td>4.9±2.0 (2)</td>
<td>0.003</td>
<td>0.0071 ± 0.0006</td>
<td>1.14</td>
<td>0.0087 ± 0.0008</td>
<td>714</td>
<td></td>
<td>4.28±1.32(4)</td>
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<tr>
<td><strong>Octupole Band Transitions</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>841</td>
<td>T+122</td>
<td>2.1±0.8 (2)</td>
<td>0.003</td>
<td>0.0077 ± 0.0007</td>
<td>1.14</td>
<td>0.0087 ± 0.0008</td>
<td>714</td>
<td></td>
<td>9.42±4.08(3)</td>
</tr>
<tr>
<td>670</td>
<td>See above</td>
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<td></td>
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<td></td>
<td></td>
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<tr>
<td>915</td>
<td>See above</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
</tbody>
</table>

Number in parentheses is power of 10 for multiplying factor. * Corrected for intensity of 963 keV radiation.
the original paper of Alder and Winther. Number two utilizes more recent calculations which include directly by a matrix diagonalization procedure. Number three uses a 10% lower \( B(E2) \) value to show the effect of an error in \( B(E2) \). The errors given with the calculated yields are from interpolation, \( dE/dx \), and target thickness uncertainties.

The best agreement between theoretical and experimental yields is for calculation number two. The ratios \( R \) are equal to unity within experimental error, indicating that the finite correction is the most valid. Multiple coulomb excitation cross sections have been checked by Graetzer and Bernstein for \( \ell = 0 \) calculations. Qualitative agreement was found with the experimental data on 2+ and 4+ level excitations, the experimental yields being smaller than the theoretical yields by about 30%. The recent finite \( \ell \) calculation employed here would give very good agreement.

The calculations above are based on wave functions for pure rotations of an axially symmetric nucleus. As has been discussed in Chapter 1.2., rotation-vibration interactions modify the reduced transition probabilities. It is noted from Table III that these interactions lead to terms in \( z/a^2 \). From the analysis discussed in the next section \( z/a^2 \) is of order \(-0.07/28 = -0.003\). Mixing effects are not expected to be large for transitions between levels in the same rotational band.

As discussed above, the total excitation cross section for a vibrational band is equal to the direct excitation cross section alone. In the gamma band the 2+ level at 1087 kev is
directly excited, and the $B(E2, 0 \rightarrow 2')$ obtained from the total intensity of the band is $0.109 \pm 0.020 \times 10^{-48}$ cm$^4$, or $4.56 \pm 0.84$ s.p. units. The total excitation of the band has been obtained by adding the intensities of 1087, 965, 869, and 1114 kev gamma rays (see Table VI). The 1087 intensity is obtained from the direct spectrum. The 965 kev gamma ray is observed in both the direct and T + 122 spectra, but the intensity in T + 122 is more accurately known and was used in the analysis. The correction for 963 kev radiation is obtained from the intensity of the 841 kev line in T + 122 and the branching ratio measured by Grodzin. The correction is only 7%. The 1087 and 965 kev intensities have been corrected for angular correlations on the basis of direct excitation of the 2+ 1087 level. The 869 and 1114 kev intensities are obtained from T + 245 and T + 122 spectra, respectively.

The experimental value for population of the 0+ level was obtained from the 563 kev gamma ray, with a correction for E0 electron transitions from measurements by Marklund et al. The intensity of the 2+ level de-excitations at 444, 689, and 811 kev yields the total 2+ level excitation. A correction for E0 transitions between 2+ levels in beta and ground state bands is included, using the data of Nathan and Hultberg. In both cases the E0 transitions are corrections of about 1%. The population of the 4+ level is obtained by assuming that the 670 and 915 kev gamma rays are entirely beta band de-excitations and adding their intensities. The error in making this assumption is taken as half of the 670 and 915 intensities.
The individual gamma ray intensities were obtained from the following spectra. The 563 kev gamma ray is less affected by 511 kev radiation in T + 122 than in the direct spectrum. The amount of 444 kev radiation is most accurately determined from the T + 245 spectrum. There is some target contamination radiation at about that energy, but the correction for this from the T + 340 spectrum lowers the intensity of the 444 radiation in T + 245 by only 5%. The uncertainty in determining the amount of background radiation in T + 340 is only a small error in the intensity of the 444 kev transition in T + 245.

The intensities of the 444 kev radiation from T + 122 and the direct spectrum give a check on the above value. The amount in the direct spectrum is much larger, but about half of it may be assigned to background radiation on the basis of the amount found in Sm$^{154}$, which has no gamma ray transition at about 444 kev. A good check on 444 kev intensity is obtained from the amount of 245 kev radiation in T + 444. Only coincidences with the 444 kev radiation and compton scattered radiation from gamma rays coincident with the 245 kev radiation of higher energy than the 444 kev radiation yield 245 kev gamma rays. The contribution of higher energy transitions is estimated at only 3%, which is much less than the uncertainty in determining the area.

The best determination of the intensity of 689 kev radiation is obtained by subtracting the amount of 670 kev radiation in T + 245 from the 689 kev radiation in T + 122. The 811 kev gamma ray appears only in the direct spectrum, and has a large uncertainty due to its low intensity above background. The
670 and 915 kev radiations are obtained from T + 245 and T + 122, respectively. The 915 kev transition intensity is very uncertain since it is not resolved from the 965 kev gamma ray and was obtained by fitting photopeak shapes to the data.

In the gamma-particle spectra the 0+ level population is easily obtained from the 563 kev intensity, but the 2+ level population is very uncertain. From the 2+ level only the 444 kev gamma ray is resolved, and it is of very low intensity. The total excitation of the 2+ level was obtained from the 444 kev intensity, and the ratio of 444 to total 2+ level intensity in the gamma-gamma spectra.

The excitation yields for the 0+, 2+, and 4+ levels individually have been calculated in the multiple coulomb excitation theory by integrating Eq. 92 over energy and angle. Each level population may be used to determine $B(E2, 0 \rightarrow 2')$ since it factors out of the cross section. The results are presented in Table VII. Column 2 lists the experimental level population (as discussed above), column 3 the calculated yield in terms of $B(E2, 0 \rightarrow 2')$, and column 4 the value of $B(E2, 0 \rightarrow 2')$, in terms of $B(E2, 0 \rightarrow 2')$, in units of $e^2 \times 10^{-48} \text{cm}^4$.

Mixing effects between beta and ground state bands have been considered and are included in the following way. Lutken and Winther have evaluated the effects of mixing due to perturbing terms in the collective Hamiltonian with the same J dependence considered here. They include these terms by modifying the interaction Hamiltonian rather than the wave functions, and
### TABLE VII  EXCITATION of BETA BAND in Sm$^{152}$

<table>
<thead>
<tr>
<th>J</th>
<th>Experimental Yield [Transitions/μC]</th>
<th>Multiple Coulomb Excitation Calculated Yield</th>
<th>B(E2,0→2')</th>
<th>Mixing Correction</th>
<th>$p^2$</th>
<th>Second-Order Perturbation Theory Calculated Yield</th>
<th>$p^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>6.03 ± 1.51 (4)</td>
<td>6.52 B(E2,0→2')(5)</td>
<td>0.093 ± 0.023</td>
<td>1.02</td>
<td>0.094 ± 0.023</td>
<td>1.43 $(\frac{B^2}{\alpha^2})(I-3Z)^2$ (6)</td>
<td>0.148 ± 0.037</td>
</tr>
<tr>
<td>2</td>
<td>10.8 ± 1.6 (4)</td>
<td>3.20 B(E2,0→2')(6)</td>
<td>0.0338 ± 0.005</td>
<td>$\frac{1}{(1+3Z)^2}$</td>
<td>0.0518 ± 0.007</td>
<td></td>
<td></td>
</tr>
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</tr>
<tr>
<td></td>
<td><strong>Total Cross Section (Gamma-Gamma Spectra)</strong></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>1.48 ± 0.21 (3)</td>
<td>2.19 B(E2,0→2')(4)</td>
<td>0.068 ± 0.016</td>
<td>1.04</td>
<td>0.070 ± 0.016</td>
<td>7.75 $(\frac{B^2}{\alpha^2})(I-3Z)^2$ (4)</td>
<td>0.067 ± 0.013</td>
</tr>
<tr>
<td>2</td>
<td>1.12 ± 0.56 (3)</td>
<td>0.90 B(E2,0→2')(4)</td>
<td>0.124 ± 0.031</td>
<td>(1) 2.00</td>
<td>0.248 ± 0.124</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(2) 1.59</td>
<td>0.197 ± 0.098</td>
<td></td>
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</tr>
</tbody>
</table>

Number in parentheses is power of 10 for multiplying factor.
obtain correction terms to the excitation amplitude. Unfortunately, these have been evaluated only in the approximation of zero magnetic quantum numbers, which is valid for $J_f = 0$, but not for $J_f = 2$ or 4. The correction to the excitation amplitude is

$$
\delta b = \frac{R_{20}(\theta, \xi)}{R_{20}(\theta, 0)} \epsilon_\beta[J_f(J_f+1) - J_i(J_i+1)]B_{J_f0}^i(q(\theta)) \\
- 6\left(\frac{4}{3} q\right)^2 \epsilon_\beta g_{00}(\theta, 0, \xi) \sum J (2J+1)^{-1/2} (2200|J0) \\
(22-11|J0) B_{J_f0}^J(q(\theta)) \tag{122}
$$

For the beta band in an even-even nucleus. The $R_{20}$ are functions of orbital integrals and $g_{00}$ is a second-order orbital integral. The other parameters have been defined previously.

The mixing corrections have been evaluated using the parameter $z_\beta = -0.064$ obtained in the next section, and are listed in column 6 of Table VIII. For excitation of the $0^+$ state, only the second term in Eq. 122 contributes. This term raises the calculated yield by 4% in the gamma-particle spectra, and by 2% in the total cross section. For the $J_f = 2$ calculation the second term is neglected. The first term is equivalent to a renormalization of $\chi$ by $\delta \chi = \frac{9}{5} q \epsilon_\beta$, which can be shown to be equal to $3z \chi$. Therefore $\chi$ becomes $\chi(1+3z)$. Since $\chi$ is proportional to the square root of $B(E2, 0 \rightarrow 2')$, this corresponds exactly to the $(1 + 3z)^2$ obtained in the formalism presented here (see Table III). In the gamma-particle spectrum the $u = 0$ approximation should be relatively valid for $J_f = 2$. The effect
of mixing on the 2+ level excitation has been considered in two cases, where the unmixed excitation amplitude is (1) calculated for \( \mu = 0 \), and (2) calculated for all \( \mu \) values. The size of the differences in the two corrections in column 6 indicates the degree to which the \( \mu = 0 \) approximation is valid. It is noted that mixing corrections are quite large for excitation of the 2+ state, but are very small for excitation of the 0+ state.

The excitation probabilities corrected for mixing are listed in column 6 in terms of \( P^2 \). This is the value of the reduced transition probability for case of no mixing (\( z = 0 \)), as is seen in Table III.

\[
B(E2, 0 \rightarrow 2') = P^2 (1 + 3z)^2
\]  

(123)

The most accurate experimental yields are for the 0+ and 2+ level excitations in the total cross section, and the 0+ level in the differential cross section. The mixing corrections are quite large for the 2+ level excitation, and may be only approximate. Thus the best value for \( P^2 \) is taken from the 0+ level excitation.

The calculation of the 0+ level excitation involves the approximation of \( \tilde{z} = 0 \) for transitions within the band. However, finite \( \tilde{z} \) values are not expected to have large effects on a single transition within a band. This can be seen in a qualitative way from a comparison of calculations 1 and 2 in Table V, where only the higher-order processes depend greatly upon \( \tilde{z} \). The 0+ level can be populated by a combination of only one rotational
First and Second Order Processes for Excitation of the Beta Band

Fig. 65
transition and one transition between bands, as indicated in Fig. 65. At lower energies $\xi$ is larger and the $0^+$ excitation cross section may be more strongly affected. The data in Table V is from a gamma-particle spectrum, where the lowest energy is 45.5 Mev, and $\xi \approx 0.06$. The total cross section data includes lower energies, and the calculated yield for $\xi = 0$ is a somewhat worse approximation than in the gamma-particle spectrum. For these reasons, and because the mixing effects are smallest, the best value of $P^2$ is from the $0^+$ level excitation in the gamma-particle spectrum, and is equal to $P^2 = 0.070 \pm 0.016 \times 10^{-48} \text{ cm}^4$.

Although the rotational transition probabilities are too large to be treated by perturbation theory, the second-order perturbation calculations are useful for understanding the lack of dependence on mixing for the $0^+$ level excitation. The $0^+$ level is populated by two second-order processes, by the excitation routes $0$ to $2$ to $0'$ and $0$ to $2'$ to $0'$. In the multiple excitation calculation, transitions within rotational bands are treated in the sudden approximation with $\xi = 0$ (degenerate in energy), and transitions between bands are obtained with finite $\xi$ (see Fig. 65). Thus the two excitation modes have the same energy and spin dependence and are equally probable. The excitation amplitudes are proportional to the square roots of $B(0 \rightarrow 2)$, $B(2 \rightarrow 0')$ and $B(0 \rightarrow 2') B(2' \rightarrow 0')$, which, from Table III, are $QP(1 + 3z)$ and $QP(1 - 3z)$. These amplitudes are the same sign, (from Eq. 48), and in the sum the $z$ term cancels. Thus mixing effects are absent.
These B(E2) values and their mixing dependence have been used to calculate the 0+ level yield by a double excitation process, and to determine P^2. However, the double E2 excitation of the 0+ level does not include the excitation sequence 0+ to 2'+ to 0+ because cross section functions have not been evaluated for intermediate levels of higher energy than the final level. If this process is about the same magnitude as the other route, than the P^2 value would be in good agreement with the P^2 calculated from the multiple coulomb excitation theory. The 0+ level excitation in the gamma-particle has a lower value for P^2, but this is probably due to over-estimates in calculated yield since the excitation probability at backward angles is too large for perturbation theory to be valid.

The excitation of the 1- level can be obtained from the intensity of 841 kev radiation in T + 122. Since the 963 kev branch is unresolved from the 965 kev gamma band transition, the branching ratio obtained by Grodzins for the relative amount of 963 kev gamma ray was used to obtain the total intensity. The B(El) calculated from a direct excitation of the 1- level is B(El) = 6.6 ± 3.0 e^2 x 10^-28 cm^2, or 1.2 x 10^-2 s.p. units. This may be compared with a calculation by Lipas, who obtains an excitation B(El) = 4.8 x 10^-4 s.p. units by treating the collective El level as a quadrupole-octupole interaction resulting from oscillations in the nuclear charge density. The experimental result obtained here is not very close to Lipas' value, possibly suggesting that the concept of a rotational band based on an octupole vibration is more valid.
The excitation of a 3- level is expected to be greater than
the 1- level excitation on the basis of the multiple excitation
theory (see Fig. 64). The 670 and 915 kev gamma rays are
possibly de-excitations from a 3- level at 1037 kev, but may also
be assigned to de-excitations from the 4+ level in the beta band.
If these two gamma rays are octupole band transitions, then

\[
\frac{B(E1, 3- \rightarrow 4+)}{B(E1, 3- \rightarrow 2+)} = \frac{1.99 \times 10^4}{2.38 \times 10^4} \left(\frac{915}{670}\right)^3 = 2.12 \pm 0.98 \quad (124)
\]

The Alaga prediction is 1.33, which is within the experimental
error. If these two gamma rays are beta band transitions, then

\[
\frac{B(E2, 4' \rightarrow 4)}{B(E2, 4' \rightarrow 2)} = \frac{1.99 \times 10^4}{2.38 \times 10^4} \left(\frac{915}{670}\right)^5 = 3.97 \pm 1.82 \quad (125)
\]

With a mixing parameter \( z = -0.064 \), the prediction is 3.60, which
is well within experimental error.

Thus either assignment is possible. The Sm\(^{152}\) decay scheme
is given in Fig. 66. We can only obtain a crude estimate of the
amount of 3- excitation. Using half of the 670 and 915 intensity
and adding the 1- level excitation, we obtain an excitation

\[ B(E3) = 0.142 \pm 0.044 \ e^2 \times 10^{-72} \ \text{cm}^6 \]

The error includes half
of the 670 and 915 kev intensities as an additional uncertainty.

3. Rotation - Vibration Interactions

As outlined in Chapter I.C., the ratios of reduced transition
probabilities for transitions from a vibrational band level to
different levels of the ground state band can be related to the
FIG. 66

\( \text{Sm}^{152} \)
admixtures of ground state and vibrational band wave functions in terms of a parameter $z$. The branching ratios, the $B(E2)$ ratios determined from them, and the $z$ values obtained are listed in Table VIII for the gamma and beta band transitions. The primed numbers denote the vibrational band states.

Two gamma band ratios are obtained from the data, from the $2^+$ and $3^+$ level de-excitations. The intensity determinations for the gamma ray transitions have been discussed in the previous section. The errors assigned to the ratios are less than for absolute intensities since only relative values are involved. It is noted from Fig. 7 and 8 that $z$ has two possible values, of different sign, and the correct one is chosen by requiring the same sign and magnitude in both cases. Unfortunately, the experimental errors for gamma band transition in this experiment are quite large in the one case and it is difficult to choose a common value of $z$. For this reason, the data of Nathan$^8_{3}$ has been included in the table. The more accurate value is from the $2^+$ level de-excitation and will be used in the analysis.

The beta band transitions have not been measured previous to this experiment. The beta band levels are not populated to any great extent in radioactive nuclide decay, and branching ratio measurements have not been possible. Thus this data represents the first direct test of beta band mixing.

Two independent branching ratios may be obtained from the beta band $2^+$ level de-excitations. This level decays to the $0^+$, $2^+$, and $4^+$ ground state band levels, by gamma rays at 811,
Table VIII

\( ^{152}\text{Sm} \) Branching Ratios

<table>
<thead>
<tr>
<th>Transitions</th>
<th>Branching Ratio</th>
<th>Energy</th>
<th>( B(E2) ) Factor</th>
<th>( z )</th>
<th>+Error</th>
<th>-Error</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Gamma Vibrational Band</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \frac{2^+}{2^+} \rightarrow 0 )</td>
<td>1.03±0.35</td>
<td>1.820</td>
<td>1.87±0.64</td>
<td>-0.045</td>
<td>-0.098</td>
<td>+0.026</td>
</tr>
<tr>
<td>( \frac{3^<em>}{3^</em>} \rightarrow 2 )</td>
<td>0.98±0.63</td>
<td>3.46</td>
<td>3.48±2.23</td>
<td>-0.218</td>
<td>-0.351</td>
<td>-0.100</td>
</tr>
<tr>
<td>( \frac{2^+}{2^+} \rightarrow 0 )</td>
<td>2.27±0.07</td>
<td>3.05</td>
<td>0.080</td>
<td>3.15</td>
<td>2.94</td>
<td></td>
</tr>
<tr>
<td>( \frac{3^<em>}{3^</em>} \rightarrow 2 )</td>
<td>0.64±0.24</td>
<td>0.479</td>
<td>-0.037</td>
<td>0.548</td>
<td>-0.0</td>
<td></td>
</tr>
<tr>
<td><strong>Beta Vibrational Band</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \frac{2^+}{2^+} \rightarrow 4 )</td>
<td>0.423±0.075</td>
<td>8.96</td>
<td>3.79±0.67</td>
<td>-0.064</td>
<td>-0.082</td>
<td>-0.045</td>
</tr>
<tr>
<td>( \frac{2^+}{2^+} \rightarrow 2 )</td>
<td>1.04±0.35</td>
<td>2.26</td>
<td>2.36±0.80</td>
<td>-0.074</td>
<td>-0.108</td>
<td>-0.014</td>
</tr>
</tbody>
</table>

*Data from O. Nathan, Nucl. Phys. 19, 148 (1960).*
689, and 444 kev. The best determination of intensities for these three transitions was discussed in the previous section.

The 444 to 689 kev ratio is the better known of the two independent ratios. However, the 689 to 811 ratio does serve as a check on the 444 to 689 ratio, and requires that \( z_\beta \) be negative. This ratio is sufficiently large that no positive value of \( z \) can be obtained (see Fig. 8). The \( z_\beta \) value from the 444 to 689 ratio is used in the following analysis. This value is less than obtained in earlier work, but at that time the contributions of contamination radiation at 420 kev were less well understood.

The two \( z \) values obtained are \( z_\gamma = -0.080 \pm 0.006 \) and \( z_\beta = -0.064 \pm 0.018 \). From Eq. 35 and 40 the contributions to the \( J^2(J+1)^2 \) term are

\[
B_\gamma = -\epsilon_\gamma^2 (\hbar \omega_\gamma) = \frac{-z_\gamma^2}{24\alpha_\gamma} \hbar \omega_\gamma = -\frac{0.290}{\alpha_\gamma^2} \tag{126}
\]

and

\[
B_\beta = -\epsilon_\beta^2 (\hbar \omega_\beta) = \frac{-z_\beta^2}{4\alpha_\beta} (\hbar \omega_\beta) = -\frac{0.702}{\alpha_\beta^2} \tag{127}
\]

The evaluation of \( B_\gamma \) and \( B_\beta \) requires the values of \( \alpha_\gamma^2 \) and \( \alpha_\beta^2 \).

Without any further experimental information the hydrodynamical model may be used to calculate \( \alpha_\gamma^2 \) and \( \alpha_\beta^2 \) (see Eq. 60). This method has been used in mixing calculations by Burde,
Rakavy, and Rakavy\textsuperscript{84} to account for deviations of ground state band lifetimes in \textit{Gd}\textsuperscript{154}, and in the earlier data from this experiment.\textsuperscript{3} This approach yields

\begin{equation}
\alpha_\gamma^2 = \frac{\hbar \omega_\gamma}{\hbar \omega_0} = 8.93
\end{equation}

(128)

\begin{equation}
\alpha_\beta^2 = 2 \frac{\hbar \omega_\beta}{\hbar \omega_0} = 11.2
\end{equation}

(129)

With these values

\begin{equation}
B_\gamma = -0.0325 \text{ kev}
\end{equation}

\begin{equation}
B_\beta = -0.0627 \text{ kev}
\end{equation}

The sum is -.095, which is about half of the value obtained from the energies of the ground state band levels, for which

\begin{equation}
B_{gs} = -0.193 \text{ kev}
\end{equation}

Thus the mixing analysis gives only a partial account of the deviations of the ground state band energies from the \textit{J}(\textit{J} + 1) dependence.

However, \(\alpha_\gamma^2\) and \(\alpha_\beta^2\) can be obtained directly from this experiment. The \(\alpha\) values are calculated from the ratio of \(B(E2)\) values for transitions within the ground state band to transitions between the bands. For gamma band excitation

\begin{equation}
\frac{B(E2, 0 \rightarrow 2)}{B(E2, 0 \rightarrow 2')} = \frac{Q_\gamma^2}{P_\gamma^2} \frac{(1 - z_\gamma/\alpha_\gamma^2)^2}{(1 + z_\gamma)^2}
\end{equation}

(130)
From the analysis in the previous section, \( B(E2, 0 \rightarrow 2') = 0.109 \pm 0.020 \, e^2 \times 10^{-48} \, \text{cm}^4 \). With \( B(E2, 0 \rightarrow 2) = 3.40 \, e^2 \times 10^{-48} \, \text{cm}^4 \) from the inelastic scattering data (see Table I),

\[
\alpha_\gamma^2 = \frac{3.40}{0.109} (1 + z_\gamma)^2 = 26.4 \pm 5.2
\]

(131)

From the analysis of beta band excitation we obtained \( P_\beta^2 = 0.070 \, e^2 \times 10^{-48} \, \text{cm}^4 \). Thus

\[
\frac{B(E2, 0 \rightarrow 2)}{P_\beta^2} = \frac{Q^2}{P_\beta^2} (1 - 3z_\beta / \alpha_\beta^2)^2 \approx \alpha_\beta^2
\]

(132)

and \( \alpha_\beta^2 \) is equal to

\[
\alpha_\beta^2 = \frac{3.40}{0.070} = 48.5 \pm 11.2
\]

(133)

Several other coulomb excitation experiments have excited levels in the beta band and obtained \( B(E2) \) values. Sheline\(^8^4\) reports \( B(E2, 0 \rightarrow 2') = 0.07 \pm 0.02 \, e^2 \times 10^{-48} \, \text{cm}^4 \) from inelastic proton scattering from the 2+ level, but this measurement is hampered by extremely low statistics. Yoshizawa et al.\(^8^5\) report a value of \( 0.08 \pm 0.02 \) for \( B(E2, 0 \rightarrow 2') \) from coulomb excitation with oxygen ions. These two numbers are in good agreement with the value found here from the multiple excitation theory and the gamma-particle spectrum results (see Table VII). Yoshizawa's
data is from a gamma-particle spectrum. Yoshizawa determines a gamma band excitation $B(E2, 0 \rightarrow 2') = 0.12 \pm 0.02 \times 10^{-48} \text{cm}^4$, also in good agreement with these data.

J. de Boer et al. report that $B(E2, 0' \rightarrow 2) = 1.9$ or $2.5 \times 10^{-48} \text{cm}^4$, from a gamma-particle spectrum. This value is equal to five times the excitation $B(E2)$ value, which is then $B(E2, 2 \rightarrow 0') = 0.44 \times 10^{-48} \text{cm}^4$. In this experiment we obtain $0.019 \pm 0.0038 \times 10^{-48}$. However, Eq. 21 in de Boer's paper seems to have a numerical error which makes his results too high by a factor of 18. Thus it seems that several experiments give good agreement with this data.

Returning to the calculation of the coefficient of $J^2(J+1)^2$ in Eq. 117, with the experimental values of $\alpha^2 = 26.4$ and $\alpha^2 = 48.5$,

$$B_\gamma = -\frac{0.290}{\alpha^2} = -\frac{0.290}{26.4} = -0.11$$

(134)

$$B_\beta = -\frac{0.702}{\alpha^2} = -\frac{0.702}{48.5} = -0.15$$

The sum is $-0.026$, which is only 14% of the actual coefficient. It is concluded that the mixing considerations do not give an adequate explanation for the deviations from $J(J+1)$ in the energy dependence of the ground state band.

It has been assumed in the calculations above that the gamma ray transitions are pure electric quadrupole. This is well-justified for the gamma band transitions, which have been
measured\textsuperscript{64,83} and found to be 98\% or more E2. For the beta band no angular correlation studies have been carried out. However, one expects no M1 radiation in transitions between vibrational states in even-even nuclei.\textsuperscript{2}

The wave function admixtures were derived under the conditions of first-order terms in centrifugal distortion. However, the nucleus could have a given equilibrium asymmetry such that the U\textsubscript{3} term in the collective Hamiltonian (E2. 14) is not small. The first-order expansion of the moments of inertia in terms of $\gamma$ would not be valid, and the entire procedure based on small $\gamma$ is incorrect. On the other hand, the first-order expansion in centrifugal distortion is a separate problem, and can still be acceptable.

The interaction of rotations and beta vibrations is considered by Davydov and Chaban\textsuperscript{25} for asymmetric nuclei. The beta dependence of the collective Hamiltonian is used, and the solution for energy levels differs from the symmetric case only in allowing finite values of $\gamma$, the asymmetry parameter. Gamma vibrations are not considered since the gamma band is also accounted for in terms of rotations of the asymmetric nucleus. A new parameter $\mu$ expresses the strength of the rotation-vibration interaction. Energies of ground state band levels are then determined from two energy ratios, the ratios of the ground state band 4+ and the gamma band 2+ level energies to the ground state band 2+ level. For Sm\textsuperscript{152} this yields $\gamma = 11.4^\circ$ and $\mu = 0.37$. These parameters are used to predict a 6+ level at 693 kev and a 0+ level at 670 kev, which are fairly close to the experimental
values of 707 and 685 keV. The 8+ level is predicted to be at 
1070 keV, which tends to rule out the 1187 keV level tentatively 
assigned as the 8+ member of the ground state band on the basis 
of a 480 keV gamma ray.

Davydov and Chaban have not given expressions for reduced 
transition probabilities between beta and ground state band 
levels, so that an extensive test of this approach cannot be 
made. The beta vibrations in this model are considered to be 
small enough to be harmonic oscillations. The $\mu^2$ parameter of 
Davydov and Chaban can be identified with the $\rho$ in Preston and 
Kiang's notation. Working backwards from $\mu = 0.37$,

$$ a_\beta^2 = \frac{2}{\rho} = \frac{2}{\mu^2} = \frac{2}{0.137} = 14.6 \quad (135) $$

This value is very different than experimental value of 48.5, 
suggesting that the agreement obtained in the modified asymmetric 
rotor model may be somewhat fortuitous. A more rigorous test 
of the model awaits calculation of the transition probabilities.
B. Samarium - 154

1. Decay Scheme

It was hoped that the beta and gamma bands in Sm$^{154}$ would be excited in order to trace the effect of mixing to a nucleus with larger deformation. However, the vibrational bands were not populated with any great intensity and neither levels nor branching ratios were determined for the beta and gamma bands. Most of the transitions observed were assigned to ground state and octupole bands, as discussed below.

In the ground state band four levels were assigned on the basis of coincidence measurements which showed that there were four gamma rays all in cascade. These assignments are further supported by the agreement of experimental yields in gamma-particle spectra with calculations in the multiple coulomb excitation theory (see Table V). The energies of the four ground state band transitions are 82, 185, 278, and 360 kev. The latter two are in some disagreement with the transitions reported by deBoer, Goldring, and Winkler$^4$ at 284 and 401 kev. However, in their work an oxide target was used, while in this data an oxide-free target gave much improved background conditions and made possible the measurement of gamma-gamma coincidences, which further support the assignments. The energies reported in this work have been further corroborated by measurements made at this laboratory by Burginyon and Greenberg$^{86}$ with an 8-kev resolution lithium-drifted germanium junction detector. The 185 kev transition has also been measured by Graetzer and Bernstein.$^{82}$ A fit of
the levels by Eq. 117 shows that a much smaller $B$ coefficient than in $\text{Sm}^{152}$ is involved, indicating less mixing with vibrational bands. The coefficients are

$$A = 13.82$$
$$B = -0.0268$$
$$C = 0.00016$$

Since the $C$ term is very small we would expect a good prediction for the $8^+$ level. The calculation gives 916 keV and the actual value is 905 keV.

The particle-gamma spectrum shows gamma rays at 474, 553(?), 630, 740, 820, 915, 1030, and 1440 keV. As mentioned earlier, the 474 is a sum coincidence. The 630 keV line appears in the $T + 278$ spectrum, marking it as coming most likely from a spin state of 4 or higher. The 740 and 915 keV transitions occur strongly in $T + 185$, indicating that they are from $2^+$ or higher spins. However, if a $2^+$ level exists at $267 + 740 = 1007$ keV, stronger transitions to the 82 keV and ground states would be expected. These do not occur either in the direct spectrum or in $T + 82$. We therefore rule out a $2^+$ level. A $4^+$ level assignment seems unlikely because no lower $2^+$ level and its strong transitions are observed.

It is possible to account for these gamma rays on the basis of an octupole band with a $3^-$ level at 1007 keV and a $5^-$ level at 1183 keV. (See Fig. 15.) The $3^-$ de-excitation to the $2^+$ level is not observed in $T + 82$, but this spectrum has poor statistics. Hansen and Nathan \cite{24} report a $3^-$ level at 1008 keV.
If the same moment of inertia is assumed for the octupole band as the ground state band a $1^-$ level would be expected at about 920 kev. Perhaps the 830 kev gamma ray found in the gamma-particle spectrum is the $1^-$ level de-excitation.

The branching ratio from the $5^-$ level can be calculated and compared to the Alaga predictions for an octupole band.

$$\frac{B(El, 5^- \rightarrow 6^+)}{B(El, 5^- \rightarrow 4^+)} = \frac{2.8}{6.0} \times \left(\frac{915}{630}\right)^3 = 0.54 \pm 0.20$$

This is not in good agreement with the Alaga prediction, which is a value of 1.20. The reason for this deviation is presently unknown.

Two gamma rays remain to be considered, at 1030 and 1440 kev. These have been assigned by Yoshizawa et al. as respectively the beta and gamma band $2^+$ level de-excitations to the ground state, from intensities in a gamma-particle spectrum. However, the more intense transitions observed from beta and gamma bands in gamma-particle spectra in Sm are not transitions to the ground state, and the actual levels may lie higher in energy. Kenefick and Sheline report levels at 1444 and 1104 kev from inelastic proton scattering, but observe no 1030 kev level. The 1030 kev transition may be the $0^+$ to $2^+$ de-excitation from a $0^+$ beta band level at about 1112 kev.

If the 1030 kev level is used to determine a $\gamma$ value in the simple asymmetric rotor model, we obtain $\gamma = 11.2^\circ$. The predictions for the $4^+$, $6^+$ and $8^+$ levels are then 272, 565, and 960 kev. These are fairly close to the actual values, which are slightly
better described by choosing $\gamma = 16.0^0$ from the 4+ to 2+ energy ratio, yielding 540 and 892 kev for the 6+ and 8+ levels. If the 1440 kev transition is assumed to be from a 2+ level, the $\gamma$ predicted is about $9^0$, giving worse predictions than above.

The decay scheme arrived at from this data is shown in Fig. 67. The decay scheme of Kenefick and Sheline obtained from inelastic proton scattering is in good agreement with the decay scheme shown in Fig. 67.

2. Reduced Transition Probabilities

The calculations of excitation yields for the ground state band were carried out in the same manner described earlier. These are listed in Table V, with Y representing the calculated yield, and R the ratio of calculated to experimental yields. Errors in calculated yields are based on extrapolation and integration energy cut-off (see Chapter IV) uncertainties.

Calculation number one used the first-order correction in $\xi$ in the first paper by Alder and Winther. Number two and three use the calculations based on matrix diagonalization with finite $\xi$ value. The first two calculations use Elbek's inelastic scattering data for B(E2). The third calculation is based on a 10% lower value, and it is seen that the ratios of theoretical to experimental yields are in better agreement than in case two. This suggests that the B(E2) value is somewhat lower than Elbek's value.

As related earlier, the B(E3) value for excitation of the 3- level in an octupole band may be obtained from the total
FIG. 67

\[
\text{Sm}^{154}
\]
excitation of the band and the perturbation theory cross section. Since only the 630, 740, and 915 intensities (see Table IX) are determined with any accuracy, we obtain a lower limit of $B(E3) = 11 \pm 3 \times 10^{-74} \text{ cm}^6$. This is in good agreement with the value of $B(E3) = 15 \times 10^{-74} \text{ cm}^6$ reported by Hansen and Nathan.\textsuperscript{39}

The Sm\textsuperscript{154} data may serve as an indication of the type of spectra to be obtained in coulomb excitation of other rare earth nuclei. Many of the gadolinium, dysprosium, erbium, and ytterbium isotopes are heavily deformed, with about the same deformation parameters as in Sm\textsuperscript{154} (where $\beta = 0.33$). These have not been investigated by coulomb excitation in the same detail as the nuclei reported here because of the problem of obtaining pure metal separated isotopic targets, but the reduction processes are being investigated and may yield adequate targets. There are experimental difficulties in that higher states are only weakly excited, and spin assignments are unknown. As brought out in Chapter I, higher charge ions than oxygen will greatly enhance excitation yields, and efforts in this direction are extremely important. Alder and Winther\textsuperscript{88} and Hansen and Nathan\textsuperscript{89} have pointed out the usefulness of gamma ray angular distributions in gamma-particle spectra for determining spins and parities. Measurements of this sort will be necessary for investigating levels in the heavily deformed nuclei.
<table>
<thead>
<tr>
<th>Eγ [keV]</th>
<th>Spectrum</th>
<th>Counts</th>
<th>a₁</th>
<th>η₁ A₁</th>
<th>a₂</th>
<th>η₂ A₂</th>
<th>μC</th>
<th>Transitions per μC</th>
<th>Excitation Mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sm¹⁵⁴</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>630</td>
<td>T + 185</td>
<td>8.7 ± 2.2(2)</td>
<td>0.003</td>
<td>0.0131 ± 0.0012</td>
<td>0.261</td>
<td>0.0253 ± 0.0023</td>
<td>795</td>
<td>4.18 ± 1.2(3)</td>
<td>E3 0.11 ± 0.03</td>
</tr>
<tr>
<td>740</td>
<td>T + 185</td>
<td>2.10 ± 0.53(3)</td>
<td>0.003</td>
<td>0.0112 ± 0.0010</td>
<td>0.261</td>
<td>0.0253 ± 0.0023</td>
<td>795</td>
<td>1.18 ± 0.33(4)</td>
<td></td>
</tr>
<tr>
<td>915</td>
<td>T + 185</td>
<td>2.00 ± 0.40(3)</td>
<td>0.001</td>
<td>0.0084 ± 0.0008</td>
<td>0.261</td>
<td>0.0253 ± 0.0023</td>
<td>795</td>
<td>1.49 ± 0.35(4)</td>
<td></td>
</tr>
<tr>
<td>82</td>
<td>T + 278</td>
<td>2.56 ± 0.40(3)</td>
<td>4.95</td>
<td>0.0152 ± 0.0014</td>
<td>0.073</td>
<td>0.0229 ± 0.0021</td>
<td>795</td>
<td>5.90 ± 1.00(4)</td>
<td></td>
</tr>
<tr>
<td>185</td>
<td>T + 278</td>
<td>2.17 ± 0.22(4)</td>
<td>0.261</td>
<td>0.0296 ± 0.0025</td>
<td>0.073</td>
<td>0.0229 ± 0.0021</td>
<td>795</td>
<td>6.02 ± 0.99(4)</td>
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</tr>
<tr>
<td>630</td>
<td>T + 278</td>
<td>5.50 ± 1.7(2)</td>
<td>0.003</td>
<td>0.0131 ± 0.0012</td>
<td>0.073</td>
<td>0.0229 ± 0.0021</td>
<td>795</td>
<td>2.48 ± 0.82(3)</td>
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<tr>
<td>Sm¹⁴⁸</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>551</td>
<td>Direct(90°)</td>
<td>4.40 ± 0.41(5)</td>
<td>0.010</td>
<td>0.0272 ± 0.0031</td>
<td>4.55</td>
<td>3.35 ± 0.54(6)</td>
<td>3.71 ± 0.60(6)*E2 0.42 ± 0.07</td>
<td></td>
<td></td>
</tr>
<tr>
<td>551</td>
<td>Direct(55°)</td>
<td>2.42 ± 0.24(5)</td>
<td>0.010</td>
<td>0.0137 ± 0.0016</td>
<td>4.62</td>
<td>3.86 ± 0.61(6)</td>
<td>E2 0.43 ± 0.07</td>
<td></td>
<td></td>
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<tr>
<td>551</td>
<td>γ-p</td>
<td>2.87 ± 0.15(4)</td>
<td>0.010</td>
<td>0.0137 ± 0.0013</td>
<td>65.6</td>
<td>3.25 ± 0.48(4)</td>
<td>0.43 ± 0.09</td>
<td></td>
<td></td>
</tr>
<tr>
<td>611</td>
<td>T + 551</td>
<td>1.10 ± 0.55(3)</td>
<td>0.002</td>
<td>0.0243 ± 0.0028</td>
<td>0.010</td>
<td>0.0159 ± 0.0019</td>
<td>68.1</td>
<td>4.21 ± 2.2(4)</td>
<td>E3 0.197 ± 0.023</td>
</tr>
<tr>
<td>630</td>
<td>T + 551</td>
<td>1.10 ± 0.55(3)</td>
<td>0.007</td>
<td>0.0237 ± 0.0028</td>
<td>0.010</td>
<td>0.0159 ± 0.0019</td>
<td>68.1</td>
<td>4.33 ± 2.3(4)</td>
<td>E2-E2 1.25 ± 0.09</td>
</tr>
<tr>
<td>914</td>
<td>γ-p</td>
<td>1.7 ± 0.7(2)</td>
<td>0.001</td>
<td>0.0084 ± 0.0010</td>
<td>65.6</td>
<td>3.08 ± 1.50(2)</td>
<td>0.062 ± 0.014</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Number in parentheses is power of 10 for multiplying factor

* Corrected for angular correlation.
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C. Samarium - 150

1. Decay Scheme

As discussed in Chapter III, there are detailed decay schemes by Groshev and Smither (Fig. 39 and 43) for the Sm\textsuperscript{150} nucleus, both obtained from detection of the de-excitation radiation following single neutron capture in Sm\textsuperscript{149}. Groshev measured direct spectra of gamma rays and internal conversion electrons. Smither's data go further, including coincidence measurements and angular correlation studies, and are therefore more complete in spin assignments and transition multipolarities.

Smither's decay scheme agrees in spins and energies (within 1 or 2 kev) with Groshev's levels at 334 (2+), 740(0+), 773(4+), 1047(2+), 1071(3-), and 1167(2+) kev. The de-excitation of the 1047 kev 2+ state is given as E2 by Smither, and M1 by Groshev. Groshev lists a low intensity 1048 kev gamma ray decaying from that level, but it is not seen in Smither's work. Smither has a 2+ level at 1193 kev, de-exciting by 859 and 1193 kev gamma rays, but Groshev does not report this level. Conversely, a 0+ level at 1256 kev in Groshev's decay scheme is not listed by Smither. Two levels at 1280 and 1360 kev reported by Groshev are assigned as 3+ and 2+ levels by Smither. Higher levels have been omitted from Fig. 39 and 43.

The direct spectrum (Fig. 40) has the intense 334 kev radiation from the first excited state and pairs of unresolved lines at about 440 and 720 kev. The 0+ and 4+ de-excitations
with energies of 407 and 439 kev are unresolved, but the fitting of gaussian-shaped photopeaks in the direct and T + 334 spectra determines that about 10% of the peak is 407 kev radiation. The determination of relative amounts of 712 and 736 kev radiation in a similar manner yields about 25% of the peak for the 712 kev gamma ray. This is corroborated by measurements of conversion electrons from coulomb excitation by Burginyon and Greenberg in which both transitions are completely resolved. Their data was taken with an incident beam energy of 65 Mev, and the corrections to a 49 Mev beam from a comparison of thick target integrations at 65 and 49 Mev yields 25% of the peak as 712 kev radiation.

The 1046 or 1047 kev level may be the 2+ member of a triplet of states corresponding to a secon-phonon of vibration, but it is very much higher in energy than the 0+ and 4+ levels. The peaks at 859 kev in the direct, gamma-particle, and T + 334 spectra may be assigned to the de-excitation from the 1193 2+ level. The 1193 kev transition is seen in the gamma-particle spectrum. Excitations of the 1165 or 1167 kev 2+ level and of the 1256 and 1279 kev levels are either absent or very weak. The 583 and 1026 kev gamma rays are assigned to transitions from the 3- level at 1358 or 1360 kev. Coincidences with 407 and 439 kev gamma rays exhibited a few very weak lines which have not been assigned in the level scheme. The decay scheme from this data is shown in Fig. 68.

2. Reduced Transition Probabilities

If the excitation of the first excited state is considered as multiple coulomb excitation of a vibrational state, (see Eq. 93), the
corrections to the $B(E2)$ value from the term $e^{-[x(\theta, \phi)]^2}$ are
5\%, yielding $B(E2, 0 \to 2) = 1.31 \pm 0.20 e^2 \times 10^{-48}$ cm$^4$. This
agrees very well with the value of Elbek et al.$^{32}$ from inelastic
scattering, $B(E2, 0 \to 2) = 1.32 \pm 0.06 e^2 \times 10^{-48}$ cm$^4$.

Using the inelastic scattering $B(E2)$ to calculate the yield in
gamma-particle spectra in the multiple excitation of a
vibrational state, we obtain a calculated yield of $1.35 \pm 0.11$
$\times 10^5$ transitions per microcoulomb. The experimental value is
$1.00 \pm 0.10 \times 10^5$. However, the excitation mechanism of harmonic
vibrations may not be valid.

The excitation of the 740 (0+), 773 (4+), 1047 (2+) and
1193 (2+) levels have been calculated for the double $E2$ excitation
process. The direct $E2$ excitation has not been included in the
1047 level excitation since no direct transition has been
observed in this or Smither's data. The direct process has been
included in the 1193 level excitation with the $B(E2)$ value for
direct excitation expressed in terms of the $B(E2, 2 \rightarrow 2')$ value
from the branching ratio of 859 to 1193 kev gamma rays.

$$\frac{B(E2, 2' \rightarrow 2)}{B(E2, 2' \rightarrow 0)} = \frac{\text{Intensity of 859}}{\text{Intensity of 1193}} \left(\frac{1193}{859}\right)^5 = R$$

(137)

$$B(E2, 2 \rightarrow 2') = R B(E2, 2' \rightarrow 0) = \frac{1}{5} R B(E2, 0 \rightarrow 2')$$

where the prime indicates the 1193 kev level. The calculations
consider only the 334 2+ level as an intermediate state in the
process. Excitations via the 1047 2+ level may also contribute,
but are probably of small value since the 1047 level is not
excited directly. The $B(E2, 0 \rightarrow 2)$ value used in double excitation calculations is $1.32 \times 10^{-48}$ cm$^4$ from Elbek et al.\textsuperscript{32}

In Table XI, $B(E2, 0 \rightarrow 2)$ is given in units of $e^2 \times 10^{-48}$ cm$^4$.

The $B(E2)$ values for excitation of the $0^+$, $4^+$, and second $2^+$ levels calculated from double excitation theory may be compared with the predictions of the vibrational model. This may be written in terms of excitation $B(E2)$ values by use of

$$B(E2, J_2 \rightarrow J_1) = \frac{2J_1 + 1}{2J_2 + 1} B(E2, J_1 \rightarrow J_2) \quad (138)$$

The predictions of the pure vibrational model are compared with the data in Table XII. The experimental values for $0^+$ and $2^+$ states are slightly less than the vibrational model prediction. The $4^+$ state excitation is in good agreement with the vibrational prediction.

The $B(E3)$ values for direct excitation of the $3^-$ states at 1071 and 1360 keV have been calculated. The 1026 keV gamma ray has not been definitely assigned to the 1360 keV level by Groshev or Smither. It is quite clear in the T + 334 spectrum. There is a contaminant radiation at 1020 keV, and the estimate from other coincident spectra in Sm\textsuperscript{150} has been subtracted from the intensity given in Table X. This is a 30% correction. Adding the 583 and 1026 intensities yields a $B(E3)$ value of $14.6 \pm 0.042 \times 10^{-74}$ cm$^6$ for the 1360 keV level. From the 736 keV intensity we obtain $B(E3) = 36 \pm 10 \times 10^{-74}$ cm$^6$ for the 1071 keV level. Hansen and Nathan\textsuperscript{39} also observe the 1071 level excitation, obtaining a somewhat lower value of
### TABLE X  Sm$^{150}$ GAMMA RAY INTENSITIES

<table>
<thead>
<tr>
<th>$E_{\gamma}$ [keV]</th>
<th>Spectrum</th>
<th>Counts</th>
<th>$\alpha_1$</th>
<th>$\eta_1 A_1$</th>
<th>$\alpha_2$</th>
<th>$\eta_2 A_2$</th>
<th>$\mu C$</th>
<th>Transitions per $\mu C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>334</td>
<td>Direct</td>
<td>5.77±0.58(6)</td>
<td>0.041</td>
<td>0.0402±0.0047</td>
<td></td>
<td></td>
<td>8.72</td>
<td>1.71±0.28(7) $^+$ 1.66±0.31(7) $^*$</td>
</tr>
<tr>
<td></td>
<td>$\gamma-p$</td>
<td>1.92±0.10(5)</td>
<td>0.041</td>
<td>0.0201±0.0025</td>
<td></td>
<td></td>
<td>95.5</td>
<td>1.04±0.16(5) 1.00±0.15(5) $^*$</td>
</tr>
<tr>
<td>407</td>
<td>$T+334$</td>
<td>1.30±0.39(3)</td>
<td>0.022</td>
<td>0.0347±0.0040</td>
<td>0.041</td>
<td>0.0262±0.0033</td>
<td>76.8</td>
<td>1.98±0.68(4)</td>
</tr>
<tr>
<td>439</td>
<td>$T+334$</td>
<td>1.19±0.12(4)</td>
<td>0.017</td>
<td>0.0330±0.0038</td>
<td>0.041</td>
<td>0.0262±0.0033</td>
<td>76.8</td>
<td>1.90±0.37(5)</td>
</tr>
<tr>
<td></td>
<td>Direct</td>
<td>6.23±1.24(4)</td>
<td>0.017</td>
<td>0.0330±0.0038</td>
<td></td>
<td></td>
<td>8.72</td>
<td>2.20±0.57(5)</td>
</tr>
<tr>
<td></td>
<td>$\gamma-p$</td>
<td>6.8±1.4(3)</td>
<td>0.017</td>
<td>0.0166±0.0021</td>
<td></td>
<td></td>
<td>95.5</td>
<td>4.36±0.95(3)</td>
</tr>
<tr>
<td>583</td>
<td>$T+334$</td>
<td>2.10±0.9(2)</td>
<td>0.003</td>
<td>0.0256±0.0030</td>
<td>0.041</td>
<td>0.0262±0.0033</td>
<td>76.8</td>
<td>4.25±1.85(3)</td>
</tr>
<tr>
<td></td>
<td>$\gamma-p$</td>
<td>2.8±1.1(2)</td>
<td>0.003</td>
<td>0.0130±0.0016</td>
<td></td>
<td></td>
<td>95.5</td>
<td>2.26±0.96(2)</td>
</tr>
<tr>
<td>712</td>
<td>$T+334$</td>
<td>1.40±0.36(3)</td>
<td>0.005</td>
<td>0.0215±0.0025</td>
<td>0.041</td>
<td>0.0262±0.0033</td>
<td>76.8</td>
<td>3.38±1.0(4)</td>
</tr>
<tr>
<td></td>
<td>$\gamma-p$</td>
<td>1.1±0.5(3)</td>
<td>0.005</td>
<td>0.0109±0.0014</td>
<td></td>
<td></td>
<td>95.5</td>
<td>1.06±0.45(3)</td>
</tr>
<tr>
<td>737</td>
<td>$T+334$</td>
<td>3.65±0.90(3)</td>
<td>0.002</td>
<td>0.0207±0.0024</td>
<td>0.041</td>
<td>0.0262±0.0033</td>
<td>76.8</td>
<td>9.14±2.7(4)</td>
</tr>
<tr>
<td></td>
<td>$\gamma-p$</td>
<td>1.1±0.5(3)</td>
<td>0.002</td>
<td>0.0106±0.0013</td>
<td></td>
<td></td>
<td>95.5</td>
<td>1.09±0.46(3)</td>
</tr>
<tr>
<td>859</td>
<td>$T+334$</td>
<td>1.38±0.27(3)</td>
<td>0.003</td>
<td>0.0170±0.0020</td>
<td>0.041</td>
<td>0.0262±0.0033</td>
<td>76.8</td>
<td>4.21±1.1(4)</td>
</tr>
<tr>
<td></td>
<td>$\gamma-p$</td>
<td>2.5±1.0(2)</td>
<td>0.003</td>
<td>0.0089±0.0011</td>
<td></td>
<td></td>
<td>95.5</td>
<td>2.94±1.22(2)</td>
</tr>
<tr>
<td>1026</td>
<td>$T+334$</td>
<td>4.50±2.0(2)</td>
<td>0.002</td>
<td>0.0145±0.0017</td>
<td>0.041</td>
<td>0.0262±0.0033</td>
<td>76.8</td>
<td>1.61±0.55(4)</td>
</tr>
<tr>
<td>1193</td>
<td>$\gamma-p$</td>
<td>3.9±1.3(2)</td>
<td>0.001</td>
<td>0.0065±0.0008</td>
<td></td>
<td></td>
<td>95.5</td>
<td>6.28±2.05(2)</td>
</tr>
</tbody>
</table>

$^*$ Corrected for angular correlation.

$^+$ Corrected for cascades from higher states.

Number in parentheses is power of 10 for multiplying factor.
### Table XI

#### Sm$^{150}$ B(E2) and B(E3) Values

<table>
<thead>
<tr>
<th>$E_\gamma$(kev)</th>
<th>Experimental Yield</th>
<th>Calculated Yield/B(E2)</th>
<th>Excitation Mode</th>
<th>$B(E2) \cdot e^2 (10^{-24})$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Total Cross Section (Direct and Gamma-Gamma Spectra)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>334$^d$</td>
<td>1.86±0.31(7)</td>
<td>1.49 (7)</td>
<td>E2</td>
<td>1.25±0.21</td>
</tr>
<tr>
<td>407</td>
<td>1.98±0.68(4)</td>
<td>3.64 B(0→2)(5)</td>
<td>E2-E2</td>
<td>0.0412±0.0150</td>
</tr>
<tr>
<td>439</td>
<td>1.90±0.37(5)</td>
<td>1.52 B(0→2)(5)</td>
<td>E2-E2</td>
<td>0.96±0.19</td>
</tr>
<tr>
<td>439$^d$</td>
<td>2.20±0.57(5)</td>
<td></td>
<td></td>
<td>1.10±0.29</td>
</tr>
<tr>
<td>583</td>
<td>4.3±1.9(3)</td>
<td>1.40(5)</td>
<td>E3</td>
<td>0.031±0.012</td>
</tr>
<tr>
<td>712</td>
<td>3.38±1.0(4)</td>
<td>6.57 B(0→2)(4)</td>
<td>E2-E2</td>
<td>0.390±0.115</td>
</tr>
<tr>
<td>737</td>
<td>9.14±2.7(4)</td>
<td>2.55 (4)</td>
<td>E3</td>
<td>0.358±0.103</td>
</tr>
<tr>
<td>859$^e$</td>
<td>1.32±0.46(5)</td>
<td><a href="4">1.32±0.42</a> x B(0→2)</td>
<td>E2-E2</td>
<td>0.0436±0.0158(+)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.145±0.054(−)</td>
</tr>
<tr>
<td>1026</td>
<td>1.61±0.55(4)</td>
<td>1.40 (5)</td>
<td>E3</td>
<td>0.115±0.040</td>
</tr>
</tbody>
</table>

#### Differential Cross Section (Gamma-Particle Spectra)

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>439</td>
<td>4.36±0.95(3)</td>
<td>5.55±0.31(3) x B(0→2)</td>
<td>E2-E2</td>
<td>0.595±0.132</td>
</tr>
<tr>
<td>712</td>
<td>1.06±0.45(3)</td>
<td>3.32±0.18(3) x B(0→2)</td>
<td>E2-E2</td>
<td>0.242±0.120</td>
</tr>
<tr>
<td>737</td>
<td>1.09±0.46(3)</td>
<td>2.92±0.15(3)</td>
<td>E3</td>
<td>0.374±0.184</td>
</tr>
<tr>
<td>583</td>
<td>2.26±0.96(2)</td>
<td>5.05±0.27(3)</td>
<td>E3</td>
<td>0.0448±0.0188</td>
</tr>
<tr>
<td>859$^e$</td>
<td>9.22±2.53(2)</td>
<td><a href="3">1.23±0.19</a> B(0→2)</td>
<td>E2-E2</td>
<td>0.0456±0.0132(+)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.0854±0.0254(−)</td>
</tr>
<tr>
<td>334</td>
<td>1.00±0.10(5)</td>
<td>1.35±0.11(5)</td>
<td>M.C.E.</td>
<td>1.32</td>
</tr>
</tbody>
</table>

$^e$Includes branching to 1193 transition.

$^d$From direct spectrum.
### Table XII

**Comparison of Sm$^{150}$ B(E2) Values with Vibrational Model Predictions**

<table>
<thead>
<tr>
<th>$E_\gamma$ [kev]</th>
<th>$J_\pi$</th>
<th>Experimental $\frac{B(E2, 2 \rightarrow J)}{B(E2, 0 \rightarrow 2)}$</th>
<th>Vibrational Model</th>
<th>Simple Asymmetric Rotor Model ($\gamma = 21.6^\circ$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>407</td>
<td>0+</td>
<td>0.031±0.011</td>
<td>0.08</td>
<td>-</td>
</tr>
<tr>
<td>712</td>
<td>2+</td>
<td>0.295±0.088</td>
<td>0.40</td>
<td>0.10</td>
</tr>
<tr>
<td>439</td>
<td>4+</td>
<td>0.774±0.160</td>
<td>0.72</td>
<td>0.53</td>
</tr>
</tbody>
</table>
25 e^2 x 10^{-74} \text{ cm}^6. Both of these 3- levels are collective; the above values representing 15 and 36 s.p. units.

The energy levels of this nucleus may be compared to simple asymmetric rotor model predictions. The ratio of energies of 2+ levels at 1047 and 334 kev determine $\gamma = 21.6$, yielding a prediction of 1020 kev for the 4+ level, instead of 773 kev. No $\gamma$ value gives the proper 4+ to 2+ ratio. The ratios of $B(E2)$ values for excitation of the 4+ and 2+ levels are listed in Table XII. They are not in as good agreement with the data as the pure vibration predictions. In the Davydov and Chaban treatment of rotation-vibration interactions, the position of 0+ and second 2+ levels determine $\gamma = 16.5^\circ$, $\nu = 0.9$, while with the 4+ and second 2+ level energies, $\gamma = 18^\circ$, $\nu = 0.7$.

However, the best test of a model is the comparison of reduced transition probabilities to experiment, and these have not been evaluated as yet. At present the data tend to indicate support for the simple vibrational model being applicable to the Sm^{150} nucleus.
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1. Decay Scheme

In Sm\(^{148}\) the energy levels higher than the first excited state (see Fig. 46) at 551 kev were not populated strongly by coulomb excitation. The de-excitation from the 4+ level to the 2+ level at 630 kev was not of sufficient intensity to appear in the direct spectrum (Fig. 44). The only peaks in this spectrum, besides the first excited state de-excitation, are lower energy transitions from isotopic impurities and higher-energy transitions from surface contaminants.

In the particle-gamma spectrum (Fig. 47) the 4+ to 2+ de-excitation appears as a shoulder on the high side of the 551 kev peak. The other peak at 914 kev is the 1- to 2+ transition from a level at 1465 kev. In the T + 551 spectrum the 4+ to 2+ transition at 630 kev is unresolved from a 3- to 2+ transition at 611 kev. The relative intensities of the two gamma rays has been taken as roughly equal in magnitude.

If Sm\(^{148}\) is a vibrational nucleus one would expect to find 0+ and 2+ levels at about twice the energy of the first level. However, these would be less strongly excited (see Eq. 89 for relative B(E2) values) and may be of too low an intensity to be detected. A possible new transition is a weak gamma ray at 1220 kev. This is found in the gamma-particle spectrum, and therefore cannot be a contamination gamma ray. It was of too low intensity to appear in coincidence spectra and therefore has not been assigned in the decay scheme.
2. Reduced Transition Probabilities

Gamma ray intensities and reduced transition probabilities are listed in Table IX. The direct excitation of the first excited state yields \( B(E2, 0 \rightarrow 2) = 0.43 \pm 0.07 \) from spectra taken at two different angles. This value is much lower than that of Elbek et al.,\(^{32}\) who obtained \( 0.89 \pm 0.10 \times 10^{-48} \text{ cm}^4 \). The use of the \( B(E2) \) value obtained here in the gamma-particle spectrum leads to a calculated yield that is in very good agreement with the experimental yield when the vibrational excitation mode is assumed. This differs from the direct excitation by only 5%. The differences between the \( B(E2) \) from this data and from Elbek's data are well outside experimental error, and suggests an error in normalization procedure.

However, for the direct spectrum taken at 90° the counter was in the same position as for Sm\(^{150}\), which gave good agreement with Elbek's data. For the direct spectrum taken at 55° and the gamma-particle spectrum, the counter was in the same position used in Sm\(^{152}\) and Sm\(^{154}\), and a common normalization factor was found in all three cases. Thus the value \( B(E2, 0 \rightarrow 2) = 0.43 \pm 0.07 \times 10^{-48} \text{ cm}^4 \) is obtained from two different counter positions which give reasonable excitation probabilities for the data from other targets. It is noted that the value listed in the data survey in ABHMW is \( 0.50 \times 10^{-48} \text{ cm}^4 \), which is close to the number found here.

The 3- level is collective, as indicated by the magnitude of the \( B(E3) \) value, being about 20 s.p. units. Hansen and Nathan\(^{39}\) obtain \( 31 \times 10^{-74} \text{ cm}^6 \), or about 34 s.p. units, but
have large uncertainties because the de-excitation gamma ray is not resolved in their experiment. The total excitation of the 1465 kev 1- level is obtained from the 914 kev intensity and the branching ratio measured by Baba et al.,\textsuperscript{29} where there is 2.4 times as much 1465 kev radiation. Inclusion of this branching raises the excitation B(El) value to 0.026 ± 0.014 e² x 10⁻²⁴ cm². This level may be collective, perhaps arising from an interaction between quadrupole and octupole vibrations, as considered by Lipas.\textsuperscript{40}

Further experimentation in Sm\textsuperscript{148} can proceed in two directions. Higher resolution is required to determine intensities of the 551, 611, and 630 kev radiations more accurately, and coulomb excitation with heavier ions than O\textsuperscript{16} would possibly excite levels not seen in this work or in beta decay studies.
VI. Summary and Conclusions

The magnitudes of the $B(E2)$ and $B(E3)$ values for excitation of low-lying states in samarium nuclei clearly show collective behavior. The $B(E2)$ values for excitation of the first excited states are 20 or more single-particle units, (see Table I) as defined in Eq. 26, or 100 or more units with the more common definition in which the $2\lambda + 1$ factor is absent. The deformed nuclei $^{152}\text{Sm}$ and $^{154}\text{Sm}$ exhibit level structures which are identified with collective rotations based on the ground state and vibrational states.

For an axially symmetric nucleus the energy of a state of spin $J$ in the ground state band is expected to be equal to $A J(J+1)$. This is found to be fairly well satisfied in $^{154}\text{Sm}$, but there are large deviations in $^{152}\text{Sm}$. These deviations are largely expressed by means of a term in $B J^2(J+1)^2$. It has been shown theoretically that such terms can arise from perturbations to the collective Hamiltonian that are customarily neglected. These perturbations account for small asymmetries on the basis of admixture of gamma band wave functions and for centrifugal distortions from admixtures of beta band wave functions. The amount of admixture can be related to the deviations of transition probabilities from vibrational bands to the ground state band from the Alaga predictions for an axially symmetric nucleus. The parameters $z_\beta$ and $z_\gamma$ are obtained, which are proportional
to $\epsilon_\beta^2$ and $\epsilon_\gamma^2$. The z values are -.064 and -.08, but the
experimental values of $\alpha$ are so large that the $\epsilon_\beta$ and $\epsilon_\gamma$ are
very small. The calculated coefficients $B_\beta$ and $B_\gamma$ are proportional
to $\epsilon_\beta^2$ and $\epsilon_\gamma^2$, rather than $z_\beta^2$ and $z_\gamma^2$, and do not account for
more than a small fraction of the ground state band value of $B$.

It is interesting to note that the values of $\alpha_\beta^2$ and $\alpha_\gamma^2$
obtained from the hydrodynamic model are much smaller than
experimental determinations. Experimentally $\alpha_\beta^2 = 48.5$ and
$\alpha_\gamma^2 = 26.4$, while from the hydrodynamic model (with $\gamma = 0$)
$\alpha_\beta^2 = 11.2$ and $\alpha_\gamma^2 = 8.93$. The hydrodynamic model assumes that
vibrations are harmonic, but this difference challenges this
assumption. The large size of $\alpha$ enhances wave function admixture
effects in transition probabilities, but suppresses energy
perturbations.

The simple asymmetric rotor model was used to compare
relative energy level spacings, but did not yield good agreement.
However, the modifications with beta vibrations included, as
carried out by Davydov and Chaban, give fairly good predictions
for the energy levels. In this model the nucleus is asymmetric,
having a finite value of $\gamma$, and has rotation-vibration interactions
due to centrifugal distortions. The rotation-vibration leads to
a new parameter $\mu$. The energy levels are calculated over a
range of $\gamma$ and $\mu$. The ground state band 2+ and 4+ level energies
and the gamma band 2+ level energy determine $\gamma$ and $\mu$ as 11.4°
and 0.37. The predictions for the ground state band 6+ level
and beta band 0+ level are 693 and 670 kev, compared with the
actual values of 707 and 685 kev.
The deviations from an energy dependence of $J(J+1)$ can then be explained on the basis of an asymmetric nucleus, with centrifugal distortions allowed. The strength of the beta vibration is essentially determined by the position of the $0^+$ level in the beta band since harmonic oscillations are assumed. The parameter $\nu^2$ can be identified with the parameter $\rho$ in Preston and Kiang's paper, so that

$$\alpha_\beta^2 = \frac{2}{\rho} = \frac{2}{\nu^2} = 14.6 \quad (139)$$

If this were used in Eq. 119 to calculate $B_\beta$, we obtain -0.130 kev which is most of the $B_{gs}$ of -0.193 kev. However, $\alpha_\beta^2 = 14.6$ is so very different from the experimental value that it is tempting to conclude that this modified asymmetric rotor model represents only a convenient method for classifying energy levels in terms of two parameters.

In a recent paper by Faessler and Greiner calculations based on a modification of the Bohr-Mottelson model have been carried out. The $a_o$ and $a_2$ coefficients of Eq. 5 are allowed to undergo small harmonic oscillations about equilibrium values. The Hamiltonian is the same as Eq. 14, but the moments of inertia are written in terms of $a_o$ and $a_2$ and serve to introduce interactions between rotation and vibration. The parameters of the theory are the unperturbed moment of inertia and the frequencies of beta and gamma vibrations. The calculations for relative transition probabilities agree well with experimental values in all deformed nuclei but the samarium region, which,
according to the authors, is because their perturbation treatment is not valid. The differences between energy levels in the beta and gamma bands are small, which implies that the oscillations in $a_0$ and $a_2$ are large, and the perturbation calculation is not correct.

The comparison of nuclear states with vibrational excitations is not well justified on the basis of level energies. The harmonic oscillator or pure vibrational model predicts the first level as $2^+$ and a degenerate triplet of levels with spins $0^+$, $2^+$, and $4^+$ at twice the energy of the first $2^+$ level. However, the triplet is never degenerate, and the average energy is somewhat higher than 2, being 2.1 and 2.6 for the Sm$^{148}$ and Sm$^{150}$ nuclei. Evidently some account must be made of anharmonic terms in the Hamiltonian, but in this thesis discussion is limited to the degree to which the pure vibrational model is accurate. The ratios of $B(E2)$ values are given in Table XIX. For Sm$^{150}$ the ratios are in better agreement with the vibrational model predictions than for the simple asymmetric rotor model. In Sm$^{148}$ the experimental error is too large to make a comparison.

As discussed in Chapter V.C., the comparison of Sm$^{150}$ energy levels with the asymmetric rotor model including beta vibrations gave different $\alpha$ and $\gamma$ parameters depending upon the levels used. Eccleshall, Hinds, and Yates$^{38}$ have found similar difficulties for some medium weight nuclei in the cadmium and palladium isotopes. They then consider the later modifications to include gamma-vibrations, but then the problem becomes meaningless because three parameters are used to fit three energy ratios.
When it is recalled (Fig. 1) how sensitive the energy of the first 2+ level is to neutron number, it is perhaps a little surprising that the 3- octupole excitations found are fairly constant in energy and in strength for each of the nuclei studied. (See Table XIII.) However, the quadrupole levels are sensitive to the static nuclear deformation and the surface interactions of extra core nucleons, and the extension of these ideas to octupole levels is not obviously valid. Hansen and Nathan have looked for 3- levels in a large range of nuclei, from $A = 106$ to 154, and found a slow variation of energy and excitation probability with total nuclear mass. These characteristics may be regarded as evidence for no occurrence of stable octupole deformations, and that the $B$ and $C$ parameters are either insensitive to small changes in nucleon number, or that both depend in the same way on these changes. These differences between collective quadrupole and octupole vibrations have been discussed by Mottelson. The quadrupole vibrations are involved with particles in unfilled shells and are therefore dependent upon the residual interactions and shell structure. The octupole vibrations are interpreted as corresponding to transitions between major shells, and are therefore not strongly dependent upon shell structure.

The comparison of excitation yields for ground state band levels has been compared with calculations in the multiple coulomb excitation theory in Table V. It is seen that the first-order correction in $\xi$ is inadequate, (calculation 1) even though $\xi$ is only about 0.06 and 0.04 in the Sm$^{152}$ and Sm$^{154}$ cases.
Table XIII

Excitation of 3- States
in Even-Even Samarium Nuclei

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>$E_3^-$</th>
<th>$B(E3) e^-^2 \times 10^{-74} \text{ cm}^6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{Sm}^{148}$</td>
<td>1162</td>
<td>$20 \pm 10$</td>
</tr>
<tr>
<td>$\text{Sm}^{150}$</td>
<td>1072</td>
<td>$36 \pm 10$</td>
</tr>
<tr>
<td>$\text{Sm}^{152}$</td>
<td>1037</td>
<td>$15 \pm 4$</td>
</tr>
<tr>
<td>$\text{Sm}^{154}$</td>
<td>1008</td>
<td>$11 \pm 3$</td>
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</table>
Calculation 2 is based on the direct inclusion of \( \delta \), and gives much better agreement with the data. However, in the Sm\(^{154}\) data much better agreement is found for a 10\% lower B(E2) value, and even then the theoretical-to-experimental ratios are high.

Systematic errors in the energy range of integration in the theoretical yields would give the largest deviation in \( R_2 \), rather than \( R_4 \), etc. It is seen that the relative errors in calculations (which include interpolation and energy cut-off uncertainties) decrease as we go to higher states. Since the angular dependence of differential cross sections is very great for the higher rotational states, uncertainties in junction detector solid angle versus scattering angle would affect yields for higher states more strongly. However, the Sm\(^{152}\) data does not deviate in the same way. A slightly larger \( \delta \) value in Sm\(^{154}\) would give better agreement, but no reason is seen for such a case. Although the standard deviation in the error for the B(E2) value reported by Elbek et al.\(^{32}\) is only 4.3\%, it seems that agreement of the multiple coulomb excitation yields for Sm\(^{152}\) levels justifies the calculation procedure, and the B(E2) value may be closer to 4.15 e\(^2\) x 10\(^{-48}\) cm\(^4\). Further experimentation to obtain yields with lower errors would be desirable.

There are several directions in which further experimentation could be profitably pursued. As shown in Chapter I, the excitation cross section is proportional to the charge and mass of the incident beam. Thus heavier ions than oxygen would be very advantageous for populating the higher levels. The new
tandem van de Graaff accelerators will be able to accelerate heavier nuclei, such as neon, argon, and elements up to and including lead, in sufficient quantities that such experiments will be not only feasible but highly profitable. Further, the complexity of the spectra obviously requires gamma ray detection with good resolution, and the recently developed lithium-drifted germanium detectors will be highly useful for separating close-lying lines and for greater accuracy in determining gamma ray intensities. Coulomb excitation will continue to be an important tool in the investigation of nuclear properties.

In conclusion, it should be emphasized that collective nuclear properties and the coulomb excitation process exist in a type of symbiotic relationship; the coulomb excitation cross sections are largest for collective levels, unlike, for example, stripping and pick-up reactions, and the accuracy to which the coulomb excitation cross sections are known allows detailed analysis of the collective level structure. In this manner data have been obtained on collective states in the samarium nuclei. A general agreement with the collective model has been known from nuclear level structure, and the work reported here has added new levels and more specific information. In addition, this is one experiment in which rotational nuclei have been studied in great detail and serves to demonstrate the usefulness and feasibility of studying complex nuclei with beams of heavy ions at energies near the coulomb barrier.
Appendix A. Detection of Inelastically Scattered Particles

1. Solid Angle of junction detectors

Eight junction detectors were used for quantitative measurements (see Fig. 59). Only two different positions are involved for scattering angles, and are denoted as A and B. The solid angle was calculated by dividing these detectors into 2 mm by 2 mm squares and using \( d\Omega = d\Delta \cos \theta/r^2 \), where \( \theta \) is the laboratory scattering angle and \( d\Omega \) is in steradians. The sensitive surfaces of the junction detector were in a plane 2.56 cm from the target.

Each unit of solid angle \( d\Omega \) at angle \( \theta \) was converted to the center-of-mass system by

\[
\sin (\theta_{\text{cm}} - \theta) = \frac{M_1}{M_2} \sin \theta
\]

\[
\frac{d\Omega_{\text{cm}}}{d\Omega} = \frac{\sin \theta_{\text{cm}} d\theta_{\text{cm}}}{\sin \theta d\theta} = \frac{\sin \theta_{\text{cm}}}{\sin \theta} \left[ 1 - \frac{M_1 \cos \theta}{\sqrt{1 - \frac{M_1}{M_2} \sin^2 \theta}} \right]
\]

where \( M_1 \) and \( M_2 \) are the masses of the projectile and target and \( \theta_{\text{cm}} \) is the center-of-mass scattering angle. The individual solid angle elements in the laboratory and center-of-mass systems are listed in Table XIV for the two different detectors. Each five lines are the elements of a vertical slice (see Fig. 58). The sums of \( \Delta\Omega \) over ranges of \( 2^\circ \) in \( \theta_{\text{cm}} \) are listed in Table XV,
Table XIV

Solid Angle of Junction Detectors

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<tr>
<th>$\pi-\theta$</th>
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Table XIV (contd.)

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*Six-degree annulus
with \( \theta_{cm} \) given as the center of the ring. Calculations of yields were made with six-degree wide annular rings, for which \( \Delta \Omega \) is given in Column 5.

2. Normalization of efficiencies

The target chamber used for gamma-particle spectra (Fig. 16) could not be completely insulated and serve as a large Faraday cup for beam integration. This is because the junction detectors were an integral part of the chamber, and one junction detector lead was grounded. Only the target was insulated, and the loss of electrons from its surface when bombarded by the beam gave incorrect current readings. The proper normalization factor was obtained by comparing the intensities of gamma rays from \( {\text{Sm}}^{152} \), \( {\text{Sm}}^{154} \), and \( {\text{Sm}}^{148} \) in direct spectra taken with the chamber used in gamma-particle spectra. Since the position of the counters were different in the two cases, the normalization factor also includes the relative efficiency. The gamma rays used in \( {\text{Sm}}^{152} \) and \( {\text{Sm}}^{154} \) spectra were the first two ground state band transitions, corrected for angular correlations from calculations by Alder.\(^{59}\) The first excited state de-excitation was used in \( {\text{Sm}}^{148} \), corrected for angular correlations from direct excitation theory.

The normalization factor \( F \) is equal to

\[
F = \frac{\eta_2 I_2}{\eta_1 I_1}
\]

where \( \eta \) is the photopeak efficiency and \( I \) is the integrator reading. Subscripts 1 and 2 refer to gamma-gamma and gamma-
particle positions. For Sm$^{152}$ $F = 0.52 \pm 0.03$, while for all targets $F = 0.49 \pm 0.03$. The difference is due to the larger physical size of the absorber used for the Sm$^{152}$ measurements. The normalization has been included in tables of intensities by writing the integrated current value given there as $F$ times the indicated current reading.

3. Multiple Coulomb Excitation Calculations

The multiple coulomb excitation of ground state rotational bands has been calculated with two different methods for correcting for finite $\xi$ values. The first (number 1 in Table V) includes a first-order correction, as given by Eq. 90. The second (number 2 in Table V) includes $\xi$ directly in the values for $P_j$. Examples of both calculations are given in Table XVI for Sm$^{154}$ at 49 Mev, where $\xi = 0.0417$ and $q = 1.920$. 
### Table XVI: Multiple Coulomb Excitation of Sm$^{154}$ Ground State Band.

<table>
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<tr>
<th>$\theta$</th>
<th>$q(\theta)/q$</th>
<th>$q(\theta)$</th>
<th>$\Delta \Omega$</th>
<th>$P_2(\xi=0)$</th>
<th>$\xi A_2$</th>
<th>$P_4(\xi=0)$</th>
<th>$\xi A_4$</th>
<th>$P_6(\xi=0)$</th>
<th>$\xi A_6$</th>
<th>$P_8(\xi)$</th>
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<th>$\xi A_8$</th>
<th>$P_{10}(\xi)$</th>
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<td>0.022</td>
<td>0.534</td>
<td>0.166</td>
<td>0.011</td>
<td>0.155</td>
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<table>
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<th>$P_2(\xi=0)$</th>
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<td>0.962</td>
<td>1.847</td>
<td>0.191</td>
<td>0.552</td>
<td>0.022</td>
<td>0.530</td>
<td>0.157</td>
<td>0.010</td>
<td>0.147</td>
<td>0.0173</td>
<td>0.0013</td>
<td>0.0160</td>
<td>0.0008</td>
</tr>
<tr>
<td>150</td>
<td>0.940</td>
<td>1.805</td>
<td>0.193</td>
<td>0.547</td>
<td>0.022</td>
<td>0.525</td>
<td>0.147</td>
<td>0.010</td>
<td>0.137</td>
<td>0.0155</td>
<td>0.0011</td>
<td>0.0144</td>
<td>0.0006</td>
</tr>
<tr>
<td>144</td>
<td>0.915</td>
<td>1.757</td>
<td>0.073</td>
<td>0.541</td>
<td>0.021</td>
<td>0.520</td>
<td>0.136</td>
<td>0.009</td>
<td>0.127</td>
<td>0.0134</td>
<td>0.0010</td>
<td>0.0124</td>
<td>0.0008</td>
</tr>
</tbody>
</table>

First-Order Correction in $\xi$:

Direct Inclusion of $\xi$:
Appendix B. Electronic Circuitry

The block diagrams of the electronic circuitry used have been described in Chapter IV. All units are based on conventional circuits but for the linear gates. The circuit diagrams for the vacuum tube and diode linear gates used in this experiment are given in Fig. 69 and 70.
HIGH LEVEL LINEAR GATE.

FIG. 69
FOUR-DIODE LINEAR GATE

FIG. 70
Appendix C. Target Preparation

Samarium targets were prepared from a mixture of equal weights of samarium oxide and lanthanum filings. The materials were placed in a tantalum crucible (Fig. 23) in a standard bell jar. The evaporation process is as follows:

1. The current is gradually raised to outgas the cylinder and its contents. This is done by continually increasing the current while keeping the pressure below $10^{-4}$ mm Hg. At about 600° C (cylinder appears dull red) further increases of current do not raise the pressure, and outgassing is considered to be completed.

2. The target and backing are then swung over the cylinder mouth. The current is slowly increased (over about 10 minutes) until the cylinder is dull orange in color (about 1100° C). This insures that the target backing is hot and that the samarium will stick to it.

3. During this period the pressure falls due to the gettering action of the hot lanthanum. This precedes the initial evaporation of samarium. A small dark spot will appear on the target backing. The current must continue to be increased at about the same rate as above or the evaporation will cease.

4. After the target surface is completely covered the water cooling is turned on. The current is increased until the cylinder is bright orange or orange-yellow (about 1200° C) and
is left there until evaporation is completed. Higher temperatures
are undesirable since the unreacted lanthanum starts to evaporate
at about 1300° C (yellow-white color). Termination of evaporation
is signalled by a slow rise in pressure due to the lanthanum
being all reacted and no longer pumping. This will be from 15
to 40 minutes after the evaporation started.

For 290 mg. Sm₂O₃ and 290 mg. lanthanum filings, about
100-120 mg. would land on a target area 7/8" in diameter,
representing an average of 25-30 mg/cm². A graph of thickness
variation over the surface for several different targets is shown
in Fig. 71. The total amount of samarium on the target can be
increased by 50% by pulverizing the slag remaining in the
cylinder, adding an equal weight of lanthanum filings, and
repeating the evaporation process. Another possibility is to
start with a larger cylinder and a greater quantity of lanthanum.
Samarium landing on the target holder can be reclaimed. The
use of a larger cylinder would undoubtedly lead to a more
uniform target thickness.
THICKNESS OF EVAPORATED SAMARIUM TARGET.

POINTS ARE FOR 4 DIFFERENT 7/8" DIAMETER TARGET.

\[
\frac{d}{d_0}
\]

DISTANCE FROM CENTER

FIG. 71
As discussed in Chapter IV, the direction-direction angular correlation of gamma rays de-exciting from states populated by direct coulomb excitation is written in terms of a particle coefficient $a_k$ and the angular correlation coefficients $A_k$ for the analogous gamma ray cascade. As in Eq. 76,

$$W(\theta_\gamma) = \sum_{\text{even } k} a_k A_k P_k (\cos \theta_\gamma)$$

where $\theta_\gamma$ is the angle between the incident beam and gamma ray directions. The correlation is attenuated by the finite size of the counter, which may be corrected for by the use of attenuation coefficients, so that

$$W(\theta_\gamma) = \sum_{\text{even } k} a_k A_k (J_k/J_0)P_k (\cos \theta)$$

The coefficients and correlations for gamma rays measured at $90^\circ$ to the beam direction are listed in Table XVII for the cases involved in the data.

For gamma-gamma coincidence measurements a triple correlation is involved between the beam direction and the two gamma ray directions. The triple correlation for three gamma rays is written as the product of Racah coefficients and spherical harmonics, summed over the quantum of the three gamma rays. For the direct coulomb excitation process instead of the first
Table XVII

Angular Correlations

**Double Correlations**

<table>
<thead>
<tr>
<th>$E_\gamma$</th>
<th>$a_2^t$</th>
<th>$A_2$</th>
<th>$J_2/J_0$</th>
<th>$a_4^t$</th>
<th>$A_4$</th>
<th>$J_4/J_0$</th>
<th>$W(90^\circ)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>334</td>
<td>0.458</td>
<td>0.357</td>
<td>0.84</td>
<td>0.011</td>
<td>1.143</td>
<td>0.53</td>
<td>0.929</td>
</tr>
<tr>
<td>551</td>
<td>0.597</td>
<td>0.357</td>
<td>0.86</td>
<td>-0.006</td>
<td>1.143</td>
<td>0.58</td>
<td>0.907</td>
</tr>
<tr>
<td>811</td>
<td>0.711</td>
<td>0.357</td>
<td>0.94</td>
<td>-0.028</td>
<td>1.143</td>
<td>0.75</td>
<td>0.871</td>
</tr>
<tr>
<td>1087</td>
<td>0.800</td>
<td>0.357</td>
<td>0.94</td>
<td>-0.049</td>
<td>1.143</td>
<td>0.75</td>
<td>0.850</td>
</tr>
</tbody>
</table>

**Triple Correlations**

<table>
<thead>
<tr>
<th>$E_\gamma$</th>
<th>$a_2^t$</th>
<th>Coeff. of $P_0$</th>
<th>Coeff. of $P_2$</th>
<th>$W(90^\circ)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>444</td>
<td>0.711</td>
<td>-0.615</td>
<td>-0.0439</td>
<td>0.964</td>
</tr>
<tr>
<td>689</td>
<td>0.711</td>
<td>-0.946</td>
<td>0.136</td>
<td>1.072</td>
</tr>
<tr>
<td>811</td>
<td>0.800</td>
<td>-0.936</td>
<td>0.142</td>
<td>1.076</td>
</tr>
</tbody>
</table>
gamma ray, the correlation is multiplied by the particle coefficient \( a_k \), which has the quantum number of the first gamma ray. The triple correlation for coulomb excitation from an initial level \( I \), to a level \( J_1 \), followed by a gamma ray cascade to levels \( J_2 \) and \( I_2 \), is given by

\[
W(a_2) = 2 \sum_{k_1,k_2,k_{12}} (-)^{\Psi} Z_1(L_1 J_1 L_1', J_1', I_1 k_1) a_{k_1}
\]

\[
Z_1(L_2 J_2 L_2', I_2 k_2) (L_{12} L_{12}', -l l | k_{12} 0)(k_{12} 0 0 | k_{12} 0)
\]

\[
X \left( \begin{array}{c} J_1 L_1 \ J_2 \ \\
\ k_1 \\
\ k_{12} \\
\ J_1' L_1' \ J_2'
\end{array} \right) ^{i -L_{12} -L_{12}' - \pi_{12} - \pi_{12}' - k_1 - k_2 + 2}
\]

\[
\sqrt{(2L_{12} + 1)(2L_{12}' + 1)(2k_1 + 1)(2k_2 + 1)} \ P_{k_{12}} (\cos \theta_{12})
\]

\[
\Psi = I_1 + I_2 + L_1 + L_1' + L_2 + L_2' + J_1 + J_2
\]

This expression is written for detection of the two gamma rays in a plane, the first at angle \( \theta_{12} \) to the beam direction, and the second at \( 0^\circ \) with respect to the beam. Subscripts 1, 12, and 2 indicate the first, second and third transitions. The multipolarity is given by \( L \) and the parity by \( \pi \). Primes indicate interfering terms. The Z and X coefficients are defined in reference 76, which also gives tables of values.

Calculations were carried out for the spin sequences
$0 \rightarrow 2 \rightarrow 4 \rightarrow 2$ and $0 \rightarrow 2 \rightarrow 2 \rightarrow 0$ for pure E2 transitions. The individual terms in the sum over k values are given in Table XVIII for the coefficients of $P_0$ and $P_2$. The $P_4$ term is neglected. Factors common to all terms have been omitted.

Attenuation of angular correlations in this case involves two coefficients since there are two gamma rays detected. The attenuation factors $(J_{k_1}^1/J_0)$ $(J_{k_2}^2/J_0)$ are included in the sum over k values.
Table XVIII

Triple Correlation Coefficients
for Pure E2 Transitions

<table>
<thead>
<tr>
<th>$k_1' k_{12} k_2$</th>
<th>Spin Sequence $0 \rightarrow 2 \rightarrow 2 \rightarrow 0$</th>
<th>Spin Sequence $0 \rightarrow 2 \rightarrow 4 \rightarrow 2$</th>
<th>$p_{k_1 k_{12}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>000</td>
<td>$-1.000 \ a_0$</td>
<td>$-0.200 \ a_0$</td>
<td>$p_0$</td>
</tr>
<tr>
<td>202</td>
<td>$0.0766 \ a_2$</td>
<td>$-0.642 \ a_2$</td>
<td>$p_0$</td>
</tr>
<tr>
<td>404</td>
<td>$-0.326 \ a_4$</td>
<td>$-0.0185 \ a_4$</td>
<td>$p_0$</td>
</tr>
<tr>
<td>022</td>
<td>$0.0766 \ a_0$</td>
<td>$-0.0401 \ a_0$</td>
<td>$p_2$</td>
</tr>
<tr>
<td>220</td>
<td>$0.0766 \ a_2$</td>
<td>$-0.0204 \ a_2$</td>
<td>$p_2$</td>
</tr>
<tr>
<td>222</td>
<td>$-0.1068 \ a_2$</td>
<td>$-0.0082 \ a_2$</td>
<td>$p_2$</td>
</tr>
<tr>
<td>224</td>
<td>$0.2125 \ a_2$</td>
<td>$0.0170 \ a_2$</td>
<td>$p_2$</td>
</tr>
<tr>
<td>422</td>
<td>$0.0834 \ a_4$</td>
<td>$0.0033 \ a_4$</td>
<td>$p_2$</td>
</tr>
<tr>
<td>424</td>
<td>$0.2125 \ a_4$</td>
<td>$-0.0172 \ a_4$</td>
<td>$p_2$</td>
</tr>
</tbody>
</table>
Appendix E. Computer Programs

The program for calculation of thick target yields with first order perturbation theory is listed below. It performs the integration of Eq. 105 over energy, where the cross section is given by Eq. 110. Total cross section functions are included in the program as a series obtained from a least squares fit to the tabulated values in ABHW. The format of the control card is obvious from an examination of the first 11 statements.
C THICK TARGET YIELD FOR $E_1$, $E_2$, $E_3$ COULOMB EXCITATION
DIMENSION SIG(200), T1(8), T2(8), T3(8)
1 READ 2, Z1, A1, Q, Z2, A2, L, EXC, EMAX, EINT, ACC
2 FORMAT (6I5,3F10.5,F10.5)
PRINT 3,EMAX,Z1,A1,Q,Z2,A2
3 FORMAT (1H1,F6.2,17H BEAM OF Z = ,I2,6H, A = ,I2,20H, WITH CHARGE STATE ,I2,27H, ON TARGET NUCLEUS OF Z = ,I2,6H, A = ,I3)
PRINT 4,L,EXC,EINT,ACC
4 FORMAT (4HO E,I1,17H EXCITATION OF A ,F6.4,11H MEV LEVEL./39H ENERGY DECREMENT FOR INTEGRATION IS ,F6.4,5H MEV./26H CUT OFF INTEGRATION AT ,F6.4,15H OF FIRST TERM. )
PRINT 9,L
9 FORMAT (1HO,l8X ,6HE(MEV),l8X,2HXI,17X,2HFE,II ,l6x,4HDEDS,17X,3HSIG X/)
   A1 = FLOATF(A1)
   A2 = FLOATF(A2)
   Z1 = FLOATF(Z1)
   Z2 = FLOATF(Z2)
   Q = FLOATF(Q)
   C = (3.761E9)/(Q*A2)
   A = 1.0 + A1/A2
   EP = A*EXC
DO 10 J = 1,200
   SIG(J) = 0.0
   XF = Z1*Z2*SQRTF(A1)*EP/12.65
   JMAX = XFIXF(EMAX/EINT)
   E = EMAX
   GO TO (100,200,300),L
10 SIG(J) = 0.0
   XF = Z1*Z2*SQRTF(A1)*EP/12.65
   JMAX = XFIXF(EMAX/EINT)
   E = EMAX
   GO TO (100,200,300),L
100 CE1 = 0.02498*A1*Z1*Z1
   DO 150 J = 1,JMAX
   XE = (E - 0.5*EP)**1.5
   X = (XF/XE)**(1.0 + 0.15625*(EP/E)*(EP/E))
   T1(1) = 4.90089789
   T1(2) = -9.00311865
   T1(3) = 6.77281409
   T1(4) = -9.67281409
   T1(5) = 7.88244605
   T1(6) = -3.59138645
   T1(7) = 0.839742757
   T1(8) = -.0768461287
   P = 1.0
   Z = 0.0
   DO 160 K = 1,8
      Z =Z + P*T1(K)
160 P = P*X
   FE1 = EXPF(Z)
   DEDS = 1.0/(0.002*E + 0.297)
   SIG(J) = CE1*(C/E)*(FE1/DEDS)
   PRINT 11,E,X,FE1,DEDS,SIG(J)
11 FORMAT (1HO ,5E20.7)
   IF (SIG(J) - ACC*SIG(1)) 170,150,150
150 E = E - EINT
   GO TO 1000
170 KL = J
   GO TO 1000
KL = JMAX
GO TO 1000

DO 250 J = 1,JMAX
XE = (E - 0.5*EP)**1.5
X = (XF/XE)*(1.0 + 0.15625*(EP/E))
T2(1) = -0.110934921
T2(2) = 0.353712238
T2(3) = -8.39590460
T2(4) = 8.18525068
T2(5) = -5.20406827
T2(6) = 1.99154310
T2(7) = -0.410058461
T2(8) = 0.0342512675
P = 1.0
Z = 0.0
DO 260 K = 1,8
Z = Z + P*T2(K)
260 P = P*X
FE2 = EXPF(Z)
DEDS = 1.0/(0.002*E + 0.297)
SIG(J) = CE2*C*(E - EP)*FE2/DEDS
PRINT 11,E,X,FE2,DEDS,SIG(J)
IF (SIG(J) - ACC*SIG(1)) 270,250,250
270 KL = J
GO TO 1000
250 E = E - EINT
KL = JMAX
GO TO 1000

300 CD = (A*Z2)**4
CE3 = 929.8*A1/(CD*Z1*Z1)
DO 350 J = 1,JMAX
XE = (E - 0.5*EP)**1.5
X = (XF/XE)*(1.0 + 0.15625*(EP/E)*EP/E)
T3(1) = -3.27113181
T3(2) = 0.0578985855
T3(3) = -1.72601394
T3(4) = -2.31470555
T3(5) = 3.27025540
T3(6) = -1.78556986
T3(7) = 0.457133405
T3(8) = -0.0441144727
P = 1.0
Z = 0.0
DO 360 K = 1,8
Z = Z + P*T3(K)
360 P = P*X
FE3 = EXPF(Z)
DEDS = 1.0/(0.002*E + 0.297)
SIG(J) = CE3*C*(E - EP)*(E - EP)*FE3/DEDS
PRINT 11,E,X,FE3,DEDS,SIG(J)
IF (SIG(J) - ACC*SIG(1)) 370,350,350
370 KL = J
GO TO 1000
1000 Y = SIG(1)
   DO 1001 K = 3,KL,2
1001 Y = Y + (4.0*SIG(K-1) + 2.0*SIG(K))
   YIELD = Y*EINT/3.0
   PRINT 5,YIELD
   5 FORMAT (61HOTHICK TARGET YIELD PER MICROCOULOMB, DIVIDED BY B VALU
   XE, IS ,E15.7)
   GO TO 1
END
VIII. References

33. K. Alder, A. Bohr, T. Huus, B. Mottelson and A. Winther, Rev. of Mod. Phys. 28, 432 (1956).
48. N. P. Heydenburg and G. M. Temmer, Phys. Rev. 93, 351 and 906 (1954); 94, 1399 (1954); 95, 861 (1954); 96, 426 (1954); 98, 1308 (1955); 100, 150 (1955); 104, 489 (1956).


61. E. Fairstein, Instrumentation and Controls Division Annual Report for Period Ending July 1, 1957, Oak Ridge National Laboratory.


86. G. A. Burginyon and J. S. Greenberg, private communication.


Using our measured value of 4 s.p. units for the quadrupole reduced transition probability to the $2^+$ member of the beta band we find that mixing of the beta and ground state band wave functions can account for only a small fraction of the 2nd order term in the energy expansion. This is a tentative conclusion as we are still analyzing existing data. We hope to extend these measurements on a tandem accelerator where $\gamma\gamma P$ triple coincidence measurements would provide a clean expt'l. determination of the necessary branching ratios.

\[ \text{Gd}^{158} \]

We excited the ground state band in this nucleus up to the $8^+$ level as well as the $2^+$ member of the gamma band confirming previous results. A search for beta band levels up to several MeV failed to detect such a band.

\[ \text{Nd}^{142}, \text{Nd}^{144}, \text{Nd}^{146}, \text{Nd}^{148} \]

We have coulomb excited all of these nuclei for the spectroscopic value and to obtain the $B(E2, 0^+\rightarrow 2^+)$ values.
The tentative decay scheme is shown here. The 340 keV transition which we assign to the $6^+ \rightarrow 4^+$ member of the ground state band has not been seen before and this $6^+ \rightarrow 4^+$ deexcitation was previously reported at 405 keV. The variation of this $\gamma$-ray with energy, the ability to predict its intensity from M.C.E. theory, and $\gamma\gamma$ coincidence measurements leads us to believe our assignment is correct. At higher incident ion energies we observed a line at 420 keV which we tentatively assign to the $8^+ \rightarrow 6^+$ deexcitation of the ground state band. Our data do not allow an unambiguous assignment of either the energy or the position of the 740 keV $\gamma$-ray in the decay scheme. We tentatively show this transition emanating from the $4^+$ level of the beta band but it could as well originate from an octipole level.

To evaluate the mixing we need experimental branching ratios from deexcitation of the $2^+$ member of the beta band. It is difficult to obtain a "good" experimental ratio from existing $\gamma P$ coincidence data as will be clear from the following considerations. The 840 keV transition is very weak limiting its usefulness. The 450 keV transition is partially obscured by the 460 keV $2^+ \rightarrow 0^+$ transition in Nd$^{146}$, a 1.03% target contaminant, and the 710 keV line by the 691 keV $2^+ \rightarrow 0^+$ transition in Nd$^{144}$, a 0.89% contaminant. Using the ratios of these contaminant lines to the 131 keV Nd$^{150}$ line in natural Nd we have corrected for the contaminant contributions to the 450 and 710 keV lines. The composite nature of the 710–740 keV peak as well as the contaminant problem could presumably be resolved by $\gamma\gamma$ coincidence measurements. Such measurements were made on the HILAC but due to a small amount of CaF$_2$ contamination inherent in the process we use to make these targets the resulting spectra were not "clean" enough to provide the necessary intensities.