BORON INDUCED TRANSFER REACTIONS

by

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1966
To my mother and father

and to Alice
ABSTRACT

Single-nucleon and deuteron transfer reactions induced by 115.9-MeV $^7$Be beams have been observed on $^{12}$C, $^{14}$N, $^{15}$N, $^{16}$O, and $^{20}$Ne targets, with the primary objectives of elucidating the reaction mechanisms and extracting information of a spectroscopic nature relevant to the interacting nuclear systems. For each target, data on the ($^7$B, $^7$He) and ($^7$B, $^7$Be) neutron and proton transfer reactions and the ($^7$B, $^7$Be) deuteron transfer reaction are presented, consisting of forward-angle energy spectra of the beryllium and boron products of these reactions and absolute differential cross section measurements for states appearing in the spectra.

All transfer reactions are observed to exhibit highly selective population of a relatively small number of final states, and evidence is presented that direct transfer represents the dominant mechanism. These reactions are further characterized by a preferential population of certain of these states, whose yields are large relative to all others in the same spectrum.

The ($^7$B, $^7$He) and ($^7$B, $^7$Be) single-nucleon transfer reaction data are consistent with the preferential population of states of a common single-particle configuration. The levels preferentially populated in the ($^7$B, $^7$Be) deuteron transfer reactions are the previously observed giant excitations of the deuteron transfer reaction and are associated with high angular momentum states of extremely pure two-particle configuration. The giant excitations of the reactions studied are interpreted on the basis of simple angular momentum arguments in terms of the capture of the transferred nucleons into $d_{5/2}$ and $f_{7/2}$ single-particle states.

A systematic comparison of equivalent ($\alpha$,d) and ($^7$B, $^7$Be) reactions reveals a high degree of similarity regarding both selective and preferential population of final states, including an unexpected similarity of the isobaric-spin configurations. The consistency of the integrated cross sections of the giant excitations of the two reactions and the equivalence of the corresponding angular distributions indicate an identical direct transfer mechanism. The mass dependence of the cross sections can be interpreted in terms of the shell structure of the respective target nuclei.

Ratios of neutron and proton transfer cross sections to analog states in the final systems of the ($^7$B, $^7$He) and ($^7$B, $^7$Be) reactions are utilized in the extraction of relative spectroscopic factors for the incident systems, and these data are used to obtain the ($^7$B$^{10}$+n) and ($^7$Be$^{10}$+p) parentage of the $^7$B$^{11}$ ground state. Calculations of the corresponding ratios and parentage based on available model wavefunctions are in good accord with the experimental results.

A particle identification system developed for this work based on the $dE/dx$ and $E$ method and a multiparameter analyzer is described. A semiempirical method for the calculation of energy loss for heavy ions has been developed and is shown to be in agreement with available stopping power data. Suggested experimental extensions of these studies are given.
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A. The Heavy-Ion Transfer Reaction

The heavy-ion transfer reaction is that in which a nucleon or cluster of nucleons is exchanged during the scattering of two complex nuclei. These reactions were first observed experimentally a little over a decade ago\(^1\) and have since been the subject of extensive theoretical and experimental study.\(^2\) Reviews of this work have been presented during this period by a number of authors\(^3-7\)

Transfer reactions have traditionally been considered a part of the broader field of heavy-ion physics. However, the emergence of the general properties of heavy-ion interactions from an ever-increasing body of experimental and theoretical information and the establishment of the qualitative characteristics of many of these reactions have resulted in a gradual integration of heavy-ion studies into the general field of nuclear structure and reaction physics. Present studies of transfer reactions, in common with companion studies involving lighter projectiles, share the separate but related objectives of elucidating the reaction mechanisms and of providing information on the structure of the nuclei involved.

There are a number of advantages inherent in the utilization of heavy projectiles in nuclear reaction studies. Because of their short mean free path in nuclear matter, heavy-ion reactions are localized in the nuclear surface and are highly selective in populating collective states strongly coupled to the ground state in scattering and particularly simple single or multi-particle or hole states in transfer reactions. Large angular momenta are characteristic of heavy projectiles, thus providing a mechanism for the formation of otherwise inaccessible states of high angular momentum. Evidence will be adduced later that an additional specific mechanism exists leading to an enhanced population of these high-spin states in transfer reactions. The complexity of the reaction products paradoxically enables a reasonably accurate specification of the reaction mechanism to be made in many cases, and the possibility of transfer-ringing several nucleons allows the examination of multi-nucleon correlations in
the nuclei involved. Finally, the features of many heavy-ion interactions may be easily understood in terms of classical analogs, which, although not detailed, furnish the physical insight required for the proper assessment of more complete theories.

It should be noted that there are attendant limitations associated with the use of heavy projectiles. The lack of detailed nuclear structure calculations for heavy nuclei has resulted in the absence of a general microscopic procedure for the extraction of absolute spectroscopic factors from experimental data. Recent evidence, both experimental and theoretical, suggests that only very restricted nuclear structure information is derivable from transfer angular distributions, at least when studied at relatively high energies. The characteristically large number of open reaction exit channels results in a general depletion of the cross section for any given process, and the resultant large number of reaction products poses formidable problems in the experimental isolation of a particular nuclear species for analysis. The high energy loss rates of complex nuclei often demand correction for energy losses in absorbing materials which are present experimentally and, more fundamentally, make it difficult to achieve adequate resolution to separate individual residual energy states. There is often an ambiguity, characteristically not present for light projectiles, in the assignment of observed final state excitations for reactions involving binary exit channels wherein both nuclei possess low-lying stable excited states.

Even in the simplest approximation, the transfer reaction remains a complicated theoretical problem which involves a nuclear system comprising at least three members, the two heavy nuclear cores and the transferred nucleon or nucleon cluster. General techniques for solving three-body nuclear problems have not as yet been developed to such an extent that they are useful in these situations. However, studies of transfer reactions have shown that these reactions are, nevertheless, often describable in terms of rather simplified nuclear reaction models. This work has also indicated that the nature of the
transfer reaction and the applicability of these models is strongly dependent on
the relation of the energy at which the reaction proceeds to the classical
Coulomb barrier. It has been observed that the most probable transfer reaction
channel at energies below the Coulomb barrier involves the transfer of a single
nucleon, with the probability of cluster transfer increasing as the energy in­
creases above the barrier.

At relatively low energies, below the Coulomb barrier, the repulsive
Coulomb potential between the reactant nuclei dominates all reaction processes.
As a result, this region can be treated quite accurately, since the Coulomb
potential is well defined. The so-called tunneling model has been remarkably
successful in reproducing the behavior of the low-energy single-nucleon transfer
reaction. At energies well above the Coulomb barrier, the effects of the short­
range nuclear forces become important, and the transfer problem is complicated
by the presence of nuclear absorptive processes. Although the situation here
is much less clear than in the low-energy case, it has recently been found that
nuclear diffraction models give a reasonably accurate description of the transfer
reaction in this region. In the intermediate energy region, in the vicinity of the
barrier, both Coulomb and absorptive effects are often important. The treatment
of this region may be further complicated by significant contributions from com­
pound nucleus formation, which are not present in either the low or high-energy
situations and which are not amenable to an equivalent simplified analysis.

The use of the terms high and low energy for heavy ions must be inter­
preted in a relative sense only. The highest energy heavy ions available at
present are produced in the linear accelerators at velocities corresponding to
an energy per incident nucleon of 10 MeV. High energy heavy ions will be
defined to have approximately this energy. Unless otherwise noted, low energies
will refer to energies below and in the vicinity of the respective Coulomb barriers.
The designation of nuclei as heavy ions is itself arbitrary. In the following dis­
cussions, heavy ions will be taken as those nuclei with atomic number greater
than that of beryllium.
B. Single Nucleon Transfer

Much of the original motivation for heavy ion studies was provided by Breit, Hull, and Gluckstern\(^{(8)}\) who pointed out the possibility of giving a semiclassical description of single-nucleon transfer reactions below the Coulomb barrier and of employing these reactions as a probe of the nucleon density at the nuclear surface. Breit and Ebel\(^{(9)}\) later presented a quantitative theory, based on this approach, which showed that the transfer of a nucleon between two nuclear surfaces depends upon the reduced widths of the nucleon bound states. This treatment constituted the first investigation of the theoretical aspects of the heavy-ion transfer reaction and marked the first attempt to formulate a realistic three-body theory in nuclear physics.

In transfer reactions proceeding at energies below the Coulomb barrier, the nuclei never approach near enough for the short-range nuclear interactions to have a direct effect, and the reaction is well described by models in which the transfer is taken into account while the nuclei scatter in the Coulomb field. In the semiclassical theory of Breit and collaborators, the motion of the nuclei before and after the transfer is treated by assuming that they travel on classical Rutherford trajectories, while the transfer itself is treated quantum mechanically. The semiclassical description of this process is characterized by the condition that the Sommerfeld parameter, \(\eta = ZZ' e^2/\hbar v\), be much greater than one. This condition is well satisfied for heavy ions at energies below the Coulomb barrier. The low-energy transfer process is now commonly referred to as the tunneling mechanism, which derives its name from the fact that the transferred nucleon must literally tunnel through the classically forbidden barrier region between the potential wells of the nuclei to which it is initially and finally bound.

Breit and collaborators\(^{(10)}\) have recently readapted the semiclassical theory to include quantal and other effects not included in their earlier work. Numerous other treatments of the tunneling mechanism have been evolved\(^{(11-13)}\) based on the formal techniques of quantum mechanical scattering theory. The
central assumption of the original tunneling model, that the transfer is essentially a perturbation of the classical trajectories, remains a common feature in these treatments. The tunneling model is thus expected to be applicable for those single-nucleon transfer reactions for which the nuclei involved do not come into contact, situations typified by energies below the Coulomb barrier and large values of the Sommerfeld parameter.

Experimental data at energies below and in the vicinity of the Coulomb barrier consist mainly of single nucleon transfer angular distributions and excitation functions. The observed angular distributions show a typical forward peak, whose position, height, and width vary systematically with energy, and the excitation functions are characterized by an initial sharp increase followed by a gradual leveling off. The tunneling model provides good agreement with the data below the barrier (14-19). The small-angle rise of the differential cross section and the low-energy increase of the total cross sections are specific predictions of the model. The peaking of the angular distributions and the saturation of the excitation functions are attributed to the onset of specifically nuclear processes which become possible when the nuclei come into contact.

These gross characteristics of the low-energy transfer data provide a striking illustration of the previously-noted surface nature of the transfer process. At large values of the classical impact parameter, corresponding to small scattering angles, the cross section decreases because the Coulomb trajectories do not deliver the transferred nucleon to a significant overlap with the final state wavefunction. At small impact parameters, or large angles, the cross section again decreases, reflecting the inception of nuclear absorption.

It has always been assumed that transfer reactions, because of their extreme surface nature, would be particularly useful in determining detailed information concerning nucleon wavefunctions at the nuclear surface. The tunneling model demonstrated the possibility of obtaining such information from low-energy single-nucleon transfer reactions in terms of the nucleon...
reduced widths. Unfortunately, these expectations have never been fully realized. Although reduced widths have been extracted from the low-energy data in some cases, these data must still be regarded as preliminary. The situation at high energies is even less well defined.

For transfer reactions proceeding at energies significantly above the Coulomb barrier the nuclear wavefunctions overlap appreciably, and the transfer process is characterized by the presence of a large number of open reaction channels, which gives rise to strong nuclear absorption. In this connection, it is customary and convenient to define absorption as any process by which particles are removed from the incident channel.

Several reaction models have recently been developed which have had some success in treating absorption in transfer reactions at high energies. These models are similar in that they represent the effects of nuclear absorption by effectively eliminating a spherical spatial region in the calculation of the transfer amplitudes. This confines the reaction zone to a small spatial region and means that only a relatively narrow band of angular momentum values contributes to the transfer process. Particles moving on trajectories corresponding to large angular momenta are transmitted with small probability for transfer because of the exponential decrease of the bound-state wavefunctions outside the nucleus. Those moving on trajectories corresponding to small angular momenta are removed by the strong nuclear absorption. This situation suggests an analogy with optical Fraunhofer diffraction by a highly opaque circular disk.

As noted above, because of the lack of a solvable three-body scattering technique, these so-called diffraction models are parameterizations of certain features of the more complex problem and, at least in their present state, must be regarded as predominantly phenomenological in nature.

The diffraction treatment of transfer reactions thus rests on the assumption that only a relatively small number of partial waves participates in the transfer process. The nuclear absorptive effects present at high energies are expected
to give rise to a cross section which oscillates with angle, due to the interference of waves emitted from opposite edges of the absorbing region. In the presence of a strong Coulomb field (large $\eta$), however, the waves in the vicinity of the absorbing region are deflected along Rutherford trajectories and must pass through an absorbing region before they can interfere. As a result of the strong nuclear absorption, little or no interference occurs, and the diffraction oscillations are correspondingly damped.

It had long been assumed that at high energies where Coulomb effects might be considered negligible (low $\eta$) the characteristic diffraction oscillations would reappear in the transfer angular distributions, and indeed, several diffraction models making this specific prediction have appeared. Measurements at Yale during the past two years have shown, however, that the oscillations anticipated in low-$\eta$ situations are not present in the experimental data. Following on this discovery, several more recent diffraction models have been evolved which succeed in predicting the required damping of the diffraction oscillations in the non-Coulombic case, although different mechanisms are proposed as responsible for the damping.

The first diffraction model to correlate a number of observations of single-nucleon transfer reactions at energies above the Coulomb barrier, in this case involving heavy targets and thus high-$\eta$ values, was that of Frahn and Venter. In this model, the transfer reaction is treated as an equivalent two-body process within the framework of their previously developed strong-absorption model for elastic scattering. The nuclear absorption and Coulomb effects are parameterized in terms of the characteristic properties of a surface reaction. The salient features of this work are the demonstration of the above-mentioned Coulomb damping for high-$\eta$ reactions and the prediction of oscillatory angular distributions for low-$\eta$ reactions. Frahn and Venter propose a damping mechanism based on a diffuse surface on the absorbing region, however, it is found that, if appropriate model parameters are used,
oscillations are still predicted for low-\(\eta\) situations such as those, noted above, for which they are not observed. In consequence, this model has been shown to be inadequate in situations such as that under consideration in the present work.

The finite-range diffraction model of Dar\(^{24,25}\) is an extension of the work of Frahn and Venter, in that it parameterizes the surface features of the transfer reaction and again shows the effects of Coulomb and diffuse-surface damping. The principal difference in the two treatments is that the finite-range model is a three-body theory which provides for an interaction between the transferred nucleon or cluster and the heavy cores to which it is initially and finally bound and thus explicitly includes nuclear structure effects. A significant prediction of this model is the persistence of the damping of the diffraction structure of transfer angular distributions in the case of negligible Coulomb interference. It is proposed that this damping is a direct result of incorporating details of nuclear structure into the formalism, and in particular, is due to specific angular momentum effects in the transfer reaction.

An alternate mechanism for the damping of transfer angular distributions in the absence of Coulomb damping has been proposed by Dodd and Greider\(^{26}\) within the diffraction formalism. They point out that all other diffraction theories neglect consideration of nuclear recoil effects, as manifested in terms of the order of the ratio of the masses of the transferred cluster and heavy core to which it is bound, and that most of these treatments employ a zero-range approximation. A prediction of the Dodd and Greider recoil model is that the usual oscillatory nature of the cross section for a strongly absorbing process disappears and is replaced by a smooth angular distribution, even without employing a diffuse nuclear surface, provided that both recoil and finite range are properly accounted for. The predicted angular distributions have the mass dependence implicit in the recoil, but no other dependence on energy, structure, or particle species. A second prediction of this model is the preferential population of residual states of high angular momentum in transfer reactions. Both
results are independent of the details of the reaction mechanism and are a reflection of the three-body nature of the transfer process.

An alternate approach to the heavy-ion transfer reaction at high energies is provided by the complex potential model, often referred to as the distorted-waves Born approximation (DWBA) method. This model was originally designed to describe scattering situations in which strong nuclear absorption is not a dominant factor, but has since been extended for use in heavy-ion reactions.\(^{(27)}\)

In the DWBA theory of direct reactions, elastic scattering is assumed to be the dominant process, and the interaction which is responsible for the transfer is treated as a perturbation of the average interaction of the reactant nuclei. The relative motion of the pairs of nuclei before and after the transfer is described by distorted waves, and the transfer is regarded as a transition between elastic scattering states. The distorted waves are generally obtained directly from an optical potential which reproduces the observed elastic scattering of the nuclei in question at the proper energy. Although the strong absorption condition is adequately accounted for by the imaginary part of the optical potential, there is at present some question of the validity of the complex potential model in the description of heavy-ion interactions. The question concerns the physical significance of the concept of an average single-particle potential, as used here, in processes involving the overlap of strongly interacting complex nuclei. It should be noted that the other three-body reaction theories discussed above are also expressed within the framework of the DWBA. However, these treatments employ analytic forms for the scattering waves and thus incorporate strong absorption without reference to such a local complex potential.

Further differences in the models which have been discussed concern the feasibility of extracting nuclear spectroscopic information from experimental data. This information cannot be obtained using either the model of Frahn and Venter or that of Dodd and Greider. The former is a two-body theory and, as such, does not explicitly include nuclear structure effects.
The latter, although a three-body theory, has been shown to yield results which are largely independent of such effects. The diffraction model of Dar is a three-body theory which is capable of making specific predictions based on details of nuclear structure. However, this information can be obtained only through a knowledge of the wavefunctions and interaction potentials of the complex nuclei involved, both of which are too imperfectly known at present to permit the extraction of, for example, absolute spectroscopic factors. The extraction of nuclear structure information using the complex potential model is, in principle, a straightforward procedure, although adequate data to provide appropriate optical model parameters for the entrance and exit channel scattering are as yet available only in isolated cases. Such data are not available for the situations considered in the present studies.

Single-nucleon transfer reactions involving energies below and in the vicinity of the Coulomb barrier or high values of the Sommerfeld parameter, or both, have been studied extensively and have been discussed above in connection with their interpretation in terms of the tunneling mechanism. However, little work has been done on such reactions proceeding at energies significantly above the Coulomb barrier and characterized by relatively small values of the Sommerfeld parameter. One of the primary reasons for this is the experimental difficulty involved in identifying reaction products and final state excitations in these reactions. This problem becomes more complex at higher energies, reflecting the opening of additional reaction exit channels with increasing energy.

Most of the early work on low-energy transfer reactions was done using activation and radiochemical techniques to identify reaction products by means of their known radioactive decay characteristics. This procedure summed contributions from several final energy states and thus precluded the investigation of transfers to specific levels in the residual nuclei. This method was later extended to include range measurements of the radioactive products. These observations have provided information on the transfer population of a
few excited states, however, the attainable resolution and accessible excitation range have been limited. The use of particle counters and kinematic recoil techniques \(^{(31, 32)}\) is adequate in these respects, but this method is limited because it is most suitable for the study of only selected final states and because it requires the detection of both products in a binary reaction channel. Only recently has adequate particle identification instrumentation become available to select isolated final nuclei and residual states for study in high-energy transfer reactions.

Single nucleon transfer reactions in high-energy low-\(\eta\) situations have recently been observed in the \(^{11}_B + ^{12}_C\) system by Sachs et al. \(^{(20)}\) and in the \(^{14}_N + ^{12}_C\) system by Birnbaum and Bromley. \(^{(21)}\) These data consist of energy spectra and angular distributions. The angular distributions, as discussed above, are smooth and fall off exponentially with increasing angle. It has also been noted that the finite-range diffraction model of Dar and the recoil model of Dodd and Greider are able to account for this absence of diffraction structure in low-\(\eta\) reactions, although for quite different reasons, and both have been found to provide a qualitative description of these data. The forward peaking and exponential decrease found in these situations correspond to the back or absorptive side of the sharply rising transfer angular distribution curve observed for the high-\(\eta\) reactions discussed above. The energy spectra are marked by the selective population of a relatively small number of final states in the residual nuclei involved. This behavior has been interpreted on the basis of a direct transfer process in which states of a single-particle nature are populated in the presence of a selective enhancement mechanism.

Greatly renewed interest has recently been focussed on the question of isobaric-spin analog states in terms of information which their study may be able to provide on the validity of the isobaric-spin quantum number in nuclear systems and on the charge symmetry and charge independence of nuclear forces. \(^{(33, 34)}\)

In general, such studies have been carried out using light projectiles, and the
relevant reactions have been sensitive to phenomena in the nuclear interior. Such measurements have characteristically shown agreement, at least in light nuclei, between the isobaric-spin predictions of cross section ratios, for example, and the experimental data. (35,36,37)

This is consistent with the view recently proposed by Robson, (38) who notes that the gradient of the Coulomb field in the nuclear interior might be expected to be sufficiently small to ensure a region of relatively pure isobaric spin. On this basis, the larger gradient in the external regions of the nucleus might be expected to induce significant mixing of isobaric-spin states which would then be observable as departures from the unmixed isobaric-spin predictions. It might be anticipated that heavy-ion transfer reactions would be particularly sensitive to such an effect, since they preferentially involve these external nuclear regions. Measurements of proton and neutron transfer reactions leading to mirror systems in the above-mentioned study of the N\(^{14}\) + C\(^{12}\) system indicate, however, that here too the isobaric spin predictions are in good accord with the experimental results.

C. Two Nucleon Transfer

During the past decade, a substantial fraction of all nuclear reaction studies have been concentrated on the observation and analysis of the single-nucleon transfer reaction. These data have provided a major source of information on the single-particle aspects of nuclear structure and parentage. In comparison, the two-nucleon transfer reaction has received relatively little study, although activity in this area has grown considerably in the past several years. As in the case of single-nucleon transfer, the two-nucleon transfer reaction has been studied most often at low energies and with light projectiles, (38,39) but has only recently been observed with both light and heavy projectiles at high energies. The two-nucleon transfer situation has been the subject of several recent reviews. (40)
Interest in reactions in which two nucleons are transferred stems from their value in the investigation of nuclei and nuclear levels which are not readily accessible with other reactions. For example, the nuclei can be removed by two nucleons from stable targets, and levels having two nucleons or holes excited can be formed which cannot appear, at least in lowest order, in single-nucleon transfer or inelastic reactions. Interest in two-nucleon transfer reactions as spectroscopic probes results from the relevant selection rules on angular momentum, parity, spin, and isobaric spin and from the fact that these reactions afford an opportunity for the study of two-nucleon parentage and correlation effects.

Early studies at the lower energies were mainly concerned with the mechanism of the two-nucleon transfer process and not with details of nuclear structure. At these energies, the mechanism appears to depend strongly on the individual reaction participants, Q values, and barriers involved. Both direct and compound system amplitudes have been found to be important.

At higher energies, reactions induced by mass 3 and 4 projectiles, and their inverses, yield angular distributions which display, in general, the oscillatory structure and forward peaking characteristic of direct surface reactions.\(^{(41-44)}\). The suggestion of a dominant reaction mechanism at high energies has led to the study of the spectroscopic nature of these reactions. However, in reactions induced by Li\(^6\) ions, there is evidence\(^{(45)}\) that a direct dissociation mechanism, rather than transfer, completely dominates the interaction, with extremely weak population of residual states from which such information might be obtained. Two-nucleon transfer with heavier projectiles has seldom been observed. In consequence, spectroscopic studies have, for the most part, involved the use of mass 3 and 4 projectiles.

If the two-nucleon transfer reactions are considered as proceeding via a direct interaction mechanism, they are similar in many respects to the corresponding single-nucleon transfer reactions induced by light (A \(\leq\) 4) projectiles.
It is therefore not surprising that the theory of two-nucleon transfer has
developed within the framework of the Born approximation along the lines
originally laid out for the analysis of the single-nucleon process. Two-nucleon
transfer was first considered by El Nadi\(^{(46)}\) who calculated the transfer cross
section in the plane-wave Born approximation. The plane-wave theory was
later extended in a number of treatments\(^{(47-49)}\) which basically represent
varying degrees of completeness in incorporating nuclear structure effects
into the formalism.

All the plane-wave calculations predict the same form for the transfer
angular distribution, which, just as for the single-nucleon case, is of an oscil-
latory nature and is characterized by the orbital angular momentum transferred
in the reaction. However, these results clearly demonstrate the essential
difference in the two reactions. In one case, the orbital angular momentum
is carried by a single nucleon, and further, only the lowest allowed value
has been found to contribute significantly to the observed angular distributions.
In the two-nucleon case, the angular momentum is carried by the pair of
nucleons, and several configurations of the two nucleons can contribute to a
given momentum transfer. The result is an angular distribution involving a
sum over the allowed angular momentum contributions, with the extent of each
contribution determined by structure-dependent weighting factors.

In analogy with the more general formulation of the single-nucleon problem
in terms of the DWBA theory, a number of such treatments have recently been
evolved for the two-nucleon transfer reaction\(^{(50-54)}\) These theories again
vary somewhat in the extent to which nuclear structure is taken into account.
In particular, provision is made for the possibility that the two nucleons will
not be captured into equivalent final states. This has the significant effect of
introducing a coherence between contributions from the various configurations
involved, which is not present in the case of single-nucleon transfer with light
projectiles or in the plane-wave treatments of two-nucleon transfer reactions.
Because of this coherence, it is expected that cross sections leading to the population of given final states may be extremely sensitive to the detailed structure of the state wave functions.

As has been stated, the body of experimental data on the two-nucleon transfer reaction is not large, thus the results of the use of these theories cannot yet be fully evaluated. The plane-wave calculations have had varying amounts of success in fitting these data, and the more detailed DWBA treatments have been applied in relatively few cases.

The two-nucleon transfer process in which a correlated neutron-proton pair is exchanged between an incident projectile and a target is commonly known as the deuteron transfer reaction. Since the neutron-proton pair may be transferred in either the singlet or triplet-spin configuration, the deuteron transfer reaction does not necessarily imply the transfer of a physical deuteron. For discussion purposes only, the deuteron transfer reaction will hereafter refer to the process in which the neutron-proton pair is transferred from the projectile to the target; the inverse reaction will be specified as required.

The deuteron transfer reaction at high energies has been examined most extensively by Harvey and collaborators in a study of the ($\alpha$,d) reaction in the light elements at energies between 40 and 50 MeV. The most striking feature of this work is the high degree of selectivity in the population of final states. In the energy spectra of the deuteron products of these reactions, only a few of the levels known to exist in the energy range covered are excited, although the excitation of many other levels not observed is not inhibited by any known selection rules, and of the levels populated some are much more intense than others. Each deuteron energy spectrum is dominated by at least one of these intense levels, whose yields are perhaps an order of magnitude greater than all others in the same spectrum. These preferentially populated levels are known as the giant excitations of the deuteron transfer reaction and have been associated with high spin states of extremely pure two-particle configuration.
A number of the giant excitations have been interpreted as corresponding to the capture of the neutron-proton pair into equivalent $d_{5/2}$ single-particle states with their spins coupled to the maximum allowed resultant and are thus representable as $(d_{5/2})^2$. Evidence that these levels have a common structure is obtained from the relationship between the $Q$ values for their formation and the mass number of the nucleus in which they are formed and from the similarity of their angular distributions. Whereas selectively populated levels associated with mixed or partially allowed components display characteristic forward-peaked oscillatory angular distributions, the preferentially populated levels all possess angular distributions which, although peaked in the forward direction, decrease monotonically with angle. The proposed identification of the $d_{5/2}$ state as the one which is involved is based on previously observed or predicted levels of this configuration which lie at the observed excitations. The coupling of the nucleon spins to the maximum resultant is based on the kinematics of the assumed surface reaction, the selection rules for the $(\alpha,d)$ reaction, and the expected preference for levels for which the wavefunctions of the nucleons in initial and final states have the maximum overlap.

The assignment of the $(d_{5/2})^2$ configuration to this group of giant excitations appears to give a consistent interpretation of the experimental data. In addition, assignments involving other two-nucleon configurations have also been made, particularly in heavier nuclei, where a number of preferentially populated levels have been associated with $(d_{5/2}f_{7/2})$ and $(f_{7/2})^2$ configurations. Confirmation of the proposed assignments has been obtained in some cases from the weak population of the giant excitations in the corresponding inelastic scattering and pick-up reactions. Moreover, the observed locations of the giant excitations correlate extremely well with the predictions of intermediate coupling shell model calculations in the case of $^{14}$N$^{(60)}$, although this is the only nucleus for which such model calculations are as yet available.
The correlation between spin and isobaric spin in the two-nucleon system results in selection rules on the spin $S$ of the transferred nucleons in the various two-nucleon transfer reactions. In particular, in the absence of an interaction which flips the spin of only one of the transferred nucleons, the ($\alpha$,d) and ($Li^6$,a) reactions involve only $S = 1$ channels, while the ($He^3$,p) and ($B^{11}$,Be$^9$) reactions involve both $S = 0,1$. Symmetry conditions impose equivalent conditions on the isobaric spin $T$ exit channels in these reactions, since singlet spin requires triplet isobaric spin, and vice versa. For example, levels populated in ($\alpha$,d) and ($Li^6$,a) reactions must have the same isobaric spin as the target ground state, i.e. $\Delta T = 0$, whereas both $\Delta T = 0,1$ transitions are allowed in ($He^3$,p) and ($B^{11}$,Be$^9$) reactions.

Deuteron transfer reactions induced by high-energy heavy ions have been studied only in isolated cases, primarily because of the lack of adequate particle identification instrumentation for the separation of the heavy products of these reactions. Such a system has recently been developed, based on particle-counter techniques originally employed for light particles, and has been used in the study of the $^{12}_C(B^{11},Be^9)N^{14}$ reaction. An interesting feature of this work is that it was carried out at an energy corresponding to approximately the same energy per incident nucleon used in the study of the equivalent ($\alpha$,d) reaction$^{(58)}$ discussed above.

Many similarities are observed in the two reactions. Specifically there appears to be equivalent selective population of final states, and the Be$^9$ product energy spectrum is dominated by a highly populated level whose excitation and angular distribution are in agreement with those of the giant excitation observed in the $^{12}_C(\alpha,d)N^{14}$ reaction. The nature of the states populated in the two reactions is expected to be similar, with the exception that $\Delta T = 1$ transitions, forbidden in the ($\alpha$,d) reaction, but allowed in ($B^{11}$,Be$^9$) and ($He^3$,p) reactions, should lead to the formation of $T = 1$ states in the $N^{14}$ residual nucleus. However, none of these
states appear to be populated with significant yield, in contrast to the situation in the equivalent \( \text{(He}^3, p) \) reaction,\(^{(62)}\) in which both \( T = 0,1 \) states are observed. There is as yet no adequate explanation for the fact that the \( \text{(B}^{11}, \text{Be}^9 \) reaction, which might be expected to be similar to the \( \text{(He}^3, p) \) reaction in this respect, instead bears more resemblance to the \( \text{(}\alpha, d\text{)} \) reaction.

It will be recalled that the reaction models of Dar and of Dodd and Greider, discussed in the previous section, are formulated in terms of nucleon cluster transfer and are thus applicable for these deuteron transfer reactions. The lack of adequate knowledge of heavy projectile and product wavefunctions, however, again precludes the use of these theories for the extraction of spectroscopic information from the heavy-ion deuteron transfer reaction data. Such is also the case with the various two-nucleon transfer reaction theories which have been described and which as yet have been applied only for such reactions induced by light projectiles. The Dar and Dodd and Greider theories are able to account for the smooth angular distributions observed in both the heavy-ion and \( \text{(}\alpha, d\text{)} \) transfer reactions and have been shown to correctly represent these data.

D. Scope of This Study

This work is an experimental study of single-nucleon and deuteron transfer reactions induced by 115.9-MeV \( \text{B}^{11} \) ions on \( \text{C}^{12}, \text{C}^{13}, \text{N}^{14}, \text{N}^{15}, \text{O}^{16} \) and \( \text{Ne}^{20} \) targets. These experiments were undertaken with the intention of studying a number of specific aspects of heavy-ion transfer reactions in the relatively unexplored energy region significantly above the Coulomb barrier. A particular motivation was the examination of the deuteron transfer reaction in some detail in an attempt to elucidate the mechanism responsible for the apparent similarity in the \( \text{(B}^{11}, \text{Be}^9 \) and \( \text{(}\alpha, d\text{)} \) reactions, including the apparent preference for the triplet-spin transition in the former reaction. At the same time, detailed information on the single-neutron and single-proton transfer
reactions was obtained in order to examine the extent to which such data could be utilized in the extraction of spectroscopic information relevant to the interacting nuclear species. The specific reactions studied in each projectile-target system were \((B^{11}, B^{10})\), \((B^{11}, Be^{10})\) and \((B^{11}, Be^{9})\), which can be interpreted as, respectively, neutron, proton, and deuteron transfers from projectile to target. The energy at which this work was carried out corresponds to an energy per incident nucleon of 10 MeV and to Sommerfeld parameters between 1 and 2, thus these reactions are to be identified with the high-energy low-\(\eta\) reactions discussed in the previous sections.

The problems presented in this work are basically a reflection of the previously-noted limitations inherent in the use of heavy projectiles in transfer reaction studies. The outstanding experimental problems are the identification of both residual nuclei and energy states and the correction for the energy loss of heavy ions in absorbing media. It has been noted that the lack of high-energy transfer reaction data is directly attributable to the limited applicability of the identification techniques which have heretofore been employed in heavy-ion studies. The high energy loss rates of heavy ions necessitate correction for the energy loss of projectiles and product nuclei in windows, targets and counters present in the experimental configuration.

For the present work, a particle identification system was developed which is capable of identifying and measuring the energy spectra of the complex products of high-energy heavy-ion reactions and which allows several such reactions to be examined at the same time. Accordingly, data on the \((B^{11}, B^{10})\), \((B^{11}, Be^{10})\), and \((B^{11}, Be^{9})\) transfer reactions, as well as on the \(B^{11}\) elastic and inelastic scattering reactions, were recorded simultaneously. A semi-empirical method for the calculation of the energy loss of heavy nuclei was developed for this work and provided a consistent means of treating the energy loss problem.
The study of the heavy-ion deuteron transfer reaction is a natural extension of the work which has been done on this reaction as induced by light projectiles. The choice of the \((B^{11}, Be^9)\) reaction as the subject of this study was prompted by several considerations. \(B^{11}\) beams of adequate quality are available, and the use of \(Be^9\) as the reaction product avoids the ambiguity inherent in most heavy-ion reactions as to whether the observed final-state excitation is to be attributed to the product or residual nucleus.

Since the \(Be^9\) nucleus is unstable against neutron emission in all of its \(T = 1/2\) excited states, all \(Be^9\) products which survive the target-detector flight time with excitations below approximately 15 MeV, where the first \(T = 3/2\) states have recently been established,\(^{63,64}\) are in the ground state. Structure in the \(Be^9\) spectrum thus necessarily reflects excitations in the residual nucleus, except for cases in which a \(T = 3/2\) state in \(Be^9\) is involved. The final excitation in the product spectrum at which this ambiguity occurs can be obtained by consideration of the structure of the target and the isobaric-spin selection rules, which allow both \(\Delta T = 0, 1\) exit channels in \((B^{11}, Be^9)\) reactions. For odd-\(A\) targets, the critical excitation is simply that of the first \(T = 3/2\) state in \(Be^9\), or 14.4 MeV. For even-\(A\) targets, it is given by the sum of this excitation and that of the first \(T = 1\) state in the residual nucleus, which is approximately 14 MeV in \(A = 4n\) nuclei and of the order of 1 MeV in \(A = 4n + 2\) nuclei.

It should be noted that the \((Li^6, \alpha)\) reaction has most of the advantages of the \((B^{11}, Be^9)\) reaction, in addition to a much higher ground-state Q value. However, the difficulties involved in the observation of this reaction at high energies have been noted. Further, only triplet-spin, \(T = 0\) channels are accessible to the \((Li^6, \alpha)\) reactions, barring spin flip of one of the transferred nucleons.

The high-energy \((B^{11}, Be^9)\) reaction has been observed previously in one case, that of the \(^{12}C(B'^{11}, Be^9)N^{14}\) reaction. This study revealed a number of
apparent similarities with the equivalent \((\alpha,d)\) reaction. Recently, a detailed study of the \((\alpha,d)\) reaction in the light elements has been carried out which indicates the preferential population of high angular momentum states of a particularly pure two-particle common configuration in the residual nuclei of these reactions. It is of interest to determine if a similar situation obtains in the corresponding \((B^{11},Be^9)\) reactions. Another purpose of the present work is then to examine the \((\alpha,d)\) and \((B^{11},Be^9)\) reactions under approximately equivalent conditions in order to investigate the apparent similarity in these two deuteron transfer reactions and to compare the reaction mechanisms.

As suggested in the interpretation of the population of the giant excitations in deuteron transfer reactions induced by both light and heavy projectiles, it is the energy per nucleon in the incident projectile, that is the angular momentum carried per nucleon, which conditions the experimental observations. The present \((B^{11},Be^9)\) reaction data at 10 MeV per incident nucleon may thus be compared directly with the 40-50-MeV alpha particle-induced \((\alpha,d)\) reaction data, both in terms of the angular distribution of the products and the relative population of residual states. In the case of the angular distributions, it is convenient to compare the differential cross sections as a function of the linear momentum transferred in the reaction, since for forward angle measurements the two reactions span complementary but overlapping regions when displayed in this manner. The targets selected for the work reported herein were therefore those for which \((\alpha,d)\) data, at energies of approximately 10 MeV per incident nucleon, were available for comparison.

The spin and isobaric spin selection rules, which allow both singlet and triplet-spin channels for the \((B^{11},Be^9)\) and \((He^3,p)\) reactions and only triplet-spin channels for the \((\alpha,d)\) reaction, have been discussed. The results of the previous investigation of the \(C^{12}(B^{11},Be^9)N^{14}\) reaction also suggest that the transfer of a deuteron in the singlet-spin, \(T = 1\) configuration is suppressed relative to that of the triplet-spin, \(T = 0\) configuration. It is of interest to
examine whether this is other than an isolated phenomenon peculiar to \( N^{14} \)
and to determine the extent to which the \((B^{11}, Be^9)\) reaction bears this further
and unexpected similarity to the \((\alpha,d)\) reaction rather than to the \((He^3,p)\)
reaction.

The potential of the particle identification system for the simultaneous
observation of several reactions allowed the concurrent study of the \((B^{11}, B^{10})\)
and \((B^{11}, Be^{10})\) neutron and proton transfer reactions. These particular re-
actions were selected because of the isobaric symmetry of their reaction
products and because the possibility exists for excitation of analog or mirror
final states in the two reactions. It should be noted that these reactions
possess the above-mentioned ambiguity regarding the assignment of final
state excitations, a fact that can be expected to condition the interpretation
of the resulting product energy spectra.

Single-nucleon transfer reactions proceeding at high energy have been
observed previously in the \( N^{14} + C^{12} \) and \( B^{11} + C^{12} \) systems. These reactions
and also deuteron transfer reactions in the \( B^{11} + C^{12} \) system have been found
to exhibit several characteristic features, which have already been given as
selective population and similar exponentially decreasing angular distributions.
A further aim of the present work is to examine the \((B^{11}, B^{10}), (B^{11}, Be^{10})\),
as well as the \((B^{11}, Be^9)\) reactions over a range of mass number, in order to
determine the nature of the final states populated and the mechanisms respon-
sible for their formation in these reactions. One of the features of the
recoil model of Dodd and Greider is the prediction of a transfer angular
distribution which is independent of all effects except the mass of the trans-
ferred particle. It is of interest to examine the angular dependence of the
single-nucleon and deuteron transfer cross sections in this respect. The
possibility of observing these cross sections as a function of energy was also
investigated.
Only restricted angular distribution data have been accumulated, however. As has been noted, previous high-energy heavy-ion studies have shown that in both single and two-nucleon transfer reactions to strongly populated states, the angular distributions display, over a wide range of reactions, a power-law dependence on linear momentum transfer or, equivalently, an exponential dependence on angle. Similar behavior has been found for the giant excitations in the ($\alpha$,d) reaction. In the present work, therefore, selected angular distribution measurements have been included to confirm this behavior, but in general attention has been concentrated on the collection of reaction product energy spectra in order to investigate relative population of individual residual states. On the assumption that the angular distributions of these states are slowly varying in nature, these data are also used to obtain total and relative cross sections.

The extraction of absolute spectroscopic information from heavy-ion transfer data using available single and two-nucleon transfer theories is at present precluded by the lack of adequate detailed nuclear structure calculations for the complex nuclei involved. The basic difficulty is that, in general, the transfer cross section is formulated in terms of a product of nuclear structure factors for the incident and final systems. An attempt has been made, however, to utilize the ratio of transfer cross sections to mirror states in the final systems to extract the ratio of structure factors for the incident systems.

In particular, recent measurements on mirror systems in the mass-energy region under consideration herein have demonstrated that the predictions of the isobaric-spin vector-coupling relationships are in good agreement with the experimental data, in other words, that isobaric spin remains a good quantum number in the external nuclear regions of importance in these heavy-ion reactions. On this assumption, it then becomes possible in non-mirror systems, for example, wherein only one pair of the reaction products are
left in mirror states following neutron and proton transfer in the same
projectile-target system, to extract ratios of spectroscopic factors from
the cross-section ratios.

A purpose of this work is to measure, for all targets, cross section
ratios for \((B^{11}, B^{10})\) and \((B^{11}, Be^{10})\) neutron and proton transfer reactions
leading to mirror states in the residual nuclei and to utilize these data to
extract various incident system spectroscopic factor ratios. Comparison
of the corresponding ratios as calculated from available wavefunctions for
the projectile and product states then permits a test of these wavefunctions,
subject to any remaining uncertainties in the validity of the approximations
inherent in the extraction of the experimental ratios from the cross section data.
The use of a variety of targets permits the same ratio to be obtained under
several reaction situations, and the internal consistency of such ratios pro-
vides a measure of the applicability of this approach. Finally, the several
ratio data will be used to examine the \((B^{10}+n)\) and \((Be^{10}+p)\) parentage of the
\(B^{11}\) ground state.

A description of the experimental apparatus and methods, including the
particle identification system and the gas target cell required by the targets
employed in this work is given in Chapter II. The experimental results are
presented and discussed in Chapter III for the neutron and proton transfer
reactions and in Chapter IV for the deuteron transfer reactions. The results
of these experiments are presented as they bear on the objectives of this
work outlined above. A semiempirical method for calculation of energy loss
for heavy ions is described in the appendix.
II. EXPERIMENTAL EQUIPMENT AND PROEDURE

A. Accelerator and External Beam Facilities

The boron-11 ($^{11}$B) beam of the Yale Heavy Ion Linear Accelerator (HILAC) was used throughout this work. This accelerator produces beams of constant velocity, corresponding to an energy of approximately 10.5 MeV per incident nucleon, which are reduced to lower energies by passage through appropriate absorbing foils. The HILAC is a pulsed-beam machine, providing a 2-msec pulse with a macroscopic duty cycle of 2 percent. The accelerator and its components have been described by Hubbard et al. A schematic diagram of the Yale HILAC is shown in Fig. (2.1).

A previously calibrated magnetic analysis system is used to provide momentum analysis of the accelerated beam. The system is a double-deflection design, which for a given experimental target area consists of two 45-degree sector magnets and three strong-focusing quadrupole magnet triplets. A rectangular energy slit placed at the focus of the first sector magnet defines the energy resolution by removing off-velocity components from the beam. Reduced energy beams are obtained by placing absorbing foils immediately before the energy slit and energizing an additional lens triplet in front of the second deflecting magnet.

For this work, a reduced energy beam was obtained by use of a 41.11 mg/cm$^2$ aluminum degrading foil. A relativistically correct form of the equations of motion of a charged particle moving in a uniform magnetic field yielded values of 115.9 MeV and 77.6 MeV for the full and reduced-energy beams (for these energies, $\beta=v/c = 0.15$ and 0.12, respectively). The energy resolution was defined by a 1/16-inch wide energy slit, the nominal resolution being 1 percent. The highest intensity beam was obtained by accelerating the $^{11}$B ions in a charge state of $+4$. The average beam intensities used in the course of this work were between 1 and 5 $\mu$A.
Figure 2.1. Schematic diagram of the Yale Heavy Ion Accelerator. The ion source appears at the right. The magnetic beam analysis system and experimental target areas are shown at the left.
TARGET AREA No.1

TARGET AREA No.2

BEAM SWITCH YARD

TARGET AREA No.3

M - 45° DEFLECTION MAGNETS MOUNTED ON SURPLUS 3" NAVAL GUN MOUNTS.

LENS TRIPLETS

ENERGY DEFINING SLITS

ENERGY DEGRADING ABSORBER WHEELS

TARGET AREA No.4

LENS TRIPLETS

10.4 MEV PER NUCLEON

89.68'

9.02'

32 INCH DIFFUSION PUMPS

STRIPPER

14.93'

10.03'

POWER AMPLIFIERS

1.5 MW PEAK EACH AT 70 MC.

POSTSTRIPPER

67 DRIFT TUBES

MAGNETIC QUADRUPOLE FOCUS

PRESTRIPPER

56 DRIFT TUBES

GRID FOCUS

COCKROFT-WALTON INJECTOR

750 KV

THE YALE HEAVY ION LINEAR ACCELERATOR
B. Scattering Chamber

The basic scattering chamber used has been described previously (68), however, it has been extensively redesigned for this work. The essential features of the chamber are shown in the drawing in Fig. (2.2). The beam enters at the right, passes through the target on the axis of the chamber, and enters a Faraday cup mounted on a port opposite the beam entrance port.

The chamber shell is a 1-inch thick aluminum cylinder with inside diameter 24 inches. As originally described, the chamber had a movable top lid, a fixed bottom lid, a single detector port in the top lid, and provision for a solid-target holder. An inner gear-driven rotating plate has been inserted in the center of the top lid, which is independently rotatable by means of a motorized chain drive. A rotatable bottom plate assembly has been mounted on the underside of the chamber. Each of these rotate under vacuum on teflon bearings with a neoprene O-ring seal, and their angular positions are indicated by verniers calibrated to 0.1 degree. This arrangement allows use of three independently movable detector or target systems. A particle detector telescope was mounted on a port in the top lid, and a monitor detector was inserted through a port in the bottom plate. Since the original chamber design provided only for solid targets, the top plate was modified to accept the gas target cell designed for this work.

Collimation of the incident beam was provided by two brass collimators located in the beam pipe and at the beam entrance port. The collimators contained circular apertures of 1/8-inch diameter and were separated by 18-5/32 inches, with the second collimator 12-5/32 inches from the target center. A brass anti-scattering collimator was located 5 inches beyond the second collimator. The beam collimation system is shown in Fig. (2.6).

Collimators with 1/8-inch circular apertures were chosen for this work after experiments with larger and smaller sizes. The final choice was a compromise between adequate experimental yield and appropriate
Figure 2.2. Schematic drawing of scattering chamber. The gas target cell is shown in position at the center of the chamber. Pumping and viewing ports are not shown.
energy and angular resolution. It was found that for work with a pulsed accelerator of the type used here, all system collimation is subject to this same compromise. The criterion for the present beam collimation system was ability to obtain meaningful angular distributions with non-restrictive data acquisition time for the lower yield portions of the distributions. It was found that apertures larger than 1/8 inch gave a relatively small increase in the amount of beam on target while smaller sizes caused large reductions in experimental yield, indicating that the effective beam diameter was approximately 1/8 inch at the position of the collimators. The angular divergence for a fully illuminated collimation system was 0.4 degree. Use of a calibration target with this system to record the beam profile revealed that the actual beam diameter at the target was very nearly 1/8 inch, indicating that the beam focusing system provided an approximately parallel beam in the vicinity of the target.

In addition to the beam entrance and exit ports, the chamber contained a number of other ports located about its circumference, as indicated in Fig. (2.3). One of these was used for a connection to a pumping bypass around the beam collimators, another was used to monitor internal pressure, a third contained a viewing window. For the present work, a rough-pumping system, inserted in another port, was added to the chamber. This system consisted of a series of baffles extending into the chamber and connected to the rough-pumping line through a needle valve, allowing fine control of pumping speed and affording protection for the fragile windows of the gas target cell and detector system. The rough-pumping line was provided with a nitrogen cold trap to decrease the deposition of pump oil residues on the windows and detectors. A mercury diffusion pump operating through the beam pipe maintained operating pressures in the range $1 \times 10^{-6}$ and $5 \times 10^{-5}$ mm Hg.

The chamber was mounted at the output of the magnetic beam analysis system in the experimental target area and was coupled to the accelerator.
Figure 2.3. Photograph of experimental target area. The accelerator beam direction is from top to bottom in this top view of the target area. The 2-ft-diameter aluminum scattering chamber appears at the center. Attached to ports around the circumference of the chamber are seen, clockwise from the top, the beam entrance pipe, pumping by-pass, particle telescope, Faraday cup and rough-pumping system. Electronic equipment appears at the bottom.
vacuum system with a flexible copper bellows, which reduced mechanical vibrations transmitted from the accelerator and pumping stations. The beam tube was of the proper length to locate the target center at the final focus of the analysis system. The alignment of the chamber, beam collimation system, and gas target cell with respect to the defined accelerator beam axis was accomplished using standard surveying and optical techniques. Fig. (2.3) is a photograph of the scattering chamber in place in the experimental target area.

The beam which passed through the target was collected in a cylindrical copper Faraday cup, 1-1/4 inches in diameter and 7-1/8 inches in length, with its entrance 13 inches beyond the target center on the beam line. The Faraday signal was used both to measure the absolute intensity of the incident beam flux and as a monitor signal for accelerator operation. A 1-k gauss magnet was used to suppress secondary electron emission from the Faraday cup.

C. Target System

The use of a gas target was dictated largely by the fact that the targets chosen for this work are available in their highest purity form in the gaseous state. Gas target utilization has the further advantage of convenience in handling and experimental application, uniform, variable, and accurately determinable target thickness, and maintenance of required target purities, specifically, by avoidance of both atmospheric and vacuum system contamination. Since the use of a differentially-pumped gas target was precluded by the necessity for relatively large beam collimators, as noted in Section B, a gas cell was designed having adequate foil windows. The design necessarily satisfied a rather stringent set of requirements, as indicated below.

Perhaps the most imposing design problem was that the gas target cell had to be utilized in the existing scattering chamber, a relatively large-
dimension chamber with the particle detection system fixed on the outer circumference. In addition, it was necessary to provide for convenient and reproducible interchange with the existing solid target assembly, thus precluding any permanent configuration not compatible with the solid target. Provision for accurately positioning the cell with respect to the beam axis and the detector system was also required.

Another set of highly correlated requirements resulted from the fact that a gas target cell does not have the point-source geometry characteristic of a thin solid target. That is, reaction products do not originate from a point defined by the intersection of the plane of the target and the beam axis, but rather from a finite source which has both length and width. The width, as is also actually the case with a solid target, is determined by the beam collimation system, which thus becomes a gas cell design criterion. The length is determined by the collimation system which defines the solid angle subtended by the detectors, therefore, detector solid angle and angular resolution are also involved in the cell design.

The gas cell had to be of adequate size to allow observation of the angular range of interest without permitting the detectors to observe either the entrance or exit of the beam from the gas volume, in order to eliminate reaction yields originating in the foil windows or window contaminants. Yet the cell had to be small enough so as not to restrict the use of small samples of isotopically enriched target gases. The collimation system thus effectively determined the cell diameter by defining a volume which did not include the windows. At the same time, the collimation system, together with the range of operating pressure chosen, had to provide for reasonable reaction yields, thus raising the question of the reaction yield-resolution compromise mentioned in the preceding section. The choice of operating pressure was also influenced by the fact that the energy losses in the target volume of both incident beam and reaction products had to be minimized in order to facilitate the conversion of observed residual energies to energies at the reaction site.
Finally, a contamination-free gas handling system was required, which provided for the maintenance and measurement of target temperature and pressure with a precision enabling reasonable determination of absolute cross sections. This system also had to provide for convenient evacuation and loading of the cell with target gases from both glass-flask and valved-bottle sources.

A gas target cell was constructed using the above requirements as design criteria. As mentioned above, the design was greatly influenced by the collimation system which determined the detector solid angle and, jointly with the beam collimation system, the active volume of the target. Using as parameters the various quantities which both determined the collimation and were dependent upon it, approximate formulae were developed which enabled construction of plots indicating the relationship of these parameters. The parameter set included collimator separation and size, target length and width, yield, angular resolution, operating pressure, and energy loss in the target. The plots resulting from this analysis were basic to the design of the cell and the determination of the collimation system, which is described in the following section.

Details of the cell design evolved are given elsewhere. The cell was constructed of brass, with a cylindrical shape, permitting convenient insertion through a 5-3/4-inch port in the top rotating plate of the scattering chamber, as shown in Fig. (2.2). The cell was attached to the plate with a neoprene O-ring seal and extended into the chamber, with the gas volume, a cylindrical volume 4 inches in diameter and 2 inches high, centered on the beam axis. The bottom plate of the cell was removable.

The beam entered the cell through a 1/2-inch diameter aperture. A 1/2-inch high window in the opposite wall of the cell covered a 60-degree angular range, as measured from target center, and permitted the beam to pass through the target to the Faraday cup and the reaction products to reach the
detectors. Both entrance and exit windows were rounded at the inner wall to accept foil windows and permit their expansion into the window apertures. The cell rotated with the plate to which it was fixed, and its angular position relative to the beam and the detectors was determined by use of the scattering chamber vernier system.

The windows were of 0.000104-inch heat-treated Havar metal foil* and were held in place on the inner wall of the cell with a vacuum-epoxy vacuum seal (Allaco Products Vak Pak, The Ealing Corporation). The gas cell was tested at pressures up to atmospheric over periods of several days, and no leaks were observed.

The cell had three valved connections, two of which were used for gas inlet and outlet; the third connected to the volume of the scattering chamber. The latter valve enabled simultaneous evacuation of the target cell and the scattering chamber without establishing a significant pressure differential across the foil windows. Needle valves were used in all cases in order to protect the windows. A glass gas handling system was employed which provided for pressure measurement and introduction of flask-contained target gases. Bottled gases were introduced directly at the inlet valved connection.

Within the designed operating pressure range, it was determined experimentally that a target gas pressure of 180 mm Hg absolute gave adequate reaction yields, and this pressure was used throughout the present work. Since tests indicated that the cell did not leak, a constant amount of gas was sealed in the cell for each experiment, instead of using the alternative method of constant-pressure gas flow. The pressure was measured with a

*Precious Metals Division, The Hamilton Watch Company. Specific gravity: 8.3 g/cc. Composition: 0.425 Co, 0.20 Cr 0.18 Fe, 0.13 Ni, 0.028 W 0.02 Mo, 0.016 Mn, 0.002 C, 0.0004 Be.
mercury manometer, which was an integral part of the gas handling system. The temperature was determined with a thermometer system embedded in the body of the cell and read externally, and the temperature of the gas was taken as that of the surrounding cell. Temperature-pressure measurements were taken at regular intervals during the experiment, and the manometer was monitored with closed circuit television in order that any abrupt pressure changes might be immediately apparent. Since the temperature and pressure variations were relatively small, it was assumed that average values were adequate for use in the data analysis. The pressure-temperature ratio was constant to within 5 percent throughout this work, and this figure has been adopted as the estimated error in this ratio.

Under these operating conditions, the energy loss of the incident 115.9-MeV $^1{B^{11}}$ beam in the Havar entrance foil was calculated to be 1.41 MeV. The energy losses in the various target gases from the entrance foil to target center, calculated using the method of the appendix, ranged from 0.8 to 1.5 MeV. Since a gas scattering experiment does not have point-source geometry, the energy loss of the beam in the target gas should take into account corrections for the variation of energy loss along the finite source. However, calculations revealed that these corrections were completely negligible, and the energy loss was taken as that to the target center.

All target gases used in this work were obtained commercially. The oxygen, nitrogen and neon targets were supplied (The Matheson Company) with analyzed purity in excess of 99.9 percent. The carbon target, in the form of methane (CH$_4$), was supplied with a nominal purity of 99 percent. The N$^{15}$ target was obtained (Isomet Corporation) in the form of enriched ammonia (NH$_3$) at 99.2 atom percent N$^{15}$. The C$^{13}$ (Merck, Sharp and Dohme of Canada, Ltd.) was in the form of enriched methane with a C$^{13}$ atom percentage of 59.1. The purity of the latter two gases was verified
by mass spectrometer analysis*. On the basis of this analysis and the stated purities of the oxygen, nitrogen, neon and methane gases, it was concluded that all target contaminations, if present, were completely negligible in this work.

The gas target cell incorporated a solid target assembly, which provided for external insertion under vacuum of a solid target into the cell volume at target center, as shown in Fig. (2.2). The target was positioned by a rod which passed through a sliding O-ring seal in the top plate of the cell, and allowed variation of target angle and withdrawal of the assembly into the upper cell body when not in use. The solid target was interchangeable, and for this work, a thin gold target was utilized for energy loss and detector response measurements.

D. Detector Assemblies

1. Particle Telescope

The dE/dx-E particle telescope utilized in this work consisted of a parallel-plate, gridded ionization chamber transmission detector followed by a semiconductor residual energy detector, as shown in Fig. (2.4). The telescope was designed to attach to a port in the top lid of the scattering chamber as shown in Fig. (2.2). An ionization chamber was used as the transmission detector for several reasons: it has an inherently high resolution, it could be made sufficiently thin for short-range particles, and it was possible to maintain a uniform detector thickness. The latter consideration was the major limitation on the use of a semiconductor transmission counter in this work, although suitable units are presently becoming available commercially.

* We are indebted to Dr. R. Paul of the Yale Isotope Separation group for this analysis.
Figure 2.4. \( \frac{dE}{dx} - E \) particle counter telescope. The ionization chamber \( \frac{dE}{dx} \) transmission detector and semiconductor residual energy detector are shown. The overall length is approximately 6 in. (From reference 70).
The particle telescope has been described previously,\textsuperscript{(70)} and details of its construction will not be given here. The particle path through the ionization chamber was parallel to the plates, as shown in Fig. (2.4). A negative repeller electrode permitted achievement of rise times fast enough for the particles under investigation, and a Frisch grid removed pulse height dependence on detector geometry. A uniform field over the particle path was obtained by having the particles enter and leave the counter volume through 0.00025-inch Mylar windows attached to removable plexiglass reentrant mounts. This eliminated loss of resolution through electron leakage to other than the collector electrode and served to define the active gas volume. The mounts used for this work defined an active path length of 2 inches. The mounts also held detector collimators, as described in the following section.

The electrodes were operated at $\pm 2\text{kv}$, which provided complete collection for this chamber geometry. Commercially available (The Matheson Company) P-10 gas (.10 methane, .90 argon) was used as the counter gas in a continuous flow system, with the pressure regulated to within 1 percent, at a flow of 5 ml/min., by means of a Cartesian manostat (Manostat Corporation, Style 8). For this work, a gas pressure of 60 inches Hg was selected in order to optimize the dE/dx and E detector resolutions. In general, E resolution decreases with increasing pressure due to straggling effects in the gas, while ionization chamber resolution is improved. At this pressure, the equivalent thickness of the chamber and windows was approximately 17 mg/cm\textsuperscript{2} aluminum, as discussed in Section G., corresponding to an energy loss of between 10 and 15 percent for the particles investigated. Detector noise, measured at the output of the preamplifier, was 3mv, peak to peak.

The resolution of the ionization chamber was determined by elastically scattering the incident beam from a thin (0.2 mg/cm\textsuperscript{2}) gold target placed in the solid target assembly of the target cell with the cell filled with gas. The average resolution was 5 percent for 115.9-MeV B\textsuperscript{11} ions, with the gold target suspended
in all gas targets at operating pressure and an ionization chamber pressure of 60 inches Hg. The resolution was independent of dE/dx counting rate up to 20k counts/min., and rates below this figure were maintained throughout this work. A typical ionization chamber response is shown in Fig. (2.5) for B\textsuperscript{11} ions elastically scattered from the gold target in neon gas. The solid line is a least squares fit to a gaussian. The response is thus very closely approximated by a gaussian.

The transmission detector signal is not actually dE/dx, but is an energy loss, ΔE. However, for a fixed path length, ΔX, the ionization chamber output signal is proportional to dE/dx, and will hereafter be referred to as such. It should also be noted that the counter actually measured dN/dx, the number of ion pairs per unit path length, which is proportional to dE/dx only if the average energy loss per ion pair is independent of energy. This has been found to be true in the case of argon.\(^{(71)}\)

The residual energy detector was held by a removable mount attached to the rear of the particle telescope, as shown in Fig. (2.4). The major consideration in the selection of this detector was the obtaining of an adequately thick depletion layer to stop the particles investigated; in this sense, 115-MeV B\textsuperscript{11} ions are equivalent to protons of approximately 7 MeV. For a given depletion depth, a detector can be chosen with, relatively speaking, high resistivity and low bias voltage, or vice versa. For the detectors tested in this work, those having relatively high maximum electric field, i.e., high bias voltage, exhibited superior resolution characteristics, and one such unit was used.

The energy detector selected was an n-type silicon-gold surface barrier detector with a resistivity of 3000 ohm-cm and an active area of 50 mm\(^2\) (Nuclear Diodes, Model ESH 4-25-8). It was operated at a bias voltage of 400v and a leakage current of 0.5 \(\mu\)A, which provided a depletion depth of 550 \(\mu\), sufficient to stop the most energetic particles analyzed. The maximum
Figure 2.5. Detector response. Shown is the response of the energy \( E \) and ionization chamber \( (dE/dx) \) detectors to \( ^{11}\text{B} \) ions elastically scattered from a thin gold target located at the center of the gas target cell filled with neon gas. The energy detector response was obtained with the ionization chamber in place at operating conditions. The solid lines are least squares fits to gaussians.
ENERGY DETECTOR
\[ \text{Au}^{197}(B^I, B^II) \text{Au}^{197} \]
\[ E_{B^II} = 115.9 \text{ MeV} \]
\[ \theta_{\text{LAB}} = 8.5^\circ \]

IONIZATION CHAMBER
\[ \text{Au}^{197}(B^I, B^II) \text{Au}^{197} \]
\[ E_{B^II} = 115.9 \text{ MeV} \]
\[ \theta_{\text{LAB}} = 8.5^\circ \]
electric field at the face of the detector under these conditions was $1.4 \times 10^4$ v/cm. Detector noise, measured at the output of the preamplifier, was 3 mv, peak to peak. The detector was cooled by an ethylene glycol refrigeration system which circulated the coolant at $0^\circ$C through copper coils attached to the detector mount. This system reduced thermal gain shifts to a negligible level and was found to be much more stable than forced air cooling.

The resolution of the energy detector was determined by the method described above for the ionization chamber. The average resolution was 1 percent for 115.9-MeV B$^{11}$ ions elastically scattered from a thin gold target suspended in all target gases used. The target cell and ionization chamber were in place at operating conditions for this measurement and absorbed 4 MeV and 15 MeV, respectively, of the incident beam energy. A typical response is shown in Fig. (2.5) for B$^{11}$ ions elastically scattered from the gold target in neon gas. The solid line is a least squares fit to a gaussian. The response is essentially gaussian with a low-energy tail due to such effects as straggling, slit scattering, incomplete energy loss in the detector, and detector radiation damage.

2. Collimation System

As discussed in Section C, the detector collimation system determines not only the detector solid angle and angular resolution, but also the active length of the gas target. The calculation of the effective solid angle in a situation such as this must reflect the fact that the reaction yield originates from a target of finite length. In scattering experiments involving gas targets, this calculation is often done in terms of a geometrical factor which relates the reaction yield to the beam intensity, target density, and scattering cross section. A calculation of this type by Silverstein$^{72}$ was used in the design of the present system and will be referred to in the following discussion.

In a gas scattering experiment in which the reaction products enter the detector through a collimation system consisting of two defining apertures,
the reaction yield, $Y$, at a laboratory angle $\theta$ can be expressed as

$$Y = \frac{n N \sigma(\theta) G}{\sin \theta}$$  \hspace{1cm} (2.1)

where $n$ and $N$ represent the total number of incident beam particles and target nuclei per unit volume, respectively, $\sigma(\theta)$ is the laboratory differential scattering cross section, $d\sigma/d\omega$, at angle $\theta$, and $G$ is the above-mentioned geometrical factor, hereafter G factor. The G factor can be considered in analogy with the solid angle in point source geometry. For a finite target, G depends both on detector solid angle in the usual sense and on the length of target seen by each element of the rear aperture, and can be thought of as a product of solid angle and effective target length. Hence, the G factor has the dimensions of length.

Since the variation of cross section over the angular range accepted by the rear aperture cannot be neglected, the G factor is a function not only of the geometry of the detector system, but also of the derivatives of the cross section with respect to angle. The G factor can be expanded in terms of a parameter given by the reciprocal of the target center-rear aperture separation and can be expressed in the form

$$G = G_0 + G_1 \frac{\sigma'(\theta)}{\sigma(\theta)} + G_2 \frac{\sigma''(\theta)}{\sigma(\theta)} + \ldots$$  \hspace{1cm} (2.2)

where the primes represent differentiation with respect to $\theta$. $G_0$ can be considered as an uncorrected G factor and $G_1, G_2$ etc., as corrections representing the deviations from point source geometry. The corrections can be thought of as reflecting the quality of the geometry, being large for poor geometry, involving large separations and small values of the expansion parameter.

For this work a collimation system consisting of two collimators with circular apertures was selected because of ease of construction and alignment and because this arrangement yields a G factor which is independent of scattering angle, a necessary condition for a fixed system. The parameters of such a system are the radii of the two apertures and the inter-collimator and target center-rear aperture separations. For the present system, the diameter of the aperture
of the front collimator was 1/16 inch, and that of the rear collimator was 1/8 inch. The collimators were separated by 12 inches, with the rear collimator located 17-5/8 inches from target center.

Since the present geometry involved large separations and was of good quality, in the sense discussed above, it was necessary to calculate only two correction terms, which, for the case of circular apertures, involve elliptic integrals. The following values were obtained for the uncorrected G factor and the two correction terms:

\[
\begin{align*}
G_0 &= 8.71 \times 10^{-6} \text{ cm} \\
G_1 &= -1.29 \times 10^{-11} \cot \theta \text{ cm} \\
G_2 &= 3.90 \times 10^{-11} \text{ cm}
\end{align*}
\]

In general, the G factor is not determined unless \( \sigma(\theta) \) and its derivatives are known, however, it could be found in this case because the correction terms were small.

If, as discussed in Chapter IV, the cross section can be expressed as \( \sigma = \sigma_0 e^{-\alpha \theta} \), then \( \sigma' (\theta)/\sigma (\theta) = -\alpha \) and \( \sigma'' (\theta)/\sigma (\theta) = \alpha^2 \). A portion of the present data was analyzed with \( G = G_0 \), neglecting corrections, and a value of \( \alpha \) was extracted to be used in calculating \( G_1 \) and \( G_2 \). This procedure involved no significant approximations because of the relative magnitudes of \( G_0 \), \( G_1 \), and \( G_2 \). The value of \( G \) so obtained, which was used in the final data reduction, was \( G = (8.72 \pm 0.8) \times 10^{-6} \) cm. The major source of error in the G factor is irregularity in the collimator apertures, and the assigned error includes an estimate of the deviation of each aperture from its measured value.

The angular aperture subtended by this collimation system, assuming a fully illuminated front collimator, was \( \pm 0.5 \) degree. The effective target length was approximately 2.5 cm, with finite beam corrections of \( \pm 2.5 \) cm. The latter figure was used only as a design criterion and was not involved in any further calculations.
The complete collimation system utilized in this work, including beam collimation, is shown in Fig. (2.6). Collimators are indicated in black. The front collimator discussed above was fabricated of brass and held by a brass mount which extended into the chamber volume. The rear collimator was made of plexiglass and was located in the exit window mount of the ionization chamber. Two brass anti-scattering collimators were held by the ionization chamber entrance and exit window mounts and prevented the entrance into the two detectors of particles which had experienced large-angle multiple scattering.

3. Monitor Assembly

A semiconductor counter was used to detect elastic scattering at a forward angle for purposes of monitoring beam integration during extended experiments and during measurement of angular distributions. The only requirement for this detector was a sufficiently thick depletion layer to stop the elastically scattered incident beam. The unit chosen was an n-type silicon-gold surface barrier detector, with a resistivity of 5000 ohm-cm and an active area of 50 mm$^2$ (Ortec, Model SC-CJ-050-500). It was operated at a bias voltage of 200 v and a leakage current of 0.5 $\mu$A, which provided a depletion depth of 500 $\mu$m. Detector noise, measured at the output of the preamplifier, was 2 mV peak to peak. The resolution of the monitor counter for 115.9-MeV $^1$B ions elastically scattered from a thin gold target was less than 1 percent.

The monitor assembly was inserted through a port in the bottom rotating plate of the scattering chamber, as indicated in Fig. (2.2), and positioned in the horizontal plane of the beam axis. The angle of observation was 15 degrees from the incident beam direction throughout this work. The collimation system was determined using a procedure similar to that used for the energy detector, with the added constraint that the effective target length be the same as for this case. The system consisted of two brass collimators, separated by 4 inches, with circular apertures 0.040-inch in diameter. The angular aperture was approximately 2 degrees.
Figure 2.6. Schematic diagram of detector and collimator assembly. The diagram is to scale with the exception of the collimators, shown in black. The particle telescope is attached to the upper lid of the scattering chamber, and the monitor assembly is inserted through a port in the bottom rotating plate.
DETECTOR AND COLLIMATOR ASSEMBLY
E. Electronics

A block diagram of the electronic instrumentation used in this work is shown in Fig. (2.7). Impedance matching circuitry is not included in this drawing. The instrumentation was located in two experimental areas which were separated by the length of the accelerator. Detection and preamplification took place in the target area, and amplification and analysis were done in the control area. Between these two areas, three data channels, or detection-analysis linkages were established: a dE/dx channel, an E channel, and a monitor channel, corresponding to the three particle detectors used.

Preamplification of all detector output signals was by conventional feedback thermionic voltage preamplifiers, which were mounted as part of the detector assemblies to minimize input capacity. The preamplifiers used with the energy and monitor detectors were modified versions of a unit designed in this laboratory for use with semiconductor detectors, and the ionization chamber preamplifier was obtained commercially (Baird-Atomic, Inc., Model 219). White cathode followers were used to couple the preamplifier outputs to the transmission cables which connected the target and control areas. These cables were approximately 250 feet in length and included the delay lines necessary for proper timing in the dE/dx and E data channels.

The linear amplifiers, single channel analyzers, and external coincidence circuitry used in this work are components of a transistorized system of modular nuclear instrumentation developed and built in this laboratory. The amplifiers were used in the delayed-output, double-delay-line-clipped mode with a clipping time of 0.75 μs and a gain in the range 30–60. The amplifiers and the multiparameter analyzer (see section F), which analyzed the dE/dx and E signals, constituted the only instrumentation required by the particle identification system developed for this work.

The amplified dE/dx and E signals were applied directly to the inputs of the multiparameter analyzer, where internal bias amplifiers specified the portions
Figure 2.7. Block diagram of electronic system. Impedance matching circuitry is not shown. The experimental target and control areas and the dE/dx, E, and monitor data channels are indicated.
of the input spectra to be analyzed. Those portions analyzed by the multiparameter analyzer were also selected by single channel analyzers and scaled for monitoring purposes. The output of an external coincidence unit was scaled for comparison with the actual number of counts stored by the analyzer, obtained by scaling the write-strobe output of the analyzer. Experimental live time was also scaled, and a ratemeter was used to record the dE/dx counting rate.

In the monitor data channel, the total monitor spectrum was recorded in a 400-channel analyzer (RIDL, Model 34-12B). As noted in the previous section, a portion of this spectrum was used to monitor the beam integration. This portion was selected by a single channel analyzer and scaled, rather than extracted directly from the multichannel analyzer, in order to avoid the dead time associated with the analyzer.

The signal from the Faraday cup was applied directly to a commercial current indicator and integrator (Elcor, Inc., Model A309A). When operated under conditions which satisfy specified requirements on high source impedance, the nominal accuracy of this instrument is given as 1 percent. The integrator was calibrated periodically during each experiment, and the same sensitivity range, or scale, was used throughout this work in order to avoid the necessity of inter-range calibration. The internal register which recorded the integration was paralleled with a scaler. The integration was monitored by detection of the elastically scattered incident beam, and final data were selected on the basis of this measurement.

The stability of the dE/dx and E data channels was monitored by observation, in the multiparameter analyzer, of a precision pulse generator signal applied to the preamplifiers. A second pulse generator was used in the monitor channel and observed in the multichannel analyzer. Stability measurements were made at 30 minute intervals, and gain drifts were compensated when necessary. In all cases, the maximum drift during a 30-minute period was of the order of 1.
channel in 200. The stability of the electronic system was not a factor in the data reduction.

F. Particle Identification System

1. Theory

Heavy-ion-induced reaction studies of the type considered in this work, involving the interaction of complex targets and projectiles at energies significantly above the Coulomb barrier, are characterized by a large number of open reaction exit channels and a plethora of reaction products, ranging to almost the mass of the compound system involved. The separation of these reaction products for independent study is perhaps the most challenging experimental problem associated with studies of this type. The approaches to the solution of this problem have been many and varied, and have been dictated largely by the nuclear species selected for study, the particular reaction involved, and the type of analysis required. The particle identification system developed for the present work was predicated on the simultaneous multi-parameter analysis of the residual energy states of several of the complex products of reactions involving heavy ions with energies of approximately 100 MeV.

Particle identification, or the determination of the charge and mass of a particle, requires, in general, the simultaneous measurement of at least two of its dynamical parameters. Identification systems utilizing emulsions, cloud chambers, magnetic spectrometers, and time-of-flight techniques have been employed to measure particle velocity, momentum, and range. In the heavy-ion field, radio-chemical and kinematic recoil techniques have been widely used to examine selected reaction products, as noted in the previous chapter. However, the method most suitable for the requirements of the present work utilizes the measurement of the stopping power, or space rate of energy loss, and the energy of the particles.
This method is based on the theoretical relationship for the stopping power for charged particles traversing matter, which can be expressed (see eqs. (A.1), (A.3), (A.4) of the appendix) as

\[- \frac{dE}{dx} = \frac{4\pi e}{m_e v^2} \cdot \frac{z^2}{(L + \Delta L)} \cdot \frac{2}{NZ} \cdot (L + \Delta L) \cdot (2.3)\]

where \(z\) and \(v\) are the charge and velocity of the moving particle, \(NZ\) is the electron density of the absorber, \(m_e\) is the electron mass, \(L\) is the stopping number per atomic electron for the particle in the absorber, and \(\Delta L\) its relativistic correction. Because of the fluctuation of the charge of a moving particle, the \(z\) in this equation is the effective ionic charge of the particle in the absorber. However, as shown in the appendix, the effective charge is closely approximated by the nuclear charge for energies above \(z^2/9\) MeV/amu, thus \(z\) will be regarded as the atomic number of the particle in this discussion. Written in terms of the particle energy \(E\), in the non-relativistic limit, Eq. (2.3) becomes

\[\frac{dE}{dx} = \frac{K m z^2}{E} \cdot \frac{2}{NZ L} \cdot (2.4)\]

where \(K\) involves only physical constants. For a given absorber, the stopping number \(L\) is given by a logarithmic dependence on particle energy, which in many practical applications may be considered constant.

Under conditions of negligible energy variation of the particle charge and stopping number, this equation may be written for a given absorber as

\[E \frac{dE}{dx} = k m z^2 \cdot (2.5)\]

The form of the stopping power formula given by Eq. (2.5) is the basis for particle identification in the \(dE/dx\) and \(E\) method. It should be noted that the \(k\) appearing in this equation is only approximately constant and may have a slight energy dependence. The importance of this effect and the extent to
which corrective measures must be taken depend on the particular application of the method. The product of E and dE/dx provides an absolute identification since no two known nuclides have the same value of mz^2. The loci of particles in dE/dx-E space are seen to approach rectangular hyperbolae characterized by the mz^2 values for each type of particle.

2. Application

The dE/dx and E method utilizes a detector system consisting of two particles counters, and various combinations of scintillation counters, proportional counters, ionization chambers, and semiconductor detectors have been utilized. The particles pass through the dE/dx detector, whose output is thus proportional to the rate of energy loss of the particles, and stop in the E detector, which provides a measure of particle energy. Particle identification is accomplished in practice by suitable combination of the signals from the two detectors. Two such methods are the plotting of one signal versus the other and the multiplication of the two signals.

In these applications of the dE/dx and E method, particles are identified as hyperbolae in a plot of dE/dx and E or as peaks in the spectrum of their product. The elemental separation of these hyperbolae or peaks is proportional to z^2 and is therefore relatively large, being more than 20 percent for all elements up to neon. The isotopic separation, however, is proportional only to the first power in the mass and is much less than the separation by charge. The fractional separation by m for adjacent isotopes and by z^2 for adjacent elements is shown for a number of particles in Fig. (2.8). In the historical development of this method, identification by element preceded identification by isotope, and the separation of the hydrogen and helium isotopes became routine prior to the first separation of the heavier isotopes. The reason for this is apparent from inspection of the figure.

A plot of dE/dx versus E is thus expected to consist of several groups of hyperbolae, representing elements, with the spacing of the groups significantly
Figure 2.8. Particle separation by mass and charge. Fractional separation is shown by $m$ for adjacent nuclides and by $z^2$ for adjacent elements.
FRACTIONAL SEPARATION

SEPARATION BY MASS AND CHARGE

\[
\frac{\Delta m}{m} = \frac{1}{m}
\]

\[
\frac{\Delta (z^2)}{z^2} = \frac{2z+1}{z^2}
\]

NUCLIDE (m,z)

H²  He⁴  Li⁶  Be⁹  B¹⁰  C¹²  N¹⁴  O¹⁶  Ne²⁰
larger than the spacing of the members of each group, representing the isotopes of each element. Isotopic separation will be maintained for each group whose fractional mass differences exceed the resolution of the identification system. Actually, efficient isotopic separation requires approximately a factor of two better resolution than the mass separations given in Fig. (2.8), a fact that can be taken as a basic criterion for the selection of a suitable detector system. For example, the fractional mass separation of the boron isotopes is about 10 percent, requiring approximately 5 percent dE/dx detector resolution.

As mentioned above, early dE/dx and E identification systems were for the isotopes of hydrogen and helium. The technique was introduced into the heavy-ion field by Quinton, Anderson, and Knox, (75) who obtained data directly from photographs of the hyperbolic loci of dE/dx and E oscillographic displays. A later system, assembled by Bromley and collaborators, (70) utilized an analog pulse multiplier (76,77) to form the dE/dx and E product directly and was the first to provide isotopic separation for isotopes heavier than those of helium. Any energy dependences of the product were compensated by adjustment of multiplier parameters with the aid of a plotting oscillograph. Voltage gates set on peaks in the product spectrum corresponding to particles selected for study permitted routing of data on these particles to individual multichannel analyzers or analyzer memory subgroups. This procedure provided for the direct and simultaneous recording of the energy spectra of up to four individual nuclear species.

For the present work, a dE/dx and E particle identification technique was developed in which the entire analog pulse multiplier system, including analyzers, the multiplier itself, and an array of associated electronics, was, in effect, replaced by a single instrument: a 20,000-channel multiparameter pulse height analyzer, built to joint Yale University-Oak Ridge National Laboratory specifications by the Tullamore Division of the Victoreen Instrument Company. (78) While retaining the facility of the multiplier system for simultaneous isotopic
identification and energy measurement for elements at least up to fluorine, the multiparameter analyzer identification system represents a significant departure from this method in many other respects, as discussed in the remainder of this section.

3. Instrumentation

The operation of the multiparameter analyzer identification system required only the addition of linear amplifiers to the analyzer itself. That is, the analyzer was capable of selection of the particles to be analyzed, suitable combination of the dE/dx and E signals to perform the identification, energy measurement, and display and storage of the resulting data. Since the multiparameter analyzer was basic to the instrumentation of the identification system, it will be described briefly in the following paragraphs.

The multiparameter analyzer (hereafter MPA) consists of two primary inputs into two independent, normally closed, analog-to-digital converters coupled to a 20,000 channel random-access magnetic core memory, arranged as a rectangular array of variable dimensions, with an individual channel capacity of $10^6$ counts. Fig. (2.9) is a simplified block diagram of the analyzer logic. Fig. (2.10) is a photograph of the MPA in operation in the experimental control area during the present experiment.

The real-time totalizing and display capability of the MPA provides for the sorting and storing in appropriate memory location of each incoming signal as it is received, thus affording immediate and quantitative availability of experimental results during data acquisition. Two long-persistence ten-inch cathode ray screens allow instantaneous display of the stored information. One screen provides a contour display of the total memory or of a portion of the memory corresponding to the intersection of a plane of variable elevation and thickness with the data surface, with intensity a linear or logarithmic function of number of counts. The second screen provides either static or dynamic display of any selected cross-section plane in memory, corresponding to a
Figure 2.9. Block diagram of multiparameter analyzer logic. (From reference 78).

Figure 2.10. Photograph of the multiparameter analyzer in place in experimental control area. The two 10-inch CRT display screens are seen at the center. Also seen are the paper tape unit and preset scalers. Components of the Yale transistorized modular instrumentation appear at the right and below the multichannel analyzer at the left.
conventional one-dimensional presentation, or isometric display of all memory planes. Channel marker and identifying facilities are included for both display screens. Examples of the contour and isometric displays are shown in Figs. (2.11) and (2.12) for data from the present work.

The input-output data handling facilities of the MPA include a paper tape punch and reader, a computer typewriter, a pen recorder, and a magnetic tape transport. The paper tape facility is shown in Fig. (2.10); the others are not shown. The standard output is onto magnetic tape. The entire memory contents can be written on magnetic tape in a computer compatible format in approximately 15 seconds.

The MPA also incorporates an internal programming system which provides for automatic control of all analyzer functions. Three preset scalers and an automatic programmer permit sequential performance of programmed functions on the basis of elapsed time, accumulated counts from any source, or an external signal, as indicated in Fig. (2.9). There are provisions for both internal and external gating signals. Each analog-to-digital converter has an associated variable-gain bias amplifier, permitting analysis of any portion of the input spectra. With the addition of post-amplifiers, the MPA can be operated as a completely self-contained particle identification system.

4. Procedure

The MPA identification system utilizes one of the two methods discussed above for combination of the dE/dx and E signals, specifically, the application of two signals to rectangular axes. The particles are thus identified by their loci in dE/dx-E space, as predicted by Eq. (2.5). It was noted in the discussion of this equation that these loci are, in many practical situations, very nearly hyperbolic, however, deviations from hyperbolic shape can occur due to energy variations in particle charge or stopping number. As will be seen, identification in the present system is independent of the particular shape of these loci. They will continue to be referred to as hyperbolae, with the understanding that true
Figure 2.11. Multiparameter analyzer contour and isometric displays. The data are from the $^{11}$B $+ ^{15}$N reaction of the present work. In the contour map at the right, bias amplifiers have been used to select the high energy portions of the spectra of the isotopes of beryllium (left) and boron (right). The ordinate is $E$ and the abscissa is $dE/dx$. Intensity is proportional to number of counts, with areas above maximum and below minimum thresholds appearing blank. An isometric display of these data is shown at the left. The beryllium and boron isotopes are in the same relative positions as in the contour display, with the $dE/dx$ axis again horizontal. The $^{11}$B elastic scattering appears as the large peak in this display and as a blank area in the contour display.

Figure 2.12. Same as Figure 2.11 for data from the $^{11}$B $+ ^{12}$C, $^{11}$B $+ ^{14}$N and $^{11}$B $+ ^{16}$O reactions of the present work.
hyperbolic shape is not a requisite for the application of the MPA system.

In the actual use of the MPA in the identification system, the dE/dx and E detector signals, after suitable amplification, were applied to the two analog-to-digital converters of the analyzer, as shown in Fig. (2.7). Analysis of a reaction event was accomplished in response to an internal slow (1-microsecond) coincidence requirement between the two signals. According to Eq. (2.5), events corresponding to each species fell in hyperbolic loci in the analyzer memory. The memory was arranged in a 200-channel (dE/dx) x 100-channel (E) configuration in order to provide the maximum isotopic and energy resolution consistent with the detector system employed.

During the course of this work, survey experiments, such as evaluation of system performance, determination of the effects of varying experimental parameters, or investigation of new targets, were carried out by analysis of the full analyzer input spectra. Data on reactions selected for study were accumulated by analysis of portions of these spectra specified with the bias amplifiers incorporated in each analyzer input. For the present work, those portions of the input spectra corresponding to beryllium and boron isotopes with excitation energies below approximately 20 MeV were analyzed. The controls of the bias amplifiers were adjusted using the channel marker and identifying features of the MPA displays and a calibrated pulse generator.

All electronic recording equipment of the identification system, including analyzers, scalers, and integrator, was operated under the automatic control of the MPA programming system. The equipment was stopped automatically at regular intervals to permit stability measurements, recording of scaled information, and analyzer read out as necessary. Simultaneous MPA dead time measurements were obtained by preset scaling of an internal dead time voltage pulse.

An extremely reliable data handling system resulted from use of the magnetic tape output facility and a multiple read out procedure. Extraction
of spectral information from the accumulated data was performed automatically by a computer technique, as described below.

5. Data Reduction

The energy spectra of particles identified by the multiparameter analyzer identification system are distributed along hyperbolae in the analyzer memory, and their availability in conventional one-dimensional form is ultimately dependent upon the data reduction process. Either manual or automatic techniques can be used for the extraction of the required spectral information.

When the hyperbolae corresponding to the isotopes of interest are well separated, the energy spectra can be obtained by manually summing the counts along each hyperbola from a contour printout of the analyzer memory. This procedure is particularly applicable when light particles, such as the isotopes of hydrogen and helium, and their associated large fractional mass separations are involved. However, for heavier particles, for which the resolution required for practical isotope separation begins to approach the resolution of the identification system, the manual method is either highly subjective or impossible.

The feasibility of manual or automatic data reduction can be determined, in a general sense, by consideration of Fig. (2.8) and applicable detector system response curves, such as shown in Fig. (2.5). More quantitative information can of course be obtained directly from the response of the identification system to the particles involved, as available from an analyzer display or printout.

As an example of the latter procedure for the present data, Fig. (2.13) shows a typical response of the identification system to $^{10}$B and $^{11}$B ions. The isotopes of boron were chosen here because, being the heaviest products analyzed in this work, they have the smallest fractional mass separation and should provide the most stringent criteria as far as resolution is concerned. This figure depicts a section of the analyzer memory consisting of those dE/dx channels which contain the boron hyperbolae for six consecutive E planes along the hyperbolae. Each E plane contains the dE/dx detector response to $^{10}$B and $^{11}$B, which, as noted in Section D, is very nearly gaussian. The area of each gaussian is the experimental
Figure 2.13. Particle identification system response to boron ions. The $^{10}\text{B}$ and $^{11}\text{B}$ ions resulted from the bombardment of a neon target by 113.6-MeV $^{11}\text{B}$ ions at a laboratory angle of 8.5°. The response is shown for five consecutive E planes of the multiparameter analyzer, with dE/dx channel displayed horizontally and the analyzer intensity axis displayed vertically. The solid curves are least squares fits to the data.
IDENTIFICATION SYSTEM RESPONSE TO BORON IONS
yield for the corresponding isotope at the energy of the plane. If reliable manual extraction of the areas of these gaussians is possible, this type of analysis can be employed. Referring to Fig. (2.13), the separation of the two gaussians in the top and bottom E planes, E = 150 and E = 155, might seem to justify the use of manual methods. However, from inspection of the intermediate E planes, where the relative heights of the two peaks change, it is clear that manual separation of the data in any consistent manner is probably not possible. Therefore, the relative population of the gaussians, as well as their separation, is a criterion in the selection of a data reduction process.

As a result of such considerations, it was decided that automatic data reduction was the only means of consistent extraction of spectral information from the present data. This decision necessarily establishes the data reduction procedure as an integral part of the experimental technique, since the type of analysis to be performed must be considered in the storage, examination, and handling of data during acquisition.

The problem involved in any automatic treatment of the data can be summarized by considering the situation which prevails in the analyzer memory during and after data acquisition. There is a hyperbola present in the memory for each species identified. According to Fig. (2.8), the elemental separation of these hyperbolae is, in general, sufficiently large relative to their isotopic separation that each element can be considered independently. Then, for each element, there is a family of gaussians corresponding to the dE/dx detector response for each isotope of that element. The area of each gaussian is the experimental yield at the energy of the plane in question. The aggregate of these areas for each isotope, for all energy planes, forms the required energy spectrum of that isotope. Thus, an automatic technique is required that will provide for computer reduction of a two-dimensional array of data to a one-dimensional energy spectrum.

The aspects of this problem which involve the fitting of specified functions to the data have been found to be particularly amenable to the method of non-linear least squares analysis. The fitting operations carried out in this work were based on the generalized least squares fitting package of Moore and Ziegler, (79) as
modified for the present application by Birnbaum\textsuperscript{(80)} for use on the IBM 7040/7094 direct coupled system at the Yale Computer Center. This method utilizes an iterative procedure due originally to Gauss to determine the best fit to the data in a least squares sense of a function, non-linear in its parameters, for which linearization in the usual manner is not feasible. Externally supplied initial estimates of the parameters of the function are used to generate corrections to these estimates, and the process is repeated until some specified convergence criterion is satisfied. Questions of convergence and other aspects of the application of this method are discussed elsewhere. (79, 80)

A brief description of the procedure adopted in the present work for the reduction of the two-dimensional matrix to conventional energy spectra will be given below. For each fitting operation, the description will include the function fitted to the data, the form of the input data, and the source of the required initial parameter estimates mentioned above.

The procedure consisted of two phases. In the first phase, for each element analyzed, the locus of each isotope was fitted to a three parameter form of the stopping power formula, given by

$$\frac{dE}{dx} = \frac{a}{E} + \frac{b}{E^{1/2}} + c$$

(2.6)

where \(a\), \(b\), and \(c\) are the parameters of the fit. This function is not unique and was used because it was found to provide an adequate description of the data, emphasizing the point made above that the shape of the loci is not critical. Experimental values of \(dE/dx\) and \(E\) were used to calculate the initial parameter estimates. The data to be fitted were obtained directly from the analyzer contour display, using the channel marker and identifier to locate the coordinates of a number of points distributed along each hyperbola.

In the second phase of the analysis, the gaussians in each plane of constant energy were fitted to a function representing a sum of gaussians. The initial estimates were obtained from the results of the first phase, and the data
were obtained directly from the magnetic tape output of the analyzer, or from an edited version of this tape. The parameters resulting from the first fit had no particular meaning other than the location of the particle loci. However, in the second phase, the least squares estimates of the parameters obtained, together with their standard deviations, had definite physical significance, and gave the location, width, and area of each gaussian and the errors associated with these quantities. The tabulation of these areas formed the required energy spectra. The standard deviations of the areas reflected not only statistical errors, but also the quality of the fit with respect to the isotopic resolution in each energy plane.

The information to be derived from an experimentally determined energy spectrum is the relative population and excitation energy of the residual states involved. This can be accomplished, again, either manually or automatically, depending on the complexity of the spectrum and the type of analysis desired. Computer techniques were utilized for this purpose for much the same reasons which prompted the choice of automatic sorting of the dE/dx and E data.

The one-dimensional energy spectra exhibited the response of the energy detector employed, which, in the present case, was essentially gaussian in nature, as noted in Section D. Each spectrum was fitted to a function representing a sum of gaussians, using a least squares technique similar to that described above. The initial estimates were obtained by inspection of the individual spectra, and the data were in the form of punched cards, which resulted from the analysis in which the spectra were obtained. The final values of the parameters of this fit were the areas, widths, and E channel locations of the peaks in the spectrum. These parameters formed the input to a computer code which converted the area of each peak to a cross section and its location to an excitation energy, using the calibration of the particle identification system, to be described. This code also calculated various other quantities of interest and presented the results in tabulated form.
6. Discussion

An identification system based on the dE/dx and E method of particle identification has been described which provides for simultaneous multiparameter analysis of the residual energy states of a number of selected nuclear species at least up to mass number 17 resulting from the interaction of heavy ions at energies near 100 MeV. This system can be compared with a group of systems which also simultaneously record energy spectra in such reactions by means of the generation of an identification signal which permits multichannel energy analysis. A member of this group, the analog pulse multiplier system described above, is a dE/dx and E system employing detection techniques similar to those of the present system and will be used as a basis for this comparison.

An identification system based on a theoretical relationship is useful, in general, only in regions in which the expression provides a valid description of the physical processes involved. For example, this criterion may apply to the future performance with heavy ions of a system which has recently come into use for lighter particles. This system is based on an empirical range-energy relationship rather than on the stopping power formula and generates a signature pulse which depends on the representation of range as a power law in the energy. The deviation of the range-energy relationship for heavier particles from a power law may limit the applicability of this technique for heavy-ion identification.

The multiplier system is based on the specific form of the stopping power formula given by Eq. (2.5), and is thus subject to the limitations which have been noted in connection with this expression. Although adjustable multiplier parameters allow for slight energy variations in this equation, the method is highly dependent upon the loci of the particles in dE/dx-E space being very nearly hyperbolic. However, for the multiparameter system, which is based on a very generalized form of the stopping power formula, the particular shape of these loci is not critical, since the least squares fit can be performed with
a function of arbitrary form. The result of this flexibility is a wider applicability of the multiparameter system with respect to both particle species and energy.

Although both methods are capable of simultaneously accumulating information on a number of reactions, the instrumentation and use of the multiparameter system is by far the less complex, requiring only amplifiers and the analyzer itself. In addition, its alignment has proven to be extremely rapid, stable, and reproducible, as is not the case for the multiplier system, with its assortment of associated electronics. Further, analysis by the multiplier system of a number of reactions equivalent to the number which is compatible with the multiparameter memory necessitates the use of more than a single multichannel analyzer, involving inter-analyzer calibration and dead time considerations. Although the multiparameter analyzer system records energy spectra in a two-dimensional form, integration of data acquisition and processing through use of a computer permits rapid, consistent, and automatic data reduction. The system is also highly amenable to real-time data reduction by addition of an on-line computer.

Further advantages of the multiparameter system involve the capabilities which were specifically designed into it for applications such as the present one. These features include immediate and quantitative data display and availability during acquisition, large data capacity, provision for fully automatic operation, and rapid and convenient read-out facilities.

G. Energy Calibration

The energy calibration of the particle identification system was utilized in the conversion of observed multiparameter analyzer locations to excitation energies. The calibration involved both the energy calibration of the analyzer and correction for energy losses of the reaction products in absorbers between the reaction site and the residual energy detector. As discussed in the previous section, this calibration procedure was a part of the automatic data reduction
technique associated with the particle identification system, and as such was coded for computer calculations.

It was not possible to obtain an independent energy calibration using standard radioactive sources, since the energies of the particles involved in this work were an order of magnitude greater than those of the highest energy alpha-particle sources available. Therefore, the beam of the accelerator and its associated magnetic analysis system formed the basis of the calibration.

The energy calibration of the multiparameter analyzer was performed by elastically scattering the full-energy $^{11}$B beam from a helium target (the Matheson Company, purity 99.99 percent) and varying the angle of observation. The use of a relatively light target enabled calibration data with adequate statistics to be obtained over the full energy range of the analyzer in a reasonable amount of time. This measurement was carried out with the ionization chamber removed from the particle telescope, so that energy loss correction was required only for the gas target cell. This was done using the method of the appendix.

The resulting calibration of the analyzer is shown in Fig. (2.14). The solid line is the least squares fit of a straight line to the calibration data and an independently determined analyzer channel of zero energy, obtained with a calibrated precision pulse generator. The relationship between the residual energy detected by the energy detector, $E$, and the analyzer channel number $C$ was obtained from the fit shown in Fig. (2.14) and is given by,

$$ C = -96.6 + 1.765 E $$

(2.7)

In the present case, the absorbers encountered by the reaction products prior to their detection included the target gas, the exit window of the target cell, the ionization chamber gas, the entrance and exit windows of the ionization chamber, and the insensitive layer of the energy detector. For this work, only the latter was considered to be negligible. For discussion purposes, these absorbers can be divided into two groups, one of which contains only the ionization chamber gas. Such a division was determined by the methods available for
Figure 2.14. Multiparameter analyzer energy calibration. The points are from the elastic scattering of 114.3-MeV $^{11}\text{B}$ ions from a helium target. The solid line is a least squares fit to these data and an independently determined analyzer zero channel (not shown).

Figure 2.15. Multiparameter analyzer excitation energy calibration. The calibration is shown for the boron and beryllium products of the $^{11}\text{B} + ^{15}\text{N}$ reaction induced by 113.5-MeV $^{11}\text{B}$ ions at a laboratory angle of 8.5°.
MULTIPARAMETER ANALYZER ENERGY CALIBRATION

He⁴(B¹²,α)He⁴
E_B^¹² = 114.3 MeV

MULTIPARAMETER ANALYZER EXCITATION ENERGY CALIBRATION

B¹¹ + N¹⁵
E_B^¹¹ = 113.5 MeV
θ_Lab = 8.5°
calculation of the associated energy loss, and two methods will be discussed for each group.

The ionization chamber gas volume is unique in that the energy loss is measured directly during the experiment and is available in both analog and digital forms for correction purposes. Calibration of the dE/dx detector thus allows the energy loss in the ionization chamber gas to be added to the residual energy either electronically during data acquisition, so that the energy analyzed is the sum of the dE/dx and E signals, or automatically by computer during data reduction. This method is particularly appropriate when the thickness of the dE/dx detector varies significantly over its area, and energy variations due to non-uniform thickness are comparable to normal straggling processes. Under these conditions the energy resolution of the sum is much better than that of the E signal alone. The second method is also applicable only for the ionization chamber, including the windows in this case, and is based on the fact that the unit can be completely removed from the particle path. The method consists of measuring equivalent thickness and using known range energy relations to compensate for the energy loss. This method was employed in the present work because of the inherent areal uniformity of the ionization chamber.

The procedure used to determine the equivalent thickness of the ionization chamber was to observe the elastic scattering of the B\textsuperscript{11} beam from a helium target, as was done in the analyzer calibration, both with the ionization chamber in place under operating conditions and removed. The energy loss at each angle was calculated by means of the analyzer calibration and compared with the range-energy relations for B\textsuperscript{11} ions in aluminum.\textsuperscript{(83)} The results for seven observation angles varied by only 1.5 percent. The value of the equivalent thickness of the ionization chamber, including windows, obtained by this method was 16.9 ± 0.8 mg/cm\textsuperscript{2} aluminum.

The equivalent thickness of the two 0.00025-inch mylar windows of the ionization chamber was independently obtained using the above procedure, and the
result was $2.17 \pm 0.2 \text{ mg/cm}^2$ aluminum. The value calculated using the density of mylar and a conversion factor for aluminum and mylar\(^{(84)}\) was $2.18 \text{ mg/cm}^2$ aluminum.

The thickness of the ionization chamber was also calculated using the method of the appendix, taking the molecular stopping power of the methane component of the gas to be a weighted sum of the stopping powers of carbon and hydrogen. The result was $16.6 \text{ mg/cm}^2$ aluminum, in agreement with the measured value.

The target cell and windows must be considered separately, as there is no direct measurement of the energy loss and the cell cannot be removed from the system. The calculation of energy losses in absorbers of this type is a general problem involved in nuclear reaction studies and is discussed in the appendix. In general, there are two methods available: application of known range-energy relations or use of a semiempirical theory of energy loss. Since a consistent set of range-energy data for the particles, energies, and absorbers involved in this work is not available, a semiempirical procedure was developed, as given in the appendix. The computer application of this method was used in the calculation of the energy losses of the reaction products in the target cell exit windows and the various target gases.

The calibration technique given above was incorporated in a computer code which related the analyzer locations to corresponding excitation energies by the systematic conversion of channel number to energies at the detector, at the entrance of the ionization chamber, and at the reaction site. The reaction energy was transformed into an excitation energy using applicable kinematic relations. The procedure involved the coding of the equations of the semiempirical method and storage in the computer memory of the analyzer calibration in the form of Eq. (2.7), the parameters of the range-energy relations, the equivalent thickness of the ionization chamber, and appropriate absorber and reaction parameters. Masses and Q values were obtained from a recent
compilation of nuclidic masses. Typical results for the particle identification system calibration are shown in Fig. (2.15) for the $^7\text{Be} + ^{15}\text{N}$ system. A similar calibration is given for each of the energy spectra presented in the following chapters.

H. Error Analysis

1. Absolute Cross Section

Absolute differential scattering cross sections were calculated using Eq. (2.1) in the form

$$\frac{d\sigma}{d\omega}_{\text{lab}} = 1.66 \times 10^{-11} \frac{zY \sin \theta}{dQ(P/T)\eta} \text{mb/sr}$$

(2.8)

where $\theta$ is the laboratory scattering angle, $z$ is the effective beam charge at the Faraday cup in units of electronic charge, $Q$ is the total integrated beam charge in coulombs, $G$ is the gas target geometrical factor in cm, $P$ and $T$ are the target gas pressure in mm Hg and temperature in °K, $Y$ is the experimental reaction yield corrected for analyzer dead time, $d$ is a constant equal to 2 for diatomic gas targets and unity otherwise, and $\eta$ is the energy counter efficiency. The counter efficiency is taken to be unity. No error is associated with the constant factor, which contains only physical constants. The remaining quantities and the experimental errors associated with their measurement are discussed below.

Effective beam charge, $z$. The energy of the $^7\text{Be}$ beam at the position of the Faraday cup was approximately 100 MeV. According to the discussion of Eq. (A.2) of the appendix, the effective charge of $^7\text{Be}$ at this energy is equal to the nuclear charge with completely negligible error. The effective beam charge is taken to be 5.0.

Total integrated beam charge, $Q$. The integrated beam charge was measured with the beam current integrator as discussed in Section E. The integration was monitored by detection of the elastically scattered beam, and this
measurement was reproducible to within 5 percent. Electron collection by
the Faraday cup and magnetically-suppressed outscattering from it were assumed
to have negligible effect on the collected beam current. The estimated error
in the integrated beam charge is 5 percent.

Scattering angle, $\theta$. The scattering angle was dependent on the alignment
of the magnetic beam analysis system, the beam collimation system, and the align-
ment of the scattering chamber and its vernier system. The vernier system was
calibrated to 0.1 degree, and as discussed in Section B, the incident beam was
nearly parallel at the position of the target. The estimated error in the scattering
angle is 0.3 degree at 8.5 degrees, and the associated error in the sine of
the scattering angle is 4 percent.

Experimental reaction yield, $Y$. The experimental yield was determined
in the automatic data reduction procedure described in Section F. The error
analysis associated with this technique reflected the quality of the isotopic
separation as well as the statistical error. The estimated error in the yield re-
sulting from counting statistics and the quality of the data reduction is 5 percent.

Another source of error in the experimental yield is the counting loss, with
major contributions from analyzer dead time, slit scattering, multiple scattering
in the gas target, energy detector outscattering, and background fluctuation. The
dead time of the multiparameter analyzer was measured directly, as described
in Section F, and varied from 3 to 10 percent. The estimated error in this
measurement is 30 percent, and the corresponding error in the yield is 3
percent. The effects of multiple scattering in the gas target tend to cancel,
however, the compensation is incomplete, largely because of the variation in
the cross section across the angular aperture of the detector. The estimated
error in the yield due to multiple scattering and slit scattering from the trans-
mission counter collimators is 2 percent.

Outscattering in the energy detector refers to situations in which particles
fail to deposit their full energy in the detector and can result from slit scattering
from the energy detector collimators, nuclear reactions in the detector, detector radiation damage, and scattering of the particle or a recoil atom out of the detector sensitive region. A quantitative estimate of the counting loss due to this effect is possible since, for a peak of given energy in the analyzer memory, the locus of outscattering events appears as a straight line of constant dE/dx and decreasing E. Scattering of the beam from a gold target at a forward angle is a convenient method of performing this measurement, because the elastic peak is isolated and strongly populated and because the low reaction yield provides an uncontaminated outscattering locus. Analysis of 500 thousand such elastic events revealed that 1.9 percent appeared in the outscattering locus. The background fluctuation was estimated to be 5 percent. The estimated error in the yield due to these contributions to the counting loss is 12 percent.

Gas target geometrical factor, G. The estimated error in the geometrical factor is 10 percent, as discussed in Section D.

Target density. The estimated error in the target density as expressed by the gas pressure-temperature ratio is given in Section C as 5 percent.

These errors are largely uncorrelated and were added in quadrature to obtain the total error. The total estimated error in the absolute magnitude of the differential cross section is 21 percent. The error in cross section ratios is determined largely by counting loss and statistics and is estimated to be 25 percent.

2. Excitation Energy

Absolute excitation energies were calculated as described in Section G. The major sources of uncertainty in energy are uncertainty in beam energy, energy straggling in the gas target cell and ionization chamber, measurement of absorber thicknesses, finite detector solid angle, and uncertainties associated with the energy detector and electronics. The magnitude of these errors will be discussed in terms of contributions to a total energy width.

Beam energy. Standard ion optics calculations yield an energy spread passed by the energy slit of 300 kev. This must be compounded with the resolution
of the magnetic beam analysis system, which is taken as 500 kev. Energy variation along the finite gas target was negligible, as discussed in Section C. The estimated energy width due to uncertainty in the beam energy is 600 kev.

Energy straggling. This refers to variations in the energy of a particle after passage through a fixed amount of absorber. This energy fluctuation can be expressed\(^{(86)}\) in terms of the charge of the particle and the electron density and length of the absorber. The straggling was calculated to be 170 kev for the target cell gas and windows and 430 kev for the ionization chamber gas and windows. The magnitude of the straggling was also obtained directly by observation of the elastic scattering of the beam with the gas cell or ionization chamber removed. The observed straggling was 180 kev for the gas cell and 370 kev for the ionization chamber. The estimated width due to straggling is 600 kev.

Absorber thickness. The measured thickness of the ionization chamber is \(16.9 \pm 0.8\) mg/cm\(^2\) aluminum. The energy loss in the gas target cell was approximately 5 MeV. The estimated width due to uncertainty in absorber thickness is 750 kev.

Finite solid angle. The finite angular aperture of the detector collimation system allowed particles having a spread in scattering angle and a corresponding spread in energy to reach the detector. A kinematic calculation was performed for an angular aperture of 0.5 degree at 8.5 degrees for the various gas targets, taking into account the circular shape of the collimator apertures. The estimated width due to finite solid angle is 400 kev.

Detector and Electronic. The uncertainty in energy for this detector for 5.5-MeV alpha particles was given as 20 kev. The estimated width for heavy ions is 200 kev.

The errors were taken to be independent and were added in quadrature to obtain the total energy width. The estimated uncertainty in the absolute values of measured excitation energies is 1.2 MeV.
In many cases it was possible, within this uncertainty, to relate an excitation energy to a known energy level. The relative values of other energies were then determined primarily by the energy calibration of the analyzer and the standard deviations of peak locations, as specified in the data reduction process. The estimated error in relative excitation energies is ± 200 kev.
III. SINGLE NUCLEON TRANSFER REACTIONS

A. Presentation of Results

1. Energy Spectra

The experimental data to be discussed in this chapter result from \((B^{11}, B^{10})\) and \((B^{11}, Be^{10})\) reactions induced by 115.9-MeV \(B^{11}\) ions on \(C^{12}, C^{13}, N^{14}, N^{15}, O^{16}\) and \(Ne^{20}\) targets. The data consist of energy spectra of the \(B^{10}\) and \(Be^{10}\) products observed at forward angles and differential cross section measurements for energy groups appearing in these spectra.

The \((B^{11}, B^{10})\) and \((B^{11}, Be^{10})\) reactions are interpreted, respectively, as neutron and proton transfers from the projectile to the target. These assignments are consistent with the fact that the allowed alternative multi-nucleon transfer mechanism, often termed heavy-particle or backward stripping, is not expected, from kinematic considerations in the incident barycentric system, to contribute significantly to the cross section at forward angles.\(^{87}\)

This effect has been observed by Bock et al.\(^{88,89}\) in angular distribution measurements in the \(N^{14} + Be^{9}\) system, as shown in Fig. (3.1). These reactions are similar to those considered in the present work in that they involve heavy ions at energies (25-30 MeV) above the Coulomb barrier and are able to proceed by alternative forward and backward transfer mechanisms. In analogy to the reactions considered in this chapter, the product observed in the reaction represented in the upper curve in Fig. (3.1) can result from either the projectile, by transfer of a nucleon to the target, or the target, by the addition of a complex cluster from the projectile. The reaction shown in the lower curve corresponds to the two-nucleon transfer reactions considered in the following chapter. The reaction mechanisms corresponding to these forward and backward transfers are shown schematically in each case. It is seen that the two transfer mechanisms contribute to the angular distribution with little interference and that the backward transfers, corresponding to heavy-particle stripping, contribute mainly in the backward direction as expected.
Figure 3.1. $^{12}C$ and $^{13}C$ angular distributions, $^{14}N + ^9Be$ reactions. The angular distributions of the $^{13}C$ and $^{12}C$ products resulting from the $^{14}N$ bombardment of $^9Be$ at energies of 30 and 25 MeV, respectively, are shown. The reaction mechanisms are indicated schematically at the left and right for each reaction. (From reference 89).
C\textsuperscript{12} AND C\textsuperscript{13} ANGULAR DISTRIBUTIONS
FROM: BOCK et al., 1965

\[ \text{Be}^9 (N^{14}, C^{13}) B^{10} (0.0) \]

\[ \frac{N^{14}}{p} \quad \frac{C^{13}}{\text{Be}^9} \quad \frac{B^{10}}{\text{O}} \]

\[ \frac{N^{14}}{\alpha} \quad \frac{B^{10}}{\text{Be}^9} \quad \frac{C^{13}}{d} \]

\[ \frac{N^{14}}{\text{He}^3} \quad \frac{B^{11}}{\text{Be}^9} \quad \frac{C^{12}}{d} \]

\[ \text{Be}^9 (N^{14}, C^{12}) B^{11} (0.0) \]

\[ 0 \quad 30 \quad 60 \quad 90 \quad 120 \quad 150 \quad 180 \]

\theta_{cm} (DEG.)
Representative energy spectra of the $^{10}\text{B}$ and $^{10}\text{Be}$ products of the $(^{11}\text{B},^{10}\text{B})$ and $(^{11}\text{B},^{10}\text{Be})$ reactions are presented in Fig. (3.2) through (3.7). The spectra corresponding to neutron and proton transfer reactions are shown together for each target, with the $^{10}\text{Be}$ spectrum in the upper half and the $^{10}\text{B}$ spectrum in the lower half of the figure. In order to demonstrate the isobaric symmetry of these reactions, ground state mirrors or analogs are aligned in the two spectra when practicable. The groups in these spectra are interpreted as the excitation of final states in the products of these reactions, and the portions of the spectra shown correspond to excitation energies up to approximately 15 MeV. The approximate range of laboratory kinetic energy included in the spectra is from 70 to 100 MeV.

As mentioned previously, the $^{10}\text{Be}$ and $^{10}\text{B}$ data for each target were recorded simultaneously. The energy spectra presented herein result from the automatic data reduction procedures described in the preceding chapter. The data points are shown as obtained from this process, and the curves in the range of excitation energy discussed are the result of a least squares computer fit of the gaussian energy detector response function to the data. The energy calibration curves were obtained independently for each spectrum by means of the calibration procedure described previously. In all cases the measurements were repeated, and the excitation energies and yields were found to be completely reproducible. The error bars shown result from the data reduction process and reflect both statistical errors and the quality of the isotopic resolution inherent in the data.

For each spectrum, the laboratory reaction energy and scattering angle are given. The small variation in the reaction energies reflects the correction applied to the incident beam energy for passage through the gas target cell. The abscissa is the multiparameter analyzer energy axis channel number, and the ordinate is the number of counts per channel.
Figure 3.2. Upper figure: $^{10}\text{Be}$ energy spectrum, $C_{^{12}\text{Be}, ^{10}\text{Be}}^{11}\text{C}$. Lower figure: $^{10}\text{B}$ energy spectrum, $C_{^{12}\text{Be}, ^{10}\text{B}}^{11}\text{C}$. The solid curves are a least squares computer fit of the energy detector response function to the data. The error bars represent both statistical errors and the quality of the isotopic resolution in a given channel.

Figure 3.3. Upper figure: $^{10}\text{Be}$ energy spectrum, $C_{^{13}\text{Be}, ^{10}\text{Be}}^{11}\text{C}$. Lower figure: $^{10}\text{B}$ energy spectrum, $C_{^{13}\text{Be}, ^{10}\text{B}}^{11}\text{C}$. The significance of the solid curves and error bars is as in Figure 3.2.

Figure 3.4. Upper figure: $^{10}\text{Be}$ energy spectrum, $C_{^{14}\text{N}, ^{10}\text{Be}}^{11}\text{O}$. Lower figure: $^{10}\text{B}$ energy spectrum, $C_{^{14}\text{N}, ^{10}\text{B}}^{11}\text{O}$. The significance of the solid curves and error bars is as in Figure 3.2.

Figure 3.5. Upper figure: $^{10}\text{Be}$ energy spectrum, $C_{^{15}\text{N}, ^{10}\text{Be}}^{11}\text{O}$. Lower figure: $^{10}\text{B}$ energy spectrum $C_{^{15}\text{N}, ^{10}\text{B}}^{11}\text{O}$. The significance of the solid curves and error bars is as in Figure 3.2.

Figure 3.6. Upper figure: $^{10}\text{Be}$ energy spectrum, $O_{^{16}\text{O}, ^{10}\text{Be}}^{11}\text{F}$. Lower figure: $^{10}\text{B}$ energy spectrum, $O_{^{16}\text{O}, ^{10}\text{B}}^{11}\text{F}$. The significance of the solid curves and error bars is as in Figure 3.2.

Figure 3.7. Upper figure: $^{10}\text{Be}$ energy spectrum, $\text{Ne}_{^{20}\text{Ne}, ^{10}\text{Be}}^{11}\text{Na}$. Lower figure: $^{10}\text{B}$ energy spectrum, $\text{Ne}_{^{20}\text{Ne}, ^{10}\text{B}}^{11}\text{Na}$. The significance of the solid curves and error bars is as in Figure 3.2.
Be$^{10}$ ENERGY SPECTRUM
C$^{12}$ (B$^{11}$, Be$^{10}$) N$^{13}$
E$_{B^{11}}$ = 113.5 MeV
$\theta_{\text{LAB}}$ = 8.5°

COUCH PER CHANNEL

EXCITATION (MeV)

CHANNEL NUMBER

B$^{10}$ ENERGY SPECTRUM
C$^{12}$ (B$^{11}$, B$^{10}$) C$^{13}$
E$_{B^{11}}$ = 113.5 MeV
$\theta_{\text{LAB}}$ = 8.5°

COUCH PER CHANNEL

EXCITATION (MeV)

CHANNEL NUMBER
Be\textsuperscript{10} ENERGY SPECTRUM
N\textsuperscript{15} (B\textsuperscript{11}, Be\textsuperscript{10}) O\textsuperscript{16}
E\textsubscript{B}\textsuperscript{11} = 113.5 MeV
\theta\textsubscript{LAB} = 8.5°

COUNTS PER CHANNEL

CHANNEL NUMBER

B\textsuperscript{10} ENERGY SPECTRUM
N\textsuperscript{15} (B\textsuperscript{11}, B\textsuperscript{10}) N\textsuperscript{16}
E\textsubscript{B}\textsuperscript{11} = 113.5 MeV
\theta\textsubscript{LAB} = 8.5°

COUNTS PER CHANNEL

CHANNEL NUMBER
Be$^{10}$ ENERGY SPECTRUM
$^{16}\text{O}$ ($^7\text{B}^+$,Be$^{10}$)F$^{17}$
$E_{^7\text{B}^+} = 113.1$ MeV
$\theta_{\text{LAB}} = 8.5^{\circ}$

COUNTS PER CHANNEL

CHANNEL NUMBER

B$^{10}$ ENERGY SPECTRUM
$^{16}\text{O}$ ($^7\text{B}^+$,B$^{10}$)O$^{17}$
$E_{^7\text{B}^+} = 113.1$ MeV
$\theta_{\text{LAB}} = 8.5^{\circ}$

COUNTS PER CHANNEL

CHANNEL NUMBER
2. Cross Sections

As discussed in the introduction, recent studies of heavy-ion single-nucleon transfer reactions in the energy range under consideration have shown that the angular distributions of final states populated in these reactions display an exponential dependence on angle or, equivalently, a power-law dependence on linear momentum transfer.

Restricted angular distribution measurements were made for several of the single-nucleon transfer reactions investigated in the present work, and the results are presented in Fig. (4.7). The transfer cross sections are found to have a common power-law dependence on the linear momentum transfer and therefore confirm the expected behavior. The results will be discussed in connection with their comparison with the deuteron transfer cross sections in the following chapter. Since it appears that only limited additional information can be obtained from further data of this type, differential cross section measurements were made only at a selected forward angle for other single-nucleon transfer reactions considered herein.

The absolute differential cross sections were obtained from the automatic data reduction process on the basis of Eq. (2.8) and represent the areas of gaussians fitted to groups appearing in the energy spectra. The differential cross section measurements are presented in the detailed discussion of the individual spectra in Section C.

B. Angular Momentum Transfer

An effect commonly observed in both single-nucleon and deuteron transfer reactions proceeding at high energies is the preferential population of states of high angular momentum or the corresponding inhibition of the formation of states of lower angular momentum. The mechanism underlying this phenomenon can thus be expected to be involved in the interpretation of the present transfer reaction data and will be discussed below in terms of the orbital angular momentum transferred in the reaction.
A qualitative explanation of the preferential population can be obtained from consideration of an extremely simplified classical model of a direct surface reaction. Such a description is suggested by the surface nature of the transfer reaction and is based on the assumption that the main contributions to such a process result from angular momentum values which correspond to classical impact parameters near the interaction radius. Such a model was originally employed in the semiclassical description of the deuteron stripping reaction, and similar arguments led to the development of the Huby triangle rule for these reactions. The salient features of this model have been given by Butler in a discussion of the relationship between the angular momentum transferred and the angular distribution in a direct interaction process.

The process considered will be the transfer of a nucleon or nucleon pair from the projectile to the target in a reaction specified by \( m_2(m_1,m_3)m_4 \). The orbital angular momentum will be written, in general, as

\[
\vec{L} = \vec{f} - \vec{r} \times r.
\]

In the limit of a pure surface reaction, the orbital angular momentum transferred to the target core is given directly by the kinematics of the reaction and may be written as

\[
\vec{L} = (m_3/m_1)\vec{L}_i - \vec{L}_f
\]

\[
= [(m_3/m_1)\vec{k}_i - \vec{k}_f] \times R \hbar
\]

where \( \vec{L} \) is the orbital angular momentum transfer and \( \vec{k}_i \) and \( \vec{k}_f \) are the relative wave numbers in the initial and final channels, respectively. \( R \) is the interaction radius, given by \( R = r_o (A_1^{1/3} + A_2^{1/3}) \), where \( A_1 \) and \( A_2 \) are the atomic masses of the projectile and target and \( r_o \) is the radius parameter.

The orbital angular momentum of the transferred nucleon or nucleons in the residual nucleus is given by \( L' = f \) for single-nucleon transfer and by \( L' = f_p + f_n \) for deuteron transfer. On the basis of this model, the enhancement
or inhibition of a transition specified by $L'$ is expected to be a function of the difference given by

$$\Delta L = |\vec{L}| - L'$$  \hspace{1cm} (3.2)

$\Delta L$ is thus a measure of the extent to which the orbital angular momentum transfer matches the orbital angular momentum value appropriate to the configuration of the transferred nucleon or nucleon pair in the final state. Small values of $\Delta L$, or good matching, imply enhancement relative to larger values of $\Delta L$.

Transitions to levels having orbital angular momentum values $L' \sim |\vec{L}|$ should be enhanced. Conversely, if the situation is such that the lowest allowed $|\vec{L}|$ value is significantly larger than $L'$, inhibition of the transition would be expected, with the greater the difference $|\vec{L}|_{\text{min}} - L'$, the stronger the inhibition. Such momentum matching effects have been demonstrated in the case of $(d,a)$ reactions and in the treatment of the $(a,d)$ reactions discussed in the introduction.

As also noted in the introduction, this phenomenon appears in a more formal fashion in the recoil reaction model of Dodd and Greider.\(^{(26)}\) Therein, the enhancement of certain angular momentum transfers and, in general, inhibition of low transfer values reflects interference between the oscillations of the bound state wavefunctions and the specific recoil factor appearing in the transfer amplitude for the reaction. In the example quoted by Dodd and Greider, involving O\(^{16}\) projectiles at 160 MeV incident energy, the relative partial cross sections for $L' = 0, 1, 2, 3$ transfers are in the ratios 1, 1.5, 3, 7.

Durand,\(^{(93)}\) in an alternative approach to the transfer problem, has also obtained these and the other results of the Dodd and Greider theory.

Of direct significance in the present situation is the fact that in surface interactions at energies of 10 MeV per incident nucleon, each nucleon delivers approximately 2–3ħ orbital angular momentum in reactions with light nuclei.
The momentum matching conditions discussed above then imply that transitions to configurations involving these $\ell$ values would be favored in these reactions.

C. $(B^{11}, B^{10})$ and $(B^{11}, Be^{10})$ Energy Spectra

The $(B^{11}, B^{10})$ and $(B^{11}, Be^{10})$ transfer reaction energy spectra are discussed separately in this section. The states corresponding to groups observed in the individual spectra are correlated where possible with their expected configurations in order to determine the nature of the mechanism involved in the population of final states in these reactions. All energy spectra were measured at the same laboratory scattering angle of $8.5^\circ$ corresponding to c.m. angles between $14^\circ$ and $16^\circ$. Differential cross sections are given for each energy group discussed.

As noted previously, these neutron and proton transfer reactions lead to the population of final states in isobaric residual nuclei. Although primary attention will be focussed in this section on the interpretation of the energy spectra in terms of the states of these nuclei, it should be noted that the $B^{10}$ and $Be^{10}$ products possess low-lying excited states which are stable against particle emission and which can also give rise to final state excitations. The population of these states reflects the $(B^{10} + n)$ and $(Be^{10} + p)$ parentage of the $B^{11}$ target ground state and will be considered in detail in Section F. Situations in which a given energy group may arise from simultaneous excitation of both product and residual nuclei will be referred to as mutual excitations in these discussions.

The only states in the product nuclei which can contribute to the observed spectra are those which lie below the lowest particle breakup threshold, since the lifetime of particle unstable states is less than the target-detector flight time. The threshold for the $(Be^{9} + n)$ breakup of $Be^{10}$ is at 6.814 MeV, thus the first four excited states in $Be^{10}$ at 3.37, 5.96, 6.18, and 6.26 MeV may be observed. The 0.72, 1.74, 2.15, 3.58, and 4.77-MeV states in $B^{10}$ may
likewise be involved, since the \((\text{Li}^6 + \alpha)\) threshold of \(\text{B}^{10}\) is at 4.463 MeV. In addition, the barrier for \(\alpha\)-particle emission may allow the 5.11 and 5.16-MeV states in \(\text{B}^{10}\) to be observed. The fact that states above these thresholds are not observed will be assumed and not commented on in each case. The experimental energy resolution is not sufficient to completely separate the ground and 0.72-MeV states in \(\text{B}^{10}\), and the \(\text{B}^{10}\) ground state group will be assumed to include the 0.72 MeV level. The 1.74 and 2.15-MeV states in \(\text{B}^{10}\) will also be regarded as one group in these discussions.

Groups which appear in the energy spectra are properly referred to in terms of their measured kinetic energy. In order to facilitate discussion of the residual excitations, an excitation energy calibration is displayed with each spectrum, and the groups will instead be identified by the excitation energies of the states to which they correspond.

The breakup of \(\text{B}^{11}\) into \((\text{B}^{10} + n)\) and \((\text{Be}^{10} + p)\) at excitations near 11 MeV has been calculated to not be a factor in the interpretation of the spectra in the excitation energy ranges considered. The fact that there is no restriction on the formation of \(T = 0, 1/2\) or 1 states in these reactions will not be commented on further.

The excitation energies quoted are from a standard compilation of energy levels, unless more recent values from literature cited in the discussion are used.

1. \(\text{C}^{12}(\text{B}^{11}, \text{Be}^{10})\text{N}^{13}\)

The \(\text{Be}^{10}\) energy spectrum from the \(\text{C}^{12}(\text{B}^{11}, \text{Be}^{10})\text{N}^{13}\) reaction at \(E_{\text{B}^{11}} = 113.5\) MeV is shown in Fig. (3.2). The only strongly populated groups appear at excitations of 0.0, 3.5, and 6.9 MeV, with differential cross sections (mb/sr at 8/5°) of 0.218, 0.753, and 0.706, respectively. This spectrum is in agreement with that obtained by Sachs et al. in a previous investigation.

The 3.5-MeV group corresponds to the 3.37-MeV state in \(\text{Be}^{10}\) and the 3.51 or 3.56-MeV states in \(\text{N}^{13}\). The 6.9-MeV group results from the mutual
excitation of these states or from a single excitation in $^{13}\text{N}$. Strong groups
are observed only at 0.0 and 3.5 MeV in the $^{9}\text{Be}$ spectrum of the corresponding
$^{12}\text{C}(^{10}\text{B},^{9}\text{Be})^{13}\text{N}$ reaction, therefore the 6.9-MeV group does not result
primarily from a state in $^{13}\text{N}$ in this region. Both of the above-mentioned
possibilities contribute to the 3.5-MeV group, since the 3.37-MeV state in $^{10}\text{Be}$
is observed in all ($^{11}\text{B},^{10}\text{Be}$) reactions considered herein (see Fig. (3.8)). The
angular momentum considerations discussed previously would suggest that the
3.56-MeV, $5/2^+$ state rather than the 3.51-MeV, $3/2^-$ state is involved in both
the 3.5-MeV and 6.9-MeV groups. Evidence from an analysis of the equiva-
lent proton transfer reaction $^{12}\text{C}(^{3}\text{He},d)^{13}\text{N}$ at $E_{^{3}\text{He}} = 25$ MeV, in which this
group was found to be predominantly $\ell = 2$, confirms such an assignment.

It is therefore probable that the 3.5-MeV group results predominantly
from $^{10}\text{Be}$ in the 3.37-MeV state and $^{13}\text{N}$ in the 3.56-MeV state and that the
6.9-MeV group is due to the mutual excitation of these states. These assign-
ments are consistent with the two-body cluster model of Phillips and Tombrello, who show that the 0.0, 2.37, 3.56, and 8.08-MeV states in $^{13}\text{N}$ have dominant
parentage ($^{12}\text{C}(0.0) + n$), while the 3.51, 6.38, 6.91, and 7.40 MeV states are
based primarily on ($^{12}\text{C}(4.43\text{-MeV}) + n$). The 0.0($1/2^-$), 2.37($1/2^+$), and
3.56 (5/2$^+$)-MeV states have been identified as the single-particle states in
$^{13}\text{N}$, while the 3.51-MeV state is expected to result from the core excitation
of a $p_{3/2}$ nucleon.

Although the strong excitation of the ground and 2.37-MeV states of $^{13}\text{N}$
would also be expected on this basis, there appears to be an inhibition of the
population of these low-spin states relative to the 3.56, 5/2$^+$ state. A similar,
although less strong inhibition is observed in the corresponding ($^{3}\text{He},d$)
reaction.

$^{12}\text{C}(^{11}\text{B},^{10}\text{B})^{13}\text{C}$

The $^{10}\text{B}$ energy spectrum from the $^{12}\text{C}(^{11}\text{B},^{10}\text{B})^{13}\text{C}$ reaction at
$E_{^{11}\text{B}} = 113.5$ MeV is shown in Fig. (3.2). A strongly populated group appears
at an excitation of 3.8 MeV, weaker groups at 0.0 and 5.8 MeV, and a group has also been resolved at 1.9 MeV. The differential cross sections (mb/sr at 8.5°) of the 0.0, 1.9, 3.8, and 5.8-MeV groups are 1.11, 0.418, 3.31, and 1.31, respectively.

The C^{13} single-particle states lie at 0.0 (1/2^-), 3.09 (1/2^+), and 3.85 (5/2^+)-MeV and are the mirrors of the N^{13} ground, 2.37, and 3.56-MeV states, respectively. The cluster model parentage predictions are (C^{12}(0.0) + n) for the 0.0, 3.09, 3.85, and 8.33-MeV states and (C^{12*}(4.43-MeV) + n) for the 3.68, 6.87, and 7.68-MeV states. On the basis of the above discussion of the C^{12}(B^{11},Be^{10})N^{13} mirror reaction, the 3.85-MeV state is expected to be strongly populated in this reaction. The group at 3.8 MeV corresponds to the population of this state and the 3.58-MeV state in B^{10}, however, the 3.58-MeV state is, in general, formed in low yield in the (B^{11},B^{10}) reactions considered herein (see Fig. (3.8)). Further evidence for the weak population of this state in transfer reactions results from the B^{11}(p,d)B^{10} data of Bachelier et al., in which the 3.58-MeV state is not observably populated and the dominant groups appear at B^{10} excitations of 0.0, 2.0, 5.0, 6.9, and 11.4 MeV. General agreement has been found between the B^{11}(p,d)B^{10} results and the present (B^{11},B^{10}) data, and similarities in the two reactions will be noted in subsequent discussions.

The group at 3.8 MeV is thus probably due primarily to the 3.85, 5/2^+ state in C^{13}. This assignment is also in agreement with the C^{12}(N^{14},N^{13})C^{13} data of Birnbaum et al. at E_{N^{14}} = 148 MeV, wherein the only strong groups appear at 0.0 and 3.8 MeV, with a relative intensity equivalent to that found in the present work. A weak group resulting from the 1.74 and 2.15-MeV states in B^{10} has been resolved, and the group at 5.8 MeV may include contributions from the mutual excitation of these states and the 3.85-MeV state in C^{13} as well as from a single excitation in C^{13}.
There again is apparently an inhibition of the low angular momentum states which might be expected to be populated. There is also no evidence for the population of the 6.87-MeV state in C$^{13}$, the formation of which would involve core excitation in a single-nucleon transfer reaction.

2. C$^{13}(B^{11}, Be^{10})N^{14}$

The Be$^{10}$ energy spectrum from the C$^{13}(B^{11}, Be^{10})N^{14}$ reaction at $E_{B^{11}} = 113.5$ MeV is shown in Fig. (3.3). Groups appear at excitations of 0.0, 3.4, 5.7, 8.8, and 12.2 MeV, with differential cross sections (mb/sr at 8.5°) of 0.250, 0.907, 0.844, 1.96, and 0.804, respectively.

The 3.4-MeV group results from the 3.37-MeV state in Be$^{40}$. The 5.7-MeV group corresponds to the population of the 5.69-MeV, 1$^-$ and 5.83-MeV, 3$^-$ states in N$^{14}$. The shell model assignments (60,99) for these states are dominant ($p_{1/2}^1s_{1/2}^1$) and ($p_{1/2}^1d_{5/2}^1$) configurations, respectively, thus their population in this reaction corresponds to the addition of $s_{1/2}$ and $d_{5/2}$ protons to the C$^{13}$ target core. The strong population of these states in the N$^{14}(\alpha,\alpha)'N^{14}$ reaction (57) further indicates their single-particle nature.

The 8.71-MeV, 0$^-$ and 8.91-MeV, 3$^-$ states in N$^{14}$ similarly have been assigned ($p_{1/2}^1s_{1/2}^1$) and ($p_{1/2}^1d_{5/2}^1$) configurations, respectively, and may be associated with the group at 8.8 MeV, although this group may also contain a contribution from the mutual excitation of the 3.37-MeV state in Be$^{10}$ and a state in N$^{14}$ near 5.7 MeV. The group at 12.2 MeV may be the mutual excitation of states in the 8.8-MeV group and the 3.37-MeV state in Be$^{10}$ or a single excitation in N$^{14}$. Significant population of states at 3.95, 6.23, 6.44, and 7.03 MeV, which would require the excitation of the target core in addition to the transfer of a nucleon, is not observed.

This spectrum and the B$^{10}$ spectrum from C$^{13}(B^{11}, B^{10})C^{14}$ to be discussed below were extracted from those obtained directly from the C$^{13}$ target by subtraction of the corresponding spectra measured with the C$^{12}$
target, normalized to the monitor counter spectrum and the measured $^{12}\text{C}$ content of the $^{13}\text{C}$ target. As a result of this procedure, the estimated errors for the higher excitations in the $^{12}\text{C}(^{11}\text{B},^{10}\text{Be})^{14}\text{N}$ and $^{13}\text{C}(^{11}\text{B},^{10}\text{B})^{14}\text{C}$ reactions are approximately twice those given in Chapter II.

$^{13}\text{C}(^{11}\text{B},^{10}\text{B})^{14}\text{C}$

The $^{10}\text{B}$ energy spectrum from the $^{12}\text{C}(^{11}\text{B},^{10}\text{B})^{14}\text{C}$ reaction at $E_{^{11}\text{B}} = 113.5 \text{ MeV}$ is shown in Fig. (3.3). A strongly populated group appears at an excitation of 6.8 MeV, and weaker groups at 0.0, 2.0, 3.6, 5.0 and 8.8 MeV. The differential cross sections (mb/sr at 8.5°) of the 0.0, 2.0, 3.6, 5.0, and 6.8-MeV groups are 0.548, 0.275, 0.331, 0.710 and 3.81, respectively.

The four groups of lower excitation may be identified as excitations in $^{10}\text{B}$ corresponding to the removal of a neutron from $^{11}\text{B}$ and formation of $^{14}\text{C}$ in the ground state, since the first excited state in $^{14}\text{C}$ is at 6.09 MeV. Relative intensities of these groups are in accord with the $^{11}\text{B}(p,d)^{10}\text{B}$ data, except that the 3.58-MeV state may be formed in slightly higher yield.

The group at 6.8 MeV corresponds to the 6.72-MeV, 3− and 6.89-MeV, 0− states in $^{14}\text{C}$ and probably results from the $^{14}\text{C}$ analog of the state observed at 8.8 MeV in the $^{12}\text{C}(^{11}\text{B},^{10}\text{Be})^{14}\text{N}$ reaction. The 8.8-MeV group may be the mutual excitation of states in the 6.9-MeV group and the 1.74 and 2.15-MeV states in $^{10}\text{B}$.

$^{14}\text{N}(^{11}\text{B},^{10}\text{B})^{15}\text{N}$

The $^{10}\text{B}$ energy spectrum from the $^{14}\text{N}(^{11}\text{B},^{10}\text{B})^{15}\text{N}$ reaction at $E_{^{11}\text{B}} = 113.2 \text{ MeV}$ is shown in Fig. (3.4). A strongly populated group appears at an excitation of 7.2 MeV, and weaker groups at 0.0, 2.0, 5.2, and 9.2 MeV. The differential cross sections (mb/sr at 8.5°) of the 0.0, 2.0, 5.2, 7.2 and 9.2-MeV groups are 0.902, 0.457, 0.835, 5.81, and 4.52, respectively.
The 2.0-MeV group results from the 1.74 and 2.15-MeV states in $B^{10}$, since the first excited state in $N^{15}$ is at 5.28 MeV. There is no significant population of the 3.58-MeV state in $B^{10}$, consistent with results of the $B^{11}(p,d)B^{10}$ reaction. (98)

The intermediate coupling shell model calculations of Halbert and French (100) predict dominant $(s^p^{10}s)$ and $(s^p^{10}d)$ configurations for the even-parity levels of $N^{15}$ below 9 MeV. The 5.28 and 5.30-MeV states are based primarily on $d_{5/2}$ and $s_{1/2}$ nucleons, respectively, coupled to an $N^{14}$, $T = 1$ first-excited-state core, while the 7.16, 7.31, 7.57 and 8.31-MeV states are expected to arise predominantly from the coupling to a $N^{14}$ ground-state core of $s_{1/2}$ (7.31 and 8.15 MeV) and $d_{5/2}$ (7.16 and 7.57 MeV) nucleons. On this basis, the 5.28 and 5.30-MeV states in $N^{15}$ are not expected to be strongly excited in this reaction, therefore, the 5.2-MeV group probably results mainly from a state near 5 MeV in $B^{10}$. The results of the $N^{14}(B^{11},Be^{19})O^{15}$ reaction to be discussed below and also the $B^{11}(p,d)B^{10}$ data confirm such an assignment.

The group at 7.2 MeV corresponds to the population of the 7.16, $5/2^+$ and 7.31, $3/2^+$ states in $N^{15}$. The 7.31-MeV state and its $O^{15}$ analog at 6.79 MeV have been observed to be strongly populated in deuteron stripping reactions on $N^{14}$, (101) indicating relatively large nucleon reduced widths for these states. The single-particle nature of the state or states appearing in the 7.2-MeV group is demonstrated by the extremely weak population of states at this excitation in the $B^{11}(O^{16}, N^{15})C^{12}$ reaction at $E_{O^{16}} = 30$ MeV. (102)

The group at 9.2 MeV may be the mutual excitation of states in the 7.2-MeV group and the 1.74 and 2.15-MeV states in $B^{10}$ or a single excitation in $N^{15}$.

The two states arising from the shell model configuration $(s^p^{11})$ have been identified with the ground, $1/2^-$ and 6.33-MeV, $3/2^-$ states in $N^{15}$, (97) although it has recently been suggested that the 6.33-MeV state and its $O^{15}$ analog at 6.15 MeV are primarily collective in nature, being based on the
6.33-MeV state in this reaction. Its relatively weak population in the $\text{B}^{11}(\text{O}^{16},\text{N}^{15})\text{C}^{12}$ reaction is also consistent with a dominant collective character.

There is agreement between this spectrum and that resulting from the $\text{C}^{12}(\text{N}^{14},\text{N}^{15})\text{C}^{11}$ reaction at $E_{\text{N}^{14}} = 148$ MeV, in which the only groups observed correspond to states at excitations of 0.0, 2.0, 5.3, 7.3, and 9.4 MeV. (21) The relative intensities of these groups in the two spectra are equivalent, except for the stronger population of the 5.2-MeV group in the $\text{B}^{10}$ spectrum, further demonstrating that this group is due primarily to an excitation in $\text{B}^{10}$.

$\text{N}^{14}(\text{B}^{11},\text{Be}^{10})\text{O}^{15}$

The $\text{Be}^{10}$ energy spectrum from the $\text{N}^{14}(\text{B}^{11},\text{Be}^{10})\text{O}^{15}$ reaction at $E_{\text{B}^{11}} = 113.2$ MeV is shown in Fig. (3.4). Groups are observed at excitations of 0.0, 3.4, 5.2, 6.9, and 10.2. The differential cross sections (mb/sr at 8.5°) of the 0.0, 3.4, 5.2, and 6.9-MeV groups are 0.200, 0.487, 0.076, and 0.838, respectively.

The group at 3.4 MeV results from the 3.37-MeV state in $\text{Be}^{10}$, since the first excited state of $\text{O}^{15}$ is at 5.17 MeV. The weak population of the 5.17 and 5.24-MeV states is in accord with observations in the $\text{N}^{14}(\text{B}^{11},\text{Be}^{10})\text{N}^{15}$ reaction, as discussed above, and the shell model description of these states. The strongly populated group at 6.9 MeV may be identified with the 6.85-MeV, 5/2+ and 6.79-MeV, 3/2+ states in $\text{O}^{15}$, which are known to have dominant single-particle configurations as in the analog situation in $\text{N}^{15}$. The dominant group in the spectrum of the $\text{C}^{12}(\text{N}^{14},\text{O}^{15})\text{B}^{11}$ reaction at $E_{\text{N}^{14}} = 148$ MeV also appears at this excitation. (21)

The 10.2-MeV group may be attributed to a mutual excitation or to a single excitation in $\text{O}^{15}$. It should be noted that the absence of this group in the
above-mentioned \( (N^{14}, O^{15}) \) reaction does not preclude identification herein as a state in \( O^{15} \), since the proton emission threshold of the \( O^{15} \) reaction product is at 7.3 MeV. There is no evidence for the formation of the 6.15-MeV, \( O^{15} \) analog of the 6.33-MeV state in \( N^{15} \).

4. \( N^{15}(B^{11}, Be^{10})O^{16} \)

The \( Be^{10} \) energy spectrum from the \( N^{15}(B^{11}, Be^{10})O^{16} \) reaction at \( E_{B11} = 113.5 \text{ MeV} \) is shown in Fig. (3.5). Isolated groups appear at excitations of 0.0, 3.4, 6.1, 9.5, and 12.5 MeV, with differential cross sections (mb/sr at 8.5°) of 0.154, 0.518, 0.642, 2.01, and 1.37, respectively.

The 3.4 MeV group results from the 3.37-MeV state in \( Be^{16} \), since the first excited state of \( O^{16} \) is at 6.05 MeV. The majority of the odd-parity states in \( O^{16} \) below 12.5 MeV have been assigned dominant \( (p_{1/2}^{-1}s) \) and \( (p_{1/2}^{-1}d) \) components in the intermediate coupling shell model calculations of Elliott and Flowers. The 6.13-MeV, \( 3^- \) state is well represented on this basis as \( (p_{1/2}^{-1}d_{5/2}) \) with a reduced width relative to the Wigner limit in the corresponding \( (d,n) \) reaction of 0.38. The 6.1-MeV group no doubt corresponds to the population of this state since the adjacent state at 6.05 MeV has been shown to be predominantly a two or more particle excitation in the calculations of Kelson.

The 9.5-MeV group cannot be uniquely identified as a single excitation in \( O^{16} \) or a mutual excitation of the 6.1-MeV state in \( O^{16} \) and the 3.37-MeV state in \( Be^{10} \). The 9.59-MeV state in \( O^{16} \), which may contribute to this group, may contain a large admixture of single-particle excitation, but appears to result mainly from a more complex configuration. The group at 12.5 MeV corresponds to the 12.44-MeV, \( 1^- \) and 12.53-MeV, \( 2^- \) states in \( O^{16} \), which have been calculated to be of dominant \( (p_{1/2}^{-1}d_{3/2}) \) configuration, although a mutual excitation can again not be excluded. It would also be tempting to identify this group with the analogs of the four low-lying \( T = 1 \) states in \( N^{16} \), at least one of which is strongly populated in the \( N^{15}(B^{11}, Be^{10})N^{16} \) reaction, to be discussed below.
The $^{15}\text{N}(^{11}\text{B},^{10}\text{Be})^{16}\text{N}$ reaction at $E_{^{11}\text{B}} = 113.5$ MeV is shown in Fig. (3.5). The 0.0-MeV group is strongly populated, and weaker groups appear at excitations of 2.0, 3.6, and 5.2 MeV. The differential cross sections (mb/sr at 8.5°) of these groups are 6.10, 2.16, 0.860, and 2.15, respectively.

The dominant group is obviously complex and corresponds to the excitation of, in addition to the ground and first excited states of $^{10}\text{Be}$, the 0.0, 0.12, 0.30, and 0.39-MeV, $T=1$ states in $^{16}\text{N}$, all of which have been shown to arise from $s_{1/2}$ and $d_{5/2}$ single-particle excitations. The 2.0-MeV group results from the 1.74 and 2.15-MeV states in $^{10}\text{Be}$ and the low-lying quartet in $^{16}\text{N}$.

The weak group at 3.6 MeV may correspond to the population of the 3.58-MeV state in $^{10}\text{Be}$. The 5.3-MeV group probably results from an excitation in $^{10}\text{Be}$ as well as from the broad state at 5.25 MeV in $^{16}\text{N}$ corresponding to the strong resonance observed in neutron scattering from $^{15}\text{N}$.

5. $^{16}\text{O}(^{11}\text{B},^{10}\text{Be})^{17}\text{F}$

The $^{10}\text{Be}$ energy spectrum from the $^{16}\text{O}(^{11}\text{B},^{10}\text{Be})^{17}\text{F}$ reaction at $E_{^{11}\text{B}} = 113.1$ MeV is shown in Fig. (3.6). Strongly populated groups appear at 0.0 and 3.4 MeV, and weaker groups at 5.5 and 8.6 MeV. The differential cross sections (mb/sr at 8.5°) of these groups are 1.06, 1.95, 0.46, and 0.75, respectively.

The intense group at 0.0 MeV corresponds to the $5/2^+$ ground state and $1/2^+$ first excited state in $^{17}\text{F}$ at 0.50 MeV, known to be the $d_{5/2}$ and $s_{1/2}$ single-particle states. The 3.4-MeV group results from the 3.37-MeV state in $^{10}\text{Be}$. Several states in both $^{10}\text{Be}$ and $^{17}\text{F}$ may be contributing to the weaker groups, and an unambiguous identification cannot be made.

This reaction was observed as a function of angle, and the results for the ground state group are shown in Fig. (4.7), as discussed in Section A. An
attempt was also made to investigate this reaction as a function of energy, however, the extremely large loss in beam intensity resulting from the introduction of the energy degrading foils precluded the possibility of obtaining an excitation function. A measurement at $E_{B11} = 77.6$ MeV indicates that the $0.0$ and $3.4$-MeV groups are again strongly populated, and that the cross section of the ground-state group attains approximately 0.8 of its full energy value.

$^{16}$O$(^1$B$^{11}$, B$^{10}$)O$^{17}$

The B$^{10}$ energy spectrum from the $^{16}$O$(^1$B$^{11}$, B$^{10}$)O$^{17}$ reaction at $E_{B11} = 113.1$ MeV is shown in Fig. (3.6). The group at 0.0 MeV is strongly populated, and weaker groups appear at excitations of 1.9, 3.4, and 5.2 MeV. The differential cross sections (mb/sr at 8.5°) of these groups are 8.0, 3.42, 0.53, and 3.70, respectively.

The intense ground-state group may again be identified with the 5/2$^+$ ground state and 0.87-MeV, 1/2$^+$ single-particle states in O$^{17}$. The 2.0-MeV group results from the 1.74 and 2.15-MeV states in B$^{10}$. The association of the weak group at 3.4 MeV and the 5.2-MeV group with states in B$^{10}$ is consistent with the results of the B$^{11}$(p,d)B$^{10}$ reaction. This reaction was observed as a function of angle and the results for the two groups of lowest excitation are shown in Fig. (4.7).

6. $^{20}$Ne$(^1$B$^{11}$, B$^{10}$)Ne$^{21}$

The B$^{10}$ energy spectrum from the $^{20}$Ne$(^1$B$^{11}$, B$^{10}$)Ne$^{21}$ reaction at $E_{B11} = 113.6$ MeV is shown in Fig. (3.7). Groups are observed at excitations of 0.0, 2.0, 4.7, and 6.6 MeV. The differential cross sections (mb/sr at 8.5°) of the 0.0, 2.0, and 4.7-MeV groups are 4.69, 3.99, and 8.80, respectively.

The dominant contribution to the intense ground state group may be from the 0.353-MeV, 5/2$^+$ first excited state of Ne$^{21}$, which is also strongly populated
in the corresponding (d,p) stripping reaction.\(^{(106,107)}\) The 2.0-MeV group results from the 1.74 and 2.15-MeV states in \(^{10}\)B. The group at 4.7 MeV may result primarily from the 4.73-MeV, \(3/2^-\) state in \(^{21}\)Ne, also strongly populated in the (d,p) reaction. The 6.6-MeV group corresponds to the mutual excitation of this state and the 1.74 and 2.15-MeV states in \(^{10}\)B.

\[ \text{Ne}^{20}(\text{B}^{11}, \text{Be}^{10})\text{Na}^{21} \]

The \(^{10}\text{Be}^+\) energy spectrum from the \(\text{Ne}^{20}(\text{B}^{11}, \text{Be}^{10})\text{Na}^{21}\) reaction at \(E_{\text{B}^{11}} = 113.6\) MeV is shown in Fig. (3.7). Groups appear at excitations of 0.0, 3.7, and 7.6 MeV, with the differential cross sections (mb/sr at 8.5°) of the former two groups being 0.984 and 2.70, respectively.

The ground state group may again result primarily from the 0.335, \(5/2^+\) first excited state in \(^{21}\)Na, the mirror of the 0.353-MeV state in \(^{21}\)Ne. The group at 3.7 MeV corresponds to the 3.37-MeV state in \(^{10}\text{Be}^+\) and the 4.18-MeV state in \(^{21}\)Na, which experimental evidence suggests to be the mirror of the 4.73-MeV state in \(^{21}\)Ne.\(^{(108)}\) The data analysis indicates that both states are being populated in this reaction. The 7.6-MeV group may be attributed to the mutual excitation of the states in the 3.7-MeV group.

7. Discussion

All \((\text{B}^{11}, \text{B}^{10})\) and \((\text{B}^{11}, \text{Be}^{10})\) neutron and proton transfer reactions considered in this work are observed to exhibit highly selective population of a relatively small number of final states in the residual nuclei involved. This selectively is demonstrated in Fig. (3.8), which is a composite of the \(^{10}\text{B}\) and \(^{10}\text{Be}\) energy spectra of all reactions discussed in this section. The yields are shown on a linear scale normalized to integrated beam current, and the scale for the \((\text{B}^{11}, \text{B}^{10})\) reactions is a factor of 5 larger than that for the \((\text{B}^{11}, \text{Be}^{10})\) reactions. The excitation energy scales shown are averages for all reactions and are therefore approximate. The groups in these spectra correspond in most cases to the excitation of individual final states in the product.
Figure 3.8. $^{10}\text{B}$ and $^{10}\text{Be}$ energy spectra from ($^{11}\text{B}$, $^{10}\text{B}$) and ($^{11}\text{B}$, $^{10}\text{Be}$) single-nucleon transfer reactions. The excitation energy scales shown are averages for all reactions and are approximate. The yields are shown on a linear scale normalized to integrated beam current, and the scale for the ($^{11}\text{B}$, $^{10}\text{B}$) reactions is a factor of 5 larger than that for the ($^{11}\text{B}$, $^{10}\text{Be}$) reactions. The selective population of final states in these reactions is evident, and the common population of states in the $^{10}\text{B}$ and $^{10}\text{Be}$ reaction products can be seen.
B$^{10}$ AND Be$^{9}$ ENERGY SPECTRA

$(B^{10}, B^{10})$ AND $(B^{11}, Be^{9})$ SINGLE NUCLEON TRANSFER REACTIONS

$E_{en} = 114$ MeV

$\theta_{lab} = 8.5^\circ$

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$^{12}(B^{11}, B^{10})^{13}_C$

$^{12}(B^{11}, Be^{10})^{13}_N$

$^{15}(B^{11}, B^{10})^{14}_C$

$^{15}(B^{11}, Be^{10})^{14}_N$

$^{16}(B^{11}, B^{10})^{15}_N$

$^{16}(B^{11}, Be^{10})^{15}_O$

$^{17}(B^{11}, B^{10})^{16}_N$

$^{17}(B^{11}, Be^{10})^{16}_O$

$^{18}(B^{11}, B^{10})^{17}_O$

$^{18}(B^{11}, Be^{10})^{17}_F$

$^{20}(B^{11}, B^{10})^{18}_Ne$

$^{20}(B^{11}, Be^{10})^{18}_Na$

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EXCITATION (MeV)
and residual nuclei, thus only a few of the levels known to exist in the range of excitation energy shown are being populated.

Another feature of these reactions illustrated in this figure is the common excitation of states in the product nuclei, reflecting the product-plus-nucleon parentage of the target ground state. For example, the formation of the 3.37-MeV state in Be\(^{10}\) and the 1.74 and 2.15-MeV states in B\(^{10}\) can be clearly seen in all spectra. Also apparent is the mutual excitation of these states and states in the residual nuclei. This effect is evidenced by the appearance of satellite peaks removed toward higher excitations from dominant groups by amounts given by the excitation energies of the product nuclear states.

The population of known states of a predominantly single-particle nature and the weak population or absence of states whose formation in these reactions would involve the excitation of the target core in addition to the transfer of a nucleon suggest that direct nucleon transfer represents the dominant reaction mechanism. The observed energy spectra are consistent with a direct interaction process wherein final states are selectively populated which have strong parentage based on the transferred nucleon coupled to the target core. It is thus proposed that the states formed in appreciable yield are of \([J_T + (\ell, s)_j]_J\) configuration, where the final orbital angular momentum \(\ell\) and spin \(s\) of the transferred nucleon couple to a resultant angular momentum \(j\), which in turn couples to the target angular momentum \(J_T\) to form the angular momentum \(J\) of the final state.

These reactions are further characterized by an effect, also evident in Fig. (3.8), which may be described as the enhancement of the population of certain final states in the residual nuclei. Since similar kinematic and mass situations are involved in all cases, the levels which are preferentially populated may be of a common configuration. Evidence for this may be obtained from a plot of the \(Q\) values for the formation of states corresponding to strong groups observed in the energy spectra as a function of the mass number of the
Figure 3.9. Dependence of reaction Q value on the mass number of the residual nucleus for levels preferentially populated in the \((B^{11}, B^{10})\) and \((B^{11}, Be^{10})\) reactions. The systematic trend indicated by the arbitrarily drawn dashed lines suggests that these levels are of a common configuration.
FORMATION Q VALUES FOR (B^{11},B^{10}) AND (B^{11},Be^{10}) REACTIONS

- (B^{11},B^{10}) LEVELS
- (B^{11},Be^{10}) LEVELS

- Q (MeV)

MASS NUMBER RESIDUAL NUCLEUS
residual nucleus, as shown in Fig. (3.9). The trend toward lower negative Q values, indicated by the arbitrarily drawn dashed lines, suggests that the transferred nucleon is being captured into the same single-particle state, which is such that it approaches the most stable configuration of the residual nucleus as the mass number increases.

In terms of the orbital angular momentum delivered per incident nucleon to the target surface in these reactions, as discussed in the preceding section, it is reasonable to expect that this state might be of \( d_{5/2} \) character. The present data are consistent with the preferential population of states of this configuration, representable in the above notation as \( [J_T + d_{5/2}]_J \). Although in no case is the identification unambiguous or the closely spaced \( J \) multiplet resolved, known levels of dominant \( d_{5/2} \) configuration lie at excitations corresponding to the observed groups in all cases.

Spin-orbit splitting strengths in the sd shell indicate that the corresponding \( d_{3/2} \) levels are expected to lie approximately 5 MeV above the \( d_{5/2} \) states. It is found that in most situations the population of these levels is not observed or is obscured by the mutual excitation inherent in these reactions. As will be discussed in the following chapter, arguments similar to the above have been used to establish the nature of the preferential population observed in equivalent deuteron transfer reactions.

D. The Transfer Cross Section

Extensive theoretical studies have been carried out on reactions involving the transfer of a single nucleon between an incident projectile and a target following the pioneering treatment of nuclear stripping reactions by Butler. (109) In large measure, these treatments have concentrated on situations appropriate to incident deuterons, although recently interest has been focused on such reactions induced by tritons, He\(^3\) ions, and alpha particles. (110) Although the past several years have been marked by an increased interest in the theoretical
aspects of the heavy-ion transfer reaction, a microscopic theory equivalent to that for the lighter particles has not emerged from these studies. A notable exception is the extensive work of Breit and collaborators\textsuperscript{(11)} on the theory of the nucleon tunneling mechanism, however, as noted in the introduction, these calculations are not applicable for the high-energy low-$\eta$ situations of the present work.

It has also been pointed out that the various other transfer reaction models which have been evolved are, at least in their present form, essentially phenomenological parameterizations of the underlying nuclear structure problem and that the wavefunctions of the heavy nuclei are not adequately established to permit the extraction of absolute spectroscopic information from the transfer cross sections predicted by these theories. As will be noted, there is an additional fundamental complication which enters when the heavier projectiles ($A > 4$) are considered, reflecting the orbital angular momentum of the transferred nucleon while still in the projectile.

The standard Born approximation theory of direct reactions will be outlined in order to display the relevant factors which enter into the expression for the single-nucleon transfer cross section. The essential difficulty involved in the case of heavy projectiles will be discussed, and lacking a precise treatment in these situations, an approximate expression will be developed for use in the analysis of the single-nucleon transfer cross section data presented in the previous section.

To establish notation, a single-nucleon transfer reaction will be described as leading from a projectile ($a$) and target ($A$) to a product nucleus ($a-1$) and a residual nucleus ($A+1$). Schematically this may be written as

\[
a[j\, t\, m_t\, m_{t_0}] + A[J\, T\, M\, M_{T_0}] \rightarrow (a-1)[j_{1\, t_1\, m_{t_1\, t_0}}] + (A+1)[J\, T\, M\, M_{T_1}]
\]
where the angular momentum and isobaric spin variables have been explicitly displayed. The differential cross section for this process is given in the Born approximation by

\[
\frac{d\sigma}{d\omega} = \frac{\mu_{i\mu_f}}{(2\pi\hbar^2)^2} \frac{k_f}{k_i} \sum |T_{fi}|^2
\]  

(3.3)

where \( T_{fi} \) is the transition amplitude for the reaction and the summation indicates an average over initial spin states and a summation over final spin states. The reduced masses and relative wave numbers in the initial and final channels are given by \( \mu_{i}, k_i \) and \( \mu_{f}, k_f \), respectively.

The transition amplitude may be written, in general, as a matrix element between product wavefunctions in the form

\[
T_{fi} = \langle \psi_{A+1}^{*} (\xi_A, \nu) \psi_{a-1}^{*} (\xi_{a-1}) | \chi_{f}^{(-)} (k_f, r_f) \rangle \langle V(\nu) | \chi_{i}^{(+)} (k_i, r_i) \psi_{A} (\xi_A) \psi_{a} (\xi_{a-1}, \rho) \rangle
\]  

(3.4)

where \( \chi^\pm \) are the scattering wavefunctions which describe the relative motions of the pairs of nuclei in the initial and final channels, \( \vec{r} \) is the corresponding relative displacement, and \( V \) is the interaction responsible for the transition from the initial to the final state. The \( \psi \)'s are the internal wavefunctions for the non-interacting separated particles, \( \xi \) denotes internal coordinates, and \( \nu \) and \( \rho \) are the position vectors of the transferred nucleon with respect to the centers of mass of the target (\( A \)) and product (\( a-1 \)) nuclei, respectively.

The nuclear overlap integrals which result from integration over internal coordinates in Eq. (3.4) are given by

\[
I_1 = \int \psi_{a-1}^{*} (\xi_{a-1}) \psi_{a} (\xi_{a-1}, \rho) d\xi_{a-1}
\]  

(3.5a)
\[ I_2 = \int \psi_{A+1}(\xi_A, \nu) \psi_A(\xi_A) d\xi_A \] (3.5b)

A convenient method of treating the transition matrix element is to evaluate the overlap integrals by carrying out a fractional parentage expansion of the wavefunctions $\psi_{A+1}$ and $\psi_A$. In such an expansion, an $n$-nucleon wavefunction is represented as an expansion in terms of the states of the first $n-1$ particles with the $n$th particle being vector coupled to these states.

If the state of the transferred nucleon is specified by $(j_l^s \ell^s t)$ while in the projectile and by $(j_l^s \ell^s t)$ after capture by the target, the overlap integrals become

\[ I_1 = \langle a | a-1 \rangle \sum_{j_l^s \ell^s t} \langle j_l^s \ell^s t | m_{n-1}^{s_1} \rangle \langle m_{n-1}^{s_1} | t \rangle \langle t | m_{n-1}^{s_2} \rangle \langle m_{n-1}^{s_2} | j_l^s \ell^s t \rangle \]

\[ I_2 = \langle A+1 | A \rangle \sum_{j_l^s \ell^s t} \langle j_l^s \ell^s t | M_{n-1}^{s_1} \rangle \langle M_{n-1}^{s_1} | T \rangle \langle T | M_{n-1}^{s_2} \rangle \langle M_{n-1}^{s_2} | j_l^s \ell^s t \rangle \] (3.6a)

\[ x \varphi_{j_l^s \ell^s t} (\nu) \sigma_{s_1} \tau_{t_1} \]

where the Clebsch-Gordon coefficients represent the angular momentum and isobaric-spin coupling and the expansion coefficients are the coefficients of fractional parentage connecting the states involved.

It is at this point that an essential simplification occurs if consideration is restricted to projectiles with $A \leq 4$. Under these circumstances it is
reasonable to assume that the transferred nucleon can be represented by an
s-wave single-particle wavefunction, that is, \( \ell' = 0 \). The consequences of
this become clear if conservation of total angular momentum in the reaction
is considered, writing schematically for the initial and final channels.

\[
L_i + J_o + [j_1 + (\ell' + s)] j_0 \equiv L_f + j_1 + [J_o + (\ell + s)] j_1
\]

(3.7)

where the vector coupling is as indicated. The orbital angular momentum trans­
fer is given, neglecting spin-flip amplitudes, by

\[
L_t = L_f - L_i = \ell' - \ell
\]

(3.8)

If \( \ell' = 0 \), only a single \(|L_t| = |\ell|\) is involved in the reaction. However,
for situations such as the present case of heavy projectiles in the p shell,
several \( L_t \) values can contribute, determined as in Eq. (3.8) by the orbital
angular momentum of the transferred nucleon in the projectile and the residual
nucleus.

Straightforward evaluation of the transition matrix element following
substitution of the overlap integrals as given in Eq. (3.6) leads, in the restricted
s-wave case, to the usual factorized expression for the transfer cross
section\(^{(112,113,114)}\) given by

\[
\frac{d\sigma}{d\omega} = \frac{\mu_{if}^2}{(2\pi \hbar^2)^2} k f k i \frac{(2J_1 + 1)}{(2J_o + 1)} C t_1 m t o C t_0 m T_o T_1 \left| F_{if}(L_t) \right|^2
\]

x \( S_{a_a \rightarrow a_1} S_{A+1, A} \)

(3.9)
where $S$ is a spectroscopic factor or relative reduced width. $F_{\text{if}}$ is proportional to the absolute square of the transfer amplitude corresponding to a particular orbital angular momentum transfer and will be referred to as the transfer function in the following.

The significance of the form of Eq. (3.9) is that the cross section has been divided into two parts. One part, the spectroscopic factors, carries the essential nuclear structure information, while the other, the transfer function, depends mainly on the kinematics of the reaction. The transfer function, usually defined to include the scattering wavefunctions, the interaction potential, and the radial wavefunction of the transferred nucleon, contains the dependence on angle and is primarily responsible for determining the angular distribution. Two common methods for the evaluation of the transfer function lead to the familiar plane wave and distorted wave expressions for the cross section. The spectroscopic factors contain the main nuclear structure information of the reaction by virtue of their definition in terms of the nuclear overlap integrals, as discussed in the following section.

When more than one $L_t$ value is allowed, this separation of the spectroscopic and transfer contributions into multiplicative factors cannot in general be carried out, and the corresponding cross section expression cannot be evaluated without detailed knowledge of the reaction mechanism and wavefunctions involved. As discussed in the introduction, a different but somewhat parallel difficulty arises in the treatment of multi-nucleon transfer processes because of the coherence between contributions from the various possible configurations of the transferred nucleons. It is thus not possible, except on the basis of a very specific set of assumptions regarding a nuclear and reaction model, to calculate the transfer function and thereby permit extraction of absolute spectroscopic factors from the measured cross section.

Lacking such detailed information concerning the heavy-ion transfer reactions considered herein, an alternative approach involving the extraction of relative spectroscopic factors has been adopted, based on a relatively
crude approximation to the above treatment. In particular, it has been assumed that an expression of the form of Eq. (3.9) will retain approximate validity for the evaluation of reaction data under approximately constant kinematic conditions and in neighboring nuclei.

As usual it is assumed that the effective transfer function so obtained is relatively insensitive to details of nuclear structure and that such information resides in the spectroscopic factors. A final assumption is that the effective transfer function remain essentially constant for reactions leading to different states in the same nucleus or for states in adjacent nuclei. It should be emphasized in support of such an assumption that, as already discussed in Section A, previous studies of heavy-ion single-nucleon transfer reactions in this energy range have shown these reactions to possess equivalent structure-less angular distributions. This behavior has been verified for several of the reactions considered herein, as discussed in Section A.

Under these assumptions, for a given initial channel leading to neutron and proton transfer final channels, the cross section ratio can be written as

\[
\frac{(d\sigma/d\omega)_n}{(d\sigma/d\omega)_p} = \frac{(k_i)_n}{(k_f)_p} \frac{(2J_1 + 1)_n}{(2J_1 + 1)_p} \frac{\left| C_{t_1}^{\text{m}_t} m_{t_1} m_{t_0} C_{T_1}^{T_0} m_t M_{T_1} \right|^2}{\left| C_{t_1}^{\text{m}_t} m_{t_1} m_{t_0} C_{T_0}^{T_1} m_t M_{T_0} \right|^2}
\]

(3.10)

The further application of this expression is dependent upon the goodness of isobaric spin for the reaction involved. In the discussion of the introduction, it has been noted that measurements in this mass-energy range on mirror reaction systems populated in equivalent neutron and proton transfer reactions, that is, where

\[
(S_{a,A-1} S_{A+1}, A)_n = (S_{a,A-1} S_{A+1}, A)_p,
\]

have indicated that Eq. (3.10) is in good accord with the experimental data.
In consequence, it is possible to evaluate the ratio of isobaric-spin Clebsch–Gordon coefficients directly for known final states and thus determine an empirical value for the ratio of spectroscopic factors. This result can then be compared with the ratios as calculated from specific model wavefunctions using, for example, the formalism developed by MacFarlane and French.\(^{(115)}\) A particularly useful application of this method is that wherein a pair of isobaric analog states are populated in the residual nuclei (A+1). In this case, the corresponding final system spectroscopic factors are necessarily equal and the cross section ratio becomes proportional to a single ratio of spectroscopic factors relating the states in the projectile and product nuclei according to Eq. (3.11). This form of the cross section ratio

\[
\frac{(d\sigma/d\omega)_{\text{n}}}{(d\sigma/d\omega)_{\text{p}}} = \frac{(k_{\text{f}})_{\text{n}}}{(k_{\text{f}})_{\text{p}}} \left| \frac{C_{m_{1}m_{0}T_{0}t_{1}t_{0}}}{C_{m_{1}m_{0}T_{0}t_{1}t_{0}}} \right|^{2} \frac{(S_{a,a-1})_{\text{n}}}{(S_{a,a-1})_{\text{p}}} \tag{3.11}
\]

will be utilized in the analysis of the single-nucleon transfer cross section data which have been presented. The choice of the specific final states and ratios to be investigated will be discussed in Section F.

E. Calculation of Spectroscopic Factors

In accord with the general interpretation of the direct interaction aspects of a reaction as those involving only a few of the nuclear degrees of freedom, the transfer reaction is characterized by the fact that only a few nucleons actively participate in the reaction. Thus, the expression for the cross section must involve overlap integrals which measure the extent to which the remaining nucleons occupy the same configuration in the initial and final states. The overlap for transfer reactions is known as the spectroscopic
factor $S$, which has been introduced above as a factor in the cross section for these reactions.

The significance of the spectroscopic factor can be examined further by consideration of its interpretation as a relative reduced width. The reduced width for the emission of a single nucleon in a transition between two nuclear states can be regarded as the product of two factors. The first, the spectroscopic factor, measures the probability that in a given initial state the nucleons will be arranged in a configuration corresponding to the final state. The second, the so-called single particle reduced width measures the probability that when this occurs the two components will actually separate. The spectroscopic factor can thus be thought of as the reduced width in units of the single particle width. The single particle width would be the proper reduced width (i.e., $S = 1$) if the final state could always be regarded as presenting an inert potential well to the transferred nucleon. The spectroscopic factor reflects the degree to which this is an inadequate description for most physical situations by taking into account the overlap of the initial and final state wavefunctions.

The calculation of spectroscopic factors thus involves the evaluation of overlap integrals wherein the nuclear states are represented by the wavefunctions of an appropriate nuclear model. The precise form of the spectroscopic factor will depend on the model being considered. In the following, the wavefunctions will be considered to be in an LS representation specified by $(\alpha LST)$, where $\alpha$ labels the quantum numbers necessary, in addition to LST, for a complete description of the state.

The spectroscopic factor in this case is expressed, in the notation of MacFarlane and French, \cite{115} as

$$S = n \sum |I(z)|^2$$  \hspace{1cm} (3.12)
where $z$ is the channel spin of the transferred nucleon ($\vec{z} = \vec{J} + \vec{s}$, for nucleon spin $\vec{s}$) and $n$ is the number of particles in the initial state identical to the transferred nucleon. The overlap integral defined by $S$ is given by

$$I(z) = \langle \Psi(\alpha LST) | \left( \Phi(\alpha^o L^o S^o T^o) + \varphi(\ell) \right) \rangle$$

where $\Psi$ is the antisymmetrized $n$-nucleon wavefunction of the initial state and $\Phi$ is an $(n-1)$-nucleon antisymmetrized wave function which is coupled to the wavefunction $\varphi(\ell)$ of the transferred nucleon.

The integral (3.13) is evaluated by a fractional parentage expansion of $\Psi$, the expansion coefficients being the coefficients of fractional parentage (cfp) connecting the states $(\alpha LST)$ and $(\alpha^o L^o S^o T^o)$. The cfp measure the degree to which $\Psi$ has the configuration of $\Phi$ coupled to a nucleon in an orbit $\ell$, that is, the degree to which $\Phi$ is a parent state of $\Psi$. Using the orthogonality of the parent states, the integral is reduced to a delta function in the quantum numbers and the channel spin, and the overlap between the basic states of this representation is given by

$$I_{xy}(z) = (-1)^{L+L_o^+L_o} \langle \alpha LST | \alpha^o L^o S^o T^o \rangle \ U(\ell L_o J^o S L_o) \ U(L^o S^o z^o J^o S) \ .$$

where $x \equiv (\alpha LST)$ and $y \equiv (\alpha^o L^o S^o T^o)$. $\langle \alpha LST | \alpha^o L^o S^o T^o \rangle$ is the cfp connecting basic states, and $U^{(116)}$ is the normalized Racah coefficient, related to the Racah coefficient $W$ and the Wigner 6-j symbol by

$$U(abcd:ef) = \left[ (2e + 1) (2f + 1) \right]^{1/2} W(abcd:ef)$$
\[
= (-1)^{a+b+c+d} \left( (2e+1)(2f+1) \right)^{1/2} \left\{ \begin{array}{c}
  a \\
  b \\
  c \\
  d \\
  e \\
  f
\end{array} \right\}
\] (3.15b)

The U coefficients result from the coupling required to specify \( J_0 \) and \( z \) in the channel spin representation.

In the usual case, the state wavefunctions are given as linear combinations of model wavefunctions, for example as

\[
\Psi_{JT} = \sum_{\alpha LS} K^J_{\alpha LS} \Psi_{\alpha LST}
\] (3.16a)

\[
\Phi_{JT} = \sum_{\alpha LS} K^J_{\alpha LS} \Phi_{\alpha LST}
\] (3.16b)

and the overlap integral becomes

\[
I(z) = \sum_{xy} K^J_x K^J_y I_{xy}(z)
\] (3.17)

Substituting in Eq. (3.16), the spectroscopic factor corresponding to \((\alpha LST) \to (\alpha L_0 S_0 T_0) + t\) is given by

\[
S = n \sum_z \left[ \sum_{xy} K_x K_y (-1)^t \right]^{J_0 + L_0 + L_0} <\alpha L_0 S_0 T_0 | J_0 S_0 T_0 > U(t L_0 J S : L_0)
\]

\[
x U(L_0 S_0 z s : J S)^2
\] (3.18)

The factor \( n \) occurs because the cfp describes the separation of the nth particle, whereas the reaction allows every equivalent particle to be emitted on the same basis. The sum over \( z \) involves at most two terms since \( s = \pm 1/2 \), and the contributions from the two channel spins add incoherently.
Equation (3.18) is the form of the spectroscopic factor used in the present analysis. The application of Eq. (3.18) to Eq. (3.11) requires two major inputs. One is the model wavefunctions in the LS representation, which provide $K_x^J$ and $K_y^{J_0}$ for the states to be investigated in the projectile and product nuclei. The other is the coefficients of fractional parentage, $<p^n|p^{n-1}>$, for the nuclear p shell.

The standard cfp for the p shell are conventionally taken to be those given by Jahn and van Wieringen\(^1^{17}\) as amended by Elliott et al.\(^1^{18}\) In this treatment, the cfp appear as coefficients in the expansion of the totally antisymmetric n-particle states of the p-shell nuclei as linear combinations of totally antisymmetric states of n-1 particles vector coupled to the states of the nth particle. The cfp are calculated using the Young–Yamanouchi representation of the permutation group and are expressed as the product of a weight factor, an orbital coefficient, and a charge-spin coefficient as

\[
<\alpha_L S T|\alpha_L S T> = w <\alpha_L |\alpha_L > <\alpha S T|\alpha S T> \tag{3.19}
\]

In this case, $\alpha$ represents the space symmetry of the orbital component of the wavefunction, which is combined with a charge-spin component of adjoint symmetry $\tilde{\alpha}$ to form the totally antisymmetric wavefunction. The weight factor $w$ is proportional to the ratio of the dimensions of the representations described by $\alpha_o$ and $\alpha$. For the present application, the required cfp are obtained as products of tabulated values of the orbital, charge-spin, and weight factors, as specified by the model wave functions employed.

The wavefunctions used herein are the intermediate coupling model wavefunctions of p-shell nuclei given by Boyarkina.\(^1^{19}\) The wavefunctions are given in the LS representation ($\alpha LS$), with $\alpha$ as described above, and are expressed as weighted linear combinations of the basic states of this representation as given in Eq. (3.16). The wavefunctions in the central region of the
p-shell considered are particularly complex, thus in view of the approximations inherent in the present treatment, only the dominant components have been used in these calculations.

The U coefficients were evaluated using a tabulation of the Racah W coefficients. The spectroscopic factors calculated according to Eq. (3.18) are given in the following section. Also given are the results of equivalent calculations made available by Kurath and carried out using his intermediate coupling p-shell wavefunctions.

F. \((B^{10} + n)\) and \((Be^{10} + p)\) Spectroscopic Factors

1. Transfer Cross Section Ratios

In this section, the comparison of measured and calculated cross section ratios for states corresponding to groups observed in the \((B^{11}, B^{10})\) and \((B^{11}, Be^{10})\) transfer reaction energy spectra will be discussed. The experimental ratios are extracted from the cross section data presented in the individual spectra discussions of Section C. The calculated ratios are obtained according to the procedure outlined in the previous two sections.

The \((B^{11}, B^{10})\) and \((B^{11}, Be^{10})\) neutron and proton transfer reactions may be written schematically for a target \(A\) as

\[
B^{11} + A = (B^{10} + n) + A \rightarrow B^{10} + (A + n) \quad (3.20a)
\]

\[
B^{11} + A = (Be^{10} + p) + A \rightarrow Be^{10} + (A + p) \quad (3.20b)
\]

This notation illustrates the isobaric symmetry of the residual nuclei \((A + n)\) and \((A + p)\) and reflects the initial and final system parentage represented by the spectroscopic factors appearing in the transfer cross section expression developed above.

*We are indebted to Dr. D. Kurath for making available his unpublished calculations.
The ratio of transfer cross sections leading to mirror or analog states in the isobaric nuclei \((A+n)\) and \((A+p)\) is given in Eq. (3.11) and may be rewritten using the above notation as

\[
\frac{(d\sigma/d\omega)_{n}}{(d\sigma/d\omega)_{p}} = \frac{(k_{t})_{n}}{(k_{t})_{p}} \frac{(C_{B^{10+n}A+n})^{2}}{(C_{Be^{10+p}A+p})^{2}} \frac{S_{B^{10+n}}}{S_{Be^{10+p}}} \tag{3.21}
\]

where the subscripts \(n\) and \(p\) denote the reactions \((B^{11}, B^{10})\) and \((B^{11}, Be^{10})\), respectively. The relative wave number in the final channel is given by \(k_{t}\). \(S_{B^{10+n}}\) and \(S_{Be^{10+p}}\) are the spectroscopic factors which represent the product nucleus-nucleon configuration of the target core. The Clebsch-Gordon coefficients indicate the isobaric spin coupling in the initial and final systems of each reaction and are given by

\[
C_{B^{10+n}} = |C_{\frac{t_{1} t_{t} t_{0}}{t_{1} t_{t} t_{0}}}|_{n}^{} = |C_{\frac{t_{1} 1/2 1/2}{t_{1} 1/2 1/2}}|_{n}^{} \tag{3.22a}
\]

\[
C_{A+n} = |C_{\frac{T_{0} t T_{1}}{M_{0} t T_{1}}}|_{n}^{} = |C_{\frac{T_{0} 1/2 T_{1}}{M_{0} 1/2 M_{1}}}|_{n}^{} \tag{3.22b}
\]

\[
C_{Be^{10+p}} = |C_{\frac{t_{1} t_{t} t_{0}}{m_{t} m_{t} m_{t}}} p = |C_{\frac{t_{1} 1/2 1/2}{m_{t} -1/2 1/2}}|_{p}^{} \tag{3.22c}
\]
\[ C_{A + p} = \left| C_{M_{T_0} M_{T_1}} \right| \]

where \( t_0, t_1, T_0, \) and \( T_1 \) are the isobaric spins of the projectile, product nucleus, target and residual nucleus, respectively. The isobaric spin of the transferred nucleon is \( t \).

For discussion purposes, Eq. (3.21) will be rewritten as

\[ \sigma = \frac{(C_{B^{10+n} A + n})^2}{(C_{Be^{10+p} A + p})^2} \]

\[ \sigma = \frac{S_{B^{10}}}{S_{Be^{10}}} \]

The symbolic notation is defined by

\[ \sigma = \frac{(d\sigma/d\omega)_n}{(d\sigma/d\omega)_p} \]

The starred cross sections are therefore given by the measured cross sections reduced by a final channel kinematic factor. The kinematic corrections are small (<10%) but are included for completeness.

In all cases, the analog or mirror states in the residual nuclei are ground states or ground-state analogs. The ratios consist of cross sections of states corresponding to groups in the \( B^{10} \) and \( Be^{10} \) energy spectra with respect to the cross section of the \( Be^{10} \) ground state in the respective \( Be^{10} \)
spectra. The groups considered are the 0.0-MeV $^{10}\text{B}$ group, corresponding to the ground and 0.72-MeV states in $^{10}\text{B}$, the 2.0-MeV $^{10}\text{B}$ group, corresponding to the 1.74-MeV and 2.15-MeV states in $^{10}\text{B}$, the 3.6-MeV $^{10}\text{B}$ group, corresponding to the 3.58-MeV state in $^{10}\text{B}$, and the 3.4-MeV $^{10}\text{Be}$ group, corresponding to the 3.37-MeV state in $^{10}\text{Be}$.

The four experimental cross section ratios obtained are therefore given by

$$
\sigma_1 = \frac{(d\sigma/d\omega)\left[B^{10}(0.0+0.72\text{ MeV})\right]}{(d\sigma/d\omega)\left[B^{10}(0.0)\right]} \tag{3.24a}
$$

$$
\sigma_2 = \frac{(d\sigma/d\omega)\left[B^{10}(1.74\text{ MeV} + 2.15\text{ MeV})\right]}{(d\sigma/d\omega)\left[B^{10}(0.0)\right]} \tag{3.24b}
$$

$$
\sigma_3 = \frac{(d\sigma/d\omega)\left[B^{10}(3.58\text{ MeV})\right]}{(d\sigma/d\omega)\left[B^{10}(0.0)\right]} \tag{3.24c}
$$

$$
\sigma_4 = \frac{(d\sigma/d\omega)\left[\text{Be}^{10}(3.37\text{ MeV})\right]}{(d\sigma/d\omega)\left[\text{Be}^{10}(0.0)\right]} \tag{3.24d}
$$

The ratios $\sigma_1$, $\sigma_2$, $\sigma_3$, and $\sigma_4$ are extracted from the data for all reactions for which groups corresponding to these states are resolved in the energy spectra.

In the calculation of the cross section ratios, two distinct situations are specified by the isobaric spin of the target ground state. The first case involves reactions proceeding from $T = T_z = 0$ targets to $T = 1/2$, $T_z = \pm 1/2$ mirror or analog states in the residual nuclei. In the second case the target has $T = T_z = 1/2$ and the residual nuclei are left in $T = 1$, $T_z = 0, 1$ analog states.
The cross section ratios given by Eq. (3.24) will be obtained by consideration of the Clebsch-Gordon coefficients in Eq. (3.23) for each of these two cases. In the following discussions, states will be referred to in a (JT) notation.

Case 1. Reactions leading to \( T_1 = 1/2, \ M_{T_1} = \pm 1/2 \) mirror or analog states in the residual nuclei from \( T_0 = M_{T_0} = 0 \) targets. The final system coefficients are given by

\[
\begin{align*}
C_{A+n} &= |C^0_{0 \ 1/2 \ 1/2} | = 1 \\
C_{A+p} &= |C^0_{0 \ -1/2 \ -1/2} | = 1
\end{align*}
\]

For \( \text{Be}^{10} \) in the (01) ground state, the \((\text{Be}^{11}, \text{Be}^{10})\) initial system coefficient is given by

\[
C_{\text{Be}^{10}+p} = |C^1_{1 \ -1/2 \ 1/2} | = (2/3)^{1/2}
\]

For \( \text{Be}^{10} \) in the (30) ground state or 0.72-MeV (10) state, the \((\text{Be}^{11}, \text{Be}^{10})\) initial system coefficient becomes

\[
C_{\text{Be}^{10}+n} = |C^0_{0 \ 1/2 \ 1/2} | = 1
\]

and

\[
\sigma_1 = \frac{S_{\text{B}^{10}(0.0)} + S_{\text{B}^{10}(0.72)}}{S_{\text{Be}^{10}(0.0)[2/3]}^{2/3}} \tag{3.25a}
\]
For $^{10}\text{B}$ in the 1.74-MeV(01) state or 2.15-MeV(10) state, the $(\text{B}^{11}, \text{B}^{10})$
initial system coefficient becomes

\[ C_{\text{B}^{10}+n}(1.74 \text{ MeV}) = | C_{0}^{1/2} 1/2 1/2 | = | - (1/3)^{1/2} | \]

\[ C_{\text{B}^{10}+n}(2.15 \text{ MeV}) = | C_{0}^{0} 1/2 1/2 | = 1 \]

and

\[ \sigma_{2} = \frac{S_{\text{B}^{10}(1.74)^{1/3}} + S_{\text{B}^{10}(2.15)}}{S_{\text{Be}^{10}(0.0)^{2/3}}} \]

Since spectroscopic factors are independent of the third component of isobaric
spin, those for the $^{10}\text{B}$, 1.74-MeV and $^{10}\text{Be}$, 0.0-MeV(01) analog states are
identical and

\[ \sigma_{2} = \frac{S_{\text{Be}^{10}(0.0)^{1/3}} + S_{\text{B}^{10}(2.15)}}{S_{\text{Be}^{10}(0.0)^{2/3}}} \]  \hspace{1cm} (3.25b)

For $^{10}\text{B}$ in the 3.58-MeV (20) state, the $(\text{B}^{11}, \text{B}^{10})$ initial system
coefficient becomes

\[ C_{\text{B}^{10}+n}(3.58) = | C_{0}^{0} 1/2 1/2 | = 1 \]

and

\[ \sigma_{3} = \frac{S_{\text{B}^{10}(3.58)}}{S_{\text{Be}^{10}(0.0)^{2/3}}} \]  \hspace{1cm} (3.25c)
Since the ground state and the 3.37-MeV state in \(^{10}\text{Be}\) both have \(T_1 = M_{T_1} = 1\),

\[
\sigma_4 = \frac{S_{\text{Be}^{10}(3.37)}}{S_{\text{Be}^{10}(0,0)}} \quad (3.25d)
\]

Case 2. Reactions leading to \(T = 1, M_T = 0, 1\) analog states in the residual nuclei from \(T_o = M_{T_o} = 1/2\) targets. The final system Clebsch-Gordon coefficients are given by

\[
C_{A+n} = |C 1/2 1/2 1 \rangle = 1
\]

\[
C_{A+p} = |C 1/2 1/2 1 \rangle = (1/2)^{1/2}
\]

The initial system coefficients are the same as for Case 1, therefore the calculated ratios for Case 2 are twice those given in Eq. (3.25). In order to permit comparison of experimental ratios to the same calculated ratios in both cases, experimental values for Case 2 are divided by 2.

Experimental values of the cross section ratios given in Eq. (3.24) were obtained in 5 cases for \(\sigma_1\), 5 cases for \(\sigma_2\), 2 cases for \(\sigma_3\), and 3 cases for \(\sigma_4\). The results are presented in Table (3.1). Also given are the average experimental ratios for all targets considered in each case and the corresponding calculated ratios. The ratios \(\sigma_1\) and \(\sigma_2\) were calculated using the Boyarkina intermediate coupling wavefunctions \(11^9\) and the method of Section E. The ratios \(\sigma_3\) and \(\sigma_4\) were calculated by Kurath \(12^1\) using his intermediate coupling wavefunctions.
Table 3.1. Comparison of experimental and calculated \( (\text{B}^{11}, \text{B}^{10}) \) and \( (\text{B}^{11}, \text{Be}^{10}) \) transfer reaction cross section ratios. All ratios are relative to \( \text{Be}^{10}(0.0) \) and are identified by \( \sigma_1 = [\text{B}^{10}(0.0)+\text{B}^{10}(0.72 \text{ MeV})] \), \( \sigma_2 = [\text{B}^{10}(1.74 \text{ MeV})+\text{B}^{10}(2.15 \text{ MeV})] \), \( \sigma_3 = \text{B}^{10}(3.58 \text{ MeV}) \), \( \sigma_4 = \text{Be}^{10}(3.37 \text{ MeV}) \). The experimental averages for all targets and the calculated ratios are also given.

<table>
<thead>
<tr>
<th>Target</th>
<th>( \sigma_1 )</th>
<th>( \sigma_2 )</th>
<th>( \sigma_3 )</th>
<th>( \sigma_4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>C(^{12})</td>
<td>4.96</td>
<td>1.90</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C(^{13})</td>
<td>3.96</td>
<td>2.01</td>
<td></td>
<td></td>
</tr>
<tr>
<td>N(^{14})</td>
<td>4.41</td>
<td>2.26</td>
<td>0.59</td>
<td>2.51</td>
</tr>
<tr>
<td>N(^{15})</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>O(^{16})</td>
<td>7.32</td>
<td>3.18</td>
<td>0.50</td>
<td></td>
</tr>
<tr>
<td>Ne(^{20})</td>
<td>4.63</td>
<td>3.26</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Avg.</td>
<td>5.06</td>
<td>2.52</td>
<td>0.54</td>
<td>3.14</td>
</tr>
<tr>
<td>Calculated</td>
<td>4.26(^a)</td>
<td>1.83(^a)</td>
<td>0.61(^b)</td>
<td>2.64(^b)</td>
</tr>
</tbody>
</table>

\(^a\)Calculated from reference 119.

\(^b\)From reference 121.

The experimental ratios are shown in Fig. (3.10) displayed relative to the respective calculated ratios as a function of the mass number of the residual nucleus. All ratios are divided by the calculated ratios, with no further normalization. The calculations are in good accord with the experimental data. The consistency of the results suggests that the method employed may be applicable in these situations.

2. \( \text{B}^{11} \) Ground State Parentage

The experimental values of the \( (\text{B}^{11}, \text{B}^{10}) \) and \( (\text{B}^{11}, \text{Be}^{10}) \) transfer reaction cross section ratios discussed above have been used to calculate relative...
Figure 3.10. Comparison of experimental and calculated \((\text{B}^{11}, \text{B}^{10})\) and \((\text{B}^{11}, \text{Be}^{10})\) transfer reaction cross section ratios. All ratios are relative to the \(\text{Be}^{10}\) ground state cross section in the respective \((\text{B}^{11}, \text{Be}^{10})\) reactions and are displayed with respect to the corresponding calculated ratio as a function of the mass number of the residual nucleus. The calculated ratio is taken to be unity in all cases.
CROSS SECTION RATIOS
(B^{11}, B^{10}) AND (B^{11}, Be^{10}) REACTIONS

$E_{B^{11}} = 114$ MeV

$\theta_{LAB} = 8.5^\circ$

$\frac{(d\sigma/d\Omega)_1}{(d\sigma/d\Omega)_2}$

- $B^{10}(0.0) + B^{10}(0.72$ MeV)
- $B^{10}(1.74$ MeV) + $B^{10}(2.15$ MeV)
- $B^{10}(3.58$ MeV)
- $Be^{10}(3.37$ MeV)

CALCULATED RATIO

MASS NUMBER RESIDUAL NUCLEUS
spectroscopic factors $S_{B^{10} + n}$ and $S_{Be^{10} + p}$ for the ground and low-lying levels in $B^{10}$ and $Be^{10}$ and to examine the $(B^{10} + n)$ and $(Be^{10} + p)$ parentage of the $B^{11}$ ground state.

The spectroscopic factors are obtained by substituting the average experimental cross section ratios given in Table (3.1) into Eq. (3.25) and solving the resulting simultaneous equations. Independence on the third component of isobaric spin permits use of spectroscopic factors for the ground and 3.37-MeV states in $Be^{10}$ to extract those for the 1.74-MeV and 5.16-MeV analog states in $B^{10}$, respectively, both of which are unresolved in the experimental data. A single value is, however, obtained for the unresolved ground and first excited states of $B^{10}$, and the results are normalized to $[S_{B^{10}(0.0) + n} + S_{B^{10}(0.72 \text{ MeV}) + n}] = 1.2$. The experimental relative spectroscopic factors are compared with the intermediate coupling predictions of Kurath[121] in Table (3.2).

Table 3.2. $(B^{10} + n)$ and $(Be^{10} + p)$ parentage of the $B^{11}$ ground state. Experimental and calculated relative spectroscopic factors are given for the ground and low-lying levels of $B^{10}$ and $Be^{10}$.

<table>
<thead>
<tr>
<th>Level (MeV)</th>
<th>$B^{10}$ (JT)</th>
<th>$Be^{10}$ (JT)</th>
<th>Relative Spectroscopic Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Experiment</td>
</tr>
<tr>
<td>0.0</td>
<td>(30)</td>
<td>0.0</td>
<td>1.20</td>
</tr>
<tr>
<td>0.72</td>
<td>(10)</td>
<td>0.0</td>
<td>0.36</td>
</tr>
<tr>
<td>1.74</td>
<td>0.0</td>
<td>(01)</td>
<td>0.48</td>
</tr>
<tr>
<td>2.15</td>
<td>(10)</td>
<td>0.13</td>
<td>1.15</td>
</tr>
<tr>
<td>3.58</td>
<td>(20)</td>
<td>3.37</td>
<td></td>
</tr>
<tr>
<td>5.16</td>
<td>3.37</td>
<td>(21)</td>
<td></td>
</tr>
</tbody>
</table>

$^a$From reference 121.
Figure 3.11. \((B^{10}+n)\) and \((Be^{10}+p)\) parentage of the \(B^{11}\) ground state. Ground and low-lying states in \(B^{10}\) and analog states in \(Be^{10}\) are shown vertically. The corresponding experimental and calculated (reference 121) relative spectroscopic factors are shown horizontally.
B$^{11}$ GROUND STATE PARENTAGE
B$^{10} + n$ AND Be$^{10} + p$

EXPERIMENT
KURATH, 1965

(JT)

3.37 (21) 5.16

(20) 3.58

(10) 2.15

0.0 (01) 1.74

Be$^{10}$

RELATIVE SPECTROSCOPIC FACTOR
The large strength for the (21) states and the weakness of the transition to the (20) state observed in the present data, as discussed in Section C, are thus predicted by the calculations. It has been pointed out previously that the $^n\text{B}^{11}\langle p,d\rangle^\text{Be}^{10}$ data of Bachelier et al. are in qualitative agreement with those of the present work regarding the relative population of these states in $^\text{Be}^{10}$.

It is interesting to note that similar results are obtained in pure $jj$ coupling wherein only four of these states are connected to the $^\text{B}^{11}$ ground state, specifically, the first (30), (10), (01), and (21) states. The relative spectroscopic factors are given in this case by $S = [(1/4)\,(2J + 1)\,(2T + 1)]$, or by $1.0\,(30) + (10),\,0.3(01)$, and $1.5\,(21)$. The change in going to intermediate coupling is that the (21) strength is split, with a higher-lying (21) state having about half the strength of the (21) state.

The relative spectroscopic factors are a measure of the ($^\text{B}^{10}+n$) and ($^\text{Be}^{10}+p$) parentage of the $^\text{B}^{11}$ ground state. This parentage is displayed in Fig. (3.11), wherein the states of $^\text{B}^{10}$ and $^\text{Be}^{10}$ are shown vertically and the corresponding experimental and calculated relative spectroscopic factors given in Table (3.2) are shown horizontally. The calculated values are in good agreement with the experimental data in all cases. Assuming from the consistency of the results presented in this section that the comparison method used is valid in these situations, these data provide an experimental test of the model wavefunctions involved in the calculated values.
IV. DEUTERON TRANSFER REACTIONS

A. Presentation of Results

1. Energy Spectra

The experimental data to be discussed result from $(^{11}\text{B},^{9}\text{Be})$ reactions induced by 115.9-MeV $^{11}\text{B}$ ions on $^{12}\text{C}$, $^{13}\text{C}$, $^{14}\text{N}$, $^{15}\text{N}$, $^{16}\text{O}$, and $^{20}\text{Ne}$ targets. The data consist of energy spectra of the $^{9}\text{Be}$ products observed at forward angles and differential cross section measurements for energy groups appearing in these spectra.

The $(^{11}\text{B},^{9}\text{Be})$ reactions are interpreted as transfers from the projectile to the target of a correlated neutron-proton pair, that is, a deuteron in either singlet or triplet configuration. It has already been noted that reference to these reactions as deuteron transfer reactions does not necessarily imply the transfer of a physical deuteron. As in the case of the single-nucleon transfer reactions discussed in the preceding chapter, this assignment is consistent with the observation that the alternate reaction mechanism involving the backward transfer of a more complex cluster is not expected to contribute significantly to the cross section in the forward direction. An example of this effect for a reaction similar to those under consideration is shown in the lower portion of Fig. (3.1). The two-nucleon transfer is seen to contribute mainly in the forward direction, with little interference from the alternate backward transfer reaction.

Representative energy spectra of the $^{9}\text{Be}$ products of the $(^{11}\text{B},^{9}\text{Be})$ reactions are presented in Fig. (4.1) through (4.6). The groups in these spectra are interpreted as the excitation of final states in the products of these reactions, and the portions of the spectra shown correspond to excitation energies up to approximately 20 MeV. The approximate range of laboratory kinetic energy included in the spectra is from 70 to 100 MeV.

The $^{9}\text{Be}$ energy spectra were recorded simultaneously with the single-nucleon transfer reaction data, and all comments made in connection with the
Figure 4.1. Be⁹ energy spectrum, C¹²(B¹¹, Be⁹)N¹⁴. The solid curve is a least squares computer fit of the energy detector response function to the data. The error bars represent both statistical errors and the quality of the isotopic resolution in a given channel.

Figure 4.2. Be⁹ energy spectrum, C¹³(B¹¹, Be⁹)N¹⁵. The significance of the solid curve and error bars is as in Figure 4.1.

Figure 4.3. Be⁹ energy spectrum, N¹⁴(B¹¹, Be⁹)O¹⁶. The significance of the solid curve and error bars is as in Figure 4.1.

Figure 4.4. Be⁹ energy spectrum, N¹⁵(B¹¹, Be⁹)O¹⁷. The significance of the solid curve and error bars is as in Figure 4.1.

Figure 4.5. Be⁹ energy spectrum, O¹⁶(B¹¹, Be⁹)F¹⁸. The significance of the solid curve and error bars is as in Figure 4.1.

Figure 4.6. Be⁹ energy spectrum, Ne²⁰(B¹¹, Be⁹)Na²². The significance of the solid curve and error bars is as in Figure 4.1.
Be$^9$ ENERGY SPECTRUM

$E_{\gamma}^n = 113.5$ MeV

$\theta_{\text{LAB}} = 8.5^\circ$
Be$^9$ ENERGY SPECTRUM

$N^{15}(B^{14},Be^9)O^{17}$

$E_{B^{14}} = 113.5$ MeV

$\theta_{LAB} = 8.5^\circ$
Be$^9$ ENERGY SPECTRUM

$O^{16} (B^{11}, Be^{9}) F^{18}$

$E_{B^{11}} = 113.1$ MeV

$\theta_{LAB} = 8.5^\circ$

COUNTS PER CHANNEL

EXCITATION (MeV)
Be$^9$ ENERGY SPECTRUM

Ne$^{20}$ (B$^{11}$, Be$^9$) Na$^{22}$

$E_{B^{11}} = 113.6$ MeV

$\theta_{LAB} = 8.5^\circ$
presentation of the $^10\text{B}$ and $^10\text{Be}$ spectra in the preceding chapter are directly applicable in this section. In particular, the reduction of the spectra to the form displayed, the significance of the error bars and the curves through the data points, and the slight variation of the laboratory reaction energies given have been discussed previously. The deuteron transfer data have also been found to be completely reproducible. The abscissa of each spectrum is again the multiparameter analyzer energy axis channel number and the ordinate is the number of counts per channel.

2. Cross Sections

As discussed in the introduction, recent studies of both single-nucleon and deuteron heavy-ion transfer reactions in the energy range under consideration have shown that the angular distributions of states populated in these reactions show no evidence of the oscillatory structure originally predicted for these low-$T_2$ situations, but instead decrease smoothly as an exponential function of angle or equivalently as a power-law function of the linear momentum transfer. Restricted angular distribution measurements were made for several of the deuteron transfer reactions investigated in the present work, and the results are shown in Fig. (4.7), in which the relative differential cross section is displayed as a function of the linear momentum transfer. Equivalent measurements made, as discussed in the previous chapter, for the single-nucleon transfer reactions are also shown.

It has also been noted that two of the contemporary transfer reaction models discussed in the introduction have been able to account for the non-oscillatory nature of the angular distributions observed in these situations. The diffraction model of Dar (24, 25) predicts a smooth variation of the transfer cross section on the basis of nuclear structure effects, specifically, the mixing of out-of-phase contributions to the cross section as a result of the orbital angular momentum transferred in the reaction. This effect is somewhat difficult to evaluate quantitatively since, as shown in the previous chapter, the orbital
Figure 4.7. Angular distributions, $B^{11}$-induced single-nucleon and deuteron transfer reactions. The relative differential cross sections of the beryllium and boron products of the indicated reactions are displayed as a function of the linear momentum transfer. The solid lines have no physical significance, and the dashed line shows the predicted $q^{-3}$ dependence of the transfer cross section.
ANGULAR DISTRIBUTIONS
B^{11} - INDUCED TRANSFER REACTIONS
E_{B^{11}} = 114 MeV

TWO-NUCLEON TRANSFER
- O^{16}(B^{11}, Be^{9})F^{18}(1.1 MeV)
- O^{16}(B^{11}, Be^{9})F^{18}(9.4 MeV)
- N^{15}(B^{11}, Be^{9})O^{17}(7.6 MeV)

SINGLE-NUCLEON TRANSFER
- O^{16}(B^{11}, Be^{10})F^{17}(0.0)
- N^{15}(B^{11}, Be^{10})O^{16}(0.0)
- O^{16}(B^{11}, B^{10})O^{17}(0.0)
- O^{16}(B^{11}, B^{10})O^{17}(1.9 MeV)
- N^{15}(B^{11}, B^{10})N^{16}(0.0)

\( \frac{d\sigma}{d\Omega} \) (RELATIVE VALUES)

MOMENTUM TRANSFER (\( f^{-2} \))
angular momentum transfer in heavy-ion transfer reactions is not a well-defined quantity. The Dar theory will not be considered further in these discussions.

In the recoil model of Dodd and Greider,\(^{(26)}\) it is proposed that the transfer cross section is essentially independent of details of nuclear structure and that a smooth angular distribution is a direct consequence of finite-range and recoil effects in the reaction. A cancellation of out-of-phase contributions in this case results from consideration of contributions to the phase of recoil terms, the order of which is the ratio of the masses of the transferred particle and the heavy cores to which it is initially and finally bound. The predicted angular distribution calculated on the assumption of an extreme diffraction model is of the form

\[
\frac{d\sigma}{d\omega} \propto q^{-3} \exp\left(-\frac{p^2 a^2}{6}\right) \tag{4.1}
\]

where \(q\) is the linear momentum transfer, given by the brackets in Eq. (3.1), \(a\) is the range of the bound-state wavefunctions, and \(p\) is the recoil momentum, given by the difference of incident and final channel wave numbers weighted by the respective recoil ratios. If \(pa \leq 1\), Eq. (4.1) reduces to

\[
\frac{d\sigma}{d\omega} \propto q^{-3} \tag{4.2}
\]

The transfer cross section thus has the dependence on the mass of the transferred particle implicit in the recoil, but no other dependence on energy, structure, or particle species, in accord with a wide variety of heavy-ion transfer data.

The \(q^{-3}\) dependence of the transfer angular distribution predicted by Eq. (4.2) is indicated in Fig. (4.7) and is shown to provide an adequate description of these data. The slight variation between the single-nucleon and deuteron transfer results reflects the mass dependence of the recoil effects. These data are also well represented by an exponential function of angle, as expected from previous studies of similar reactions.\(^{(20)}\) Since only very restricted additional information can be obtained from further angular distribution data of this type in the mass-energy region under consideration, differential cross section
measurements for the other deuteron transfer reactions investigated were made only at a selected forward angle.

The behavior of the deuteron transfer cross sections illustrated in Fig. (4.7) has been used in the extraction of total cross sections from these limited measurements for comparison with other work. The actual dependence of the angular distribution on the linear momentum transfer is found to be $q^{-2.7}$. The procedure adopted is to employ appropriate kinematic relationships to convert this dependence to an angular dependence, which has been shown to be well represented by

$$\frac{d\sigma}{d\omega} = \sigma_0 \exp(-\alpha \theta)$$  \hspace{1cm} (4.3)

where $\theta$ is the c.m. scattering angle. The resulting slope and intercept parameters $\alpha$ and $\sigma_0$, respectively, are used to obtain the integrated cross section from

$$\sigma = \int_{4\pi} (d\sigma/d\omega) d\omega = 2\pi \sigma_0 \int_0^{\pi} \sin\theta \ e^{-\alpha \theta} \ d\theta$$  \hspace{1cm} (4.4)

which may be evaluated directly. Errors due to extrapolation to small and large angles are expected to be small due to the $\sin\theta$ and exponential modulation, respectively.

This approach is expected to be applicable on the basis of the close approximation by a common dependence on linear momentum transfer and angle found for the angular distribution data of these and similar reactions. Total cross sections for the reactions represented in Fig. (4.7) have been obtained using both this procedure in conjunction with a single forward-angle measurement and by direct integration of the complete angular distribution plotted as a function of angle. A comparison of these values shows no relative error and an absolute error of less than 5 percent, which is negligible compared to the estimated error in the individual cross section measurements, as given in Chapter II. The results of the application of this method to the deuteron transfer reactions considered in the present work are given in Section D.
Figure 4.8. $^{9}\text{Be}$ energy spectra, $^{16}\text{O}^{11}\text{Be}^{9}\text{F}^{18}$. The yield is shown on a logarithmic scale, and the c.m. scattering angle is given for each spectrum. The intensity of strong groups in these spectra is seen to decrease systematically as the angle is increased.
Be$^9$ ENERGY SPECTRA
O$^{16}$ (Be$^{9}$, Be$^{9}$) F$^{18}$
$E_{e''} = 113.1$ MeV
The absolute differential cross sections were obtained, as for the single-nucleon transfer reactions, from the automatic data reduction procedure on the basis of Eq. (2.8) and represent the areas of gaussians fitted to groups in the energy spectrum. These measurements are presented in the detailed discussion of the individual spectra in the following section. Since the angular distributions of states populated in these reactions are expected to be slowly varying, the differential cross sections in a given spectrum are expected to be a measure of the relative total cross sections. Therefore, in situations in which absolute total cross sections are not required, the differential rather than the integrated values will be compared. Energy spectra measured at four scattering angles for one of the reactions considered herein are shown in Fig. (4.8). The systematic decrease in the intensity of major groups in these spectra as the angle is increased illustrates the expected behavior.

B. \( (B^{11}, Be^9) \) Energy Spectra

The \( (B^{11}, Be^9) \) deuteron transfer reaction energy spectra are discussed separately in this section. States corresponding to groups observed in the individual spectra are correlated where possible with their expected configuration in order to determine the nature of the mechanism involved in the population of final states in these reactions. A detailed comparison is made with corresponding \( (\alpha, d) \) reactions in cases where such data are available. Deuteron energy spectra \(^{57,59}\) from \( (\alpha, d) \) reactions at energies of 40-50 MeV on \( C^{12}, N^{14}, N^{15}, O^{16}, Ne^{20} \) and \( Mg^{24} \) targets are shown in Fig. (4.9).

All energy spectra were measured at the same laboratory scattering angle of 8.5°, corresponding to c.m. angles between 13° and 15°. Differential cross sections are given for each energy group discussed. As noted above, the smoothly varying angular distributions characteristic of heavy-ion transfer reactions at these energies make integration of these cross sections unnecessary before comparison.
Figure 4.9. Deuteron energy spectra from ($\alpha$, d) reactions. Spectra are shown from ($\alpha$, d) reactions on $^{12}$C, $^{14}$N, $^{16}$O, $^{20}$Ne, $^{24}$Mg (from reference 59) and $^{15}$N (from reference 57) targets. The laboratory scattering angle and incident energy are shown for each spectrum.
Since all Be\(^9\) product nuclei formed in \(T = 1/2\) excited states decay to the ground state before detection, groups appearing in the spectra correspond to specific levels in the residual nuclei. The ambiguity resulting from the excitation of \(T = 3/2\) states in Be\(^9\) does not occur in the excitation energy ranges considered. The breakup of Be\(^{10}\) into (Be\(^9\)+n) at 6.814 MeV has been calculated to not be a factor in the interpretation of those portions of the spectra discussed.

Following the notation adopted in the previous chapter for the single-nucleon transfer reactions, groups appearing in the spectra will be identified by the excitation energy of the corresponding states. The excitation energies quoted are from a standard compilation of energy levels,\(^{(94)}\) unless more recent values from literature cited in the discussion are used.

1. \(^{12}\text{C}^{11}(\text{Be}^9)\text{N}^{14}\)

The Be\(^9\) energy spectrum from the \(^{12}\text{C}^{11}(\text{Be}^9)\text{N}^{14}\) reaction at \(E_{\text{Be}^{11}} = 113.5\) MeV is shown in Fig. (4.1). Strongly populated groups appear at \(\text{N}^{14}\) excitations of 9.0 and 12.7 MeV. Several groups appear at excitations above 14 MeV, and a weak group between 5 and 7 MeV has been resolved into two peaks. No other groups are excited appreciably above background. In particular, the ground state and 3.95-MeV second excited state are weakly populated, and there is no evidence of the 2.31-MeV, \(T = 1\) first excited state. There is no indication that the other known \(T = 1\) states in \(\text{N}^{14}\), which lie between 8 and 11 MeV, are being populated, although several of these levels would be obscured by the highly populated group at 9.0 MeV. This spectrum is in agreement with that obtained by Sachs et al. in a previous study.\(^{(20)}\)

The \(^{12}\text{C}(\alpha,d)\text{N}^{14}\) reaction has been observed at \(E_\alpha = 53\) MeV,\(^{(58)}\) and the deuteron energy spectrum is given in Fig. (4.9). The \(\text{N}^{14}\) energy levels identified in the \((\alpha,d)\) reaction are compared with the groups observed in the present work in Table (4.1). Also given are the \((\alpha,d)\) integrated cross sections, the \(^{11}\text{Be}(\text{Be}^9)\) differential cross sections \((8.5°)\), and the cross sections
Table 4.1. Comparison of $^\text{14}_\text{N}$ levels identified in $^\text{12}_\text{C} (\alpha,d)^{14}_\text{N}$ and observed in $^\text{12}_\text{C} (\text{B}^{11},\text{Be}^{9})\text{N}^{14}_4$. Also shown are absolute and relative cross sections.

<table>
<thead>
<tr>
<th>Energy $(\alpha,d)$ (MeV)</th>
<th>Cross Section (mb)</th>
<th>$J^\pi$</th>
<th>Energy $(\text{B}^{11},\text{Be}^{9})$ (MeV)</th>
<th>Cross Section (mb/sr)</th>
<th>Relative Cross Section $(\alpha,d)$ $(\text{B}^{11},\text{Be}^{9})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>.98</td>
<td>$1^+$</td>
<td>0.0</td>
<td>.059</td>
<td>.15</td>
</tr>
<tr>
<td>3.95</td>
<td>.31</td>
<td>$1^-$</td>
<td>4.0</td>
<td>.022</td>
<td>.05</td>
</tr>
<tr>
<td>5.10</td>
<td>1.58</td>
<td>2</td>
<td>5.8</td>
<td>.158</td>
<td>.24</td>
</tr>
<tr>
<td>6.44</td>
<td>1.69</td>
<td>$3^+$</td>
<td>6.5</td>
<td>.25</td>
<td>.26</td>
</tr>
<tr>
<td>7.03</td>
<td>.32</td>
<td>2</td>
<td>10.7</td>
<td>.038</td>
<td>.14</td>
</tr>
<tr>
<td>8.47</td>
<td>1.08</td>
<td>$5^+$</td>
<td>12.7</td>
<td>.341</td>
<td>.41</td>
</tr>
<tr>
<td>9.00</td>
<td>5.67</td>
<td>1</td>
<td>15.2</td>
<td>.202</td>
<td>.30</td>
</tr>
<tr>
<td>12.30</td>
<td>2.74</td>
<td>$4^+$</td>
<td>16.3</td>
<td>17.1</td>
<td>17.7</td>
</tr>
</tbody>
</table>

$^a$ From reference 58.

$^b$ Estimated from reference 59.

for these reactions relative to the 8.47-9.0-MeV doublet. The $(\alpha,d)$ 15.1-MeV cross section was estimated from the data of Fig. (4.9). All levels excited appreciably above background in the $(\alpha,d)$ reaction are observed in the $(\text{B}^{11},\text{Be}^{9})$ reaction. The $^\text{14}_\text{N}$ excitation energies of all groups in the $(\text{B}^{11},\text{Be}^{9})$ spectrum lie within 100 keV of levels identified in the $(\alpha,d)$ spectrum and will
hereafter be associated with these levels, as indicated in Table (4.1). The only apparent difference in the two spectra is that groups corresponding to states below 9 MeV are more intense relative to the level at this excitation in the \((\alpha, d)\) reaction. The 8.47 and 9.0-MeV levels are not resolved in the \((B^{11}, Be^9)\) spectrum.

In recent theoretical treatments of the \(N^{14}\) nucleus, Warburton and Pinkston\(^{99}\) have used experimental data as a guide to give shell model assignments for most of the states up to 11 MeV, and True\(^{60}\) has arrived at very similar results from a two-particle intermediate coupling calculation based on two particles in \(p, s,\) or \(d\) orbitals around a closed \((s_{1/2})^4 (p_{3/2})^8 C^{12}\) core. These shell model assignments are in excellent agreement with the results of the \((\alpha, d)\) reaction as well as with the \((B^{11}, Be^9)\) data. All levels formed in relatively high yield below an excitation of 9 MeV have been assigned to states of dominant two-particle configuration, specifically, 0.0 \((p_{1/2}^1 d_{1/2}^1)\), 5.83 MeV \((p_{1/2}^1 d_{5/2}^1)\), 6.44 MeV \((s_{1/2}^1 d_{5/2}^1)\), and thus correspond to the capture of the neutron-proton pair into adjacent shells. Conversely, dominant core excitation or hole states, for example, those at 3.95 and 7.03 MeV, are weakly populated.

The strongly populated levels at 9.0, 12.76 and 15.1 MeV have been associated\(^{58}\) with two-particle states of high angular momentum which have been calculated to lie at about these excitations. The 12.76-MeV level is thought to be a state of strongly mixed \((d_{3/2}^1 d_{5/2}^1)\) and \((p_{1/2}^1 f_{7/2}^1)\) configuration. The 9.0 and 15.1-MeV levels have been identified as the giant excitations of this reaction and have been associated with extremely pure two-particle states of \((d_{5/2}^2)\) configuration and \((d_{5/2}^2 f_{7/2}^1)\) configuration, respectively. The giant excitations of this and the other deuteron transfer reactions considered herein will be discussed in Section C.
2. $^\text{14}\text{N} (\text{B}^{11}, \text{Be}^9) \text{O}^{16}$

The Be$^9$ energy spectrum from the $^\text{14}\text{N} (\text{B}^{11}, \text{Be}^9) \text{O}^{16}$ reaction at $E_{\text{B}^{11}} = 113.2$ MeV is shown in Fig. (4.3). Strongly populated groups appear at $O^{16}$ excitations of 14.7, 16.2, and 20.5 MeV, and weaker groups at 6.1, 11.1, and 13.0 MeV. The $^\text{14}\text{N} (\alpha, d) O^{16}$ reaction has been observed at $E_{\alpha} = 48$ and 42 MeV, \((56, 59)\) and the deuteron energy spectrum is given in Fig. (4.9). The $O^{16}$ levels identified in the $(\alpha, d)$ reaction are compared with the groups observed in the present work in Table (4.2). Also given are the $(\alpha, d)$ integrated

Table 4.2. Comparison of $O^{16}$ levels identified in $^\text{14}\text{N}(\alpha, d)O^{16}$ and observed in $^\text{14}\text{N}(\text{B}^{11}, \text{Be}^9)O^{16}$. Also shown are absolute and relative cross sections.

<table>
<thead>
<tr>
<th>Energy $^{(\alpha, d)}$ (MeV)</th>
<th>Cross Section (mb)</th>
<th>$J^{\pi}$</th>
<th>Energy $^{11}\text{B}$ (MeV)</th>
<th>Cross Section (mb/sr)</th>
<th>Relative Cross Section $(\alpha, d)$ $(^\text{11}\text{B}, \text{Be}^9)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>.65</td>
<td>0$^-$</td>
<td>0.0</td>
<td>.073</td>
<td>11</td>
</tr>
<tr>
<td>6.1</td>
<td>2.03</td>
<td>3$^-$</td>
<td>6.1</td>
<td>0.073</td>
<td>36</td>
</tr>
<tr>
<td>7.0</td>
<td>1.23</td>
<td>1$^-$</td>
<td>7.2</td>
<td>0.011</td>
<td>36</td>
</tr>
<tr>
<td>8.9</td>
<td>.76</td>
<td>2$^-$</td>
<td>9.0</td>
<td>0.038</td>
<td>13</td>
</tr>
<tr>
<td>11.0</td>
<td>0</td>
<td>0$^-$</td>
<td>11.2</td>
<td>0.075</td>
<td>13</td>
</tr>
<tr>
<td>12.3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>13.1</td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>13.6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>14.33</td>
<td>2.2</td>
<td>4$^+$</td>
<td>14.7</td>
<td>0.386</td>
<td>1.00</td>
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<tr>
<td>14.74</td>
<td>3.5</td>
<td>6$^+$</td>
<td>16.2</td>
<td>0.220</td>
<td>1.00</td>
</tr>
<tr>
<td>16.16</td>
<td>2.9</td>
<td>5$^+$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>17.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$^{a}$ From references 56 and 59.

cross sections, the $(^\text{11}\text{B}, \text{Be}^9)$ differential cross sections $(8.5^0)$, and the cross sections for these reactions relative to the 14.33-14.74-MeV doublet. All levels excited appreciably above background in the $(\alpha, d)$ reaction are observed in the
(B\textsuperscript{11}, Be\textsuperscript{9}) reaction. The O\textsuperscript{16} excitation energies of all groups in the (B\textsuperscript{11}, Be\textsuperscript{9}) spectrum lie within 200 keV of levels identified in the (α, d) spectrum and will hereafter be associated with these levels, as indicated in Table (4.2). The only apparent differences in the two spectra are the relative intensity of the group corresponding to a state near 13 MeV and the stronger inhibition in the (B\textsuperscript{11}, Be\textsuperscript{9}) reaction of lower-lying levels relative to the 16.2-MeV level. The 14.33 and 14.74-MeV levels are not resolved in the (B\textsuperscript{11}, Be\textsuperscript{9}) spectrum.

All O\textsuperscript{16} excited states observed below 12 MeV have been accounted for as having dominant (p\textsubscript{1/2}\textsuperscript{-1}d) and (p\textsubscript{1/2}\textsuperscript{-1}s) components,\textsuperscript{(104)} therefore, as also observed in the deuteron transfer reaction leading to N\textsuperscript{14}, the population of these states corresponds to the capture of the two nucleons into adjacent shells, specifically, 6.13 MeV (p\textsubscript{1/2} d\textsubscript{5/2}), 7.12 MeV (p\textsubscript{1/2} s\textsubscript{1/2}), 8.88 MeV (p\textsubscript{1/2} d\textsubscript{5/2}), and 10.95 MeV (p\textsubscript{1/2} s\textsubscript{1/2}). Calculations suggest that the 9.59-MeV state, not observed in this reaction, arises predominantly from a multi-particle or core excitation.\textsuperscript{(105)}

The appearance of a level in the (α, d) reaction at 13.1 MeV has been taken as evidence for a T = 0 state in this region. The stronger relative population of this level in the (B\textsuperscript{11}, Be\textsuperscript{9}) reaction can thus not be definitely attributed to the population of a T = 1 state, although the first such states in O\textsuperscript{16} lie at about this excitation. The 14.3, 14.7, and 16.2-MeV levels have been identified as the giant excitations of this reaction and have been associated with the (d\textsubscript{5/2})\textsuperscript{2} triplet. The group at 20.5 MeV may also correspond to the population of a similar high angular momentum state of dominant two-particle configuration.

3. N\textsuperscript{15} (B\textsuperscript{11}, Be\textsuperscript{9})O\textsuperscript{17}

The Be\textsuperscript{9} energy spectrum from the N\textsuperscript{15} (B\textsuperscript{11}, Be\textsuperscript{9})O\textsuperscript{17} reaction at $E_{B^{11}} = 113.5$ MeV is shown in Fig. (4.4). Strongly populated groups appear at O\textsuperscript{17} excitations of 7.6 and 9.0 MeV. The only other groups observed are at
0.0, 0.9, 3.8, and 5.4 MeV. There are no unresolved groups at excitations above 10 MeV, possibly reflecting the relatively low (13 MeV in $^{9}\text{Be}^{+}$) $^{9}\text{Be}^{+}$+n breakup threshold for this reaction. The first $T = 3/2$ states in $^{17}\text{O}$ also appear in this region.

The $^{15}\text{N}(\alpha,d)^{17}\text{O}$ reaction has been observed at $E_\alpha = 47$ MeV,\textsuperscript{(57)} and the deuteron spectrum is given in Fig. (4.9). The groups appearing in the $(\alpha,d)$ and $(B_{11}^{11},Be^{9})$ reactions are compared in Table (4.3). Also given are the

<table>
<thead>
<tr>
<th>Energy ($\alpha,d$) Cross Section (mb)</th>
<th>$J^\pi$</th>
<th>Energy ($B_{11}^{11},Be^{9}$) Cross Section (mb/sr)</th>
<th>Relative Cross Section $(\alpha,d)$ ($B_{11}^{11},Be^{9}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>1.18</td>
<td>$5/2^+$</td>
<td>0.090</td>
</tr>
<tr>
<td>0.8</td>
<td>.26</td>
<td>$1/2^+$</td>
<td>0.911</td>
</tr>
<tr>
<td>3.85</td>
<td>(7/2$^-$)</td>
<td>3.8</td>
<td>0.033</td>
</tr>
<tr>
<td>5.7</td>
<td></td>
<td>5.4</td>
<td>0.229</td>
</tr>
<tr>
<td>7.6</td>
<td>6.4</td>
<td>11/2$^-$</td>
<td>7.6573</td>
</tr>
<tr>
<td>9.0</td>
<td>3.1</td>
<td>9/2$^-$</td>
<td>9.0409</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>10.7</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>12.3</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>14.3</td>
</tr>
</tbody>
</table>

\textsuperscript{a} From reference 57.

$(\alpha,d)$ integrated cross sections, the $(B_{11}^{11},Be^{9})$ differential cross sections (8.5$^0$), and the cross section ratios for these reactions relative to the 7.6-MeV level. With the exception of the group near 5.5 MeV, which apparently results from the population of several states between 4.5-6 MeV, groups appear in the two reactions at $^{17}\text{O}$ excitation energies which lie within 100 keV and will hereafter be associated as indicated in Table (4.3). The two spectra are, below
the 9.0-MeV level, in complete agreement.

The ground \((5/2^+)\), 0.87 \((1/2^+)\), and 5.08 \((3/2^+)\)-MeV states in \(^{17}\text{O}\) are expected to be of relatively pure configuration resulting from the coupling of \(d\) and \(s\) neutrons to an \(^{16}\text{O}\) core. \(^{122}\) Groups appear at these excitations, although the latter has other contributions. The formation of these states in this reaction thus involves the capture of a \(p_{1/2}\) proton with, in order of increasing excitation, \(d_{5/2}\), \(s_{1/2}\), and \(d_{3/2}\) neutrons. The strongly populated states corresponding to groups at 7.6 and 9.0 MeV have been identified as the giant excitations of this reaction and have been associated with the \((d_{5/2})^2\) doublet.

4. \(^{16}\text{O}\)(\(^{11}\text{B},^{9}\text{Be}\))\(^{18}\)

The \(^{9}\text{Be}\) energy spectrum from the \(^{16}\text{O}\)(\(^{11}\text{B},^{9}\text{Be}\))\(^{18}\) reaction at \(E_{\text{B}^{11}} = 113.1\) MeV is shown in Fig. (4.5). An intense group appears at an \(F^{18}\) excitation of 1.1 MeV, and the only other strongly populated groups appear at 4.4, 6.3, and 9.4 MeV. There are low-lying \(T = 1\) states in \(F^{18}\) at 1.0, 3.1, and 6.2 MeV. The former and latter would be obscured by other groups, however, the 3.1-MeV state is not observed. The next \(T = 1\) states lie in the region of 13 MeV, and none appears to be populated appreciably above background.

The \(^{16}\text{O}\)(\(\alpha,d\))\(^{18}\) reaction has been observed at 48 and 52 MeV, \(^{57,59}\) and the deuter ax spectrum is given in Fig. (4.9). The \(N^{14}\) energy levels identified in the \((\alpha,d)\) reaction are compared with the groups observed in the present work in Table (4.4). Also given are the \((B^{11},^{9}\text{Be})\) differential cross sections \(^{8.5}\) and cross sections relative to the 1.1-MeV group. The \((\alpha,d)\) relative cross sections and the 9.4-MeV cross section have been estimated from the data of Fig. (4.9). The broad group at 6.3 MeV in the \((B^{11},^{9}\text{Be})\) spectrum possibly corresponds to the 6.12 and 6.76-MeV levels observed in the \((\alpha,d)\) reaction. The group in the \((B^{11},^{9}\text{Be})\) spectrum at 4.4-MeV is not strongly populated in the \((\alpha,d)\) reaction, and in fact, is the only major
Table 4.4. Comparison of $^18F$ levels identified in $^{16}O(a,d)^{18}F$ and observed in $^{16}O(B^{11},Be^9)^{18}F$. Also shown are absolute and relative cross sections.

<table>
<thead>
<tr>
<th>Energy $(\alpha,d)$ Cross Section (MeV)</th>
<th>$J^\pi$</th>
<th>Energy $(B^{11},Be^9)$ Cross Section (MeV)</th>
<th>Relative Cross Section $(\alpha,d)^b (B^{11},Be^9)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.10</td>
<td>11.2</td>
<td>5$^+$</td>
<td>1.1</td>
</tr>
<tr>
<td>2.05</td>
<td></td>
<td>2.5</td>
<td>0.72</td>
</tr>
<tr>
<td>3.68</td>
<td></td>
<td>4.4</td>
<td>3.69</td>
</tr>
<tr>
<td>4.25</td>
<td></td>
<td>6.3</td>
<td>3.66</td>
</tr>
<tr>
<td>6.12</td>
<td></td>
<td>7.6</td>
<td>.142</td>
</tr>
<tr>
<td>6.76</td>
<td></td>
<td>9.4</td>
<td>.513</td>
</tr>
<tr>
<td>7.13</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7.65</td>
<td>2.9$^b$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9.44</td>
<td>11.4</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a From reference 59.
b Estimated from reference 59.

discrepancy observed in the $(\alpha,d)$ and $(B^{11},Be^9)$ reactions considered herein.

It is possible that this group may correspond to the excitation of a $T = 1$ state in $^{18}F$, although none has been reported in this region. The other strongly populated groups do, however, lie within 100 keV of levels identified in the $(\alpha,d)$ spectrum and will hereafter be associated with these levels, as indicated in Table (4.4).

The intense group observed near 1 MeV in the $(\alpha,d)$ spectrum has been associated with the 1.1-MeV, 5$^+$ member of the closely spaced quartet lying in this region and identified as the $(d_{5/2})^2$ giant excitation of this reaction. The ground state of $^{18}F$ has been calculated to be of highly mixed character composed of $(d_{5/2}), (d_{3/2})$, and $(s_{1/2})$ components. This state does not appear to be populated in the $(B^{11},Be^9)$ reaction, and its population relative to the 1.1-MeV
state is found to be 1:40 in the \((\alpha,d)\) reaction at 18.6\(^o\) \((124)\). Although some calculations have been carried out for the levels in \(\text{F}^{18}\) near 1 MeV, higher-lying states are at present not well understood theoretically \((122)\). The strongly populated level at 9.4 MeV is thought to be a giant excitation of \((d_{5/2} f_{7/2})\) configuration.

This reaction was observed as a function of angle, and the results are shown in Figs. (4.7) and (4.8), as discussed in Section A. As noted in the preceding chapter, the measurement of an excitation function was precluded by the extremely large loss in beam intensity due to the introduction of the energy degrading foils. A measurement at \(E_{B11} = 77.6\) MeV indicates that the 1.1-MeV level is again strongly populated and that the cross section for this state attains approximately 0.9 of its full energy value.

5. \(\text{Ne}^{20}(\text{B}^{11}, \text{Be}^9)\text{Na}^{22}\)

The \(\text{Be}^9\) energy spectrum from the \(\text{Ne}^{20}(\text{B}^{11}, \text{Be}^9)\text{Na}^{22}\) reaction at \(E_{B11} = 113.6\) MeV is shown in Fig. (4.6). Strongly populated groups appear at \(\text{Na}^{22}\) excitations of 1.53, 7.6, and 9.5 MeV. There are no other strong excitations, in particular, the ground state and \(T = 1\) state thought to lie at 0.66 MeV do not appear to be populated significantly.

The \(\text{Ne}^{20}(\alpha,d)\text{Na}^{22}\) reaction has been observed at \(E = 45\) MeV, \((59)\) and the deuteron spectrum is given in Fig. (4.9). The \(\text{Na}^{22}\) levels identified in the \((\alpha,d)\) reaction are compared with the groups observed in the present work in Table (4.5). The two spectra are in agreement below 5 MeV. The 5.95 and 6.62-MeV levels are not resolved in the \((\text{B}^{11}, \text{Be}^9)\) spectrum. The \(\text{Na}^{22}\) excitation energies of all other groups in the \((\text{B}^{11}, \text{Be}^9)\) spectrum lie within 100 keV of levels identified in the \((\alpha,d)\) spectrum and will hereafter be associated with these levels, as indicated in Table (4.5). The two spectra are in agreement except for a relatively larger population of the levels of higher excitation relative to the 1.53-MeV level in the \((\text{B}^{11}, \text{Be}^9)\) reaction.
Table 4.5. Comparison of Na$^{22}$ levels identified in Ne$^{20}$($\alpha$,d)Na$^{22}$ and observed in Ne$^{20}$(B$^{11}$,Be$^{9}$)Na$^{22}$. Also shown are absolute and relative cross sections.

<table>
<thead>
<tr>
<th>Energy (MeV)</th>
<th>Cross Section (mb)</th>
<th>$J^\pi$</th>
<th>Energy (MeV)</th>
<th>Cross Section (mb/sr)</th>
<th>Relative Cross Section ($\alpha$,d) (B$^{11}$,Be$^{9}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.</td>
<td>.15</td>
<td>$3^+$</td>
<td>1.5</td>
<td>.245</td>
<td>.07</td>
</tr>
<tr>
<td>1.53</td>
<td>2.06</td>
<td>$5^+$</td>
<td>2.9</td>
<td>.088</td>
<td>1.00</td>
</tr>
<tr>
<td>2.98</td>
<td>.54</td>
<td></td>
<td>4.7</td>
<td>1.133</td>
<td>weak</td>
</tr>
<tr>
<td>3.74</td>
<td></td>
<td></td>
<td>6.4</td>
<td>.180</td>
<td>weak</td>
</tr>
<tr>
<td>5.29</td>
<td></td>
<td></td>
<td>7.4</td>
<td>.305</td>
<td>.52</td>
</tr>
<tr>
<td>5.95</td>
<td></td>
<td></td>
<td>7.9</td>
<td>1.268</td>
<td>1.09</td>
</tr>
<tr>
<td>6.62</td>
<td>.51</td>
<td></td>
<td>9.5</td>
<td>.425</td>
<td>1.73</td>
</tr>
<tr>
<td>7.46</td>
<td>1.07</td>
<td>$6^-$</td>
<td>9.0$^b$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$^a$ From reference 59.

$^b$ Estimated from reference 59.

This level has been identified as the $(d_{5/2})^2$ giant excitation of this reaction, and the 7.46-MeV level is thought to be a giant excitation of $(d_{5/2}f_{7/2})$ configuration. The group at 9.5 MeV may also correspond to the population of a similar high angular momentum state of dominant two–particle configuration.

6. C$^{13}$ (B$^{11}$,Be$^{9}$)N$^{15}$

The Be$^{9}$ energy spectrum from the C$^{13}$ (B$^{11}$,Be$^{9}$)N$^{15}$ reaction at $E_{B^{11}} = 113.5$ MeV is shown in Fig. (4.2). The strongly populated group at an N$^{15}$ excitation of 12.6 MeV has been resolved into two peaks at 12.0 and 12.9 MeV. Weaker groups appear at 0.0, 5.3, 7.6, and 9.8 MeV. The differential cross sections (mb/sr at 8.5$^9$) of these groups, in order of increasing excitation, are 0.068, 0.081, 0.182, 0.211, 0.355, and 0.788.
The ground, 1/2\(^-\) and 6.33-MeV, 3/2\(^-\) states in \(N^{15}\) are expected to result from the shell model configuration \((s^4_p^{11})\). (97) Although the ground state is well represented by a \(p_{1/2}\) neutron coupled to the \(N^{14}\) ground state, it has recently been suggested that the 6.33-MeV state is primarily of a collective nature related closely to the \(2^+\) state at 6.92 MeV in \(O^{16}\). (103) There is no evidence for the population of the 6.33-MeV state in this reaction.

Halbert and French have carried out intermediate coupling shell model calculations for the even-parity states of \(N^{15}\) and have been able to correlate the seven experimentally observed levels below 9 MeV with states calculated to have dominant \((s^4_p^{10}s)\) or \((s^4_p^{10}d)\) configurations. (100) The states at 5.28 and 5.30 MeV are based primarily on \(d_{5/2}\) and \(s_{1/2}\) nucleons, respectively, coupled to an \(N^{14}, T = 1\) first-excited-state core and are weakly populated in this reaction. The 7.16, 7.31, 7.57 and 8.31-MeV states are expected to result predominantly from the coupling to a \(N^{14}\) ground state core of \(s_{1/2}\) (7.31 and 8.31 MeV) and \(d_{5/2}\) (7.16 and 7.57 MeV) nucleons. The groups observed at 7.6 and 9.8 MeV may result from the excitation of the 7.57-MeV, \(7/2^+\) and 9.83-MeV, \(7/2\) states. On the basis of the systematics of the giant excitations, discussed in the following section, the strong group observed near 13 MeV may correspond to the population of a high angular momentum state of \((d_{5/2})^2\) configuration.

This spectrum was extracted from that obtained directly from the \(C^{13}\) target, as noted in the preceding chapter, by weighted subtraction of a \(Be^9\) spectrum resulting from the \(C^{12}(B^{11},Be^9)N^{14}\) reaction. The estimated errors, especially for the higher excitations, are approximately twice those given in Chapter II, and for this reason no excitation energy or cross section measurements were made above 13 MeV.

7. Discussion

All \((B^{11},Be^9)\) deuteron transfer reactions considered in this work are observed to exhibit highly selective population of a relatively small number of
final states in the residual nuclei involved. This selectivity is demonstrated in Fig. (4.10), which is a composite of the Be$^9$ energy spectra of all reactions discussed in this section. The yields are shown on a linear scale normalized to integrated beam current, and the scale is a factor of 2 smaller than that for the (B$^{11}$,Be$^{10}$) reactions shown in Fig. (3.8). The excitation energy scale shown is an average for all reactions and is therefore approximate. The groups in these spectra correspond in most cases to the excitation of individual final states in the residual nuclei, thus only a few of the levels known to exist in the range of excitation energy shown are being populated.

The population of states of a two-particle nature and the weak population or absence of states whose formation in these reactions would involve the excitation of the target core in addition to the transfer of a neutron-proton pair suggest that direct two-nucleon transfer represents the dominant reaction mechanism. The observed energy spectra are consistent with a direct interaction process wherein final states are selectively populated which have strong parentage based on the transferred nucleons coupled to the target core. It is thus proposed that the states formed in high yield are representable in jj-coupling notation as \[ j_{T} + (j_{n}j_{p})J \] , where the final angular momenta \( j_{n} \) and \( j_{p} \) of the neutron and proton couple to a resultant \( J' \), which then couples to the angular momentum \( J_{T} \) of the target to form the angular momentum \( J \) of the final state.

The deuteron transfer spectra are in all cases dominated by one or more levels whose population is enhanced relative to that of other levels in the same spectrum, an effect which is also evident in Fig. (4.10). These levels are associated with high angular momentum states of extremely pure two-particle configuration and are identified as the giant excitations of these reactions. As noted in the preceding chapter, it is reasonable to expect the preferential population of such high-spin states in heavy-ion transfer reactions at these energies. The detailed nature of the giant excitations, in particular the proposed assignment of the angular momenta \( j_{n}, j_{p} \), and \( J' \) appearing in the above representation of these states, will be considered in the following section.
Figure 4.10. Be$^9$ energy spectra from ($B^{11}$, Be$^9$) deuteron transfer reactions. The excitation energy scale is an average for all reactions and is approximate. The yields are shown on a linear scale normalized to integrated beam current. The selective population of final states in these reactions can be seen.
Be$^9$ ENERGY SPECTRA
(B$^9$,Be$^9$) DEUTERON TRANSFER REACTIONS
$E_{in}=114$ MeV
$\theta_{lab}=8.5^\circ$

C$^{12}$(B$^9$,Be$^9$)N$^{14}$
C$^{13}$(B$^9$,Be$^9$)N$^{15}$
N$^{14}$(B$^9$,Be$^9$)O$^{16}$
N$^{15}$(B$^9$,Be$^9$)O$^{17}$
O$^{16}$(B$^9$,Be$^9$)F$^{18}$
Ne$^{20}$(B$^9$,Be$^9$)Na$^{22}$

EXCITATION (MeV)
The spin and isobaric-spin selection rules for \((B^{11}, Be^9)\) reactions allow the neutron-proton pair to be transferred in either the triplet or singlet-spin configuration and the corresponding excitation of states via both \(\Delta T = 0,1\) transitions. There is no unambiguous evidence for the population of \(\Delta T = 1\) states in these data, indicating that the transfer of the singlet-spin, \(T = 1\) configuration may be inhibited relative to the triplet-spin, \(T = 0\) configuration.

A systematic comparison of the energy spectra of equivalent \(B^{11}\) and alpha-particle induced deuteron transfer reactions shows that these reactions possess a high degree of similarity regarding both selective and preferential population of residual states. All levels formed in high yield in the \((\alpha, d)\) reactions are observed in the \((B^{11}, Be^9)\) reactions with approximately the same relative population. Further comparisons of these reactions will be made in Section D.

C. Preferential Population of Final States

One of the significant features of the \((B^{11}, Be^9)\) reactions considered in the present work is the preferential population of levels which have been referred to as the giant excitations of the deuteron transfer reaction. This phenomenon has also been observed in a study of the \((\alpha, d)\) reaction in the light elements by Harvey and collaborators, and a correlation between the giant excitations observed in equivalent \((B^{11}, Be^9)\) and \((\alpha, d)\) reactions has been made in the discussions of the preceding section.

It has been proposed\(^{(59)}\) that the giant excitations are of a common basic configuration formed by coupling the transferred neutron and proton to the target core, that the two nucleons are captured into \(d_{5/2}\) or \(f_{7/2}\) single-particle states, and that a final state angular momentum of 5-7 is favored. A group of these levels, populated in residual nuclei of mass number 14-26 and including the majority of those discussed in the preceding section, have accordingly been identified with a configuration which is given in the \(jj\)-coupling notation used above as
\[ J_T + \left( d_{5/2} \right)^2 \] _J, indicating that the two nucleons are captured into \( d_{5/2} \) states with their angular momenta coupled to the maximum allowed resultant. The basis for this assignment has been outlined in the introduction. Similar arguments may be advanced for the \((B^{11}, Be^9)\) reactions and are given below.

Evidence for the common configuration of the giant excitations results from consideration of a plot of formation Q value as a function of mass number, similar to that shown previously for the single-nucleon transfer reactions. Such a plot for the levels preferentially populated in the \((B^{11}, Be^9)\) reactions is shown in Fig. (4.11). The systematic decrease of the negative Q values with increasing mass as indicated by the arbitrarily drawn dashed lines suggests that the nucleons are captured into the same states, which are such that they are at high excitations in the lighter nuclei but approach the ground states in the heavier nuclei as the lower shells become filled. The proposed assignments given in the preceding section are indicated in the figure.

The prediction of a \((d_{5/2})^2\) configuration for the two nucleons follows from the expected preference for the \(d_{5/2}\) state noted in the previous discussion of the single-nucleon transfer reactions. The appearance of giant excitations at 1.1 MeV in \(F^{18}\) (see Table (4.4)) and at the ground state in \(Al^{26}\) in the \((\alpha,d)\) reaction on \(Mg^{24}\) (see Fig. (4.9)), both known \(5^+\) levels, strongly suggests that capture is taking place into \(d_{5/2}\) states. Shell model calculations\(^{60}\) also place a pure \((d_{5/2})^2\) level near the observed giant excitation in \(N^{14}\).

Coupling of the nucleon angular momenta to the maximum resultant follows from consideration of the LS representation\(^{(126)}\) of the jj multiplet given by \((d_{5/2})^2\), wherein this member of the multiplet is that for which the orbital angular momenta of the nucleons are parallel. This value of orbital angular momentum would be expected to be favored by the dynamics of a grazing collision at the target surface. Detailed two-nucleon transfer calculations\(^{(53,54)}\) also suggest that the population of states of this dominant configuration should be favored, although due to coherence effects this may in some cases be a necessary and not a sufficient condition.
Figure 4.11. Dependence of reaction Q value on the mass umber of the residual nucleus for levels of proposed \( (d_{5/2})^2 \) and \( (d_{5/2} f_{7/2})_6 \) configurations preferentially populated in \( (B^{11}, Be^9) \) reactions. The systematic trend indicated by the arbitrarily drawn dashed lines suggests that each group of levels is of a common configuration.
FORMATION Q VALUES FOR \((B^{11}, Be^{9})\) REACTIONS

- \((d_{5/2})^2\) LEVELS
- \((d_{5/2} f_{7/2})_6\) LEVELS
Support for the \((d_{5/2})^2\) assignments comes from the fact that all members of the multiplet formed by the coupling of the nucleon and target angular momenta have been observed in both \((B^{11}, Be^9)\) and \((\alpha, d)\) reactions on \(C^{12}, N^{14}, N^{15}, O^{16}\) and Ne\(^{20}\) targets. This configuration has also been associated with the ground state of Al\(^{26}\) and may also be the nature of a state near 13 MeV corresponding to a strong group observed in the \((B^{11}, Be^9)\) reaction. Other assignments involving \((d_{5/2} f_{7/2})_6\) and \((f_{7/2})^2\) configurations have been made for groups of giant excitations appearing at higher excitations. A number of levels of proposed \((d_{5/2} f_{7/2})_6\) configuration have been observed in both \((\alpha, d)\) and \((B^{11}, Be^9)\) reactions, and those for the latter are included in Fig. (4.1). A group of levels appearing in the spectra of \((\alpha, d)\) reactions on heavier targets in the sd shell has been identified with the \((f_{7/2})^2\) configuration.

Evidence of an indirect nature for the proposed character of the giant excitations can be obtained from a study of the corresponding inelastic scattering reaction, since single-particle and collective excitations rather than levels of dominant two-particle configuration should be strongly populated in this case. As noted previously, elastic and inelastic scattering data were recorded simultaneously with the transfer data in the present work. The \(B^{11}\) energy spectra from elastic and inelastic reactions on \(N^{14}\) and \(O^{16}\) targets are shown in Fig. (4.12). The arrows indicate the positions of the giant excitations, and it can be seen that the relative population of these levels compared to that in the transfer reactions is not appreciable.

D. Comparison of the \((B^{11}, Be^9)\) and \((\alpha, d)\) Reactions

The data which have been presented in this chapter are derived from \((B^{11}, Be^9)\) and \((\alpha, d)\) deuteron transfer reactions proceeding at energies of 115 MeV and 40-50 MeV, respectively. In each case the deuteron's share of the incident kinetic energy is approximately 20 MeV, thus the linear momentum carried by the deuteron and the orbital angular momentum delivered to the target surface
Figure 4.12. $B^{11}$ energy spectra, $N^{14}(B^{11},B^{11})N^{14}$ and $O^{16}(B^{11},B^{11})O^{16}$. The solid lines connect the experimental data points, and the arrows indicate the positions of the giant excitation levels in the $N^{14}$ and $O^{16}$ nuclei.
are equivalent in the two reactions. In addition, the separation energy of the deuteron from the projectile is comparable, being approximately 24 MeV for the alpha particle and 16 MeV for $^6$Be. Since it has been shown that angular momentum as well as parentage considerations play a major role in conditioning experimental observations in the deuteron transfer reaction, these data may be compared directly on the basis of the similarity of the incident systems and should provide direct and complementary information on the reaction mechanism.

The systematic comparison of the $^6$Be and deuteron product energy spectra in Section B indicates that the ($\alpha,d$) and ($^6$Be,$^6$Be') reactions are characterized by an equivalent selective and preferential population of final states. Levels populated significantly in the ($\alpha,d$) reactions are in all cases observed in the corresponding ($^6$Be,$^6$Be') reactions and with approximately the same relative yields. In particular, all ($\alpha,d$) giant excitations have been correlated with those observed in the present work, and the mass variation of the total cross sections of these levels in the two reactions has been found to be equivalent, as discussed below.

The observed similarity of the ($^6$Be,$^6$Be') and ($\alpha,d$) reactions extends to the isobaric-spin nature of the final states. Of the two spin-isobaric spin configurations of the neutron-proton system, $S = 1$, $T = 0$ and $S = 0$, $T = 1$, the latter is not allowed and indeed has not been identified in the ($\alpha,d$) reaction. Although the singlet-spin, $T = 1$ channel is allowed in the ($^6$Be,$^6$Be') reaction, there is no unambiguous evidence in the present data which indicates that the two nucleons are being transferred in this configuration. This result is unexpected, since the ($^6$Be,$^6$Be') reaction might a priori be anticipated to resemble more closely the ($^3$He,p) reaction, which shares the same selection rules and wherein the $T = 1$ channel has been found to be highly effective, as noted in the introduction.

An obvious explanation would be that the inhibition of the singlet-spin, $T = 1$ channel is related to the two-nucleon parentage of the $^6$Be ground state.
An attempt has been made to determine the nature of this effect by calculating the spectroscopic factors for the two isobaric-spin configurations, using the \( \langle p^n | p^{n-2} p^2 \rangle \) two-nucleon fractional parentage coefficients\(^{(118)}\) and the Boyarkina wavefunctions\(^{(119)}\) in a method similar to that described in the preceding chapter. The results do not indicate that the triplet-spin, \( T = 0 \) configuration should be favored on this basis, in particular, the spectroscopic factor for \( \text{Be}^9 \) plus a nucleon pair in an \( S = 0, T = 1 \) configuration has been found to be a factor of 1.8 larger than that for the \( S = 1, T = 0 \) configuration, contrary to experimental observations. As discussed previously, more detailed calculations are precluded by the presence of a coherence effect, characteristic of two-nucleon transfer reactions, involving not only the parentage factors but also the spatial overlap of the two nucleons in their initial and final states in the projectile and final nuclei.

It has been pointed out in the preceding section that the mechanism proposed for the formation of the giant excitations is consistent with both the \((\alpha, d)\) and \((B^{11}, \text{Be}^9)\) data, and similarities regarding the excitation energy and relative population of these levels in the two reactions have been discussed previously. A more quantitative comparison involving total cross sections has also been carried out, as indicated in Table (4.6). Levels preferentially populated by the \((\alpha, d)\) and \((B^{11}, \text{Be}^9)\) reactions which have been identified as giant excitations of \( (d_{5/2}^2) \) and \( (d_{5/2} f_{7/2}) \) configurations in the discussions of Section B are given, together with the proposed spin and parity assignments. The multiplets given result from the coupling of the nucleon resultant angular momentum \( J' \) to the target angular momentum \( J_T \) to form levels of spin \( (J' + J_T) \) to \( (J' - J_T) \). The \((\alpha, d)\) integrated cross sections are from Tables (4.1) through (4.5). Those for \((B^{11}, \text{Be}^9)\) have been calculated as described in Section A from the data given in these tables.

The total cross sections given in Table (4.6) are shown as a function of the mass number of the residual nucleus in Fig. (4.13). The contributions
Figure 4.13. Comparison of absolute total cross sections for \((\text{Be}^{11}, \text{Be}^{9})\) and \((\alpha, d)\) giant excitations. The integrated cross sections are displayed as a function of the mass number of the residual nucleus, and the proposed \(d_{5/2}^2\) and \(d_{5/2 \ 7/2}^f\) configurations are shown. The contributions from the multiplet of levels resulting from the nucleon and target angular momentum coupling are summed for each nucleus. The error bar indicates the estimated absolute error in the \((\text{Be}^{11}, \text{Be}^{9})\) measurements, and the \((\alpha, d)\) data (references 57, 58, 59) are multiplied by a factor of 1/15.
TOTAL CROSS SECTIONS
DEUTERON TRANSFER REACTIONS

\((d_{5/2})^2\) LEVELS
\((d_{5/2} f_{7/2})_6\) LEVELS

\(\sigma(\alpha,d) \times 1/15\)
Table (4.6). Comparison of total cross sections of levels in \((\alpha, d)\) and 
\((B^{11}, Be^9)\) deuteron transfer reactions. The proposed spin, parity, and 
configuration assignments are also given.

| Final Nucleus | Energy Level (MeV) | \(J^\pi\) | Total Cross Section (mb) \((\alpha, d)\) | \((B^{11}, Be^9)\) | Proposed Configuration 
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>N(^{14})</td>
<td>9.0</td>
<td>5(^+)</td>
<td>5.67</td>
<td>0.450</td>
<td>((d_{5/2})^2)</td>
</tr>
<tr>
<td></td>
<td>15.1</td>
<td>6(^-)</td>
<td>2.0(^c)</td>
<td>0.184</td>
<td>((d_{5/2} f_7/2))</td>
</tr>
<tr>
<td>N(^{15})</td>
<td>12.9</td>
<td></td>
<td>0.653</td>
<td></td>
<td></td>
</tr>
<tr>
<td>O(^{16})</td>
<td>14.33</td>
<td>4(^+)</td>
<td>2.2</td>
<td>0.307</td>
<td>((d_{5/2})^2)</td>
</tr>
<tr>
<td></td>
<td>14.74</td>
<td>6(^+)</td>
<td>3.5</td>
<td>0.172</td>
<td>((d_{5/2})^2)</td>
</tr>
<tr>
<td></td>
<td>16.16</td>
<td>5(^+)</td>
<td>2.9</td>
<td>0.304</td>
<td>((d_{5/2})^2)</td>
</tr>
<tr>
<td>O(^{17})</td>
<td>7.6</td>
<td>11/2(^-)</td>
<td>6.4</td>
<td>0.427</td>
<td>((d_{5/2})^2)</td>
</tr>
<tr>
<td></td>
<td>9.0</td>
<td>9/2(^-)</td>
<td>3.1</td>
<td>0.304</td>
<td>((d_{5/2})^2)</td>
</tr>
<tr>
<td>F(^{18})</td>
<td>1.10</td>
<td>5(^+)</td>
<td>11.2</td>
<td>0.668</td>
<td>((d_{5/2})^2)</td>
</tr>
<tr>
<td></td>
<td>9.44</td>
<td>6(^-)</td>
<td>2.9(^c)</td>
<td>0.367</td>
<td>((d_{5/2} f_7/2))</td>
</tr>
<tr>
<td>Na(^{22})</td>
<td>1.53</td>
<td>5(^+)</td>
<td>2.06</td>
<td>0.151</td>
<td>((d_{5/2})^2)</td>
</tr>
<tr>
<td></td>
<td>7.46</td>
<td>6(^-)</td>
<td>1.07</td>
<td>0.191</td>
<td>((d_{5/2} f_7/2))</td>
</tr>
</tbody>
</table>

\(a\) From reference 59.

\(b\) From references 57, 58, 59.

\(c\) Estimated from reference 59.

from the multiplets given in the table are summed for each nucleus. The error 
bar shown indicates the estimated absolute error in the \((B^{11}, Be^9)\) measurements, 
as given in Chapter II. The \((B^{11}, Be^9)\) and \((\alpha, d)\) results are in good agreement, 
and the consistency of the data indicates that an identical mechanism is responsible 
for the formation of the giant excitations in these reactions. The fact that the 
\((B^{11}, Be^9)\) cross sections are consistently a factor of 15 lower than those for the
(α,d) reaction is a reflection of the larger number of open exit channels, hence greater competition, in the former reactions. This effect is characteristic of heavy-ion reactions in general and is also demonstrated by the fact that the (B\(^{11}\), Be\(^9\)) cross sections represent an extremely small fraction of the average geometric cross section of 1.4 barns for these reactions.

It is reasonable to expect a constant cross section for the levels of proposed \( (d_{5/2})^2 \) configuration for target nuclei which have no particles in the 
\( d_{5/2} \) shell, in simple terms of number of holes available for the capture process. On this basis the cross section of such levels would be expected to decrease as the 
\( d_{5/2} \) shell begins to fill for targets of mass number greater than 16. This prediction is in good accord with the data of Fig. (4.13). The (B\(^{11}\), Be\(^9\)) cross sections for all targets up to and including O\(^{16}\) lie within experimental error from their average value of 0.6 mb. That for Ne\(^{20}\) is reduced by a factor of 3 from this value. The cross sections for the levels of proposed \( (d_{5/2}f_{7/2}) \) configuration are approximately constant, as might be anticipated on this basis.

The Be\(^9\) and deuteron angular distributions corresponding to two of the giant excitations populated in equivalent (α,d) and (B\(^{11}\), Be\(^9\)) reactions on an O\(^{16}\) target are shown in Fig. (4.14). The variation of the cross sections of the 1.1-MeV and 9.4-MeV levels in F\(^{18}\) is shown as a function of the linear momentum transfer, which has been noted to be the relevant quantity in these considerations. In the angular ranges covered, it can be seen that the two reactions span equivalent but overlapping ranges of momentum transfer. The nearly smooth behavior of the data when displayed in this manner provides further evidence for the similarity of the reaction mechanisms. It has already been pointed out that this behavior is characteristic of all heavy-ion transfer reactions of this type and that the angular distributions of all (α,d) giant excitations have been found to be equivalent.
Figure 4.14. $^{9}\text{Be}$ and deuteron angular distributions. The variation of the cross sections for the formation of the 1.1 and 9.4-MeV giant excitations in $^{18}\text{F}$ in $(\alpha, d)$ (from reference 59) and $(^{11}\text{B}, ^{9}\text{Be})$ deuteron transfer reactions on an $^{16}\text{O}$ target is shown as a function of linear momentum transfer.
ANGULAR DISTRIBUTIONS
DEUTERON TRANSFER REACTIONS

$^{16}_O(\alpha, d)^{18}_F$

$E_\alpha = 52$ MeV

- $1.1$ MeV LEVEL
- $9.4$ MeV LEVEL

$^{16}_O(B^{11}, Be^9)^{18}_F$

$E_{B'} = 113$ MeV

- $1.1$ MeV LEVEL
- $9.4$ MeV LEVEL

$d\sigma/d\Omega$ (mb/sr)

MOMENTUM TRANSFER ($f^{-1}$)
The above comparisons of the \((\alpha, d)\) and \((\text{B}^{11}, \text{Be}^{9})\) deuteron transfer reactions indicate that an identical mechanism is involved in the population of final states in these reactions. If such is the case, strong evidence exists that the \((\alpha, d)\) reaction proceeds predominantly by a direct transfer rather than a knockout mechanism, since the probability of the complex \(\text{Be}^{9}\) nucleus emerging from a process other than direct transfer, especially in view of its low internal binding energy, is extremely small.
V. SUMMARY

Single-nucleon and deuteron transfer reactions induced by 115.9-MeV B\textsuperscript{11} ions have been observed with C\textsuperscript{12}, C\textsuperscript{13}, N\textsuperscript{14}, N\textsuperscript{15}, O\textsuperscript{16}, and Ne\textsuperscript{20} targets, with the primary objectives of elucidating the reaction mechanisms and extracting information of a spectroscopic nature relevant to the interacting nuclear systems.

A particle identification system based on the dE/dx and E method has been developed for this work which is capable of identifying and measuring simultaneously the energy spectra of several nuclear species at least up to mass number 17 resulting from the interaction of complex nuclei at energies near 100 MeV. The system employs a high resolution dE/dx-E particle telescope, a 20,000 channel multiparameter pulse height analyzer, and automatic data reduction techniques. It has been demonstrated that this system is capable of providing a complete, consistent set of reaction information for use in detailed analysis of heavy-ion transfer data in terms of the appropriate entrance and exit channel scattering situations. A semiempirical method for the calculation of the energy loss of heavy ions has been developed for this work and has been shown to be in good accord with available experimental stopping power data.

For each projectile-target incident system investigated, data were recorded simultaneously on (B\textsuperscript{11}, B\textsuperscript{10}), (B\textsuperscript{11}, Be\textsuperscript{10}), and (B\textsuperscript{11}, Be\textsuperscript{9}) reactions, which have been interpreted as neutron, proton, and deuteron transfers, respectively, from the projectile to the target. The data which have been presented herein consist of the energy spectra of the boron and beryllium products of these reactions observed at forward angles and absolute differential cross section measurements for final energy states corresponding to groups appearing in the spectra.

All single-nucleon and deuteron transfer reactions have been observed to exhibit highly selective population of a relatively small number of final
states in the residual nuclei involved. The population of known states of a predominantly single-particle nature in the neutron and proton transfer reactions and of a two-particle nature in the deuteron transfer reactions, as well as the weak population or absence of states whose formation in these reactions would involve the excitation of the target core in addition to the transfer process, have suggested that direct transfer represents the dominant reaction mechanism. These reactions are further characterized by the preferential population of certain states, whose yields are large in relation to those of all other states appearing in the same energy spectrum.

It has been proposed that states are selectively populated in the \((B^{11}, B^{10})\) and \((B^{11}, Be^{10})\) single-nucleon transfer reactions which have strong parentage based on the transferred nucleon coupled to the unexcited target core and which are thus representable in \(jj\)-coupling notation as \([J_T + (l,s)]_J\), where \(j\), \(J_T\), and \(J\) are the angular momenta of the transferred nucleon, target nucleus, and final state, respectively. Consideration of the nature of the observed preferential population has indicated that a state of common single-particle structure is involved in all reactions studied. The present data have been found to be consistent with this state being of \(d_{5/2}\) configuration, as expected on the simplest classical representation of the transfer process at these incident energies.

Selected angular distribution measurements for both single-nucleon and deuteron transfer reactions to strongly populated final states have shown that the transfer cross section decreases smoothly as an exponential function of angle or alternatively as a power-law function of the linear momentum transfer, in accord with recent observations of other heavy ion systems in this energy region. A recently proposed reaction model, which predicts an inverse power-law dependence, independent of all reaction and structure effects except the mass of the transferred particle, has been found to provide an adequate qualitative description of these data. This model is expressed
within a diffraction formalism, consistent with the strong nuclear absorption present in low-\( \eta \) situations such as those under consideration, but specifically includes finite range and nuclear recoil effects to obtain results essentially independent of specific structure phenomena.

It has been proposed that states are selectively populated in the \( (B^{11}, Be^{9}) \) deuteron transfer reactions which have dominant parentage based on the transferred neutron-proton pair coupled to the unexcited target core and which can be represented as \( [J_T + (j_n, j_p), J, J] \), where the neutron and proton angular momenta \( j_n \) and \( j_p \) couple to a resultant \( J' \) which in turn couples to the target angular momentum to form the final state \( J \). The preferentially populated levels in these reactions are the previously observed giant excitations of the deuteron transfer reaction and have been associated with high angular momentum states of extremely pure two-particle configuration. A group of giant excitations populated in all reactions investigated in the present work has been associated with a common \( (d_{5/2})^2 \) configuration on the basis of reaction systematics, the weak population of these states in corresponding inelastic scattering reactions observed in this work, and angular momentum arguments similar to those advanced for the single-nucleon transfer situations. Equivalent arguments have been applied to recent \( (\alpha, d) \) reaction data on light-element targets where similar states have been identified. Another giant excitation group of proposed \( (d_{5/2} f_{7/2})^6 \) structure lying at higher excitations has been observed in both reactions.

A systematic comparison of equivalent \( (\alpha, d) \) and \( (B^{11}, Be^{9}) \) reactions proceeding at energies of 10 MeV per incident nucleon has revealed that these reactions possess a high degree of similarity regarding both selective and preferential population of final states. The observed similarity has been found to extend in an unexpected fashion to the isobaric-spin configurations, as evidenced by the fact that no unambiguous evidence has been found in the \( (B^{11}, Be^{9}) \) data which indicates that the neutron-proton pair are transferred in the allowed singlet-spin, \( T = 1 \) configuration, which is forbidden and not observed in the
(α,d) reaction. An attempt has been made to investigate the nature of this effect by calculating the two-nucleon parentage of the B\textsuperscript{11} projectile nucleus ground state, however, the calculations have predicted a preference for the singlet rather than the triplet-spin configuration, contrary to experimental observations. No satisfactory explanation is available as yet for the apparent dominance of the S = 1, T = 0 transfer channel in the (B\textsuperscript{11},Be\textsuperscript{9}) reaction.

The consistency of the results of a detailed comparison, over a range of mass number, of the integrated cross sections of giant excitations populated in (α,d) and (B\textsuperscript{11},Be\textsuperscript{9}) reactions and the equivalence of the corresponding angular distributions when displayed as a function of linear momentum transfer has provided evidence for an identical direct transfer mechanism. It has been shown that the mass dependence of the cross sections can be interpreted in terms of the number of target holes available for the capture of the two nucleons into the proposed configurations. A suggested extension of the present work would involve a coincidence correlation study of the decay of the giant excitations in order to establish their assignments unambiguously. Excitation function measurements would also be particularly useful in testing the angular momentum arguments advanced to explain the formation of these levels.

Ratios of neutron and proton transfer cross sections to analog or mirror states in the final systems of the (B\textsuperscript{11},B\textsuperscript{10}) and (B\textsuperscript{11},Be\textsuperscript{10}) reactions have been utilized in the extraction of relative spectroscopic factors for the incident B\textsuperscript{11} systems, on the assumption of the validity of the isobaric spin predictions for these ratios. These data have subsequently been used to obtain the (B\textsuperscript{10}+n) and (Be\textsuperscript{10}+p) parentage of the B\textsuperscript{11} ground state. Calculations of the corresponding ratios and parentage based on available intermediate coupling shell model wavefunctions have been found to be in good accord with the experimental results. The validity of this approach has been supported by the consistency of the data obtained in reactions over a range of target nuclei. An extension of this method would involve a calibration of the relative spectroscopic information
in terms of known reaction situations. Such data were not available for comparison with the results which have been obtained in this work.

The present studies have demonstrated the utility of heavy-ion interactions as spectroscopic probes. This has become possible only through the development of instrumentation which permits isolation and study of individual residual nuclei and energy states and which has not previously been available. It is anticipated that the equipment and procedures developed in this work will be directly applicable in the research programs of the higher energy higher resolution electrostatic accelerators soon to become available in this and other laboratories. These studies have shown that the heavy-ion transfer reaction will be particularly useful in populating states of special character, including high-spin, single-particle, and multi-particle configurations, in order that they may be subjected to more intensive study.
APPENDIX. SEMIEMPIRICAL CALCULATION 
OF ENERGY LOSS FOR HEAVY IONS

A. Introduction

The energy loss of charged particles in matter often becomes a factor in the analysis of experimental data resulting from nuclear reaction studies, particularly those utilizing gas or thick solid targets. Corrections for energy losses of incident beam particles and reaction products in foil windows, targets, gas columns, and counters may be required for the determination of beam energies and the proper reduction of detected residual energies to reaction energies. Such corrections are especially important for heavy ions because of their high energy loss rates.

Direct recourse to experimental data for energy loss information is in most cases not possible, because of the extreme scarcity of such measurements, notably for heavy ions with energies above 5 MeV per nucleon. For situations in which corrections are necessary but not available, some consistent theoretical or semiempirical procedure for obtaining energy loss must be adopted. A semiempirical method for the calculation of energy loss for heavy ions was developed for this work and is described in this appendix.

For purposes of this discussion only, the term heavy ion will refer to particles with atomic number greater than that of the alpha particle. A general form of the stopping power formula is discussed in Section B and is used in Section C to formulate a semiempirical relation for heavy ions in terms of proton stopping cross section parameters. In Section D, a range-energy relation based on the calculation of equivalent thickness is obtained. Practical application of the method and comparison with available data is discussed in Section E.
B. The Stopping Power Formula

In passage through an absorbing medium, charged particles lose energy predominantly through ionization and excitation of the atoms of the medium. The purpose of any theoretical description of this process is to predict the average energy loss per unit path length, a quantity known as the stopping power of the material for the ion, and designated dE/dx. There have been many theoretical treatments of this problem, however, all results can be expressed in a common form, with their differences confined to a dimensionless quantity known as the stopping number, B. The stopping number is defined in terms of the stopping power by

\[ - \frac{dE}{dx} = \frac{4\pi(\gamma z)^2}{m_e v^2} \frac{e^4}{NB} \]  

(A.1)

where \( z \) is the atomic number of the ion, \( v \) is the velocity of the ion, \( N \) is the number of atoms per unit volume of the absorbing medium, \( m_e \) is the mass of the electron, and \( \gamma \) denotes the effective charge parameter. Further consideration of the stopping power formula will follow a brief discussion of the effective charge parameter and the behavior of the charge of an ion moving through matter.

As an ion passes through an absorbing medium, its charge fluctuates due to electronic capture and loss processes as the ion collides with atoms of the absorber. The charge of a moving ion is thus not the nuclear charge \( z \), but must be taken as an effective charge. The effective charge can be represented by \( \gamma z \), \( \gamma \leq 1 \), where \( \gamma \), the effective charge parameter, is defined as the ratio of the instantaneous charge of the ion to its nuclear charge. A knowledge of the heavy ion effective charge parameter is required in many practical considerations, for example, determination of the number of beam ions from their total collected charge, the use of charge-dependent
particle identification techniques, and the present application, calculation of energy loss in absorbers.

Although the processes involved in the capture and loss of electrons by a moving ion are, in general, so intricate as to defy precise description, it may be assumed that an ion will begin to capture electrons when its velocity decreases to approximately that of an orbital electron in its own atom. Charge data have shown that this is true for protons and alpha particles, which have been found to maintain their full nuclear charge down to energies of approximately 1 MeV/amu. According to this simple viewpoint, the significant quantity for the determination of the effective charge parameter, \( \gamma \), for heavy ions is the ratio of ion velocity to the orbital velocity of the electron being captured, which, for the energies considered here, is expected to be the first \( K \) electron of the ion.

Such a relationship has, in fact, been demonstrated by Northcliffe, in a study of the energy loss of heavy ions in aluminum.(127) These measurements indicate that there is apparently a universal dependence of \( \gamma \) on the above-noted ratio parameter for all heavy ions. An empirically determined formulation of this relationship is given by

\[
\gamma^2 = 1 - k \exp(-2v/v_k)
\]  

(A.2)

where \( (v/v_k) \) is the ratio of ion velocity to the Bohr orbital velocity of the first \( K \) electron of the ion and the constant is given by \( k = 1.85 \). Essential agreement with this prediction for the effective charge parameter has been found for heavy ions in other solid materials and in a number of gases.(128,129) Available experimental information thus indicates that the effective charge parameter is strongly dependent on the velocity and nuclear charge of the moving ion, and nearly independent of the atomic number, physical state, density, and mass of the absorbing medium.
The results of the above discussion pertinent to further application to the energy loss formula is that the principal difference between the passage of protons or alpha particles and heavy ions through matter at energies above 1 MeV/amu is the variation in the charge of the heavy ion. Significant variation from the full nuclear charge will occur for heavy ions at energies above 1 MeV/amu, dependent almost entirely on the elemental species and energy of the ion.

The first calculation of the stopping number, B, defined by Eq. (A.1) was carried out by Bohr (130) using nonrelativistic classical mechanics. The first wave mechanical treatment was given by Bethe, (131) and this result has proven to be a valid description of the energy loss process. The stopping number is given in Bethe's theory as

\[ B = Z \ln \frac{2m_e v^2}{I} = ZL \]  

(A.3)

where \( Z \) is the atomic number of the absorbing medium, \( I \) is the mean excitation energy of the atomic electrons of the medium, \( m_e \) and \( v \) are as defined in connection with Eq. (A.1), and \( L \) is the stopping number per atomic electron. Further refinements in the theory were given by Bloch, (132) who pointed out that the mean excitation energy is expected to be approximately proportional to \( Z \). This relation is often expressed in terms of the Bloch constant, \( K \), as

\[ I = KZ \]

Each of these authors later extended his treatment to relativistic ion velocities, and each obtained the same relativistic correction to the stopping number, given in terms of \( \beta = v/c \) as

\[ \Delta B = Z \left[ - \ln (1 - \beta^2) - \beta^2 \right] = Z \Delta L \]  

(A.4)

where \( \Delta L \) is the relativistic correction to the stopping number per atomic electron.
On the basis of these results, the electronic stopping power can be expressed, in general, by the formula

\[
- \frac{dE}{dx} = \frac{4\pi e^4}{m_e} N Z \frac{z^2}{v^2} \gamma^2(v, z) \left[ L(v, (\gamma z), Z) + \Delta L(v) \right]
\] (A.5)

where the dependence on the presumed most important variables is displayed explicitly. Thus, the calculation of stopping power is reduced to the equivalent problem of calculating the parameters \(L\) and \(\gamma\). The basis of a semiempirical theory is the expression of the relative relationship of these parameters for different ions and media. Differences in these theories then lie in the choice of comparison criteria, e.g., different ions in the same material, the same ions in different materials, etc.

C. Semiempirical Stopping Power for Heavy Ions

The formulation of the present semiempirical method for the calculation of heavy-ion energy loss will be discussed in terms of the comparison criteria on which it is based. Since the dependence of the stopping number per atomic electron, \(L\), and the effective charge parameter, \(\gamma\), on ion velocity is complicated and not precisely known, a logical first criterion is the comparison of ions having the same velocity. An equivalent requirement expressed in terms of energy is that the ions have the same value of energy per unit mass, \(E_m\). In this case, Eq. (A.5) can be written as

\[
\frac{dE}{dx} = KNZ \frac{z^2}{v^2} \gamma^2(z) \left[ L(\gamma z, Z) \right]
\] (A.6)

As a second criterion, the comparison will be made in the same material, i.e., for the same \(Z\). The relative stopping power of a given medium for different ions moving with the same velocity is then given by:
\[
\left(\frac{dE}{dx}\right)_2 / \left(\frac{dE}{dx}\right)_1 = \left[\frac{z_2}{z_1}\right]^2 \left[\frac{\gamma(z_2)}{\gamma(z_1)}\right]^2 \left[\frac{L(\gamma z_1)}{L(\gamma z_2)}\right]. 
\]  
(A.7)

where 1 and 2 denote two different ions. Assuming that the Bethe theory is an adequate description of the energy loss process, \(^{(134)}\) the parameter \(L\) is independent of \(z\) (see Eq. (A.3)), and Eq. (A.7) reduces to

\[
\left(\frac{dE}{dx}\right)_2 = \left(\frac{dE}{dx}\right)_1 \left[\frac{z_2}{z_1}\right]^2 \left[\frac{\gamma(z_2)}{\gamma(z_1)}\right]^2 
\]  
(A.8)

Since \(dE/dx\) and \(\gamma\) are rather well known for protons, a third criterion will be the choice of the proton as a reference ion associated with the subscript 1 in this equation. The subscript 2 will be taken to denote a heavy ion.

For energies above 1 MeV/amu, \(\gamma = 1\) for protons, and Eq. (A.8) becomes

\[
\left(\frac{dE}{dx}\right)_{\text{ion}} = \left(\frac{dE}{dx}\right)_{\text{proton}} \gamma_{\text{ion}}^2
\]  
(A.9)

The simplicity of this relationship is marred somewhat by the presence of the effective charge of the ion, a quantity which is at best only imperfectly known. It is well to investigate under what conditions this factor can be removed from the theory with negligible effect upon the accuracy of the results.

It has been found from direct consideration of charge-state population data\(^{(135)}\) that the square of the effective charge parameter has dropped from unity to \(\gamma^2 = 0.97\) for \(O^{16}\) ions with \(E_m = 7\) MeV/amu. According to the above discussion of effective charge, this value of \(\gamma^2\) obtains for all ions with equal values of the ratio parameter, \((v/v_k)\). Fig. (A.1) shows the \(E_m\) values plotted as a function of ion atomic number for which \(\gamma^2 = 0.97\), in other words,
Figure A.1. Effective charge parameter, $\gamma$, as a function of ion atomic number and energy. The solid line is closely approximated by $E_m = \left( z_{\text{ion}} \right)^2 / 9$. 
EFFECTIVE CHARGE PARAMETER FOR HEAVY IONS

$\gamma^2 < 0.97$

$\gamma^2 > 0.97$
for which a 3% error in the stopping power results from deviation of the ion charge from the nuclear charge. The lower, unshaded portion of this plot indicates the area in energy-charge parameter space for which the effective charge parameter can be removed from Eq. (A.9) with less than 3% error in the stopping power theory. A fourth criterion of this method is then that it is applicable with less than 3% error for ions whose atomic number and energy are such that they fall in the lower portion of Fig. (A.1). This criterion is satisfied for ions with energies given approximately by

\[ E_m > \frac{z_{\text{ion}}^2}{9} \]  

(A.10)

It should be noted that this criterion could also have been arrived at through consideration of Eq. (A.2).

Under the conditions discussed above, the stopping power for any ion can be represented in terms of the proton stopping power as

\[ \left[ \frac{dE}{dx} \right]_{\text{ion}} = z_{\text{ion}}^2 \left[ \frac{dE}{dx} \right]_{\text{proton}} \]  

(A.11)

These conditions can be summarized as

1. The ion and the proton have the same \( E_m \) value.
2. The stopping power for ion and proton are in the same absorbing medium.
3. The \( E_m \) value has a lower limit given by 1 MeV/amu or \( (z_{\text{ion}})^2/9 \) MeV/amu, whichever is greater.

The critical values of (3) for several representative heavy ions are as follows: 1.8 MeV/amu for Be, 2.7 MeV/amu for B, 3.9 MeV/amu for C, 5.4 MeV/amu for N, and 7.0 MeV/amu for O.

The stopping power of any ion can now be obtained through the application of a consistent set of proton stopping power data to Eq. (A.11). The
proton stopping power is often given in terms of the stopping cross section, \( \epsilon \), related to the stopping power by \( \epsilon = -N^{-1} \frac{dE}{dx} \). Values of the proton stopping cross section for 26 absorbing media have been given by Whaling, with later additions and changes by Demirlioglu, in the form

\[
\epsilon = \frac{0.24 \, Z}{E_m} \left[ \ln\left(E_m/Z\right) + c(Z) \right] \times 10^{-15} \text{ ev-cm}^2
\] (A.12)

where \( c \) is a parameter dependent upon the absorbing medium, represented here by a functional dependence on \( Z \). In this formalism, the proton stopping power becomes

\[
\left[ \frac{dE}{dx} \right]_{\text{proton}} = \frac{0.24 \times 10^{-24} N_A Z}{A E_m} \left[ \ln\left(E_m/Z\right) + c(Z) \right] \text{ MeV/amu mg/cm}^2
\] (A.13)

where \( N_A \) is Avogadro's constant and \( A \) is the atomic weight of the absorbing medium. Replacing the proton stopping power in Eq. (A.11) with this value, the stopping power for any ion in any medium for which the parameter \( c \) is known is given by

\[
\left[ \frac{dE}{dx} \right]_{\text{ion}} = 0.24 \times 10^{-24} \frac{Z^2_{\text{ion}}}{A A E_m} \left[ \ln\left(E_m/Z\right) + c(Z) \right] \text{ MeV/amu mg/cm}^2
\] (A.14)

It should be noted that this is the electronic stopping power for the ion. The specifically nuclear stopping power has been neglected in these considerations since a heavy ion loses a negligible amount of energy through collisions with nuclei of the absorbing medium at the energies at which this method is expected to be applicable.
D. Range-Energy Relations

In practical applications, it is usually not the stopping power for an ion which is required, but rather the range of an ion for a given energy loss, or the energy loss of an ion in a given range, i.e., a range-energy relation. Neglecting lateral displacements of the ion in the absorber and statistical fluctuations in the energy loss process, which are important only for very low ion velocities when the nuclear energy loss process is significant, the range is simply given by integration of the reciprocal of the stopping power. Thus, range-energy relations can be developed by numerical integration of Eq. (A.14).

In the present case, however, an alternative procedure based on the calculation of equivalent thickness has been adopted. The motivation for this approach was provided by the availability of a consistent, complete set of empirical range-energy relations for heavy ions in aluminum recently published by Northcliffe. (83) This technique consists of the conversion of absorber thickness to an equivalent aluminum thickness and the application of these range-energy relations.

In terms of Eq. (A.14), the ratio of the stopping powers for a heavy ion in two absorbing media, denoted by 1 and 2, is given by

\[
\frac{\left[ \frac{dE}{dx} \right]_2}{\left[ \frac{dE}{dx} \right]_1} = \frac{A_1 Z_2}{A_2 Z_1} \left[ \frac{\ln(E_m/Z_2) + c_2}{\ln(E_m/Z_1) + c_1} \right]
\]

(A.15)

The equivalent thickness of an absorber will be defined as the thickness of a reference medium in which the ion loses an amount of energy equal to its energy loss in the absorber. The equivalent thickness of medium 2, \( R_1 \), is given in terms of its actual thickness, \( R_2 \), as

\[
(R_1)_{\text{solid}} = \frac{R_2 A_1 Z_2}{A_2 Z_1} \left[ \frac{\ln(E_m/Z_2) + c_2}{\ln(E_m/Z_1) + c_1} \right] \text{ mg/cm}^2
\]

(A.16a)
For a given gas absorber, $R_2$ can be expressed as a constant times the atomic weight of the gas.

$$
(R_1)_{gas} = k d \frac{A_1}{Z_1} \left( \ln(\frac{E_m}{Z_1}) + c_1 \right) \frac{mg}{cm^2}
$$

(A.16b)

where $k$ is a constant containing the physical state of the gas in terms of its temperature and pressure, and $d$ is a constant equal to 2 for diatomic gases and unity otherwise. The equivalent thickness of compounds can be similarly expressed, since the molecular stopping power of a compound can be computed as a weighted sum of the stopping powers of its constituent atoms to a very good approximation.

If the reference medium is taken to be aluminum, the use of Eq. (A.16), together with range-energy relations for heavy ions in aluminum, is insufficient to calculate the energy loss of any heavy ion in any absorbing medium, again, providing that $c$ is known.

E. Application

To facilitate the practical application of this procedure, the range-energy relations of Northcliffe have been fitted to a three-parameter power series in the energy of the form:

$$
R = a + b E_m + c E_m^2
$$

(A.17)

where $R$ is the range in mg/cm$^2$ and $E_m$ is the energy in MeV/amu of the ion. The results of this fitting operation are shown in Fig. (A.2) for several representative heavy ions, and the resulting parameters are listed in Table (A.1). The accuracy of these parameters is equivalent to the data from which they were obtained. Ranges for other isotopes of the elements shown in Fig. (A.2) are in the ratio of the masses, as can be shown by a straightforward application of Eq. (A.8), e.g., $R_{B^{10}} = (m_{B^{10}}/m_{B^{11}}) R_{B^{11}}$. 
Figure A.2. Range-energy relations for heavy ions in aluminum. The curves are least squares fits to the data (reference 83) of the function:

\[ R = a + bE + cE^2, \]

where \( R \) is the range in mg/cm\(^2\) (Al), \( E \) is the energy in MeV/amu, and the parameters \( a, b, c \) are as given in Table A.1.
RANGE OF HEAVY IONS IN ALUMINUM

RANGE (mg/cm²)

ENERGY (MeV/amu)

Be⁹
B¹⁰
C¹²
N¹⁴
O¹⁶
Ne²⁰
Table A.1. Empirical range-energy relation for heavy ions. \( R = a + b E + c E^2 \), where \( R = \text{range in mg/cm}^2 \) (Al) and \( E = \text{energy in MeV/amu} \).

<table>
<thead>
<tr>
<th>Nuclide</th>
<th>( a )</th>
<th>( b )</th>
<th>( c )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Be(^9)</td>
<td>.070</td>
<td>2.715</td>
<td>.705</td>
</tr>
<tr>
<td>B(^{11})</td>
<td>.385</td>
<td>2.249</td>
<td>.544</td>
</tr>
<tr>
<td>C(^{12})</td>
<td>.439</td>
<td>1.855</td>
<td>.404</td>
</tr>
<tr>
<td>N(^{14})</td>
<td>.466</td>
<td>1.737</td>
<td>.340</td>
</tr>
<tr>
<td>O(^{16})</td>
<td>.482</td>
<td>1.696</td>
<td>.290</td>
</tr>
<tr>
<td>F(^{19})</td>
<td>.533</td>
<td>1.776</td>
<td>.266</td>
</tr>
<tr>
<td>Ne(^{20})</td>
<td>.529</td>
<td>1.702</td>
<td>.220</td>
</tr>
</tbody>
</table>

As an example of the application of this semiempirical method, consider the calculation of the energy loss of a heavy ion in passing through a foil window or gas target. The range in aluminum of the incident ion of given \( E_m \) value is calculated from Eq. (A.17), and the equivalent thickness of the foil or target is obtained from Eq. (A.16). The difference of these two quantities is used in the inversion of Eq. (A.17) to obtain the \( E_m \) value of the emergent ion. It has been found that this procedure easily lends itself to computer calculation through the coding of Eqs. (A.16) and (A.17) and computer storage of the parameters of Table (A.1) and appropriate absorber constants.

A comparison of the results of this method with available heavy-ion stopping power data \(^{128,129,133}\) is shown in Fig. (A.3). The curves are from the indicated experimental data, and the points are absolute values of the corresponding stopping powers calculated using the present theory. According to the discussion in connection with Fig. (A.1), this method is expected to be applicable for boron and carbon ions with \( E_m \) values above...
Figure A.3. Stopping power for heavy ions in various materials. The points are from the present theory, and the curves are from the indicated experimental data (references 128, 129, 133). The theory is expected to be valid for $E \gtrsim (z_{ion})^2 / 9$ MeV/amu.


**STOPPING POWER FOR HEAVY IONS**

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**THEORY vs EXPERIMENT**

- **B'' ION**
- **B'' ION**
- **C'' ION**

**THEORY**

- NORTHCLIFFE
- MARTIN et al.
- ROLL et al.

---

**Diagram Details**

- **x-axis**:
  - \( E_m (\text{MeV/amu}) \)

- **y-axis**:
  - \( -\frac{dE_m}{dx} \text{ (MeV/amu/mg/cm}^2) \)

---

**Examples**

- **B'' in C**
- **B'' in O_2**
- **B'' in Al**
- **B'' in Ni**
- **B'' in Ni**

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**Legend**

- **Solid line**: Theory
- **Dashed line**: Experiment

---

**Axes Labels**

- **x-axis**: Stopping Power for Heavy Ions
- **y-axis**: Linear scale for energy loss per unit path length.
2.7 and 3.9 MeV/amu, respectively. Departures from the data of Martin et al. (128) for C_{12} in N_{2} and from that of Northcliffe (133) for B_{11} in C occur at approximately these values. There is good agreement at all energies with Northcliffe's data for boron ions in aluminum and nickel. There are departures from the data of Roll and Steigert (129) for B_{11} in O_{2} at E_{m} = 6 MeV/amu, however, there is also some discrepancy between other portions of these data and the more complete data of Northcliffe.

F. Summary

A semiempirical procedure for the calculation of the energy loss of heavy ions in absorbing media has been developed. This method utilizes the Bethe theory of the energy loss of charged particles in matter and employs empirical expressions for proton stopping cross sections and heavy-ion aluminum range-energy relations. The method is expected to be applicable for heavy ions of atomic number z in the energy range above E_{m} = z^2/9 MeV/amu. Agreement has been found in this range with available experimental heavy-ion stopping power data for boron and carbon ions in various gas and solid absorbers. The accuracy of the method is that of the empirical relations employed. Practical application of this procedure has been found to be amenable to calculation by computer.
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