THE COLLECTIVE STRUCTURE OF TUNGSTEN NUCLEI

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ABSTRACT
THE COLLECTIVE STRUCTURE OF TUNGSTEN NUCLEI
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The neutron pairing character of states in $^{178, 180, 181, 182, 184}$W and $^{182, 184}$W has been investigated using the (p,t) reaction at an incident proton energy of 21 MeV. Excitation energies and angular distributions were obtained for all states populated up to approximately 2.5 MeV of excitation. Excited $J^P = 0^+$ states, many previously unreported, were populated in $^{178}$W at 997 keV (9%), 1356 keV (3%), and 1643 keV (1%); in $^{180}$W at 1516 keV (7%), 1095 keV (2%) and 2356 keV (4%); in $^{182}$W at 1135 keV (14%), 2520 keV (2%), 2552 keV (1%), and 2725 keV (3%); in $^{184}$W at 995 keV (8%); where the numbers in parentheses are the yields given as a percentage of the ground state yields. Strong $L = 0$ transitions were also seen in the $^{183}$W(p,t)$^{181}$W reaction to states at 454 keV and 1864 keV of excitation.

The first excited $0^+$ states in these isotopes can be divided into two groups according to their moments of inertia. Their properties in $^{184}$W and $^{182}$W and those of all of the tungsten ground states can be well described in terms of a pairing vibration scheme arising from an energy gap in the neutron single-particle level scheme occurring at $N = 108$. For the lower mass isotopes, the first excited $0^+$ states in $^{178}$W and $^{180}$W as well as the 2725 keV state in $^{182}$W are found to have nearly the same experimental (p,t) reaction Q-value. Their structure may, therefore, be interpreted as the ground state of the target.
nucleus coupled to two holes in a group of single-particle levels somewhat below, and decoupled from, the Fermi surface. The decoupling can be explained by assuming a reduced pairing matrix element between oblate and prolate single-particle orbitals. A specific prediction of this structure model, that the $L = 0$ transition to the 1864 keV state in $^{181}$W should not be blocked by the unpaired neutron, is found to be fulfilled.

The systematics of transitions to the $J^e(n) = 2(\pi)$ member of the $K = 2$ gamma vibrational bands in these nuclei are discussed with particular attention to the determination of the main two-neutron configurations involved in the population of these states. Although model calculations indicate four possible configurations that could play a substantial role, the conclusion from the strength systematics of the even-mass targets and the results of the $^{183}$W(p,t)$^{181}$W experiment, is that only the $(1/2[510],5/2[512])$ two-neutron hole configuration is important.
In Memory of my Father

HERBERT SVEND MORTENSEN

1930 - 1977
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Chapter 1

INTRODUCTION
1.1 INTRODUCTION

Nuclei in the rare earth region have mass numbers between 150 and 192 so that on average there are about 175 nucleons. The number of possible configurations available to such a system is enormous ($\sim 10^6$ below 5 MeV excitation) so that the study of the structure of these nuclei even at low excitation energies might seem impossibly difficult. It has been found, however, that certain states possess what is termed "collectivity": the wavefunction of such states are composed of sums of many single-particle configurations, with special phase relationships existing between the terms.

Two types of collective states important in nuclei in the rare earth region can be defined operationally:

(1) Those states which are strongly excited in inelastic nucleus-nucleus scattering. These states must possess collectivity related to the promotion of single nucleons from the ground-state configuration to higher single-particle levels, since in a microscopic picture the inelastic scattering operator is a one-body operator. Large electromagnetic couplings to the ground states must also be in evidence for these states for the same reason.

(2) Those states which are strongly populated in direct reactions involving the transfer of a pair of like nucleons. These states must possess collectivity related to the addition or removal of pairs of
particles, which is thus called "pairing collectivity."

In the case of even-even rare earth nuclei, the ground state is dominated by the pairing collectivity. Reasonable semi-quantitative estimates (Br 72) indicate that for two-nucleon transfer reactions, states other than those in the ground-state rotational band should be populated with only a very small fraction of the ground-state strength (a few percent). Recently, however, (p,t) and some (t,p) reactions on several rare earth and actinide nuclei have found J = 0⁺ states which are populated with 10% to 15% of the ground state strength (Bj 66, De 70, Fl 71, De 72, El 72, Ma 72a, Ma 72b, Fl 73, Oo 73). To explain such large pairing collectivity in excited states would normally require extremely detailed calculations, since special circumstances must be at work to so fragment the pairing strength. As will be shown, however, the systematic study of a series of isotopes with (p,t) and (t,p) reactions can reveal much about the dependence of collective states upon the underlying single-particle structure so long as the microscopic structure does not change appreciably with neutron number. Such systematic studies have been performed on Sm (Hi 65, Bj 66, De 70, Mc 70), Yb (Oo 73), Os (Sh 76) and Gd (Fl 73) isotopes. The interpretation, however, is clouded in all cases but Yb by the changing shape of the nuclei with neutron number, bespeaking a change in the underlying single-particle structure. The shapes of tungsten nuclei, in contrast, are all very similar. Inelastic alpha particle scattering experiments on 182W, 184W, and 186W show only a very small shape change
(Le 75) while estimates from rotational band structures of the odd isotopes show no large unexpected change in deformation between $^{177}_{\text{W}}$ and $^{183}_{\text{W}}$ (Be 74). Thus, tungsten nuclei form an excellent system for such a systematic study.

There have been many experimental studies of the isotopes of tungsten over the past ten years. The neutron single-particle structure has been probed with spectroscopic studies of the odd-mass isotopes using $(d,p)$, $(d,t)$ (Ca 71, Ca 72) and $(^3\text{He},\alpha)$ (Ki 73) reactions as well as numerous gamma-ray measurements. The $^{186}_{\text{W}}(p,d)^{185}_{\text{W}}$ study of Ascuitto, King, and McVay (As 72c, Ki 73, As 74, Ki 74) provided some of the first convincing evidence for multiple-step inelastic effects in one-nucleon transfer as well as testing details of the structure of $^{185}_{\text{W}}$.

The even-mass isotopes have not been neglected, with studies using $(d,d')$ (Gu 71), $(d,p)$ and $(d,t)$ reactions (Ca 72). The $^{186,184,182}_{\text{W}}(p,t)$ reactions to states low in excitation have been investigated (Ma 72) as has $^{\text{NATURAL}}_{\text{W}}(p,t)$ (Oo 73) and the gamut of $(t,p)$ reactions (Ca 73, Ca 76b, Ga 77). Again, the work of King et al. (As 72c, Ki 72, Ki 73, Ki 74) on the $^{186}_{\text{W}}(p,t)^{184}_{\text{W}}$ reaction must be mentioned as an early but definitive study of the effects of multiple-step inelastic excitation in $(p,t)$ transitions. In addition, the more recent work of Hanson et al. (Er 74, Ha 75, Ha 76) in which these effects in heavy-ion-induced two-nucleon transfer reactions were investigated, added much to our understanding of the structure of the ground-state bands of tungsten nuclei.
The experimental part of the present study seeks to extend the previous (p,t) measurements to states other than those belonging to the ground state band and to extract the transition strength systematics for the whole range of tungsten nuclei accessible with the (p,t) reaction. This particular reaction was chosen for several reasons:

1. The multiangle magnetic spectrograph is ideally suited for measuring (p,t) reactions with the good energy resolution which is crucial for spectroscopic studies of nuclei in the rare earth region.

2. The two neutrons in the triton are in a relative angular momentum zero state. They therefore form a good "pair" so that their transfer directly probes the pairing collectivity of the states.

3. The use of light ions in particle transfer studies allows many calculational simplifications to be made, obviating the use of the more complex reaction codes necessary to describe heavy ion induced reactions.

The study consists of (p,t) reaction measurements on all five of the naturally occurring tungsten isotopes (A = 186, 184, 183, 182, and 180) at 21 MeV bombarding energy, all performed with the same apparatus and analyzed in a consistent manner. In each case studied, excited 0^+ states (0^+_n) were found to be populated with about 10% of the ground state strength. However, during the course of the study, it became apparent that the characteristics of some supposedly highly collective states
actually changed considerably with mass number, implying that they must be populated via a very limited number of configurations.

Simple models were used to provide a framework for understanding the systematics of these states. The strong \((p,t)\) excited 0\(^+\) states in the low-mass tungsten isotopes, all with the same experimental reaction Q-value, can be explained by dividing the single-particle orbitals into two noninteracting groups according to the sign of their mass quadrupole moments \((Gr \, 71)\). The excited 0\(^+\) states then gain their \((p,t)\) strength through pairing collectivity involving orbitals decoupled from those involved in the ground state. The existence of these states adds considerable weight to the concept of this division of the single-particle orbitals into two groups, which has been under considerable debate for several years \((Be \, 72, \, Ch \, 72, \, Gr \, 71, \, Mo \, 77, \, Ri \, 72)\). Moreover, their constant reaction Q-value provides an excellent signature which the models which seek to explain the division on a microscopic basis should be able to reproduce.

In order to explain \((p,t)\) and \((t,p)\) reaction data on the high-mass isotopes, the concept of pairing vibrations was applied. In these cases, the fragmentation of the pairing strength was attributed to an energy gap in the single-particle spectrum which partially decouples the levels above the gap from levels below the gap. \((Such \, a \, decoupling \, previously \, had \, been \, seen \, only \, in \, the \, Yb \, isotopes.) Through the use of such models the essential features of the data can be understood, although the schematic nature of
the models precludes detailed agreement between model predictions and experimental data. Even so, the present study demonstrates the first successful application of the pairing vibration model to deformed nuclei.

In addition to constituting a detailed study of the collective neutron pairing states, the present study also provides spectroscopic information on states in tungsten nuclei to about 2.5 MeV excitation. Such data are valuable because they directly provide excitation energy data, confirming and sometimes supplementing level schemes inferred from gamma-ray measurements.
Chapter 2

THE STRUCTURE OF RARE EARTH NUCLEI: AN OVERVIEW
2.1 SINGLE-PARTICLE STRUCTURE

The study of nuclear structure is the study of extremely complex many-body systems. Nuclei in the rare earth region have over one hundred nucleons, all interacting through some forces, usually taken to be two-body forces, giving a Hamiltonian for particle $i$:

$$H_i = -\frac{\hbar^2}{2M_i} \nabla_i^2 + \sum_{i \neq j} V_{ij}(\vec{r}_i, \vec{r}_j) \quad (2.1)$$

Since the two-body forces $V_{ij}$ are extremely complicated and not fully understood at present, it might seem that the calculation of the structure of complex nuclei is impossible. However, it has become apparent that a major part of the sum total of all forces acting on an individual nucleon in a nucleus can be represented by a simple potential and that the same potential can be used for all of the nucleons in a nucleus (Co 71). Thus

$$V'_{ij} = V(\vec{R}) + V_{ij} \quad (2.2)$$

where $V_{ij}$ is a (hopefully) small residual interaction. This rather surprising fact has been well exploited in nuclear physics in the "nuclear shell model." One form of the potential that has been extremely useful is the spherical Woods-Saxon type:

$$V(\vec{R}) = V(R) = V_0(1 + \exp[(R - R_0)/a])^{-1} \quad (2.3)$$

which has a shape between that of a harmonic oscillator potential and a square-well potential. With the inclusion of a spin-orbit potential of the form $\vec{L}_i \cdot \vec{s}_i$ (where $\vec{L}_i$ is the orbital angular
momentum of the nucleon "i" and $\bar{s}$ is its spin angular momentum) and a spherical Coulomb potential, as well as the application of phenomenological residual interactions, the model has been found to reproduce many of the properties of light nuclei as well as of nuclei in the lead, calcium, and zirconium regions.

Attempts have also been made to calculate the structure of the ground states of actual multinucleon systems using self-consistent methods such as Hartree-Fock and its many variations. These attempts have met with some success, especially for light nuclei ($A \leq 40$) (Ri 68), although many severe approximations are necessary due to the complexity of the problem (Va 73). One interesting feature of these calculations is that if the self-consistent field is allowed to vary from a spherical shape, nuclei far from closed shells usually take on a deformed prolate shape with $R^L/R^S \approx 1.2$ where $R^L$ is the length of the longer (symmetry) axis and $R^S$ that of the shorter axis (Va 73).

In the event of a deviation from sphericity, the spherical shell model is no longer adequate to describe the single-particle structure. The Nilsson model (Ni 55) takes the next logical step, and calculates the single-particle levels in a harmonic oscillator well deformed in the $B_2$ direction, where the nuclear surface is parametrized by $R = R_0 (1 + B_2 Y_{20})$. Such a deformation splits the $M$ ($z$-projection of the orbital angular momentum) degeneracy and induces interactions among states of the same value of $M$. Normally, for convenience, however, only the
couplings between levels in one major shell are included.

The deformed shell model can be pushed to near the limit of complexity by the inclusion of inter-shell coupling and deformation of the Woods-Saxon well given not only by $\beta_2$, but also higher-order deformations $\beta_4$, $\beta_6$, $\beta_8$, ... Such a calculation was performed in the course of this study where $\Delta N = 4$ mixing (major shell mixing up to 4 shells away) and $\beta_2$, $\beta_4$ and $\beta_6$ deformations were included. The calculation was performed with the code DEF2NT of Ascuitto and Sorenson using potential well parameters suitable for the lead region (As 72b), and experimentally determined deformation parameters for the tungsten nuclei taken from the literature (Le 75). The results of such calculations are shown in Figure 2-1 where on the left hand side are the levels as calculated for a spherical well. The value of $\beta_2$ was then varied until the appropriate value for $^{184}$W was reached, when $\beta_4$ was varied from zero to its proper value. The resulting single-particle spectrum is shown to the right and also in Figure 2-2.

The question of how well such a calculation reproduces the experimental data is addressed in Figure 2-2. The calculated single-particle orbits for neutrons are shown on the left and the orbits as extracted by Ogle et al. from experimental data (Og 71) on the right. The general agreement is seen to be quite good although individual levels differ in energy by 100-300 keV. The levels in each case are labelled by their 'asymptotic quantum numbers' which are the designation of the dominant component
as a function of $\beta_2$ and $\beta_4$ deformation.

The calculated spectrum for $\beta_2 = 0.221$, $\beta_4 = -0.087$ is shown to the right along with the shapes of the Fermi surfaces for $^{184}$W and $^{180}$W from BCS calculations as described in Section 2.2.

Figure 2-2. The single-particle level structure calculated as described in the text is shown at left while that extracted from experimental data by Ogle et al. (Og 71) is shown at right. The relative energy scale of the "experimental" values are from the reference, while the zero of the scale (which was not given) was set to give some average agreement with the calculated values. The labels indicate the asymptotic quantum numbers, $K[NN_{2A}]$, for the orbitals.
SINGLE-PARTICLE LEVEL STRUCTURE

CALCULATED "EXPERIMENTAL"
of the level in a cylindrical harmonic oscillator basis for the case of very large $\beta_2$ deformation. The quantum numbers are given as $K[N N Z \Lambda]$ where $K$ is the projection of the single-particle total angular momentum along the symmetry axis, $N$ is the total number of oscillator quanta, $N_Z$ is the number of oscillator quanta along the symmetry axis, and $\Lambda$ is the projection of the orbital angular momentum. On some occasions, this is also written $K[\pi N N Z \Lambda]$ where $\pi = +$ or $-$, the parity of the level; this designation is redundant, however, since $\pi = +$ if $N$ is even and $\pi = -$ if $N$ is odd. Of these asymptotic quantum numbers, however, only $K$ and $\pi$ are good quantum numbers, and $K$ only in the limit that Coriolis coupling is ignored (see section 2.2.3).
2.2 COLLECTIVE STRUCTURE:

2.2.1 Pairing

The Nilsson model has been found to be inadequate in explaining the collective structure of deformed nuclei so that other residual forces between the particles in the Nilsson levels must be included. The pairing interaction (or more correctly, the monopole pairing interaction) is the primary interaction used, with a Hamiltonian of the form

$$H_{\text{total}} = H_{\text{Nilsson}} + H_{\text{pairing}}$$

where

$$H_{\text{pairing}} = \sum_{\mu \nu} G_{\mu \nu} A_{\mu}^{+} A_{\nu}$$

and $A_{\mu}^{+}$ creates a zero-coupled nucleon pair in the orbit $\mu$ and $G_{\mu \nu}$ is the strength of the interaction between orbits $\mu$ and $\nu$. In most calculations, the interaction strength is taken to be the same for all pairs of orbits so that $G_{\mu \nu} = G$, a procedure known as the "constant pairing matrix element approximation." The use of such an interaction, which is the same between all orbitals regardless of their single-particle energies, is clearly an oversimplication since the interaction between nucleon pairs in orbitals differing greatly in energy should be reduced. To compensate for this overly large coupling, it is conventional to truncate the space of single-particle orbits allowed to interact via the pairing force to 30 or so valence orbitals (As 72b).

In only a few cases can the problem so formulated be solved
exactly; for most cases the approximate solution of Bardeen, Cooper, and Schriffer, the so-called BCS solution, is applied:

\[ |\Phi_{BCS}\rangle = \prod_{\nu} (U_{\nu} + V_{\nu} A_{\nu}) |0\rangle \]

where \( U_{\nu}^2 + V_{\nu}^2 = 1 \) \hspace{1cm} (2.5)

and \( \sum_{\nu} V_{\nu}^2 = \) the number of particles

It should be noted that the above solution does not possess a definite number of particles, each term differing from the others by at least two particles. The "quasiparticle transformation" is usually applied through a canonical transformation of the form:

\[
\alpha_{\nu}^- = U_{\nu} \alpha_{\nu} + V_{\nu} \alpha_{\nu}^+ \\
\alpha_{\nu}^+ = U_{\nu} \alpha_{\nu}^+ - V_{\nu} \alpha_{\nu}
\]

where \( \alpha_{\nu}^+ \) creates a nucleon in the orbital \( \nu \) and \( \overline{\nu} \) represents the time reverse of \( \nu \) (i.e. \( A_{\nu}^+ = a_{\nu}^+ a_{\nu}^+ \)). It can then be shown that \( |\Phi_{BCS}\rangle \) is the vacuum for the quasiparticle destruction operator and that the energy of the state is a minimum if

\[
V_{\nu}^2 = \frac{1}{2} (1 + e_{\nu}^2 / e_{\nu}^2 + \Delta^2 )
\]

\[
V_{\overline{\nu}}^2 = \frac{1}{2} (1 - e_{\nu}^2 / e_{\nu}^2 + \Delta^2 ) \hspace{1cm} (2.7)
\]

where

\[
\Delta \equiv G \sum_{\nu} U_{\nu} V_{\nu} \\
e_{\nu} \equiv E_{\nu} - \lambda
\]

and \( E_{\nu} \) is the single-particle energy of orbital \( \nu \), and where \( \lambda \) is set by the requirement that the average number of particles in the calculation equal the actual number of particles. The quantity \( V_{\nu}^2 \) is then recognized as the probability of occupation
of the level $v$ by a nucleon pair, the "fullness"; and similarly $U_\nu^2$ as the "emptiness". The Fermi surface is now no longer sharp (some levels full and others empty) but diffuse with an energy spread of about $2\Delta$, as depicted in Figure 2-3.

After the single-particle structure is set, the only free parameters in the BCS theory are the number of levels chosen to be considered valence particles (32 in all calculations in this work) and the value of the pairing matrix element $G$. In this work the value of the pairing matrix element has been chosen so that the BCS gap parameter $\Delta$, is equal to the experimental odd-even mass difference $P_N$ as determined from the Nilsson-Prior prescription. Such a choice is well justified in the literature (Ni 61).

The applicability of the constant pairing matrix element approximation has been questioned in recent years both on intuitive as well as experimental grounds. Under the assumption of a short-range force between orbitals, some pairs of orbitals which have large mass overlaps should interact much more strongly than other pairs whose mass overlap is small. In particular, Griffin, Jackson, and Volkov (Gr 71), in an effort to explain some (p,t) data in the actinide region (Ma 70), suggested that the valence orbitals should be broken into two subgroups, oblate orbitals ($dE_v/dB_2^2 > 0$) and prolate orbitals ($dE_v/dB_2^2 < 0$) (see Figure 2-1). The overlap of two prolate orbitals (polar orbits about the prolate nucleus) will be of the same order of magnitude as the overlap of two oblate orbitals (equatorial orbits) but both will be much large than the overlap between an oblate and a prolate orbital.
Figure 2-3. To the left is schematically shown the structure of a nucleus in the Nilsson model. The levels below the dotted line are full and those above, empty. Upon application of the pairing force via the BCS theory, the result is as shown at right where the Fermi surface is diffuse with an energy spread of about $2\Delta$. The curve actually plots the values of $V^2$ and $U^2$, the "fullness" and the "emptiness" of each level while the dotted line represents the energy value of the chemical potential, $\lambda$. 
Single-Particle Binding Energy (to the continuum→)

NO PAIRING

Levels Full

Levels Empty

WITH PAIRING

$V_2$

Levels Full

$-U_2$

Levels Empty

Lev. Part. Full
Thus, to some extent, the oblate orbitals and prolate orbitals form separate subspaces which interact weakly with each other.

The suggestion of Griffin et al. was incorporated in calculations of $0^+$ states in actinide nuclei by van Rij and Kahana (Ri 72) and placed perhaps on a more sound theoretical footing by Bes, Broglia, and Nilsson (Be 72) with their concept of quadrupole pairing fields. In no case, however, was the agreement between calculations and experiment sufficient to completely support the assumption of reduced oblate-prolate pairing matrix elements, thereby leaving the matter unsettled (Ch 72).

2.2.2 Rotational Bands

The finite deformation of certain nuclei has a profound effect on their energy spectra; whereas spherical nuclei cannot rotate in a quantum mechanical sense, the breaking of spherical symmetry leads to well developed rotational bands. A useful description, where the motion of the individual nucleons is the same in a body-fixed frame of reference, whether or not the nucleus as a whole rotates, is known as the strong coupling model, or adiabatic model, and was developed by Bohr and Mottelson (Bo 52, Bo 53). Within this model, if a nucleus is both axially symmetric and reflection symmetric (as in the case for a nucleus with $\lambda \geq 2$ deformations where $\lambda$ is even), then the total wavefunction for a state of angular momentum $J$, total angular momentum projection $M$, and intrinsic angular momentum projection $K$ is given by:
\[ |JMK\rangle = \left[ \frac{2J + 1}{16\pi^2(1 + \delta_{K0})} \right]^{1/2} (D^J_{M,K}|K\rangle + (-)^{J-K} D^J_{M,-K} R_1|-K\rangle) \] (2.8)

where \( D^J_{M,K} \) is a rotation D-matrix, \( |K\rangle \) is the intrinsic wavefunction, and \( R_1 \) denotes a rotation of 180° about the 1-axis.

The energies of the states are given by

\[ E = \frac{n^2}{2I} [J(J + 1) - 2K^2] + E^{(0)}_K \] (2.9)

where \( I \) is the moment of inertia of the band and \( E^{(0)}_K \) is the energy eigenvalue of \( |K\rangle \) in the body-fixed system. For each intrinsic state \( |K\rangle \), there then exists a rotational band of energy spacing \( J(J + 1) \) with angular momentum \( J = K, K = 1, K = 2, \ldots \) except for the case of \( K = 0 \) bands where the form of the wavefunction \( |JMK\rangle \) evidently demands that only even values of \( J \) are allowed for positive parity intrinsic states.

### 2.2.3 Other Residual Interactions

One of the principal results of the BCS theory is that the lowest excited states of even-even nuclei (neglecting the members of the ground-state rotational band) should occur at an excitation energy above \( 2\Delta \), about 1.5 MeV for tungsten. The existence of rotational bands with bandheads of \( J^n = 0^+, 2^+, \) and \( 2^- \) inside the superconducting gap then seems to require that these states possess collectivity due to some correlating force besides the pairing force. In one interpretation, the \( J = 0^+ \) and \( 2^+ \) bandheads arise from two of the modes of vibration of a \( \beta_2 \) deformed liquid drop (Da 68); the \( J^n=0^+ \) state being called a
"beta vibration" corresponding to vibrations preserving the axial symmetry, and $J^m = 2^+$ "gamma vibrations" arising from vibrations causing deviations away from axial symmetry. The $J^m = 2^-$ state in this picture comes from a coupling of the octupole vibrational and collective rotational degrees of freedom, which will not be discussed here. These hydrodynamic explanations adequately describe only the most general properties of these collective states, but the terms beta, gamma and octupole vibrations are still often used as convenient labels.

Of the many interactions used to describe the beta and gamma vibrations, the most fully explored is a quadrupole-quadrupole interaction of the form (Be 63, Be 66, Be 67)

$$H_{qq} = -\frac{1}{2} \chi \sum_{\mu \nu} q_{\mu \nu} (a_\mu \circ a_\nu + a_\mu \circ a_\nu)^2$$  \hspace{1cm} (2.10)

where the $q_{\mu \nu}$ are the matrix elements of an operator $f(r)Y_{20}(\hat{\rho})$ where usually $f(r) = r^2$, although other forms have also been used (As 72a, As 72B). This interaction can lead to collective particle-hole states which can be strongly excited in inelastic scattering, just as collective mass vibrations should be. The use of this interaction has proved quite successful when combined with various calculational techniques such as the random-phase approximations (RPA) and quasi-boson approximation. One general result of these calculations is that the structure of excited $J^m = 0^+$ states involves not just broken-pair configurations (particle-hole types) but also configurations involving the
excitation of zero-coupled pairs of nucleons (pairing type). Mikoshiba, et al. (Mi 67) published an early paper on the subject in which they attempted to calculate the interplay of the pairing configurations and the broken-pair configurations in the structure of excited $0^+$ states in rare earth nuclei. Their calculations indicated that excited $0^+$ states were not, in general simply beta vibrations (from the particle-hole mode) in the spirit of the hydrodynamic oscillation model, but much more complicated states arising from the interplay of the beta vibrational and pairing degrees of freedom. In fact, the precise character of excited $0^+$ states were found to be extremely dependent upon the underlying single-particle structure; thus no discussion of even highly collective states can ignore the single-particle microscopic structure.

2.2.4 **Coriolis Coupling**

One other residual interaction known to be important in explaining the details of one-neutron transfer reactions in tungsten nuclei, is the Coriolis force (Ca 71, Ca 72, Li 73, So 75). This term in the nuclear Hamiltonian, which is neglected in the strong coupling model, couples the single-particle degrees of freedom with the collective rotations (hence it is sometimes called rotation-particle coupling). Microscopically, the term has the effect of inducing interactions between states of the same $J^\pi$ but differing by one unit of $K$. In addition, the interaction has diagonal matrix elements for $K = 1/2$ bands leading
to deviations from the usual \( J(J + 1) \) rotational band energy spacings \((D_a 68)\). For simplicity, however, the effects of Coriolis coupling have been neglected in this study.
Chapter 3

TWO-NEUTRON TRANSFER REACTIONS:

A PROBE OF COLLECTIVE STRUCTURE
3.1 REACTION DYNAMICS

3.1.1 The Distorted-Wave Born Approximation (DWBA)

Currently used methods of calculating direct nuclear transfer differential cross sections fall into three rather distinct categories corresponding to increasing calculational and conceptual difficulty; the DWBA, the Coupled-Channels Born Approximation (CCBA), and the multiple-step sequential approximation. The simplest of the three is the DWBA in which the transfer process is broken into three distinct steps:

(1) The projectile and target nucleus approach each other under the influence of their Coulomb and nuclear interactions.

(2) The transfer takes place through a first-order process (e.g., in two-nucleon transfer, both nucleons are transferred at the same time, not sequentially).

(3) The outgoing particle and the residual nucleus move apart under the influence of their nuclear and Coulomb fields.

Within the DWBA, the differential cross section for the stripping reaction $A(a,b)B$ where $a = b + x$ and $B = A + x$ is given by:

$$
\frac{d\sigma}{d\Omega} = \frac{\mu_{a} \mu_{b}}{(2\pi h^2)^2} \frac{k_b}{k_a} \sum_{LSJM} |T_{LSJM}^{DW}|^2
$$

(3.1)

where

$$
T_{LSJM}^{DW} = \int d^3\rho \int d^3\rho' \psi_b(\rho') \Psi_a((-\rho') + [A/(A + 2)]\bar{R}) f_{LSJM}(\bar{\rho}, \bar{R})
$$

(3.2)
and \( \mu_a \) and \( \mu_b \) are the reduced masses of the \((A + a)\) and \((B + b)\) systems, respectively. \( k_a \) and \( k_b \) are the wave numbers of the projectile and outgoing particles, respectively, and \( \psi \) are the wavefunctions of the incoming (+) and outgoing (-) distorted waves. The coordinates employed are defined in Figure 3-1.

The total form factor \( f_{\text{LSJM}}(E, R) \) can be written

\[
f_{\text{LSJM}}(E, R) = \int d^3\xi \, d^3\zeta \, d^3r \, \psi_B^{*}(\xi, \mathbf{r}_1, \mathbf{r}_2, \zeta) \phi_b(\zeta) V_f(\zeta)
\]

where \( A, B, a, b \) represent all quantum numbers necessary to define the various nuclear states, \( \xi \) are all coordinates in addition to \( \mathbf{r}_1 \) and \( \mathbf{r}_2 \) for the nucleus \( B \), and \( \zeta \) are all coordinates in addition to \( \mathbf{r}_1 \) and \( \mathbf{r}_2 \) for nucleus \( a \).

After the suitable distorted waves \( \psi_a^{(+)} \) and \( \psi_b^{(-)} \) are found (see Section 3.1.2) the problem reduces to calculating the matrix element using Eq. 3.3. For \((p,t)\) or \((t,p)\) reactions, three calculational simplications are usually made:

1. The interaction \( V(\rho) \) is taken to be a scalar interaction \( V(\rho) = V(p) \).
2. Only the relative s-state of the two transferred nucleons in the triton is considered since the d-state contribution to the matrix element is known to be small for most (strong) transitions (Au 70).
3. The zero-range approximation is made. This has the physical meaning that both nucleons are transferred
Figure 3-1. Coordinate systems used in the DWBA calculations of two-nucleon transfer reaction $A(a,b)B$ where $a = b + n_1 + n_2$ and $B = A + n_1 + n_2$. The location of the various centers-of-mass are shown in the figure.
c.m. of $A + n_1 + n_2 = B$

□ c.m. of $b + n_1 + n_2 = a$

△ c.m. of $n_1 + n_2$

\[
(A/A+2) \bar{R} + \rho
\]

\[
\bar{R} + ((a-2)/a) \bar{\rho}
\]
together, being emitted from the nucleus $a$ and picked up by the nucleus $B$ at the same point. Mathematically, this has the effect of collapsing the six-dimensional integral into a three-dimensional integral. The use of the zero-range approximation for light-ion induced transfer reactions is well justified in the literature, the primary effect being an overall renormalization of strength of the calculated cross sections (Au 69).

Under these assumptions, the transition matrix element can be written

$$T_{LSJM_L} = (2L + 1)^{-1} \left< \psi_b^* (R) \phi_{LM_L} (R) \right| \psi_a (R) \right>(3.4)$$

where the two-neutron form factor is

$$\phi_{LM_L} (R) = i^{-L} \gamma_{LM_L} (R) U_L (R) \quad (3.5)$$

and

$$U_L (R) = \sum_{\nu \nu} K(\mu, \nu, L, S, J) B(A, B; \nu, \nu; J) U_{\mu \nu L} (R) \quad (3.6)$$

where $\mu$ and $\nu$ are single-particle basis states (usually spherical shell model states), $\gamma_{LM_L} (R)$ is a spherical harmonic, and $K$ is a $j-j$ to $1-s$ coupling transformation coefficient. The radial information for a given two-particle configuration $[\mu \otimes \nu]_L$ is contained in the "radial form factor" $U_{\mu \nu L} (R)$ while the $B$ coefficients contain the structural information of the state to be populated as well as angular-momentum coupling information. Since the $B$ coefficients can be related to the amplitude of the configuration $[\nu \otimes \nu]_L$ in nucleus $B$, they are known as "spectroscopic
amplitudes." In particular, it can be shown that

\[ B(A,B;\nu,\nu;J) = \langle \phi_B^{JM} \left\{ \frac{[a^+(\nu)a^+(\nu)]}{(1 + \mathcal{S}(\mu\nu))^{1/2}} |\phi_A\right\} \rangle \]

(3.7)

where the brackets \([\ ]\) and \(\{\}\) denote vector coupling.

Equations 3.1 and 3.6 illuminate an extremely important aspect of two-particle transfer reactions; \(T^{DW}\) is a sum over many terms \([u\otimes v]_L\) and since \(\sigma(\theta) \sim |T^{DW}|^2\), what is being probed is not the properties of the individual terms, but the correlation aspects of the nuclear states connected by the two-nucleon transfer.

Details of the methods used in this study to calculate the radial form factors will be given in Section 3.2.1 while specific equations for the spectroscopic amplitudes in several models will be given in Section 3.2.2.

3.1.2 The Optical Model

In order to calculate the distorted waves \(\psi(z)\) for input to the DWBA calculation, potentials must be chosen to describe the interaction between the projectile and target and between the ejectile and residual nucleus. The potentials employed in the present work include a real potential of depth \(V_0\) and shape given by a spherically symmetric Woods-Saxon well with radius \(R_0 = r_0A^{1/3}\) and diffuseness \(a\). Also used were spin-orbit potentials \((V_{so})\) of the form \(\tilde{L} \cdot \tilde{S}\) as well as the Coulomb interactions, each with their own radius and diffuseness parameters.

The removal of flux through nuclear reactions is simulated
by the use of imaginary potentials, giving rise to what is called the "optical model." Normally used are such potentials as a volume absorption \( W \) in the shape of a Woods-Saxon well and a surface absorption term \( W_D \) with the shape of the derivative of a Woods-Saxon well.

Since the interactions simulated by the optical model should be manifest in the elastic scattering angular distribution, the optical model parameters are normally chosen to fit the elastic scattering of the \( a + A \) system and the \( b + B \) system, at the appropriate bombarding energy. For the present study, the proton parameters were taken from the global set of Becchetti and Greenlees (Be 69) and the triton parameters as extracted from triton elastic scattering on a number of nuclei at 15 and 20 MeV bombarding energy (Be 71) (Table 3-1). The effects of substantial changes in the values of the optical model parameters upon the DWBA results were not investigated during the course of this study, but small changes (5-10\%) were found to have little effect upon the shapes of the DWBA angular distributions.

3.1.3 Multistep Processes

In certain circumstances, one or more nuclear reaction channels are so strong that they must be explicitly included in the reaction calculation, rather than simulated via the optical model. Such a circumstance occurs in strongly deformed nuclei, such as tungsten, where the members of the rotational bands are strongly coupled to each other via inelastic
### TABLE 3-1

**OPTICAL MODEL PARAMETERS**

<table>
<thead>
<tr>
<th></th>
<th>$V$ (MeV)</th>
<th>$r_0$ (fm)</th>
<th>$r_c$ (fm)</th>
<th>$a$ (fm)</th>
<th>$W$ (MeV)</th>
<th>$W_D$ (c) (MeV)</th>
<th>$r_I$ (fm)</th>
<th>$A_I$ (fm)</th>
<th>$V_{so}$ (MeV)</th>
<th>$r_{so}$ (fm)</th>
<th>$A_{so}$ (fm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w+p^{(a)}$</td>
<td>-57.0</td>
<td>1.17</td>
<td>1.25</td>
<td>0.75</td>
<td>-1.9</td>
<td>35.2</td>
<td>1.32</td>
<td>0.64</td>
<td>-24.8</td>
<td>1.01</td>
<td>0.75</td>
</tr>
<tr>
<td>$w+t^{(b)}$</td>
<td>-161.4</td>
<td>1.20</td>
<td>1.25</td>
<td>0.72</td>
<td>-25.0</td>
<td>-</td>
<td>1.30</td>
<td>0.84</td>
<td>-10.0</td>
<td>1.20</td>
<td>0.72</td>
</tr>
</tbody>
</table>

(a) Reference Be 69  
(b) Reference Be 71  
(c) Convention used in Reference Ku 69
scattering. In such a case the DWBA is no longer adequate to describe the transfer process since excited states may be populated not only by a direct particle transfer reaction, but also by inelastic scattering (or transfer) to an intermediate state, followed by transfer (or inelastic scattering). Some of the possible routes for populating members of the ground-state band are shown in Figure 3-2. The code utilized in the present work to calculate such multistep contributions was a version of the CCBA code LISA of Ascuitto (As 72a, As 72b) which includes inelastic couplings to all orders between members of a rotational band (the $0^+$, $2^+$, and $4^+$ members of the GSRB in this case) and to first-order the two-neutron transfer process (see Figure 3-2).

The explicit inclusion of inelastic excitations has an effect upon the elastic scattering angular distributions so that the optical model parameters used in a CCBA calculation should in general be different from those used in a DWBA calculation. Following reference As 72b, the optical model parameters used in this work are taken from proton and triton elastic scattering on spherical nuclei where CCBA effects are small. The inclusion of inelastic couplings when such spherical optical model parameters have been used has been shown to adequately reproduce the elastic and inelastic differential cross sections (Ki 73).

The third of the calculational procedures involves the inclusion of sequential particle transfer such as a $(p,d)$ reaction followed by a $(d,t)$ reaction and is currently of great interest.
Multiple step routes included in the calculations. The wavy lines represent inelastic couplings to all orders while the solid lines represent (p,t) transitions to first order only.
MULTIPLE-STEP ROUTES

(A)

(A-2)
(Pi 74, Sh 77, Tu 78). Such effects seem to be important primarily in cases where the probability of direct transfer to the final state is especially small (Se 75, Ig 72, Ba 70) and so except for very weakly excited states, the effect is usually ignored. The possibility exists, however, that the inclusion of sequential particle transfer effects will be found to be necessary to reproduce the absolute cross sections of even strong two-nucleon transfer transitions.
3.2 NUCLEAR STRUCTURE INPUT

3.2.1 Radial Form Factors

The calculation of the radial form factors $U_{\mu \nu L}(R)$ requires that a coordinate transformation be performed on the wavefunctions of the two transferred nucleons from a reference frame fixed in nucleus A to a frame fixed in nucleus B. A further transformation must be made to coordinates describing the motion of the center-of-mass of the two nucleons ($R$) and their relative motion ($\tilde{r}$). Two methods of accomplishing these transformations have been used in the present study. The code DEF2NT takes the single-particle wavefunctions (which have been calculated on a cylindrical harmonic oscillator basis) as fixed in B. and expands them on a spherical harmonic oscillator basis. The required transformation is then carried out analytically using Moshinsky brackets for each of the terms in the spherical harmonic oscillator expansion.

The other method employed was used in calculations with the two-nucleon transfer option in the DWBA code DWUCK (Ku 69). The single-particle wavefunctions were taken from Davidson (Da 68) as calculated for a deformed harmonic oscillator with deformation $\beta_2 = 0.3$ and the transformations performed numerically using the Bayman-Kallio method (Ba 67). In these calculations the individual single-particle levels were bound at one-half the experimental two-neutron separation energy ($S_{2N}$) so that no energy was allowed in the nucleons' relative motion.
3.2.2 Spectroscopic Amplitudes

The spectroscopic amplitudes contain the dependence of the transition amplitude upon the structure of the state populated in the residual nucleus. In this section, specific equations for these in the adiabatic approximation will be given for two types of states, members of the GSRB, and pure Nilsson two-hole states, as well as simplified expression for the differential cross section $\sigma(\theta)$.

(1) Ground State Band (Br 73)

$B(J_f M_f K_f, J_i M_i K_i; \nu_1 \nu_2; L) = B(JMK,000;\nu_1 \nu_2; L)$

$= (1 + \delta_{\nu_1 \nu_2})^{-\frac{1}{2}} (2J + 1)^{-\frac{3}{2}} \sum_{\omega > 0} [1 + (-)^{J + L + 1}]$\n
$x (-)^{J + \frac{1}{2} + \omega} <j_{1\omega_1} j_{2\omega_2} |j_0>$

$x \sum_{i} (U_{i}^{(A-2)} V_{i}^{(A)} W_{j_{1\omega_1}}^{i} W_{j_{2\omega_2}}^{i}) \delta_{J L}$ \hspace{1cm} (3.8)

where

$u \equiv (n,l,j,\omega)$ Shell Model State

$i \equiv (\omega[NN_{\Lambda}]$ Physical (Nilsson) state

and the $W_i$ are coefficients in the transformation

$a_+^{\mu} = \sum_{i} W_i^{\mu} a_i^{\mu}$ \hspace{1cm} (3.9)

The conceptually important information to note is that the contribution of each single-particle level "$i$" is weighted in Equation 3.8 by the factor $U_{i}^{(A-2)} V_{i}^{(A)}$ where the factor $U$ measures the emptiness of orbital $i$ in the ground-state of the residual
nucleus and $V$ measures the fullness of the orbital in the target nucleus ground state. This factor is peaked near the center of the Fermi surface (where $U_i = V_i$) so that the orbitals near the middle of the Fermi surface are the primary contributors to the population of the members of the ground-state band in $(p,t)$ reactions.

A simplified expression can be obtained for the ground-state cross section (Be 72) by making use of the so-called 0s approximation (Br 68, Br 71) and neglecting the difference in binding energy of the various orbitals:

$$
\sigma(J_y^+) = \left[ \sum_i \sqrt{\sigma_i(\theta)} \ u_i^{(A-2)} v_i^{(A)} \right]^2
$$

(3.10)

where $\sigma_i(\theta)$ is the differential cross section for the $L = J$ pickup of a neutron pair from orbital "i".

(2) For the case of pickup leading to a pure Nilsson two-hole state $(i)^{-1} (k)^{-1}$ of spin $J$ and intrinsic angular momentum projection $K$, the expression for the spectroscopic amplitude has been given by Garrett et al. (Ga 71):

**Pure Nilsson Two-Hole State**

$$
B(JMK,000;\mu_1\mu_2;L) = [2/(2J + 1) (1 + \delta_{K0})]^{\frac{3}{2}} \delta(K,\omega_i+\omega_k)
$$

$$
\times (<j_1^i\omega_1^i,j_2^i\omega_2^i|JK> W_{i1}^{\mu_1} W_{i2}^{\mu_2})
- <j_1\omega_k^i,j_2\omega_i^i|JK> W_{i1}^{\mu_1} W_{i2}^{\mu_2}) \delta_{\mu_1 \mu_2}
$$

(3.11)

The above expression makes it possible to calculate the differential cross section for the pickup of either a neutron pair.
(i = k) or of two non-paired neutrons (i ≠ k). Within the 0s approximation and neglecting Q-effects as before leads to:

\[ \sigma(J^+) = \left[ \sum_{i<k} \sqrt{\varphi_{ik}(\theta)} (a_{ik} V_i V_k + b_{ik} U_i U_k) \right]^2 \]  

where J is any state, whose structural information is contained in \( a_{ik} \) and \( b_{ik} \) and \( \varphi_{ik}(\theta) \) is calculated from Equation 3.11.

These expressions will be used later in the present work in discussions of the various models employed to systematize and explain the experimental data.
Chapter 4

DATA ACQUISITION AND REDUCTION
4.1 EXPERIMENTAL PROCEDURE

The protons used to induce the reactions discussed in this work were obtained from the Yale MP Tandem accelerator. Hydrogen ions from a duoplasmatron ion source were passed through a charge exchange canal and then accelerated to a kinetic energy of 21 MeV before momentum selection. In order to reduce slit scattering, the beam was defined by a slit of dimension 0.050" x 0.005" which was located immediately after the momentum-selecting 90° bending magnet, some 35 feet before the target. An anti-scatter roller slit of dimension 0.040" was placed about 8" before the target. The beam, of approximately 100 nA intensity, was focused and steered such that ~75% of it passed through a small hole (0.012" x 0.065") in a ZnS-coated screen placed at the geometrical center of the magnetic torus of the multiangle spectrograph (Ki 73, Ma 73). After focusing, the target replaced the screen and photographic emulsions were used to detect the reaction tritons after momentum analysis by the spectrograph. The approximate target thicknesses and the total charge accumulated for each experiment are listed in Table A-1. In each case, except where noted, data were taken at 23 angles, from 5° to 87.5° and from 92.5° to 167.5° in 7.5° steps.

The targets, with the notable exception of that of $^{180}$W, were fabricated by direct vacuum deposition of a sub-oxide of WO$_3$ onto 20 μg/cm$^2$ carbon backings. The separated-isotope WO$_3$ was obtained from Oak Ridge National Laboratory (ORNL) while the natural target was composed of high-purity natural WO$_3$. The
<table>
<thead>
<tr>
<th>Nominal Target Mass</th>
<th>Multigap Run No.</th>
<th>Total Charge Collected ($\times 10^3$µC)</th>
<th>Approximate Thickness of W ($\mu g/cm^2$)</th>
<th>Isotopic Composition (%)</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>180</td>
<td>126</td>
<td>30</td>
<td>50</td>
<td>&lt;0.05</td>
<td>Only 1st 6 angles</td>
</tr>
<tr>
<td>182</td>
<td>118</td>
<td>10.1</td>
<td>130</td>
<td>94.3</td>
<td>Only 1st 6 angles</td>
</tr>
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<td>183</td>
<td>132</td>
<td>20</td>
<td>70</td>
<td>3.5</td>
<td>Only 1st 6 angles</td>
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<tr>
<td>184</td>
<td>119</td>
<td>8</td>
<td>160</td>
<td>1.8</td>
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</tr>
<tr>
<td>186</td>
<td>129</td>
<td>12.5</td>
<td>120</td>
<td>0.5</td>
<td>Only 1st 6 angles</td>
</tr>
<tr>
<td>Natural</td>
<td>133</td>
<td>10</td>
<td>60</td>
<td>26.3</td>
<td>Only 1st 6 angles</td>
</tr>
</tbody>
</table>

* From spectrum estimated to be maximum of 2%

** Not seen in spectrum
sub-oxide of WO$_3$ was made by resistance heating the samples under vacuum in a tantalum boat until the samples changed from their usual bright yellow color to deep purple. During the subsequent evaporation of the sub-oxide onto the carbon backings, all but the central portions of the backings were masked to minimize heat-induced stresses. The isotopic analyses as given by ORNL for all of the targets are given in Table 4-1.

The target of $^{180}$W, an isotope which is only 0.13% naturally occurring, was made by J. G. Tracey of ORNL by direct deposition of metallic $^{180}$W on a 30 µg/cm$^2$ carbon backing in an isotope separator. This resulted in a remarkably isotopically pure target of metallic $^{180}$W with only a thin oxidized layer. The upper limit of 2% $^{182}$W contamination of the target listed in Table 4-1 was determined from the ($p$,t) results since no direct isotopic analysis was possible.

The photographic emulsions used for detecting the tritons were obtained from Ilford, Ltd. In view of the low ionizing power of tritons, the most sensitive emulsions available ($K$ 5) were used. The plates were developed in Kodak D5 developer for thirty minutes at a temperature of 5°C and fixed in Kodak Rapid Fixer with Hardener for two hours at 10°C. The reduced temperature during development and fixing gave good contrast between the tracks and the background, although longer developing and fixing times as well as stronger solutions than usual were required because of the decreased chemical activity. Additionally, for the $^{NAT}$W and $^{186}$W
target experiments, .005" thick acetate foils were placed in front of the emulsions during exposure to slow down the tritons in order to increase their energy loss in the emulsions and thereby make the tracks more evident.
4.2 DATA REDUCTION

After development, the photographic plates were scanned in 0.5mm strips by microscopists using a 20x microscope and dark-field illumination. The raw data were reduced with the computer program ALLPLOT which corrects for the spectrograph’s changing solid angle with plate position, performs a laboratory to center-of-mass conversion of the yields, and then plots and tabulates the data with respect to reaction Q-value. The conversion from distance along the plate to reaction Q-value requires that the magnetic field (B) and the plate distance (D) to radius-of-curvature (ρ) conversion for the gap in question be known. The D to ρ conversion has been measured and found to be independent of B to a good approximation (Ko 69). All of the results reported in this work were obtained using the latest D to ρ calibration, CALIBM. Since the fringe fields in each gap are different and, in general, unknown, an empirical approach - known as the effective field method - was employed to determine the effective magnetic field in each gap. For each gap, the plate position of a peak well separated from the others (usually the \( 0^+ \) or \( 2^+ \) states) was determined. Then, assuming a reaction Q-value for the peak, an effective field for the gap was determined and used to calculate the reaction Q-value corresponding to each bin, assuming the nominal reaction masses. In all cases, the effective field so calculated differed from the nominal value of the magnetic field by less than 0.1%.

Two main contaminant peaks were seen in the experiments.
These arose from the $^{170}(p,d)^{16}_{0}(g.s.)$ reaction with oxygen in the target and from the $^{13}C(p,d)^{12}C$ reaction with the carbon backing. In principle, incident deuterons or tritons of the same magnetic rigidity can be distinguished by the brightness of their tracks; in practice, however, this distinction is difficult and was judged not worth the extra scanning time. In no case, however, were the tritons confused with deuterons from the $W(p,d)$ reactions because of the great difference in magnetic rigidity of these two reaction products.

Representative spectra for the separated-isotope target experiments are shown in Figures 4-1 to 4-5 for laboratory angle of 20°. Peaks assigned to the various nuclei are numbered for later identification. In these spectra, the data are compressed into one millimeter bins and the yields of the reaction groups corresponding to the population of the ground-state band are reduced in magnitude by a factor of five. In each case the resolution was of the order of 20 keV, arising primarily from finite beam spot size (10 keV), straggling in the target (5 keV) and spectrograph aberrations.

In most cases the peak yields were determined by summing the counts as taken from the ALLPLOT output and subtracting the apparent background. Overlapping peaks were deconvoluted by eye or through the use of the Yale interactive version of the program AUTOFIT (Ma 73) which fits peak positions and heights with a reference peak shape taken from a selected peak in the measured
Figures 4-1 to 4-5. Triton spectra at a laboratory angle of 20° for the separated-isotope W(p,t) experiments. The data are shown in 1 mm bins. The peaks are numbered for identification purposes and correspond to the numbers in Tables 5-4 to 5-8 and to those in Appendix C.
$^{186}W(p,1)^{184}W$

$E_p = 21 $ MeV

$\Theta = 20^\circ$
$^{182}\text{W}(p,\gamma)^{180}\text{W}$

$E_p = 21 \text{ MeV}$

$\Theta = 20^\circ$
\(^{180}\text{W}(p,t)^{178}\text{W}\)

\(E_p = 21\) MeV

\(\Theta = 20^\circ\)
spectrum. In particular, most of the yields for the high-lying states in the complicated $^{180}\text{W}$ spectra were analyzed with AUTOFIT. The errors quoted in the differential cross sections reflect statistical errors as well as approximate fitting errors where appropriate, but do not include effects due to scanning reproducibility, estimated good to 5%, not possible errors in the absolute cross sections.
4.3 **ABSOLUTE CROSS SECTION MEASUREMENTS**

The absolute cross sections for the $^{184, 182, 180}_{\text{W}}(p,t)$ reactions were measured in a separate experiment which utilized a solid state detector-telescope and on-line computer data acquisition to simultaneously detect and isolate elastically scattered protons and reaction tritons. Proton elastic scattering angular distributions were measured for each target at a bombarding energy of 21 MeV to establish an absolute cross section scale (see Section 5.1) and then both triton and proton spectra were taken at various lab angles. The $0^+$ and $2^+$ members of the ground state band were unresolved in these experiments, so their summed yield was compared to the elastic proton yield to establish the absolute $(p,t)$ summed cross section.
Chapter 5

PRESENTATION OF DATA
5.1 Q-VALUES

The (p,t) experiment on the natural-tungsten target provided accurate relative Q-values for the lowest \( L = 0 \) transitions for the \( ^{186,184,183,182}\text{W}(p,t) \) reactions, shown in Table 5-1. The values obtained assume a Q-value of \(-4.470\text{ MeV}\) for the \( ^{186}\text{W}(p,t)^{184}\text{W}(g.s.) \) transition (Ma 65) and are believed to be accurate to within \( 5\text{ keV} \) relative to one another. The \( L = 0 \) transition in the \( ^{183}\text{W}(p,t)^{181}\text{W} \) reaction is to the \( \frac{1}{2}^- \) [510] level in \( ^{181}\text{W} \), which occurs at an excitation energy of \( 458\text{ keV} \) (Da 71). Thus the Q-value to the ground state in \( ^{181}\text{W} \) is determined to be \(-5.769\text{ MeV}\), in disagreement with the value of \(-5.810\text{ MeV}\) quoted in Reference 00 72 but in excellent agreement with the value of \(-5.770\text{ MeV}\) appearing in the 1965 Mass Table (Ma 65).

As \( ^{180}\text{W} \) is so rare in nature, the ground-state Q-value for the \( ^{180}\text{W}(p,t)^{178}\text{W}(g.s.) \) reaction could not be obtained from the natural-tungsten target experiment (predicted yield \( \leq 10 \) counts). The \( ^{13}\text{C}(p,d)^{12}\text{C}(g.s.) \) peak in the spectra from the separated-isotope experiment was therefore used to determine the effective field and then the Q-value of the ground-state transition to \( ^{178}\text{W} \) determined to be \(-6.844 \pm 0.015\text{ MeV}\).
### TABLE 5-1

(p,t) Q-VALUES FOR LOWEST L = 0 TRANSITIONS

<table>
<thead>
<tr>
<th>Residual Mass</th>
<th>Q&lt;sup&gt;(a)&lt;/sup&gt; (MeV)</th>
<th>Previous Values (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>184</td>
<td>-4.470</td>
<td>-4.470</td>
</tr>
<tr>
<td>182</td>
<td>-5.107</td>
<td>-5.120</td>
</tr>
<tr>
<td>181</td>
<td>-6.227</td>
<td>-7</td>
</tr>
<tr>
<td>180</td>
<td>-6.247</td>
<td>-6.271</td>
</tr>
<tr>
<td>178</td>
<td>-6.844 ± .015</td>
<td>-6.843</td>
</tr>
</tbody>
</table>

(a) Assuming Q = -4.470 for 186<sup>W</sup> (p,t) 184<sup>W</sup> (0<sup>+</sup>), errors ±.005 MeV unless indicated.

(b) Reference Ma 65

(c) Reference Oo 73

(d) Reference Ca 76
5.2 **RELATIVE AND ABSOLUTE CROSS SECTION SCALES**

Table 5-2 lists the relative differential cross sections for the lowest \( L = 0 \) transitions as determined from the natural target experiment. Only statistics and scanning reproducibility introduce uncertainty into the relative cross section values for the \(^{186}\text{W}(p,t)\) and \(^{184}\text{W}(p,t)\) reactions. The relative triton yields from the \(^{183}\text{W}\) and \(^{182}\text{W}\) in the target are less certain due to their nearly equal \( Q \)-values. These differ by only 20 keV while the experimental resolution was 25 keV, necessitating separation of the two peaks out of the composite, using detailed peak fitting routines.

The absolute cross section scales for the \(^{184},^{182},^{180}\text{W}(p,t)\) reactions were determined using the proton scattering method described in the previous chapter. Figure 5-1 shows the measured proton elastic scattering angular distribution on \(^{180}\text{W}\) as well as the normalized optical model prediction using the proton parameters of Table 3-1. The agreement is completely satisfactory. Table 5-3 lists the results for each target so studied; also included are the values of \( \text{FACTOR} \) adopted for each multigap run, where \( \sigma(0) \, (\mu b/sr) = \text{(yield)}/\text{FACTOR} \). The quoted errors for \( \text{FACTOR} \) are all estimated absolute errors and vary between 10% and 15%.
### TABLE 5-2

**RELATIVE CROSS SECTIONS FOR LOWEST L = 0 TRANSITIONS FROM $^{\text{nat}}_{\text{W}}(p,t)$**

<table>
<thead>
<tr>
<th>Residual Mass</th>
<th>Yield @ 20°</th>
<th>$\frac{d\sigma}{d\Omega}$ 20°</th>
<th>Yield @ 27°</th>
<th>$\frac{d\sigma}{d\Omega}$ 27°</th>
</tr>
</thead>
<tbody>
<tr>
<td>184</td>
<td>1163 ± 34</td>
<td>1.12 ± .04</td>
<td>2267 ± 48</td>
<td>1.27 ± .04</td>
</tr>
<tr>
<td>182</td>
<td>1112 ± 34</td>
<td>1.00</td>
<td>1915 ± 44</td>
<td>1.00</td>
</tr>
<tr>
<td>181</td>
<td>326 ± 30</td>
<td>0.29 ± .04</td>
<td>355 ± 40</td>
<td>0.40 ± .05</td>
</tr>
<tr>
<td>180</td>
<td>1160 ± 40</td>
<td>0.57 ± .08</td>
<td>1748 ± 40</td>
<td>1.07 ± .10</td>
</tr>
</tbody>
</table>
Figure 5-1. Elastic scattering angular distribution for 21 MeV protons on $^{180}$W plotted as ratio to Rutherford. The error bars are smaller than the data points. The calculation is an optical model prediction using the proton parameters of Table 3-1 and was normalized to the data.
$\sigma/\sigma_{\text{RUTHERFORD}}$

$^{180}\text{W}(p,p)$

$E_p = 21$ MeV
### TABLE 5-3

**ABSOLUTE \((p,t)\) CROSS SECTIONS**

<table>
<thead>
<tr>
<th>Residual Mass</th>
<th>(d\sigma/d\Omega) ((0^+ + 2^+)) ((\mu)b/sr)</th>
<th>Multigap</th>
<th>Run No.</th>
<th>Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>184</td>
<td>-</td>
<td>129</td>
<td>14.9 ± 1.8</td>
<td></td>
</tr>
<tr>
<td>182</td>
<td>- 322 ± 32</td>
<td>119</td>
<td>12.9 ± 1.3</td>
<td></td>
</tr>
<tr>
<td>181</td>
<td>-</td>
<td>133</td>
<td>23.9 ± 3.6</td>
<td></td>
</tr>
<tr>
<td>180</td>
<td>- 181 ± 14</td>
<td>118</td>
<td>13.8 ± 1.8</td>
<td></td>
</tr>
<tr>
<td>178</td>
<td>809 ± 75</td>
<td>126</td>
<td>6.7 ± 1.0</td>
<td></td>
</tr>
</tbody>
</table>
5.3 SPECTROSCOPIC RESULTS

5.3.1 General Comments

The spectroscopic results for each of the individual experiments will now be discussed with particular attention to the ground-state bands, the two rotational bands presumably built on the quadrupole mass vibrations (the beta \( K^\pi = 0^+ \) and gamma \( K^\pi = 2^+ \) bands) and the octupole vibrational band \( (K^\pi = 2^-) \). The various states populated are enumerated in Tables 5-4 through 5-8 and the complete set of angular distributions for these states are given in Appendix C.

Spin assignments are made to low-lying states by the comparison of their excitation energies with previous level scheme determinations and by comparisons of their angular distribution shapes with those of levels of known spin. For \( 0^+ \) states, the highly characteristic oscillatory angular distributions make their spin assignment unambiguous, even for weakly populated states. Other than the above exceptions, no attempt has been made to assign spins, as the angular distribution for \((p,t)\) reactions on deformed nuclei are not characteristic of the transferred angular momentum due to complications reflecting the presence of multistep processes.

The general features of the spectra can be seen to change smoothly but markedly with mass number. Whereas the \( \text{^{186}W(p,t)^{184}W} \) spectra show many rather strongly populated states between 1 and 3 MeV excitation, the \( \text{^{180}W(p,t)^{178}W} \) reaction leads to much less excited state strength. This feature can be understood as arising
from two factors, the effect of the Coulomb barrier and the effect of changing angular momentum matching conditions. The Coulomb barrier for tritons on tungsten is about 12 MeV, which corresponds to an excitation of 2.5 MeV in $^{178}$W, 2.7 MeV in $^{180}$W and still higher in the higher mass isotopes. Thus, the reduction in overall population of excited states for the low-mass tungstens can be understood as a barrier effect. In addition, angular momentum matching also plays a role. The best matching for $^{178}$W at 1 MeV excitation is $L = 0$, and increases by approximately one unit of angular momentum for each MeV more positive Q-value. A value of $L = 3$ is reached for the $^{186}$W(p,t)$^{184}$W(g.s.) transition. Thus, higher spin states ($J \geq 3$) should be more strongly populated in the higher mass isotopes.

To investigate the overall effects of these two factors, typical-configuration (the (2f 7/2)$^2$ configuration) two-particle pickup DWBA calculations were performed for the $^{182}$W(p,t)$^{180}$W reaction for various Q-values, and angular momentum transfers of 0 and 2 units. These calculations utilized the code DWUCK (Ku 69) and the optical model parameters of Table 3-1 with bound-state radius parameters; $r_0 = 1.25$ fm and $a = 0.60$ fm. The angular distribution shapes so calculated were found to change only slightly with Q-value while their magnitudes varied considerably. Figure 5-2 presents the calculated relative cross sections for $L = 0$ and 2 transitions as a function of Q-value, both normalized to 1 at $Q = -6.271$ MeV. The positions of the various ground-state transitions on the Q-value scale are shown for orientation as well as the
Relative strengths of (p,t) reactions for transferred angular momentum 0 and 2 as a function of reaction Q-value. The calculations employed the optical parameters of Table 3-1 and a typical-configuration form factor \((2f_7/2)^2\). Also shown are the Q-values for the various ground-state (p,t) transitions.
Q-VALUE DEPENDENCE OF (p,t) TRANSITIONS

RELATIVE STRENGTH

Q-VALUE (MeV)

-4 -5 -6 -7 -8 -9

L=0  DIRECT ROUTES ONLY

L=2
Q-value corresponding to the triton having only the Coulomb separation energy. The figure demonstrates quantitatively the results previously discussed qualitatively.

5.3.2 The Nucleus $^{184}\text{W}$

The spectrum for the $^{186}\text{W}(p,t)^{184}\text{W}$ reaction taken at a laboratory angle of 20° is shown in Figure 4-1 where the peak numbers correspond to those in the table of excitation energies, Table 5-4 and to the angular distributions in Appendix C-1. Only the low-lying states of $^{184}\text{W}$ were investigated, up to an excitation energy of 1.2 MeV, although a complete spectrum is shown. A search was made for $J^\pi = 0^+$ states excited with greater than 10% of the ground-state strength below an excitation energy of 3 MeV. Only the known state at 995 keV fitted these criteria.

5.3.3 The Nucleus $^{182}\text{W}$

The spectrum for the $^{184}\text{W}(p,t)^{182}\text{W}$ reaction taken at a laboratory angle of 20° is shown in Figure 4-2 where the peak numbers correspond to those in Table 5-5 and to the angular distributions in Appendix C-2. All peaks below 3 MeV excitation were analyzed and angular distributions extracted. Doublets were unfolded by eye where possible, or with the program AUTOFIT where necessary. The ground-state rotational band (GSBR) is strongly in evidence as well as the bandhead of the well-known first excited $K^\pi = 0^+$ band (#4), the $J^\pi = 2^+$ bandhead (#5) and the $4^+$ member (#8) of the $K^\pi = 2^+$ gamma band, and the $3^-$ member of the $K^\pi = 2^-$ octupole band (#7). The $2^+$ member of the first-excited $K^\pi = 0^+(0^+_1)$ band (#6)
is found at a higher excitation energy than the $\gamma$ and is considerably more weakly excited.

Besides the well established $0^+$ state at 1138 keV, three other $0^+$ states were located, at 2520 keV (#27), 2552 keV (#28) and an especially strongly populated one at 2725 keV (#31). The assignment for the 2520 keV state is somewhat uncertain as the peak appears to be an unresolved doublet.

5.3.4 The Nucleus $^{181}$W

A sample spectrum for the $^{183}\text{W}(p,t)\,^{181}\text{W}$ reaction taken at a laboratory angle of 20° is shown in Figure 4-3. The identifying peak numbers correspond with those in Table 5-6 and the angular distributions in Appendix C-3. All states below 2 MeV excitation were analyzed.

The most prominent peak in the spectrum corresponds to a strong $L = 0$ transitions (#5). This transition presumably leads to the $J^\pi = \frac{1}{2}^-$ bandhead of the band built on the $\frac{1}{2}^- - [510]$ intrinsic state ($\frac{1}{2}^- - [510]$) since the odd neutron of the $^{183}\text{W}$ ground state is assigned to that single-particle state (Ar 66). Based on the level schemes of References Da 71 and Ca 72, the $\frac{1}{2}^- - [510]$ state at 458 keV should not be completely resolved from the $3/2^-$ state at 450 keV and barely resolved from the $7/2^-$ state at 476 keV. The highly oscillatory nature of the angular distribution, however, unambiguously identifies peak #5 as due to an $L = 0$ transition and not $L = 2$ or $4$ transition which would be required for populating the $3/2^-$ and $7/2^-$ states, respectively. The maximum possible contribution
from the $3/2^-$ and $7/2^-$ states to the value of the differential cross section at the second maximum of peak #5 may be estimated by noting that they would fill in the minimum in the angular distribution at 12.5°, where the differential cross section is 4.2 $\mu$b/sr. The two states are then seen to add at most 5 $\mu$b/sr to $\sigma(\theta)$ at the second maximum of peak #5 where $\sigma(\theta) = 280$ $\mu$b/sr, a negligible contribution. Besides the $L = 0$ transition to the bandhead of the $\frac{1}{2} - [510]$ band, $L = 2$ transitions to the next two members of its rotational band are seen, the $3/2^-$ and $5/2^-$ being peaks #7 and #8 respectively.

Conspicuously absent from the spectrum is a strong peak corresponding to an $L = 0$ transition to the $\frac{1}{2} - [521]$ state, which would be a doublet with the $L = 4$ transition to the $7/2^+ 7/2^- [514]$ state. Peak #4 corresponds to the correct excitation energy for this doublet, but its angular distribution shows very little oscillation, requiring that its $L = 0$ contribution be small.

Assuming that the odd $\frac{1}{2}^-$ neutron is only weakly coupled to the gamma vibration, the pickup strength of the $2_2^+$ should split between two states, a $3/2^-$ and a $5/2^-$. These states would be formed by the two possible couplings of the angular momentum of the odd particle with that of the gamma vibration. They should also occur at approximately the same excitation energy above the $\frac{1}{2} - [510]$ state, as the $2_2^+$ is above the ground states of neighboring even-mass nuclei. Such states are not seen. Instead, a strong $L = 2$ transfer to the $5/2^+ 5/2^- [512]$ state at 365 keV (#3) is in
evidence with a strength comparable to that of the gamma vibrational bandheads in neighboring even-mass tungsten nuclei. The implications of these observations will be discussed in section 8.2.

A strong $L = 0$ transition is located at an excitation energy of 1864 keV and is interpreted as arising from a transition to a state formed by the coupling of the odd neutron to the excited $0^+$ state in $^{180}$W at 1516 keV. The occurrence of another state at 1892 keV with a very forward-peaked angular distribution required that the two peaks be fitted out of the composite, resulting in a somewhat larger uncertainty in the values of the experimental differential cross sections at forward angles.

5.3.6 The Nucleus $^{180}$W

Table 5-7 lists the states populated in the $^{182}$W($p,t$) reaction where the identifying peak numbers are consistent with the designations in the sample spectrum (Figure 4-4) and the angular distributions in Appendix C-4. Considerable emphasis was placed upon the extraction of spectroscopic information since the structure of $^{180}$W previously had not been extensively probed with particle transfer reactions and its level scheme not well established (Gr 75). The $J^{\pi} = 2^+$ state reported at 1118 keV in a ($d,d'$) experiment (Gu 71) is consistent with our observation of a state at 1120 keV (#9) with an angular distribution similar to that for the known $J^{\pi}K = 2^+2$ state in $^{180}$W at 903 keV and is therefore assigned $J^{\pi}K = 2^+2$. The state at 1321 keV (#12) is similarly identified with one member of a doublet seen in the ($d,d'$) study.
at 1319 keV and is assigned $J^P = 4^+ 2$ on the basis of a comparison with the $^{186}\text{W}(p,t)^{184}\text{W}(4^+ 2)$ transition angular distribution shape. The 1083 keV state has an angular distribution similar to that of the $3^-$ state populated in the $^{186}\text{W}(p,t)^{184}\text{W}$ reaction and has been previously identified (Gu 71).

Four non-$L = 0$ transitions corresponding to excitation energies in $^{180}\text{W}$ of 746 keV (#4), 831 keV (#5), 1012 keV (#6) and 1034 keV (#7) were observed at the limit of detection, about 2 $\mu$b/sr peak cross section. Due to the difficulties of observation, however, their excitation energies are assigned larger uncertainties ($\pm 10$ keV) and in fact their assignment as states in $^{180}\text{W}$ must be considered tentative. No evidence for the $0^+ 0$ state suggested in Reference Gr 66 to lie at 903 keV was found. In addition, the $(d,d')$ work on $^{180}\text{W}$ reported no excited $0^+ 0$ or $2^+ 0$ states, although the present work shows a very strongly populated $0^+ 0$ state at 1516 keV excitation, with an associated $2^+$ band member at 1589 keV. Also, three other weakly excited $0^+$ states were located, at 1695, 1824, and 2356 keV.

5.3.7 The Nucleus $^{178}\text{W}$

The $^{13}\text{C}(p,d)^{12}\text{C}(g.s)$ peak from the target backing obscures the spectrum above 2 MeV excitation at forward angles, as shown in the sample spectrum in Figure 4-5. The states below 2 MeV excitation energy populated in the reaction are enumerated in Table 5-8 and their angular distributions displayed in Appendix C-5. Within this small excitation region, three excited $0^+$ states were located. The lowest at 997 keV (#4) is the most strongly
populated with 9% of the ground state yield. The others, at 1356 keV (#8) and 1643 keV (#11) excitation energy, are populated with 3% and 1% of the ground state yield, respectively. The assignment of 0+ for the 997 keV state is consistent with the suggestion in References Ga 70 and Ca 76 of a 0+ state at 1001 keV; the 2+ and 4+ members of the rotational band also agree with the previous assignments. As discussed in the above references, the 3− member of the octupole band lies only 9 keV from the 2+, resulting in a composite peak (#6) which was indeed wider than the other peaks in the spectrum.
TABLE 5-4  
LEVELS POPULATED IN $^{186}W(p,t)^{184}W$

<table>
<thead>
<tr>
<th>State #</th>
<th>$J^\pi_K$ (a)</th>
<th>Excitation Energy (b) (keV)</th>
<th>Previous Experiments (a) (keV)</th>
<th>$d\sigma/d\Omega$ MAX (ub/sr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$0^+0$</td>
<td>0</td>
<td>0</td>
<td>740</td>
</tr>
<tr>
<td>1</td>
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<td>109</td>
<td>111</td>
<td>144</td>
</tr>
<tr>
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<td>$4^+0$</td>
<td>364</td>
<td>364</td>
<td>25</td>
</tr>
<tr>
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<td>$6^+0$</td>
<td>744</td>
<td>748</td>
<td>5 (c)</td>
</tr>
<tr>
<td>4</td>
<td>$2^+2$</td>
<td>901</td>
<td>903</td>
<td>37</td>
</tr>
<tr>
<td>5</td>
<td>$0^+0$</td>
<td>995</td>
<td>1002</td>
<td>57</td>
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<tr>
<td></td>
<td>$3^+2$ (d)</td>
<td></td>
<td>1006</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>$2^+0$</td>
<td>1121</td>
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<td>22</td>
</tr>
<tr>
<td></td>
<td>$2^-2$ (d)</td>
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<td></td>
</tr>
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<td></td>
<td>$4^+2$</td>
<td></td>
<td>1134</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>$3^-2$</td>
<td>1214</td>
<td>1221</td>
<td>8</td>
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</tbody>
</table>

(a) From Reference Ki 73 unless otherwise indicated
(b) Excitation energies ±10 keV
(c) Upper limit
(d) Strengths expected to be vanishingly small for unnatural parity states
<table>
<thead>
<tr>
<th>State #</th>
<th>$J^E_K$</th>
<th>Excitation Energy (keV)</th>
<th>Previous Experiments (keV)</th>
<th>$\frac{d\sigma}{d\Omega}_{MAX}$ (µb/sr)</th>
</tr>
</thead>
<tbody>
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<td>0</td>
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</tr>
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<td>100</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>$4^+$</td>
<td>330</td>
<td>329</td>
<td>26</td>
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<tr>
<td>3</td>
<td>$6^+$</td>
<td>690</td>
<td>680</td>
<td>6</td>
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<tr>
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<td>$0^+$</td>
<td>1135</td>
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<td>$2^+$</td>
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<td>7</td>
<td>$3^-$</td>
<td>1372</td>
<td>1374</td>
<td>7</td>
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<tr>
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<td>$(5)^-$</td>
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<td>5</td>
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<td>$1810^d$</td>
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<td>1824</td>
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<td>$2^+$</td>
<td>1853</td>
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<td>1960.4</td>
<td>20</td>
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<td>16</td>
<td>$(5,6)^-$</td>
<td>1961</td>
<td>1960.9</td>
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</tr>
<tr>
<td>21</td>
<td>$(3^-)$</td>
<td>2209</td>
<td>2209</td>
<td>15</td>
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<tr>
<td>22</td>
<td></td>
<td>2251</td>
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<td>4</td>
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<td>23</td>
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<td>2278</td>
<td>$2274^d$</td>
<td>14</td>
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<tr>
<td>24</td>
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<td>2311 D</td>
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**TABLE 5-5**

Levels populated in $^{184}_W(p,t)^{182}_W$
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<th>Excitation Energy (b) (keV)</th>
<th>Previous Experiments (a) (keV)</th>
<th>dE/dΩ)_{MAX} (ub/sr)</th>
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<tbody>
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(a) Assignments and excitation energies from Nucl. Data Sheets 14 (1975) unless otherwise indicated.

(b) Excitation energies ±5 keV below 2 MeV, ±10 keV above 2 MeV

(c) Assignment this work.

(d) From references Gu 71, Ki 73, Je 77

D Doublet

M Multiplet
<table>
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<th>State #</th>
<th>$J^\pi K[NN_z\Lambda]$ (a)</th>
<th>Excitation Energy (keV)</th>
<th>Previous Experiments (keV)</th>
<th>$d\sigma/d\Omega_{\text{MAX}}$ (ub/sr)</th>
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<td>$11/2 + 9/2[624]$</td>
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<td>113</td>
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(a) From Reference Ca 72 unless otherwise indicated
(b) Calculated assuming 5/2 - [512] state at 365 keV excitation, error ±5 keV
(c) Assignment this work
(d) From Reference Ca 71
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TABLE 5-7 (cont'd)
LEVELS POPULATED IN $^{182}W(p,t)^{180}W$

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<th>Previous Experiments (keV)</th>
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<td>32</td>
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<td>2400</td>
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(a) From reference Gr 75 unless indicated
(b) Error ±5 keV below 2 MeV, ±10 keV above 2 MeV
(c) Assignment this work
(d) Doublet with $^{184}W (p,t) ^{182}W$ (1135 keV)
(e) States near lower limit of observation, error ±10 keV.

D Doublet
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<th>Previous Experiments (keV)</th>
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<td>6</td>
<td>(2$^+2 + 3^−2$)</td>
<td>1117 D</td>
<td>(1111 + 1120)</td>
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(a) From reference Ca 76 unless otherwise indicated

(b) Excitation energies ±5 keV

(c) Assignment this work

(d) Suggested in Reference Ca 76

D Doublet

M Multiplet
Chapter 6

SYSTEMATICS OF COLLECTIVE STATES:

GROUND-STATE BANDS
The next three chapters are devoted to the systematic properties of the major bands, with particular attention being paid to the extraction of microscopic structural information through the study of such systematics. Chapter 6 deals with transitions between members of the same superfluid band; the ground state bands in the even-even nuclei and the "favored" band in the odd-mass nucleus case where the one quasiparticle is left undisturbed. Chapter 7 then explores the systematics of the excited $K^\pi = 0^+$ bands while Chapter 8 contains a discussion of the gamma vibrational band.
6.1 ANGULAR DISTRIBUTION SHAPES FOR MEMBERS OF THE GROUND-STATE BAND

Figure 6-1 displays the angular distributions for transitions to the ground-state rotational bands (GSRB). The shapes of the angular distributions for ground-state transitions are in each case very similar while the shapes of the $L = 2$ transitions vary considerably, although smoothly, with mass. In contrast, as discussed in the previous chapter, one-step DWBA calculations predict similar shapes for all masses. It has been shown, however, (As 72b, Ki 72, Ki 73) that the inclusion of inelastic excitation effects in the transfer process is necessary to describe the relative magnitudes and shapes of the angular distributions for transitions to the members of the GSRB and $K^\pi = 0^+_1$ band in the $^{186}_{\mathrm{W}}(p,t)^{184}_{\mathrm{W}}$ reaction at 18 MeV bombarding energy. During this study, a similar calculation was performed for the $^{182}_{\mathrm{W}}(p,t)^{180}_{\mathrm{W}}$ reaction at 21 MeV according to the procedure outlined in section 3.1.3. The results are shown in Figure 6-2 as solid lines and the DWBA predictions as dotted lines. Only one normalization constant was used for all of the curves. The shape and magnitude of the ground-state angular distribution is described equally well by both the DWBA and CCBA calculations, indicating that multistep inelastic effects for strong $L = 0$ transitions are minimal. The slow drop of the $2^+_g$ angular distribution from $0^\circ$ to $50^\circ$ is not reproduced by the DWBA, but is well described by the CCBA calculation. This lack of oscillatory structure in $(p,t)$ transitions to $2^+_0$ states seems to be a general feature in
Figure 6-1. Angular distributions for the $0^+$, $2^+$, and $4^+$ members of the ground-state rotational band for $^{186, 184, 182, 180}_{\text{W}}$($p, t$) reactions at $E_p = 21$ MeV. The error bars contain statistical errors only and are smaller than the data points if not shown.

Figure 6-2. Angular distributions for the $0^+$, $2^+$, and $4^+$ members of the GSRB for the $^{182}_{\text{W}}$($p, t$)$^{180}_{\text{W}}$ reaction at $E_p = 21$ MeV. The CCBA predictions are shown by the solid lines where only one normalization constant was used. The DWBA predictions, similarly normalized, are shown as dotted lines. Both calculations employed the optical parameters of Table 3-1 and form factors as described in the text.
the rare earth region (e.g. As 72b, Ki 73).

One obvious deficiency of the CCBA calculation is the over-prediction of the magnitude of the $^{2+}_g$ angular distribution. We find, however, that the magnitude depends sensitively upon the value of the pairing matrix element, which was chosen so that $A = P_N$. Thus, in the presence of a large energy gap in the single-particle spectrum, there can be a rather sensitive dependence of this magnitude on the energy of only a few orbitals near the middle of the Fermi surface. Further, in the nuclear structure calculation, since the deformation parameters of $^{180}$W are not known, the same values as for $^{182}$W were used. In light of these facts, the results presented here are considered satisfactory.

The ability of the calculation to reproduce the shapes of the $^{2+}_g$ angular distributions in both the $^{186}$W(p,t)$^{184}$W and the $^{182}$W(p,t)$^{180}$W reactions is very gratifying, yet the reproduction of the data without physical insight into the cause of the changing shapes is not enough. This insight was gained through studies of $(^{12}$C, $^{14}$C) reactions on the $A = 186, 184,$ and 182 isotopes of tungsten by Hanson et al. (Er 74, As 75, Ha 75, Ha 76) where the shapes of the $^{2+}_g$ angular distributions were also observed to change with target mass. These data, in conjunction with detailed CCBA analyses, showed that although the strengths of the multiple-step transitions remained rather constant through the tungsten isotopes, the probability of the direct $L = 2$ transfer changed considerably. The direct route was found to be weak for the
\( ^{186}W(p,t)^{184}W(2^+_g) \) transition due to an efficient cancellation between the contributions from the various orbitals near the Fermi surface (the contributions of each orbital being weighted by a UV factor as described in Section 3.2.2). As the target mass decreases, the direct quadrupole transfer strength increases, although staying small, as different orbitals become important in the transfer process due to the changing UV factors. The net effect thus, is a changing ratio of (two-step route to \( 2^+_g \))/(direct route to \( 2^+_g \)). Since these two amplitudes add coherently and have different angular dependences, the result is changing angular distribution shapes as a function of mass.
6.2 ANALOG TO GROUND-STATE TRANSITION: ODD MASS TARGET

6.2.1 Angular Distribution Shapes

In the adiabatic approximation, the wavefunction of an odd-mass nucleus is $|JM_K> = D_{MK}^J|K>$ where the intrinsic wavefunction in the BCS theory is given by $|K> = a_K^+|\phi_{BCS}>$. Such states are known as "one quasiparticle states" since their structure is described as one quasiparticle in orbit $K$, coupled to the BCS vacuum $\phi_{BCS}$ (which is also the quasiparticle vacuum). The spectroscopic amplitudes $B$ as given in Equation 3.8 then depend upon the "intrinsic form factor" $<K|A^+_\nu|K'>$ (where $A^+_\nu$ is a two-quasiparticle creation operator) which now becomes

$$<K|A^+_\nu|K'> = <\phi (A + 2)|a_K^+ a_{K'}^+|\phi_{BCS}(A)>$$  \hspace{1cm} (6.1)

Under the assumption that $A^+_\nu$ operates only on the core ($\phi_{BCS}$) and not on the quasiparticle in orbits $K$ and $K'$, then

$$<K|A^+_\nu|K'> = <\phi (A + 2)|A^+_\nu|\phi_{BCS}(A)\delta_{K, K'}$$  \hspace{1cm} (6.2)

where the first factor is recognized as the intrinsic form factor for the neighboring even-mass target nucleus transition. Thus, within the model, there should exist "favored" $(p, t)$ transitions in odd-mass nuclei analogous to transitions in the neighboring even-mass nuclei, but with the slight added complication of the odd quasiparticle. Angular distributions for such transitions from the $^{183}$W ground state, the one quasiparticle (q.p.) $\frac{1}{2} - [510]$ state, to the one q.p. $\frac{3}{2} - [510]$ state in $^{181}$W via $L = 0$, and via $L = 2$ to the $3/2$ and $5/2$ members of the rotational band are
shown in Figure 6-3. The angular distributions for \( ^{182}W \rightarrow ^{180}W \) transitions to the \( 0^+ \) and \( 2^+ \) members of the GSRB are also shown for comparison.

Both the \( 3/2 \) and \( 5/2 \) members of the \( \frac{1}{2} - [510] \) band can be reached via direct \( L = 2 \) transitions since the angular momentum of the odd quasiparticle can couple with \( L = 2 \) to give \( 3/2 \) or \( 5/2 \). Thus, the quadrupole transfer strength should split between these two states. In the adiabatic limit, the splitting follows the rule

\[
\frac{\sigma(\theta) (3/2^-)}{\sigma(\theta) (5/2^-)} = \frac{2}{3}
\]

at all angles (see Appendix B). Experimentally, although the summed cross sections for the first six angles give the correct ratio \( (3/2^-) \) to \( (5/2^-) \) of \((132 \pm 6) : (184 \pm 8) = (2.15 \pm .14) : 3\), the shapes of the two angular distributions are not in fact the same, having very different behavior at forward angles (Figure 6-3). Such a difference can be attributed to different multiple-step routes populating the two states. This extremely interesting, but complicated, situation will be more fully explored in Appendix F.

### 6.2.2 Blocking

One test of the assumption behind Equation 6.2, that the presence of the odd particle does not modify the intrinsic form factor, is to compare the differential cross sections for the \( L = 0 \) transitions \( ^{183}W(p,t) ^{181}W(\frac{1}{2} - [510]) \) and \( ^{182}W(p,t) ^{180}W(0^+) \) which, according to Equation 6.2, should be the same. In fact, although their shapes are very similar, their absolute magnitudes
Figure 6-3. Angular distributions for the $^{182}\text{W}(p,t)^{180}\text{W}$ reaction at $E_p = 21$ MeV to the $0^+$ and $2^+$ members of the GSRB and for the $^{183}\text{W}(p,t)^{181}\text{W}$ reaction at $E_p = 21$ MeV to the $\frac{1}{2}^-, 3/2^-$, and $5/2^-$ members of the $\frac{1}{2}^- [510]$ band. The solid lines are drawn to connect the data points for the $^{182}\text{W} \rightarrow ^{180}\text{W}$ transitions while the same lines, but renormalized by a factor of 0.6, are shown on the $^{183}\text{W} \rightarrow ^{181}\text{W}$ angular distributions for comparison.
differ by about 40%, the odd-mass target transition being weaker. Such as effect is well documented in superfluid nuclei (e.g. Hi 72, Hi 75, Sh 76) and has been given the name "blocking." As discussed previously, the (p,t) transition to the ground state (or ground state analog) of a superconducting nucleus involves the coherent removal of a neutron pair from many levels. When a single particle occupies a level, there can be no pair pickup from that level, thus reducing the total pickup strength. Although blocking effects of factors of two in (p,t) reactions are not unknown (see, for example Sh 76), the large value seen here merits explanation. In an attempt to provide such an explanation, the standard set of calculated single-particle levels was used with the code DEF2NT, in conjunction with the DWBA code DWUCK, to calculate the strength of the $^{182}_{\text{W}} \rightarrow ^{180}_{\text{W}}$ ground-state transition. Precisely the same calculation was then performed, except that the $\frac{1}{2} - [510]$ level was effectively blocked by artificially respecifying its binding energy to place the level well above the Fermi surface. Its UV factor was then such that it contributed negligibly to the result. The calculation predicts a (p,t) blocking effect of 45%, in reasonable agreement with experiment. The calculated values of $\Delta$ did not, however, agree well with the experimental values of $P^N$ for either $^{181}_{\text{W}}$ nor $^{183}_{\text{W}}$, reflecting not unexpected deficiencies in the single-particle spectrum used. When the calculation was repeated, with the value of the pairing matrix chosen so that $\Delta = P^N$, a blocking effect of 36% was predicted, again in good agreement with the experimental value of 40%.
The size of the blocking effect may seem surprisingly large since the ground states are populated by coherent pair pickup from about 30 orbitals. In fact, two effects conspire to give this large blocking effect. First, in all cases the orbital blocked is the one which in the neighboring even nuclei has the largest UV factor and so would normally contribute a reasonable fraction of the total strength. Secondly, in the present case the blocked orbital is the $\frac{1}{2} - [510]$ which has a large intrinsic $L = 0$ pair pickup strength. Figure 6-4 shows the relative probability of $L = 0$ (and $L = 2$) pair pickup from each of the orbitals included in the full calculation. The values were determined by calculating transfer form factors for each of them individually with DEF2NT and matching on a Hankel function tail. The resulting form factors were then used as input to the code DWUCK for the $^{182}\text{W}(p,t)^{180}\text{W}(Q = -7.8 \text{ MeV})$ reaction. The strength was taken to be the value of the differential cross section at the second maximum. Also shown on the figure are the values of the UV factors appropriate for the $^{182}\text{W} \rightarrow ^{180}\text{W}$ transition. As discussed, the $\frac{1}{2} - [510]$ orbital, located at a binding energy of 5.9 MeV, is seen to have a large UV factor and carries a large intrinsic pickup strength besides, so that its blocking leads to a considerable reduction in the available $L = 0$ strength.
Figure 6-4. L = 0 and L = 2 transfer strengths for the orbitals shown in Figure 2-1, calculated as described in the text. The solid curves give the UV weighting factors appropriate to the $^{182}W(p,t)^{180}W$ reaction. Also indicated for orientation purposes is the level occupied by the unpaired neutron in the $^{183}W$ ground state, the $\frac{1}{2} - [510]$ orbital. The signs of the L = 2 transfer strengths indicate the signs of their form factors at the nuclear surface.
L = 0 TRANSFER STRENGTHS
FOR VARIOUS NILSSON ORBITALS

L = 2 TRANSFER STRENGTHS
6.3 CROSS SECTION SYSTEMATICS OF GROUND STATES

6.3.1 Pairing Rotation Limit

In the pairing-rotation model (Br 73) the ground states of neighboring even-even superfluid nuclei are described by a single intrinsic wavefunction, $\phi_{\text{BCS}}$, but with various orientations in particle-number space; $\psi = e^{iN\phi_{\text{BCS}}}$ where $N$ is the number of particles and $\phi$ is the gauge angle in this space. Within this model, using the $0s$ approximation, the $(p,t)$ cross sections between ground states of even isotopes are given by

$$
\sigma(\theta)^{0s} \alpha (2\Delta)^2 \sigma(\theta)^{(0)}
$$

(6.4)

where the last term contains only the kinematics of the reaction under consideration. To compare with experiment, the last factor can be taken from the DWBA calculations of Section 5.3 while $\Delta = P_N$. The value of $G$ is expected to change only slowly with mass so that a constant value is used. Table 6-1 gives the results obtained; the predictions are clearly not fulfilled. Not only do the pairing-rotation values change by only 10% rather than by 50% as found experimentally, but the prediction is that the $^{182}_{\text{W}} \rightarrow ^{180}_{\text{W}}$ transition should be the strongest while it is actually the weakest. Thus, the pairing-rotational limit does not reproduce the data.

6.3.2 Full Form Factor DWBA Calculations

In a further attempt to reproduce the ground-state cross sections, the BCS form factor for each ground-state transition was calculated with the code DEF2NT, using the deformation parameters and pairing matrix elements shown in Table 6-2. These
TABLE 6-1
GROUND-STATE \((p,t)\) STRENGTHS IN THE PAIRING-ROTATIONAL LIMIT

<table>
<thead>
<tr>
<th>Mass</th>
<th>(P_N) (^{(a)})</th>
<th>(P_N) (AVERAGE) (^{(b)})</th>
<th>(f(Q)) ((p,t)) (^{(c)})</th>
<th>((P_N^2 f(Q))_{REL.}) (^{(d)})</th>
<th>EXPT. (^{(e)})</th>
</tr>
</thead>
<tbody>
<tr>
<td>186</td>
<td>.797</td>
<td>.758</td>
<td>1.05</td>
<td>713</td>
<td>740</td>
</tr>
<tr>
<td>184</td>
<td>.720</td>
<td>.765</td>
<td>1.05</td>
<td>720</td>
<td>720</td>
</tr>
<tr>
<td>182</td>
<td>.809</td>
<td>.807</td>
<td>1.00</td>
<td>750</td>
<td>500</td>
</tr>
<tr>
<td>180</td>
<td>.805</td>
<td>.818</td>
<td>0.92</td>
<td>698</td>
<td>744</td>
</tr>
<tr>
<td>178</td>
<td>.832</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(a) Calculated using the Nilsson-Prior Prescription (Ni 63), masses from Reference Ma 65.
(b) Average of two neighboring values
(c) Kinematic factor from figure 5-2
(d) Pairing-rotational prediction, normalized to the \(^{184}W (p,t) \quad ^{182}W\) (g.s.) transition
(e) Taken to be the differential cross section at the second maximum \(=27^\circ\)
were then used as input to the code DWUCK along with the optical model parameters of Table 3-1. The resulting values of the differential cross section at the second maximum are listed in Table 6-2 and shown in Figure 6-5. The predicted curve is arbitrarily normalized to the result of the $^{184}$W(p,t)$^{182}$W transition. The results of the calculations show a slowly increasing strength with decreasing mass, in reasonable agreement with the $^{186}$, $^{184}$, $^{180}$W (p,t) results. The decreased strength of the $^{182}$W(p,t)$^{180}$W transition, however, is not reproduced. To investigate whether this problem can be traced to deficiencies in the underlying single-particle spectrum, the calculations were repeated including only the 13 orbitals shown in Figure 2-2 as valence orbitals. The energies of these were adjusted to agree with the "experimental" scheme shown on the right of Figure 2-2. The relative strengths of the ground-state transitions so calculated were identical to those shown in Figure 6-5. It is thus concluded that the explanation for the failure of the calculation to correctly reproduce the trends can be traced to the assumptions inherent in the BCS theory itself. The BCS wavefunction does not correspond to a state with a definite number of particles and is, in fact, a wave-packet in particle-number space. Therefore, the solution for a given nucleus has some of the properties of neighboring even nuclei mixed in. Thus, the BCS technique can be expected to describe only the average properties of a series of isotopes. Moreover, the applicability of the BCS theory to a given situation is directly related to the degeneracy of the orbits;
### TABLE 6-2

**W(p,t)W(g.s.) CROSS SECTION CALCULATIONS**

#### Structure Input

<table>
<thead>
<tr>
<th>Mass</th>
<th>$B_2$ (a)</th>
<th>$B_4$ (a)</th>
<th>G (b)</th>
<th>P_N</th>
</tr>
</thead>
<tbody>
<tr>
<td>186</td>
<td>0.209</td>
<td>-0.088</td>
<td>0.143</td>
<td>0.797</td>
</tr>
<tr>
<td>184</td>
<td>0.221</td>
<td>-0.087</td>
<td>0.140</td>
<td>0.720</td>
</tr>
<tr>
<td>183</td>
<td>0.219</td>
<td>-0.069</td>
<td>0.157</td>
<td>0.772</td>
</tr>
<tr>
<td>182</td>
<td>0.219</td>
<td>-0.069</td>
<td>0.146</td>
<td>0.809</td>
</tr>
<tr>
<td>181</td>
<td>0.219</td>
<td>-0.069</td>
<td>0.150</td>
<td>0.772</td>
</tr>
<tr>
<td>180</td>
<td>0.219</td>
<td>-0.069</td>
<td>0.143</td>
<td>0.805</td>
</tr>
<tr>
<td>178</td>
<td>0.219</td>
<td>-0.069</td>
<td>0.138</td>
<td>0.832</td>
</tr>
</tbody>
</table>

#### Kinematic Input and Results

<table>
<thead>
<tr>
<th>Residual Mass</th>
<th>Q-Value (c) (MeV)</th>
<th>$d\sigma/d\Omega$ (d) (ub/sr)</th>
<th>SECOND MAX (ub/sr)</th>
<th>EXPT. (ub/sr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>184</td>
<td>-4.470</td>
<td>681</td>
<td>740 ± 75</td>
<td></td>
</tr>
<tr>
<td>182</td>
<td>-5.120</td>
<td>720</td>
<td>720 ± 70</td>
<td></td>
</tr>
<tr>
<td>181</td>
<td>-6.228</td>
<td>646</td>
<td>320 ± 42</td>
<td></td>
</tr>
<tr>
<td>180</td>
<td>-6.271</td>
<td>836</td>
<td>500 ± 75</td>
<td></td>
</tr>
<tr>
<td>178</td>
<td>-6.843</td>
<td>855</td>
<td>744 ± 75</td>
<td></td>
</tr>
</tbody>
</table>

(a) $^{186, 184}_W$ deformations from Reference Le 75, scaled to $r_0 = 1.25F$. $^{182}_W$ deformation from Reference Ap 70, other deformations taken to be the same as for $^{182}_W$.

(b) Value chosen so that $\Delta = P_N$.

(c) From Reference Ma 65.

(d) Normalized to $^{184}_W (p,t) ^{183}_W$ experimental result.
Figure 6-5. Cross sections for the ground-state transitions for the tungsten isotopes in \((p,t)\) reactions at \(E_p = 21\) MeV. Experimental values are shown with error bars reflecting both statistical and relative cross section uncertainties. The curve gives the results of calculations as described in the text.
only in the limit of very large degeneracies is the BCS technique expected to properly describe the system. In the case of deformed nuclei, the single-particle levels are only doubly degenerate, and even worse, the gap in the single-particle spectrum at \( N = 108 \) further reduces the effective degeneracy in the vicinity of the Fermi surface. Thus, in such cases, the BCS theory provides an inadequate nuclear structure model, requiring that more sophisticated methods be used, such as an exact diagonalization of the pairing force.
Chapter 7

SYSTEMATICS OF COLLECTIVE STATES:

EXCITED $K^* = 0^+$ BANDS
7.1 GENERAL CONSIDERATIONS

Of all of the many $J^m = 0^+$ states in deformed even-even nuclei, the ground state exhibits the greatest pairing collectivity. (It is this fact that makes that particular state the ground state.) This same collectivity is also responsible for the strong two-nucleon transfer transitions which connect the ground states. In the BCS theory, the ground state contains essentially all of the pairing strength so that excited $0^+$ ($0^+_n$) states should be populated with only a few percent of the ground-state strength in two-nucleon transfer reactions (Be 66, Br 73). Thus, the observation of substantial excited $L = 0$ strength is worth of note. There are, however, known circumstances in which $0^+_n$ states can demonstrate large $(p,t)$ or $(t,p)$ strength:

1. Near closed shells where the inter-shell spacing is larger than the pairing interaction. In such a case the single-particle levels below and above the gap are nearly non-interacting so that the pairing correlation can produce strong two-neutron transfer transitions by the addition or removal of nucleon pairs from levels above or below the gap, thereby populating different states. The concept of pairing vibrations (Be 66) is useful in such cases and has been used with success on the Zr, Ni, Ca, and Pb nuclei.

2. For deformed nuclei where a gap in the energy spacing of the Nilsson orbits occurs, the levels above and
below the gap can to a certain extent decouple, thereby splitting the $L = 0$ transfer strength. Such an occurrence has been seen experimentally in the Yb (Oo 70) and perhaps in the Cm (Fl 77) isotopes.

(3) When a rapid change in ground-state shape occurs, an excited state of the residual nucleus may be more closely related to the target nucleus ground state than is the residual nucleus ground state leading to strong $L = 0$ transitions to $0^+_n$ states. The Sm isotopes are known to demonstrate such an effect (Bj 66, Br 73).

(4) In the Actinide nuclei, there occurs a region of non-uniform distribution of oblate and prolate single-particle levels around the Fermi surface. Excited $L = 0$ transitions of 15% to 20% of the ground state strength have been seen experimentally (Ma 70, Ma 72a, Ca 72b, Fr 74), and are interpreted as being due to the partial decoupling of the prolate orbitals which compose the bulk of the Fermi surface from the oblate orbitals, which lie below the surface (Gr 71, Be 72, vR 72).

With the exception of (3), the above splitting of the $L = 0$ strength arises from the decoupling of one group of single-particle orbits from another. Without this splitting, the ground state would collect almost all of the available pairing collectivity and hence almost all of the $L = 0$ two-neutron transfer strength discussed previously. Inherent in the discussion so far, however,
is the assumption that the $L = 0$ transitions for $(p,t)$ reactions occur through the pickup of zero-coupled neutron pairs, both neutrons being in the same Nilsson orbit. Certain excited $0^+$ states, especially beta vibrations, however, should include broken-pair configurations $(\nu,\nu)_{J^\pi K} = 0^+0^+$ where $\nu$ and $\nu$ are different Nilsson levels. Within the space of the 13 orbitals shown in Figure 2-2, only two broken-pair configurations can be involved in $J^\pi K = 0^+0^+$ states. Moreover, it is calculated that the probability of pickup for these configurations is smaller than the probability of pair pickup by factors of 5 to 20 on the average (see Appendix E). In light of these considerations, the effect of broken-pair configurations on the population strength of $0^+$ levels in this mass region will be ignored. Therefore, in the discussions that follow, substantial excited $L = 0$ strength is interpreted in terms of the splitting of the pairing strength due to the decoupling of groups of levels from each other.

Figure 7-1 shows the Q-value corrected $L = 0$ $(p,t)$ strengths plotted versus excitation energy in the residual nucleus. Table 7-1 tabulates the relevant information and Figure 7-2 displays the angular distributions for the first excited $0^+$ states $(0_1^+)$ their associated $2^+$ states, and the $2_\gamma^+$ states. In each case, the $0_1^+$ state is populated with 10% to 15% of the ground-state strength. A further notable feature of the data is the discontinuity for the even nuclei in the excitation energy of the $0_1^+$ state in $^{180}$W where it has a value of 1.5 MeV rather than the 1.0 to 1.1
TABLE 7-1
L = 0 STRENGTHS

<table>
<thead>
<tr>
<th>Residual Mass</th>
<th>Excitation Energy (keV)</th>
<th>$do/d\Omega$ SECOND MAX. (ub/sr)</th>
<th>$S_0^+(a)$ (p,t) (ub/sr)</th>
<th>Percent of g.s. Strength</th>
</tr>
</thead>
<tbody>
<tr>
<td>184</td>
<td>0</td>
<td>740</td>
<td>705</td>
<td>100</td>
</tr>
<tr>
<td>182</td>
<td>0</td>
<td>720</td>
<td>686</td>
<td>100</td>
</tr>
<tr>
<td>181</td>
<td>454</td>
<td>320</td>
<td>320</td>
<td>100</td>
</tr>
<tr>
<td>180</td>
<td>0</td>
<td>500</td>
<td>500</td>
<td>100</td>
</tr>
<tr>
<td>178</td>
<td>0</td>
<td>744</td>
<td>809</td>
<td>100</td>
</tr>
<tr>
<td>1643</td>
<td>9</td>
<td>17</td>
<td></td>
<td>2</td>
</tr>
<tr>
<td>1516</td>
<td>10</td>
<td>15</td>
<td></td>
<td>3</td>
</tr>
<tr>
<td>1695</td>
<td>10</td>
<td>15</td>
<td></td>
<td>3</td>
</tr>
<tr>
<td>1824</td>
<td>6</td>
<td>10</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>997</td>
<td>67</td>
<td>97</td>
<td></td>
<td>12</td>
</tr>
<tr>
<td>1356</td>
<td>22</td>
<td>37</td>
<td></td>
<td>5</td>
</tr>
</tbody>
</table>

(a) $do/d\Omega$ SECOND MAX. after Q-Value correction

(b) Doublet, upper limit given
Relative strengths of $L = 0$ transitions, after Q-value correction, vs. excitation energy in the residual nucleus. The favored $L = 0$ transitions have been reduced in size by a factor of 10 for graphical purposes.

Angular distributions for the first two members of the first excited $K^\pi = 0^+$ band, and for the lowest $2^+\gamma$ states. The error bars reflect statistical errors only. The values for the $2^+$ in the $^{178}$W residual nucleus also contain contributions from the $3^-$ state which was unresolved from it.

Difference in energy between various $J^\pi K = 0^+\gamma$ states and their associated $2^+\gamma$ states for the ground-state bands and $0^+_1$ bands in the even tungsten isotopes. Note the suppressed zero.
$^186_W(p,t)^{184}W$

$^184_W(p,t)^{182}W$

$^183_W(p,t)^{181}W$

$^182_W(p,t)^{180}W$

$^180_W(p,t)^{178}W$

EXCITATION ENERGY (MeV)
$E(2^+)-E(0^+)$ (keV) vs. $A$
MeV as in the other isotopes. Moreover, the energy spacing between the $0^+_1$ states in $^{180}$W and $^{178}$W and their associated $2^+$ states are substantially different from those of the ground states or $0^+_1$ states in the higher mass isotopes (Figure 7-3 and Table 7-1), reflecting different deformations and therefore substantially different structures for these states. The first excited $0^+$ states therefore can be divided into two distinct groups corresponding to states in $^{178}$, $^{180}$W and in $^{182}$, $^{184}$, $^{186}$W.
7.2 \textbf{0}_1^+ \text{ STATES IN THE HIGH-MASS ISOTOPES}

The first excited $0^+$ states in $^{182}\text{W}$ and $^{184}\text{W}$ are strongly populated in $(p,t)$ with 16% and 8% of their respective ground state strengths and, as previously discussed, must therefore possess special properties. The calculations of Mikoshiba et al. (Mi 67) predicted that the $0^+_1$ state in $^{182}\text{W}$ is primarily a neutron pair vibrational state and not a beta vibration. Further, $(d,d')$ experiments on $^{182},^{184}\text{W}$ showed a much weaker coupling between the ground and $0^+_1$ bands than would be expected for beta vibrations (Gu 71). However, RPA calculations of Ascuitto and Sorenson (As 72b) with conventional pairing and surface-peaked quadrupole-quadrupole forces fit both the $B(E2; \text{g.s.}\rightarrow 2^+_1)$ values and the ratio of the $(p,t)$ strength of the $^{186}\text{W}(p,t)^{184}\text{W}(0^+_1)$ transition at $E_p = 18$ MeV to the ground state transition strength. No detailed comments were made, however, as to the main configurations involved in the excited $0^+_1$ state.

As has been noted before (Ba 73), there exists an energy gap in the single-particle level scheme of tungsten above 108 neutrons. The existence of such a gap is demonstrated in Figure 7-4 where the two-neutron separation energy ($S_{2N}$) for the even-mass tungsten isotopes is plotted against the mass numbers. The general trend of the plot is to lower values of $S_{2N}$ with increasing mass. This trend is explained by the fact that although the potential well deepens somewhat, as more neutrons are added it requires less energy to remove a neutron pair since the
Figure 7-4. The two-neutron separation energies ($S_{2N}$) for the various tungsten isotopes. The two parallel lines are to emphasize the smooth variation of $S_{2N}$ vs. mass above and below $N = 108$ ($A = 182$) and the striking discontinuity at $N = 108$. 
TWO-NEUTRON SEPARATION ENERGIES FOR THE TUNGSTEN ISOTOPES

S\textsubscript{2N} (MeV)

MASS NUMBER
Fermi surface is moving towards zero single-particle energy. The sudden change in $S_{ZN}$ between $A = 182$ and $A = 184$ bespeaks a larger than average energy spacing between the $N = 108$ and $110$ neutron single-particle levels. The "experimental" single-particle level scheme of Figure 2-2 also exhibits an energy gap of about 1 MeV above the $9/2 - [624]$ orbital ($N = 108$).

The existence of a gap in the single-particle energy spectrum should lead to a partial decoupling of the levels above and the levels below the gap, splitting the $L = 0$ strength. The extent of the decoupling depends upon the ratio of the size of the gap to the energy spread of the Fermi surface, about $2\Delta$. Since $\Delta = P_N$, we find $R = (\text{GAP})/(2\Delta) = 0.67$ for the present case, whereas for the $N = 104$ gap in Yb, $R = 0.60$ and in Sn, $R = 0.45$. Thus the decoupling in the case of tungsten should be even greater than in Yb or Sn where the effect has been previously observed (Br 73). This decoupling is graphically demonstrated in Figure 7-5 where the Fermi surfaces for various even-mass tungsten isotopes are shown. The energies of the single-particle levels are taken from the "experimental" values of Figure 2-2 and the pairing force constant chosen so that $\Delta = P_N$. As seen from the figure, the $A = 182$ nucleus has the levels below the gap almost completely full ($\sum U^2 = 0.6$) and the levels above the gap almost completely empty ($\sum V^2 = 0.6$), the pairing force just barely being able to overcome the gap. Then, to a reasonably good approximation, the ground states of the $A > 184$ tungsten nuclei as formed by adding
for each of the even-mass tungsten isotopes. The calculations were performed within the framework of the BCS theory where the (constant) pairing matrix element was chosen so that $\Delta = P_N$. The energies of the single-particle orbits are those of the "experimental" values of Figure 2-2. The fact that in $^{182}\text{W}$ the orbitals above the gap are almost completely empty and the orbits below the gap almost completely full suggests the applicability of the pairing vibrational model.
CALCULATED FERMI SURFACES FOR THE TUNGSTEN ISOTOPES

$E_{s.p.}$ (MeV)

186  184  182  180  178
neutron pairs to orbitals above the gap, while those of the A < 182 nuclei result from the removal of pairs from orbitals below the gap. A model developed to describe such a scheme is the "pairing vibration model" of Bes and Broglia (Be 66) based on a suggestion by Bohr and Mottelson (Bo 64). This model is only schematic, however, so that although it may contain much of the essential physics, it may well be inadequate to explain the experimental details. In the next section the major results of the pairing vibration model will be discussed (much of the discussion taken from References Br 73 and Bo 75), while Section 7.2.3 will consider the application of the model to tungsten nuclei.

7.2.1 The Pairing Vibration Model: General Description

The schematic pairing vibration model is most useful in situations where there exists a gap in the single-particle energy level scheme at \( N = N_0 \) large enough to overcome the pairing force. The vacuum of the model is taken to be the ground state of the \( N = N_0 \) nucleus, including ground-state correlations if necessary. Two operators are then defined; \( A_r \) which (to first approximation) removes a neutron pair from the levels below the gap, and \( A_a \) which adds a neutron pair to the levels below the gap. It should be noted that these creation and destruction operators are but a linear combination of those previously defined, \( A^+_{\nu} \) and \( A^-_{\nu} \), except that now the space of single-particle levels those operators act on has been divided into two subspaces, each with their own destruction and creation operators. The \( 0^+ \) states within this model are labelled by
\( (N_r, N_a) \) where \( N_r \) is the number of removal quanta and \( N_a \) are the number of addition quanta needed to form the state from the vacuum. In the approximation in which these quanta are non-interacting (harmonic approximation), simple energy relationships describe the relative binding energies of the nuclei in the various states. Figure 7-6 shows these relationships, where the energy scale is the total energy relative to that of the ground-state energy of the \( N_0 \) nucleus, after subtraction of a term linear in the neutron number. The coefficient of the linear term is chosen as per the convention of Reference Bo 75 so that the (1,0) and (0,1) states both correspond to the same phonon energy.

As shown on Figure 7-6 also, the pairing vibration model in the harmonic approximation can describe the relative intensities for two-nucleon transfer reactions. The relevant matrix elements are (Bo 75):

\[
\langle N_r, N_a + 1 | A_a^+ | N_r, N_a \rangle = \sqrt{N_a + 1}
\]

\[
\langle N_r - 1, N_a | A_r | N_r, N_a \rangle = \sqrt{N_r}
\]

so that the relative intensities are

\[
(N_r, N_a) \rightarrow (N_r, N_a + 1) \quad \sigma(\varepsilon) = (N_a + 1)x
\]

\[
(N_r, N_a) \rightarrow (N_r + 1, N_a) \quad \sigma(\theta) = (N_r + 1)y
\]

where \( x \) and \( y \) can be different, reflecting the differences in structure, degeneracy, level density, etc. between orbitals above and orbitals below the gap.

Selection rules for two-particle transfer also come from
Figure 7-6. The predictions of the harmonic pairing vibration theory for the relative energies of $0^+$ states and for the $(t,p)$ and $(p,t)$ cross sections. The vacuum is taken to be the $N_0$ nucleus while all of the states are labelled by $(N_r, N_a)$ as described in the text. The figure is taken from Reference Br 73.
the model. For instance, a \((t,p)\) transition to the first excited \(0^+\) state in an \(N > N_0\) nucleus is not allowed since it would require changing both quantum numbers. Similarly, \((p,t)\) transitions to excited \(0^+\) states in the \(N > N_0\) nuclei are not allowed. It is extremely interesting to note that within the model, one, and only one, excited \(0^+\) state can be excited in both \((p,t)\) and \((t,p)\) reactions, the \((1,1)\) state in the \(N = N_0\) nucleus.

Anharmonic effects to a certain extent can be included within the framework of the model, but with some difficulties. The expression for the energies of the states can be generalized to include a first-order interaction between phonons by adding the term

\[
\delta E(N_r,N_a) = \frac{1}{2} V_{rr} N_r (N_r - 1) + V_{ra} N_r N_a + \\
+ \frac{1}{2} V_{aa} N_a (N_a - 1)
\]

where \(V_{rr}\), \(V_{ra}\), and \(V_{aa}\) are phenomenological parameters. The modifications necessary to include such effects in the expression for the transition strengths are not known, however. Thus, in the presence of large anharmonicities, the pairing vibration model loses its appeal as a detailed nuclear structure model, but, hopefully, still retains its ability to organize the data into a conceptually simple pattern.

7.2.2 The Pairing Vibration Model: Application to the Tungsten Isotopes

The experimental results for the tungsten isotopes within the framework of the pairing vibration model are shown in
Figure 7-7. In the figure, the zero of the energy scale has been set by assuming equal energies for addition and removal phonons, giving a phonon energy of 575 keV. The predicted locations of the 0^+ states in the harmonic approximation are shown as dotted lines in the figure while the numbers give the values of the (p,t) or (t,p) differential cross sections at the second maximum, the (p,t) values for E_p = 21 MeV and the (t,p) values E_t = 15 MeV. In parentheses are also shown the values for the inverse reaction.

The 180_pW(t,p)_182_W(0^+g) data were from Reference Ga 77, the 182_pW(t,p) from Ca 73, and the 184_pW(t,p) results from Ga 77. The 180_pW(t,p)_182_W(0^+_1) value is a preliminary value from work done by R. R. Betts and the author (Be 78), a sample spectrum for which is shown in Figure 7-8. Not shown in Figure 7-7 are the results of the (t,p) transitions to the 0^+_1 state in the A = 184, 186, and 188 isotopes, which are extremely small (given as 2.8 ub/sr and less than 1.5 and 6 ub/sr, respectively) as is predicted by the model. The 0^+_1 states in 180_pW and 178_pW are also omitted because, as discussed before, they fall into a different structural category. These states will be discussed in Section 7.3.

The energy spectrum of the tungsten isotopes shown in Figure 7-7 shows large anharmonicities, although the excitation energy of the 0^+_1 state in 182_pW is well described. If a first order interaction is allowed between the phonons where V_{rr} = V_{aa} and V_{ra} = 0 (see Equation 7.3), as shown by dots, the energies of the ground states are well described, but the excitation energies of the
Figure 7-7. The experimental tungsten isotope spectrum within the framework of the pairing vibration model. The solid lines give the experimental energies of the various states while the dotted lines are the predictions of the model in the harmonic approximation and the series of dots the prediction when a first order phonon-phonon interaction is allowed ($V_{rr} = V_{aa} = 610$ keV). The numbers give the experimental strengths of the transitions while the values for the inverse reactions are shown in parentheses.

Figure 7-8. Sample spectrum from the $^{180}$W($t,p)^{182}$W reaction at $E_t = 15$ MeV and a laboratory angle of $26.25^\circ$ from Be 78. The figure shows the very strong population of the first excited $0^+$ state (labelled in the figure as $0_2^+$) relative to that of the ground state.
EXPERIMENTAL W-ISOTOPE SPECTRUM

ENERGY (1 PHONON UNIT = 575 keV)

- Experiment
- Harmonic Approximation
- First Order Interaction

744 ~100 110 314 (700)
500 (283) 273 (720)

176 178 180 182 184 186 188
$^{180}W(t,p)^{182}W$

$E_t = 15$ MeV

$\theta = 26 \frac{1}{4}^\circ$

COUNTS PER $\frac{1}{2}$ mm STRIP

PLATE DISTANCE (cm)
$0^+_1$ states in $^{184}$W and $^{186}$W are predicted to be too high by several MeV. Thus, the agreement between experiment and the model predictions of the relative energies is only marginally acceptable.

In contrast to the failure of the model to predict the correct energies of the states, the model predictions of which states should be strong and which should be weak in (p,t) and (t,p) transitions are very well fulfilled. Most notable are the results for the $^{182}$W($0^+_1$) state which is populated with about 15% of the ground-state strength in the (p,t) reaction and about 40% of the ground-state strength in the (t,p) reaction. The detailed numerical predictions of these strengths are not borne out, however, again reflecting the large anharmonic effects; the transitions to excited states are always weaker than the ground-state transitions. (Such a reduction in strength is also seen in the $^{112}$Sn(t,p)$^{114}$Sn reaction where the gap occurs above $^{114}$Sn, the excited $0^+$ state is excited with 11% of the ground-state strength.) Further limits on the applicability of the pairing vibration model to the present case are evident from the fact that the $0^+_1$ states in $^{182}$W and $^{180}$W are at least weakly populated in inelastic scattering whereas the model calls for zero strength. This result comes from the fact that within the model the $0^+_1$ states are built out of the ground states by the excitation of pairs of neutrons across the gap, while the inelastic scattering operator creates particle-hole pairs. An admixture of proton beta-vibrational strength in the states however, could explain this result. A much more serious limitation
is seen in the results of the $^{183}\text{W}(d,t)^{182}\text{W}$ reaction where the $0^+_1$ state is populated with 23% of the ground-state strength ($K\ell 73$). The model would describe the ground state of $^{183}\text{W}$ as the vacuum coupled to one particle in the $\frac{3}{2}[510]$ orbital above the gap. The removal of the particle should then populate only the vacuum, the $^{182}\text{W}$ ground state. A similar argument holds for the $^{183}\text{W}(d,p)^{184}\text{W}(0^+_1)$ transition which has 23% of the ground-state strength. In addition, the occurrence of blocking in the $^{183}\text{W}(p,t)^{181}\text{W}(\frac{3}{2}[510])$ transition indicates that the $\frac{3}{2}[510]$ orbital does in fact take part in the ground-state transition in the $^{182}\text{W}(p,t)^{180}\text{W}$ reaction, in disagreement with the (first order) model predictions.

One interesting prediction of the model is that there should exist an excited $0^+$ state at 1.0 MeV excitation in $^{180}\text{W}$, with the designation $(2,1)$. Such a state would not be excited in $(p,t)$ but would be strong in $(t,p)$ transitions. The prediction is difficult to check directly since $^{178}\text{W}$ is unstable, but a serious effort to locate the state through either $\alpha,\text{xn}$ reactions or an especially high-statistics $^{182}\text{W}(p,t)^{180}\text{W}$ experiment, or even an alpha-transfer stripping reaction on Hf is called for.

Even though the limitations of the model as applied to the tungsten isotopes are apparent, the ability of this very simple model to organize the data is impressive. Unfortunately, there is no simple way to include anharmonic effects in the model so that a more detailed discussion in the framework of the pairing vibration model would not be useful. The use of another model, which includes
the phonon-phonon interactions, is clearly called for. Such a model can be developed with the use of the Interacting Boson Approximation of Arima and Iachello (Ar 75, Ar 76) and no doubt will be found in the future to be of great value in explaining these data in a quantitative manner.
7.3 **$O_1^+$ STATES IN THE LOW-MASS ISOTOPES**

Before discussing the $O_1^+$ states in $^{180}$W and $^{178}$W, it will be helpful to review their properties:

1. They both are excited strongly in $(p,t)$, with 13% and 12% of their respective ground state strengths.

2. They possess similar moments of inertia which are substantially different from those of the $O_1^+$ states in the higher-mass tungsten isotopes and of all of the ground states.

3. They occur at very different excitation energies, 1516 and 997 KeV respectively.

The last property would seem to preclude any reasonable possibility of relating the structure of these two states with a simple model. When the $L = 0$ transfer strength versus experimental Q-value is plotted as in Figure 7-9, however, a new simplicity emerges. Both states occur at nearly the same Q-value, -7.8 MeV, which corresponds to a two-neutron separation energy ($S_{2N}$) of 16.3 MeV. In addition, substantial $L = 0$ strength is seen in $^{182}$W at nearly the same Q-value. This constancy of $S_{2N}$ can be explained through the assumption that in populating the constant-Q states (Q_c states) neutron pairs in each case are removed from the same single-particle level or group of levels. Since the single-particle potential well does not change significantly with a small change in mass, each state should have the same $S_{2N}$. These Q_c states, being two neutron holes in a particular group of
Figure 7-9. Relative strengths of $L = 0$ transitions after Q-value correction vs. experimental Q-values. The favored $L = 0$ transitions have been reduced in size by a factor of 10 for graphical purposes.

Figure 7-10. Neutron single-particle level scheme calculated as described in Chapter 2. The orbitals occupied by the unpaired nucleon for various isotopes are shown along with the spin-parity of those levels. The purpose of the figure is to point out the clustering of single-particle levels near a binding energy of 8 MeV.
\[ \delta(p,t) \text{ (arbitrary units)} \]

\[ {}^{186}_{}\text{W}(p,t){}^{184}_{}\text{W} \]

\[ {}^{184}_{}\text{W}(p,t){}^{182}_{}\text{W} \]

\[ {}^{183}_{}\text{W}(p,t){}^{181}_{}\text{W} \]

\[ {}^{182}_{}\text{W}(p,t){}^{180}_{}\text{W} \]

\[ {}^{180}_{}\text{W}(p,t){}^{178}_{}\text{W} \]

Q-VALUE (MeV)
levels, coupled to the target ground state, can be called pair vibrations in a macroscopic picture. These pair vibrations, however, have a qualitatively different structure from the pair vibrations in the high-mass isotopes (Mo 77).

The calculated single-particle level scheme is displayed again in Figure 7-10 in order to point out the grouping of orbitals below the Fermi surface at a single-particle binding energy of about 8 MeV. If the $Q_C$ states can be described as two neutron holes in this group of orbitals, coupled to the ground state of the target nucleus, then to first order $S_{2N} = 16$ MeV. Under normal circumstances, however, these orbitals should be strongly coupled via the pairing interaction to those participating in the Fermi surface. The excited states would then display the normal few percent of the ground-state $(p,t)$ strength. A close look at the microscopic nature of the single-particle levels, however, shows that while the Fermi surface is composed almost entirely of oblate levels, four out of the six levels in the group are prolate (the $\frac{1}{2}[510]$, the $3/2[651]$, the $5/2[512]$, and the $5/2[642]$ orbitals). Under the assumption of a reduced pairing matrix element between oblate and prolate orbitals, as discussed in Section 2.2, these four prolate orbitals would be somewhat decoupled from the Fermi surface so that the excited $0^+$ state could display substantial $(p,t)$ strength. A calculation of the $(p,t)$ strength of an excited $0^+$ state formed by coupling a dineutron hole in the four orbitals to the ground state of $^{182}$W predicts $\sigma(0^+_0)/\sigma(0^+_g) = 0.10$.
in reasonable agreement with the data (see Appendix D for a description of the calculational procedure). The two transitions, to the ground state of the residual nucleus through a coherent removal of a neutron pair from the oblate orbitals (resulting in a net lowering of the Fermi surface), and to the $Q_c$ states through the removal of the dineutron from the decoupled prolate orbitals, can be schematically pictured as in Figure 7-11.

This model for the $Q_c$ states makes a strong prediction for the $^{183}_{\text{W}}(p,t)^{181}_{\text{W}}$ reaction. As discussed in Section 6.2, the odd neutron causes the $\frac{3}{2} - [510]$ level to be blocked, decreasing the strength of the favored $L = 0$ transition to the $\frac{3}{2}[510]$ state in $^{181}_{\text{W}}$. The situation is schematically represented in Figure 7-12 where the favored $L = 0$ transition (the analog to the ground-state transition in even-even nuclei) is shown at the top. The transition involves the coherent removal of a dineutron from the oblate orbitals, leaving the unpaired neutron and the neutron pairs in the prolate orbitals completely undisturbed. The strength of the transition is less than that of the neighboring even target nuclei since the $L = 0$ pickup strength of the blocked orbital is not available in the transition. The transition shown at the bottom of the figure, where the odd particle and the oblate orbitals are left undisturbed and a dineutron is removed from the group of prolate orbitals, however, should not be blocked.

As Figure 7-10 shows, the strengths of the $Q_c$ states vary smoothly, including the odd-mass target case, although the strengths
Figure 7-11. Schematic representation of two $L = 0$ transitions. At the top, the transition to the ground state involves a coherent removal of neutron pairs from the oblate orbitals, lowering the Fermi surface. At the bottom, a neutron pair is (coherently) removed from the prolate levels, leaving the Fermi surface undisturbed and hence populating an excited state of the residual nucleus.

Figure 7-12. Figure similar to 7-11 but for an odd-neutron nucleus such as $^{183}$W. The favored $L = 0$ transition shown at the top is reduced in strength due to the blocking of an oblate orbital by the odd nucleon. The transition to the excited state formed by coherently removing a neutron pair from the prolate orbitals should not be blocked.

Figure 7-13. $L = 0$ strengths for favored transitions (open dots) and $Q_c$ transitions (closed dots) for the tungsten nuclei. The value for $^{182}$W was obtained by summing the strengths of the three close-lying $0^+$ states.
OBLATE LEVELS

PROLATE LEVELS

\[(p, t)\] transition to ground state

\[(p, t)\] transition to excited \(O^+\) state
OBLATE LEVELS

PROLATE LEVELS

analog to ground state transition (blocking included)

analog to excited $O^+$ state transition (blocking has no effect)
(p,t) Strengths for L=0 Transitions

\( \xi(p,t) \) (arb. units)

Favored L=0

Q_c States

RESIDUAL MASS

178 180 181 182 184
of the ground states vary considerably. Thus, the $L = 0$ transition to the $(Q_c \otimes \frac{1}{2}[510])$ state in the $^{183}$W(p,t)$^{181}$W reaction is in fact not blocked, in accordance with the model. The strength of the $Q_c$ states does decrease with increasing mass in contrast to the model which predicts a constant strength, but the reduction in strength can, perhaps, be accounted for by assuming that as their excitation energy increases, the $Q_c$ states mix with other $0^+$ states.
7.4 J = 2 STATES IN K = 0\(^+\) BANDS

The strengths and 0\(^+\) to 2\(^+\) energy differences for all 
J\(^\pi\)K = 2\(^+\)0 states located are listed in Table 7-2. The strengths of
the transitions are taken to be the sums of the differential cross
sections at the first six angles, from 5\(^\circ\) to 42\(\frac{1}{2}\)\(^\circ\) in 7\(\frac{1}{2}\)\(^\circ\) steps,
divided by the appropriate Q-dependence factor from Figure 5-2.
Also shown are the ratios of the strengths of the 2\(^+\) states to
those of their associated 0\(^+\) states. The S(2\(^+\))/S(0\(^+\)) ratios
for the 0\(^1\) bands in \(^{180}\)W and \(^{178}\)W are seen to be very different
from the ratios for other K\(^\pi\) = 0\(^+\) bands, being about a factor of
3 or 4 larger than those of the ground state bands and a factor
of 9 larger than the ratios for the 0\(^1\) bands in \(^{184}\)W and \(^{186}\)W.
Such a result can in fact be qualitatively understood in the present
model for the Q\(_c\) bands in \(^{178}\)W and \(^{180}\)W. As has been discussed
by Hanson et al. (Ha 75, Ha 76) and as can be seen in Figure 6-4,
the L = 2 direct transfer strength is unusually weak in tungsten
due to both cancellations between contributions from various or-
bitals as well as due to the small quadrupole transfer strength
of each of the orbitals near the Fermi surface. In comparison,
the quadrupole transfer strength of the four prolate orbitals
in the group at 8 MeV single-particle binding energy is calculated
to be quite large, giving S(2\(^+\))/S(0\(^+\))Q\(_c\) = \(6 S(2^+)/S(0^+)\)\(_{G.S.}\)
for the direct route. Although the result is suggestive, it is not
possible to make a direct comparison with experiment due to the
effect of multistep inelastic routes also populating the 2\(^+\)0 states.
### Table 7-2

$J^\pi_K = 2^+\ 0^+$ Strengths

<table>
<thead>
<tr>
<th>Final A</th>
<th>$\text{Ex}(2^+)-\text{Ex}(0^+)$ (keV)</th>
<th>$\sum \frac{d\sigma}{d\Omega}$ (ub/sr)</th>
<th>$S^{2+}_{(p,t)}$</th>
<th>$S^{2+}/S^{0+}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ground State Bands</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>184</td>
<td>111</td>
<td>423</td>
<td>325</td>
<td>.96</td>
</tr>
<tr>
<td>182</td>
<td>100</td>
<td>623</td>
<td>515</td>
<td>.75</td>
</tr>
<tr>
<td>180</td>
<td>104</td>
<td>492</td>
<td>502</td>
<td>1.00</td>
</tr>
<tr>
<td>178</td>
<td>106</td>
<td>718</td>
<td>845</td>
<td>.96</td>
</tr>
<tr>
<td>First Excited $0^+$ Bands</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>184</td>
<td>119</td>
<td>69(e)</td>
<td>58(e)</td>
<td>1.1(e)</td>
</tr>
<tr>
<td>182</td>
<td>136</td>
<td>41</td>
<td>42</td>
<td>.38</td>
</tr>
<tr>
<td>180</td>
<td>$74 \pm 6$</td>
<td>146</td>
<td>304</td>
<td>4.7</td>
</tr>
<tr>
<td>178</td>
<td>$86 \pm 5$</td>
<td>176</td>
<td>289</td>
<td>3.0</td>
</tr>
</tbody>
</table>

(a) Excitation energy different between $2^+$ state and its associated $0^+$ state

(b) Sum over six laboratory angles $5^\circ$ to $42^\circ$ in $7^\circ$ steps

(c) $S^{2+}_{(p,t)} = (\sum \frac{d\sigma}{d\Omega})/Q$ - dependence factor from Figure 5-2

(d) Ratio of $S^{2+}_{(p,t)}$ to $L = 0$ strength from Table 7-1

(e) Doublet, upper limit given

(f) Consistent with value of 82 keV from Reference Ca 76.
Chapter 8

SYSTEMATICS OF COLLECTIVE STATES:

OCTUPOLE AND GAMMA VIBRATIONAL STATES
8.1 THE OCTUPOLE VIBRATIONAL BANDS

The $3^-$ member of the $K^{π} = 2^-$ octupole band was located in each of the nuclei studied, except for $^{178}\text{W}$ where it formed an unresolved doublet with the $2^+_γ$ of the gamma vibrational band. In each case, it was weakly excited, and followed no simple pattern in strength or excitation energy as seen in Table 8-1.

8.2 GAMMA VIBRATIONAL BANDS

The $J^πK = 2^+2$ gamma vibrational states represent a most interesting but confusing study, especially with regards to $^{182}\text{W}$ and $^{184}\text{W}$. The conclusions of Gunther et al. (Gu 71) from their $(d,d')$ study were that the $2^+2$ states in $^{182}\text{W}$ and $^{184}\text{W}$ were well developed gamma vibrations with little mixing between them and the ground or $0^+_1$ bands. In addition, in this study, the $^{183}\text{W}(d,p)^{184}\text{W}$ and $^{183}\text{W}(d,t)^{182}\text{W}$ cross sections to the $2^+$, $3^+$, $4^+$, and $5^+$ members of the gamma bands were well reproduced by DWBA calculations using RPA-calculated structure for the gamma vibrations with no mixing with $K = 0$ bands. In contrast, the calculations of Kumar and Baranger (Ku 68), which were successful in platinum and osmium nuclei predict large mixings between the ground, $0^+_1$, and gamma bands. The recent measurement of the quadrupole moments of the $2^+_γ$ in $^{186}\text{W}$ and $^{184}\text{W}$ by O'Brien et al. (OB 77) showed a near-zero quadrupole moment moment for the $2^+_γ$ in $^{184}\text{W}$ and a trend reasonably reproduced by
### TABLE 8-1

**L = 2,3 TRANSITIONS**

<table>
<thead>
<tr>
<th>Final Mass (J^πK = 2^+_2 GAMMA)</th>
<th>Excitation Energy (keV)</th>
<th>( \sum \frac{d\sigma}{d\Omega} ) (( \mu )b/sr)</th>
<th>( S_{(p,t)} ) (b)</th>
<th>( \frac{S}{S(0^+)} ) (c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>184</td>
<td>903</td>
<td>110 ± 20</td>
<td>94</td>
<td>.13</td>
</tr>
<tr>
<td>182</td>
<td>1219</td>
<td>175</td>
<td>179</td>
<td>.26</td>
</tr>
<tr>
<td>180</td>
<td>1118</td>
<td>76</td>
<td>106</td>
<td>.21</td>
</tr>
<tr>
<td>178</td>
<td>1111</td>
<td>74(d)</td>
<td>123(d)</td>
<td>.15(d)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Final Mass (J^πK = 3^-2 OCTUPOLE)</th>
<th>Excitation Energy (keV)</th>
<th>( \sum \frac{d\sigma}{d\Omega} ) (( \mu )b/sr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>184</td>
<td>1214</td>
<td>22</td>
</tr>
<tr>
<td>182</td>
<td>1372</td>
<td>27</td>
</tr>
<tr>
<td>180</td>
<td>1083</td>
<td>68</td>
</tr>
<tr>
<td>178</td>
<td>1120</td>
<td>74(d)</td>
</tr>
</tbody>
</table>

(a) Sum over six laboratories, 5° to 42\(^o\) in 7\(^o\) steps.

(b) \( S_{(p,t)}^{2^+} = \frac{\sum \frac{d\sigma}{d\Omega}}{Q} \) - dependence factor from Figure 5-2

(c) Ratio of \( S_{(p,t)}^{2^+} \) to \( L = 0 \) ground state strength from Table 7-1

(d) Doublet, upper limit given
the Kumar-Baranger calculations.

To this morass of conflicting conclusions will now be added the (p,t) systematics for populating the $2^+_Y$. The shape and population systematics of the $2^+_Y$ gamma are displayed in Figure 7-2 and Table 8-1. The strength of the $2^+_Y$ is largest in the $^{184}_W\rightarrow^{182}_W$ transition and decreases as the residual mass changes. To investigate this changing strength, the $L = 2, \Delta K = 2$ transfer strength was calculated for every two-neutron configuration within the space of the single-particle orbitals shown in Figure 2-2. The spectroscopic amplitudes were calculated with Equation 3.11 and utilized the Nilsson coefficients from Davidson (Da 68) with $\mu_5 = 0.45$ and $\delta = 0.3$. The relative strengths so calculated are shown in Table 8-2 where the "sign" is the sign of the total bound-state form factor at the nuclear surface. It is immediately obvious that only four configurations have significant pickup strength, the ($\frac{3}{2}[510], 3/2[512]$), the ($\frac{3}{2}[510], 5/2[512]$), the ($\frac{1}{2}[521], 5/2[512]$), and the ($3/2[512], 7/2[503]$) configurations. An estimate of the total available $L, \Delta K = 2,2$ strength can be obtained by summing these intrinsic strengths weighted with the appropriate $V$ factors (from the BCS calculation with parameters taken from Table 6-2), as per the $0s$ approximation, Equation 3.12, while neglecting the backward-going amplitudes $b_{ik}U_{ik}$. Figure 8-1 gives the relative cross sections so obtained and the excitation energy for each of the four main configurations. At the bottom of the figure are the experimental excitation energies
**TABLE 8-2**

$J^\pi K = 2^+ 2$ PICKUP STRENGTHS

<table>
<thead>
<tr>
<th>Configuration</th>
<th>$\frac{d\sigma}{d\Omega}$</th>
<th>Sign (a)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1/2[510] \otimes 3/2[512]$</td>
<td>660</td>
<td>+</td>
</tr>
<tr>
<td>$\otimes 5/2[512]$</td>
<td>164</td>
<td>-</td>
</tr>
<tr>
<td>$1/2[521] \otimes 3/2[512]$</td>
<td>10</td>
<td>-</td>
</tr>
<tr>
<td>$\otimes 5/2[512]$</td>
<td>147</td>
<td>-</td>
</tr>
<tr>
<td>$3/2[512] \otimes 7/2[503]$</td>
<td>334</td>
<td>+</td>
</tr>
<tr>
<td>$\otimes 7/2[514]$</td>
<td>3</td>
<td>-</td>
</tr>
<tr>
<td>$5/2[512] \otimes 9/2[505]$</td>
<td>15</td>
<td>+</td>
</tr>
<tr>
<td>$1/2[660] \otimes 5/2[642]$</td>
<td>1</td>
<td>+</td>
</tr>
<tr>
<td>$3/2[651] \otimes 7/2[633]$</td>
<td>1</td>
<td>-</td>
</tr>
<tr>
<td>$5/2[642] \otimes 7/2[624]$</td>
<td>8</td>
<td>-</td>
</tr>
<tr>
<td>$7/2[633] \otimes 11/2[615]$</td>
<td>2</td>
<td>-</td>
</tr>
</tbody>
</table>

(a) Sign of form factor outside of nuclear surface
Figure 8-1. $J^H K = 2^+ 2$ strengths for the pickup to various two-neutron-hole configurations. The $y$-axis is the excitation energy of the two-quasiparticle configuration while the $x$-axis is the residual mass from the $(p,t)$ reaction. Shown above each point is the relative pickup strength of the configuration as described in the text. The horizontal bars at the bottom of the figure display the excitation energies of the $2^+_Y$ states while above the bars are the relative total available pickup strengths and below are the relative experimental strengths (all arbitrarily normalized to the $^{182}$W result).
J^{\pi} K = 2_+^2$ STRENGTHS

\[
\frac{d\sigma}{d\Omega} = (V_1 V_2)^2
\]

ENERGY (MeV)

\begin{align*}
\text{Theory} & : 1.7 \\
\text{Expt} & : 0.5
\end{align*}

\[
\begin{array}{c}
\text{Residual Mass} \\
184 & 182 & 180 & 178
\end{array}
\]

\[
\begin{array}{c}
1.0 & 0.5 & 0.1 & 0.06 & 0.03 & 0.003
\end{array}
\]

\[
\begin{array}{c}
0.90 & 0.43 & 0.10 & 0.10 & 0.29 & 0.01
\end{array}
\]

\[
\begin{array}{c}
0.32 & 0.34 & 0.33
\end{array}
\]

\[
\begin{array}{c}
\text{Symbols:} \\
\bullet & 1/2[510] \otimes 3/2[512] \\
\times & 1/2[510] \otimes 5/2[512] \\
\circ & 1/2[521] \otimes 5/2[512] \\
\Theta & 3/2[512] \otimes 7/2[503]
\end{array}
\]
of the $2^{+}_\gamma$ states with the relative experimental strengths below and the calculated total available strength above. The experimental value for $^{178}W$ is an upper limit. The trend of decreasing strength from residual nuclei of mass 182 to 178 is reproduced, but the prediction of a relative strength of 1.7 for the $^{186}\rightarrow^{184}$ transition is not in agreement with the experimental value of 0.5. It should be remembered, however, that the $V$ factors used in these calculations have been taken from BCS calculations, the validity of which are in doubt due to the gap in the single-particle spectrum.

Hamamoto (Ha 73) has performed an RPA calculation which shows the ($\frac{3}{2}[510]$, $\frac{3}{2}[512]$), the ($\frac{5}{2}[510]$, $\frac{5}{2}[512]$), and the ($\frac{3}{2}[512]$, $\frac{7}{2}[514]$) to be the major two-neutron configuration involved in the gamma vibration, with proton configurations constituting less than 20% of the wavefunction. Considering these configurations to deposit their entire available $L$, $\Delta K = 2,2$ strength in the $2^{+}$ state, the calculation predicts relative strengths for these states in the residual nuclei $^{178}:^{180}:^{182}:^{184}$ of $0.1:0.3:1.0:1.9$ in comparison with the experimental values of $0.7$ (upper limit):$0.6:1.0:0.5$, in no better agreement than the previous calculation.

The question of which neutron configurations really take part in the gamma vibration would be left in doubt but for two observations:
(1) The decreasing transfer strength to the $2^+_\gamma$ as the residual mass goes from 182 to 178 indicates that the configurations involved are primarily orbitals near the center of the Fermi surface, where the V factors change considerably with changing mass.

(2) The $^{183}\text{W}(p,t)^{181}\text{W}$ experiment does not show two strong bandheads of $J^\pi K = 3/2^- - 3/2^-$ and $5/2^- - 5/2^-$ near the Q-value of the $^{182}\text{W}(p,t)^{180}\text{W}(2^+_\gamma)$ transition. These two states could be formed by coupling the odd $\frac{1}{2}[510]$ particle to the gamma vibration.

These results lead to the conclusion that the $(\frac{1}{2}[510],X)$ configurations are of paramount importance in the population of the $2^+\gamma$ states with (p,t) reactions on the even-mass tungsten isotopes, in agreement with the conclusion of Hamamoto. The identification of the specific orbital X which contributes to the $(\frac{1}{2}[510],X)_{2^+_2}$ configuration also comes from the $^{183}\text{W}(p,t)^{181}\text{W}$ results where the second strongest peak in the spectrum with $S(p,t) = 219 \text{ ub/sr}$ (sum of the differential cross sections of the first six angles, divided by the Q-dependence factor) is the $5/2^- - 5/2[512]$ state whereas the strength for the $^{182}\text{W}(p,t)^{180}\text{W}(2^+_\gamma)$ transition is 179 ub/sr. Thus, this one L = 2 transition in the odd-mass target case contains most, if not all, of the transfer strength normally populating the gamma vibration in even-mass tungsten nuclei. The conclusion is therefore that the pickup of the $(\frac{1}{2}[510], 5/2[512])_{2^+_2}$ two-neutron configuration supplies almost
all of the gamma vibrational pickup strength for the even-mass (p,t) reactions.

The fact that basically only one configuration is important in the (p,t) population of the $2^+_\gamma$ states in even-mass tungsten nuclei (even though these states are strongly populated) suggests that broad systematic studies of the population of $2^+_\gamma$ states with (p,t) and (t,p) reactions (Ca 73b) tell us little about the structure of gamma vibrations in general. This is not to say, however, that the anomalously strong (t,p) population of the $2^+_\gamma$ state in $^{184}$W noted by Casten and Garrett (Ca 73b) is trivial to explain, but only that the detailed structure of the $2^+_\gamma$ state is very important. One experiment which would be useful in determining which configurations are important in (t,p) transitions to these states would be the $^{183}$W(t,p)$^{185}$W experiment.
Chapter 9

SUMMARY AND CONCLUSIONS
9.1 LIMITATIONS OF THE STUDY

In any experimental study of nuclear structure it is not enough to present data without a framework within which to understand, at least qualitatively, the results obtained. This systematic study of a series of related nuclei has demonstrated that, without recourse to detailed calculations, the pairing character of states can be understood in terms of relatively simply models. When such simple models are applied to a system as complex as rare earth nuclei, however, the limitations of the models must be carefully considered. In the present case, only the neutron pairing degree of freedom is being probed; whereas many other collective degrees of freedom certainly enter into the structure of many of the states described here. For example, the proton pairing degree of freedom has been neglected as well as the beta-vibrational (quadrupole-quadrupole) interaction among quasiparticles. Other more exotic degrees of freedom, such as spin-quadrupole, have also been suggested to explain the existence of more than one $0^+_n$ state within the superconducting pairing gap of some nuclei (e.g. References. Ab 72, Be 69b, Pu 72). None of these have been considered here.

In view of these obvious shortcomings, the fact that the present models give reasonable and understandable interpretations to the $(p,t)$ and $(t,p)$ data may at first seem fortuitous. Upon reflection, however, we realize that the effect of other collective degrees of freedom may modify the structure of particular states but that they will not destroy the neutron pairing degree of freedom itself.
The effect may then be to modify the model results, perhaps changing the energy of a given state or fragmenting the \( L = 0 \) strength, but the overall results may very well remain the same. With these points in mind, conclusions will now be drawn.
9.2 PAIRING VIBRATIONS

The tungsten nuclei have been seen to be a showcase for pairing vibrations, with both types in evidence. The ground states and first excited $0^+$ states in $^{182}$W and the heavier isotopes fit surprisingly well into a pairing vibration scheme. This model predicts strong $(p,t)$ and $(t,p)$ transitions to the $0^+_1$ state in $^{184}$W, and almost vanishingly small $(t,p)$ transitions to these states in $^{186}$W and $^{188}$W. The model also makes a prediction for $^{180}$W, where there should exist an excited $0^+$ state at 1.0 to 1.1 MeV which should not be excited in $(p,t)$. Since no such state was in evidence in the $(p,t)$ data presented here, the location of a $0^+$ state at this excitation energy would lend considerable weight to the model. Such a state might be located with the $^{181}$Ta$(p,2\gamma)^{180}$W reaction via the detection of the coincident gamma transitions such as $0^+_n \rightarrow 2^+_g \rightarrow 0^+_g$ or perhaps directly with the $^{176}$Hf$(^6\text{Li},d)^{180}$W alpha transfer reaction.

The low-mass isotopes were found to possess excited $0^+$ states characterized by the same experimental $(p,t)$ Q-value and were hence called $Q_c$ states. In $^{178}$W and $^{180}$W, where their associated $2^+$ states could be unambiguously located, the $0^+$-to-$2^+$ energy spacings were found to be significantly different from that of the $0^+_1$ bands in the higher-mass isotopes, and from those of all of the ground bands. Their very different structure was further emphasized by the larger ratio of $S(2^+)$ to $S(0^+)$. The structure of these $Q_c$ states was interpreted to be the ground-state of the target.
nucleus coupled to two holes in a group of prolate orbitals somewhat below the Fermi surface which are partially decoupled from the oblate orbitals which constitute the bulk of the Fermi surface. The model provides an explanation for both the constant Q-value and the large $S(2^+)/S(0^+)$ ratio but does not provide a simple explanation of the larger moment of inertia of these bands since the removal of two neutrons from prolate levels should decrease the moment of inertia. The model does, however, make the strong prediction that the excited $L = 0$ transition in the $^{183}_{\text{W}}(p,t)^{181}_{\text{W}}$ reaction should not be blocked by the unpaired nucleon, a prediction that is borne out by experiment. One further prediction of the model must be mentioned, the $2.725$ MeV $0^+$ state in $^{182}_{\text{W}}$, which is one of the $Q_c$ states, should not be populated in the $(t,p)$ reaction since it is two-hole coupled to the $^{184}_{\text{W}}$ ground state. It is experimentally difficult to check this particular prediction due to protons from the $(t,p)$ reaction on light contaminants in the $^{180}_{\text{W}}$ target which obscure the spectrum at forward angles (Be 78).

These two types of pairing vibrations, although very different in structure, have one very important common cause; the decoupling of one group of single-particle orbitals from another. This decoupling leads to the systematics in excitation energy or Q-value as well as supplying substantial $L = 0$ strength to the excited $0^+$ state.
9.3 **TRANSITION TO J^π = 2^+ STATES**

The angular distribution shapes for transitions to the 2^+ states in the ground state rotational bands were found to change with target mass. The origin of this effect can be traced to the changing location of the Fermi surface which changes the individual contribution of specific levels to the total strength. The direct quadrupole transfer strength was found to be weak in the high-mass isotopes and to increase as the target mass decreases while the multiple-step inelastic transition strength remains rather constant. The changing angular distribution shapes then result from an interference between the two routes.

Excited J^K = 2^+0 and 2^+2 states were also investigated although few definite conclusions could be drawn. In particular, the strong coupling between these two states reported in several works and predicted by Kumar and Baranger, could not be reasonably checked with the present results, since the (p,t) reaction is not in general sensitive to such couplings. It is of some interest, however, that in ^{182}W where the two states are nearly degenerate and significant mixing is expected, the 2^+2 state was populated with almost 10 times the strength of the 2^+0 while in ^{184}W the ratio was found to be only 2.

Figure 9-1 shows the strengths for the transitions to the 2^+0, 2^+0_1, and 2^+2 states as well as the sum of the total strength. The very strong mass dependence is striking although it must be realized that the effects of multiple step processes are considerable.
Figure 9-1. Transfer strengths to $J^\pi = 2^+$ states for the tungsten isotopes. The transfer strength is defined as the summed cross section over the first six angles approximately corrected for kinematic effects. Also plotted are the sums of the $J^\pi K = 2^+ 2$ and $2^+ 0_1$ results, and the sum total of all three of the transitions.
9.4 GENERAL COMMENTS

The study of the systematics of the tungsten isotopes with the (p,t) reaction has illuminated new and interesting facets of the neutron pairing degree of freedom without recourse to detailed and complex structure calculations. What is surprising, perhaps, is that in the midst of potentially incredible complexities, several clear and beautiful patterns emerge, allowing a glimpse of the simplicity of nature. Such glimpses are the true rewards of research.
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Sign Conventions for Radial Form Factors

Two sign conventions are currently in use for the radial part of single-particle bound-state wavefunctions expanded on a spherical basis set. One convention specifies that the value of the wavefunction at infinity be greater than zero, the other convention that the wavefunction near zero be positive. Since two-nucleon transfer calculations depend upon the relative magnitudes and phases of the various spherical configurations, expressions and function values must always be kept in the same convention. This appendix lists several standard papers in the field and gives their sign conventions.

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Wavefunction Positive at Infinity


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APPENDIX B

Splitting of \((p,t)\) Transfer Strength on Odd-Mass Nuclei

The transition matrix element for the \((p,t)\) reaction
\((J_i \ K_i \ a_i) \rightarrow (J_f \ K_f \ a_f)\) is given by

\[
M(J_i \ K_i \ a_i, J_f \ K_f \ a_f; R) = (2J_i + 1)^{-1} \times \sum_{J M_i} \sum_{M_f} |<J_i \ M_i \ K_i \ a_i | A^+(R) | J_f \ M_f \ K_f \ a_f>|^2
\]

where \(\alpha\) represents any additional quantum numbers necessary to specify the state. Upon application of the Wigner-Eckhart theorem, this may be written

\[
M = (2J_i + 1)^{-2} \sum_{J \ M_i} \sum_{M_f} |<J_i \ M_i \ K_i \ a_i | A^+ | J_f \ M_f \ K_f \ a_f>|^2
\]

where the brackets denote vector coupling

\[
[A^+_J | J_f \ K_f \ a_f >]_{J_i \ M_i} = \sum_{M M_f} |<J_f \ M_f \ J \ M | J_i \ M_i > A^+_M | J_f \ M_f \ K_f \ a_f>
\]

Reference As72a gives the expression for the required matrix element in equation B-2 within the adiabatic approximation.

\[
f_{J a_i a_f} = \sum_{J M_i} |<J_i \ M_i \ K_i \ a_i | A^+_J | J_f \ K_f \ a_f>|^2
\]

\[
= (-)^{J + K} [(1 + \delta_{K_i 0}) (1 + \delta_{K_f 0})]^{-\frac{1}{2}} \{<J_i -K_i \ J \ K | J_f -K_f> \times \sum_{J_f} f_{J}^{\text{intr}}(a_i \ K_i \ a_f \ K_f) \delta(K = K_i - K_f) + (-)^{J_f + K_f} <J_i -K_i \ J \ K | J_f \ K_f> \times \sum_{J_f} f_{J}^{\text{intr}}(a_i \ K_i \ a_f \ -K_f) \delta(K = K_i + K_f)\]
For transition within a superfluid band \( (\alpha_f = \alpha_i = \alpha) \) and \( (K_i = K_f = K) \) the second term \( (K = 0) \) can be neglected.

\[
M = \sum_J |(-)^J <J; -K | J_f -K > f^{\text{intr}}_J (aK aK)|^2
\]

\[
= \frac{(2J_f + 1)}{(2J_i + 1)} \sum_J |(-)^J <J_f K | J \ 0 \ | J_i K_i > f^{\text{intr}}_J (aK aK)|^2
\]

For \( J_i = \frac{1}{2} = K \) only one value of \( J \) can contribute due to angular momentum coupling and parity considerations. In particular,

\[
\frac{M (J_f = 3/2)}{M (J_f = 5/2)} = \frac{2}{3}
\]
APPENDIX C

Compilation of Angular Distributions

The angular distributions for states populated in the various (p,t) experiments are shown. Each angular distribution is labelled "A - B" where A is the mass number of the residual nucleus and B is a peak number (consistent with the numbers in Tables 5-4 through 5-8 and Figures 4-1 through 4-5). The error bars shown are statistical with estimated fitting errors, where appropriate. When not shown, the error bars are smaller than the points.
σ(Θ_{cm}) (μb/sr) vs Θ_{cm} (degrees)
APPENDIX D

Calculations of the \((p,t)\) Strength of \(Q_c\) States

Within the \(0s\) approximation the \((p,t)\) differential cross section for the ground state is given by (Be 72)

\[
\sigma(0^+_g) = \left( \sum \sqrt{\sigma_v} U_v V_v \right)^2
\]

where \(\sigma_v\) is the differential cross section for pickup of a dineutron from the single-particle orbital \(v\) and \(V_v\) and \(U_v\) are the fullness and emptiness factors for the level \(v\). The differential cross section for an excited \(0^+\) state \((0^+_n)\) given by

\[
|0^+_n > = \left( \sum V_v A_v^n \alpha^+_v + \sum V_v B_v^n \alpha^+_v \right) |0 >
\]

where \(A_v^+\) is the quasiparticle creation operator for the state \(v\), \(|0 >\) is the ground state wavefunction and \(A_v^n\) and \(B_v^n\) are numerical coefficients which give the structure of the state, is

\[
\sigma(0^+_n) = \left[ \sum \sigma_v \left( A_v^n V_v^2 + B_v^n U_v^2 \right) \right]^2
\]

Assuming that the \(Q_c\) states consist of two (quasiparticle) holes in the four prolate levels which are nearly degenerate in energy, a reasonable assumption would be that the dineutron hole is shared equally among the four levels, implying \(A_v^n = \frac{1}{4}\) for \(v = \) (one of the four prolate levels). If the backward-going amplitudes \(B_v^n\) are neglected, then

\[
\sigma(Q_c) = \left( \sum \sigma_v \left( \frac{1}{2} V_v^2 \right) \right)^2
\]

The values of \(\sigma_v\) can be taken from figure 5-4 and the \(V\)-factors
from the BCS calculation for $^{182}_{\text{W}} (p,t)$, giving the result:

$$\frac{\sigma(Q_c)}{\sigma(0_g^+)} = 0.1$$

This value could be somewhat higher or lower depending upon the values of $A^n$ used, but represents a reasonable estimate of the $(p,t)$ strength of the $Q_c$ states.
**APPENDIX E**

**Calculation of Broken-Pair Configuration \((p, t)\) Strength of 0\(^+\) States**

Within the space of the 13 orbitals shown in Figure 2-2a, only two seniority non-zero (broken-pair) two-neutron configurations can couple to give \(J^\pi K = 0^+0\); these are the \((1/2[510], 1/2[521])\) and \((7/2[503], 7/2[514])\) configurations. Their intrinsic \((p, t)\) strengths as calculated with Equation 3.11 using Nilsson coefficients from Davidson (Da 68) are 1.1 and 4.8 arbitrary units, respectively, as opposed to a value of 22 for the average intrinsic dineutron pickup strength of the 13 orbitals near the Fermi surface calculated using the same prescription. It is thus concluded that broken-pair two-neutron pickup is not important in populating the low-lying \(J^\pi = 0^+\) states in tungsten, especially when it is considered that the \(1/2[521]\) and the \(7/2[503]\) are deep lying hole states while the \(1/2[510]\) and \(7/2[503]\) are above the Fermi surface.

The above argument makes no mention of the proton degree of freedom which may, or may not, play a role in the structure of any given excited state.
APPENDIX F

The Favored L = 2 Transitions in the $^{183}_{W(p,t)}^{181}_{W}$ Reaction

While the DWBA predicts that the shapes of the angular distributions for transitions to the 3/2 1/2 - [510] and 5/2 1/2 - [510] states will be the same, experimentally a large difference at forward angles is apparent. Such a difference can be traced to the differences between the multiple-step routes which contribute to the population of these states. The various routes will be discussed in this appendix after a brief description of the formalism necessary to explain the differences. In the following discussion, only two-step processes will be considered.

For an operator

$$T^L_k = \left[ a^{L1}_k \times b^{L2}_k \right] L$$

where $a^L_k$ and $b^L_k$ are irreducible tensor operators which operate on the same coordinates, it can be shown that (Des63),

$$\langle J K \alpha || T^L || J' K' \alpha' \rangle = (-)^{J+L+L'} (2L + 1)^{1/2} \sum_{jk \alpha''} \left\{ L_1 \ L_2 \ L \right\} x$$

$$x \langle J K \alpha || a^{L1} || j k \alpha'' \rangle \langle j k \alpha'' || b^{L2} || J' K' \alpha' \rangle$$

(F.1)

With this expression, and expression for the two individual reduced matrix elements, the two-step contributions to the population of the states can be investigated.

The inelastic form factor for a transition of multipolarity $L$ is (Ki 73)
(J K α|Q^L|J' K' α') = (2J + 1)^{1/2} (-)^{J-J'} \langle J K L 0|J' K' > V_{L0} (F.2)

where the ΔK = 0 contribution (the single-particle contribution) has been neglected and where V_{L0}, defined in Reference Ki 73, contains the radial information of the reduced matrix element.

For the case of two-nucleon transfer, neglecting the K = 0 contribution gives

(J K α|A^L_α||J' K' α') = (2J + 1)^{1/2} (-)^{J-J'} intr \langle J K L 0|J' K' > f_{L} (α K a' K') (F/3)

(see Equation B.4).

Considering only the two-step routes which involve an inelastic excitation of multipolarity two and a transfer of multipolarity zero (since f_0^{intr} >> f_2^{intr}; see Ki 73) leads to the following total reduced matrix elements for transitions to the 3/2^- and 5/2^- states:

RME(3/2-) = -(4/5)^{1/2}(f_2^{intr} + V_{20} f_0^{intr}) + . . . (F.4)

RME(5/2-) = +(6/5)^{1/2}(f_2^{intr} + V_{20} f_0^{intr}) + . . .

So that in this approximation, the shapes are still the same, the only difference being in the magnitudes of the cross sections.

When the next higher order terms are included, however, the situation changes:
\[ RME(3/2-) = -(4/5)^{1/2}(f_{2}^{\text{intr}} + V_{20}f_{0}^{\text{intr}} + a V_{20} f_{2}^{\text{intr}}) + \ldots \]  
\[ RME(5/2-) = +(6/5)^{1/2}(f_{2}^{\text{intr}} + V_{20}f_{0}^{\text{intr}} + b V_{20} f_{2}^{\text{intr}}) + \ldots \]  

If \( a = b \) then the two angular distributions would have similar shapes, however this is not the case. Instead it is found

\[
\begin{align*}
a &= 13/(5\sqrt{14}) \\
b &= -(6\sqrt{3})/(5\sqrt{7})
\end{align*}
\]

so that for one state, the last term will add constructively to the rest of the form factor while for the other state, it will give destructive interference.

The observation of different angular distribution shapes for what in the DWBA would be considered simple \( L = 2 \) transfers, then, should allow the direct investigation of multiple-step inelastic routes in \((p,t)\) reactions. Such a study, although not undertaken here, should constitute a very interesting and rewarding project. It should be noted that the case considered here, where the ground state of the target nucleus has \( J = 1/2 \), is optimum for such a study since the direct \( L = 2 \) transfer strength splits between only two states.