A Study of Two Particle-One Hole States in $^{209}\text{Pb}$ by Means of the $(d,p)$ Reaction

by

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1971
To my mother, my father
and to Mary
ABSTRACT

The Pb\(^{208}\)(d,p)Pb\(^{209}\) reaction was studied at a deuteron energy of 20.0 Mev using the Yale MP Tandem Accelerator and Multigap Spectrograph. The purpose was to determine the extent to which two particle-one hole (2p-1h) states in Pb\(^{209}\) could be excited by this reaction; and then to use this information to study the shell closure of the Pb\(^{208}\) ground state and the fragmentation of the single-particle strength in Pb\(^{208}\). In addition to the strong transitions to the single-particle states, there were observed over 35 weak transitions of cross section 10-100 \(\mu\)b/sr, leading to states of 2p-1h character in the excitation region 2.0 to 4.5 Mev in Pb\(^{209}\). These transitions had angular distributions which exhibited typical stripping patterns and were assumed to occur by a direct neutron transfer mechanism. They were assigned \(l\)-values and spectroscopic factors by a DWBA analysis of the angular distributions.

The qualitative agreement between the model calculations and the experimental results supports the interpretation of these transitions as one-step neutron transfers proceeding in some cases via single-particle admixtures in the final states and in most cases (at least for the transitions to states up to 4.0 Mev excitation) via correlations in the Pb\(^{208}\) ground state. While the measured 2p-2h admixtures in the Pb\(^{208}\) ground state were found to have individual amplitudes squared of the order about 1\%, the good agreement of the predictions of the RPA calculations with our results implies that the RPA description of the Pb\(^{208}\) ground state is essentially correct in so far as comparison is possible and hence the Pb\(^{208}\) ground state departs from a pure doubly closed shell configuration. No evidence for fragments of positive parity single particle states of the \(N=126-184\) shell is found below 4.0 Mev. The major fraction (60\%) of the negative parity \(1j_{15/2}\) state resides in the well known level at 1.42 Mev, but additional fragments are now found between 3.0 and 4.0 Mev excitation. There is also evidence that strength from the \(N>184\) shell is found below 4.0 Mev. Our results provide additional evidence that 2p-1h states in Pb\(^{209}\) are qualitatively well described in terms of a hole coupled to the two-particle excitations of Pn\(^{210}\) or a particle coupled to the core excitations of Pb\(^{208}\), although the details of the structure such as the configuration mixing are not yet reproduced by present particle-core calculations.
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VII. SUMMARY
I. INTRODUCTION

It is found experimentally that nuclei containing 2, 8, 20, 28, 50, 82, and 126 protons or neutrons are particularly stable. These magic numbers have been shown to correspond to completely filled shells in analogy with those observed in atomic physics. Indications of such shell closure come from a number of properties observed in these nuclei. For example, the first excited states in nuclei containing magic numbers of nucleons are found to lie at excitations which were anomalously large compared to those observed in other nuclei (SG53). The nucleon binding energies for these nuclei are also found to be unusually large while the binding energies of the nuclei which have a magic number of nucleons plus one are found to be unusually small. In the shell-model description, the low-lying states of nuclei one nucleon removed from a doubly magic nucleus are predicted to correspond to single-particles (holes) in shell-model orbits outside (inside) a doubly closed core. Historically the observation that the shell model described well such states in the light and medium weight nuclei was considered a strong argument for the validity of the model and the concept of independent particle motion. However, experimental information concerning such states in the heavy nuclei was not available until recently and the question has remained open as to whether the simple structure predicted by the shell model would be found in these heavy
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nuclei. With the advent of new accelerators and techniques it became possible to make precision studies of the structure of heavy nuclei and in particular to obtain information concerning the shell-model structure in the region of the doubly-magic Pb\(^{208}\) nucleus (\(Z = 82, N = 126\)). These studies revealed simple structures in the nuclei adjacent to Pb\(^{208}\) which are remarkably well described as single-particle and hole states. Indeed, the shell model predictions agree so well with the experimental results that the Pb region has become the leading example of the model's validity (BW68, ST69).

The first studies of the Pb region were primarily of an exploratory nature and interest was focussed on the more prominent structure in the experimental spectra. These studies established the general features of the level structure of the Pb nuclei which for the most part can be understood in terms of simple model descriptions. Experimental studies have now reached the point where they are important for the investigation of the less prominent structure which is indicative of deviations from the simple pictures of the structure involved. Such studies in the Pb region are particularly useful from a theoretical point of view because of the demonstrated success of the simple shell model as a zero-order description of the structure. Here corrections can be applied to the shell model in a perturbative manner and their effects on the structure can be noted and compared with the experimental result more easily than
in other mass regions where the structure is expected to be more complex. The model calculations which make these corrections, for example, predict that the \( {\text{Pb}}^{208} \) core deviates from a closed shell configuration and contains admixtures of higher order configurations of the character two particle-two hole (2p-2h), four particle-four hole (4p-4h), etc. Similarly, the single-particle and hole states in the adjacent nuclei are not expected to be completely pure configurations but rather are predicted to be fragmented because of mixing with nearby states. The possible experimental detection and measurement of the deviations from simple shell-model predictions are then important because they provide a direct test of nuclear models which attempt to improve upon the simple shell model. While there exists some experimental information concerning these deviations, much more information is needed to permit conclusions concerning the success of the model calculations in the description of the details of the structure. The purpose of this study is to provide such information.

This dissertation reports a detailed study of the \((d,p)\) reaction on the doubly magic \( {\text{Pb}}^{208} \) nucleus. In addition to the strong transitions to the well-known neutron single-particle states, there were observed weaker transitions to states in the excitation region 2.0 to 4.0 Mev in \( {\text{Pb}}^{209} \). These weaker transitions, leading to states of 2p-1h character in \( {\text{Pb}}^{209} \), were of primary interest since their excitation via a direct transfer reaction implies that the
structure of either the $^{208}_{\text{Pb}}$ ground state or the 2p-1h final state in $^{209}_{\text{Pb}}$, or both, deviates from the simple shell-model configurations. The model calculations predict that some of these 2p-1h states should be populated in this reaction and the experimental results can be used to test these predictions. The interpretation of the experiment data is complicated however by the possibility that the excitation of the 2p-1h state results from a more complex reaction mechanism than a simple direct transfer process and consideration must be given to such an interpretation.

Previous studies of neutron transfer reactions on $^{208}_{\text{Pb}}$ have reported the weak excitation of 2p-1h states, but under experimental conditions where either the low beam energies or the small yields to these states prohibited detailed investigation. In this study a systematic investigation of the weak transitions was undertaken with the A.W. Wright Nuclear Structure Laboratory MP Tandem Van de Graaff Accelerator and broad-range magnetic multigap spectrograph. The high resolving power and rapid data acquisition of the spectrograph permitted the identification of over 30 weakly-excited levels up to 4.3 MeV of excitation in $^{209}_{\text{Pb}}$ whose associated proton groups were simultaneously recorded at 23 angles in the angular range $0^\circ$-$167^\circ$. The high energy of the incident deuterons ($E_d = 20.0$ MeV) made the proton angular distributions distinctive enough to permit the identification of the orbital momentum, $l$, carried by the
transferred neutron under the assumption of a direct transfer process. The analysis of the angular distributions was performed using the Distorted Wave Born Approximation (DWBA) reaction codes which were used to assign \( \ell \)-values and spectroscopic factors. The predictions of the model calculations were then compared to these results of analysis.

It will be shown in the following text that the interpretation that the weakly excited \( 2p-1h \) states are populated in the \((d,p)\) reaction by a direct transfer reaction is consistent with the experimental results. Based on this interpretation we will conclude that there is evidence of admixtures in the \( \text{Pb}^{208} \) ground state and that these admixtures are roughly of the magnitude and character predicted by model calculations. Furthermore, there is found evidence that the \( l_j 15/2 \) single-particle state in \( \text{Pb}^{209} \) is fragmented and mixed with at least three \( 2p-1h \) states located at higher excitation, that single-particle strength of a shell-model state which is unbound in \( \text{Pb}^{209} \) is located in several bound \( 2p-1h \) states at lower excitation, and that the \( 2p-1h \) level structure of \( \text{Pb}^{209} \) appears to be qualitatively well understood in terms of a weak-coupling model.

In Chapter II the first two sections provide a summary of the results of previous experimental and theoretical studies of relevance to this study, and the last section defines the scope and purpose of the present study in the context of these previous results. Some of the questions concerning reaction mechanisms which are of
importance in this study are discussed in Chapter III. The experimental apparatus and method are described in Chapter IV and the experimental results are presented and analyzed in Chapter V. The discussion of the results of the analysis and the comparison of theoretical predictions to these results are presented in Chapter VI. Finally in Chapter VII the results and conclusions of this study are summarized.
The nuclear structure in the Pb region is characterized by three distinctive types of structure which have been deduced from the experimental studies made in this mass region. The first of these is the remarkable shell model structure observed in the vicinity of the doubly magic Pb$^{208}$. Although the shell model structure is more easily seen in the single-particle and hole states at low excitations, the shell model continues to be applicable for states of more complex character at higher excitation where one might not have expected much simplicity. The second is the collective structure whose properties can not be well understood in the framework of simple shell model, but which is observed to persist from even nucleus to even nucleus in this region. The third is the particle-vibration structure observed in the odd nuclei and believed to be the combination of the two types of structure discussed above; that is, the weak coupling of single-particle shell model structure to the collective excitations (vibrations) of the even cores. A remarkable amount of the experimental data in this region can be well explained, at least in a qualitative manner, by the application of the simple shell model, together with a weak coupling scheme. This point has been stressed in several discussions of the structure in the Pb region (EW68, ST69).

In the first section we shall review some of the
experimental results which are characteristic of the lead region (particularly the structure of Pb\(^{208}\) and Pb\(^{209}\)) together with their interpretation. The purpose of the discussion is to show the extent to which the shell model provides a good description of the nuclear structure in the Pb region, and the direction and character of the recent model calculations which attempt to improve upon the shell model. The second section of this chapter will discuss very briefly the more formal aspects of these theoretical models with particular emphasis on those calculations relevant to this Pb\(^{208}\)(d,p)Pb\(^{209}\) study. The final section of the chapter will discuss the scope and purpose of the present investigation.
A. Review of Experimental Results

1. General Systematics

Within the framework of the shell model the ground state of the doubly magic Pb\(^{208}\) should contain closed shell configurations of neutrons and of protons, and the spectra of adjacent nuclei should have states corresponding to single particles in orbits outside the Pb\(^{208}\) core and holes in orbits inside the core. The shell model orbits predicted in the mass region near the doubly magic Pb\(^{208}\) are shown in Figure II-1. Information about the single-particle (hole) states can be obtained from transfer reactions which deposit (remove) a single nucleon in a reaction on Pb\(^{208}\). Stripping reactions, such as the (d,p) reaction, will give information about the unoccupied single-particle orbits above the nuclear Fermi sea. Pickup reactions such as the (p,d) reaction, will give information about the occupied single-particle orbits below the Fermi sea. The transfer reactions not only locate the single-particle states, but also measure the purity of the state in terms of a spectroscopic factor, \(S\); i.e., the overlap of the wave function of the actual final state and the target ground state with one nucleon added or removed. The spectroscopic factor is explicitly given by the expression:

\[
S = \left| \langle \psi_{f} \mid a_{jm}^{+} \mid \psi_{o} \rangle \right|^2, \tag{A.1}
\]

for the case of a stripping reaction, where \(a_{jm}^{+}\) is the single-particle creation operator, and \(\psi_{o}\) and \(\psi_{f}\) are
Predicted ordering of the proton and neutron shell model orbits in the mass region $A=208$. (BW60)
the wave functions of the initial and final states. For a Pb²⁰⁸ target and assuming completely closed shells the deviation of a spectroscopic factor from unity in a stripping reaction and 2j+1 in a pickup reaction indicates the extent to which the actual final state deviates from a pure single-particle or hole configuration.

The results of the single-nucleon transfer reaction studies on Pb²⁰⁸ are shown in Figure II-2. The energies listed are those observed experimentally. The only probable discrepancy between the level ordering predicted by the shell model and the observed one involves the location of the 1f¹⁵/² state (See Section II.A.3.). The numbers shown in parentheses are the measured spectroscopic strength for these states obtained from a survey of many experimental results. For 19 of the 24 states, 80% of the expected spectroscopic strength is found in a single level. In the case of the other 5 states, less than 80% is found and the missing single-particle strength is expected to be found mixed into states located at higher excitation energies. Such a clear demonstration of single-particle and hole orbits has not been found elsewhere in the periodic table. In most cases, spectroscopic strengths of single-particle states are found to be spread over a number of nuclear levels, and it is rare even in a magic nucleus to find more than one or two fragments of single-particle states with over 50% of the strength.

Having now had an overall view of the experimental
Figure II-2

Observed ordering of single-particle and hole orbits, and average values of the experimental spectroscopic factors.
evidence for single-particle (hole) structure in the nuclei adjacent to Pb\textsuperscript{208}, it is of interest to pursue the matter further to determine how far the simple shell model can go towards describing the details of structure observed. For this purpose the structure of Pb\textsuperscript{208} and Pb\textsuperscript{209} will be considered in more detail.

2. Structure of Pb\textsuperscript{208}

a. Shell Structure

We consider first the excitation spectrum of Pb\textsuperscript{208}. In the framework of the simple shell model one would expect the first excited levels to be formed by the simple promotion of a particle (proton or neutron) from below the nuclear Fermi sea to orbits above it. Since the neutrons have a smaller shell gap than the protons (See Figure II-1) one would expect the lowest of such particle-hole (lp-lh) states to result from the \((g_{9/2})(p_{1/2}^{-1})\) neutron configuration at an unperturbed energy of 3.44 Mev. It is very encouraging to find that states with spins of 4\textsuperscript{−} and 5\textsuperscript{−} are observed experimentally at 3.20 and 3.48 Mev in Pb\textsuperscript{208}. A complete level scheme can be constructed in this way forming lp-lh configurations of both neutrons and protons to give a zero-order description of the Pb\textsuperscript{208} excitation spectrum.

We might not expect so simple a picture to be very realistic, since we have not yet included the effects of the residual interactions which will remove the degeneracies...
of states with the same configuration and different spins, and which will also mix the states of same spin and parity. Because of the large level densities, it might well have been the case that the mixing could be so thorough as to render a simple shell model picture useless. That is to say, the mixing could have spread the lp-lh configurations over such a large energy range that the interpretation of the spectrum in terms of a simple shell-model picture would not be conceptually useful. However, the shell model calculations (KB69) which include these residual interactions predict that except for the few collective excitations, most of the configurations should exist close to their unperturbed energies in states with simple wave functions containing only a few important configurations. It is a success for the shell model that this is what is observed experimentally in the transfer reaction (BT67, MG70, DH68) and analog state studies (ZP68, SH67, KS68). We illustrate this in Figure II-3 where we compare the predictions of the simple shell model to the results of the Pb\textsuperscript{207} (d,p)Pb\textsuperscript{208} (BT67) and Bi\textsuperscript{209} (d,He\textsuperscript{3})Pb\textsuperscript{208} (MC70) reaction studies. The target nuclei Pb\textsuperscript{207} and Bi\textsuperscript{209} have been shown to be essentially pure 3p\textsubscript{1/2} and 1h\textsubscript{9/2} configurations and thus these two reactions preferentially excite lp-lh states of the character neutron configuration \( (3p_{1/2})^{-1} (nlj) \)\textsuperscript{\textsubscript{j+1/2}} and proton configuration \( (1h_{9/2}) (nlj)^{-1} \)\textsuperscript{\textsubscript{j\textsubscript{π}}} (where \(|j-9/2| < J < |j+9/2|\), respectively, and clearly show the distribution of the neutron particle and proton hole
Comparison of zero-order p-h centroid position to the observed distribution of p-h configurations observed in the $\text{Pb}^{207}(d,p)\text{Pb}^{208}(\text{BT67})$ and $\text{Bi}^{209}(d,\text{He}^3)\text{Pb}^{208}(\text{MC70})$ studies.
strength about the centroid position. The data indicate that the simple shell-model picture is a good zero-order approximation for the structure up to about 6.00 Mev excitation.

b. Collective Excitations

There are also important aspects of the $^{208}\text{Pb}$ spectrum which are not well described by the shell model. Figure II-4 shows the collective excitations in the Pb region which persist from nucleus to nucleus at approximately the same excitation energies. In the even nuclei they are single levels (with the exception of $^{210}\text{Pb}$) and in the odd nuclei they fragment into several levels interpreted as the weak coupling of either a single particle or hole to these collective excitations. These collective levels in $^{208}\text{Pb}$ are believed to be superpositions of many $1p-1h$ configurations ($\text{CP60, CO66}$) and possess transition strengths which are measured to be several times the single-particle unit. (The $3^-$ state at 2.615 Mev, for example, has a $B(E3)$ of 30 to 45 times the single-particle strength ($\text{ZP68}$)). Shell model calculations ($\text{CP60}$) cannot reproduce well either the low excitation energies or the large deexcitation strengths of these observed levels without introducing correlations into the $^{208}\text{Pb}$ ground state wave function. That is to say, the closed shell is broken by the admixture of configurations of the type $2p-2h, 4p-4h,$
Figure II-4
Collective Levels in the Pb Region (ST69)
etc., into the ground state wave function. The results of calculations, (GG66, AG69) made in the Random Phase Approximation (RPA) which allow for the introduction of such correlations are in better agreement with experiment than are those of the shell model especially in the prediction of the transition rates; there remain, however, several disturbing discrepancies. For example, the predicted B(E3) of the 3– state (2.615 Mev) is still underestimated by about a factor of two. Figure II-5 compares the shell model and RPA calculations to the collective experimental excitations in the spectrum.

c. Ground State Correlations

Whereas the enhanced electromagnetic transitions of the collective states of Pb208 provide an indication of the presence of p–h type correlations in the ground state, the observation of enhanced transitions in the two nucleon transfer reactions on Pb208 (B068, BH66) imply the presence of particle paired (2p) type correlations in the ground state. Motivated by the evidence for pairing vibrations, two particle (2p) RPA calculations (VG70) have been performed to predict the level structure of nuclei that are two particles removed from Pb208. These RPA calculations also imply shell breakage in the Pb208 ground state. Figure II–6 illustrates graphically the magnitude of the deviations from a closed shell configuration predicted by the P–h and 2p RPA calcula-
Figure II-5

Comparison of theoretical calculations of Pb$^{208}$ excitation spectrum to the experimental levels

(a) RPA calculations of Gillet, et al. (GG66),
(b) Experimental levels from inelastic (proton) scattering (SV66), and (c) Shell Model Calculations of Carter, et al. (CP60).
tions (GG66, VG70) in terms of occupation fractions (probability that a particle will be found in a specific shell model orbit). Both calculations predict occupation fractions of the order of 2-5% in the largest cases. No one shell model orbit is predicted to dominate, but rather all orbits are predicted to contribute approximately equally to the correlations. That is because the 2p-2h, 4p-4h, etc, configurations that admix into the ground state wave function are made up of many of the shell model orbits where no individual configuration is found to have an especially large amplitude.

The two calculations overlap to a certain extent so that the occupation fractions cannot simply be added together without double counting in some instances. This arises since the two treatments consider two different modes of admixtures into the ground state which are not mutually exclusive. They both consider the shell model as the dominant lowest order term in a perturbation series. In the p-h RPA the particle-hole correlations in the ground state provide the next higher order term. On the other hand, in the 2p RPA it is the particle-pair correlations in the ground state that provide the next higher order term. The occupation fractions resulting from the inclusion of both types of higher order terms must come from a calculation in which the two modes are treated simultaneously (VG70).

Although the predicted individual correlations are small they nevertheless are significant and should be
Figure II-6

2n RPA OCCUPATION FRACTIONS
YUKAWA FORCE RESULTS
○ NEUTRONS
● PROTONS

p-h RPA OCCUPATION FRACTIONS
○ NEUTRONS
● PROTONS

SINGLE PARTICLE ENERGY (MeV)

OCCUPATION FRACTION

-16 -14 -12 -10 -8 -6 -4 -2 0

-16 -14 -12 -10 -8 -6 -4 -2 0
detectable experimentally. We have already mentioned the coherent effects of correlations which contribute to the enhanced electromagnetic transitions of the collective excitations of $^{208}$Pb and the enhanced two nucleon transfer reactions on $^{208}$Pb. Indeed, these observations provided the motivations for the present theoretical treatments. But while these provide a measure of the coherent effects of correlation, they do not measure directly the magnitude of the individual correlations. Information of this type can come from single-nucleon transfer reactions on $^{208}$Pb. A measure of the amount of specific $2p-2h$ configurations in the ground state can be obtained by studying the transitions to $2p-1h$ and $1p-2h$ states in the $A=209$ and $A=207$ nuclei. The population of a specific $2p-1h$ state of $^{209}$Pb, for example, via a $(d,p)$ reaction (under the assumption that the reaction mechanism is a direct transfer and that the $2p-1h$ final state has no single-particle admixtures) implies that the ground state must have $2p-2h$ admixtures; and the strength of the transition provides a measure of the admixture. While there is experimental data concerning the correlated effects, coming from the study of the enhanced electromagnetic and two nucleon transitions, there is very little experimental data concerning the direct measurement of individual correlations. These measurements are one of the goals of this experimental study and will be discussed in greater detail in Section II.B.2.
3. Structure of $^{209}\text{Pb}$

a. Single-particle states

The single-particle states of $^{209}\text{Pb}$ have already been discussed in the general context of the shell structure which characterizes the Pb region. Here we consider in more detail the experimental results concerning these states. In particular we are interested in the spectroscopic factors that have been measured experimentally and how accurately their values can be determined in practice.

These states have been studied by the $(d,p)$ (JD69, DH67, CR68, MP67, EK69) and $(t,d)$ (IP69) reactions in a number of investigations at a number of bombarding energies. The spectroscopic factors obtained in several of the $(d,p)$ studies have recently been collected and compared by Macfarlane (MA69) who was interested in determining the ability of the conventional DWBA analysis to give reliable spectroscopic factors. These spectroscopic factors are shown in Figure II-7 where they are plotted as a function of the bombarding energy at which the reaction was studied.

Several things can be noted from this comparison. First, the absolute spectroscopic factors obtained in the different studies show significant variations and are stable to no better than about $\frac{1}{4}$ the full single particle strength. These variations in the values of the spectroscopic factors deduced from experiment to experiment reflect to a large extent some of the uncertainties which plague the present DWBA analysis of reaction data. (This subject will
Figure II-7
Neutron Spectroscopic Factors in Pb$^{209}$ (MA69).
be discussed in some detail in the following chapter
(Section III. C.3.) It is sufficient to point out at this
time that the results shown in the figure indicate that
the uncertainties in the reaction theory severely limit
the knowledge of the detailed structure of these states
and prevent the detection of deviations from a pure
configuration which are less than about 25% of the full
strength.

The second thing which can be observed from the
comparison of the spectroscopic factors is that all seven
states have spectroscopic factors which are within 25% of
the "expected" value of unity. More recent measurements
of these spectroscopic factors (BK69, IE69) are in
agreement with the results obtained in the studies discussed
above; an exception is the $l_j^{15/2}$ state at 1.424 Mev
which was found to have a value of 0.50-0.75 of the full
single-particle strength. This smaller value for the
spectroscopic factor is now believed to be the better
measurement in view of recent lifetime measurements made
by Ellegaard, et al. (EK69) who found that the $15/2^-$ state
at 1.424 Mev has a large decay strength to the ground state
which is attributed to a significant admixture of the core-
coupled state ($^{208}\text{Pb}(3)^- \times g_9/2)_15/2$. Possible states
containing the missing $l_j^{15/2}$ strength have been suggested
(EK69), but no positive evidence for the location of such
strength has yet been found experimentally. Thus, to
summarize, the results of the stripping reaction studies
indicate that all the states have spectroscopic factors which are consistent with the interpretation of pure single-particle configurations, except the $1j_{15/2}$ state which appears to have at least 25% of the full strength separated from the main component.

The question of what the "expected" spectroscopic factors for the single-particle states should be is a very interesting one. Single-particle strength can be removed from a single level by a number of mechanisms. We discuss these in two broad categories. The first of these will be the reduction of single-particle strength because of interactions with nearby states of the same spin and parity which cause the strength to fragment over a number of bound states. The second category will include the reduction of single-particle strength because of the interactions which will cause the strength to be spread to distant states which lie in the continuum. The distinction between the two types of strength reduction is important within the framework of sum rules which have been established. In the analysis of direct reactions on a target $A$, it is usually assumed that the sum rule of Macfarlane and French (MF60) should be satisfied by the bound states in the $(A+1)$ and $(A-1)$ systems (or at least by well defined resonances). To be more explicit the sum rule states that (for a given shell-model orbital) the sum of single-particle strength observed in the stripping reaction to the bound states in the $(A+1)$ nucleus should equal to one. (Here we have
divided the normal expression of the sum rule by \( 2J+1 \). We consider, for example, the case of neutron stripping and pickup on \( \text{Pb}^{208} \) where for simplicity it is assumed that the \( \text{Pb}^{208} \) ground state is a closed core and the \( 2g_{9/2} \) single-particle state in \( \text{Pb}^{209} \) is a pure configuration. Then in the stripping reaction to the \( 2g_{9/2} \) state in \( \text{Pb}^{209} \) the single-particle strength measured would be unity and that measured in the pickup reaction would be zero satisfying the sum rule. If the \( 2g_{9/2} \) configuration were now spread over several bound states in \( \text{Pb}^{209} \), the sum rule would still be valid and yield 1.0 when all the fragments of single-particle strength are summed. On the other hand, if the single particle strength is carried away by states that lie in the continuum, then the sum rule clearly will not be satisfied since this strength will not be included in the sum which is only over the bound states.

The existence of fragmentation of single-particle strength in \( \text{Pb}^{209} \) because of interaction with nearby states has already been illustrated in the case of the \( 1j_{15/2} \) "single-particle" state at 1.424 Mev excitation. The \( 15/2^- \) state, as has been noted previously, appears to have a significant admixture of the particle-vibration state \( (\nu \text{Pb}^{208}(3^-) \times g_{9/2})_{15/2^-} \) which accounts for the large decay strength of this state to the ground state. Hamamoto (HA69) has recently made a model calculation for this state and has obtained the wave function,
\[ |_{15/2}^{->} = c_1 |0^+ 1j_{15/2} > + c_2 |^{3^-} x 2g_{9/2}^{15/2} > + c_3 |^{3^-} x 1i_{11/2}^{15/2} >, \]

with
\[ c_1 = 0.645; c_2 = 0.349; c_3 = 0.005, \]

where this indicates that the state contains only about 65% of the single-particle configuration in agreement with the recent measurements of the spectroscopic factor. This wave function can be used to calculate the reduced transition matrix element to the ground state and yields,

\[ B(E3) (15/2^{+} \rightarrow 9/2^{+}) = 5.7 \times 10^4 \text{ e}^2 \text{ fm}^2, \]

which is in good agreement with the experimentally determined value of \( (7.2 \pm 2.0) \times 10^4 \text{ e}^2 \text{ fm}^2 \) (EK69). Fragments of the missing single-particle strength are predicted to be found at around 3.4 MeV and 3.5 MeV (HA69), but as noted previously they have not been experimentally located.

The other single-particle states are of positive parity and are not expected, from simple shell model arguments, to fragment to such a large degree because of the absence of nearby bound positive parity states with which they can mix strongly. The closest positive parity states are estimated on the basis of these arguments to be located at about 4.0 MeV excitation. These states are unbound and thus the fragmentation of the single-particle strength because of these states falls in the province of
the second category of strength reduction.

Various mechanisms for the loss of single-particle strength to states which lie in the continuum have been proposed. To mention a few; (1.) the effects of hard core correlations of the Bruckner theory (which have been estimated by Brandow (BR67)) give a depletion of 15-20%, (2.) the coupling to vibrational states in the core (calculated by Bertsch and Kuo (BK68)) gives a depletion of 15-25%, and (3.) the coupling of the ground state channel to excited state channels (calculated by McKeller (MC70)) gives a depletion of about 10%. In all these calculations some, or all, of the depleted single-particle strength is transferred to states in the continuum.

Recently a somewhat different approach was taken by Hamamoto (HA70) who performed a calculation which allowed for the prediction of admixtures of single-particle strength into explicit particle-vibration states of $^{209}$Pb which are unbound. The predictions for some of these states are listed in Table II-l to illustrate the magnitudes of the fragmentation predicted.

### Table II-l

<table>
<thead>
<tr>
<th>Single-Particle Component</th>
<th>Admixed Probability</th>
<th>Main Configuration</th>
<th>Calculated Excitation Energy (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g_{9/2}$</td>
<td>0.09</td>
<td>$3^{-} x j_{15/2} &gt; 9/2^{+}$</td>
<td>4.32</td>
</tr>
<tr>
<td>$d_{5/2}$</td>
<td>0.03</td>
<td>$4 x g_{9/2} &gt; 5/2^{+}$</td>
<td>4.22</td>
</tr>
<tr>
<td>$d_{3/2}$</td>
<td>0.04</td>
<td>$4 x g_{9/2} &gt; 3/2^{+}$</td>
<td>4.28</td>
</tr>
<tr>
<td>$s_{1/2}$</td>
<td>0.13</td>
<td>$4 x g_{9/2} &gt; 1/2^{+}$</td>
<td>4.59</td>
</tr>
</tbody>
</table>
The particle-vibration states formed by the weak coupling of a single particle to the collective core excitations of Pb$^{208}$ will be described in more detail in the next section. It can be noted that the single-particle admixture predicted in these states are some 3-13% of the single-particle strength.

From the results of the calculations noted above, it is clear that up to about 25% or more of the full single-particle strength may be expected to be transferred to unbound states. Whether this is an accurate estimate of the reduction is hard to say since there is little, if any, experimental information about such strength in the continuum. Indeed, it is very difficult to obtain such information and it is safe to say that all this possible strength, spread out over an excitation of perhaps 5-50 Mev can never be found. The most that might be realistically expected, would be the detection of the larger fragments of strength mixed into the unbound states of the type which Hamamoto has studied. Such information would be valuable in assessing how well the calculations work in this limited sense and thus perhaps give an indication of how far to trust the estimates of the total strength mixed in the continuum. The experimental detection of even the larger fragments of single particle strength mixed into unbound states is a difficult task. Such information could possibly come from the study of the Pb$^{208}$($n,\gamma$) reaction. The use of the (d,p) reaction to obtain such information is limited by the lack of a suitable theoretical method for the
analysis of stripping to unbound states and thus can give
at best only an order of magnitude indication of such
strength. If the estimates of the amount of strength
transferred to states in the continuum are correct, then a
look should be taken at some of the long accepted practices
in the usual DWBA analyses and interpretations of reaction
data.

Another matter of some interest involves the neutron
single-particle states of the next shell \( n > 184 \) which are
unbound in \( \text{Pb}^{209} \) and which are expected to be found some­
where in the excitation region about 8.0-10.0 Mev. In
figure II-3 the location and level ordering of the orbits
of this shell are estimated by extrapolating the results
obtained in the theoretical studies of heavier nuclei
where these orbits are bound (SG66). It can be seen that
the single-particle orbits \( 2h_{11/2}, 4p_{3/2} \), and \( 3f_{7/2} \) are
predicted to be the lowest lying unbound orbits. Experi­
mentally, there is very little if anything known about
these states.

These single-particle states are of interest since
they might be expected to mix into states of lower
excitation in \( \text{Pb}^{209} \) which are bound. An example of this
has been calculated by Hamamoto (HA69) in her treatment
of the 11/2\(^-\) member of the (\( \text{Pb}^{209}(3^-) \times g_{9/2} \)) particle­
vibration multiplet. In this calculation the wave
function for the 11/2\(^-\) state predicted at 2.61 Mev was
found to be composed primarily of the configuration
Figure II-8

Estimated level ordering of the unbound neutron single-particle states in Pb$^{209}$ based on predictions of the level ordering in heavier nuclei (CERN).
(\(\text{Pb}^{208}\ (3^-) \times g_{9/2}\)) with a small admixture of the configuration \((\text{Pb}^{208}\ (0^+) \times h_{11/2}\)) whose amplitude was given as \(\sqrt{0.027}\). The admixtures of these single-particle configurations (such as the \(2h_{11/2}\)) into bound states at lower excitation can be detected in the studies of the \((d,p)\) reaction. In this reaction, states which normally could not be excited in a direct transfer reaction will be excited via the small admixture of single-particle configuration. Again we have the case of single-particle strength spreading over large energy ranges, except in this instance it is the strength in the continuum mixing into states which are bound.

\[\text{b. Two Particle-One Hole States}\]

The success of the simple shell model in predicting the general features of the particle-hole structure of the higher excitations in \(\text{Pb}^{208}\) prompts us to examine such a description for \(\text{Pb}^{209}\). From the shell model, the higher excitations in \(\text{Pb}^{209}\) following the single-particle \((1p)\) states should be the two particle-one hole \((2p-1h)\) states formed by the promotion of a particle from below the Fermi sea to an orbit above it. For example, the configuration \(( (g_{9/2})^2 \ J ( p_{1/2})^{-1})_{J^+} \) can be formed by the promotion of a neutron into the \(2g_{9/2}\) orbit leaving a hole in the \(3p_{1/2}\) orbit. An entire level scheme of \(2p-1h\) states can be formed in this way promoting both protons and neutrons from filled orbits below the Fermi sea to single-particle
orbits above it and constructing a zero-order description of the Pb$^{209}$ spectrum in the same way the spectrum of Pb$^{208}$ was developed and described in the last section.

Let us consider in more detail the neutron configuration ((g$_{9/2}$)$^2$ (p$_{1/2}$)$^{-1}$) which would be expected to have the lowest excitation energy. The two (g$_{9/2}$) neutrons can couple to spins of 0$^+$, 2$^+$, 4$^+$, 6$^+$, and 8$^+$. The configuration ((g$_{9/2}$)$^2$ (p$_{1/2}$)$^{-1}$))$_{1/2}^-$ is expected to the lowest lying state because of the pairing effect and is predicted to be found at 2.20 MeV. The other configurations with the two (g$_{9/2}$) neutrons coupled to 2$^+$, 4$^+$, 6$^+$, and 8$^+$ are expected in zero-order to be located at 3.37 MeV. If residual interactions are introduced these states are shifted in energy and no longer should be degenerate.

To estimate the effects of these residual interactions we can look at the spectrum of Pb$^{210}$. In the most naive picture one might expect the ground state and low lying spectrum to be described by the two (g$_{9/2}$) neutrons coupled to 0$^+$ and then to the other possible spin values. Indeed this description is not too far from being correct since the ground state and lowest excited states (which have the level ordering 2$^+(0.795$ MeV), 4$^+(1.092$ MeV), 6$^+(1.190$ MeV), and 8$^+(1.271$ MeV)) are found experimentally (BI70) to consist of the (g$_{9/2}$)$^2$ configuration. The situation however is a bit more complex with the inclusion of configurations such as (i$_{11/2}$)$^2$, (j$_{15/2}$)$^2$, etc. Since the low lying states of Pb$^{210}$ are known, we can depart from a strict simple shell
model description for the 2p-1h states of Pb$^{209}$ and use this information to construct a level spectrum for Pb$^{209}$ which includes at least some of the effects of the residual interactions. For example, instead of using the configuration \((g_{9/2})^2 \cdot (d_{1/2}^{-1})_{1/2} \cdot 1/2^{-}\) to predict the location and character of the lowest 2p-1h state we can use the description \((Pb^{210}(0^+) \times d_{1/2}^{-1})_{1/2}^{-}\). Similarly we can use the observed energies of the low lying Pb$^{210}$ states and the Pb$^{207}$ hole states to predict the location of the other 2p-1h states of the character \((Pb^{210}(J^π) \times (nlj)^{-1})\). These predicted positions are shown in Figure II-9 together with 2p-1h states of a different character which are also expected and which we now consider.

The collective excitations in the Pb region as we have already seen in our discussion of Pb$^{208}$, do not fit into the framework of a simple shell model description. The dominant collective excitation as discussed previously (See Figure II-4) appears as a single level in the even nuclei but split up into several levels in the odd nuclei. This has been interpreted as weak coupling of the odd particle (hole) to the collective excitations of the core. Thus, for example, in Bi$^{209}$ the septuplet of collective levels observed at about 2.6 Mev excitation is thought to consist of members of the multiplet formed by the coupling of the 1$h_{9/2}$ proton to the Pb$^{208}$ octupole vibration (HW66), and the doublet observed in Pb$^{207}$ is thought to comprise the states formed by the coupling of the 3$p_{1/2}$ hole to the
Figure II-9

Zero-order weak coupling description of the 2p-1h states in Pb\(^{209}\),
same octupole vibration of $\text{Pb}^{208}$ (IW66). It is expected then that weak coupled states of the type $(\text{Pb}^{208} (J^m) \times (nlj))$ should be found in $\text{Pb}^{209}$ in which the collective states of $\text{Pb}^{208}$ couple to the various neutrons outside the core (MO67). These states are also of the form 2p-lh because the collective states in $\text{Pb}^{208}$ are of a 1p-lh character being composed of a superposition of many 1p-lh configurations. The zero-order positions of these 2p-lh states have been shown in figure II-9 together with those formed by the coupling of the various hole states to the particle excitations of $\text{Pb}^{210}$. These two sets of basis states then form the description of the 2p-lh level structure of $\text{Pb}^{209}$.

The question now arises as to what experimental evidence there is to test such a description of the 2p-lh states in $\text{Pb}^{209}$. Two transfer reactions have been studied recently which strongly populate certain kinds of 2p-lh states in $\text{Pb}^{209}$. These are the $\text{Pb}^{207} (t,p) \text{Pb}^{209}$ (BH68, FI69) and the $\text{Pb}^{210} (p,d) \text{Pb}^{209}$ (FI69) reactions. The results are shown graphically in Figure II-10 (FI69).

The $\text{Pb}^{207}(t,p)\text{Pb}^{209}$ reaction strongly populates states of the configuration,

$$[\text{Pb}^{207}(1/2^-) \times [(n_{11}l_{11})(n_{21}l_{21})] J J_{\pm \frac{1}{2}},$$

where $J$ represents the various spins to which the two transferred neutrons can couple. However, if the $\text{Pb}^{207}$ ground state is taken to be a pure $3p_{1/2}^{-1}$ configuration
Table: Excitation Energy (MeV) for Pb$^{210}$ and Pb$^{207}$

<table>
<thead>
<tr>
<th>L = 1</th>
<th>L = 3</th>
<th>L = 6</th>
<th>L = 0</th>
<th>L = 2</th>
<th>L = 4</th>
<th>L = 6</th>
<th>L = 8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1/2^-$</td>
<td>$3/2^-$</td>
<td>$5/2^-$</td>
<td>$7/2^-$</td>
<td>$9/2^-$</td>
<td>$11/2^-$</td>
<td>$13/2^-$</td>
<td>$15/2^+$</td>
</tr>
</tbody>
</table>

Figure II-10

Graphic Display of the spectroscopic factors obtained in the Pb$^{210}$(p,d)Pb$^{209}$ study (FI69) and the cross sections measured in the Pb$^{207}$(t,p)Pb$^{209}$ study (FI69) of 2p-1h states in
and is assumed to be a spectator in the reaction then the states populated strongly in the reaction may be assumed to be of the character \((p_{1/2}^{-1}) \times \text{Pb}^{210}(J^\pi)\) where \(J^\pi\) are the spins of the low lying even parity states in \(\text{Pb}^{210}\). That is to say, the reaction is identical to the \(\text{Pb}^{208}(t,p)\text{Pb}^{210}\) reaction except that there is a passive \(p_{1/2}^{-1}\) spectator, and whereas in the \(\text{Pb}^{208}(t,p)\text{Pb}^{210}\) reaction a single level is populated in the \(\text{Pb}^{207}(t,p)\text{Pb}^{209}\) reaction two levels are populated corresponding to the two ways of coupling the \(p_{1/2}^{-1}\) hole to the state of \(\text{Pb}^{210}\).

For example, in the \(\text{Pb}^{208}(t,p)\text{Pb}^{210}\) reaction an \(L = 4\) transition was observed to the level at 1.092 Mev in \(\text{Pb}^{210}\) and in the \(\text{Pb}^{207}\) reaction two \(L = 4\) transitions were observed to levels at 3.026 Mev and 3.203 Mev corresponding to the states of spin \(9/2^-\) and \(7/2^-\) formed by coupling of \(4^+\) to \(\frac{1}{2}^-\). The sum of the cross sections measured for these two transitions was equal, within experimental errors, to that measured for the \(L = 4\) transition to \(\text{Pb}^{210}\). The actual situation is a little more complicated than the one just outlined since the \(2p-1h\) states in \(\text{Pb}^{209}\) can mix and thus more than two states can have pieces of the configuration \((\text{Pb}^{210}(J^\pi) \times p_{1/2}^{-1})\); however, in all cases it was found that the sum of the cross sections for any one \(L\) transfer to the various \(2p-1h\) states was equal to the cross section measured for the corresponding \(L\) transfer in the \(\text{Pb}^{208}(t,p)\text{Pb}^{210}\) study. The results of the \(\text{Pb}^{207}(t,p)\text{Pb}^{209}\) reaction for the strongly populated states in the excitation
region up to 3.5 Mev are shown listed in Table 11-2.

The $^{210}\text{Pb}^2(p,d)^{209}\text{Pb}$ reaction strongly populates states with the configurations $(\text{Pb}^{210}(0^+) \times (n\ell j)^{-1})_j$, where $n\ell j$ correspond to the neutron orbits in the shell $N = 82-126$. The results for the more strongly populated states observed in this study are shown in Table II-3. It will be noted that the single-hole strength is significantly fragmented in some cases because of interactions with other nearby states. The spectroscopic sum rule, however, is found to be satisfied for all the single-hole states except the $2f^1_7$ hole state whose strength was found at higher excitation.

Taken together the results of the two studies show a number of things about the level structure of the $2p$-$1h$ states in $^{209}\text{Pb}$. First it shows that the simple model description for these states is in good agreement with what is observed experimentally. In particular, the predicted centroid positions of the configurations of character $(\text{Pb}^{210}(J^+) \times n\ell j^{-1})$ were found to be in good agreement with those observed in the studied. A second point is that a significant amount of mixing is observed between the $2p$-$1h$ configurations which make it possible to obtain information about $2p$-$1h$ states of different basic configuration. For example, in the excitation region around 2.0-3.0 Mev there were observed more states than could be accounted for in terms of the basis states built only on $^{210}\text{Pb}$ states coupled to holes, and these extra states appear to be the $2p$-$1h$ states whose main configura-
### TABLE II-2

<table>
<thead>
<tr>
<th>Level No.</th>
<th>$E_x$ (keV)</th>
<th>L</th>
<th>$J^\pi$</th>
<th>$g(209)/g(210)$</th>
<th>$\Sigma g(209)/g(210)$</th>
<th>Centroid Relative to 2152 KeV Level (keV)</th>
<th>$E_x(210)$ (keV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1$^a$</td>
<td>2152</td>
<td>0</td>
<td>$1/2^-$</td>
<td>1.02</td>
<td>1.02</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4$^a$</td>
<td>2737</td>
<td>2</td>
<td>$5/2^-$</td>
<td>0.25</td>
<td>0.38</td>
<td>697</td>
<td>795</td>
</tr>
<tr>
<td>5$^a$</td>
<td>2868</td>
<td>2</td>
<td>$5/2^-$</td>
<td>0.38</td>
<td>1.02</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6$^a$</td>
<td>2902</td>
<td>2</td>
<td>$3/2^-$</td>
<td>0.38</td>
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<td></td>
<td></td>
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<tr>
<td>7$^a$</td>
<td>3028</td>
<td>4</td>
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<td>1.00</td>
<td>955</td>
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<td>3026</td>
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<td>$(9/2^-)$</td>
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<tr>
<td>8</td>
<td>3072</td>
<td>6</td>
<td>$(11/2^-,13/2^-)$</td>
<td>0.43</td>
<td></td>
<td>1.00</td>
<td>1056</td>
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<tr>
<td>12</td>
<td>3309</td>
<td>6</td>
<td>$(11/2^-,13/2^-)$</td>
<td>0.58</td>
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<td></td>
<td>1187</td>
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<tr>
<td>10</td>
<td>3100</td>
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<td>$(15/2^-,17/2^-)$</td>
<td>0.41</td>
<td></td>
<td></td>
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<tr>
<td>13</td>
<td>3432</td>
<td>9</td>
<td>$(15/2^-,17/2^-)$</td>
<td>0.30</td>
<td></td>
<td>1.07</td>
<td>1198</td>
</tr>
<tr>
<td>15$^a$</td>
<td>3561</td>
<td>9</td>
<td>$(15/2^-,17/2^-)$</td>
<td>0.36</td>
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<td></td>
<td>1268</td>
</tr>
</tbody>
</table>

$^a$ Seen in both (t,p) and (p,d) reactions.

### TABLE II-3

<table>
<thead>
<tr>
<th>Level No.</th>
<th>$E_x$ (keV)</th>
<th>$J^\pi$</th>
<th>$E_x$ (keV) Relative to 2152 KeV Level in $^{207}$Pb$^b$</th>
</tr>
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<tbody>
<tr>
<td>1$^a$</td>
<td>2153</td>
<td>$1/2^-$</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>2320</td>
<td>$3/2^-$</td>
<td>749 (2128)</td>
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<tr>
<td>6$^a$</td>
<td>2906</td>
<td>$3/2^-$</td>
<td>749 (2128)</td>
</tr>
<tr>
<td>9</td>
<td>3077</td>
<td>$3/2^-$</td>
<td>749 (2128)</td>
</tr>
<tr>
<td>3</td>
<td>2463</td>
<td>$5/2^-$</td>
<td>570 (2128)</td>
</tr>
<tr>
<td>4$^a$</td>
<td>2741</td>
<td>$5/2^-$</td>
<td>570 (2128)</td>
</tr>
<tr>
<td>5$^a$</td>
<td>2873</td>
<td>$5/2^-$</td>
<td>570 (2128)</td>
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<tr>
<td>7$^a$</td>
<td>3031</td>
<td>$7/2^-$</td>
<td>2048 (2128)</td>
</tr>
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<td>14</td>
<td>3499</td>
<td>$7/2^-$</td>
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<td>3902</td>
<td>$7/2^-$</td>
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<tr>
<td>24</td>
<td>3995</td>
<td>$13/2^+$</td>
<td>1633 (2128)</td>
</tr>
</tbody>
</table>

$^a$ Seen in both (t,p) and (p,d) reactions.

$^b$ Data normalized.
tions are based on the core excitations of Pb\textsuperscript{208} coupled to a particle. In this specific example, the states at 2.320 Mev and 2.463 Mev were identified as the \( \frac{3}{2}^- \) and \( \frac{5}{2}^- \) members of the \((\text{Pb}\textsuperscript{208}(3^-) \times g\frac{9}{2})\) multiplet.

Hence the studies of the \text{Pb}\textsuperscript{207} \((t,p)\text{Pb}\textsuperscript{209}\) and \text{Pb}\textsuperscript{210} \((p,d)\text{Pb}\textsuperscript{209}\) reactions have established the locations and primary configuration of a large number of 2p-1h states in the excitation region up to 4.0 Mev in Pb\textsuperscript{209}. From these studies it appears that the 2p-1h level structure is well described qualitatively in terms of the zero-order weak coupling scheme involving single particles and holes coupled to the excitations of Pb\textsuperscript{208} and Pb\textsuperscript{210}, respectively. Given the simplicity of the model, and the large densities of states involved, it is rather remarkable that this description works as well as it does.

The next logical step is a more realistic treatment of the residual interactions to reproduce the configuration mixing that is observed experimentally. It should be pointed out that there are several types of configurations which will be involved in these calculations. These are the single-particle states of the shell \( N = 126-184 \), the 2p-1h states of the Pb\textsuperscript{208} core excitations coupled to particles, the 2p-1h states of the Pb\textsuperscript{210} excitations coupled to holes, and finally the single-particle states of the shell \( N>184 \) which are unbound in Pb\textsuperscript{209} but which nevertheless can mix into bound states below. Thus the 2p-1h states in the excitation region 2.0-4.0 Mev which are of interest in this
study might be made up primarily of the two types of basis states along with possible small admixtures of single-particle configurations from either the shell $N = 126-184$ or the shell $N > 184$.

Calculations of the $^{209}$Pb $2p$-$1h$ structure have recently been made by Hamamoto (HA69, HA70) and by Bes and Broglia (LA70) who have emphasized different aspects of the structure. Hamamoto has taken the particle-vibration states of character ($^{184}$Pb $^3\Sigma$) $\times$ nlj) and evaluated the effects on these states of interactions with the other states in the spectrum. The results of this calculation are the locations and wave functions of the particle-vibration states. While Hamamoto's calculation has the shortcoming of not including in all cases the effects of the $2p$-$1h$ states of character ($^{210}$Pb $^3\Sigma$) $\times$ nlj$^{-1}$, it does calculate the admixtures of single-particle configurations of both the bound orbits and those that are unbound. The calculations of Bes and Broglia take $2p$-$1h$ states built on both $^{208}$Pb and $^{210}$Pb and attempt to describe the $^{209}$Pb level structure up to 4.0 MeV of excitations. Their treatment allows for mixing of these $2p$-$1h$ configurations as well as mixing with the bound single-particle states. The admixture of single-particle configurations which are unbound is not included. Both calculations take into account the effect of ground state correlations.
1. Summary of Theoretical Models

1. Shell Model Calculations

In the nuclear shell model it is assumed that the nuclear analogy of an atomic Hartree-Fock calculation has been performed. A set of single-particle wave functions \( \phi_i(x) \) and energies \( E_i \) is derived which are the eigenstates of some self-consistent potential \( V_{\text{sm}}(x) \). The basic nuclear many-body wave functions are the Slater determinants of these single-particle wave functions. It is assumed that the potential is local and remains unchanged over some range of neighboring nuclei. The potential is also assumed to be spherically symmetric and to involve a one body spin-orbit term, that is,

\[
V_{\text{sm}}(x) = V_0(x) + V_{ls}(x) \ell \cdot \bar{\sigma}
\]

It is further assumed that there is an effective residual interaction left over after the self-consistent potential has been constructed.

In practice no serious attempt to solve such a self-consistent calculation for the potential is made, but instead, a phenomenological potential is chosen. The short range of the nuclear forces suggests that the potential roughly follow the shape of the density distribution, and thus as a first approximation we might consider a square well potential. Still simpler and qualitatively similar, is the harmonic oscillator potential, which is therefore extensively used and serves the role in nuclear physics.
comparable to the Coulomb potential in atomic physics. The harmonic oscillator potential, however, does not give rise to the observed magic numbers (BM69). The potential which has a more realistic radial dependence (HO63) is the Wood-Saxon potential,

$$V(r) = V_0 \left[ 1 + \exp\left( \frac{r-R}{a} \right) \right]^{-1},$$  \hspace{1cm} B.2

where $R$ is the radius of the nucleus and is usually taken to be $R = r_0 A^{1/3}$, and $a$ is the diffuseness of the well. This potential modifies the calculated sequence of orbits, but still does not account for the spacing between major shells. It is only after a relatively strong spin-orbit term is added to the central potential that the spacing of the major shells are produced (HJ49, MA49, WJ55). When a spin orbit term of strength about 25-30 times the Thomas term is introduced, gaps between those orbits corresponding to the observed magic numbers are predicted.

The sequence of single-particle levels that results when the spin-orbit term is added to the central potential has been confirmed by a large body of experimental evidence. This information, as mentioned previously, can be most easily obtained from stripping and pickup reactions on magic nuclei. The situation in the vicinity of the doubly magic Pb$^{208}$ ($Z = 82, N = 126$) has already been illustrated in figure II-1 and II-2 which respectively show the predicted single-particle and hole orbits and those observed experimentally. It can be seen from the figures that the shell
model provides a very good description of the observed low lying excited states in the nuclei adjacent to Pb$^{208}$. Similar results in other mass regions have confirmed the fact that the shell model provides a good description of the low-lying excited states of nuclei adjacent to doubly magic nuclei. In view of this demonstrated success of the shell model, it is of interest to include more complex nuclear structure within the same framework.

In the simplest picture, the excited states of doubly magic nuclei are expected to be states whose configurations are obtained by exciting a single particle from the highest completely filled orbits into the lowest unfilled orbits. It is found however that such a theoretical spectrum obtained by ignoring residual interactions is almost always in drastic disagreement with that seen experimentally. For example, in $^{16}$O ($Z = 8, N = 8$) the lowest negative parity state is predicted at about 10 Mev and the lowest positive parity state at about 20 Mev, while experimentally there are found seven states (four of negative parity and three of positive parity) in the excitation region 6.0 Mev to 10 Mev. From this example and those for other doubly magic nuclei, it became apparent that if the experimental spectra were to be reproduced strong configuration mixing because of the residual interactions would be required.

The calculation which include the possibilities of configuration mixing use a Hamiltonian of the form;
\[ H = \sum_i \left[ -\frac{\hbar^2}{2m} \nabla_i^2 + V_{sm}(x_i) \right] + \sum_{i>j} V_{RIS}(x_{ij}) \]  
\[ H = H_0 + H_1 \]

where,
\[ H_0 \phi_K(x_1 \cdots x_A) = \varepsilon_K \phi_K(x_1 \cdots x_A) \]  
with
\[ \phi_K(x_1 \cdots x_A) = (A!)^{-\frac{1}{2}} \text{det}_K \phi_i(x_j) \]  
and
\[ \varepsilon_K = \sum_{i=1}^A \varepsilon_i \]

The single-particle wave functions \( \phi_i(x) \) and \( \varepsilon_i \) are given in principle from the equation,
\[ -\frac{\hbar^2}{2m} \nabla_i^2 + V_{sm}(x) \phi_i(x) = \varepsilon_i \phi_i(x) \]

but in practice well known functions (such as the harmonic oscillator function) are usually used and the single-particle energies are deduced from reaction data. The symbol \( K \) denotes a specific configuration; i.e., distributions of the particles over the states. The total wave function \( \psi(x_1 \cdots x_A) \) of the system is then taken to be a linear combination of the determinantal wave functions,
\[ \psi(x_1 \cdots x_A) = \sum_K C_K \phi_K(x_1 \cdots x_A), \]  
and a solution to the calculation is obtained when the \( C_K \) coefficients have been determined.
The residual interaction $v_{RES}^{x_i}$ used in the calculation has been treated in several ways. Essentially three different approaches have been taken. Either one can choose a very simple semi-phenomenological residual two-body interaction and fit the interaction parameters to the data (e.g. Elliott and Flowers, (EF55)), or one can parameterize the problem in terms of the two-body matrix elements themselves (e.g. Talmi and Unna, (TU62)), or finally one can attempt a simplified reaction matrix calculation (e.g. Kallio and Kolltveit (KK64)). For details concerning the methods of calculation and their results, the reader is referred to the references listed above.

It is found that these calculations, in general, are able to explain reasonably well the positions of the odd parity states in the doubly magic nuclei. The extent of the agreement that is obtained can be found in the calculations of Elliot and Flowers (EF55), and Kallio and Green (KG66) for $^{16}_O$, Horie and Yokozawa (HY63) for $^{40}_{Ca}$, and Carter, et al., for $^{208}_{Pb}$ (CP60). The calculations, however, are not able to describe well the location of all the states in the spectra. There are notable exceptions such as the $1^{-}$ state at 9.59 Mev in $^{16}_O$ and the $3^{-}$ state at 2.6 Mev in $^{208}_{Pb}$ whose predicted excitation energies are much higher than the observed ones. In particular, the positions and wave functions of the collective levels of $^{208}_{Pb}$ are not well reproduced by the calculations of Carter, et al. (See figure II-5).
The positive parity states, which are 2p-2h configurations in these calculations, are even more poorly described by these shell model calculations than are the negative parity states. The model predicts states which are much too high in excitation and which possess electromagnetic transition rates which are much smaller than that observed (IR67).

Better agreement with experiment for both the negative and positive parity states is obtained when we relax the requirement of doubly closed shell configurations. The calculations which include shell breakage and hence introduce correlations into the ground state wave function will be discussed next.

2. RPA Calculations

Thus far we have discussed calculations formulated to terms of single-particle representations in which it is assumed that the nucleons, apart from being bound in a shell-model potential, are essentially free. It is not clear that such representations are best suited for the description of a highly correlated many-body system. This is strongly suggested by the results of calculations which use this "free" particle representation where many shell-model configurations are found to be necessary to describe the spectra adequately. Thus consideration is given to calculations which describe the system in terms of "quasi-particles", where each quasi-particle is compro-
mised of a subset of many shell-model configurations. This formalism has the potential to describe the excited states of the nucleus in terms of perhaps a relatively few quasi-particle states.

In order to illustrate what is meant here let us consider a simple example. In the Goldhaber-Weneser model (GW55) of vibrations it is assumed that the vibrations can be described as harmonic oscillator bosons, where the excited state $|\alpha\rangle$ is described by the creation operator $\omega_\alpha^+$; i.e.,

$$|\alpha\rangle = \omega_\alpha^+ |0\rangle,$$

where the ground state wave function is denoted here by $|0\rangle$. Since these operators are bosons they must obey the usual commutation rules,

$$\left[ \omega_\alpha, \omega_\beta^+ \right] = \left[ \omega_\alpha^+, \omega_\beta \right] = 0$$

If the Hamiltonian $H$ is chosen here such that $H|0\rangle = 0$, then the energies of the excited states will be given in relation to the ground state; i.e.,

$$H|\alpha\rangle = H \omega_\alpha^+ |0\rangle = H \omega_\alpha^+ |0\rangle = E_\alpha |\alpha\rangle = E_\alpha \omega_\alpha^+ |0\rangle$$

When the Hermitian conjugate of this expression is taken a second equation is obtained. These two "equations of motion"
$$[H, \omega_\alpha^+] = E_{\alpha} \omega_\alpha^+$$

$$[H, \omega_\alpha^-] = -E_{\alpha} \omega_\alpha^-$$

...together with the definition of the ground state $|0\rangle$ as the quasi-particle vacuum (i.e. $\omega_\alpha |0\rangle = 0$ for all $\alpha$) describe the vibrational part of the spectrum. What is left to be done is to choose the form of the operators which will satisfy the equations of motion and at the same time provide a description of the nucleus.

We now consider the more general problem. Let us assume that there is a set of operators, $Q_\alpha$, which give configurations in a usual single-particle representation and which satisfy the equation,

$$[H, Q_\alpha] = \sum_\beta M_{\alpha \beta} Q_\beta,$$

where the matrix $M$ has eigenvectors $X_{\alpha \beta}$ and corresponding eigenvalues $E_\alpha$. The quasi-particle operators, $Q_\alpha^+$, can then be defined as,

$$Q_\alpha^+ = \sum_\beta X_{\alpha \beta} O_\beta^*,$$

which, as in the example, will have the ground state $|0\rangle$, as their vacuum,

$$Q_\alpha |0\rangle = 0 \text{ for all } \alpha,$$

and the excited states $|\alpha\rangle$ will have their energy in terms of the ground state. The equations of motion and other properties of the operators (such as orthonormalization...
conditions, etc.) can then be used to determine the matrix elements and define the quasi-particle operators explicitly.

The problem resolves itself into the selection of a set of operators, $0_\alpha$, which satisfy equation (B.13). Except for the simplest set of operators it is not possible to find a set which satisfies the equation exactly and in general some approximations must be made. This can be illustrated by writing the expression for the commutator for a specific choice of operators, $0_\alpha$,

$$\left[ i, 0_\alpha \right] = \sum_\beta M_{\alpha \beta} 0_\beta + \text{other operators}, \quad \text{B.15}$$

where the expression is exact. Since $H$ is generally not in the set, $\{ 0_\alpha \}$ we can not expect the commutator to be entirely in the set. What is done in practice is to choose a set of operators which on physical grounds appear to be most reasonable and then to approximate by discarding those terms which are not in the set $\{ 0_\alpha \}$. This elimination of the non-linear terms in the commutator defines the "Random Phase Approximation". These nonlinear terms contribute matrix elements which are assumed to have completely random phases and to give no contributions on the average.

For the case of a spherical ground state and excited states of $p-h$ character, there are several choices for the operators, $0_\alpha$, which might be made. First, we can take,

$$\{ 0_\alpha \} = \{ A_{\alpha a}^+ \}, \quad \text{B.16}$$
where $A_{Aa}^+$ is the particle-hole creation operator defined as the product of a single-particle creation and annihilation operators; i.e.,

$$A_{Aa}^+ = a_{Aa}^+ a_{Aa}, \quad B.17$$

where the upper case Latin subscript denotes a single-particle orbit above the Fermi surface and the lower case subscript denotes one below the Fermi surface. Thus the operator $A_{Aa}^+$ will create the configuration composed of a particle in the orbit $A$ and a hole in the orbit $a$. This choice of operators is called the Tamm-Dancoff approximation and is equivalent to the shell model calculations of configuration mixing discussed in the last section. An essential point here is that this choice of operators does not introduce correlations into the ground state and when treating a doubly magic nucleus for example, the ground state is still regarded as a doubly closed shell configuration.

A second choice of the operators, $0_a$, could be the set

$$\{0_a\} = \{A_{Aa}^+, A_{Aa}\}, \quad B.18$$

where here not only are particles created above the Fermi level and annihilated below, but we now also have particles annihilated above and created below the Fermi surface. We now for the first time allow the possibility for the ground state to have particles above and holes below the
Fermi surface; i.e., p-h correlations in the ground state. The solution of this problem is called the p-h RPA and it will be discussed in more detail in a separate section.

There are higher classes of operators which might be chosen in treating correlations of p-h character (for example $A^+_{Aa} A^+_{Bb}$ and $A^+_{Aa} A^+_{Bb}$) but very little work has been done with such operators and they will not be discussed further here.

The calculations discussed above have been concerned with the description of p-h type excitations in spherical nuclei where the quasi-particle operators have been given terms of the p-h operators $A^+_{Aa}$. There is no reason, however, for the quasi-particle formalism to be restricted to the description of excited states in the same nucleus as the ground state. For example, quasi-particle operators acting on the ground state of the nucleus A could be used to generate the excited states in adjacent nuclei. The operator set, $\{0^+_\gamma\}$, that will give the description of the spectrum of the $(A+2)$ nucleus could be chosen to be,

$$\{0^+_\gamma\} = \{F^+_A F^+_{ab}\},$$

where $F^+_A$ and $F^+_{ab}$ are taken to be the particle pair-creation operators

$$F^+_A = a^+_A a^+_B$$
$$F^+_{ab} = a^+_a a^+_b$$

where the single-particle operators subscripts have the
same meaning as before. We see here that this set of operators also allows for the possibility of ground state correlations by virtue of the operators $F_{ab}$ which create particles below the Fermi sea. The solution of this problem has been called the two particle (2p) RPA and it also will be discussed at greater length below.

Having now outlined in a very schematic fashion some of the more basic assumptions and the formalism underlining the RPA calculations, we briefly consider the application of the p-h RPA and 2p RPA calculations in the Pb region with particular emphasis on those results pertaining to predictions of ground state correlations of Pb$^{208}$ which can be detected experimentally.

a. Particle-Hole RPA

To arrive at the particle-hole equation two assumptions are made. First, we assume that the actual ground state $|0\rangle$, differs from the Hartree-Fock determinental wave functions $|1\rangle$ in only a perturbative manner. Second, we assume that the generator of the excitations, $\omega^+_\alpha$, to consist only of certain particle-hole operators. The basic set of operators are taken to be the particle-hole operators $a^+_A a^+_a$ and $a^+_a a^+_A$. The quasi-particle operators take the form

$$Q^+_\alpha = \sum_{Aa} \left( X^{aa}_{Aa} a^+_A a^+_a + Y^{aa}_{Aa} a^+_a a^+_A \right),$$

where to emphasize the particle-hole character of these basic operators we have written the single-particle operator
product. To be correct, the particle-hole operators which give properly normalized and antisymmetrized states when acting on the ground state should be used (GS66, AG69, VG70).

We obtain the RPA equations by substituting our operators $Q^+_\alpha$ into the equations of motion and using the method of "linearization of the equations of motion" (also called the "quasi-boson approximation") which has been discussed in a more general context in the last section. The state $|\alpha\rangle = Q^+_\alpha |0\rangle$ will then be an eigenstate of $H$ with eigenvalue $E^\alpha$, and the ground state will be determined by the condition that it be the quasi-boson vacuum.

The operators $a_\alpha^+ a_\alpha$ destroy a particle below the Fermi sea and create one above the Fermi sea, and the operators $a_\alpha^+ a_a$ destroy one above and create one below. Thus the coefficient $X_{\alpha A}$ has the same meaning as in standard shell model calculations; however, the $Y_{\alpha A}$ coefficients will be non-zero only if the ground state wave function exhibits correlations; i.e. if it has higher p-h configurations mixed into the closed shell configuration.

The particle-hole RPA calculations for $^{208}$Pb performed by Gillet, Green, and Sanderson (GG66) show a distinct improvement over the Tamm-Dancoff (TD) calculations. This has been illustrated in figure II-5 where the prediction of the RPA and TD calculations are compared with the experimental spectrum. It was found that most of the levels were not changed much compared to the TD calculation, except for the collective states where the
RPA effects were significant and the predictions were in much better agreement with the experimental data. The introduction of ground state correlations not only improved the energy spectrum, but even more impressively improved the excite to ground states transition amplitudes. While there are still some disturbing discrepancies in these transition rates (notably the deexcitation strength of the $3^-$ state at 2.6 Mev which is measured to be about twice as strong as what is predicted) the calculations in general indicate strong evidence for the p-h RPA and ground state correlations. It should be mentioned that the occupation fractions predicted for Pb$^{208}$ for orbits above the Fermi sea are small and thus seemingly justify the quasi-boson approximation of the RPA calculations (AG69).

Once correlations are introduced into the ground state wave function it is possible to predict population of more complex states in a transfer reaction. That is, since the correlations are configurations of the type $2p-2h$, $4p-4h$, etc, in the ground state function, it should be possible to populate states of $2p-1h$, $4p-3h$, etc, character in the residual nucleus by a simple direct stripping reaction. For example, the states in Pb$^{209}$ of the character $[\text{Pb}^{208}(J^\pi) \times (n_1^lj)^A_ja]$ can be reached by stripping into the hole orbit $ja$ present in the target ground due to the correlations. The spectroscopic factor for this state can be written as (VG70):
which reduces to

\[ S_{a}^{\lambda \pi JT_{Z}; A} = \left| \langle 0 | \left\{ Q(\lambda; \pi JT_{Z}) a_{A} \right\} ja a_{a}^{+} | 0 \rangle \right|^{2}, \]  

where the Y coefficients are tabulated by Gillet, et al. (GG66). The (d,p) reactions to these \( 2p\rightarrow 1h \) states then measures directly the amount of p-h correlations in the ground state and can be used to directly test the coefficients obtained in the RPA calculations if the reaction process is a direct transfer and no higher order reaction processes are involved.

**b. Two-Particle RPA**

To arrive at the two nucleon equation two assumptions are made. First, as in the case of the p-h RPA, the ground state \( |0\rangle \) is assumed to differ from the Hartree-Fock determinental wave function only in a perturbative manner. Second, only certain particle-pair (hole-pair) operators are assumed to make up the generator of excitations \( 0^{+}(\nu^{+}) \) for the \( A+2 \) (A-2) system. The basic set of operators which make up \( 0^{+} \) are particle-pair creation operators \( a_{a}^{+} a_{b}^{+} \) and \( a_{a}^{+} a_{b}^{+} \), while \( \nu^{+} \) is made up of the particle-pair annihilation operators \( a_{a} a_{b} \) and \( a_{A} a_{A}^{+} \). The subscripts, as before indicate single-particle orbits above the Fermi sea when upper case letters are used and orbits below the Fermi
sea when lower case letters are used. The quasi-particle operators then take the form,

$$0^+ = \sum_{A \leq B} V_{AB} a^+_A a^+_B + \sum_{a \leq b} W_{ab} a^+_a a^+_b$$  \hspace{1cm} B.24

for the \(A+2\) system and,

$$p^+ = \sum_{A \leq B} V^*_{AB} a^+_A a^+_B + \sum_{a \leq b} W^*_{ab} a^+_a a^+_b$$  \hspace{1cm} B.25

for the \((A-2)\) system. We have again written out the particle-pair operators in terms of the product of the single-particle operators to emphasize their character.

The form of the operator which give properly normalized and antisymmetrized particle-pair (hole-pair) states can be found in the paper of Vary and Ginocchio (VG60) where the details of the formalism and method of application are discussed.

Similiar to the p-h RPA calculations, the results of the 2p RPA calculations imply the presence of correlations in the ground states of the nucleus \(A\). In specific the operator \(a^+_a a^+_b\) creates particles below the Fermi sea and thus non-zero values for the coefficient \(W_{ab}\) implies admixtures in the ground state. It should be noted here that while the type of correlations treated in the 2p RPA are of a different character than those treated in the p-h RPA the two types of correlation are not mutually exclusive, but have an overlap. Thus one must be careful not to overestimate the amount of correlations as would be the case if the predicted correlations obtained in the two calculations were simply added together.
Recently Vary and Ginocchio (VA70) have performed 2p RPA calculations for nuclei in the Pb region. In their first paper (VG70), the vibration spectra of the nuclei Pb\textsuperscript{206} and Pb\textsuperscript{210} were calculated. In these calculations the single-particle energies levels and the form of the residual interactions were taken to be the same as those used by Gillet, et al. in the p-h RPA calculations of Pb\textsuperscript{208}. The low lying spectra of both Pb\textsuperscript{206} and Pb\textsuperscript{210} appeared to be well described in terms of what were called one boson and two boson states. The one boson states were those resulting from the addition or removal of paired particles, while the two boson states were taken to be states described by the coupling of the pair addition and removal mode to a collective mode of the Pb\textsuperscript{208} core. In general it was found that the calculated structure of the low lying states agreed with the experimental results. The results for the two-nucleon transfer reaction on Pb\textsuperscript{208} are of particular importance for testing the details of these calculations, since they sample the coherent effects of the particle pairing correlations in the ground state. The theoretical predictions are difficult to compare with the experimental results, however because of reaction theory uncertainties in absolute normalization. The comparison of the predicted relative strengths for some of the transitions in the Pb\textsuperscript{208}(p,t)Pb\textsuperscript{206} to those observed are found to be in better agreement than that found in the comparison with shell-model predictions.
The study concludes that the comparison of the 2p RPA calculations with experiment provides strong evidence both for the 2p RPA calculations and ground state correlations.

The particle-pair correlations predicted in the Pb$^{208}$ ground state should make it possible to predict the population of more complex states in the adjacent nuclei by direct stripping reaction, just as the p-h correlations made this possible. Again to be specific, we consider the Pb$^{208}$ (d,p)Pb$^{209}$ reaction. The 2p-1h states in Pb$^{209}$, as we have seen (Section II.3.), are not restricted to only those formed by the coupling of particle to Pb$^{208}$ core excitations. We may also couple holes to the particle excitation of Pb$^{210}$ of the form (Pb$^{210}(\lambda\nu^\tau) \times (n1j)_b^{-1}$) $^{ja}$. The 2p-1h states of this character can be populated through the type of 2p-2h correlation treated in the 2p-RPA by stripping into the hole orbit of the correlation. Thus the 2p RPA can predict spectroscopic factors for the (d,p) reaction to these 2p-1h states which have the general form (VG70):

$$S_{a}^{\lambda_{n}\mu_{J}T_{z}; b} = |<0 | \{0(\lambda_{n}\mu_{J}MT_{z}) a_{b}^{+} \}^{ja} a_{a}^{+} | 0 > |^{2},$$

which reduces to

$$S_{a}^{\lambda_{n}\mu_{J}T_{z}; b} = (1 + \delta_{ab}) \frac{2J + 1}{2J + 1} \frac{2 J + 1}{2J + 1} W_{ab}^{\lambda_{n}\mu_{J}T_{z}} 2^{2},$$

where the coefficients $W_{ab}$ have been computed in the 2p RPA calculation for Pb$^{210}$. The spectroscopic factors measured
for these 2p-1h states in Pb\textsuperscript{209} are then a direct measure of the coefficient \( W \) and thus the magnitude of a specific correlation in the Pb\textsuperscript{208} ground state if, as noted before, the reaction process can be assumed to be a direct transfer process.

3. Particle-Vibration Coupling Calculations

The particle-vibration coupling model attempts to describe the nuclear level structure of odd nuclei where there is reason to believe that the structure might be interpreted as valence particle weakly coupled to a collective excitation (vibration) of the core. The experimental evidence for such particle-vibration coupling in the Pb region is perhaps best illustrated by the \( (\text{Pb}^{208}(3^-)\times h_{9/2}) \) multiplet observed in Bi\textsuperscript{209} which is (Section II.B.3) interpreted as the coupling of the odd \( h_{9/2} \) proton to the octupole vibration of the Pb\textsuperscript{208} core. The small splitting of the observed multiplet indicates that the coupling is weak in this case and thus it is a good zero-order approximation to take the wave function as the simple product of the wave function of the \( h_{9/2} \) proton and the \( 3^- \) vibrational excitation of Pb\textsuperscript{208}. Such a description implies that members of the multiplet should be populated according to their spins by a \( (2J+1) \) rule (M067) in an inelastic scattering experiment. This is in fact what has been observed experimentally (HW66, M067) supporting the validity of this approximate description.
The Hamiltonian which describes the particle-vibration multiplet can be separated into three parts:

\[ H = H_p + H_c + H_{\text{couple}}, \]

where \( H_p \) is the Hamiltonian of the odd particle, \( H_c \) is that of the internal motion of the core, and \( H_{\text{couple}} \) is the interaction Hamiltonian between particle and core. The coupling arises from the fact that the energy of the odd particle is changed when the core deforms from a spherical shape. When the deformations are small the energy change is assumed to be proportional to the deformation and the multipole moment of the odd particle. The coupling Hamiltonian can be chosen to be,

\[ H_{\text{couple}} = K(r_p)(2\lambda+1)^{\frac{1}{2}}(\alpha_{\lambda} m_{\lambda}(p))_0, \]

where we have taken the leading order term in the expansion of the deformation and of the multipole moment. The angular momentum of the vibration is denoted here by \( \lambda \) and the factor \((2\lambda+1)^{\frac{1}{2}} \) is introduced to be consistent with the conventional normalization. The radial form factor \( K(r) \) takes the form (M067),

\[ K(r) = -r \frac{dV(r)}{dr}, \]

where \( V(r) \) is the single-particle potential.

If the observed splitting of the multiplet is small, then the coupling is weak and the problem can be treated by a perturbation calculation. The contributions to the energy of the particle-vibration states are of second order.
in the coupling Hamiltonian and may be represented in the form of the four graphs shown in figure II-11. Graph a represents the effect on the particle-vibration member \( I = j_2 \) because of the admixture of the single-particle state \( j_2 \). The admixture in this case can be written as \( (M_{067}) \):

\[
| (j_1 \lambda)_I > = | (j_1 \lambda)_I > + \epsilon | j_2 = I > , \quad B.31
\]

where

\[
\epsilon = \frac{\langle j_1 \lambda j_2 | H_{\text{couple}} | j_2 \rangle}{E_2 - E_1 - \hbar \omega_\lambda} . \quad B.32
\]

and the corresponding shift in the energy is,

\[
\delta E(I) = -\epsilon^2 (E_1 - E_2 - \hbar \omega_\lambda) \delta(I,j_2) \quad B.33
\]

The effect on the energy because of the process represented by Graph d is of the same form, except in this case \( j_2 \) represents the single-particle component below the Fermi sea, and arises as the consequence of the p-h correlations induced in the core by the odd particle. Graph b represents the effects arising from small coupling of the configuration \( |((j_2 \lambda)j_1, \lambda)_I > \); consisting of two phonons and one particle, \( j_2 \), to the member \( |(j_1 \lambda)_I > \) of the multiplet. The remaining graph (Graph c) represents the physical effect of the vibration disassociating into a particle-hole pair, the hole reassociating with the odd particle while the remaining particle now becomes the odd particle, such an effect gives rise to an exchange force between the odd particle and the
Figure II-11
Diagrams representing the contributions to the particle-vibration states which are in second order in the coupling Hamiltonian (MD67).
vibration. Each of the processes represented by these graphs makes contributions to the particle-vibration members determining their final configuration and excitation energy with respect to their zero-order energy.

Calculations of the type outlined above have recently been applied by Hamamoto (HA69, HA70) to the odd mass nuclei adjacent to Pb$^{208}$. In these calculations the single particle Hamiltonian, $H_p$, uses an appropriate Wood-Saxon well, with a spin-orbit term, to specify the single-particle potential $V(v)$. The core Hamiltonian, $H_c$, satisfies the equation,

$$H_c \hat{\psi}_N = \omega_N \hat{\psi}_N \quad \text{B.34}$$

where $\hat{\psi}_N$ is the wave function of the core Pb$^{208}$ in its $N^{th}$ state, and $\omega_N$ is the energy of the $N^{th}$ state as observed experimentally. The interaction Hamiltonian, $H_{\text{couple}}$ is of the form discussed before where the amplitude of the vibrational motion $\alpha_\lambda$ is determined experimentally from the inelastic cross section. The total wave function $\psi$ can be written as,

$$\psi_{\text{IM}} = r^{-1} \sum_{Nl_n} \sum_{J_n} R_{\text{IM}}(r) \left[ \mathcal{Y}_{l_n J_n} \hat{\psi}_N \right]_{\text{IM}} \quad \text{B.35}$$

where $\mathcal{Y}_{l_n J_n}$ is the spin-angular momentum function of the odd particle. After substitution of the various parts of Hamiltonian and the total wave function into the Schroedinger equation for the system one can obtain,
which represents a set of coupled equations where the energy $E$ can be obtained as a solution of an eigenvalue problem when all the parameters are fixed. Although it is possible, in principle, to solve a set of any number of these coupled equations, it was found convenient by Hamamoto to evaluate the admixtures of some configurations by the perturbation method. The coupled equations were used to obtain a solution only in the cases of strong mixing. In the other cases the admixtures and energy shifts were calculated by a perturbation method which evaluated the contributions of the four graphs shown in figure II-11.

In the lowest order of the coupling Hamiltonian two graphs of figure II-11 (Graphs a and c) can contribute to the cross section of the stripping reaction on the target $^{208}\text{Pb}$, leading to a final state (namely, a state in $^{209}\text{Pb}$ or $^{209}\text{Bi}$) which is approximately a particle-vibration state (HAN69). As discussed above these two graphs express (1) the effects of single-particle admixtures into the final state, and (2) the $2p-2h$ admixtures (correlations) in the $^{208}\text{Pb}$ ground state. Two features of Hamamoto's calcula-
tions make it especially applicable for the treatment of stripping reactions to $2p-1h$ particle vibration states. The first is that although the number of configurations included are finite, the evaluation includes, for a given configuration of the core, contributions from all orbits of the odd particle which have the same angular momentum but differ in the radial quantum number (number of nodes). In this way, the contributions to particle-vibration states from states which lie in the continuum can be evaluated. In $^{209}\text{Pb}^{*}$, for example, this means that contributions to the $(^{208}\text{Pb}(3^-(g_{9/2}^9))_3^3/2)$ state from both the $3p_{3/2}$ orbital below the Fermi sea and the $4p_{3/2}$ orbital above the neutron separation energy are included. The second feature is that the single-particle wave functions admixed into the particle-vibration multiplets are calculated at the energies at which they are observed. This gives the correct radial shape for the single-particle wave functions, which is very important for correctly calculating the magnitude of the cross section of the stripping and pickup reaction. The calculated radial form factors for the single-particle admixture can be used in the standard DWBA codes to predict the cross sections to these states.

C. Scope of the Present Investigation

This dissertation is a study of the excitation of $2p-1h$ states of $^{209}\text{Pb}$ by means of the $^{208}\text{Pb}(d,p)^{209}\text{Pb}$ reaction. From the previous discussion of model calcula-
tions in this chapter, it is clear that 2p-1h states are expected to be weakly populated either through single-particle admixtures in the final state, or through 2p-2h correlations in the Pb^{208} ground state, or both. Some experimental evidence for the excitation of 2p-1h states has been reported in previous neutron stripping studies, however, no systematic study of these excitations has been undertaken and essentially no spectroscopic information concerning these states has been reported.

The purpose of the present experimental study is to establish by careful investigation of the weak transitions (1.) which of the 2p-1h states of Pb^{209} are excited. (2.) whether the proton angular distribution associated with these levels show the usual stripping patterns, and if so, (3.) whether a DWBA analysis of these angular distribution will yield nuclear structure information. The experimental data will provide us with the number of 2p-1h states that are populated by the (d,p) reaction, and whether the transitions observed to these states are consistent with the interpretation of a simple neutron transfer reaction. This latter point is important since it is not clear from calculations how important the contributions from reaction processes which are more complex than the simple one-step stripping will be for the case of weak transitions. Recent calculations (AS70, GL69) indicate that in some cases where the one-step process is going through a small fragment of single-particle excitation mixed into a pre-
dominantly particle-vibration state, the two-step process (i.e., an inelastic excitation of the vibration and then a stripping onto the excited core) may compete. It is of interest then to see whether it will be possible to rule out or at least set a limit on the relative importance of two-step processes in the weak transitions. This is important for providing information about such reaction mechanisms, but of more importance in this study, for determining whether it is reasonable to analyze these transitions on the assumption that they are one-step transfer processes. These questions of reaction theory will be discussed in greater detail in the next chapter.

The motivation of the study is to obtain information concerning both the level structure of Pb\textsuperscript{209} and the ground state of Pb\textsuperscript{208}. In particular this study has the potential of providing information about the distribution of single particle strength in Pb\textsuperscript{209} as well as further information about the locations and character of 2p-1h states not reported previously. It should also be able to obtain information about the deviation of Pb\textsuperscript{208} from a closed shell configuration by observing transitions to 2p-1h states in Pb\textsuperscript{209} which can be populated via stripping into those orbits below the Fermi level which are not entirely filled due to 2p-2h correlations in the ground state. The experimental data can thus provide a test of model calculations for both the Pb\textsuperscript{209} level structure and the Pb\textsuperscript{208} ground-state wave function.
III. REVIEW OF THE (d,p) REACTION THEORY

A. General Considerations

A stripping reaction such as $A(d,p)_3$ reaction is a three body problem, because we must deal with a core $A$, the lighter outgoing particle $p$, and the captured particle $n$ ($B = A + n$). The situation is made more difficult because of the possibility of internal excitation of the core. This problem has not been solved and a model must be constructed to describe the reaction. That used most frequently in recent years has been embodied in the Distorted Wave Born Approximation (DWBA) theory of reactions. The first section of this chapter will outline briefly how the expression of the differential cross section of a $(d,p)$ reaction can be obtained from DWBA. This theory had been successful in describing reaction data in certain circumstances, but fails badly in other cases. The limits of validity of the conventional spectroscopic factor analysis using DWBA will be discussed in the second section of the chapter. The final section will deal with some of the attempts to deal more realistically with the treatment of certain weak transitions where the conventional analysis is not thought to be completely valid.

B. DWBA Theory of the (d,p) Reaction

The details of the formalism and theoretical development of the DWBA theory of reactions has been well described
in the literature (e.g. GL63, SA64, ORNL 3240 (BD62)) and will not be discussed here at any great length. It is of interest, however, to outline some of the basic assumptions in the derivation of the expression for the differential cross section since we will have occasion to refer to some of these in later discussions.

The theory describes the direct reaction from an initial channel to a final channel which in this case will be deuteron and proton channels respectively. The wave function for the reaction in both channels must satisfy the equation,

\[ H\psi = E\psi \quad \text{B.1} \]

where the total Hamiltonian characterizing the reaction in the deuteron and protons channels can be written as,

\[ H = H_d + T_d + V_P + V_n = H_p + T_p + V_P + V_{pn} \quad \text{B.2} \]

The deuteron channel is described by \( H_d \) which is the Hamiltonian for the internal structure of the target \( A \) and of the deuteron, by \( T_d \) which is the kinetic energy of the relative motion of \( d \) and \( A \), and by \( V_P \) and \( V_n \) which represent the interactions of the proton and neutron (of the deuteron) with all the nucleons in the target \( A \). The proton channel is similarly described where \( V_{pn} \) denotes the proton-neutron interaction. The wave functions \( \psi_d \) and \( \psi_p \) which satisfy the equations,
will be used to denote the plane wave solutions in the entrance and exit channels.

The exact transition matrix element connecting the initial and final channels can be obtained from the general theory of scattering (GG53) and for the (d,p) reaction can be written as,

\[ T(d,p) = \langle \chi_p^{-} \left| V_{pn} + V_p^E \right| \psi_d^{(+)} \rangle, \]  

where the subscripts and superscripts on the exact total wave function \( \psi \) denote boundary conditions. The subscript \( d \) indicates that plane waves of deuterons are found at infinity, and the superscript \( (+) \) refers to the boundary condition of outgoing spherical waves. It has been found convenient to rewrite the matrix element in terms of elastic scattering wave functions (or "distorted waves"), \( \chi_p \) where the interactions between the proton and the residual nucleus \( B \) are included. The wave function \( \chi_p \) satisfies the equation,

\[ (H_d + T_d + U_{pB} - E)\chi_p = 0, \]  

where \( U_{pB} \) is the optical model potential which describes the elastic scattering. In terms of these proton distorted waves the transition matrix element can be written as,

\[ T(d,p) = \langle \chi_p^{-} \left| V_{pn} + (V_p^E - U_{pB}) \right| \psi_d^{(+)} \rangle, \]  

where the expression is still exact.
At this point two approximations are usually made to obtain a form of the transition amplitude which can be used in calculations. Since the exact wave function is not known it must be approximated; and is replaced by the distorted waves $\chi_d^{(+)}$ specified by the optical model potential $U_{dA}$ which describes the elastic scattering of deuterons on the target A. This is the Born Approximation and is justified if the elastic channel is only weakly coupled to the other channels. The second approximation which is usually made, is to set, $V^p$, the proton interaction with the nucleus A equal to $U_{PB}$ the proton optical model potential in equation (B.6); that is,

\[(V^p - U_{PB}) = v \approx 0\]  

B.7

While the difference between $V^p$ and $U_{PB}$ should be small, it is clear that they can not be exactly equal since $V^p$ is a real quantity and $U_{PB}$ has an imaginary component. The imaginary part is necessary to account for the inelastic channels which have not been included in our choice of wave functions. The interaction term represented by $v$ is important since it allows for the population of a final state by means of a two step process; i.e., an inelastic scattering followed by stripping onto an excited core and vice versa. The assumption that this term is zero is equivalent to the assumption that only one step processes are important. We will return to this possibility of two step processes in a discussion in a later section (Section
III. C., D.). The final form of the transition amplitude after these two approximations have been made then involves the transition between two elastic scattering states,

\[ T(d,p) \equiv \langle \chi_p^(-) | V_{pn} | \chi_d^+(+) \rangle, \]  

and it is this matrix element which is used in standard DWBA calculations.

Separating the wave functions \( \chi_p \) and \( \chi_d \) into two parts describing the internal wave functions of the interacting nuclei and the wave functions of the relative motion in the corresponding optical potentials, we can write the transitions amplitude for the reaction \( A(d,p)B \) as:

\[ T(d,p) = \int dr_d \int dr_p \chi_p^(-)(k_p, r_p) \langle B | V_{pn} | A, d \rangle \chi_d^+(+) (k_d, r_d), \]  

where \( r_d \) and \( r_p \) are the displacements from the nucleus A and B respectively. The functions \( \chi_p^(-) \) and \( \chi_d^+(+) \) are the distorted waves generated by solutions to the Schrödinger equation. The wave functions can be expanded in terms of spherical harmonics, for example \( \chi_p \) can be expanded as;

\[ \chi_p^(-)(k_p, r_p) = \frac{4\pi}{k_p r_p} \sum_{L=0}^{\infty} \sum_{M=-L}^{L} Y_{L}^{M}(r_p) Y_{L}^{M}(k_p) f_{L}(k_p r_p) \]  

where each partial wave function \( f_{L} \) satisfies the radial Schrödinger equation,

\[ \left[ \frac{d^2}{dr_p^2} + \frac{k_p^2 - \frac{L(L+1)}{r}}{r} - \left( \frac{2m}{\hbar^2} \right) (U_{PB} + U_{R}) f_{L}(k_p r_p) = 0 \right], \]  

with the optical model potential \( (U_{PB}) \).
The factor

\[ \langle B, p \mid V_{pn} \mid A, d \rangle = \int d \xi B^* p V_p n A d, \]  

is the matrix element of the interaction causing the transition. It is taken between the internal states of the initial and final channels and is integrated over all coordinates except \( r_p \) and \( r_d \). The factor contains all the nuclear structure information, as well as reaction information such as angular momentum selection rules and type of reaction.

This matrix element (B.12) can be expanded in terms which correspond to the transfer to the nucleus of a definite angular momentum, \( j \), which is separable into an orbital part \( l \) and a spin part \( s \). If the spins of the initial and final nuclei are \( J_A \) and \( J_B \) and the spins of the proton and deuteron are \( S_p \) and \( S_d \), then we can define

\[ j = J_B - J_A \quad s = s_p - s_d \quad l = j - s (= L_d - L_p), \]

where \( L_p \) and \( L_d \) are angular momenta in the proton and deuteron channels. The expression for the matrix then can be written as;

\[ \langle J_B M_B, s_p m_p \mid V_{pn} \mid J_A M_A, s_d m_d \rangle = \sum_{l s j} \xi_{l s j} \mu_{l s j} \left( \bar{r}_p, \bar{r}_d, \bar{p}_B, \bar{d}_A \right) \]

\[ \times \left( s_p - m_p \right) \langle J_A M_A, M_B - M_A \mid J_B M_B \rangle \langle s_d m_d - m_p \mid s, m_d - m_p \rangle \]

\[ \times \langle l s m, m_d - m_p \mid J_B M_B \rangle, \]

where \( m = M_B - M_A + m_p - m_d \), and where the expression above
does not include the factors arising from isospin (See SA64). In the (d,p) reaction G is proportional to the wave function of the captured neutron times the product of the proton-neutron interaction and the deuteron internal wave function,

\[ G_{lsj m}(\vec{r}_p, \vec{r}_d) \propto \phi_{s}(\vec{r}_n) V_{pn}(\vec{r}_{pn}) \delta_{d}(\vec{r}_{pn}) \], \hspace{1cm} B.14

where \( \vec{r}_{pn} \) is scalar.

The neutron wave function is introduced when the residual nucleus state \( \hat{\mathbf{J}}_{B} \) is expanded in terms of the eigenstates of the target; i.e.

\[ \phi_{J_{B}M_{B}}(\vec{r}_n, \zeta_n \zeta_A) = \sum_{j_{\mu}} \phi_{J_{A}M_{A}}^{*}(\zeta_A) \phi_{j_{\mu}}(\vec{r}_n, \zeta_n) \langle J_{A} j_{\mu} J_{M_{A}} \mid J_{B} M_{B} \rangle \]. \hspace{1cm} B.15

Because of the character of \( V_{pn}(\vec{r}_{pn}) \) only one term in the expansion over \( J_{A}^{*} \) will give a non-zero contribution to the matrix element (B.14), namely \( J_{A}^{*} = J_{A}, M_{A}^{*} = M_{A} \) where \( J_{A} \) is the ground state. That is to say, only those states of the residual nucleus whose parentage is based on the target ground state have a non-zero transition probability. (It is of interest to note here that those states of the residual nucleus whose parentage is based on the excited states of the target (i.e. \( J_{A}^{*} \neq J_{A} \)) could possibly have been populated had the interaction term \( \nu \) (allowing for a two step process) been included.) The bound neutron wave function can further be expanded in terms of spherical harmonics:

\[ \phi_{j_{\mu}(\vec{r}_n, \zeta_n)} = \sum_{l,m} \phi_{l m}^{*}(\theta_n) \chi_{l m}(\zeta_n) \phi_{j_{\mu}}(\vec{r}_n) \chi_{l}(\zeta_n) \]. \hspace{1cm} B.16
where $\psi_l$ is the radial part of the wave function and $\chi_s$ is the spin wave function. The radial wave function is assumed in most standard calculations to be proportional to a shell model wave function of the orbit $(nlj)$, i.e.,

$$\psi_l(r) = \sqrt{S_{Nj}} U_{Nlj}(r), \quad \text{(B.17)}$$

where $N$ is the radial quantum number, and $S_{Nj}$ (the normalization factor) is called the spectroscopic factor. The spectroscopic factor can be shown to be a measure of the overlap of the final state and the initial state plus neutron and thus contains valuable nuclear structure information. It is not clear that the choice of a normalized shell-model wave function for the bound-state radial wave function is always a good approximation and in certain cases it is believed to be quite poor. The procedures for specifying the bound state radial wave function will be discussed in the next section.

The scalar product $V_{pN}(r_{pn})\psi_d(r_{pn})$ in the factor $G_{lsj}$ can be assumed to be approximately equal to;

$$V_{pN}(r_{pn})\psi_d(r_{pn}) \approx D(r_{pn}) \equiv D_0 \delta(r_p - (A/B)r_d) \quad \text{(B.18)}$$

This is known as the "zero-range" approximation and has the physically meaning that the proton is emitted from the same point at which the deuteron was absorbed. This assumption makes it possible to evaluate the transition amplitude (B.9) by reducing the 6 dimensional integral (over $r_p$ and $r_d$) to a 3 dimensional integral. It has been
shown that this approximation is usually reasonable for deuteron stripping for energies and Q values ordinarily encountered (BA66).

The factor G can also be written as a product of two coefficients,

$$G_{l',s',l,m}(\vec{r}_p,\vec{r}_d) = A_{l,s} f_{l,s,j,m}(\vec{r}_p,\vec{r}_d)$$ \hspace{1cm} \text{(B.19)}

where the factor is separated into a spectroscopic coefficient $A_{l,s}$ (containing the spectroscopic factor, $S_{nlj}$, and the proton-neutron interaction strength, $D_0$) and a form factor coefficient $f_{l,s,j}$ which contains the angular dependence. It is usually in terms of these coefficients that the transition amplitude (B.9) is written,

$$T(d,p) = \sum (2l+1)^{1/2} A_{l,s} f_{l,s,j} \langle J_A^M | A'_B^{-M} | J_B^M \rangle \beta_{s,j}^{l,m}(k_d,\vec{r}_d)$$ \hspace{1cm} \text{(B.20)}

where $\beta_{s,j}^{l,m}(k_d,\vec{r}_d)$ is defined as;

$$\beta_{s,j}^{l,m} = (2l+1)^{-1/2} \frac{1}{d_{p}} |d_{p}^{l} \times (-)^{(+) } (\vec{k}_p,\vec{r}_d) f_{l,s,j,m}(\vec{r}_p,\vec{r}_d)$$ \hspace{1cm} \text{(B.21)}

The coefficient $f_{l,s,j}$ contains both the zero-range approximation (i.e. $\delta(r_p-(A/B)r_d)$) which reduces the integral of $\beta_{s,j}^{l,m}$ to a three dimensional integral, and the bound particle form factor $U_{l,j}(r)$.

For computation purposes the distorted waves are inserted into equation (B.21) in the form of the partial-wave expansion of equation (B.10) and the angular inte-
gations of (B.27) are done analytically, yielding (SA64),

$$\beta_{LM}^{t} = \sum_{i} \frac{L_{d}^{L_{p}} - t - i \sigma_{p}^{L_{d}} + i \sigma_{p}^{L_{p}}}{m_{d}^{L_{d}L_{p}}} \left[ \frac{2 \pi (2L_{p} + 1)}{2L_{d} + 1} \right]^{L_{d}L_{p}} \beta_{L_{d}L_{p}}^{t} j$$  \hspace{1cm} (B.22)

where

$$\beta_{L_{d}L_{p}}^{t} \cdot \frac{M_{B}}{M_{d}^{L_{d}L_{p}}} \cdot \frac{4 \pi}{k_{d}} \cdot \frac{1}{k_{d}} \int_{L_{d}}^{L_{p}} (k_{d}, r) U_{L_{d}L_{p}}^{t} j(r) f_{L_{d}L_{p}}^{t} (k_{d}^{M_{d}^{L_{d}L_{p}}}, r) \ dr,$$  \hspace{1cm} (B.23)

for the case of the A(d,p)B reaction.

The explicit expression for the A(d,p)B reaction with transfer of angular momentum $j$ is;

$$T(d,p) = 4 \pi \frac{M_{A}}{M_{B}} \frac{k_{p}}{k_{d}} \sum_{J_{d}J_{p}M_{d}M_{p}} \left[ L_{d}^{L_{p}} - t \right] \left[ \frac{2L_{p} + 1}{2L_{d} + 1} \right] D_{0}(s) s_{p}^{m_{p}} s_{d}^{m_{d}}$$

where $a(s)$ involves the overlap of the proton and neutron in forming the deuteron.

The differential cross section can now be written as;

$$\frac{d\sigma}{d\Omega} = \frac{m_{d}^{m_{p}}}{(2\pi h^{2})^{2}} \frac{k_{p}}{k_{d}} \frac{1}{(2J_{p} + 1)(2s_{d} + 1)} \sum_{M_{d}M_{p}} T(d,p)^{2}$$  \hspace{1cm} (B.25)

which can be reduced to,
\[
\frac{d\sigma}{dQ} = \frac{m_{dp}}{(2\pi\hbar^2)^2} \frac{k_p}{k_d} \frac{(2J_B+1)}{(2J_A+1)(2s_d+1)} \sum_{jm} \left| \sum_{ls} A_{ls} \beta_{lj}^m \right|^2
\]

If only one \( l \) and \( s \) are allowed this can further be reduced and written as a more familiar expression,

\[
\frac{d\sigma}{dQ} = \frac{2J_B+1}{2J_A+1} \left[ \frac{A_{ls}}{(2s_d+1)} \sigma_{ls} \right]^2
\]

where the reduced cross section is now,

\[
\sigma_{lsj} = \frac{m_{dp}}{(2\pi\hbar^2)^2} \frac{k_p}{k_d} \sum_{jm} \left| \beta_{lj}^m \right|^2
\]

C. Limitations of the Conventional DWBA Analysis

In the preceding section we have outlined how the differential cross section for the \((d,p)\) reaction is obtained within the framework of the usual DWBA formalism. This approach makes a number of approximations in the course of arriving at the final expression for the reaction cross section, and it is not clear that all the approximations will remain valid for all the cases encountered experimentally. In particular in this \( \text{Pb}^{208}(d,p)\text{Pb}^{209} \) study we encountered weak transitions to certain nuclear levels where there is reason to believe that the application of the conventional DWBA analysis is not valid. It is therefore of interest to discuss briefly what some of the specific limitations of this analysis might be.
1. Assumptions of DWBA

In the DWBA treatment of transfer reactions several assumptions are made. Here will be discussed four of these which we consider to be of particular relevance to our experimental study. First, it is assumed that the transition takes place directly between the entrance elastic channel and the exit channel. Only the transferred particle is treated explicitly, while all other particles of the "core" are regarded as passive. Second, it is assumed that the optical-model wave functions used as the distorted waves of the initial and final channels are correct in all relevant regions of configuration space. Third, it is assumed that the transfer process is so weak that it may be treated in first order. Using these assumptions we obtain the expression for the transition amplitude (Equation B.9) which can be written as:

\[ T(d,p) = \int d\vec{r}_p |d\vec{r}_d \chi_p^{(-)}(\hat{k}_p, \vec{r}_p) \rangle \langle \psi_B | \hat{V} \hat{f}_A \psi_d \rangle \chi_d^{(+)}(\hat{k}_d, \vec{r}_d) \]  

where the notation has been defined previously. A fourth assumption is usually made for the choice of the radial form factor of the bound neutron which is defined in principle by the overlap integral defined by the brackets in the expression above. A "separation-energy" approximation is usually used to specify the form factor of the nucleon; here the radial form factor in the nuclear interior is taken to be a shell model single-particle wave function and this
is joined smoothly to an appropriate exponentially decaying tail in the nuclear exterior.

The calculations of the differential cross section with these assumptions, and the comparison of the results with the experimental cross sections in order to obtain a spectroscopic factor comprises a conventional DWBA analysis. These analyses have had considerable success but within certain well defined limits. That is, they are adequate for the extraction of relative spectroscopic factors of strong transitions (where the strength is $>1/10$ of the single-particle strength) leading to states which are bound but not too tightly bound (MA69). In the remaining part of this section, we will discuss in more detail the question of the limitations in the DWBA analysis by investigating the circumstances under which its assumptions break down. We will see that there are cases where one or both of the first two assumptions above are violated. The third assumption is probably valid for most transfer reactions, and the fourth assumption is valid only in certain cases.

2. Discussion of Assumptions

The first assumption will be invalid when a transition takes place between states, one of which is not a parent of the other. This is illustrated for a (d,p) reaction in figure III-1. In this example, the lower daughter states $\phi_o(A \ell)(n)$ and $\phi_o(A \ell')'(n)$ of the (A+1) nucleus have
Figure III-1
Schematic representation of a two-step process in the (d,p) reaction
as their parent the ground state $\psi_0(A)$ of the target, however, the higher lying daughter state $\psi_1(A)\tilde{\psi}(n)$ has as its parent the excited state $\tilde{\psi}_1(A)$ of the target. The usual DWBA can treat the transitions to the two low lying states, but not the transition to the third state which proceeds through the intermediate state $\tilde{\psi}_1(A)$. When the states of the nucleus $A$ have large inelastic cross sections, and if such states are the partial parents of the states of interest in the nucleus $(A+1)$ that are produced in a transfer reaction then the second-order processes going through intermediate states may become important. Of course, in real nuclei the parentage of any state will generally not be pure, and each state of nucleus $(A+1)$ will have parentage relationship with both the ground and excited states of nucleus $A$. Thus, in practice, one can populate states of the residual nucleus both through the direct transfer process as well as indirectly through the two-step transfer process. Whether in fact, these indirect modes of production compete with the direct mode depends on the fraction of parentage that is based on an excited state and on the strength with which this excited state is produced in the inelastic scattering with the incoming and outgoing particles. The cross section to a final state depends on (1.) the nuclear structure of the initial and final states, (2.) the neutron form factors for both the stripping to the ground state and the excited state, and (3.) the effect of the interference between the direct and indirect modes.
Calculations are now in progress (AS70) which attempt to treat this problem and estimate the importance of these two-step processes. Previous calculations (AS70, GL69) have found results which indicate that such processes can be important, especially when weak transitions are being considered. (See Section III.D.2.) The comparison of these calculations with more experimental data is necessary, however, before any conclusions concerning the importance of second-order process to the reaction cross section can be drawn.

The second of the above assumptions involves the choice of the distorted waves for use in the entrance and exit channels. These distorted waves, as noted previously are generated by optical model potentials determined by fits to elastic scattering and polarization data. Aside from the ambiguities and uncertainties in the optical model parameters which arise in practice there remains a more serious criticism of the procedure. That is, the elastic scattering measurements only determine the asymptotic form of the distorted waves (namely, that "outside" the nucleus), and it is a major extrapolation to use the simple optical-model wave functions in the nuclear interior. The importance of the interior wave function to the reaction cross section has been pointed out by Austern (AU63) and by Hooper (HO66) who have emphasized the examination of the partial-wave decomposition of the transition amplitude. To illustrate this let us consider what can be learned from such an examination.
We let $L_d$ and $L_p$ be the angular momenta in the deuteron and proton channels respectively, and let $l$ be the transferred orbital momentum in the reaction $A(d,p)B$. The transition amplitude (equation B.23) can be written as a superposition of partial-wave amplitudes,

$$\beta_{L_dL_p} = \int \frac{k_p}{k_d} \frac{F_t^* (k_p, r) U_{lj}(r) F_{L_d}(k_d, \frac{M_A}{M_B} r)}{L_p} dr,$$

where $F_{L_d}^*$ and $F_{L_d}$ are the partial distorted waves and $U_{lj}$ is the neutron form factor. The angular momentum must obey the conservation laws,

$$\overline{L_d} + \overline{l} = \overline{L_p},$$

and,

$$(-1)^{L_d+l} = (-1)^{L_p},$$

which, of course, establish the connection between the angular momenta in the deuteron and proton channels.

The reliability of the stripping calculation depends on the extent to which the fitting procedure of the elastic scattering determines those partial waves $F_L$ which are important in the stripping process. It is found (AU61, SB67) that the elastic scattering channels ($A,d$) and ($B,p$) are dominated by only a few partial waves,

$$L_{do} \approx k_d R$$

and

$$L_{po} \approx k_p R$$

which correspond to a grazing collision at the nuclear
surface \( R \approx r_o A^{1/3} \). This means that only the few partial waves near \( L_d \) and \( L_p \) are determined well from an analysis of elastic scattering data. In the case of the transfer reaction, the behavior of the partial wave amplitude \( \beta_{L_d L_p}^{j} \) is determined primarily by the overlap of the distorted waves \( F_{L_d} \) and \( F_{L_p} \), since \( U_{L_d} \) varies more slowly with \( r \) than do the wave functions \((AU70)\). Therefore when the angular conservation laws are satisfied so as to involve those angular momenta of the deuteron and proton channels favored in elastic scattering that is,

\[
\Delta L = |L_d - L_p| = |k_d R - k_p R| \approx l, \tag{C.6}
\]

the transfer cross sections are dominated by the partial waves that are well determined in the elastic scattering. Under these circumstances there is angular momentum matching and there are relatively small contributions to the cross section from the low partial waves \( L < L_o \) which are not well determined. If on the other hand, an \( l \) transfer is considered such that,

\[
\Delta L_o = |k_d R - k_p R| \gg l \tag{C.7}
\]

or

\[
\Delta L_o = |k_d R - k_p R| \ll l
\]

then an angular momentum mismatch occurs and relatively large contributions to the cross section must come from the low partial waves of either the deuteron or proton channel which have not been well-determined. This is important
since there are several reasons to believe that the low partial waves (corresponding to the wave functions in nuclear interior) specified by the elastic scattering optical model potential are not adequate for use in stripping calculations. One feature that is usually neglected in the simple phenomenological optical model is the non-locality of the potential which has been shown to affect the low partial waves. Further, it is not clear that it is sufficient to include only the elastic parts of the scattering wave functions. That is, if a particular inelastic transition is strong, the population of the excited state becomes large enough that the de-excitation back to the elastic channel becomes significant. This process can be described by a coupled channel calculation which produces changes in the predicted low partial waves (MA69). Here we see that inelastic processes are also an important factor in the excitation of state produced in a single-step process.

Thus to summarize, DWBA is reliable for the transitions where there is an angular momentum match, however, in the transitions where there is angular momentum mismatching DWBA becomes less reliable.

The fourth assumption deals with the choice of the radial wave function (or radial form factor) of the bound particle. The bound particle form factor is defined as the overlap integral of the initial and final nuclear state wave function where the integration is only over the
internal coordinates of the initial core nucleus A. It can be written (Equation B.15) as:

$$\phi_{AB}^{\ell j} = \langle \psi_{J_B}^{(A+1)} | \psi_{J_A}^{(A)} \rangle = \sum_{j_m} \langle J_A \_l M_A \_m | J_B \_R M_B \rangle Y_{\ell j}^{AB}(r) \phi_{\ell j}^{AB}(r)$$

where $Y_{\ell j}^{AB}(r)$ contain the spin-angle functions. The radial form factor $\phi_{\ell j}^{AB}(r)$ is unnormalized and the only thing known about it is inferred from the Schroedinger equation satisfied by $|\psi_{J_A}>$ and $|\psi_{J_B}>$. The form factor must have the asymptotic form of a decaying exponential with decay constant determined by the separation-energy ($E_B - E_A$) of the final state; i.e.,

$$\phi_{\ell j}^{AB}(r) \xrightarrow{r \to \infty} e^{-\kappa r}$$

where

$$\kappa = \frac{2m_r}{\hbar^2} (E_B - E_A)$$

and $m_r$ is the reduced mass of the neutron. Further specification of the form factor is necessarily model dependent.

It is possible to introduce shell-model wave functions for $|\psi_{J_A}>$ and $|\psi_{J_B}>$ in equation (C.8) and calculate $\phi_{\ell j}^{AB}(r)$ directly. This is not adequate, however, because the shell-model wave functions are only accurate in the nuclear interior and not at or outside the surface where the transfer reactions occur. To remedy this we would try to include enough shell-model orbits (nlj) to adequately describe the tail of the form factor by
defining the form factor as an expansion,

\[ \Phi_{l_j}(r) = \sum_n \sqrt{S_{n_l_j}^{AB}} U_{n_l_j}(r), \]

de where the \( U_{n_l_j}(r) \) orbits are the complete set underlying the shell model and the sum is over the radial quantum numbers, \( n \). The normalized shell model wave functions are related to the form factor through the normalization constants, \( S_{n_l_j}^{AB} \), called the spectroscopic factors. This procedure for describing the form factor is, however, a difficult task in practice, because the shell model calculations become very large and little work in this direction has been done. If the sum is only over one \( n \) (i.e., a single shell model wave function is taken) then the tail of the form factor is determined not by the separation-energy, but by the shape of the single-particle potential.

There is one situation where the choice of a single shell-model wave function for the radial form factor is correct. This occurs when the target nucleus is a closed shell and the final state can be regarded as a single-particle orbit \( (n_l_j) \) outside this closed core. Then only one value of \( n \) contributes, the spectroscopic factor is unity, and the form factor simply becomes \( U_{n_l_j}(r) \). However, in practice one usually encounters situations where the single-particle excitation is spread over several states. The use of simple shell-model wave functions for these states will not be correct in general, since the
shell-model functions do not have the correct asymptotic form in general.

To remedy this problem the phenomenological separation energy prescription discussed previously has been used extensively to compute form factors for use in stripping calculations. In this prescription the form factor is approximated by a single-particle wave function calculated in a shell-model potential whose depth is adjusted to bind the particle by an amount equal to the observed separation energy of the final state. This method thus ensures that the form factor has the correct asymptotic form both at the nuclear surface and outside in regions important in the transfer reaction. The separation energy prescription has been demonstrated to provide quite acceptable results when the single-particle strength is fragmented over a relatively small energy range. Each state is treated on an individual basis and the spectroscopic factor is considered to give a good measure of the fraction of the single-particle strength located in that particular state.

When the single-particle strength is fragmented over a large energy range, the approximation is less obvious. In particular when a small fragment of single-particle strength is remote from the single-particle centroid position it becomes clear that the approximation is poor. While this prescription makes certain that the form factor has the correct asymptotic behavior, it almost certainly results in an incorrect normalization. In these cases a
more realistic form factor which includes the details of the nuclear structure must be obtained. Such calculations have been performed and their results are discussed in some detail in the next section (Section III.D.1.).

It should be pointed out that even when the strength in the single-particle excitation is concentrated in a finite energy-range, the form factor calculated by the separation energy approximation has uncertainties which concern not only the behavior in the nuclear interior, but also the overall normalization (spectroscopic factor). These uncertainties contribute largely to the limitations of the accuracy of the DWBA analysis even in the circumstances when the approximation should be good.

3. Results of Conventional DWBA Analyses

Based on the preceding discussion, the limitations on the validity of the usual DWBA may be summarized as follows; the conventional DWBA analysis should be adequate for those transitions (1.) which are strong with large spectroscopic factors (>1/10 full strength) and are not likely to be effected significantly by possible two-step processes, (2.) which have angular momentum matching and the contributions to the cross sections come primarily from the partial waves which have been well determined from the elastic scattering, and (3.) which lead to states which are bound, and for which the form factors produced by the separation energy prescription are reasonable. Before considering the relevancy of the
conventional DWBA analysis to weak transition where there is reason to believe that the method is not completely reliable, let us consider what results are obtained in circumstances where the analysis should be applicable.

Recently MacFarlane (MA69) collected and compared the spectroscopic factors obtained from the analyses of several single-particle transfer reaction studies on the doubly magic Pb\textsuperscript{208} nucleus. The strong transitions to the single-particle and hole states in the nuclei adjacent to Pb\textsuperscript{208} are ideally suited for analysis by DWBA and thus a comparison of the results obtained in several studies at different energies should give an indication of the accuracy of the DWBA method. Typical of what is found in these studies are the results of the Pb\textsuperscript{208}(d,p)Pb\textsuperscript{209} reaction which have already been illustrated in Figure II-7. From the comparison of the spectroscopic factors obtained in all the transfer reaction studies on Pb\textsuperscript{208} (which include the transitions to the single-particle states in Pb\textsuperscript{209} and Bi\textsuperscript{209} and to the single-hole states in Pb\textsuperscript{207}) the following observations can be made: (1.) the absolute spectroscopic factors $S$ vary considerably from one experimental determination to the other and are stable to no better than 25% of the available single-particle strength. (2.) The variations from one experiment to another tend to diminish at bombarding energies below the Coulomb barrier. (3.) The relative spectroscopic factors vary a little less from experiment to experiment than do the absolute spectroscopic factors, however the variations are
The absolute spectroscopic factors of the single-particle (hole) state are found to fall within 25% of the "expected" value of unity.

Perhaps the most significant conclusion to be drawn from these observations concerns the relative spectroscopic factors. It appears from the comparison of these results that the relative spectroscopic factors obtained in the practical application of the DWBA method can be trusted to only about 25%. The discrepancies might be attributed primarily to the uncertainties in the description of the distorted waves (particularly the low partial waves) and the uncertainties in the single-particle potential which sensitively affect the behavior and normalization of the form factors; although other effects such as the deuteron breakup, the polarization of the deuteron, the inadequate treatments of the finite-range of the p-n interaction, and the non-locality of the optical potentials may also be important. In regard to the absolute spectroscopic factors, the uncertainties must be considered significantly greater. Not only are the uncertainties mentioned above in regard to the relative spectroscopic factors applicable, but in particular the uncertainties in the exact shape of the single-particle potential makes the overall normalization of the spectroscopic factors equally uncertain. In most of the transfer reaction analyses single-particle potentials were used which gave spectroscopic factors which were consistent with the "expected" values of unity. From the
nuclear structure calculations which have been discussed in Section II.A.3. it is clear that the reduction of the single-particle strength by 25-40% (MC70) because of interactions with states in the continuum would not be a complete surprise. Thus the fact that the single-particle and hole states have absolute spectroscopic factor consistent with unity (within the associated uncertainties of 25%) should not, at least at this stage of our understanding be considered conclusive.

Having now seen limits on the reliability of the results of DWBA analysis under the best of circumstances, let us consider how reliable may be the results of DWBA analysis under less favorable circumstances where one or more of its basic assumptions is either doubtful or violated. The next section discusses the treatment of weak transitions within the DWBA framework.

D. Treatment of Weak Transitions

As discussed in the last section, one or more of the assumptions made in the usual DWBA may not be valid for all transitions encountered experimentally. In particular, in the case of weak transitions; (1.) the contributions to the cross section from two-step processes may not be negligible, and (2) the form factors calculated in the separation energy prescription clearly represent poor approximations. The importance of these effects must be determined from more detailed studies of the reaction
processes. In the discussion below we will consider some of the recent studies directed towards (1.) the calculation of more realistic form factors for use in the analysis of weak transitions, and (2.) the calculation of cross sections for two-step processes.

1. Bound State Form Factor
   a. Introduction

   The form factor for a A(d,p)B reaction has already been defined in equation (B.15). For convenience it is reproduced here,

   \[ \phi_{\ell j}^{AB} = \int d^3r \, \phi_B^{*}(r, x) \phi_A(r) = \sum_{j m_{1} m_{2}} \langle J_{B} A_{m_{1}} | J_{A} m_{2} \rangle \gamma_{\ell j}^{AB}(x) \phi_{\ell j}^{AB}(x) \]

   D.1

   The question is that of specifying the radial form factor \( \phi_{\ell j}(x) \). In the simple case where the target is a closed shell core and the nucleon is stripped into an unfilled single-particle state, the specification of \( \phi_{\ell j}(x) \) by the separation energy prescription, as we have discussed, is a rather good approximation. However, in more complicated cases this approximation may not be very good.

   As a specific example of the difficulties which arise with more complex configurations, let us consider the addition of a further nucleon to a closed shell-plus-one target. If the nucleon enters into the same orbit as the target nucleon outside the core, the resulting \( j^2 \) configuration forms states of spin \( J_B \) ranging from 0 to 2j (where
only even spins are allowed by the Pauli principle if the nucleons are identical). These states will have different energies depending on the final spin $J_B$ because of the residual interactions between the two nucleons. On the other hand, the shell model picture leads us to believe that the second nucleon is in the same orbit as the initial nucleon bound to the core, that is, it has the same wave function as the first one, independent of $J_B$. In the shell model calculation, the effect of the residual interaction on the total energy of these two nucleons and its dependence on $J_B$ has been taken into account; however, the effect on the radial wave function of the two nucleons has not since it is not necessary for the description of the nuclear structure. In the stripping reaction it is the radial wave function of the nucleon that is important and the use of a single-particle wave function is not correct since it does not contain the modifications arising from the effect of the residual interaction.

b. Results of Some Calculations

If the effects of residual interactions are included explicitly in the description of the system, the Schroedinger equation can be written as:

$$(E_B - E_A - T_N - V_{nA}^0)\varphi_{AB}(x) = \langle B \mid \psi_{nA}^{\text{RES}} \rangle_A,$$

where $T_N$ is the kinetic energy of the transferred neutron $n$ relative to the core $A$, $V_{nA}^0$ is the shell model potential,
\( V_{\text{RES}}^{nA} \) is the residual interaction. The use of this equation results in a set of coupled equations which can be solved in principle to determine the radial form factor \( \phi_{AB} \). Several calculations have been performed recently (HU68, PP68, PP69, HA69) solving suitably truncated versions of these coupled equations for the realistic form factors and the results of some of these calculations will now be discussed.

Consider first a transition that is weak because it involves a pickup from a nearby empty orbit; e.g. the transition \( \text{Ni}^{58}(p,d)\text{Ni}^{57} \) to the \( 1f_{5/2} \) single-particle state at 0.78 Mev in \( \text{Ni}^{57} \). Hutton (HU68) and Philpott et al. (PP68) have calculated form factors for this transition by approximate solutions of equation (D.2). In both calculations the computed realistic form factor is significantly displaced towards large radii as compared to the form factor computed by the separation-energy (S.E.) prescription; in consequence it is noticeably larger near and outside the nuclear surface which is the region of greatest importance in transfer reactions. This increases the calculated DWBA cross section and thus correspondingly reduces the spectroscopic factor obtained. Hutton in his analysis of the 22 Mev deuteron data of Sherr, et al. (SR64) found that the separation energy method overestimates the \( 1f_{5/2} \) spectroscopic factor by about 50%.

As a second example, we consider a transition that is weak because it involves a stripping into a nearby-full orbit; e.g. the transition \( \text{Ca}^{40}(d,p)\text{Ca}^{41} \) to the \( 3/2^+ \) state.
at 2.04 Mev in Ca$^{40}$. Pinkston, et al. (PP69) have calculated a realistic form factor for this transition and have compared their results to those obtained when the form factor is calculated with the S.E. prescription. It was found that the effect is opposite to that encountered in the first example. The realistic form factor was pulled significantly towards a smaller radius and consequently is much smaller than the S.E. form factor in the critical region near the nuclear surface. This results in smaller contributions to the DWBA cross section and a corresponding increase in the spectroscopic factor obtained. It is interesting to note here that the S.E. method used previously to analyze these data yielded an underestimate of the spectroscopic factor and hence indicated a smaller degree of shell breakage of the Ca$^{40}$ core than is now implied from the realistic form factor analysis. In the realistic form factor calculation two forms of the residual interaction (surface-delta and Gaussian) were used and found to affect sensitively the shape of the computed form factor and, in consequence, the cross section. When the Gaussian form was used (probably the more reasonable form) a spectroscopic factor was found which was about 3 times larger than that obtained with the S.E. prescription.

The results in the two examples above are typical of what has been found in other studies which have investigated the effects of using improved form factors in the analysis of weak transitions. In the paper of
Philpott, et al. (PP68) some of the general trends observed are discussed. It appears a general result that the angular distribution shapes are relatively insensitive to the form factor shapes as long as the form factors have the correct asymptotic behavior (e.g. PS65). This is because the dominant contributions to the reaction usually come from the region of the nuclear surface and beyond. (There are of course special cases which might be exceptions to this rule.) On the other hand, the magnitudes of the predicted cross sections appear to be quite sensitive to whether realistic form factors are used, and the change can be as large as a factor of 2 to 3. These changes in the cross section reflecting use of a more realistic form factor appear to be understandable in terms of simple concepts (PP68).

The model used to compute the realistic form factor is dependent on the assumption that the independent-particle model provides a good zero-order approximation to the motion of the captured nucleon. It is then a reasonable approximation to take as the wave function of the nucleon transferred into (or out of) a shell-model orbit (nlj) the zero-order single-particle wave function in the shell-model potential well. This is also the same philosophy of the binding energy approximation (PS65, PP68) where the same single-particle radial wave function is used for the transitions to all the fragments of single-particle excitation. As has often been stated (e.g. PS65), this binding energy procedure gives the wrong asymptotic behavior,
but one would suppose that the main effect of the residual interaction between the nucleons is to modify the tail to satisfy the asymptotic conditions. Thus the bulk of the form factor in the interior retains the single-particle character of the independent motion and its behavior is dominated by the depth of the shell-model potential and by the size of the nucleus.

Based on these simple arguments it is possible to relate the form factor calculated with the S.E. prescription to the realistic form factor. If the separation energy of the state is less than the zero-order single-particle energy (as for example the $1d_{3/2}$ transition in the $\text{Ca}^{40}(d,p)\text{Ca}^{41}$ reaction) the realistic form factor will more nearly represent that of a particle in a potential well bound by more than the separation energy. The wave function will then be contracted and the magnitude of the tail reduced compared with that given by the separation energy method, even though the form of the tail will be the same. Consequently the cross section is decreased and the spectroscopic factor increased. On the other hand, if the separation energy of the state is larger than the zero-order single-particle energy (as for example the $1f_{5/2}$ transition in the $\text{Ni}^{58}(p,d)\text{Ni}^{57}$) the opposite is true. The cross section increases and the spectroscopic factor obtained is decreased when the realistic form factor is used.

It was in this spirit that Pinkston and Satchler (PS65) suggested a phenomenological method for the specification of a more reasonable form factor for use in analyzing such
transitions. In their method the depth of the potential well is chosen close to that obtained for pure single-particle levels, but with a radius that is determined by binding the particle at the separation energy of the final state. The effect of this method is to make the wave function in the interior more nearly approximate the realistic wave function. In practice the radial wave function is pulled towards the nuclear surface for the cases where the separation energy is smaller than the zero-order single-particle energy, and just the opposite in the cases where the separation energy is larger. Thus the method produces exactly the same effect as produced by the inclusion of the residual interaction in the calculation of the form factors. Prakash and Austern (PA69) discussed a similar prescription, where it was suggested that the well depth be chosen and fixed, and alterations of both the diffuseness and the radius be made to produce the correct binding energy. It should be pointed out that the routine application of the above phenomenological methods should not be made since they have well defined restrictions (PS65, PA69), and indeed the methods were not suggested to take the place of the complete form-factor calculations. However, they can be used to give, in many cases, estimates of the magnitude of spectroscopic factors which are certainly better than those obtained using the conventional separation energy prescription.
2. Two-Step Reaction Mechanism

a. Introduction

The problem of including excitation of the core in the transfer reaction has been investigated by several authors (PS64, IA66, KD66, AS69). Penny and Satchler (PS64) proposed evaluating the amplitude for the (d,p) reaction from

\[ T_{d,p} = \sum_{p^*,d^*} |\psi^{(-)*}_{p^*,d^*}(A+1)| V_{np} |\psi_d^*(A)\rangle |\psi_{d'}^{(+)}_d^*| \chi_{p^*d^*} \chi_{d'd} \]  

where \( \psi^{(+)}_{d'd} \) is a generalized distorted wave in the channel \( d' \) found by solving the coupled equations for the inelastic scattering by \( A \) with an incident wave in the channel \( d \), and \( \psi^{(-)}_{p^*d^*} \) is similarly defined for the proton \( (A+1) \) system. However, a numerical solution to the full problem has never been obtained. Iano and Austern (IA65) considered the problem from a similar point of view, but solved it only to the first order with respect to the inelastic transitions. That is to say, their generalized distorted waves correspond to the DWBA approximation for the inelastic processes. Kozlowsky and De-Shalit (KD66) and Levin (LE66) have also worked on the problem. They too treat the inelastic effects only in first order, and only in the exit channel. Recently Ascuitto and Glendenning (AG70) have treated the problem from a different point of view in which they were able to treat inelastic effects to all orders in both entrance and exit channels. In the discussion below we will summarize very briefly some of the general
findings of these calculations.

b. Results of Some Calculations

In the work of Iano and Austern (IA66) the interference of the indirect amplitudes with the direct transition amplitudes for deuteron stripping on deformed nuclei were investigated. In particular, calculations were performed for the reactions \( \text{Mg}^{24}(d,p)\text{Mg}^{25} \) and \( \text{U}^{238}(d,p)\text{U}^{239} \). The indirect transitions considered were those arising via intermediate rotational excitations of both the target and product nuclei. It was found that; (a) the indirect amplitudes are small; (b.) the indirect amplitudes are affected more strongly by the intermediate excitations in the deuteron channel than by the intermediate excitations in the proton channel; (c.) the indirect amplitudes generally flattened the angular distributions of the reaction protons; and (d.) the ratio of the strength of the indirect transitions to that of the direct transitions was apparently independent of the mass of the target. While the indirect amplitudes were found to be small they produced measurable effects in the differential cross sections when they add coherently to the direct amplitude. The effects change both the shapes of the angular distributions and the magnitudes of the differential cross sections. Changes in the spectroscopic factors obtained when the indirect amplitudes were included amounted to about 50% in some cases.

In the calculations of Kozlowsky and De-Shalit, the reaction \( \text{Ni}^{62}(\text{He}^3,d)\text{Ou}^{63} \) was treated in some detail and the
stripping process in which the target nucleus is excited through its interaction with the outgoing particle in the exit channel was considered. The direct and indirect transitions were considered separately and it was found that the angular distribution in the stripping with core excitations was very similar to that of ordinary stripping to single-particle levels. The absolute magnitude of the cross section however, depends critically on the assumed radius of the region in which the reaction occurs. It was concluded that an unambiguous interpretation of the experimental results was not possible and the experimental data were consistent with both the interpretation that the reaction proceeded via a dominant single particle configuration in the excited state Cu$^{63}$, and the interpretation that the reaction proceeded via inelastic excitation of the N$^{62}$ core.

The calculations of Ascuito and Glendenning (AS70) were outlined for the (d,p) reaction,

$$d + A \rightarrow p + (A+1) \quad \text{D.4}$$

It was assumed that the transfer process is weak; that channels of the configuration d+A are coupled only weakly to those of p+(A+1) and this coupling was treated only in first order. First attention was focussed on a typical deuteron channel d'; the symbol d' is used to label all the quantum numbers needed to define the channel, such as the states of the nucleus A, and the angular momentum of the deuteron. The equation describing the motion in this channel is,
in which the various terms on the right represent the feeding of the channel \( d' \) by inelastic processes from other channels \( d'' \). Equation (D.5) together with those describing the other included channels constitute the usual system of coupled equations for inelastic scattering.

Next, attention was focussed on a typical proton channel \( p' \). Again this channel is fed by inelastic processes leading from other proton channels, but in addition, it is fed by the transfer reaction from the various deuteron terms in the equation for the proton motion. Accordingly the equation can be written,

\[
(T_{p'} - V_{p'p'} - E_{p'}) W' = - \sum_{p'' \neq p'} V_{p'p''} W'' + \sum_{d'} \rho_{d'}.
\]

Here \( \rho \) represents the source of the protons in the channel \( p' \) as the result of the stripping reaction in \( d' \). Of course \( \rho \) will depend upon the solutions of the deuteron equation.

The results of calculations of this type for the \((t,p)\) reaction have recently been reported on by Glendenning (GL69, AS70). In these calculations an idealized example of \( \text{Ni}(p,t)\text{Ni} \) was treated to illustrate the importance of indirect processes. It was found that the angular distributions for a transition involving a core-excitation was essentially indistinguishable from that predicted for a direct transition. Furthermore, the polarizations of the emergent protons were also predicted to be the same.
was also found that the magnitude of these indirect processes was relatively large so that an inclusion of a 10% admixture of a configuration of core-excitation parentage into a dominant configuration which can be reached by direct transition could result in a cross section which might range from 0.4 to 1.6 of the result previously obtained without the admixture.

To summarize, it appears from the calculations discussed above that there is no way to distinguish between an indirect transition and a direct transition on the basis of differences in the predicted angular distribution. That is to say, if a final state is thought to be primarily of a core excited parentage, then it will be difficult to determine whether the weak excitation of the level results as a consequence of an indirect transition or a small admixture of a configuration which can be populated directly. On the other hand, the discussion above indicates that even relatively small amounts of indirect amplitude are capable of modifying significantly the cross sections of transitions which are largely of a direct nature and thus perhaps provides a way of estimating the importance of the indirect processes.
IV. EXPERIMENTAL METHODS

The deuteron elastic scattering and (d,p) reaction experiments on Pb$^{208}$ were studied with the Wright Laboratory MP Tandem Van de Graaff Accelerator and Multigap Magnetic Spectrograph. The following sections will briefly describe the accelerator and beam handling systems, the multigap spectrograph, and the data acquisition, reduction, and analysis procedures.

A. Accelerator and Beam Handling

The physical layout of the Wright Laboratory MP Tandem Van de Graaff Accelerator faculty used to produce deuteron beams for use in these studies is shown in Figure IV-1. A beam of negative deuterium ions is first produced by a duoplasmatron ion source, and then inflected into the beam tube through a 20° bending magnet. The initial 300 KeV source beam is focussed by an Einzel lens, positioned by a set of x-y steerers, and accelerated to the center of a terminal at high positive potential (10 Mv in this experiment) located at the accelerator center, where the two electrons are stripped from the deuterium ions by collision with oxygen molecules constantly supplied to a stripper canal. The newly formed positive ions are then further accelerated away from the terminal to ground potential at the opposite end of the accelerator tank. A magnetic quadrupole lens at this high energy end of the
Physical layout of the Wright Nuclear Structure Laboratory Accelerator Facility

Figure IV-1
accelerator focuses the beam onto a set of slits which form the object for the subsequent beam optics. These slits were set to a spacing of 20 Mils (0.5 mm) during these studies. The beam was then deflected by a 90° flat field precision analyzing magnet and focussed on the image slits whose horizontal aperture was usually set to equal that of the object slits. The switching magnet after the image slits then deflected the beam into the spectrograph beam line.

The exact energy of the beam is determined by the 90° analyzing magnet. The magnetic field (measured by a nuclear magnetic resonance (NMR) probe) is related to the particle energy by a calibration based on the measurements of well-known (p,n) threshold energies (OP69). To ensure beam stability the terminal voltage is regulated by a feedback system between the image slits and the corona current regulator (controlling the current drawn off the terminal through the corona points).

The energy spread of the beam is determined by the dispersion of the analyzing magnet and the slit settings. The system typically produces beams with energy spreads of 0.03-0.05% and has demonstrated beam spreads of \( \leq 0.02\% \) in the measurement of the \( ^{12}(p,p')^{12} \) resonance at 14.233\( \pm \)0.008 Mev (LP69).

The analyzed beam enters the spectrograph beam line and is focussed by a magnetic quadrupole doublet to a crossover point at half the distance between the switching magnet
and the spectrograph center by another doublet positioned just before the spectrograph. A set of x-y magnetic steerers is placed symmetrically about the cross-over point.

The spectrograph experiments require the placement of a small well-shaped beam spot on target without the use of collimation after the beam has been deflected into the spectrograph beam line. The magnetic spectrograph is a powerful spectroscopic tool because of its capability of obtaining a clean spectrum with excellent energy resolution. The use of intermediate collimation is undesirable since it produces slit-edge scattering and other background contributions which not only make the energy resolution worse, but also create a particle background on which the spectrum lies. The absence of collimation makes it necessary to demand two specific inter-related requirements on the beam optics. First, we must produce (with the focusing elements in the spectrograph beam line alone) a beam spot of small vertical dimension on the target at the spectrograph center, and, second, we must set up an optical system which minimizes the beam spot motion over long periods of time, and which makes the beam spot independent of accelerator instabilities. The size of the beam spot and the limits on the allowed beam spot motion are determined by the energy resolution required in the particular experiment performed.

The energy resolution observed in the spectra obtained
in the spectrograph is directly related to the vertical dimension of the beam spot on the target. For the \((d,p)\) reaction on \(^{208}\text{Pb}\) at \(E_d = 20.0\) Mev, for example, a 40 mil (1.0 mm) high beam spot would contribute approximately 30 kev to the energy resolution. Therefore since 10 kev resolution was desired, it was necessary to produce a beam spot of about 10 mils (0.25 mm) vertical dimension. (There are of course other things which contribute to the energy resolution, such as beam energy spread, target energy losses, etc; but in magnetic spectrograph experiments the beam spot size is of crucial importance). The only requirement on the horizontal dimension was that the beam spot remain within a region defined by a circle of 125 mil diameter at the multigap center. This insured that the spectra taken at the different angles had the same solid angle (See discussion Section IV.B). Thus a beam spot of dimension 10 x 125 mils at the center of the multigap was finally required in these studies. From the discussion above it should be clear why we also required that there be no movement of the beam spot, especially in the vertical direction.

In practice, these requirements were met with careful alignment and focus of the beam through the various beam transport elements. This was done by maintaining the object and image slit dimensions in the vertical dimension at 20 mils, and by making certain that the beam was placed along the axes of the aligned quadrupole elements. To
facilitate the beam handling, beam current monitors are installed at various places along the transport system, such as at the object and image slits. In addition, the beam passed through a set of slits in the multigap immediately before striking the target. These slits were used to scrape the edges of the well-focussed beam (not define or collimate the beam) and in practice collected <10% of the total beam currents. The beam was focussed in the spectrograph with the use of a thin aluminum "flat" painted with zinc sulfide. The flat had a small opening (12 x 165 mil) located at the center of the spectrograph through which the beam was focussed. Since the usual target orientation to the beam was 45°, the zinc sulfide flat displayed a 12 x 125 mil slit to the beam. The slit was accurately located at the center of the spectrograph with the use of a calibration standard designed for this purpose (K070). The first crude focussing was done visually by observing the fluorescence of the zinc sulfide on a television monitor. The final focussing was accomplished by minimizing the current collected on the aluminum flat and maximizing the current passing through the slit. In practice, the focus was considered acceptable when less then 10% of the beam current was monitored on the flat, and very little or no beam observed on the television monitor. The beam could either be collected by a faraday cup located inside the spectrograph or one located outside. The neutron background observed
when 20.0 Mev deuterons were collected by the internal faraday cup was so great (producing a dense background of proton recoil tracks in the nuclear emulsion used) that it was necessary to pass the beam through the spectrograph and to collect the beam externally. This has now become the standard operating procedure in all experiments.

The above procedure of alignment and focus has now been demonstrated on numerous occasions to produce a suitable beam spot which shows little or no motion (≤ 5 mil) over long periods of time independent of accelerator stability. The data presented in this thesis provides an example of the excellent resolution that can be obtained during a long exposure (≈15 hours); this would have been impossible without a small stationary beam spot during the entire exposure.

B. Multigap Spectrograph

1. Physical Description

The Yale broad range multigap (multiangle) magnetic spectrograph is similar in design to existing multigaps at MIT(EB63) and at Oxford (MH62). It is an extension of the Browne-Buechner single gap (angle) spectrograph (BB56) allowing for data to be taken simultaneously at many angles. A detailed description of this instrument can be found in a laboratory internal report (KO70). In the following discussion only a general description will be given.
Figure IV-2 is a simplified drawing of the principles of operation and physical layout of the multigap spectrograph. From the top view we are able to see the toroidal magnetic circuit made of forty-eight 7.5° cast-iron wedges; 23 of which are thinner than the others in order to leave gaps between them. The segments are clamped together to form a solid toroid (stainless steel spacers maintaining the gaps). Each gap is 5/8 inch wide and contains a magnetic field produced by the coils on the two adjacent poles. Thus we have 23 "single gap" spectrographs located at every 7.5° spanning the angular range 0°-162.5°. The toroid can be rotated to any of three orientations to place each gap at three different angles with respect to the beam. Table IV-1 shows the various sets of angles at which data can be taken, corresponding to the three orientations.

From the side view (figure IV-2) it is possible to see the characteristic magnet pole face geometry of the Browne-Buechner design; this is identical for each gap. A beam can enter through one of three holes in the toroid and strike the target at the center. The particles produced by a reaction in the target leave the scattering chamber through slits on the chamber wall to enter the uniform magnetic field of each gap where the charged particles are momentum analyzed and then exit to strike a photographic emulsion in the focal surface of the magnet. The pole face radius is 31.0 inches, and the range of
Figure IV-2

Schematic layout of the Multigap Spectrograph
energy covered in one exposure is 2.5:1 corresponding to 85 Mev protons at the largest radius and 34 Mev at the smallest radius with the field of 15 kilogauss, for example. The instrument has shown resolving powers (i.e. \[ RP = \frac{E\text{(Mev)}}{\Delta E\text{(Mev)}} \] of 2200 in practice, that is to say, in a proton spectrum the proton group of 22.0 Mev would have a peak width (FWHM) of 10 kev.). The spectrograph uses one inch wide emulsions of 48 inch total length in each gap. A hydrogen nuclear magnetic resonance (NMR) probe in gap #15 (at about 110°) is used to measure the magnetic field and also to stabilize the field by acting as an input to a feedback system controlling the current in the coils. The magnetic field in the gaps is stable to better than 1 part in \( 10^4 \) over long periods of time.

A more detailed side view of the spectrograph is shown in figure IV-3. In the drawing is shown the vacuum sphere which encloses the entire spectrograph. Shown also is the physical layout of the scattering chamber, toroid, and photographic emulsion mounts. A more detailed drawing of the mechanism for holding the photographic or "nuclear" emulsions is shown in figure IV-4A. The nuclear emulsions are loaded into plate holders where they are deformed by a spring mechanism to lie along a machined hyperbolic surface reproducing the focal surface of the magnet. The plate holders are placed on an indexing box where index marks placed at equally spaced intervals along the nuclear emulsions are made by exposure to a light source. The
YALE MULTIGAP SPECTROGRAPH

Figure IV-3

- Near Probe
- Liquid Nitrogen Coils
- Diffusion Pump Port
- Zone Defining Slits
- Magnet Coils
- Plate Holder Assembly
- Slits
- Magnet Pole Face
- Beam Entry
- Beam Tube
- Roller Slit Assembly
- Target Assembly
- Base Plate
<table>
<thead>
<tr>
<th>Position #1 Gap (Degrees)</th>
<th>Position #2 Gap (Angle)</th>
<th>Position #3 Gap (Degrees)</th>
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<td>22</td>
</tr>
<tr>
<td>23</td>
<td>172.5</td>
<td>23</td>
</tr>
</tbody>
</table>
holders are loaded in the rotating spline assemble (figure IV-4A) in the spectrograph. The rotating splines can hold three plate holders simultaneously allowing for three separate plate exposures without opening the multigap spectrograph to atmospheric pressure for reloading.

Figure IV-4B shows the details of the scattering chamber within the spectrograph. We have the option of using a set of "roller" slits consisting of four ellipsoidal cylinders which define an opening which can be adjusted to intercept the edges of the beam before it strikes the target; the roller slits were always used in this work. The target assembly shown in the drawing can hold three targets, although in practice one position is usually filled by the above mentioned zinc sulfide flat used to focus the beam at the correct spectrograph center. The reaction particles leave the scattering chamber through slits mounted on the chamber wall. A shutter ring on the chamber wall can be rotated to three positions to (1.) close the slits, (2.) open the normally used slits which have an $\alpha$ of $2.6^\circ$, or (3.) open the so-called Rutherford slits where $\alpha$ is modified by a $\csc^4(\theta/2)$ factor. The symbol $\alpha$ denotes the maximum vertical angular divergence of the particle trajectory from the central ray (See figure IV-2). The Rutherford slits are installed only in the forward quadrant and allow one to obtain an angular distribution with reasonably good statistics even when the cross section drops by several decades from $0^\circ$ to $90^\circ$. 
Figure IV-4A

Detailed views of the Photo-Plate Assembly and Multigap Scattering Chamber
The slits on the chamber wall give the vertical definition of the solid angle. The horizontal definition is given by slits fixed on the cast iron mounts which support the nuclear emulsion holders (See figure IV-4A). These two slit edges are approximately 2.5 inches from the emulsions along their entire 48 inch length, thus defining a zone of particles of constant width (7/16 inch), but changing solid angle since the distance from the target to the slits increases as the radius of curvature becomes larger. The solid angle at 90° deflection is about 3.6 x 10^{-4} steradians.

2. Experimental Properties

There are several properties and parameters of the multigap spectrograph which must be known to analyze the raw data accumulated on the nuclear emulsions. These all relate eventually to the measurements of excitation energies and reaction cross sections.

To establish excitation energies we must be able to measure the energies of the various particle groups observed on the nuclear emulsions. In a magnetic spectrograph this means that we must establish a relation between the location of the particle group on the nuclear emulsion and the particle energy. This is done through the intermediate quantity \( \rho \), the radius of curvature of the particles in the magnetic field. From Figure IV-2 it can be seen that there
exist a unique relationship between $D$, the distance along the nuclear emulsion and $\rho$, the radius of curvature. This relation is usually established empirically and is expressed for this spectrograph by the expansion:

$$\rho = \sum_{i=1}^{6} A_i D^i ,$$  \hspace{1cm} \text{(B.1)}

where $A_i$ are the calibration coefficients determined from experimental measurement. The coefficients for each gap of the Yale spectrograph were established by an elastic scattering experiment and are discussed elsewhere (K070). The calibration is usually considered a permanent property of the instrument and is not expected to change unless modification of the toroid, the orientations of the emulsion surfaces, or beam spot position is made. The radius of curvature is then related to the energy of the particle through the magnetic rigidity of the particle, which is defined as the product of the magnetic field, $H$, and the radius of curvature. The energy can be expressed as,

$$E = M c^2 \left[ 1 \left\{ 1 + \left( \frac{ZeHp}{M c^2} \right)^2 \right\}^{1/2} - 1 \right] ,$$  \hspace{1cm} \text{(B.2)}

where $M$ and $Ze$ are the mass and charge of the particle.

The accuracy of the energy determination depends upon (1.) how well the particle group can be located on the emulsions, (2.) how good the $\rho$-$D$ calibration is, and (3.) how well the magnetic field, $H$, can be measured. The peak position can be measured to $\pm 0.25$ mm when the scanning of the emulsions has been done in $0.5 \times 12.0$ mm steps. This
corresponds to about ±3 kev in this experiment. The accuracy of the p-D calibration is a rather complex question since the calibration has been established in a certain field region and there is no guarantee that at higher fields, for example, the calibration will be the same. However, it is usually assumed, that the calibration shape is not a function of magnetic field. It was found in this study that the calibration gave Q-values for known contaminant levels which were in agreement with published values to better than ± 5 kev. The determination of the actual fields in the gaps is not a simple problem for several reasons. The field is measured in one gap (Gap #15) by a hydrogen NMR probe. However, because of small inhomogeneities of the field in the gap, the magnetic field which affect the particles may not be the same as that measured. In fact, particles in the same gap which have different momenta may experience slightly different fields. Furthermore, because of the design of the instrument, the fields need not necessarily be the same from gap to gap. These various effects have been found to be quite small in most cases. A more complete discussion of these effects in a multigap spectrograph and the way in which they are corrected for in data analysis can be found in the literature (CO68). Generally speaking, an exact determination of the field at every point of the pole face is usually not sought, but rather an effective field for each gap is found empirically using well known lines observed in the
spectra and these fields are then used in the data analysis. It is believed that excitation energies can be reliably established to within \( \pm 5 \) kev using effective fields.

To establish cross sections from the observed number of tracks in the emulsions we must know the solid angles of the spectrograph. The solid angles for the normal and Rutherford slits were calculated from the geometry and the values were checked experimentally by elastic scattering measurements obtained at bombarding energies which should give Rutherford scattering cross sections. The details of these calculations and the experimental measurements can be found in a laboratory internal report (MA70). The geometric expression for the relative solid angle along the nuclear emulsion as a function of \( \rho \) (the radius of curvature) was found to be:

\[
\frac{d\Omega}{d\Omega_{\text{sr}}} = \frac{1.256 \times 10^{-5}}{0.791 \, \sec(2 \tan^{-1}\rho - 25.6^\circ) + \rho (\pi - 2 \tan^{-1}\rho)}
\]

The results of the experimental measurements gave solid angles consistent with those calculated by the above expression to better than 10\%. The absolute solid angles were thought to be established to better than 20\%.

C. Data Acquisition and Reduction

1. Experimental Data

A study of the \( \text{Pb}^{208}(d,p)\text{Pb}^{209} \) reaction at a deuteron
energy of 20.0 Mev was made in the multigap spectrograph. The proton groups corresponding to states in Pb$^{209}$ up to 5.0 Mev excitation were recorded in 23 gaps spanning the angular region 5° to 172°. The experiment was divided into several parts to study the structure of Pb$^{209}$ which was both strongly and weakly excited in the reaction. One short exposure was made to observe the well-known strongly populated single-particle states. Two long exposures at different toroid orientations (carousel positions) were made to observe the more weakly populated states of Pb$^{209}$. The short exposure consisted of a charge collection of 298 micro-coulombs taken with beam currents of 100-150 nano-amperes. The two long exposures had charge collections of 3800 micro-coulombs and 5966 micro-coulombs and were also taken with beam currents of 100-150 nano-amperes. The detected protons had energies between 17.0 and 22.0 Mev. The same magnetic fields in the multigap, as measured by the NMR probe, were used in all three exposures. In addition to the (d,p) data a complete angular distribution of elastically scattered deuterons was measured at $E_d = 20.0$ Mev for use in determining optical model parameters. To establish absolute cross sections, a separate run in a separate scattering chamber was made using solid state detectors.
2. Targets

Two targets were used in the series of experiments. The targets consisted of 99% isotopically pure Pb$^{208}$ (obtained from the Oak Ridge National Laboratory) evaporated on thin ($\approx 10 \, \mu g/cm^2$) carbon foils. The two targets used were approximately 250 $\mu g/cm^2$ thick and contributed approximately 7 kev to the energy resolution when they were oriented at 45° to the beam in the experiment. The carbon backings on the targets were responsible for large peaks in the spectra due to the $^{12}(d,p)^{13}$ and $^{13}(d,p)^{14}$ reactions which obscured levels of interest in Pb$^{209}$ at several angles. Although self-supporting targets would have been preferred, targets of the thickness desired were found to be difficult to make and extremely fragile. It was found, in fact, that contaminants other than Carbon were of more concern to us in the analysis, and that the carbon levels turned out to be useful in certain aspects.

3. Spectrograph Exposures

The protons from the $(d,p)$ reactions were recorded on 50 micron Ilford K2 and K5 nuclear emulsions on glass plates $\frac{1}{2}$ inch wide and 24 inches long. Two plates were used to cover the entire 48 inch range of exposure. The emulsions were covered with approximately 25 mils of aluminum foil to stop deuterons and to reduce the energy of the protons. The emulsions were developed with Kodak
D-19 Developer and Rapid Fixer at a temperature of approximately 5°C. The K2 emulsions were found to be only marginally suitable for the detection of protons and have not been used in subsequent proton experiments.

The emulsions were microscopically scanned for proton tracks in 0.5 x 12.0 mm steps. The protons of interest should pass completely through the emulsions in the direction corresponding to entering the emulsion on the side facing the target and exiting on the other side. This was the criterion used to identify the proton tracks. A small background of tracks of different lengths and orientations in the emulsion was observed. These tracks were rejected and only tracks with the length 75-150 microns which deviated by less than 30° from the correct direction parallel to the plate length were counted. The rejected tracks were thought to be due to protons which scattered in the aluminum foil, from zone defining slits, etc, or due to proton recoils from neutron collisions in the emulsion. As the emulsions were counted, the number of proton tracks per strip were recorded on both count sheets and IBM porta-punch cards. These cards were used to produce more permanent cards for use in computer codes at various steps of the analysis.

The deuterons in the elastic scattering were recorded on 50 micron Ilford K2 nuclear emulsions. No absorber foils were used. The Rutherford slits were used and the collected charge was 48.59 micro-coulombs. The plates
were developed and scanned in the same manner as described for the protons above.

4. Scattering Chamber Experiments

The target thickness was determined from the elastic scattering of 6.0 Mev deuterons which were assumed to obey the Rutherford scattering relation. The data were acquired in the Laboratory 30 inch Ortec Scattering chamber using silicon surface barrier detectors with conventional electronics (consisting of pre-amplifier, amplifier, and multichannel analyzer). Yields were taken at five angles in the angular range 50° to 90°. At each angle the target thickness \( N' (\text{Gm/cm}^2) \) was calculated from the expression:

\[
\text{Yield} = nN' \left( \frac{A_0}{\text{G.Wt.}} \right) \frac{d\sigma}{d\Omega}
\]

where \( n \) is the number of particles passing through the target. \( A_0 \) is Avogadro's Number, G.Wt. is the gram molecular weight of the target nuclei, \( \frac{d\sigma}{d\Omega} \) is the Rutherford cross section, and \( d\Omega \) is the solid angle. The value of \( N' \) at the five angles were averaged and a value of 270±20 \( \mu\text{g/cm}^2 \) was obtained, where the error assigned corresponds to the uncertainties in the absolute charge collection (+5%), the absolute solid angle (+4%), and the angle of the target orientation (+3%). The errors arising from statistics were ≤1%, the errors due to deviation from the Rutherford scattering cross section (as measured by the R.M.S. deviations
of the ratio of $\text{Yield}(\theta)/d\sigma_{\text{Ruth}}(\theta)$ were ≤0.5% and the errors due to uncertainties in the detector angles were <1%. The target thickness determined was in good agreement with alpha-gauge (BB65) measurements which gave 300±30 µg/cm².

The same target was then used to measure the angular distributions of the $(d,d_0)$ scattering and $(d,p)$ reaction at 20.0 Mev. The elastic deuterons and the proton groups associated with the single-particle levels in $^{209}$Pb were simultaneously detected and their yields observed at 15 angles between 25° and 120°.

In addition to the detector used in the Rutherford scattering a second detector located immediately behind the first was used to take this data. The outputs of the two detectors were summed and a coincident condition set to route the summed outputs to different quadrants of the multichannel analyzer depending on whether the particle was stopped in the first detector or passed through into the second. The first detector was chosen to have a thickness which stopped all deuterons, thus the non-coincidence spectrum corresponded to the deuteron spectrum, and the coincidence spectrum to the proton spectrum.

The cross sections for both the $(d,d_0)$ and $(d,p)$ reactions were obtained by inserting the observed yields into expression (C.1), where the target thickness is assumed to be the same as that determined from the Rutherford scattering discussed above. The cross sections so determined were estimated to have errors of about ±10%.
This value reflects the uncertainties in the target thickness arising both from the uncertainties in the initial determination by the Rutherford scattering and the uncertainty in the exact location of the beam spot on the target at the two different energies. The other sources of errors such as charge collection, target orientation, detector orientation, etc. are also included.

D. Data Analyses

1. Analysis Procedure

The analysis of the data taken in multigap spectrographs has been described in detail elsewhere (CO68) and will only be outlined here with emphasis on those aspects of the procedure which were of particular importance in this study. The raw spectra were first plotted and lines from reactions on known target contaminants tentatively identified. These contaminants are usually well studied nuclei such as $^{12}\text{C}$, $^{13}\text{C}$, $^{14}\text{N}$, $^{16}\text{O}$, etc, whose presence in almost any target might be expected. The lines arising from these contaminants as well as those from known levels of the nucleus under investigation are then used to determine the effective magnetic field for each gap in a self calibration procedure. These effective fields are then fed into a computer program Q-Plot together with all the spectra. This program makes use of the fact that the apparent $Q$-value for the reaction leading to a nuclear
level must be the same for every observation angle. The mass of the final nucleus, $M_f$, is specified and the spectra are plotted as the number of events versus reaction Q-value. A change in the apparent Q-value of a level from angle to angle implies that it has been assigned to an incorrect final nucleus $M_f$ in the Q-equation. The spectra are plotted one above the other on a large sheet of paper and automatically aligned. Thus if a group of peaks is found to be along a straight vertical line it must correspond to a level of the nucleus of mass $M_f$. Levels in nuclei of different $M_f$ will show a shift as one goes back in angle and the rate of shift will be characteristic of the mass of the final nucleus. Of course, for heavy nuclei the levels in final nuclei which differ by $A+1$, for example, can in certain cases appear to lie along the same vertical line, so that even the levels showing no shift must be checked to determine whether they belong to contaminants.

The Q-plots proved to be extremely useful for establishing the levels in Pb$^{209}$ and identifying the levels of the various contaminants. Found in the spectra were levels due to the contaminants C$^{12}$, C$^{13}$, N$^{14}$, O$^{16}$, Na$^{23}$, and Si$^{30}$ in the target. No evidence for the presence of Pb isotope contaminants (Pb$^{204}$, Pb$^{206}$, or Pb$^{207}$) in the target was found. The Q-plots were of particular importance in analyzing the long (d,p) exposures where levels due to contaminants such as Na$^{23}$, and Si$^{30}$, which were present in the target in only small amounts, could be seen in the Q-plot
moving through the Pb$^{209}$ levels of interest as a function of angle. The small contaminant levels were of some concern since the weak transitions to the contaminant levels were in many cases of the same magnitude as the states being studied in Pb$^{209}$. It was important to identify all the weakly populated contaminant levels and trace them through the spectra, so as to ensure that the angular distributions of the weak transitions to levels of Pb$^{209}$ were free of contributions from contaminants. All peaks in the spectra were identified as either levels of Pb$^{209}$ or some contaminant. A level in Pb$^{209}$ was considered established if it could be observed unambiguously at six angles. Once having established the assignments to Pb$^{209}$ the yields could be extracted.

The yields were obtained by summing the tracks in a peak and subtracting a background which was determined in a near-by region free of peaks. This job was performed by the peak fitting computer program *Autofit* (SP65, CO68, MS70). The program takes a reference peak shape and fits it to the raw data. A dense spectrum consisting of 20 or more overlapping peaks can be fitted simultaneously. The peak positions and area under each peak are the output of the program. The peak positions are labelled by their level number and are put into a program which computes the average Q-value for each level. The yields are similarly labelled by their level number and are the input to the program *Crosec* which adjusts the yields by a multiplicative
factor and makes solid angle corrections for kinematics and location along the emulsions; giving as the output the absolute cross sections. The multiplicative factor in this program can either be read into the program as (1.) a simple factor which has been found to relate the relative cross sections to the absolute cross sections, or (2.) calculated by the program if the target thickness and charge collection are read into the program.

2. Cross Section Normalization

The angular distributions obtained in the multigap spectrograph (both the elastic scattering and (d,p) reaction distributions) were normalized to the absolute cross sections measured in the scattering chamber. This was done by adjusting simultaneously the two angular distributions until the best overlap was obtained. The uncertainties in this procedure was estimated to be about 10% in the case of the deuteron elastic scattering and about 5% for the (d,p) reaction where five angular distributions corresponding to single-particle states in Pb$^{209}$ were compared to determine the best normalization. The absolute sections assigned by this method were compared to the cross sections calculated from the target thickness based on the alpha-gauge measurements of the target used in the multigap experiments. The two methods produced results that were in agreement to 10% in all cases. We believe that our absolute cross sections have been
established to $<20\%$ in this study.
V. EXPERIMENTAL RESULTS AND ANALYSIS

A. Presentation of Experimental Results

Typical proton spectra obtained in the study of the strong and weak transitions of the \( \text{Pb}^{208}(9d,p)\text{Pb}^{209} \) reaction are shown in figure V-1, V-2, and V-3. The first figure shows the seven strongly populated single-particle states whose cross sections were observed to be in the range of 1-10 mb/sr. The second and third figures (figures V-2 and V-3) show the weakly populated levels of \( \text{Pb}^{209} \) in addition to the strong transitions. The weak transition cross sections are on the order of 10-100 \( \mu \text{b/sr} \). Energy resolutions of about 0.05-0.07\% (10-15 keV) were observed in these studies. The levels of \( \text{Pb}^{209} \) are labelled either numerically or by their neutron configurations; groups arising from lighter mass contaminants are labelled appropriately. The energy levels established up to 5.2 MeV excitation are listed in Table V-1 where they are compared with the results of other studies. The ground state Q-value was determined to be 1.710\(+0.015\) MeV compared to 1.719\(+0.014\) MeV obtained from the known masses (MT65).

Table V-1 contains a number of levels not reported in previous studies of this reaction. In particular, the levels above 4.3 MeV represent new level assignments. A great number of the states populated up to 4.3 MeV excitation can be associated with levels strongly populated by the \( \text{Pb}^{207}(t,p)\text{Pb}^{209} \) (FI69, FI70) and \( \text{Pb}^{210}(p,d)\text{Pb}^{209} \)
A typical proton spectrum obtained in the short exposure (charge=298μC) studying the strongly populated single-particle states.

Figure V-1

$^{208}\text{Pb}(d,p)^{209}\text{Pb}$

$E_d = 20.0 \text{ MeV}$

$\theta = 53.75^\circ$
A typical proton spectrum obtained in the (d,p) long exposure (charge=5966 uc) studying the weakly populated states in Pb⁹⁰ (Figures V-2 and V-3)
Figure V-3

$^{208}\text{Pb}(d,p)^{209}\text{Pb}$

$E_d = 20.0 \text{ MeV}$

$\theta = 53.75^\circ$
<table>
<thead>
<tr>
<th>Level No.</th>
<th>Present Study</th>
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\(^\dagger\) Sn(209) taken to be 3.937±0.010 MeV (Nuclear Data Sheets 1963)

a) Reference (EK69)
b) Reference (JD69)
c) Many references including (EK69, JD69, MP67)
d) Reference (IB68)
e) Reference (IF70, FI69)
f) Reference (IF70, FI69)
g) Reference (KB62)
h) Reference (BSG1)
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j) Reference (FC62)
(FI69, FI70) reactions, verifying the 2p-1h character of these states as suggested by shell model considerations.

The angular distribution of the proton groups associated with the seven single-particle states are shown in Figure V-4. The cross sections for the transitions to these states were found to be in agreement with a previous measurement at 20.3 Mev (MP67) in all cases except the \(2g_{9/2}\) and \(1i_{11/2}\) states which were found in this study to be larger by 20% and 30% respectively. The angular distributions of 34 weak transitions are shown in Figures V-5 to V-7. These correspond to the levels populated in the excitation region 2.0 to 4.3 Mev in \(^{209}\text{Pb}\). Above this excitation the level density becomes too great for the accurate extraction of quantitative information.

The experimental errors associated with the differential cross sections are shown in the figures. The errors associated with the strongly populated states (figure V-4) are based on the estimated scanning reproducibility (+5%) and statistics. Those associated with the weakly populated states (figures V-5 to V-7) were more difficult to assign because of backgrounds and level densities. The errors shown reflect the uncertainties in the determination of backgrounds and the unfolding of overlapping peaks, as well as the estimated uncertainties in scanning and statistics. The curves drawn in the figures are fits to the data obtained in the DWBA analysis to be discussed in a following section.
B. Analysis of Angular Distributions

1. Optical Model Potentials

   a. General Considerations

   In the DWBA analysis of direct reactions it is necessary to specify the distorted-waves in the entrance and exit channels. These distorted waves are solutions to the Schrödinger equation for the elastic scattering in each channel, using an effective nucleon-nucleus potential to describe the interaction. The potential form that has been found to be the most satisfactory in describing the elastic scattering data has been that of the optical model potential. The optical model assumes that the nucleon-nucleus interaction can be replaced by a one-body potential which acts on the incident nucleon. The individual interactions are averaged out and replaced by a potential that depends only on the radial distance $r$ from the projectile to the nucleus. Many discussions concerning the origin, justification, and application of the optical model can be found in the literature (BE40, FP54, BA52, PB62, BF58) and they will not be discussed in any detail here.

   The success of this model rests primarily on its ability to describe elastic scattering data over a large range of incident nucleon energies and target mass regions by the smooth systematic variation of the parameters which characterize the model. There have been several comprehensive optical model analyses of elastic scattering which have established the systematic trends of the various para-
Annular distributions of the strong transitions to the seven single-particle states in Pb.
Figure V-5

Angular distributions of the weak transitions, leading to levels 1 through 17
Angular distributions of the weak transitions
Figure V-7

Angular distributions of the weak transitions leading to levels 35 and 39
meters in the model. It is in this context that the optical model becomes an extremely useful means for the parameterization of a large body of experimental information. In principle, one can consult these optical model studies to find what parameters to use for specification of the optical potentials (describing the distorted waves) for use in a reaction calculation. In practice, the optical model studies have not reached the point where they are entirely free of ambiguities, and elastic scattering data is usually obtained when a reaction is studied, so that the appropriate optical model potential can be determined independently. Even so it is generally believed that the potential so determined must be roughly in agreement with generalized optical potential of the comprehensive optical model studies if it is to be considered meaningful. Since the reaction cross sections depend sensitively on the potentials used to describe the distorted waves, it is clear that the spectroscopic information extracted in an analysis depends heavily on the reliability of the optical model parameters chosen. In the following section the optical model parameters chosen for this study will be discussed.

b. Optical Model Parameters

The optical potential used in the analysis is of the general form:
V(r) = -V_0(1+e^x)^{-1} \cdot \left[ \frac{a}{(1-e^{-x})^{1/2}} \right] \cdot \frac{d}{dx} \cdot \left[ \frac{1}{(1+e^{x'})^{1/2}} \right] \cdot \frac{a_0}{\hbar c} \cdot \left[ \frac{d}{dr} \right] \cdot \left[ \frac{1}{(1+e^{x''})^{1/2}} \right]^{-1}

where,

x = \frac{(r - r_0 A^{1/3})}{a_0}

x' = \frac{(r - r_1 A^{1/3})}{a_1}

x'' = \frac{(r - r_s A^{1/3})}{a_s}

to which there is added the Coulomb potential for a uniformly charged sphere,

V_{coul}(r) = \begin{cases} \frac{2\pi e^2}{2R} \left( 3 - \frac{r^2}{R^2} \right) & r < R \\ \frac{2\pi e^2}{r} & r > R \\ \end{cases}

where the radius of the sphere is R = r_c A^{1/3}.

The deuteron parameters used in this study were determined by a fit to the elastic scattering cross section measured at 20.0 Mev. A least squares fit to the data was made with the computer code ABACUS-II (AU65) starting with the optical model parameters derived by Satchler (HO66) from an analysis of 21.5 Mev deuterons on the Pb isotopes. This parameter set had been found to give a fit to 20.3 Mev elastic scattering on Pb^{208} in the study of Muellhelner, et al., and was likewise found to give a good fit to the data obtained in this study. When the parameters were allowed to vary and the data searched upon, very little change in the parameters was observed. The potential derived
Comparison of the optical model fits to the measured deuteron elastic cross section.
### TABLE V-2

Optical Model Parameters

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</table>

^a Optical model analysis of this work

^b Reference (HO66)

^c Reference (PE64)

^d Reference (PE63)

^e Reference (AC70)
in this search and the Satchler potential are listed in Table V-2 and designated DI and DII respectively; the corresponding theoretical curves are shown compared to the experimental cross sections in Figure V-8. The spin-orbit coupling term was set to zero in both parameter sets. The fitting criterion used in the elastic scattering analysis was defined as the minimum of the quantity $\chi^2$ given by:

$$\chi^2 = \frac{1}{N} \sum_{i=1}^{N} \left( \frac{F(\theta_i) - f(\theta_i)}{e(\theta_i)} \right)^2$$

where $F(\theta_i)$ and $f(\theta_i)$ are the experimental and theoretical cross sections at angle $\theta_i$, where $e(\theta_i)$ is the absolute error in $F(\theta_i)$, and $N$ is the number of points considered. The values of $\chi^2$ for the two sets are listed in Table V-2. Since both sets gave acceptable fits to the data and indeed differed only slightly, it was decided to use parameter set DII taken from the literature. It will be shown later that the two parameter sets give essentially indistinguishable results when used in the DWBA calculation of the (d,p) cross sections.

The proton parameters used in the analysis were those of the parameter set derived by Perey (PE64) in an optical model study of 17.0 Mev proton elastic scattering data on several heavy nuclei including Pb$^{208}$ and Bi$^{209}$. The potential set was chosen because it was determined from elastic scattering at an energy near the range of proton energies observed in our experiment (17.0-22.0 Mev), and because it was consistent with the general potential form...
indicated for heavy nuclei from various other optical model studies (e.g. RA65, PE63). This parameter set with the real depth adjusted for 20.0 Mev protons was used in this analysis and is listed in Table V-2 as PI. A survey of the literature had revealed that several other potential forms have been suggested from optical model analyses of large bodies of elastic scattering data. Since our proton parameters must be obtained by some extrapolation procedure, it was of interest to determine what effect the use of different proton parameter sets taken from the literature might have on the calculated (d,p) cross sections. The two other parameter sets considered are listed as PII and PIII in Table V-2. Parameter set PII is a set determined by Perey (PE63) in a study of elastic scattering data taken in the energy range 9-22 Mev on nuclei in the mass region Al to Au. It differs from PI only in the imaginary part of the potential where the imaginary diffuseness $a_I$ for PII is 0.47 fm while $a_I$ for PI is increased to 0.76 fm. The larger diffuseness for heavier nuclei has been indicated in several studies (BE65, RA65, PE64) and this was one of the reasons parameter set PI was chosen for the analysis. Parameter set PIII is the potential set derived by Ascuitto (AC70) in the fit of 24.5 Mev proton scattering on Pb$^{206}$. This parameter set includes both volume and surface absorption terms in the imaginary part of the potential which have been found to be necessary for the description of higher energy (30.0-40.0 Mev) elastic scattering data (SA67, BH68).
Proton potentials of the form characterized by PI and PII have been used most frequently in the recent analyses of 
(d,p) and (p,d) reactions (MP67, EK69, JD69). The proton potential characterized by PIII has been used in the analysis of several (p,t) and (t,p) studies (BF68, 
HC70). The effect these different parameter sets have on the predicted (d,p) cross sections will be investigated in Section V.B.2.b.

2. DWBA Analysis of Reaction Data
a. General Considerations

The distorted-wave code DWUCK (KU70) was used throughout the analysis. It employs a wood-Saxon form for the real potential, V, of the incident, emitted, and captured particles and has options for imaginary potentials of both volume and surface absorptive character as specified by a Wood-Saxon, and Wood-Saxon derivative form. There are options for spin-orbit terms in the scattering, although they were not employed in this analysis. A spin orbit potential of the Thomas type whose strength can be adjusted as an independent parameter is included for the captured particle. The program adjusts the real well depth of the captured particle to reproduce the empirical separation energy and the form factor is calculated as an eigenstate of this well. There are additional options available in the program some of which will be discussed in more detail in the remainder of this section. These include the options for the use of
externally calculated form factors in the calculation of the reaction cross section, and the application of finite range and nonlocality corrections to the cross sections.

The application of DWBA analysis to experimental data has been discussed in detail in many places in the literature (e.g. LS64). The analysis here follows these conventional lines unless otherwise noted. The expression for the \((d,p)\) cross section can be written as,

\[
\frac{d\sigma}{d\Omega} = N_o \frac{2J_f+1}{2J_o+1} \left( S_{lj} \frac{\sigma_{lj}(\theta)}{2j+1} \right),
\]

where \(N_o\) is the usual zero-range normalization factor taken here to be 1.5, \(J_o\) and \(J_f\) are the spins of the initial and final states, and \(j\) is the total angular momentum transferred by the neutron. The cross section \(\sigma_{lj}(\theta)\), is calculated by \textsc{DWUCK}, and \(S_{lj}\) is the spectroscopic factor determined by a comparison between the DWBA cross section and the experimental cross section. When the target ground state has a \(0^+\) spin and parity and the final spin is known this expression reduces to,

\[
\frac{d\sigma}{d\Omega} = N_o S_{lj} \sigma_{lj}(\theta)
\]

The conservation laws for this case are

\[
J_f = j \quad \text{and} \quad \pi_f = (-1)^l,
\]

where \(\pi_f\) is the parity of the final state.

The analysis of the strong and weak transitions observed in the study were treated separately. In both cases the conventional DWBA analysis was made for the assignment
of $l$-values, and the extraction of spectroscopic factors, but the emphasis in the two analyses was different. In the case of the strong transitions to the well known single-particle states which are ideally suited for DWBA analysis, the aim of the analysis was to obtain accurate values of the spectroscopic factors for those states. To this end some effort was spent in investigating the relatively small effects the uncertainties in the optical potentials and neutron well might have on the spectroscopic factors obtained. The effects of including finite-range and nonlocality corrections in the calculations were also studied. In the case of the weak transitions to states of $2p$-$1h$ character the very fact that these states are excited by this reaction is of significance. The $2p$-$1h$ states of Pb$^{209}$ are not expected to be excited in a $(d,p)$ reaction unless (1.) there are small admixtures of configurations in either the initial or final states, through which the state can be populated by simple stripping, or (2.) the reaction proceeds by a two-step reaction mechanism. The reaction theory of the weak transitions to such states is not entirely understood and the method of analysis is uncertain. In particular, it is not even known whether the two-step processes are expected to play an important role in the excitation of these states. On the other hand, nuclear structure calculations predict that some of the $2p$-$1h$ states should be excited by a simple stripping reaction. Thus in the analysis of this study the reactions were assumed to be of a simple one-step nature and the transitions analyzed with the conventional
DWBA theory for $l$-values and spectroscopic factors. The assumption is consistent with the observation that the angular distributions of these transitions have typical stripping type patterns which could be fit reasonably well by the usual DWBA predictions. Transitions to known $2p-1h$ states in $\text{Pb}^{209}$ were found to give $\ell$-values in agreement with spin and parity assignments made in other reaction studies. Encouraged by these results, fits were made to the angular distributions of the remaining transitions and $\ell$ assignments made. The analysis of these transitions with the conventional DWBA theory is not believed to be completely correct and in particular is thought to result in spectroscopic factors subject to some error. Theoretical studies indicate, however, that these spectroscopic factors are meaningful and can be used to provide a reasonable estimate of the true values. In the remaining portion of this section some of the specific aspects of the application of the analysis to the data will be discussed.

b. Optical Model Uncertainties

The optical model parameters used in this study were determined, as discussed previously, in the conventional manner, by the analysis of appropriate elastic scattering data. There remain, however, ambiguities and uncertainties in these parameters whose effect on the calculated reaction cross sections will be investigated below. Before proceeding some general comments about these potentials should
be made. First, there is evidence for the need of spin-orbit coupling terms in both the proton (PE63, SA67) and deuteron (RA63, PP66, HM68) optical potentials. Inclusion of such terms in the present calculations of (d,p) cross section for the transitions to the single-particle states in Pb$^{209}$ resulted in only slight changes in shapes and changes in magnitudes of $\leq 5\%$. In the analysis to be presented here no spin-orbit terms were used in the proton or deuteron optical potentials. Second, the phenomenological potentials are only the local equivalents of the true nonlocal potentials in the sense that they represent the same scattering. The inclusion of nonlocality in the potentials will change the wave functions in the nuclear interior, thereby affecting the reaction cross sections in some cases. The corrections for one kind of nonlocality that is often postulated can be accurately computed using the so-called local-energy approximation. This correction which results in a change in magnitude of the (d,p) cross sections with little or no alteration in the shape of the calculated angular distributions will be discussed in more detail in a following subsection (Section V.B.2.d).

It is of interest now to investigate what effect the uncertainties in the determination of the elastic deuteron and proton optical potentials might have on the predicted (d,p) cross sections. In particular we are interested in strongly populated single-particle levels in Pb$^{209}$. The deuteron parameter set was determined by a fit to elastic scattering data taken at 20.0 Mev. It was
observed to be in good agreement with a parameter set derived by Satchler (H066) in a study of 21.3 MeV scattering data in the Pb region. A comparison of the (d,p) cross sections calculated with the two deuteron sets DI and DII showed differences of less than 2% in magnitudes with no observed changes in shapes.

The three proton parameter sets discussed in Section V.B.1.b. were used to calculate (d,p) cross sections to the seven single-particle states in Pb$^{209}$. The resulting distributions for $l = 2, 4, \text{ and } 6$ transitions (leading to the $3d_{5/2}$ state at 1.565 MeV, $2g_{9/2}$ state at 0.00 MeV, and the $1i_{11/2}$ state at 0.779 MeV respectively) are shown in figure V-9. The calculations were done in the local zero-range approximation and the deuteron potential set DII and neutron well DH (Table V-3) were used. It can be seen that the (d,p) cross sections calculated with PII and PIII remain in agreement with that calculated using PI to better than 10-15% in the region on the maxima. The shapes change appreciably only in very forward angles ($\leq 20\%$) and in the back angles ($\geq 90\%$). No single parameter set was found to give a marked improvement in the fits to the experimental distributions. Since some changes in magnitudes are noted in these predictions, spectroscopic factors for the single-particle states are extracted using these three proton potentials and are compared (Section V.3.a.) to determine whether significant differences are noted.
Figure V-9
Comparison of the DWBA curves calculated using different proton optical model potentials.
c. Neutron Form Factor Uncertainties

**Single-Particle States**  The neutron form factors used for the single-particle states were calculated in the separation energy prescription by the computer code DWUCK. The single-particle potential in the calculation was taken to be a Wood-Saxon well of radius $r_0A^{1/3}$ and diffuseness $a_0$ with an additional spin-orbit term of strength $\lambda$ time the Thomas factor; i.e.,

$$V(r) = -V_0 \left[1 - 2\lambda \left(\frac{\hbar}{2mc}\right)^2 (\mathbf{L} \cdot \mathbf{S}) \frac{1}{r} \frac{d}{dr}\right] \left(1 + e^x\right)^{-1},$$

where

$$x = \left(\frac{r - r_0A^{1/3}}{a_0}\right).$$

The form factors calculated by this prescription should be quite good for these states. There are some uncertainties however, in the exact shape of the Wood-Saxon well to use for the potential.

In the calculations to be reported here the neutron well parameters $r_0 = 1.23$ fm, $a_0 = 0.65$ fm, and $\lambda = 25$ were used. These geometric parameters were obtained from the Dost and Hering (DH63) and Oxford (JD69) studies of the $^{208}\text{Pb} (d,p)^{209}\text{Pb}$ reactions at incident deuteron energies below the Coulomb barrier. In these studies the neutron parameters ($r_0$ and $a_0$) were varied until spectroscopic factors close to unity were obtained for the "single-particle" states populated. Since the distorted waves should be simply the Coulomb waves at these energies, these analyses should be free of many of
the ambiguities which plague higher energy studies. Most reaction studies have found that the use of these values of the parameter (or values quite similar to these) give reasonable agreement with the data. These parameters, however, do not reproduce the observed level ordering and binding energies of the observed neutron "single-particle and hole states". The neutron well which will reproduce the observed level ordering of both particle and hole states is quite different from the Dost and Hering well. The studies of Rost (RO63) and Batty (BA70) find that these parameter values are typically of the magnitude of $r_o \approx 1.35$ fm, $\alpha_o \approx 0.80$, and $\lambda \approx 35.0$. The use of these parameters in the analysis of reaction data, however, give spectroscopic factors which are much smaller than are expected. Recently the level ordering of the neutron single-particle levels alone were fit by Zaidi and Darmodjo (ZD67). In their study they excluded the $1j_{15/2}$ level at 1.424 Mev from consideration since there is strong evidence that this level is significantly fragmented. The parameters obtained ($r_o=1.19$ fm, $\alpha_o=.075$ fm, and $\lambda=20.5$) were found to yield reasonable results in the analysis of recent (t,d) (IB68) and (d,p) ( JD69) studies. The parameters, however, do a rather poor job of reproducing the level ordering and binding energies of the neutron single-hole states. A more detailed discussion of the current status regarding the single-particle potential of Pb$^{208}$ can be found in a review article by Batty (BA70).

In view of these uncertainties in the exact shape of
the single-particle potential it was of interest to use the parameter sets discussed above in the DWBA calculations to study their effect on the fits and spectroscopic factors. In particular, four sets were chosen to study. These include the Dost and Hering (DH) parameters, the Rost (RO) parameters, and the Zaidi and Darmodjo (ZD) parameters discussed above. Also considered was a parameter set identical to the DH potential, except for the spin-orbit strength (λ=32). This potential labeled CO gives a slightly better fit to the level ordering than does the DH potential. The parameter sets are listed in Table V-3. The predicted DWBA cross sections using these four potential for the \( l = 2, 4, \) and 6 transitions (leading to the \( 3d_{5/2}, \) \( 2g_{9/2}, \) and \( 1i_{11/2} \) states of \( \text{Pb}^{209} \)) are shown in figure V-10. The optical parameters DII and PI were used and the calculations were done in the zero range-local version of the calculation. The most obvious observation is the large cross sections predicted by the Rost potential. The other three potentials give by and large the same shape and magnitudes to about 5% and thus it appears that for all practical purposes they can be regarded as the same.

Two Particle-One Hole States  The neutron form factors used in the analysis of the transitions leading to the \( 2p-1h \) states in \( \text{Pb}^{208} \) also were obtained using the separation energy prescription. The neutron parameters DH in Table V-3 were used throughout. For the transitions to states which are unbound (\( E_x > 3.940 \text{ Mev} \)), the states were assumed to be
Table V-3

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*Well depth adjusted to give a binding energy equal to the observed separation energy.
Comparison of DWBA curves calculated using different neutron well parameters
bound by 25-100 kev and treated in the normal manner.

The use of the separation energy approximation for the calculation of the neutron form factor, for the transitions to the 2p-1h states is clearly not a very good approximation. The errors resulting in the form factors calculated in this manner have been discussed in detail in Section III.B., and C. Although the prescription does ensure that the wave functions have the correct exponential behavior in the nuclear exterior, it almost certainly results in an incorrect normalization which strongly affects the spectroscopic factors extracted. Furthermore, there is no guarantee that the shape of the form factor in the interior is the same as the true shape which introduces further differences in the reaction cross sections to these states. In these cases, it is clear that the nuclear structure of the final (and/or initial) state must be taken into account in the specification of the bound state form factor for use in reaction calculations. Calculations of this type are still at a preliminary stage, although the results thus far look very promising (Section III.C.). From previous studies of transitions to states of the character populated in this study it was observed that while the predicted angular distribution obtained using form factors calculated with the S.E. prescription had roughly the correct shapes, the magnitudes of cross sections were in disagreement with that obtained using realistic form factors by as much as a factor of 2 to 4. Thus it
appears that it should be possible to assign \( l \)-transfers in this study, but that the spectroscopic factors have uncertainties of the order of at least a factor of 2. (See discussion Section III.C.).

The \( 2p-1h \) states could be populated in a direct transfer reaction by two possible processes, either through small fragments of single-particle excitation mixed into the final \( 2p-1h \) state or through \( 2p-2h \) correlations in the target ground state. Most of the \( 2p-1h \) states in the excitation range of interest are of negative parity (See Section II and Section V.A.) and thus must be populated by an odd \( l \)-transfer. Since the major neutron shell \((N=126-184)\) is composed of positive parity orbits (except for the \( 1j_{15/2} \) orbit), this means that the stripping into the \( 2p-1h \) states must be proceeding via orbits of the adjacent shells; i.e. the shell \( N=82-126 \), corresponding to levels below the nuclear Fermi sea, and the shell \( N>184 \) corresponding to the unbound single-particle states of \( \text{Pb}^{209} \). Thus it is possible to populate a final state by stripping a neutron into one (or both) of two shell model orbits of the same \( l\), but different radial quantum number, \( n \). For example, a final state of spin-parity \( 1/2^- \) can in principle be populated by stripping into either (or both) the \( 3p_{1/2} \) or \( 4p_{1/2} \) orbits. The predicted DWBA cross sections for the transitions of \( l=1, 3, \) and 5 leading to orbits of different radial quantum number, \( n \), are shown in figure V-11. The calculations were done in ZR-L (See next section for
Comparison of the DWBA curves calculated for stripping into shell-model orbits of the same $t, j$ but of different radial quantum numbers.
explanation of this notation) with optical parameters $D_{ll}$ and $P_{l}$, and neutron well $DH$ (Table V-2). From the figure it can be seen that the shapes are quite similar and distinctive. If a radial cut off is used the predicted angular distributions are found to be almost indistinguishable. The magnitude of the angular distribution corresponding to the larger $n$ was adjusted in the figure to illustrate the similarity in predicted shapes. The cross sections for the transitions to orbits of larger $n$ (corresponding to orbits in the $N>184$ shell) were found to be uniformly about a factor of 2 larger than that predicted by the smaller $n$. This is just what might be expected because the wave function for the shell-model orbit of larger $n$ extends further out into the nuclear exterior which is important in the stripping process.

While it is true that it is only the negative parity $2p-1h$ states which are expected from shell model considerations to be found in the excitation region studied, the existence of positive parity states can not be ruled out a priori. Thus in the analysis positive $\ell$-transfers were also considered possible. This presents difficulties since the angular distributions of transfers which differ by a single unit of orbit angular moment are quite similar. This is illustrated in figure V-12. In practice it was difficult to distinguish between $\ell = 2$ and 3, and $\ell = 4$ and 5 transitions. This will be discussed in more detail in the section dealing with the analysis.
Comparison of the DWBA curves predicted for stripping into orbits which differ by one unit of orbital angular momentum
d. Finite-Range and Nonlocality Corrections

Most distorted-wave calculations of stripping make use of the so-called zero-range approximation. Although it is possible with available codes to do an exact finite-range computation, the effects can usually be incorporated into a zero-range calculation by using the local energy approximation (BG64, PS64). This modulates the bound-state wave function by the multiplicative factor calculated in DWUCK (KU70) as;

\[ \Lambda(r) = \left[ 1 - \frac{2}{\hbar^2} \frac{m_p m_n}{m_d} R^2 (V_d(r) - V_p(r) - V_n(r) - B_d) \right]^{-1}, \]

where \( V_i(r) \) is the optical potential for particle \( i \), \( B_d \) is the binding energy of the deuteron, and \( R^2 \) is the range (defined in Ref(BG64) from which we take \( R=0.620 \text{ fm} \)).

It is believed that the optical potentials should be regarded as local equivalences of the non-local potentials which give the same scattering. Studies have shown (PE62, AU65) that the wave functions associated with non-local potentials are identical to those of the local potential in nuclear exterior, but are changed in the nuclear interior compared with those obtained with the equivalent local potential. For the type of non-local potentials usually assumed, this effect can be taken into account in the distorted wave calculations with local potentials by the use of another local-energy approximation factor to modify each distorting wave (PS65),

\[ W(r) = \left[ 1 - \left( \frac{p^2 m_1}{4 \hbar^2} \right) V_1(r) \right]^{-\hbar}, \]
where \( m^* \) is the reduced mass of the particle, \( \beta \) is the non-locality range, and \( V_4(r) \) the equivalent local potential. In our calculations we have used \( \beta_d = 0.54 \) and \( \beta_p = 0.85 \). Similar effects are expected for the neutron bound states. While there is no reason to believe that the local-energy, approximation is not valid for bound states, it is not clear whether the local Saxon well used is equivalent to the true non-local well. For this reason this correction has not been included. If it had been included with a nonlocality parameter of \( \beta_n = 0.85 \) the spectroscopic factors would have been decreased by 20%. This effect on the spectroscopic factors has been noted in previous \((d,p)\) studies on \( \text{Pb}^{208} \) (MP67).

The effects of making the zero-range (ZR) approximation with the local (L) distorting potential compared to the complete finite-range (FR) calculation with nonlocal (NL) distorting potentials is shown in figure V-13. The FR-NL calculations of the cross section are noted to decrease by 5-15%, where the effects become greater for the larger \( l \) transfers. The shapes change only slightly where again more of a difference is noted for the larger \( l \) transfers. Also, shown in the figure V-13 is a ZR-L calculation with a radial cut off \( R_{CO} = 8.5 \) fm. Several studies in the Pb region have noted (WS70, PH69) that better fits are obtained when sharp cut offs on the stripping radial integrals are used to completely remove contributions from the interior. In this study we observed similar results where better fits especially for the transfers \( l = 6 \) and 7 were obtained when
Comparison of the DWBA curves calculated in zero-range local (ZR-L), finite-range nonlocal (FN-NL) and in zero-range local with a radial cutoff at $R_{co} = 8.5 \text{fm}$.

Figure V-13

\[ E_d = 20.0 \text{ MeV} \]
Figure V-13

Comparison of the DWBA curves calculated in zero-range local (ZR-L), finite-range nonlocal (FN-NL) and in zero-range local with a radial cutoff at $R_{co} = 8.5\text{fm}$.

$Pb^{208}(d,p)Pb^{209}$

$E_d = 20.0\text{ MeV}$
Comparison of the DWBA curves calculated in zero-range local (ZR-L), finite-range nonlocal (FN-NL) and in zero-range local with a radial cutoff at \( R_{co} = 8.5 \text{ fm} \).
Comparison of the DWBA curves calculated in zero-range local (ZR-L), finite-range nonlocal (FR-NL) and in zero-range local with a radial cutoff at $R_{co} = 8.5$ fm.

Figure V-13
the cutoff was used. This may be an indication that other kinds of nonlocality are present \cite{SC67, RA67} and that greater reductions in the wave functions in the interior are necessary. Possible evidence for increased damping of this kind has been found both for \((d,p)\) and \((p,d)\) reactions \cite{Y067, PP68}.

The use of finite-range and nonlocality corrections for the weak transitions to the 2p-1h states were also investigated. The change in the cross sections when these corrections were included in the calculations resulted in approximately a 10\% increase in cross sections compared to the ZR-L calculation with essentially no change in shapes. In view of the large uncertainties in the predicted cross sections arising from the form factor ambiguities, these corrections (FR-NL) were not included in the analysis of these levels.

C. Results of Analysis

1. Single-Particle States

   a. Fits and Spectroscopic Factors

   The theoretical DWBA fits to the angular distributions of the seven single-particle levels in \(\text{Pb}^{209}\) are shown in Figure V-4. The DWBA curves were computed in the zero-range local (ZR-L) version of the calculation using the parameters summarized in Table V-4. The quality of the fits are typical of what has been found in other studies of this reaction at these energies \cite{JD69, MP67}. In general the basic shapes of the angular distributions are well predicted, but some
Table V-4

<table>
<thead>
<tr>
<th>Parameter Set</th>
<th>( V_0 ) (MeV)</th>
<th>( r_0 ) (fm)</th>
<th>( a_0 ) (fm)</th>
<th>( W ) (MeV)</th>
<th>( W_D ) (MeV)</th>
<th>( r_I ) (fm)</th>
<th>( a_I ) (fm)</th>
<th>( r_c ) (fm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deuterons</td>
<td>100.0</td>
<td>1.14</td>
<td>0.89</td>
<td>0.0</td>
<td>13.80</td>
<td>1.33</td>
<td>0.75</td>
<td>1.30</td>
</tr>
<tr>
<td>Protons</td>
<td>52.0</td>
<td>1.25</td>
<td>0.65</td>
<td>0.0</td>
<td>7.5</td>
<td>1.25</td>
<td>0.76</td>
<td>1.25</td>
</tr>
<tr>
<td>Neutrons</td>
<td>(a)</td>
<td>1.23</td>
<td>0.65</td>
<td>( \lambda = 25 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Optical Model Parameters used in the analysis

(a) Well depth adjusted by DWUCK to give the binding energy of the neutron
of the details of the diffraction structure are not always so well reproduced. This has been observed in other studies on the Pb isotopes (JD69, BT67, MP67) in which for example the theoretical $\ell=2$ transitions at forward angles, where there are pronounced maxima predicted at about 150°, do not reproduce the data. The poorer fits of the larger $\ell$ transitions ($\ell=6$, and $\ell=7$) have also been noted in these studies.

The spectroscopic factors for the single-particle states were obtained assuming the $j$-values suggested by the shell model and are listed in Table V-5. The values obtained using both the zero-range local (ZR-L) and finite-range nonlocal (FR-NL) versions of the calculations are compared in the table. The inclusion of finite-range nonlocality corrections ($R_{FR} = 0.621$ fm, $\beta_d = 0.54$ fm, and $\beta_p = 0.85$ fm) resulted in only slight changes in shape (Section V.B.2.d) and an increase of 5-15% in the magnitude of the spectroscopic factors when the largest changes were observed for the larger $\ell$ transfers. All the states are observed to have spectroscopic factors within 20% of unity with the exception of the $1j_{15/2}$ state with a spectroscopic factor of about 0.50. It must be remembered that these spectroscopic factors have uncertainties which arise because of the statistics and curve fitting ($\approx 5\%$), because of uncertainties in absolute cross section ($\approx 15\%$), and because of the DWBA uncertainties discussed previously ($\approx 20\%$).

b. Discussion

One of the more interesting questions concerning the analysis of the single-particle states is why the angular distributions for these states are not better fit by the
Table V-5

Spectroscopic Factors obtained in ZR-L and FR-NL with optical model parameters PI, and DII and the neutron well DII.

<table>
<thead>
<tr>
<th>nlj</th>
<th>ZR-L</th>
<th>FR-NL</th>
</tr>
</thead>
<tbody>
<tr>
<td>2g_{9/2}</td>
<td>.72</td>
<td>.80</td>
</tr>
<tr>
<td>l_{11/2}</td>
<td>.83</td>
<td>.98</td>
</tr>
<tr>
<td>l_{15/2}</td>
<td>.47</td>
<td>.55</td>
</tr>
<tr>
<td>3d_{5/2}</td>
<td>.88</td>
<td>.94</td>
</tr>
<tr>
<td>4s_{1/2}</td>
<td>.95</td>
<td>1.00</td>
</tr>
<tr>
<td>2g_{7/2}</td>
<td>.95</td>
<td>.98</td>
</tr>
<tr>
<td>3d_{3/2}</td>
<td>.90</td>
<td>.95</td>
</tr>
</tbody>
</table>
DWBA predictions. Indeed, it would seem that the DWBA theory would be especially well suited to treat the transitions to these states which are some of the best examples of single-particle structure. Since the theoretical angular distributions are quite sensitive to the distorted waves used in the calculations, one might first suspect that these have not been properly specified. This has recently been investigated in the Oxford study (JD69) of the Pb$^{208}$(d,p) Pb$^{209}$ reaction. In the Oxford study of this reaction at a number of bombarding energies both above and below the Coulomb barrier, it was noted that better fits to the angular distributions could be obtained if a potential for the distorted waves of the deuterons was used that was quite different from the potential determined from elastic scattering data. The spectroscopic factors obtained for these states were found to be closer to the expected values of unity than those obtained in the DWBA analyses using standard deuteron potentials. The potential used had an unusually large surface absorption term ($W \approx 40$ Mev) and while it predicted the general trend of the elastic scattering cross section it did not reproduce the detailed oscillations observed. The use of a non-standard optical model potential in the analysis of the (d,p) transitions while of interest is not particularly satisfying since it constitutes a violation of one of the assumptions of the conventional DWBA theory.

In the Oxford study and in previous studies no investi-
gations were reported concerning the sensitivity of the predicted (d,p) cross sections to the choice of proton potential. In most recent studies (JD69, MP67, EK69) the proton potential of the form of PI or PII (Table V-3) was taken from the literature and used in the analysis. It was of interest to determine whether the use of different standard proton potentials obtained from the literature would result in either better fits to the data or significant changes in the values of the spectroscopic factor obtained. Thus in addition to the proton parameters set finally used in the analysis (Table V-4) DWBA curves were calculated with two other proton parameter sets. They were compared to the experimental angular distributions of the seven single-particle states and spectroscopic factors were extracted. The fits to the data of curves calculated with all three protons parameter sets are shown in Figure V-14. Although a better fit to some angular distributions might be observed with the predictions of one parameter set, no one parameter set was observed to give a noticeably better fit to the angular distributions of all the transitions. The sensitivity of the spectroscopic factors to the choice of proton potential is shown in Table V-6. The deviations of the values based on PII and PIII from the values determined with set PI were on the average within ±10%. There is the exception of the \(1j_{15/2}\) state where the spectroscopic factors deviated by about 20%. It was also observed that the spectroscopic factors obtained using the parameter set PII were on the whole about 10% smaller than those obtained
Fits to the angular distributions of the seven single-particle states by the DWBA curves calculated with different proton parameters.
Table V-6

Comparison of spectroscopic factors obtained in using the same neutron form factor and the same deuteron parameter, but in varying the proton parameters. The calculations were done in ZR-L and $R_{CO} = 0.0 \, fm$.

<table>
<thead>
<tr>
<th>nlj</th>
<th>PI</th>
<th>PII</th>
<th>PIII</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>2g9/2</td>
<td>.72</td>
<td>.67</td>
<td>.80</td>
<td>.73</td>
</tr>
<tr>
<td>1i11/2</td>
<td>.83</td>
<td>.72</td>
<td>.83</td>
<td>.79</td>
</tr>
<tr>
<td>1j15/2</td>
<td>.47</td>
<td>.53</td>
<td>.60</td>
<td>.53</td>
</tr>
<tr>
<td>3d5/2</td>
<td>.88</td>
<td>.78</td>
<td>.89</td>
<td>.85</td>
</tr>
<tr>
<td>4s1/2</td>
<td>.95</td>
<td>.90</td>
<td>1.03</td>
<td>.96</td>
</tr>
<tr>
<td>2g7/2</td>
<td>.95</td>
<td>.84</td>
<td>0.91</td>
<td>.90</td>
</tr>
<tr>
<td>3d3/2</td>
<td>.90</td>
<td>.80</td>
<td>0.90</td>
<td>.87</td>
</tr>
</tbody>
</table>
using PI and PIII. The averaged spectroscopic factors agreed very well with those obtained using parameter set PI alone in the analysis.

It is also of interest to examine the dependence of the fits and spectroscopic factors on the choice of neutron well. The shapes of the predicted angular distributions were found to be essentially independent of the neutron well chosen and for all practical purposes could be considered as identical. The spectroscopic factors based on the four different neutron wells (Section V.B.2.c.) are compared in Table V-7. The neutron well of Rost (RO) gave spectroscopic factors which were about a factor of 2 to 3 times smaller than the expected values (of about unity) obtained using the other three wells. These results of the Rost potential have been observed in other reaction studies (Pb\textsuperscript{208}(\Delta t)Pb\textsuperscript{207} (PH68), and Pb\textsuperscript{208}(t,d)Pb\textsuperscript{209} (IB69)) and constitute something of a mystery. That is, the potential parameters are based on a fit to the level ordering and binding energies of the observed neutron single-particle and hole states and thus might be thought to provide a good description of the single-particle potential $\alpha\text{Pb}\textsuperscript{208}$. Yet the spectroscopic factors obtained using this potential are significantly smaller than what are indicated by the experimental spectra and nuclear structure calculations. The neutron potentials DH, CO, and ZD (See Section V.B.2.) which give more "reasonable" spectroscopic factors are used in most analyses of neutron stripping reactions on Pb\textsuperscript{208}. Spectroscopic factors
Comparison between spectroscopic factors obtained by various neutron bound state wells. The calculations were done in ZR-L with $R_{CO}=0.0$. The potentials are listed in Table (V-2).

<table>
<thead>
<tr>
<th>$n\ell j$</th>
<th>Neutron Wells</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>DH</td>
</tr>
<tr>
<td>$2g_{9/2}$</td>
<td>.72</td>
</tr>
<tr>
<td>$1i_{11/2}$</td>
<td>.83</td>
</tr>
<tr>
<td>$1j_{15/2}$</td>
<td>.47</td>
</tr>
<tr>
<td>$3d_{5/2}$</td>
<td>.88</td>
</tr>
<tr>
<td>$4s_{1/2}$</td>
<td>.95</td>
</tr>
<tr>
<td>$2g_{7/2}$</td>
<td>.95</td>
</tr>
<tr>
<td>$3d_{3/2}$</td>
<td>.90</td>
</tr>
</tbody>
</table>

* $r=1.233$ fm for this level (see text)
obtained using these potential in the DWBA calculations were found to be identical to about ±10%. These results are in agreement with results of the (d,p) study at Oxford (JD69), but are in disagreement with the observed changes of >15% in certain cases in the (t,d) study at Los Alamos (IB69) where spectroscopic factors obtained using a DH type potential and a ZD potential were compared.

The two values listed in Table V-6 for the 15/2- state for the Zaidi and Darmodjo well (ZD) correspond to the values extracted using the conventional separation-energy prescription and the Pinkston-Satchler prescription (PS65) to compute the neutron form factor. When the standard ZD well is used an uncommonly deep well is needed to bind the particle at the observed excitation energy of 1.424 Mev. It will be recalled that the ZD well was determined by a fit to the observed binding energies of the other six single-particle levels (Section V.B.2.). In this well \(r_0 = 1.19\) fm, \(a_0 = 0.75\) fm, \(\lambda = 20.5\) and \(v \approx 50.8\) Mev) the \(1j_{15/2}\) level is predicted to be found at 3.50 Mev excitation. In the Pinkston-Satchler prescription the radius of the well is increased until a well depth of 50.8 Mev results, binding the \(1j_{15/2}\) level at 1.424 Mev excitation. The spectroscopic factor decreases by about 10% when the larger radius (1.19 fm \(\rightarrow\) 1.233 fm) is used to specify the well.

To summarize, the spectroscopic factors obtained in the analysis of this experiment (Table V-4) do not appear to be affected by more than about ±15% by the uncertainties
in the specification of either the proton potential or the neutron single-particle potential.

The spectroscopic factors obtained in the present investigation are compared with those found in other recent studies in Table V-7. Since in most of these studies the analysis was done in the zero-range local approximation, the results we obtained in the zero-range local analysis of our data are listed in the Table. For the purpose of consistency the comparison is restricted to those analyses which used standard optical model potentials for the distorted waves and conventional neutron wells (DH or CO of Table V-3). Several things can be noted from the comparison. The spectroscopic factors obtained in the studies below the Coulomb barrier are found to be within about 10% of unity. Indeed, the neutron wells of these analyses were chosen to produce these values (DH63). In the studies near and above the Coulomb barrier the spectroscopic factors decrease on the whole to values about 10-25% smaller than unity. In these calculations the same neutron well as used in the analysis below the Coulomb barrier was used and the distorted waves used were generated by optical model potentials determined by fitting elastic scattering data. Also listed in Table V-8 are the spectroscopic factors obtained in the (t,d) study (IB69) on Pb$^{208}$. The spectroscopic factors obtained in studies near and above the Coulomb barrier are observed to show fluctuations of about ±20%.

It is found in general that spectroscopic factors of about 0.80 to 0.95 are obtained for all states except the
<table>
<thead>
<tr>
<th>$^n l_j$</th>
<th>Dost &amp; Herring (8 Mev)</th>
<th>Oxford (12 Mev)</th>
<th>Copenhagen (18.7 Mev)</th>
<th>Oxford (20.0 Mev)</th>
<th>This Study PI (20.0 Mev)</th>
<th>Los Alamos (t,d) 20.0 Mev</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2g_{9/2}$</td>
<td>1.07</td>
<td>1.04</td>
<td>0.78±.1</td>
<td>0.66</td>
<td>.72</td>
<td>0.76</td>
</tr>
<tr>
<td>$1^{11/2}$</td>
<td>---</td>
<td>1.07</td>
<td>0.96±.2</td>
<td>0.75</td>
<td>.83</td>
<td>0.86</td>
</tr>
<tr>
<td>$1^{15/2}$</td>
<td>---</td>
<td>---</td>
<td>0.53±.2</td>
<td>0.71</td>
<td>.47</td>
<td>0.49</td>
</tr>
<tr>
<td>$3d_{5/2}$</td>
<td>1.14</td>
<td>0.98</td>
<td>0.88±.1</td>
<td>0.62</td>
<td>.88</td>
<td>0.84</td>
</tr>
<tr>
<td>$4s_{1/2}$</td>
<td>1.11</td>
<td>0.96</td>
<td>0.88±.1</td>
<td>0.70</td>
<td>.95</td>
<td>0.82</td>
</tr>
<tr>
<td>$2g_{7/2}$</td>
<td>0.98</td>
<td>1.03</td>
<td>0.78±.1</td>
<td>0.81</td>
<td>.95</td>
<td>0.86</td>
</tr>
<tr>
<td>$3d_{3/2}$</td>
<td>1.01</td>
<td>0.98</td>
<td>0.88±.1</td>
<td>0.88</td>
<td>.90</td>
<td>0.79</td>
</tr>
</tbody>
</table>
2g_{9/2} and the 1j_{15/2} and these have values of 0.70-0.80 and 0.45-0.70 respectively. If we had listed the spectroscopic factors we had obtained using proton parameter set PII (Table V-6) which was used in the analysis of the Oxford and Copenhagen data there would have been a slight improvement in the agreement with the values obtained in these studies.

When finite range and nonlocality corrections are included, the spectroscopic factors are increased by 5-15% and brought more in line with the value obtained at studied below the Coulomb barrier. A comparison between the spectroscopic factors obtained in this study in the finite-range nonlocality analysis (Table V-4) and those obtained in the low energy studies (DH63, JD69) shows that all levels are in agreement and consistent with unity to 10% except for the 2g_{9/2} states which has a spectroscopic factor of 0.80 in this study. The 1j_{15/2} state which is not observed in the studies below the Coulomb barrier is found to have a spectroscopic factor of 0.55 in our study.

2. Two Particle-One Hole States
   a. Fits to the Angular Distributions

The angular distributions for the 34 weak transitions are shown in Figures V-5 to V-7. The theoretical curves are the DWBA predictions made in the zero-range local (ZR-L) version of the calculation with the same potential
parameters used to analyze the transitions to the single-particle states (Table V-4). Transitions to unbound states were analyzed as though they were bound by 25-100 kev. All \( t \) transfers were considered possible and the DWBA cross sections calculated for all \( t \) transfers (\( t = 0 \) to 8) were compared to the experimental angular distributions and the best fits chosen. In many cases \( t \) values differing by only one unit of orbital angular momentum were found to fit the data almost equally well. Other experimental information was then used to eliminate one of the two \( t \)-values when possible. Spectroscopic information was obtained using the most probable \( t \) value based on this experiment and the results of other studies.

In the simplest picture of the \((d,p)\) reaction a final state is populated by stripping into a single shell model orbit specified by quantum numbers \( n\ell j \). In our case for the weak transitions it is possible in principle to populate some levels by stripping into two orbits of the same \( \ell j \), but which differ in radial quantum number \( n \). In our analysis we assume that the reaction proceeds via either of two possible orbits and determine \((2\ell + 1)S\) values for each. For example, a \( \frac{1}{2}^- \) state in \( \text{Pb}^{208} \) can be populated by stripping into either the \( 3p_{3/2} \) or \( 4p_{1/2} \) orbit and the spectroscopic factor for each transition is extracted. In the figures showing the angular distributions the DWBA curve for only one of the transfers is shown. The predicted curves for the transfer into orbits of different radial quantum numbers have been illustrated in a previous
section (Section V.B.2, figure V-10) and shown to have small differences in shapes.

The solid DWBA curves in the figures (figures V-5, V-7) represent fits corresponding to the indicated $t$ assignment. The dashed curves imply tentative $t$-assignments. They are also used to represent fits which would not have been made on the basis of this data alone, but which are in agreement with spin assignments of the levels made in other studies. When two curves are shown, the solid curve is to be regarded as the preferred fit based on either the results of other studies or on shell model arguments. The $t$-assignment of each level is discussed later in this section. The assigned $t$-value, the extracted value of $(2J+1)S$, and the maximum cross section $\left(\frac{\partial \sigma}{\partial \Omega}\right)_{\text{max}}$ for each transition are listed in Table V-9.

The agreement of the DWBA curves with the data range from fairly good to poor. In several cases it can be observed that the data do not fall as rapidly as the DWBA predictions at large angles. This might be attributed, at least in part, to the small cross sections at these angles and the attendant systematic errors in the extraction of small yields from the spectra. At these large angles compound reaction contributions to the cross sections could possibly become significant in some cases. What is somewhat surprising is the observation of rather distinct "stripping patterns" of the angular distributions for almost all the transitions studied which might not necessarily have been
<table>
<thead>
<tr>
<th>Level (Mev)</th>
<th>t_n</th>
<th>Level (Mev)</th>
<th>t_n</th>
<th>Level (Mev)</th>
<th>t_n</th>
<th>Level (Mev)</th>
<th>t_n</th>
<th>Level (Mev)</th>
<th>t_n</th>
<th>Level (Mev)</th>
<th>t_n</th>
<th>Level (Mev)</th>
<th>t_n</th>
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<th>t_n</th>
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<th>t_n</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.000</td>
<td>4</td>
<td>0.799</td>
<td>6</td>
<td>1.424</td>
<td>7</td>
<td>1.565</td>
<td>2</td>
<td>2.033</td>
<td>0</td>
<td>1.152</td>
<td>1</td>
<td>2.320</td>
<td>1</td>
<td>(2.424)</td>
<td>4</td>
<td>2.461</td>
<td>(3)*</td>
<td>2.493</td>
<td>4</td>
<td>2.537</td>
<td>2</td>
</tr>
<tr>
<td>0.000</td>
<td>4</td>
<td>0.799</td>
<td>6</td>
<td>1.424</td>
<td>7</td>
<td>1.565</td>
<td>2</td>
<td>2.033</td>
<td>0</td>
<td>1.152</td>
<td>1</td>
<td>2.320</td>
<td>1</td>
<td>(2.424)</td>
<td>4</td>
<td>2.461</td>
<td>(3)*</td>
<td>2.493</td>
<td>4</td>
<td>2.537</td>
<td>2</td>
</tr>
<tr>
<td>0.000</td>
<td>4</td>
<td>0.799</td>
<td>6</td>
<td>1.424</td>
<td>7</td>
<td>1.565</td>
<td>2</td>
<td>2.033</td>
<td>0</td>
<td>1.152</td>
<td>1</td>
<td>2.320</td>
<td>1</td>
<td>(2.424)</td>
<td>4</td>
<td>2.461</td>
<td>(3)*</td>
<td>2.493</td>
<td>4</td>
<td>2.537</td>
<td>2</td>
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<tr>
<td>0.000</td>
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<td>1.424</td>
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<td>2.493</td>
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<td>2.493</td>
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<td>2.537</td>
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*Fit based on spin assignment of level made in previous studies (see discussion).*
expected for these weak transitions. It was also noted that, as in the case of the strong transitions to the single-particle states, better fits could be obtained when a radial cutoff was used in the DWBA calculation.

The $l=1$ transitions are rather distinctive and relatively easy to identify because of their maxima at about 25° and 60°. Likewise the $l=6$ and $l=7$ transitions were equally distinctive by virtue of their peaking at rather large angles and the slow decrease in cross section with increasing angle. The assignment of $l = 2, 3, 4$ or 5, however, were more difficult since in many cases two adjacent $l$ transfers fit the angular distribution equally well. In particular it was difficult to choose between an $l = 2$ or 3 and an $l = 4$ or 5 assignment. In many cases the choice was made simply by the fact that the level populated corresponded to a state whose spin and parity had been established in another study.

Previous studies indicate that, disregarding the single particle $4s_{1/2}, 2g_{7/2}$ and $3d_{3/2}$ states, essentially all the states in the excitation region 2.0 to 4.3 Mev excitation are of negative parity. In fact, except for the $1i_{13/2}$ strength seen in the $^{210}\text{Pb}^{(p,d)}^{209}\text{Pb}$ study (FI69, IF70), there is no experimental evidence for the existence of 2p-1h positive parity states up to 4.3 Mev excitation. Only two other positive parity states have been identified in this region. These two states at 3.708 Mev and 4.100 Mev have been observed in the $^{207}\text{Pb}^{(t,p)}^{209}\text{Pb}$ reaction (FI70) and have been interpreted as being the lowest lying 3p-2h
states in the Pb\textsuperscript{209} excitation spectrum. The absence of positive parity states is not too surprising since based on simple shell model arguments these states are not expected to be found below about 4.0 Mev. This would imply that the ambiguous \( l = 2 \) or 3 and \( l = 4 \) or 5 angular distributions observed are most likely \( l = 3 \) and \( l = 5 \) transfers respectively. These arguments will be used later in the interpretation of these transitions, however, in Table V-9 both \( l \) values will be considered as possible for the transitions to levels of unknown spin and parity.

There are two cases that are worthy of special comment at this time. The first case concerns the four observed \( l = 4 \) or 5 angular distributions associated with levels 5, 8, 12, and 13 in the excitation region around 3.0 Mev. In all four cases the angular distributions are essentially identical and were found not only to fit somewhat better by an \( l = 4 \) transfer, but were found to be very similar to the \( l = 4 \) transitions observed in the 2\( g_9/2 \) and 2\( g_7/2 \) single-particle levels. On the other hand, two of the four transitions lead to levels strongly populated in the Pb\textsuperscript{207} (t,p)Pb\textsuperscript{209} reaction and identified with spin-parity 9/2\textsuperscript{-} or 11/2\textsuperscript{-}. These assignments would imply \( l = 5 \) transitions in the (d,p) reaction which is also more consistent with shell model predictions. Thus it appears that \( l = 5 \) assignments for all four transitions is preferred. This would indicate that the maximum in the \( l = 5 \) predicted angular distribution should be displaced towards smaller angles.
The second case concerns the nine observed transitions to the level #23-31 in the excitation region 3.90 to 4.14 Mev. Since the neutron separation energy is about 3.94 Mev, these levels either are bound only slightly or are unbound. The angular distributions for these transitions are remarkably similar especially in the first 90°. At back angles one or two of the distributions do not fall as rapidly as do the rest, however, on the whole the similarity is rather striking. The predicted $l = 3$ DWBA curves fit the data best. However, the observed angular distributions for the $l = 2$ transitions to the $3d_{5/2}$ and $3d_{3/2}$ single-particle states were also very similar to the transitions to these nine slightly bound and unbound levels. Thus both $l = 2$ and $l = 3$ assignments to these transitions must be regarded as possible. Since these states are near the excitation where positive parity states might be expected, shell model arguments can not be used to rule out either assignment.

b. Discussion of Individual Transitions

This section is devoted to a discussion of each of the 2p-1h states observed in this experiment. The present results are combined with the results of other measurements to obtain $l$-values, spins, parities, and spectroscopic factors to the extent that can be determined at present. Since the results of the Pb$^{207}(t,p)$Pb$^{209}$ study (IF69, IF70) and the Pb$^{210}(p,d)$Pb$^{209}$ study (IF69, IF70) are used
extensively in the discussion of the spins and parities of these levels, for brevity they will be denoted as the (t,p) and (p,d) studies respectively. In quoting the results of the Pb \( ^{207} \) (t,p)\( ^{209} \) Pb study the strength of a transition is given as the ratio of its cross section to that observed in the Pb \( ^{208} \) (t,p)\( ^{210} \) Pb reaction to the ground \(+\) + + + \) state and the low lying 2, 4, 6, and 8 excited states of Pb \( ^{210} \). (For more details see Section II.A.3.).

**Level 1 -- 2.152 Mev**

This level was observed in the (t,p) reaction to be populated by an \( L=0 \) transition with a cross section equal to the Pb \( ^{208} \) (t,p)\( ^{210} \) Pb ground state transition. Based on this data the level was assigned a \( J \) of \( \frac{1}{2} \) and the configuration \( \left( \text{Pb} \left[ \left( \begin{array}{c} 0 \cr \frac{1}{2} \end{array} \right) \times p_x \right. \right)_{\frac{1}{2}} \). This assignment was confirmed by the results of the (p,d) reaction which populated the level by an \( =1 \) transfer with cross section \( ^{208} \) Pb \( \rightarrow ^{207} \) Pd \( \rightarrow ^{207} \) transitions (WS68). The level has been seen in other single neutron transfer reactions on Pb \( ^{208} \) (JD69, EK69, IB69), but too weakly to obtain an angular distribution. Estimates of the spectroscopic factor for this state assuming a pure \( \left( \text{Pb} \left[ \left( \begin{array}{c} 0 \cr \frac{1}{2} \end{array} \right) \times 3p_{\frac{1}{2}} \right)_{\frac{1}{2}} \) configuration were reported by the Pb \( \rightarrow ^{208} \) (d,p)\( ^{209} \) Pb \( \rightarrow ^{209} \) Pb study (EK69) to be \( S=0.005 \) and by the Pb \( \rightarrow ^{208} \) (t,d)\( ^{209} \) Pb \( \rightarrow ^{209} \) Pb study (IB68) to be \( S=0.060-0.090 \). In the present (d,p) study the experimental angular distribution was obtained and fit by an \( =1 \) transfer. The fit back to 75 is fairly good. At back angles the maxima of the distribution are well reproduced by the DWBA curve, but the
predicted curve falls much more rapidly than do the data. Spectroscopic factors of \( S(3p_{1/2}^{-1}) = 0.0067 \) and \( S(4p_{1/2}) = 0.0036 \) are obtained.

Level 2 — 2.319 Mev

This level was not observed in the (t,p) reaction. It was observed in the (p,d) reaction to be excited via an \( l = 1 \) transition. Based on the sum rule argument that the \( 3p_{1/2}^{-1} \) strength had been exhausted in the transition to the 2.152 Mev state, the level was given a tentative \( 3/2^- \) assignment and a spectroscopic factor of \( S = 0.56 \) obtained. In the (d,p) reaction the angular distribution associated with this level is fit by an \( l = 1 \) transfer assignment and, similar to the \( l = 1 \) transition to the level at 2.152 Mev, agrees with DWBA predictions for the forward angles, but does not fall as rapidly in the back angles as does the predicted curve. Spectroscopic factors of 
\( S(3p_{3/2}^{-1}) = 0.0074 \) and \( S(4p_{3/2}) = 0.0045 \) are obtained if spin is assumed to be \( (3/2^-) \).

Level 3 — 2.424 Mev

This level has not been observed previously in any other study. The observed angular distribution does not allow for an \( l \) assignment.

Level 4 — 2.461 Mev

The level was not observed in the (t,p) reaction, but
was seen in the (p,d) reaction with an $l=3$ angular distribution. A (p,d) spectroscopic factor of $S(2f_{5/2}^{-1}) = 0.81$ was obtained when the level was assigned a tentative $J^m$ of $5/2^-$ based on shell model arguments and the sum rule for the $2f_{5/2}^{-1}$ strength. The transition to this level in the (d,p) reaction is too weak to obtain a good angular distribution and thus prohibited an $l$ assignment for the transition. If an $l=3$ transition to a $(5/2^-)$ state is assumed spectroscopic factors of $S(2f_{5/2}^{-1}) = 0.0012$ and $S(3f_{5/2}) = 0.0007$ are obtained.

**Level 5 — 2.592 Mev**

The (t,p) and (p,d) reactions both excited this level but too weakly to obtain spectroscopic information. The angular distribution obtained in the (t,d) study (JD69) has recently been analyzed by Hamamoto (HA69) and found to be consistent with an $l=5$ transition to the $11/2^-$ member of the $(3^-xg_{9/2})$ multiplet containing a small admixture of $h_{11/2}$ single-particle excitation. The level has also been observed in previous (d,p) studies (JD69, EK69), but not analyzed for $l$-value or spectroscopic factor. In the present (d,p) study an angular distribution consistent with an $l=4$ or 5 assignment is observed, where the $l=4$ fit the data at forward angles is noted to be better. However, based on other similar angular distributions where $l=5$ transfers are required, as well as shell model arguments that rule out additional positive parity states
in this region, it is believed that the $l=5$ assignment is probably correct. This ambiguity has already been discussed (See previous section) (If an $l=4$ fit is assumed a spectroscopic factor of $S(2g_{7/2}) = 0.0274$ is obtained.) When the $l=5$ transfer is assumed spectroscopic factors of $S(1h_{9/2}^{-1}) = 0.0825$ and $S(2h_{11/2}) = 0.0185$ are obtained. The spectroscopic factor obtained for stripping into the $1h_{9/2}^{-1}$ orbital is an order of magnitude larger than what might be expected from theoretical calculations even allowing for the uncertainties in the reaction theory while the value for the spectroscopic factor for stripping into the $2h_{11/2}$ orbital is quite reasonable. On the basis of this a tentative $(11/2^-)$ assignment for the level is indicated.

**Level 6 — 2.738 Mev**

A $J^\pi$ assignment of $5/2^-$ was made for this level based on the $L=2$ transition ($J^\pi = 3/2^-, 5/2^-$) observed in the $(t,p)$ reaction and the $l=3$ transition ($J^\pi = 5/2^-, 7/2^-$) observed in the $(p,d)$ reaction. The $(t,p)$ transition had a cross section of strength $S = 0.25$ compared to the $^{208}\text{Pb}(t,p)^{210}\text{Pb}$ ($2^+$) transition. The spectroscopic factor obtained in the $(p,d)$ study was $S(2f_{5/2}^{-1}) = 4.76$. In the $(d,p)$ reaction the angular distribution of the transition to this level is fit by an $l=3$ transfer and spectroscopic factors of $S(2f_{5/2}^{-1}) = 0.0027$ and $S(3f_{5/2}) = 0.0013$ are obtained.
Level 7 — 2.866 Mev

This level was observed in the \( (t,p) \) study to be populated by an \( L=2 \) transfer \( (J^n=3/2^-, 5/2^-) \) with a strength of \( S(2^+) = 0.38 \). It was also observed in the \( (p,d) \) reaction where it was found to be populated by an \( \ell=3 \) transition \( (J^n=5/2^-, 7/2^-) \). Based on the two studies the level is assigned a \( J^n \) of 5/2^-). We observed an angular distribution associated with this level consistent with an \( \ell=3 \) transition. Spectroscopic factors of \( S(2f_{5/2}^{-1}) = 0.0010 \) and \( S(3f_{5/2}^{-1}) = 0.0005 \) are obtained.

Level 8 — 3.026

This level (3.028 Mev) has been reported in the \( (t,p) \) study to show an angular distribution analyzed to be an \( L=4 \) transfer \( (J^n=7/2^-, 9/2^-) \) with strength 0.56 of the \( \text{Pb}^{208}(t,p)\text{Pb}^{210}(4^+) \) transition. A level at 3.031 Mev has been reported in the \( (p,d) \) study to be an \( \ell=1 \) transition with spectroscopic factor \( S(3p_{5/2}^{-1}) = 0.010 \). The angular distribution for this level observed in the \( (d,p) \) reaction can be fit by an \( \ell=4 \) or 5 transfer assignment where the \( \ell=5 \) assignment is preferred (See previous section). The \( \ell=5 \) assignment is consistent with the 9/2^- spin assignment of the \( (t,p) \) study. A spectroscopic factor of \( S(1h_{9/2}^{-1}) = 0.0090 \) is obtained. The disagreement with the \( (p,d) \) results may imply the existence of two closely spaced levels.
Level 9 -- 3.052 Mev

This level has been observed only in previous (d,p) (KD69, EK69) and (t,d) (IB68) studies where no \( l \) assignments were made. In this study an angular distribution fit by an \( l=7 \) transfer is observed. Spectroscopic factors of \( S(1j_{15/2}) = 0.0520 \) and \( S(1j_{13/2}) = 0.1230 \) are obtained. In view of the proximity of this level to the \( 1j_{15/2} \) state at 1.424 Mev known to be fragmented by 30-40\%, the level is assigned the spin of \( (15/2^-) \).

Level 10 -- 3.075 Mev

A level has been observed at 3.072 Mev in the (t,p) study to be populated by an \( L=6 \) transfer \( (J^n=11/2^-,13/2^-) \) with a strength 0.43 of the \( {^{208}}\text{Pb}(t,p){^{210}}\text{Pb}(6^+) \) transition. In the (p,d) study a level at 3.077 Mev was populated via an \( l=1 \) transition and assumed to contain the great bulk of the \( 3p_{3/2}^{-1} \) strength \( S(3p_{3/2}^{-1}) = 2.62 \). Since each transition goes rather strongly, it appears that two closely spaced levels are being excited. The present level at 3.075 Mev is observed to be populated by an \( l=1 \) transfer and hence is probably the same one observed in the (p,d) study. Spectroscopic factor of \( S(3p_{3/2}^{-1}) = 0.0021 \) and \( S(4p_{3/2}) = 0.0012 \) are obtained.

Level 11 -- 3.203 Mev

This level (3.203 Mev) was observed to have an \( L=4 \)
angular distribution \( J^\pi = 7/2^-, 9/2^- \) in the \((t,p)\) study with cross section 0.45 the strength of the \( \text{Pb}^{208}(t,p) \text{Pb}^{210}(4^+) \) transition. Based on the \((d,p)\) results for the other \( L=4 \) transfer observed in the \((t,p)\) reaction, which gave a \( J^\pi \) of 9/2\(^-\) to the level at 3.028 Mev, this level is expected to be 7/2\(^-\) member of the \( (\text{Pb}^{210}(4^+) \times p_{1/2}^-) \) doublet. The \((d,p)\) transition to this level has an angular distribution fit by an \( l=3 \) transfer confirming the 7/2\(^-\) spin assignment. Spectroscopic factors of \( S(2f_7/2^-) = 0.0012 \) and \( S(3f_7/2^-) = 0.0007 \) are obtained.

**Level 12 -- 3.309 Mev**

This level has been observed in the \((t,p)\) study and was reported to be populated by an \( L=6 \) transfer \( (J^\pi=11/2^-, 13/2^-) \) with a strength 0.58 of the \( \text{Pb}^{208}(t,p)\text{Pb}^{210}(6^+) \) transition. In the \((d,p)\) study we observed this level to have an angular distribution consistent with an \( l=4, 5 \) assignment contrary to the tentative \( l=7 \) assignment for this level made in a previous \((d,p)\) study (EK69). The results of the \((t,p)\) study imply that an \( l=5 \) assignment for the transfer in the \((d,p)\) reaction is correct and hence an assignment of 11/2\(^-\) spin-parity can be made for this level. A spectroscopic factor of \( S(2h_{11/2}^-) = 0.0050 \) is obtained.
This level has been observed only in single neutron stripping reactions. We observe an angular distribution for this level consistent with an \( t = 4, 5 \) assignment. Based on the shapes of other \( t = 5 \) transfers observed in this study an \( t = 5 \) assignment for this level is preferred. Spectroscopic factors of \( S(1h_{9/2}) = 0.0466 \) and \( S(2h_{11/2}) = 0.0087 \) are obtained for the possible \( 9/2^- \), \( 11/2^- \) spin assignments. Based on the rather large size of the spectroscopic factor for the transfer into \( 1h_{9/2}^- \) orbit implying the existence of an unusually large \( 2p-2h \) admixture into \( \text{Pb}^{208} \) ground state, the spin \( 9/2^- \) spin assignment is considered less probable than the \( 11/2^- \) assignment.

**Level 14 -- 3.389 Mev**

In the \((t,p)\) reaction a level was observed at 3.384 Mev, but with a yield too small to extract spectroscopic information. In the present \((d,p)\) reaction it is also observed to be too weakly populated for assignment of a unique \( t \)-value. The angular distribution is suggestive of an \( t = 1 \) transfer and if a tentative \( t = 1 \) assignment is made \((2j+1)S \) values of \((2j+1)S(3p^{-1}) = 0.0102\) and \((2j+1)S(4p) = 0.0054\) are obtained.

**Level 15 -- 3.414 Mev**

This level has only been observed in this \((d,p)\)
In the (t,p) study a level at 3.432 Mev was populated via an L=8 transfer ($J^\pi=15/2^-, 17/2^- $) with a strength of 0.30 of the $^{208}_{\text{Pb}}(t,p)^{210}_{\text{Pb}}(8^+)$ transition. It is observed in the (d,p) reaction to be weakly populated and it is not possible to assign an $I$ value for the transfer. If an $I=7$ fit is made, consistent with the possible $J^\pi=15/2^-$ spin assignment indicated by the (t,p) results, a spectroscopic factor of $S(1j_{15/2^1})=0.0067$ is obtained. The fit however is poor.

**Level 17 -- 3.490 Mev**

A level was observed in the (p,d) reaction at an excitation energy of 3.499 Mev and its angular distribution fit by an $I=3$ transfer. A spin assignment of $7/2^-$ was assumed on the basis of shell model and sum rule arguments, and a spectroscopic factor of $S(2f_7/2^-)=0.16$ was obtained. The present level observed in the (d,p) reaction is also excited via an $I=3$ transfer and is probably the same level seen in the (p,d) study. If a spin of $7/2^-$ is assumed, spectroscopic factors of $S(2f_7/2^-)=0.0013$ and $S(3f_7/2^-)=0.0007$ are extracted.

**Level 18 -- 3.556 Mev**

In the (t,p) study a level at 3.561 Mev was populated
via an \( L=8 \) transfer \((J^\pi=15/2^-, 17/2^-)\) with a strength of 0.36 the \( \text{Pb}^{208}(t,p)\text{Pb}^{210}(8^+) \) transition. A level was also populated at excitation 3.562 Mev in the \((p,d)\) reaction and was reported to be an \( t=1 \) transfer \((J^\pi= 1/2^-, 3/2^-)\) with a spectroscopic factor of \( S(3p_{3/2}^-)= 0.010 \). The \((d,p)\) reaction populates a level at this energy whose angular distribution is fit by an \( t=7 \) transfer and is therefore consistent with the possible 15/2^- spin assignment suggested by the strong \((t,p)\) results. A spectroscopic factor of \( S(1j_{15/2})= 0.0280 \) is found.

**Level 19 — 3.615 Mev**

This level at 3.615 Mev had only been observed in this \((d,p)\) study and no \( t \) assignment was possible.

**Level 20 — 3.656 Mev**

A possible spin assignment of \( J^\pi= 3/2^-, 5/2^- \) was made for a level at 3.659 Mev by virtue of the observed \( L=2 \) transition in the \((t,p)\) reaction. The cross section for this transition is about 0.10 the strength of the \( \text{Pb}^{208}(t,p)\text{Pb}^{210}(2^+) \) transition. In the \((p,d)\) study a level also at 3.659 Mev is determined to be the strongest \( t=6 \) transition in the deuteron spectrum containing about 90\% of the \( l_{13/2}^- \) strength. The \((d,p)\) reaction to this level has an angular distribution which is best fit by an \( t=3 \) assignment, although an \( t=2 \) assignment cannot be ruled out completely. The \( t=3 \) assignment is consistent with the
spin assignment of $5/2^-$ suggested by the (t,p) results and based on this assignment spectroscopic factors of $S(2f_{5/2}^{-1})=0.0070$ and $S(3f_{5/2})=0.0044$ are found. It appears that there must be two closely spaced levels at this energy of spins $5/2^-$ and $13/2^+$. 

**Level 21 — 3.681 Mev**

The level has been observed only in this (d,p) study. It is assigned an tentative $\ell$-value of 1 based on the fit and a possible spin of $1/2^-$ and $3/2^-$. If a $1/2^-$ spin assignment is assumed spectroscopic factors of $S(3p_{1/2}^{-1})=0.0048$ and $S(4p_{1/2})=0.0029$ are obtained.

**Level 22 — 3.717 Mev**

This level appears only in the spectra of the single neutron stripping reactions (JD69) and in this study was observed to have an angular distribution fit by an $\ell=7$ assignment with spectroscopic factors of $S(1j_{15/2})=0.0270$ and $S(1j_{13/2})=0.0600$. Based on the arguments that the missing $1j_{15/2}$ is expected to be found in this excitation region and the fact that little is known even about the predicted location of the $1j_{13/2}$ orbit of the next shell a tentative spin assignment of $(15/2^-)$ is made.

**Level 23 — 3.904 Mev**

This level was excited by all three reactions. No
L-assignment was made reported in the (t,p) study. An \( l=3 \) assignment was made for the angular distribution associated with this level in the (p,d) study and a spectroscopic factor of \( S(2f_{7/2}) = 0.11 \) was obtained from the analysis. In the (d,p) reaction the angular distribution for this level is found to be identical to that observed for the next eight levels studied. These levels were observed to have cross sections about 5 to 10 times larger than most of the weak transitions thus far discussed. An \( l=3 \) assignment to these angular distribution was found to give the best fit and was taken as the assignment, but an \( l=2 \) value cannot be ruled out definitely (See previous section). The \( l=3 \) assignment for this level is in agreement with the results of the (p,d) study. If a spin-parity of \( 7/2^- \) is assumed spectroscopic factors of \( S(2f_{7/2}^{-1}) = 0.0192 \) and \( S(3f_{7/2}) = 0.0113 \) are obtained. The size of the spectroscopic factor for stripping into the \( 2f_{7/2}^{-1} \) orbit implies rather large correlations in the Pb\(^{208}\) ground state involving the \( 2f_{7/2}^{-1} \) configuration whose presence are not predicted by theoretical calculation nor indicated by the transitions to \( 7/2^- \) states at lower excitation. Thus it would appear that if this level and the next eight levels are of \( 7/2^- \) spin-parity then it is more probably that the transitions are proceeding via \( 3f_{7/2} \) single-particle configurations in the final states rather than through the ground state correlations.
Level 24 — 3.947 Mev

In the (t,p) study this level was observed but no spectroscopic information reported. The (p,d) study reports the excitation of a level at 3.937 Mev which could correspond to this level. It was found to be populated by an \( l=6 \) transfer and to have spectroscopic factor of 0.91. This \( l \) assignment however, is in disagreement with the \( l=3 \) assignment made in the present (d,p) study. When the \( l=3 \) assignment is made and a spin \( 7/2^- \) assumed a spectroscopic factor of \( S(3f_{7/2})=0.0161 \) is obtained. It appears that two different levels are being excited in the (d,p) and (p,d) reactions.

Level 25 — 3.985 Mev

The transition to this level is identified as an \( l=3 \) transfer and a spectroscopic factor of \( S(3f_{7/2})=0.0178 \) is obtained if a \( 7/2^- \) spin is assumed.

Level 26 — 4.008 Mev

This level is also found to have an angular distribution best fit by an \( l=3 \) assignment. A spectroscopic factor of \( S(3f_{7/2})=0.0070 \) is obtained if a \( 7/2^- \) spin is assumed.

Level 27 — 4.022 Mev

This level was seen in all three transfer reactions.
The transition to this level was identified as an $L=4$ transfer ($J^\pi = 7/2^-, 9/2^-$) in the $(t,p)$ study. In the $(p,d)$ study the level at 4.024 Mev was reported as populated via an $l=1$ transition ($J^\pi = 1/2^-, 3/2^-$) and found to have a spectroscopic factor of 0.0300. In this $(d,p)$ study the transition to the level is fit by an $l=3$ transfer consistent with the $7/2^-$ spin assignment suggested by the $(t,p)$ study. A spectroscopic factor of $S(3f_{7/2}) = 0.0038$ is obtained.

**Level 28 -- 4.075 Mev**

A level at 4.074 Mev was excited in the $(t,p)$ study, but no $L$ assignment was reported. In the $(d,p)$ study the transition to this level is identified as an $l=3$ transfer and a spectroscopic factor of $S(3f_{7/2}) = 0.0030$ is obtained if a $7/2^-$ spin assignment is assumed.

**Level 29 -- 4.096 Mev**

A level at 4.100 Mev was populated via a transfer identified as $L=3$ ($J^\pi = 5/2^+, 7/2^+$) in the $(t,p)$ study with a strength 0.40 of the $^{208}\text{Pb}(t,p)^{210}\text{Pb} (3_1^-)$ transitions. The transition to the level was best fit by an $l=3$ transfer, but was consistent with an $l=2$ assignment. If the spin is assumed to be $7/2^-$ a spectroscopic factor of $S(3f_{7/2}) = 0.0070$ is obtained. If an $l=2$ is assumed a spectroscopic factor $S(3d_{5/2}) = 0.0120$ is obtained both the $l=2$ and the $l=3$ fits to the data are shown.
Level 30 -- 4.112 Mev

A level at 4.119 Mev was observed in the (p,d) study and reported to be excited via an $t=1$ transfer ($S(3p_{3/2}^{-1}) = 0.05$). This level may correspond to our level seen at 4.112 Mev, but our assignment of an $t=3$ transfer ($5/2^-, 7/2^-$) to the transition to the level is not in agreement with the possible spin assignments of the (p,d) study. If we assume a $7/2^-$ spin assignment for the state, a spectroscopic factor of $S(3f_{7/2}) = 0.0125$ is obtained.

Level 31 -- 4.137 Mev

Levels at 4.140 Mev in (t,p) study and 4.145 Mev in the (p,d) study were reported, but no spectroscopic information was given. In the (d,p) reaction the present level is populated by a transition identified as an $t=3$ transfer. If a $7/2^-$ spin assignment is assumed a spectroscopic factor of $S(3f_{7/2}) = 0.0078$ is obtained.

Level 32 -- 4.166 Mev

A level at 4.169 Mev was populated by the (t,p) reaction and the transition identified as an $L=6$ transfer ($J^\pi=11/2^-, 13/2^-$). In the (d,p) study the angular distribution for this transition could not be fit on the basis of the data alone. If an $t=5$ transition is assumed (shown in the plot of the data), consistent with the $11/2^-$ assignment suggested by the (t,p) results, a spectroscopic factor
of $S(2h_{11/2}) = 0.0023$ is obtained.

**Level 33 -- 4.211 Mev**

This level was observed in the (p,d) study and reported to be a $t=3$ transfer with a spectroscopic factor of 0.91. In the (d,p) study the level is populated but no angular distribution is obtained.

**Level 34 -- 4.239 Mev**

A level was seen in the (p,d) study at 4.222 Mev which may correspond to this level. It was reported to be populated by an $t=3$ transfer with a spectroscopic factor of 2.08. In the (d,p) study this level is weakly populated and no angular distribution is obtained.

**Level 35 -- 4.295 Mev**

This level was seen in the (t,p) study but no spectroscopic information was reported. In the Pb$^{208}$ plus neutron total cross section studies (KB62, PA50) a state was reported to be excited at an incident energy of 0.350 Mev ($E_x = 4.289$ Mev) by an $t=1$ neutron and the level was assigned to be $3/2^-$. In the (d,p) reaction this level is populated by a transfer that appears to be of low $t$-value. Both the $t=0$ and $t=1$ theoretical curves fit the general trend of the angular, but neither fit the rather indistinct oscillatory structure. The experimental angular distribu-
tion is found to be very similar to that observed for the transition to the $4s_{1/2}$ single-particle state at 2.033 MeV implying an $\ell=0$ assignment. However, an $\ell=1$ assignment cannot be ruled out, especially in view of the results of the neutron studies, and thus both $\ell$ values are considered possible. Spectroscopic factors of $S(4s_{1/2}) = 0.2444$ and $S(4p_{3/2}) = 0.0931$ are obtained when spin assignment of $1/2^-$ and $3/2^-$ are assumed, respectively.

Level 39 -- 4.464 MeV

A level at 4.452 MeV was excited in the $(t,p)$ reaction which could correspond to this level seen in the $(d,p)$ reaction. No angular distribution for the level was reported in the $(t,p)$ study. In the Pb$^{208}$ plus neutron studies (KB62, PA50) a state at an incident energy of 0.525 MeV ($E_x = 4.463$ MeV) was excited by an $\ell=2$ neutron and identified as a $d_{5/2}^+$ state. The transition to this level in the $(d,p)$ reaction showed an angular distribution which is essentially identical to that observed for the previous level (#35) at 4.295 MeV and thus was also assigned possible $\ell$-values of 0 and 1. Spectroscopic factors of $S(4s_{1/2}) = 0.1093$ and $S(4p_{3/2}) = 0.0724$ are obtained when spins of $1/2^-$ and $3/2^-$ are assumed respectively. From a comparison of the excitation energies reported in the neutron study and in the $(d,p)$ study one is tempted to conclude that a single level is excited by the two reactions however the poor fit to the $\ell=2$ angular distribution in the $(d,p)$
study leaves this identification in doubt.

c. Summary of the Results

A summary of the conclusions reached in the discussions in the previous section of the weak transitions to the 2p-1h states are listed in Table V-10. In the table are listed the excitation energies, $\ell$-values, spin and parity assignments, and spectroscopic factors found in this study. The $\ell$-values listed correspond to the assignments made on the basis of fits to the angular distributions which were consistent with both previous experimental results and the shell model arguments considering the parity of 2p-1h states below 4.0 Mev excitation. To be more specific, the $\ell=1$ and $\ell=7$ assignments were made on the basis of fits alone. In the case of the angular distributions fit both by an $\ell=2$ and 3 assignment and by an $\ell=4$ and 5 assignment, the spin and parities of the final states assigned in previous studies were used to rule out one of the $\ell$-values. In all cases (below 3.9 Mev) the $\ell$-values were determined to be odd (i.e. either $\ell=3$ or $\ell=5$). Only the two transitions to the states at 2.592 Mev and 3.365 Mev which were both fit by an $\ell=4$ or 5 assignment remained after a comparison to the results of other studies. These were both given tentative $\ell=5$ assignments based on the arguments that no positive parity 2p-1h states are expected at these excitation energies. It should be noted that in no instance were the $\ell$-assignments made in this study found to be inconsistent with the results of other studied. This
is an important point since it was not clear at the outset that the analysis of the weak transitions with the conventional DWBA theory would yield reliable $l$-assignments. It should also be noted that on the basis of this study the difficulties in distinguishing between $l$-transfers which differ by one unit of orbital angular momentum appear to have been due to the rather large errors associated with the measured cross sections and the similarities in the predicted curves which are a characteristic of the Pb region at this deuteron energy, rather than some property of the reaction mechanism of the weak transitions. This question, of course, can only be answered when better theoretical treatments of the transitions are calculated.

The difficulties of the present DWBA theory to predict accurately the angular distributions of even the strong transitions to the single-particle states have been pointed out previously.

The spin assignments in Table V-10 correspond to spins indicated from the results of other studies when such information was available and in other cases they correspond to tentative assignments made on the basis of shell model arguments. In a number of cases spin assignments were made based on the $l$-assignments of this study which ruled out one of two possible spins assigned by other studies.

The spectroscopic factors listed in Table V-10 correspond to the values extracted under the assumption that the 2p-1h states are populated by stripping into a single
shell-model orbit. In most cases the final state can in principle be populated either by stripping into an orbital below the Fermi sea or by stripping into an orbital whose centroid is found in the continuum. In these cases spectroscopic factors for both are given. In the case of the $l=7$ transitions where it appears probable that the transition is proceeding via $1j_{15/2}$ single-particle admixtures, and the case of $11/2^-$ states where the only reasonable explanation for the population of these states is through admixtures of $2h_{11/2}$ single particle strength it is possible to eliminate other orbits and only one spectroscopic factor is extracted. However, in most cases there appears to be no way of eliminating the possibility of stripping to these states via either $2p-2h$ ground state correlations or admixtures of single particle strength from the next shell ($N>184$) without comparison with theoretical calculations.

The spectroscopic factors obtained in this study as has been stressed previously are subject to errors due to the uncertainties in the neutron form factors used in the calculations. In the discussion of weak transitions in Section III.C, it was pointed out that the results of studies of transitions where realistic form factors were employed had indicated general trends that might be used to help determine the true spectroscopic factors from those obtained from analyses where the separation energy approximation had been used to specify the form factors. From these studies it was found that when the separation
energy approximation was used for transitions leading to states which had a separation energy that was smaller than the zero-order single particle energy as is the case when the neutron is stripped into an orbit below the Fermi level (for example, the transition to the 1/2^- state at 2.152 Mev by a transfer into the 3p_{1/2}^- orbit), the use of a more realistic form factor would result in an increase in spectroscopic factor extracted by perhaps as much as a factor of two. The opposite was indicated to be true when the neutron is stripped into a state via an orbit whose zero-order energy is smaller than the separation energy of the state. In these cases the spectroscopic factor will be decreased by an amount which from previous studies is indicated to be again perhaps as large as a factor of two. This means that when considering the spectroscopic factors for the states which are populated via correlations in the Pb\textsuperscript{208} ground state, the values in the table represent a lower limit and that the "true" spectroscopic factor could be larger by as much as a factor of two. In the case when the state is populated through single particle admixtures from the states in the continuum (N>184) the spectroscopic factors listed are inflated and the values must be regarded as upper limits on the true values which are indicated to be smaller by perhaps a factor of two.
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* Fit based on spin assignment of level made in previous studies (see discussion).
VI. DISCUSSION

A. General Comments

In this chapter the nuclear structure information obtained from this (d,p) study will be discussed. For the purpose of this discussion the experimental results will be divided into three categories and will be presented in the following three sections of this chapter. The categories are, respectively, the strong transitions leading to the single-particle states, the weak transitions leading to the bound states in the excitation region 2.0 to 3.9 Mev, and the weak (but slightly stronger) transitions leading to the unbound states in the excitation region 3.9 to 4.3 Mev. In each case we first will discuss the nuclear structure information that can be inferred from the experimental results without reference to model calculations. We then compare the results of model calculations with the experimental data and discuss the information concerning nuclear structure implied by the comparison. Conclusions concerning the nuclear structure, and in some cases concerning the model calculations, will be discussed at the end of each section. In the discussion of the weak transitions the experimental results will be interpreted under the assumption that the reaction process is that of a direct transfer. In Section (VI.C.3.b.) the possible interpretation of these transitions as two-step processes will be discussed. In this discussion it will
be shown that the assumption appears to be justified and that there is no clear evidence that the two-step reaction process is the main process in the weak excitation of the states studied here.

Before proceeding with discussion of the experimental results some brief comments will be necessary concerning the models involved. The predictions resulting from three different calculations will be compared with the experimental data. Because these three calculations have emphasized different aspects of the nuclear structure reflected in the \((d,p)\) reaction data, the calculations in many cases do not make overlapping predictions. Indeed, no one of the calculations has included all the nuclear structure effects which might be important in obtaining a complete description of the results and in each case the comparison of the model calculations to the experimental results is meaningful. The calculations have been presented in a previous section (Section II.B.); in this section we wish to emphasis briefly those aspects of nuclear structure each calculation treats and that information which can be expected from the comparison of each of the calculations with the experimental results.

The particle-hole (p-h) RPA calculations of Gillet, et al. (GG66, AG69) which fitted the excitation spectrum of \(^{208}\text{Pb}\) predict 2p-2h, 4p-4h, etc., correlations of p-h character to be found in the \(^{208}\text{Pb}\) ground state wave function. In the two-particle (2p) RPA calculations of Vary and Ginocchio (VG70), for the level spectra of nuclei two
mass units from Pb\textsuperscript{208}, there also are predicted correlations but of a particle-pairing (2p) character. Presence of 2p–2h correlations in the Pb\textsuperscript{208} ground state would make it possible to populate certain otherwise inaccessible 2p–1h states in Pb\textsuperscript{209} in a (d,p) reaction on Pb\textsuperscript{208} by stripping into a partially filled orbit below the Fermi level. In these calculations no predictions concerning the detailed structure of the 2p–1h states in Pb\textsuperscript{209} are made, however, and some additional assumptions concerning the structure of these final states must be made in order to permit the prediction of spectroscopic factors which can be compared to the experimentally determined values. In this comparison we test whether the predicted correlations in the Pb\textsuperscript{208} ground state can adequately describe the population of 2p–1h states observed in this (d,p) study.

Calculations by Hamamoto (HA69) concerning particle-vibrations states in Pb\textsuperscript{209} may be also tested directly through comparison with our experimental data. In these calculations the particle-vibration multiplets in Pb\textsuperscript{209} based on the weak coupling of the odd neutron orbits to the core excitations of Pb\textsuperscript{208} are studied. Among the various perturbations which remove the particle-vibration multiplet degeneracies are those resulting in admixture of single-particle wave function amplitudes through which the states can be populated in a (d,p) reaction. These calculations have the unique feature of computing these single-particle admixtures without reference to the radial quantum number, and hence include all the single-particle orbits of the
same \( t_j \), but different radial quantum number \( n \). This makes it possible to evaluate the admixture of a single-particle configuration whose centroid position is unbound in a particle-vibration state. Also included in the calculation is an estimate of the effect of the odd particle in the residual nucleus on the ground state correlations of the \( \text{Pb}^{208} \) core; this contribution is also present in the single-particle component. It follows that in a stripping reaction on \( \text{Pb}^{208} \) the admixed amplitude of the single-particle component reflects both the amplitude for stripping into an orbit above the Fermi level (and which may be unbound) as well as the amplitude for stripping into an orbit which is below the Fermi level and which is not entirely filled because of the correlations in the ground state. In the comparison of these calculations with the experimental results we will determine whether this weak coupling treatment can reproduce (1.) the observed excitation energies of the particle-vibration states, and (2.) the distribution of single-particle strength among the particle-vibration (2p-1h) states.

The weak coupling calculations of the \( \text{Pb}^{209} \) level structure made by Bes and Broglia (BB70, FI70) will be also compared to the experimental results. In these calculations the possibility of 2p-1h states based on collective excitations of \( \text{Pb}^{208} \) coupled to neutron single-particle orbitals in addition to two particle excitations of \( \text{Pb}^{210} \) coupled to neutron single-hole orbitals are included. Collective excitations of both \( \text{Pb}^{208} \) and \( \text{Pb}^{210} \) are generated
by quasi-boson operators which have been determined from RPA calculations somewhat similar to those of Gillet, et al. (GG66) and of Vary and Ginocchio (VG70), except that a different form for the residual interaction was used. The weak-coupling calculation of the Pb$^{209}$ two particle-one hole level structure is similar to that of Hamamoto except that it includes the perturbation effects to 2nd and 3rd order on the weak coupled states, and the possibility of mixing between the 2p-1h states based on both 2p-1h basis sets. It includes, moreover, the possibility of admixtures of single-particle states of the major neutron shell ($N=126-184$), but not the single-particle states which are unbound ($N>184$). Thus in the comparison of the results of the calculations of Bes and Broglia with the experimental data, we test both the predicted level structure of Pb$^{209}$ (i.e. the excitation energies of the 2p-1h states, the distribution of single-particle strength, etc), and the predictions of the population of the 2p-1h states via correlations in the Pb$^{208}$ ground state.

B. Single-Particle States

1. Discussion

The results of the analyses of the data concerning the transitions to the single-particle states were consistent with those obtained in previous studies. The spectroscopic factors for all single-particle states, with the single exception of the $1j_{15/2}$ state at 1.424 Mev, were found to be consistent with a unit value within the typical
uncertainties associated with even the most reliable extraction of spectroscopic factors (i.e. about 20%). The spectroscopic factor for the $15/2^-$ state was found to be 0.55 and thus this is the only state which the present results indicate to be fragmented by more than 20%.

2. Predictions of Model Calculations

In the framework of a simple shell model all these states are expected to have unit spectroscopic factors. Model calculations which have included the effects of residual interactions, however, predict that the spectroscopic factors should be less than unity because of (1.) the presence of correlations in the Pb$^{208}$ ground state and (2.) the fragmentation of single-particle strength because of interactions with states at higher excitations in Pb$^{209}$.

The correlations in the Pb$^{208}$ ground state predicted by the RPA calculation (GG66, VG70) are of such a nature that only about 5% of the single particle strengths are predicted to be reduced because of their presence. The predicted spectroscopic factors for the seven single-particle states which have been considered in the RPA calculations of Gillet, et al., and of Vary and Ginocchio are listed in Table VI-1. This table illustrates the effect the correlations have on the magnitude of the spectroscopic factors. Thus as long as there are uncertainties of at least 20% in the extraction of spectroscopic factors,
Table VI-1

Predicted Single-Particle Spectroscopic Factors

<table>
<thead>
<tr>
<th>n£j</th>
<th>p-h RPA</th>
<th>2p RPA</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Gillet, et al.</td>
<td>Vary and Ginocchio</td>
</tr>
<tr>
<td>2g9/2</td>
<td>.970</td>
<td>.977</td>
</tr>
<tr>
<td>1i11/2</td>
<td>.982</td>
<td>.973</td>
</tr>
<tr>
<td>1i15/2</td>
<td>.997</td>
<td>.981</td>
</tr>
<tr>
<td>3d5/2</td>
<td>.997</td>
<td>.989</td>
</tr>
<tr>
<td>4s1/2</td>
<td>.992</td>
<td>.992</td>
</tr>
<tr>
<td>2g7/2</td>
<td>.994</td>
<td>.984</td>
</tr>
<tr>
<td>3d3/2</td>
<td>.998</td>
<td>.989</td>
</tr>
</tbody>
</table>
value of $5.7 \times 10^4 \text{ e}^2\text{ fm}^2$ obtained using the calculated wave functions for the $15/2^-$ state and the $9/2^+$ ground state, is found to be in good agreement with the experimentally measured value of $B(E3)\equiv(7\pm2) \times 10^4 \text{ e}^2\text{ fm}^2$ (EK69).

The fragmentation of the positive parity single-particle states has also been calculated by Hamamoto. These states are predicted to mix with 2p-1h states located at about 4.0-4.5 Mev excitation. The predicted fragmentation because of admixture of these more distant states, is significantly less than that predicted for the $1j_{15/2}$ state, and the spectroscopic factors for the positive parity states are predicted to be only about 3-10% less than unity (HA70). Thus, as in the case of the effects of correlations, the predicted deviations from unity are smaller than can be reliably measured.

Further information concerning the purity of the single-particle states will necessarily come from a search for the possible fragments of single-particle strength at higher excitation. This matter will be discussed in the next two sections (Sections VI.C., and VI.D.).

3. Conclusions

Thus, in summary, the predictions of the recent theoretical calculations which imply that the single-particle strength is reduced because of correlations in the $\text{Pb}^{208}$ ground state and fragmentation in $\text{Pb}^{209}$ are consistent with the experimental spectroscopic factors obtained in this study. However, because of the large uncertainties in
the experimental spectroscopic factors only the \( l_j_{15/2} \) provides any indication of the success of these calculations since simple shell model predictions are also consistent with the experimental spectroscopic factors.

C. Two Particle-One Hole States

1. Discussion

There are 22 weak transfers observed leading to levels up to 3.9 Mev excitation in Pb\(^{209}\). From simple shell model arguments, which have been presented previously (Section II.A.3.), these levels are expected to be of 2p-1h character. The excitation energies and primary configurations of a number of these states have been established experimentally in the studies of the Pb\(^{207}(t,p)\)Pb\(^{209}\) and Pb\(^{210}(p,d)\)Pb\(^{209}\) reactions (FI69) which populates strongly the configurations

\[
(Pb^{210}(J^\pi) \times p_{1/2}^{-1})_{J+1/2} \text{ and } (Pb^{210}(0^+) \times (n1j)^{-1})_{J},
\]

respectively. When the excitation energies of the states excited in the (d,p) reaction are compared with those found in these (t,p) and (p,d) studies, it is noted that the (d,p) reaction excites over 70% of the number of levels excited in either the (t,p) or (p,d) reactions. From this comparison there appears to be little doubt that many states of predominantly 2p-1h character are being populated in the (d,p) reaction. From the (t,p) and (p,d) studies it is possible to obtain information concerning
the spin, parity, and dominant configuration of a substantial number of levels; this, in turn, can be used to provide understanding of the (d,p) transfers to these levels.

Apart from those levels in the Pb\(^{209}\) spectrum which were observed in the (t,p) and (p,d) studies, there are a number which have been seen only in neutron stripping reactions, and this then provides a clue to their primary character. That is, since the (t,p) and (p,d) reactions strongly populate the 2p-1h states with configurations based on Pb\(^{210}\) and its excited states coupled to holes, then the 2p-1h levels of Pb\(^{209}\) not populated by these two reactions are probable candidates for states based on another coupling scheme (e.g., the core excitations of Pb\(^{208}\) coupled to the various particles or (Pb\(^{210}\)(J\(^n\)) \times P3/2^{-1}), etc.).

There are a number of points which may be noted from the (d,p) results. As already mentioned, there is evidence that the \(1j_{15/2}\) single-particle strength is fragmented and mixed into states in the 2.0 to 3.9 Mev excitation region. In the present study there have been observed three transitions which have angular distributions consistent with an \(l=7\) transfer assignment; the corresponding final states are strong candidates for the recipients of the missing \(1j_{15/2}\) strength. These states and the corresponding spectroscopic factors are shown in Table VI-2 together with the \(1j_{15/2}\) "single-particle" state at 1.424 Mev and its corresponding spectroscopic factor. The level
Table VI-2

The $1j_{15/2}$ Single-Particle Strength

<table>
<thead>
<tr>
<th>Excitation Energies (Mev)</th>
<th>Spectroscopic Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ZR-L</td>
</tr>
<tr>
<td>1.424</td>
<td>0.470</td>
</tr>
<tr>
<td>3.052</td>
<td>0.052</td>
</tr>
<tr>
<td>3.556</td>
<td>0.028</td>
</tr>
<tr>
<td>3.716</td>
<td>0.027</td>
</tr>
<tr>
<td></td>
<td>0.567</td>
</tr>
</tbody>
</table>
at 3.556 Mev has been reported previously in the Pb$^{207}$(t,p) Pb$^{209}$ study and given the possible spin assignments (15/2$^-$, 17/2$^-$). The two levels at 3.052 Mev and 3.716 Mev have been seen in other (d,p) and (t,d) studies, but the corresponding transfer reaction data have not been analyzed for l-values. In a previous (d,p) study (EK69) the level at 3.310 Mev was suggested as a possible candidate for the recipient of some of the missing l$j_{15/2}$ strength, however, in this study the angular distribution for the corresponding transition was found to be best fitted with an l=5 transfer assignment. The sum of the spectroscopic factors for the 4 states listed in Table VI-2 is 0.67 and thus it is not clear that the entire strength has been observed. Because of the ambiguities in the DWBA analyses and the attendant uncertainties in the extraction of spectroscopic factors, the amount of the l$j_{15/2}$ strength that remains unaccounted for, if any, could be as much as 30%.

From the experimental analyses it was found that a few of the transitions were consistent with even l-assignment which could indicate the possibility of transitions via fragments of the positive parity single-particle states. However from simple shell model arguments, there should be no positive parity 2p-1h states in this excitation region with which any of the single-particle states could mix. These crude predictions are reinforced in both the weak coupling calculations of Hamamoto (HA69, HA70) and those of Bes and Broglia (BB70). Based on these considerations it was concluded that it was improbable that any of the
transitions led to final states in this region which had fragments of the positive parity single-particle states. In the $^{207}$Pb(t,p)$^{209}$Pb study (FI70) there was reported a level at 3.708 Mev which was identified to have a possible assignment of either $5/2^+$ or $7/2^+$ and interpreted to be the 3p–2h state of configuration ($^{210}$Pb$^-$ x p$_{1/2}^-1$).

This state might have been expected to have an admixture of either the $3d_{5/2}$ or $2g_{7/2}$ single-particle strength, however, no evidence for the excitation of this level was observed in this (d,p) study.

The experimental results indicate that in addition to the $1j_{15/2}$ single-particle admixtures in the 2p–1h states, there is evidence that single-particle strength from the next neutron shell ($N>184$) which lies in the continuum may be found in some of the 2p–1h states in this region. The best example of this comes from the observation of transitions to $11/2^-$ states in the spectrum. Three $l=5$ transitions were observed to such states. The state at 3.309 Mev has been reported, in the (t,p) study, to have possible assignment of $11/2^-$ or $13/2^-$ thus the $l=5$ assignment in the (d,p) study appears to establish a definite $11/2^-$ assignment for this level. The other two $l=5$ transitions to the levels at 2.592 Mev and at 3.365 Mev were given tentative $11/2^-$ spin assignments based on spectroscopic factor arguments which indicated unreasonable large values if the $9/2^-$ assignment were assumed. In addition, there is reason to believe that the state at 2.592 Mev is the $11/2^-$ member of the ($^{208}$Pb$^-$ x g$_{9/2}$) multiplet. The theoretical
calculations of Hamamoto (HA69) have predicted an admixture of $h_{11/2}$ single particle configuration into this particle-vibration state which has been found to predict cross sections consistent with previous (d,p) and (t,p) results for the transitions to this state.

In these transitions to the $11/2^-$ states, it would appear that the reaction is proceeding through small fragments of $2h_{11/2}$ single-particle configuration whose centroid position is unbound and expected at about 8 Mev of excitation. The alternative explanation would be that the reaction proceeds through 2p-2h correlations in the $^{208}\text{Pb}$ ground which include those involving the $1h_{11/2}$ orbit which is found about 8 Mev below the Fermi energy. The possibility of the presence of a significant amount of this shell-model orbit in the correlations seem so improbable simply on the basis of the location of the orbit in the nuclear well that it appears safe to rule out this mechanism as the explanation of these transitions (AG69, VG70). From these arguments, together with Hamamoto's prediction concerning the 2.592 Mev state, it appears that the transitions to the $11/2^-$ states are most reasonably explained as transitions through small fragments of $2h_{11/2}$ single-particle configuration mixed into predominantly 2p-1h states.

In the case of the other transitions to levels in this excitation region, the explanation of their excitation via the (d,p) reaction on the basis of the experimental results
and simple shell-model arguments is not possible. The transitions may be proceeding either through small admixtures of single-particle states of the next higher shell \((N>184)\) or through \(2p-2h\) correlations in the ground states. The evidence that the \(2h_{11/2}\) single-particle strength is fragmented into the bound \(2p-1h\) states opens up the possibility that other unbound single-particle states mix with the bound \(2p-1h\) states. In the next neutron shell \((N>184)\) the shell-model orbits \(2h_{11/2}, 4p_{3/2}, 3f_{7/2}, 1j_{13/2}, 2h_{9/2}, 1k_{17/2}, 3f_{5/2}\) and \(4p_{1/2}\) might be expected (See Figure II-8). The lowest-lying unbound single-particle states which can mix with bound \(2p-1h\) states are the \(2h_{11/2}\), the \(3f_{7/2}\), and the \(4p_{3/2}\) orbits and thus some contributions to the \((d,p)\) cross sections of the transitions to levels of spin \(3/2^-, 7/2^-\) and \(11/2^-\) might reflect admixtures of these configurations. Indeed, we have already adduced some evidence for admixtures of \(2h_{11/2}\) single-particle configuration.

On the other hand, the \(3p_{1/2}^{-1}, 2f_{5/2}^{-1}, 3p_{3/2}^{-1}\), and \(2f_{7/2}^{-1}\) orbits are the most probable candidates for construction of \(2p-2h\) configurations which might mix into the ground states of \(Pb^{208}\) and thus could be expected to provide cross sections for \((d,p)\) transitions to \(2p-1h\) states of spins \(1/2^-\), \(3/2^-\), \(5/2^-\), and \(7/2^-\) in \(Pb^{209}\). No simple arguments can be advanced concerning the importance of these correlations in the ground state beyond that the enhanced electromagnetic transitions rates of the collective excitations of \(Pb^{208}\) and the enhanced transitions observed
in the two nucleon transfer reactions on Pb\textsuperscript{208} imply that correlations are needed to explain these experimental results. It would be expected that the single-particle orbits nearest the Fermi energy would play the most important roles in the correlations. It can be noted that except for the 11/2\textsuperscript{−} state, most of the other states (i.e., of spins 1/2\textsuperscript{−}, 3/2\textsuperscript{−}, 5/2\textsuperscript{−}, and 7/2\textsuperscript{−}) can be populated in principle via either single-particle admixtures, or ground state correlations, and thus a comparison of theoretical predictions with the experimental data must be made in order to answer the question of how these states are being excited.

2. Predictions of Model Calculations
   a. RPA Calculations

In the theoretical paper of Vary and Ginocchio (VG70) spectroscopic factors predicted for the (d,p) reaction to 2p-1h states in Pb\textsuperscript{209} arising from the correlations predicted in the Pb\textsuperscript{208} ground state are presented. These are based on both the p-h RPA calculations of Gillet, et al (GG67) and the 2p RPA calculations of Vary and Ginocchio (VG70). The final states were taken to be levels constructed in a zero-order weak coupling model with the possibility of the various single-particle states of Pb\textsuperscript{209} coupling to the collective excitations of Pb\textsuperscript{208} and the various single-hole states of Pb\textsuperscript{207} coupling to the two-particle excitations of.
The simple coupling scheme that results has been illustrated previously in Figure II-9. The level structure of Pb$^{209}$ is expected to be more complex because of the mixing of states of the same spin and parity. The spectroscopic factors listed in Table VI-3 give the predicted value for stripping in a "pure" particle (hole)-vibration configuration in Pb$^{209}$. It should also be noted that while there are good physical arguments why the two sets of basis states built on the excitations of Pb$^{208}$ and Pb$^{210}$, respectively, should be used to describe the 2p-1h level structure of Pb$^{209}$, the two sets are redundant since the actual states in Pb$^{209}$ should in principle be completely described by one or the other of the two sets. This will result in "over counting" the number of available shell-model configurations in certain cases where there is a large overlap in the configurations of the two sets and thus case must be taken in the interpretation of the spectroscopic factors listed.

Since mixing between the weak coupled states is expected, and, indeed has been observed, it is difficult to compare the values calculated to spectroscopic factors obtained in this experiment. It is possible, however, to use the results of the Pb$^{207}$(t,p)Pb$^{209}$ and Pb$^{210}$(p,d)Pb$^{209}$ studies to establish roughly the nuclear structure of the 2p-1h states in Pb$^{209}$ up to about 3.0 Mev and construct wave functions for these states by using the spectroscopic factors and cross sections measured in these two reaction studies. The wave functions so constructed for seven
Table VI-3

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<tr>
<td>(</td>
<td>g_{9/2} x^3_{1}\rangle)</td>
</tr>
<tr>
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<td>g_{9/2} x^5_{1}\rangle)</td>
</tr>
<tr>
<td>(</td>
<td>l_{11/2} x^3_{1}\rangle)</td>
</tr>
<tr>
<td>(</td>
<td>g_{9/2} x^4_{1}\rangle)</td>
</tr>
<tr>
<td>(</td>
<td>l_{11/2} x^5_{2}\rangle)</td>
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<tr>
<td>(</td>
<td>l_{11/2} x^5_{1}\rangle)</td>
</tr>
<tr>
<td>(</td>
<td>l_{15/2} x^3_{1}\rangle)</td>
</tr>
<tr>
<td>(</td>
<td>l_{15/2} x^3_{1}\rangle)</td>
</tr>
<tr>
<td>(</td>
<td>l_{11/2} x^4_{1}\rangle)</td>
</tr>
<tr>
<td>(</td>
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</tr>
<tr>
<td>(</td>
<td>l_{11/2} x^2_{1}\rangle)</td>
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<tr>
<td>(</td>
<td>l_{15/2} x^2_{1}\rangle)</td>
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Table VI-3 (Continued)

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<th>Final State</th>
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<td>$</td>
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<tr>
<td>$</td>
<td>f_{5/2}\times10^+_{-1}\rangle$</td>
</tr>
</tbody>
</table>
states below 3.0 Mev of excitation are shown in Table VI-4. In this excitation region there are observed one 1/2\(^-\) state, three 3/2\(^-\) states, and three 5/2\(^-\) states (F169). The 1/2\(^-\) state at 2.152 Mev was interpreted to be a pure \((\text{Pb}^{208}_2(0^+) \times p_{1/2}^{-1})\) configuration. Two of the three 3/2\(^-\) states were interpreted to reflect the configurations \((\text{Pb}^{208}_2(0^+) \times p_{3/2}^{-1})\) and \((\text{Pb}^{210}_2(2^+) \times p_{1/2}^{-1})\), while the third level was assumed to be the 3/2\(^-\) number of the \((\text{Pb}^{208}_2(3^-) \times g_{9/2})\) multiplet. The mixing among the three configurations, as determined from the experiments, is shown in the Table VI-4. In the case of the three 5/2\(^-\) states the configurations \((\text{Pb}^{210}_2(0^+) \times f_{5/2}^{-1})\) and \((\text{Pb}^{210}_2(2^+) \times p_{1/2}^{-1})\) are thought to account for two of the three levels, while the third is again assumed to be a member of the \((\text{Pb}^{208}_2(3^-) \times g_{9/2})\) multiplet. Their mixing is shown in the Table VI-4.

In this table are compared the spectroscopic factors predicted from the RPA results using these wave functions with those obtained experimentally. In comparing these spectroscopic factors, it should be remembered that the absolute experimental spectroscopic factors are subject to uncertainties which can be as large as a factor of two where the values listed should be considered as lower limits on the true spectroscopic factors. (See Section V.C.2.c.). It should also be noted that the construction of the wave functions was crude and that it contains not only the errors that are associated with the extraction of spectroscopic factors and strengths in the \((p,d)\) and \((t,p)\)
studies, but it also ignores the problem of double counting which contributes small errors. Despite this, the agreement illustrated in Table VI-4 is remarkably good; the predictions not only appear to be of the right order of magnitude but also appear to be in agreement with the relative strengths observed.

In the case of the other weak transitions observed in the present study it is difficult to compare the predictions with the experimental results since the structure of the final states is not well known; thus only certain general observations can be made. Out of the large number of the weak coupled states expected in the excitation region up to 4.0 Mev. only a relatively few are predicted to have spectroscopic factors large enough for the transitions to have been observed in the present experiments. This is illustrated graphically in Figure VI-I where the predicted spectroscopic factors are compared with the measured spectroscopic factors. (The lines in the figure indicate the weak coupled states with predicted spectroscopic factors less than 0.002.) Generally speaking, for each state predicted to have a spectroscopic factor larger than 0.002 there is observed a level within about 200 kev which could possibly correspond to the prediction. Whether this is coincidental or not must wait for a theoretical treatment of the Pb$^{209}$ level structure.

The $7/2^-$ member of the $(\text{Pb}^{208}(3^-) \times g_{9/2})$ multiplet
<table>
<thead>
<tr>
<th>J^π</th>
<th>E_x (MeV)</th>
<th>(2, d)</th>
<th>(t, p)</th>
<th>Wave Functions</th>
<th>SpectroscopicFactors</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>λ</td>
<td>S_{exp}</td>
<td>L</td>
<td>( \sigma(209) / \sigma(210) )</td>
</tr>
<tr>
<td>1/2^-</td>
<td>2.152</td>
<td>1</td>
<td>2.00</td>
<td>0</td>
<td>1.02</td>
</tr>
<tr>
<td>3/2^-</td>
<td>2.320</td>
<td>1</td>
<td>0.56</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3/2^-</td>
<td>2.906</td>
<td>1</td>
<td>0.26</td>
<td>2</td>
<td>0.38</td>
</tr>
<tr>
<td>3/2^-</td>
<td>3.077</td>
<td>1</td>
<td>3.16</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5/2^-</td>
<td>2.463</td>
<td>3</td>
<td>0.81</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5/2^-</td>
<td>2.740</td>
<td>3</td>
<td>4.06</td>
<td>2</td>
<td>0.25</td>
</tr>
<tr>
<td>5/2^-</td>
<td>2.876</td>
<td>3</td>
<td>1.02</td>
<td>2</td>
<td>0.38</td>
</tr>
</tbody>
</table>
Figure VI-1

Comparison of the predicted spectroscopic factors (VG70) of pure weak coupled states to the spectroscopic factors measured for the 2p-1h states in this study.
predicted to be seen in the reaction, is not observed. However, the Pb$^{210}$(p,d)Pb$^{209}$ study indicates that this state should be located at 2.563 Mev and thus would lie too close to the strongly populated 3d$^{3/2}$ single particle state at 2.537 Mev to have been seen in this study. The only marked exceptions to the agreement appear to be the 7/2$^-$ and 9/2$^-$ states at 3.203 Mev and 3.026 Mev which are believed to be members of the (Pb$^{210}$(4$^+) \times p_{1/2}^{-1}$) doublet (FI69). These states are predicted by Vary and Ginocchio to have essentially zero spectroscopic factors yet transitions to these two states were observed. In particular, the 9/2$^-$ state was found to have relatively large spectroscopic factor. It must be remembered, however, that when two configurations which can be populated in the reaction mix, the cross sections for the transitions to the final states depend on the phases of the configurations and thus the apparent disagreement of the predicted spectroscopic factors with the measured values may be resolved when a theoretical description of the wave functions for these states is calculated.

These calculations have not made predictions concerning several important segments of the experimental results obtained. First, since the Pb$^{209}$ level structure has not been calculated no comment on the fragmentation of the 1j$^{15/2}$ strength can be made. On the other hand, in these RPA calculations no predictions concerning correlations containing the 1h$^{11/2}$ shell-model orbital have been reported with which to compare the three $l$=5 transitions to 11/2$^-$.
states. The results of the RPA calculations (See Figure II-6) indicate that correlations containing the $1h_{11/2}$ orbit are very small and hence the predicted cross sections for the population of the $11/2^- 2p-1h$ states via correlations are also very small. Therefore these results tend to confirm our earlier simple arguments that the $11/2^-$ states of $^{209}\text{Pb}$ are populated in the $(d,p)$ reaction via admixtures of $2h_{11/2}$ single-particle configuration.

b. Particle-Vibration Coupling Calculation

Recently Hamamoto has studied the $(\text{Pb}^{208}(3^-) \times \frac{9}{2}^-)$ particle-vibration multiplet in $^{209}\text{Pb}$ and has also investigated the fragmentation of single-particle strength among the various other multiplets in the spectrum. Since in the $(d,p)$ reaction the particle-vibration states which contain fragments of single-particle configurations are populated, the results of the present study can be used to test both the treatment of the particle-vibration states and the predictions of the single-particle fragmentation.

In Table VI-5 are listed the predicted excitation energies of the $(\text{Pb}^{208}(3^-) \times \frac{9}{2}^-)$ septuplet with the mixing probabilities (HA69) of the single-particle components to be found in their wave functions. The comparison is limited to four states since no levels corresponding to the $7/2^-, 9/2^-, 13/2^-$ members of the multiplet were found. The level seen at 2.42 Mev in the present study could possibly be either the $9/2^-$ or $13/2^-$
Table VI-5

The \( ^{208}\text{Pb} (^{3-}) \times g_{9/2} \) Multiplet

<table>
<thead>
<tr>
<th>Spin ( J^\pi )</th>
<th>Excitation Energy (MeV)</th>
<th>Spectroscopic Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Experiment</td>
<td>Theory</td>
</tr>
<tr>
<td>( 3/2^- )</td>
<td>2.319</td>
<td>2.37</td>
</tr>
<tr>
<td>( 5/2^- )</td>
<td>2.461</td>
<td>2.35</td>
</tr>
<tr>
<td>( 7/2^- )</td>
<td>(2.563)*</td>
<td>2.66</td>
</tr>
<tr>
<td>( 9/2^- )</td>
<td>---</td>
<td>2.46</td>
</tr>
<tr>
<td>( 11/2^- )</td>
<td>2.592</td>
<td>2.61</td>
</tr>
<tr>
<td>( 13/2^- )</td>
<td>---</td>
<td>2.59</td>
</tr>
<tr>
<td>( 15/2^- )</td>
<td>3.052</td>
<td>3.21</td>
</tr>
</tbody>
</table>

* Observed in the \(^{210}\text{Pb} (p, d)^{209}\text{Pb} \) reaction (IF70).
state, however, it was too weakly excited to permit assignment of an l-value to the corresponding transfer. The \( \frac{7}{2}^- \) member of the multiplet, as remarked previously, has been tentatively identified in the \( \text{Pb}^{210}(p,d)\text{Pb}^{209} \) study to be at 2.563 Mev excitation, and would not be seen in the \( (d,p) \) study because it would be obscured by the strong population of the \( 3d_{3/2} \) single-particle state which is located nearby in energy. As can be noted from the comparison of the excitation energies, the predicted energies are in good agreement with the observed energies.

The single-particle mixing probabilities, as noted previously, contain the amplitude for all the single particle configurations of the same \( l_j \), by different radial quantum numbers, \( n \). Included are the shell-model orbits both above the Fermi energy and unbound and those below the Fermi energy into which the neutron can be stripped. In the case of the \( \frac{11}{2}^- \) and \( \frac{15}{2}^- \) states, where only one shell-model orbit might be thought to be involved, the direct comparison of the mixing probability with the experimental spectroscopic factor is meaningful. However, in the cases of the \( \frac{3}{2}^- \) and \( \frac{5}{2}^- \) states where two shell-model orbits may be involved (i.e. the orbits \( 3p_{3/2} \) and \( 4p_{3/2} \), and \( 2f_{5/2} \) and \( 3f_{5/2} \) the comparison may not be too meaningful if both orbitals are important (See Section 3.a. of this chapter). If, however, it is assumed that with the exception of the \( \frac{11}{2}^- \) and \( \frac{15}{2}^- \) states the particle-vibration states are populated primarily by stripping into those shell-model orbits below the Fermi
energy which are not completely filled because of the correlations, then the mixing probabilities can be compared to the spectroscopic factors obtained in this study. This comparison is shown in Table VI-5. In all cases the mixing probabilities are found to be about a factor of two to four times larger than the spectroscopic factors found in this study. It should again be noted that the listed experimental spectroscopic factors for the 3/2^- and 5/2^- states are expected to be increased by a factor of two or more while the spectroscopic factor for the 11/2^- state should be decreased by perhaps as much as a factor of two (Section V.C.2.c.) Thus we find that the predicted single-particle mixing probabilities are in better than order of magnitude agreement with the experimental spectroscopic factors and indeed well reproduce the relative strengths by which these states are excited.

The rather good agreement of the mixing probability of the 11/2^- state with the measured spectroscopic factor is significant since it implies that a good description of this transition can be given by the stripping of the neutron via admixture of the 2h_11/2 single-particle state located in the continuum. It is of interest that two other states in the spectrum at slightly higher excitation, identified as 11/2^- states, are also observed to be strongly excited relative to the other weak transitions and indicative of even more 2h_11/2 single-particle strength in the bound 2p-1h states.

A comparison can also be made of the predicted frag-
mentation of the $l_j$ single-particle strength. This is done in Table VI-6 where the mixing probabilities predicted by Hamamoto are compared to the spectroscopic factors of the levels identified in this study as possibly containing $l_j$ single-particle strength. The levels at 3.050 Mev, 3.556 Mev, and 3.716 Mev had angular distributions which were well fitted by an $l=7$ transfer. The transfer to the level at 3.430 Mev had an angular distribution which did not allow for an assignment of $l$-value, however the state populated appears to correspond to the level at 3.432 Mev seen in the Pb$^{207}$($t$,p)Pb$^{209}$ study and identified in the study as a possible 15/2$^-$ or 17/2$^-$ state. If it is assumed to be a 15/2$^-$ state and the corresponding angular distributions is fitted by an $l=7$ transfer in the ($d$,p) reaction the small spectroscopic factor of 0.006 is obtained. From the comparison in the Table it follows that the calculations do not appear to reproduce the observed distribution of the strength where in particular the large fragment of single-particle strength predicted at about 3.20 Mev is not observed experimentally.

c. RPA-Weak Coupling Calculations

Theoretical calculations of the 2p-1h level structure of Pb$^{209}$ in the excitation region 2.0 to 4.0 Mev have been made by Bes and Broglia (BB70). In these calculations the possibility of the mixing of states based on the collective core excitations of Pb$^{209}$ coupled to single-
Table VI-6

Distribution of $1j_{15/2}$ Single-Particle Strength

<table>
<thead>
<tr>
<th>Experiment $E_x$ (MeV)</th>
<th>Theory $E_x$ (MeV)</th>
<th>Primary Configurations</th>
<th>$S$ (norm.)</th>
<th>$S$ (unnorm.)</th>
<th>$S$ (norm.)*</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.424</td>
<td>1.41</td>
<td>$(0^+ \times 1j_{15/2})$</td>
<td>0.645</td>
<td>0.460</td>
<td>0.800</td>
</tr>
<tr>
<td>3.052</td>
<td>3.21</td>
<td>$(3^- \times g_{9/2})$</td>
<td>0.255</td>
<td>0.052</td>
<td>0.092</td>
</tr>
<tr>
<td>(3.432)</td>
<td>3.18</td>
<td>$(5^- \times g_{9/2})$</td>
<td>0.041</td>
<td>0.006</td>
<td>0.010</td>
</tr>
<tr>
<td>3.556</td>
<td>3.47</td>
<td>$(3^- \times i_{11/2})$</td>
<td>0.022</td>
<td>0.022</td>
<td>0.049</td>
</tr>
<tr>
<td>3.716</td>
<td>--</td>
<td>---</td>
<td>0.027</td>
<td>0.045</td>
<td></td>
</tr>
</tbody>
</table>

* Spectroscopic factors normalized to satisfy sum rule limit.
particles with the states based on Pb\textsuperscript{210} and its excitations coupled to single-hole is considered. The results of these calculations have been compared with the results obtained in the experimental studies of the Pb\textsuperscript{210}(p,d)Pb\textsuperscript{209} and Pb\textsuperscript{207}(t,p)Pb\textsuperscript{209} reactions in previous papers (BB70, FI70) and with some exceptions were found to give a reasonably good description of the experimental results for states up to 4.0 Mev of excitation in Pb\textsuperscript{209}. In these calculations Bes and Broglia have also considered the spectroscopic factors of the 2p-1h states excited in the (d,p) reaction on Pb\textsuperscript{208}, where the states are predicted to be populated either through single-particle admixtures (in these calculations only the \(1j_{15/2}\) single-particle configuration is important) or through correlations in the Pb\textsuperscript{209} ground state (BB70).

The results of these calculations will first be compared with the experimental results on the low lying 2p-1h states in Pb\textsuperscript{209} for which there exists the most experimental information. In Table VI-7 the predicted wave functions of the seven low lying states in Pb\textsuperscript{209} are compared to those deduced from the experimental results of the Pb\textsuperscript{210}(p,d)Pb\textsuperscript{209} and Pb\textsuperscript{207}(t,p)Pb\textsuperscript{209} studies (Section VI.C.2.a.). It follows that the theoretical wave functions are in rather good agreement with those inferred from the experimental studies. In particular, the $1/2^-$ state and the three $3/2^-$ state appear to be well described. In the case of the $5/2^-$ states, the (Pb\textsuperscript{210}(0+) x f\textsubscript{5/2}^-) and
<table>
<thead>
<tr>
<th>$J^\pi$</th>
<th>$E_x$</th>
<th>Empirical Wave Functions</th>
<th>$E_x$</th>
<th>Theoretical Wave Functions</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1/2^-$</td>
<td>2.152</td>
<td>$\sqrt{1.00}(0^+x_{p1/2}^{-1})$</td>
<td>2.220</td>
<td>$\sqrt{1.00}(0^+x_{p1/2}^{-1})$</td>
</tr>
<tr>
<td>$3/2^-$</td>
<td>2.319</td>
<td>$0.14(0^+x_{p3/2}^{-1})$ + $\sqrt{0.85}(3^-x_{9/2})$</td>
<td>2.360</td>
<td>$0.12(0^+x_{p3/2}^{-1})$ - $0.01(2^+x_{p1/2}^{-1})$ + $0.83(3^-x_{9/2})$</td>
</tr>
<tr>
<td>$3/2^-$</td>
<td>2.906</td>
<td>$0.07(0^+x_{p3/2}^{-1})$ + $\sqrt{0.95}(2^+x_{p1/2}^{-1})$</td>
<td>2.980</td>
<td>$0.06(0^+x_{p3/2}^{-1})$ + $0.94(2^+x_{p1/2}^{-1})$ + $0.10(3^-x_{9/2})$</td>
</tr>
<tr>
<td>$3/2^-$</td>
<td>3.075</td>
<td>$0.78(0^+x_{p3/2}^{-1})$</td>
<td>3.190</td>
<td>$0.81(0^+x_{p3/2}^{-1})$ - $0.04(2^+x_{p1/2}^{-1})$ - $0.03(3^-x_{9/2})$</td>
</tr>
<tr>
<td>$5/2^-$</td>
<td>2.463</td>
<td>$0.14(0^+x_{f5/2}^{-1})$ + $\sqrt{0.85}(3^-x_{9/2})$</td>
<td>2.370</td>
<td>$0.05(0^+x_{f5/2}^{-1})$ + $0.002(2^+x_{p1/2}^{-1})$ + $0.02(3^-x_{9/2})$</td>
</tr>
<tr>
<td>$5/2^-$</td>
<td>2.737</td>
<td>$0.99(0^+x_{f5/2}^{-1})$ + $\sqrt{0.42}(2^+x_{p1/2}^{-1})$</td>
<td>2.790</td>
<td>$0.92(0^+x_{f5/2}^{-1})$ - $0.02(2^+x_{p1/2}^{-1})$ - $0.03(3^-x_{9/2})$</td>
</tr>
<tr>
<td>$5/2^-$</td>
<td>2.866</td>
<td>$0.13(0^+x_{f5/2}^{-1})$ + $\sqrt{0.63}(2^+x_{p1/2}^{-1})$</td>
<td>2.880</td>
<td>$0.02(0^+x_{f5/2}^{-1})$ + $0.97(2^+x_{p1/2}^{-1})$ - $0.05(3^-x_{9/2})$</td>
</tr>
</tbody>
</table>
(Pb$^{210}_{1/2} (2^+)$ $\times$ p$^{-1}$)$^{198}$ configurations are not predicted to be mixed as strongly as is found experimentally. This has been noted and investigated in some detail in the earlier discussion of these calculations (BB70). Thus while the description of these low lying 2p-1h states in terms of a particle (hole)-vibration coupling scheme appears to be correct in principle, the details of the configuration mixing are not reproduced.

In Table VI-8 are listed the predicted spectroscopic factors for the Pb$^{210}$ (p,d) Pb$^{209}$ (FI69, IF70) and Pb$^{208}$ (d,p) Pb$^{209}$ reaction compared to those observed experimentally for these states. Also shown are the ratio of the experimental cross sections to predicted cross section for the Pb$^{207}$ (t,p) Pb$^{209}$ reaction (FI69, FI70), where the data have been normalized to the predicted cross section to the state at 2.152 Mev. It will be observed that the predictions agree rather well with the (p,d) results, except for the underestimate of the spectroscopic factor of the 5/2$^-$ state at 2.866 Mev by about an order of magnitude. In the case of the (t,p) predictions there is good agreement except for the 5/2$^-$ state at 2.738 Mev whose cross section is underestimated by a factor of six. As already noted, there does not appear to be sufficient predicted mixing of the configurations of these two 5/2$^-$ states. For the (d,p) results the agreement is reasonably good except for the cases of the 3/2$^-$ and 5/2$^-$ states at 2.319 Mev and 2.461 Mev with dominant configuration (Pb$^{208}_{3^-} (3^-)$ $\times$ g$_{9/2}$) which are predicted to have spectroscopic factors almost an order of magnitude larger than what is observed. However,
<table>
<thead>
<tr>
<th>Excitation Energy (Mev)</th>
<th>$^{210}$Pb$(p,d)^1$ Spectroscopic Factors</th>
<th>$^{207}$Pb$(t,p)^2$ Spectroscopic Factors</th>
<th>$^{208}$Pb$(d,p)^3$ Spectroscopic Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Exp</td>
<td>Theory$^2$</td>
<td>Exp</td>
</tr>
<tr>
<td>$1/2^-$</td>
<td>2.152</td>
<td>2.223</td>
<td>2.15</td>
</tr>
<tr>
<td>$3/2^-$</td>
<td>2.319</td>
<td>2.36</td>
<td>0.50</td>
</tr>
<tr>
<td>$3/2^-$</td>
<td>2.902</td>
<td>2.979</td>
<td>0.20</td>
</tr>
<tr>
<td>$3/2^-$</td>
<td>3.077</td>
<td>3.19</td>
<td>2.60</td>
</tr>
<tr>
<td>$5/2^-$</td>
<td>2.461</td>
<td>2.37</td>
<td>0.61</td>
</tr>
<tr>
<td>$5/2^-$</td>
<td>2.738</td>
<td>2.786</td>
<td>4.75</td>
</tr>
<tr>
<td>$5/2^-$</td>
<td>2.866</td>
<td>2.983</td>
<td>1.02</td>
</tr>
</tbody>
</table>

1. Ref(IF70)
2. Ref(FI70)
3. Present Work
4. Ref(BB70, FI70)
the relative strengths of the transitions to these two states is well predicted. It should also be noted that the $3/2^-$ state at 3.077 Mev which is weakly observed in the experiment is not predicted to be excited. Thus while these $2p-1h$ states are predicted to be populated by means of a direct transfer reaction proceeding via Pb$^{208}$ ground state correlations, the magnitude of the predicted spectroscopic factors are only in order of magnitude agreement with the measured spectroscopic factors.

The predicted excitation energies and spectroscopic factors of the members of the (Pb$^{208}(3^-) \times g_{9/2}$) multiplets are compared to the experimental values obtained in this study in Table VI-9. The comparison, as noted previously, is limited to only four states. The predicted excitation energies are similar to those obtained by Hamamoto, and are in rather good agreement with the measured energies. The spectroscopic factors of the $3/2^-$ and $5/2^-$ members, as already discussed, are overestimated by about an order of magnitude. The spectroscopic factor of the $11/2^-$ member is underestimated by more than an order of magnitude and thus supports the interpretation that this state is being populated through an admixture of $2h_{11/2}$ single particle configuration which has not been included in this calculation. The spectroscopic factors of the $15/2^-$ member appears to be overestimated by about a factor of three and will be discussed in the context of the distribution of the $1j_{15/2}$ single-particle strength. Thus as in the case of other low-lying $2p-1h$ states, the predicted magnitudes of the
Table VI-9

The \([\text{Pb}^{208}(3^-) \times g_{9/2}]\) Multiplet

<table>
<thead>
<tr>
<th>(J^\pi)</th>
<th>Excitation Energy (MeV)</th>
<th>Spectroscopic Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Experiment</td>
<td>Theory</td>
</tr>
<tr>
<td>3/2^-</td>
<td>2.319</td>
<td>2.36</td>
</tr>
<tr>
<td>5/2^-</td>
<td>2.461</td>
<td>2.37</td>
</tr>
<tr>
<td>7/2^-</td>
<td>(2.563)*</td>
<td>2.61</td>
</tr>
<tr>
<td>9/2^-</td>
<td>---</td>
<td>2.47</td>
</tr>
<tr>
<td>11/2^-</td>
<td>2.594</td>
<td>2.68</td>
</tr>
<tr>
<td>13/2^-</td>
<td>---</td>
<td>2.51</td>
</tr>
<tr>
<td>15/2^-</td>
<td>3.052</td>
<td>3.22</td>
</tr>
</tbody>
</table>

* Observed in the \(\text{Pb}^{210}(p,d)\text{Pb}^{209}\) reaction (IF70).
spectroscopic factors are overestimated (except for the notable and important $11/2^-$ state), but the relative strengths are in agreement with the experimental results.

In Table VI-10 are compared the predicted distribution of the $lj_{15/2}$ single-particle strength with that observed experimentally. Also shown are the primary configurations predicted for each state containing this strength. The distribution of the $lj_{15/2}$ strength as can be noted from the comparison is not well reproduced by the theory. In particular the state at 3.556 Mev does not have the large spectroscopic factor predicted. The comparison of the predictions to the $(t,p)$ and $(p,d)$ result had already indicated that sufficient mixing of the $15/2^-$ members of the $(\text{Pb}^{208}(3^-) \times g_{9/2}), (\text{Pb}^{210}(8^+) \times p_{1/2}^{-1})$ and the $(\text{Pb}^{208}(3^-) \times i_{13/2})$ multiplets had not been obtained in the calculations. The $(d,p)$ result confirms the difficulty of the model to treat properly these $15/2^-$ configurations and specifically to predict the observed mixing with the single-particle configuration.

In Figure VI-2 all the predicted spectroscopic factors for the levels up to 4.0 Mev of excitation are compared graphically to those observed in this study. It is difficult to draw any conclusions from the comparison at excitations above 3.0 Mev except to stress that of the rather large number of weakly coupled states predicted in this region only a relatively few are predicted to have significant spectroscopic factors and indeed this is what
Table VI-10

Distribution of $1j_{15/2}$ Single-Particle Strength

<table>
<thead>
<tr>
<th>Experiment $E_x$ (MeV)</th>
<th>Theory $E_x$ (MeV)</th>
<th>Primary Configuration</th>
<th>$S$ (norm.)</th>
<th>$S$ (unnorm.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.424</td>
<td>1.41</td>
<td>$\sqrt{.81}</td>
<td>0^+ x j_{15/2} &gt;$</td>
<td>0.650</td>
</tr>
<tr>
<td>3.052</td>
<td>3.22</td>
<td>$\sqrt{.49}</td>
<td>3^- x g_{9/2} &gt; + \sqrt{.50}</td>
<td>3^- x i_{11/2} &gt;$</td>
</tr>
<tr>
<td>3.432</td>
<td>3.32</td>
<td>$\sqrt{.98}</td>
<td>210^{(8^+)} x p_{1/2} &gt;$</td>
<td>0.001</td>
</tr>
<tr>
<td>3.556</td>
<td>3.47</td>
<td>$\sqrt{.50}</td>
<td>3^- x g_{9/2} &gt; + \sqrt{.48}</td>
<td>3^- x i_{11/2} &gt;$</td>
</tr>
<tr>
<td>3.715</td>
<td>3.85</td>
<td></td>
<td>0.002</td>
<td>0.027</td>
</tr>
</tbody>
</table>

* Spectroscopic factors normalized to satisfy sum rule limit.
Figure VI-2

Comparison of the predicted spectroscopic factor of the 2p-1h states (BB70a) to those obtained in this Pb$^{208}(d,p)$Pb$^{209}$ study
is observed. Furthermore, within about 100 kev of most levels predicted to be seen in the (d,p) reaction, there is observed a level. However, a closer look at the comparison reveals a number of disturbing discrepancies. For example, the states at 3.026 Mev and 3.203 Mev identified in the (t,p) study to be of the configuration \( \text{Pb}^{210}(4^+) \times \text{P}_{1/2}^{-1} \text{g}_{7/2}^{-}, \text{g}_{9/2}^{-} \) were observed in the experiment, while theoretically they are predicted to have zero spectroscopic factor. In particular the \( 9/2^- \) state at 3.026 Mev is found to have a spectroscopic factor of relatively large magnitude. Another example is the set of three \( 11/2^- \) states which have no corresponding predictions and thus appear to contain single-particle strength from the states which are unbound. In Table VI-II the comparison of the spectroscopic factors is shown explicitly where the theoretical level of the correct spin nearest an observed level was arbitrarily identified with the experimental level.

3. Conclusions
a. Comments on Nuclear Structure

Based on comparison of the experimental results obtained in this study with those obtained in other reaction studies it is concluded that a substantial number of states of predominantly 2p-1h configuration in \( \text{Pb}^{209} \) are excited in the (d,p) reaction. It is not possible, on the basis of the experimental results alone, to identify
### Table VI-11

Comparison of Experimental and Theoretical Results

<table>
<thead>
<tr>
<th>Level No.</th>
<th>Experiment</th>
<th>Theory</th>
<th>Spin</th>
<th>$J^π$</th>
<th>$I$</th>
<th>$S_{exp}$</th>
<th>$S_{th}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.152</td>
<td>2.22</td>
<td></td>
<td>$1/2^-$</td>
<td>1</td>
<td>0.0067</td>
<td>0.0090</td>
</tr>
<tr>
<td>2</td>
<td>2.319</td>
<td>2.40</td>
<td></td>
<td>$(3/2^-)$</td>
<td>1</td>
<td>0.0074</td>
<td>0.0530</td>
</tr>
<tr>
<td>3</td>
<td>2.424</td>
<td>(2.49)</td>
<td></td>
<td>$(9/2^-)$</td>
<td>(3)</td>
<td>0.0012</td>
<td>0.0100</td>
</tr>
<tr>
<td>4</td>
<td>2.461</td>
<td>2.90</td>
<td></td>
<td>$5/2^-$</td>
<td>(3)</td>
<td>0.0185</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>2.592</td>
<td>2.68</td>
<td></td>
<td>$(11/2^-)$</td>
<td>(5)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>2.738</td>
<td>2.79</td>
<td></td>
<td>$5/2^-$</td>
<td>3</td>
<td>0.0027</td>
<td>0.0060</td>
</tr>
<tr>
<td>7</td>
<td>2.866</td>
<td>2.98</td>
<td></td>
<td>$5/2^-$</td>
<td>(3)</td>
<td>0.0010</td>
<td>0.0040</td>
</tr>
<tr>
<td>8</td>
<td>3.026</td>
<td>3.21</td>
<td></td>
<td>$9/2^-$</td>
<td>(5)</td>
<td>0.0090</td>
<td>0.0010</td>
</tr>
<tr>
<td>9</td>
<td>3.052</td>
<td>3.22</td>
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<td>$(15/2^-)$</td>
<td>7</td>
<td>0.0520</td>
<td>0.1700</td>
</tr>
<tr>
<td>10</td>
<td>3.075*</td>
<td>3.21</td>
<td></td>
<td>$(3/2^- , 13/2^-)$</td>
<td>1</td>
<td>0.0021</td>
<td>0.0000</td>
</tr>
<tr>
<td>11</td>
<td>3.203</td>
<td>3.22</td>
<td></td>
<td>$7/2^-$</td>
<td>(3)</td>
<td>0.0012</td>
<td>0.0003</td>
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<tr>
<td>12</td>
<td>3.309</td>
<td>3.28</td>
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<td>$11/2^-$</td>
<td>(3)</td>
<td>0.0000</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>3.365</td>
<td>3.32</td>
<td></td>
<td>$11/2^-$</td>
<td>(5)</td>
<td>0.0087</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>3.389</td>
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</tr>
<tr>
<td>15</td>
<td>3.414</td>
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<td></td>
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<td></td>
</tr>
<tr>
<td>16</td>
<td>3.430</td>
<td>3.32</td>
<td></td>
<td>$(15/2^-)$</td>
<td>(7)</td>
<td>0.0060</td>
<td>0.0010</td>
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<tr>
<td>17</td>
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<td>$(7/2^-)$</td>
<td>(3)</td>
<td>0.0017</td>
<td>0.0020</td>
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<tr>
<td>18</td>
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<td>3.47</td>
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<td>$(15/2^-)$</td>
<td>7</td>
<td>0.0280</td>
<td>0.0340</td>
</tr>
<tr>
<td>19</td>
<td>3.675</td>
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<td></td>
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<td></td>
</tr>
<tr>
<td>20</td>
<td>3.650*</td>
<td>3.41</td>
<td></td>
<td>$5/2^- , 13/2^+$</td>
<td>3</td>
<td>0.0070</td>
<td>0.0240</td>
</tr>
<tr>
<td>21</td>
<td>3.681</td>
<td>3.83</td>
<td></td>
<td>$(11/2^-)$</td>
<td>1</td>
<td>0.0048</td>
<td>0.0160</td>
</tr>
<tr>
<td>22</td>
<td>3.716</td>
<td>3.85</td>
<td></td>
<td>$(15/2^-)$</td>
<td>7</td>
<td>0.0270</td>
<td>0.0020</td>
</tr>
</tbody>
</table>

* Experimental results indicate that two closely spaced levels of the spins and parities listed are found at these excitations.
the reaction process by which these states are excited except in a few specific cases. In particular, the three \( \ell = 7 \) transitions are interpreted as the excitation of 2p-1h states of assignment via admixtures of \( 1j_{15/2} \) single-particle configuration and transitions to the states of \( 11/2^- \), \( 15/2^- \) assignment are interpreted as proceeding via admixtures of \( 2h_{11/2} \) single-particle configuration. In other cases the excitation of the 2p-1h states in \( \text{Pb}^{209} \) could reflect either admixtures of single-particle configurations in the final state, correlations in the ground state of the target, or a more complex reaction process than a direct transfer; there appears to be no way to rule out any of these mechanisms of excitation without reference to predictions of specific structure calculations.

Based on the comparisons of the predictions of the three model calculations considered which assume a direct transfer mechanism there are several comments concerning structure that can be made. The spectroscopic factors predicted for the low lying 2p-1h states below 3.0 Mev based on the RPA calculations of Gillet, et al, and of Vary and Ginocchio were found to be in good agreement with those obtained from the experimental data (Table VI-4). Similarly, the predicted spectroscopic factors calculated by Bes and Broglia for these states were found to be in rough order of magnitude accord with the experimental values in all cases except the \( 3/2^- \), and \( 5/2^- \) states of primary configuration \( (\text{Pb}^{208}(3^-) \times g_{9/2}) \) which were pre-
dicted to have spectroscopic factors an order of magnitude larger than that observed (Table VI-9). It should be emphasized that by good agreement we imply that the predicted spectroscopic factors were within a factor of two of the experimental values. This, it will be recalled (Section V.C.2.c.) is roughly the magnitude of the estimated uncertainty in the extraction of spectroscopic factors from the weak transitions using the conventional DWBA analysis. Since in both calculations the non-zero spectroscopic factors result as a consequence of the predicted presence of correlations in the Pb$^{208}$ ground state, the good description of the (d,p) results by these calculations (especially in the calculations of Gillet, et al., and Vary and Ginocchio) implies that the predicted magnitude and character of the correlations are consistent with experimental measurements.

Information concerning the Pb$^{208}$ ground state correlations which can be obtained from the comparison of these calculations with the experimental results for states at higher excitation is inconclusive since the detailed level structure of these states is not yet known well enough. Even in the Bes and Broglia calculations, where the level structure of Pb$^{209}$ is predicted, it is difficult to associate the predicted levels with those observed and difficult to assess whether discrepancies reflect incorrect descriptions of the 2p-1h states in Pb$^{209}$ or in the predicted correlations in the Pb$^{208}$ ground state. It is possible, however, to comment on the 7/2$^-$ and 9/2$^-$ states at 3.203 Mev and
3.026 MeV which have been shown to be composed of the 
\((\text{Pb}^{210}(4^+) \times p_{1/2}^{-1})\) configuration. These states are 
predicted in both calculations to have either zero or very 
small spectroscopic factors, yet they are observed in the 
\((d,p)\) reaction to have spectroscopic factors of 0.0012 
and 0.0090 respectively. Without further examples of this 
type it is difficult to determine whether these results 
have any implications concerning the predicted correlations. 
It should be noted in summary, while it is difficult to 
compare the predictions to these states at higher excitation, 
that with the exception of the two states mentioned above 
the predictions on the whole are not inconsistent with the 
experimental results.

Based on the strength of the transitions to the three 
\(11/2^-\) states identified in the spectrum, it has been 
suggested that these states are being populated via admixture 
of \(2h_{11/2}\) single-particle configuration. The results 
of the three model calculations are consistent with such an 
interpretation. In Hamamoto’s particle-vibration coupling 
calculations the \(11/2^-\) member of the 
\((\text{Pb}^{208}(3^-) \times g_{9/2})\) 
multiplet is predicted to have an admixture of \(h_{11/2}\) single-
particle component. If this admixture is assumed to be an 
essentially pure \(2h_{11/2}\) configuration and the predicted 
mixing probability compared to the spectroscopic factor 
obtained in this study good agreement it found. No 
predictions concerning other \(11/2^-\) states have been reported 
and it would be extremely interesting to see whether the
wave function of the $11/2^-$ member of the $(\text{Pb}^{208}(3^-) \times i_{11/2})$ multiplet, for example, would contain an admixture of $h_{11/2}$ configuration which would be in agreement with the 3.365 Mev state observed. The other two calculations tend to support this interpretation of the excitation of the $11/2^-$ states, by their predictions of very small cross sections to these states via ground state correlations.

There is no clear indication from the experimental data that other single-particle strength from the next shell ($N>184$) is found in the bound 2p-1h states. Hamamoto has calculated the admixtures of single-particle components in some of the 2p-1h states, but it is difficult to determine from the comparison of Hamamoto's calculation with the experimental data whether the final states are being populated via admixtures of unbound single-particle states. The only predictions which can be compared to the experimental results involve the $3/2^-$ and $5/2^-$ states of the $(\text{Pb}^{208}(3^-) \times g_{9/2})$ multiplet. However, the comparison of the results of these calculations to the experimental spectroscopic factors is not unambiguous since in Hamamoto's calculation the single-particle admixture includes (1.) contributions of the unbound shell-model orbit into the final state and also (2.) the shell-model orbit below the Fermi level which can be populated because of the correlations induced in the Pb$^{208}$ ground state. The nuclear structure information is therefore obscured by the manner in which the results of the calculations are presented. The mixing probability is not equivalent to a spectroscopic factor obtained as we have
done under the assumption that the neutron is stripped into one of the two shell model orbits. The test of Hamamoto's calculations would be the use of the computed neutron radial form factor in a reaction calculation that would generate the predicted angular distribution which could be compared directly to the experimental angular distributions. If these states are assumed to be populated primarily via correlations in the ground state (as suggested by the other two calculations), then the mixing probabilities can be compared to the spectroscopic factors obtained in this study. The mixing probabilities for the single-particle component in the $3/2^-$ and $5/2^-$ states are predicted to be 0.0370 and 0.0040 respectively compared to the measured spectroscopic factors of 0.0074 and 0.0012 respectively. While the ratio of strength predicted is approximately correct the mixing probabilities are about a factor of four to five times too large. These predictions can be compared with the predictions of Bes and Broglia who perform a somewhat similar weak coupling calculation where single-particle admixtures from states in the continuum are not included, who also overestimate the strength of these transitions, and who also obtain the right ratio of strengths. Thus from the limited comparison of Hamamoto's predictions with the experimental results it is not possible except for the $11/2^-$ state, to draw any definite conclusions concerning the magnitude of single-particle admixtures from the N>184 shell into the low-lying 2p-1h states beyond that the magnitudes of these admixtures if any are quite small.
From the comparisons of the model predictions with the experimental results it is possible to draw some conclusions concerning how well the model calculations describe the 2p-1h level structure of Pb$^{209}$. Both of the weak coupling calculations (of Hamamoto, and of Bes and Broglia) predict excitation energies for the members of the (Pb$^{208}(3^-) \times g_{9/2}$) multiplet which are in good agreement with experimental energies. The states of spin $3/2^-, 5/2^-, 7/2^-, 11/2^-$, and $15/2^-$ in the spectrum which have been tentatively identified as members of the multiplet are found to lie within 100 kev of the predicted energies and have the observed level ordering predicted by the calculations.

The lack of success of these calculations at higher excitation probably reflects the larger level densities and the increased likelihood of configuration mixing that can then take place. This is illustrated in this (d,p) study by the failure of both the Hamamoto and the Bes and Broglia calculations to describe well the distribution of the observed $1j_{15/2}$ single-particle strength. In particular it is noted that (1.) both calculations indicate that a significantly larger fraction of the strength is located in the particle-vibration states than is observed, and (2.) neither of the calculations predict the way in which the strength is divided among the 2p-1h states. Further illustration of the difficulties of the Bes and Broglia model calculations can be found in the comparison of the predictions to the results of the Pb$^{207}(d,p)$Pb$^{209}$ and Pb$^{210}(d,p)$Pb$^{209}$.
studies (BB70, FI70) where the 2p-1h states are populated strongly and the interpretation is less ambiguous than the weak transitions studied in this Pb$^{208}$(d,p)Pb$^{209}$ reaction. The conclusions of these studied are that while the model calculations give a reasonable qualitative picture of the structure below 5 Mev, the detailed level structure is not predicted. The principle discrepancies which have been pointed out (FI70) are the small amount of splitting calculated for the (Pb$^{210}$($j^n$) x P$^{1/2-}$) doublets (about 10 kev for each doublet, as compared to the observed splitting of 100 kev to 200 kev) and the failure of the model to predict the observed mixing for the 5/2$^-$ and 15/2$^-$ states. These conclusions are confirmed by the (d,p) results in the cases where comparison of the predictions to the experimental data was possible and unambiguous.

Thus in summary, we have found from the study of the weak transitions to the bound 2p-1h states that the interpretation that the predominantly 2p-1h states in Pb$^{209}$ are populated in the (d,p) reaction by means of a direct transfer process is consistent with the reaction data. In some cases the 2p-1h states are interpreted as having been populated through admixtures of single-particle configurations in the final states. Specifically, these are the 2p-1h states of 15/2$^-$ and 11/2$^-$ assignment which contain fragments of the 1j$_{15/2}$ and 2h$_{11/2}$ single-particle strength. In most cases the 2p-1h states are interpreted as having been populated through correlations in the Pb$^{208}$ ground
state. The most conclusive evidence for this interpretation comes from the transfers to the 2p-1h states below 3.0 Mev excitation whose structure is rather well known, however, the evidence from the transfers to states at higher excitation, while not without some ambiguities, is not inconsistent with our interpretation. It is also concluded on the basis of the results of this study and those of other reaction studies, that the description of the 2p-1h level structure in terms of a particle (hole)-vibration coupling scheme is qualitative correct even though the details of the configuration mixing and excitation energies have not yet been reproduced by the present model calculations.

b. Comments on Reaction Mechanism

Thus far in this discussion we have made only brief mention of the possibility of the excitation of the final states by a reaction mechanism more complex than a simple direct transfer. In this study the 2p-1h states were assumed to be populated by a direct one-step stripping process and the experimental data analyzed accordingly. The assumption was not without some basis since structure calculations indicate that the 2p-1h states considered in this study should be populated by direct stripping, and indeed, the experimental angular distributions observed for the transitions to these states are consistent with such an interpretation of the reaction mechanism. However, as pointed out in the discussion of the reaction theory (Section III.C.2.), theoretical calculations of two-step
reaction processes indicate that their contributions to the cross sections may not be negligible, especially in the case of weak transitions leading to states whose parentage is based on core excited states of the target (such as some of the 2p-1h states in this study). It is therefore both interesting and important to determine whether any evidence for such two-step processes can be observed in the experimental data of this study. Since no calculations have been performed which consider the two-step processes in the \( \text{Pb}^{208}(d,p)\text{Pb}^{209} \) reaction to the 2p-1h states in \( \text{Pb}^{209} \), our conclusions considering this question must necessarily be based on rather general arguments.

The results of two-step reaction calculations (GL69) appear to indicate that one-step and two-step modes of excitation have very similar angular distributions. Therefore, the fact that the transitions observed in this study had angular distributions which were reasonably well fit by DWBA predictions cannot in itself be considered evidence for a one-step process. But perhaps a more interesting result of these calculations is the prediction that when a state is populated via both a direct and indirect mode, they interfere and can modify the angular distribution significantly from that predicted for a direct transfer (AG70, IA66). If such interference were taking place in the transitions observed in this study then we should have seen experimental angular distributions which deviated from the DWBA predictions. It was found, however,
that with the possible exception of the $L=1$ transitions to the states at 2.152 Mev and 2.319 Mev the observed angular distributions showed no drastic deviation from the predicted DWBA curves. The best examples of such agreement might be illustrated by the angular distributions of the transitions leading to the three states containing fragments of the $1j_{15/2}$ strength. These states were measured to contain only about 2-10% of the single-particle configuration through which they could be directly populated, where the remaining configuration is of particle-vibration character which can be excited only via a two-step process. Yet these angular distributions are all very well fit by the DWBA prediction of an $L=7$ transfer (See Figure V-7, V-8).

The conclusion to be drawn from this is that either the magnitude of the possible two-step process is small and not detectable in this study or the interference between the two-step and one-step process is such that it does not noticeably affect the angular distribution.

Another question which must be considered is whether the states expected to be excited by a two-step process are observed in this study. Of the collective levels in Pb$^{208}$ the $3^-$ octupole vibration is the most strongly excited in inelastic scattering and thus particle-vibration states in Pb$^{209}$, based on the $3^-$ core excitation, would be expected to be the most strongly excited by means of a two-step process. Hence we consider the experimental results for
the states of the \( \text{Pb}^{208}(3^-) \times g_{9/2} \) multiplet. In the \((d,p)\) reaction only four states which might be identified as members of the septuplet were observed. A fifth level in this excitation region could possibly be considered another member, however its identification must be regarded as more tentative. The levels identified as the \(3/2^-\) and \(5/2^-\) members of the multiplet have maximum cross sections of 45\(\mu\)b/sr and 7\(\mu\)b/sr respectively, while the \(11/2^-\) and \(15/2^-\) members have maximum cross sections of 150\(\mu\)b/sr and 65\(\mu\)b/sr respectively. It would be extremely interesting to see whether the calculations of the two-step process could predict such magnitudes, and perhaps more importantly offer an explanation of why the other members of the multiplet are not observed. On the other hand, we have shown that the interpretation of these states in terms of a direct transfer mechanism is reasonable. The \(3/2^-\) and \(5/2^-\) states are interpreted as being populated via ground state correlations, and the \(11/2^-\) and \(15/2^-\) states are interpreted as being populated via fragments of single-particle configuration. Of the other states, only the \(7/2^-\) member is predicted to be observed in this study and there is reason to believe that this state lies near the strongly populated \(3\Delta_{3/2}\) single-particle state and therefore cannot be detected.

There is an additional comment to be made about the \(15/2^-\) state of this multiplet. Both model calculations, that of Bes and Broglia and that of Hamamoto, predict that
a larger fraction of the $1j_{15/2}$ strength should be found in this state than is observed. In fact, the predictions also put more strength in the other 2p-1h states than is observed. It is an interesting possibility in the case of the $15/2^-$ states that an interference between one-step and two-step processes could decrease the cross sections and account for the disagreement between the model predictions and the experimental results. This could also account for the apparent failure of the sum of the spectroscopic factors over the four $l=7$ transitions to satisfy the sum rule limit. In this interpretation it would have to be assumed that the interference modifies only the magnitude of the cross section and not the angular distribution since as discussed previously the angular distributions are well fit by DWBA predictions of a one-step process. It must be noted, however, that it is also possible and perhaps even more probable that the uncertainties in the DWBA analysis and, in particular, the fact that the large value of the $t$-transfer produces a condition of angular momentum mismatching what account for the apparent missing strength. Therefore until this situation is investigated by another reaction, such as the $(t,d)$ reaction, for example, where angular momentum mismatching is not such a problem for $t=7$ transitions, these experimental results cannot be considered as evidence for a two-step reaction mechanism.

One further comment can be made concerning the states populated in this study which have configurations based on
$^{210}_{\text{Pb}}$ excited states coupled to the single-hole states of $^{207}_{\text{Pb}}$. These states would not be expected to be excited by the two-step process of the type which we have discussed since these final states do not in any simple sense have parentage based on the core excitations of the target nucleus. It is possible to conceive of a two-step mechanism of the type $^{208}_{\text{Pb}}(d,t)\,^{207}_{\text{Pb}}(t,p)\,^{209}_{\text{Pb}}$; however, simply from the magnitudes of the individual transfers it might be estimated that such a two-step transition would be small. Thus, while it is possible that these states can be excited by some two-step mechanism, the possibility of the excitation of these states with the magnitude observed in this study appears to be rather remote. On the other hand, these states are predicted by the model calculations to be populated in the $(d,p)$ reaction by stripping via the $^{208}_{\text{Pb}}$ ground state correlations. Examples of the states of this character which were observed are the $1/2^-$ state at 2.152 Mev of configuration $(^{210}_{\text{Pb}}(0^+)\times p_{1/2}^{-1})$, the $3/2^-$ state at 3.077 Mev of configuration $(^{210}_{\text{Pb}}(0^+)\times p_{3/2}^{-1})$, and the $5/2^-$ state at 2.740 Mev of configurations $(^{210}_{\text{Pb}}(0^+)\times f_{5/2}^{-1})$ and $(^{210}_{\text{Pb}}(2^+)\times p_{1/2}^{-1})$. From the comparison of the predictions of RPA calculations with the experimental results (Table VI-4) it follows that these transitions to these states are reasonably well explained on a one-step basis.

Based on the discussion of the reaction mechanism in this subsection, it is clear that no obvious evidence of two-step processes has been observed. It may well be that the excitations actually involve both one-step and two-step
processes, however, it is not possible at present to draw conclusions concerning this until calculations which include both possibilities are performed. Based on the present experimental data and its analysis (with its attendant uncertainties) we conclude that two step mechanisms are not dominant in the excitation of 2p-1h states in the \( \text{Pb}^{208}(d,p) \) reactions.

D. Unbound States

1. Discussion

Transitions to 12 unbound levels in the excitation region 3.90 Mev to 4.46 Mev were studied in this experiment. The angular distributions of the first 9 levels (#23-31) in the excitation region 3.90 to 4.14 Mev were found to be very similar and could be fitted reasonably well by either an \( l=2 \) or an \( l=3 \) transfer assignment. Two other transitions leading to states at 4.295 Mev and 4.464 Mev appear to correspond either to an \( l=0 \) or an \( l=1 \) transfer, while the last transition, leading to the state at 4.166 Mev, was too weak to permit an \( l \)-assignment. These transitions were observed to have cross sections which ranged between factors of 2 to 10 times larger than that observed for the transitions to the 2p-1h states at lower excitation (i.e. below 3.90 Mev).

A comparison of these \( (d,p) \) results obtained in this study with the results obtained in other reaction studies (Table V-1) shows that two of the 9 levels (at 3.904 Mev
and 4.022 Mev) can be given tentative 7/2− assignments and one level at 4.096 Mev can be given a tentative 5/2+ assignment based on the results of the Pb210 (p,d)Pb209 and Pb207(t,p)Pb209 studies. Some of the other 9 levels populated appear to correspond to levels seen in the Pb210(p,d)Pb209 study when the excitation energies are compared. However, when the ℓ-values assigned in the two studies are compared there appear to be disagreements which are not easily resolved.

For example, the levels excited at 3.947 Mev and 3.985 Mev in the (d,p) study might be identified with the levels excited at 3.937 Mev and 3.995 Mev in the (p,d) study. However, in the (d,p) study these states are assigned transfers of ℓ=2 or 3 while in the (p,d) study they are assigned ℓ=6 transfers. It seems improbable, at least on the basis of the (d,p) results, that an ℓ=6 transfer would be mistaken for an ℓ=2 or 3 transfer. Similarly, the states populated at 4.022 Mev, 4.075 Mev, and 4.112 Mev in the (d,p) study might be identified with the states populated at 4.024 Mev, 4.084 Mev, and 4.119 Mev in the (p,d) study. However, in the (d,p) study the transitions are identified as ℓ=2 or 3 transitions while in the (p,d) study they are identified as ℓ=1 transfers. In this case the difference in the ℓ-assignment is less and thus there may be some question as to whether one of the assignments is in error. It is found in the (d,p) study, however, that an ℓ=1 transfer assignment does not give a
very good fit to the experimental angular distribution since
the predicted curve has its first maximum precisely at the
angle at which there appears to be a minimum in the angular
distribution. It should also be mentioned that the \((t,p)\)
reaction populates one of the three levels mentioned above
(i.e. at 4.022 Mev) and gives possible spin values of
\((7/2^-, 9/2^-)\) to this level. Therefore the \((d,p)\) and
\((t,p)\) results appear to indicate an assignment of \(7/2^-\) for
this state at 4.022 Mev, in disagreement with the possible
\((1/2^-, 3/2^-)\) implied by the \((p,d)\) results. The conclusions
appear to be that (1.) either different levels are being
excited by the \((d,p)\) and \((p,d)\) reactions, or (2.) the
angular distributions for these levels in the \((d,p)\) study
are modified in some manner because the levels are unbound
preventing the possibility of identification of the \(l\)-
transfer by the standard methods. Although both explanations
are possible, the observation of transitions to states at
higher excitations (i.e. the states at 4.164 Mev, 4.295 Mev,
and 4.464 Mev) which show different angular distributions
make the former conclusion more probable. In this discussion,
we will proceed on the assumption that the disagreement in
\(l\)-value assignments between the two studies indicates that
different states are excited.

Based on the \(l\)-assignments made in the \((d,p)\) study it
is possible to speculate on the character of the final states
populated. If the \(l\)-assignments of the transitions to some
of the 9 levels are taken to be \(l=2\), then these final states
can have spins of $3/2^+$ or $5/2^+$ and it is possible that the
transitions are proceeding via small admixtures of $3d_{3/2}$
or $3d_{5/2}$ single-particle fragments. It is at just about
4.0 Mev that the first positive parity 2p-1h states in
Pb$^{209}$ are expected in a zero-order weak coupling model
(See Figure II-9). A simple counting of the possible
$3/2^+$ and $5/2^+$ states predicted in this region gives about
6 to 8 states with which the $3d_{3/2}$ and $3d_{5/2}$ single-particle
states can mix. This could explain the sudden increase in
the cross section of the transitions to levels above about
4.0 Mev excitations since the $t=2$ transfers have rather
large cross sections and even a small admixture into a level
would give it substantial strength in a (d,p) reaction. If
a number of these levels are $5/2^+$ states, it is somewhat
surprising that the $5/2^+$ member of the configuration
($Pb^{210}(3^-)$ x $p_{1/2}$) expected at 4.100 Mev and thought to
be excited by the $Pb^{207}(t,p)Pb^{209}$ reaction does not mix
with the other nearby $5/2^+$ states.

If some of the transitions to these levels are
assumed to be $t=3$ transitions then final states of spins
$5/2^-$ and $7/2^-$ can be populated. The relatively strong
excitation of these transitions compared to the transitions
to the $5/2^-$ and $7/2^-$ two particle-one hole state at lower
excitation (i.e. below 3.9 Mev) appear to rule out the
possibility that these levels are populated via ground
state correlations. The other possibility is that the
states are populated via admixtures of the $3f_{5/2}$ and
$3f_{7/2}$ single-particle configurations. Based on the expected
locations of these two single-particle orbits (See Figure II-8) the $3f_{7/2}$ single-particle strength is perhaps more probable to be found at this excitation. The presence of even small fragments of $3f_{7/2}$ configuration in these $2p$-$1h$ states would give significant cross sections for the population of these states in a $(d,p)$ reaction since the radial form factor of the $3f_{7/2}$ orbit extends quite far out into the nuclear exterior. This could also explain, at least in a qualitative manner, why these levels starting at 3.9 Mev have cross sections noticeably larger than the $2p$-$1h$ states at lower excitation. If these states are $7/2^-$ states, however, it is difficult to understand why so little mixing takes place with the nearby states observed in the Pb$^{210}(p,d)$ Pb$^{209}$ study which are identified as the $7/2^-$ states containing the $2f_{7/2}^{-1}$ strength. Of the 9 levels populated in the $(d,p)$ reaction only one state (at 3.904 Mev) was reported to be populated by an $l=3$ transfer in the $(p,d)$ study. The absence of strong mixing between the states excited in the $(d,p)$ reaction and the $7/2^-$ states excited in the $(p,d)$ reaction does not necessarily prove that the states observed in the $(d,p)$ study are not $7/2^-$ states. It could simply indicate that states of the configuration (Pb$^{208}(0^+) \times 3f_{7/2}$) do not strongly mix with states of the configuration (Pb$^{210}(0^+) \times 2f_{7/2}^{-1}$); that is, that hole states fragment by mixing with holes coupled to Pb$^{210}$ core states and particle states fragment by mixing with particles coupled to Pb$^{208}$ core states. To answer the question it will be necessary to obtain more experimental information as well as more detailed theoretical calculations.
To summarize, from the comparison with the results of other studies it appears that the group of 9 states populated in the excitation region 3.9 to 4.14 Mev contains both negative and positive parity members. The excitation of these states in the Pb$^{208}$ (d,p) Pb$^{209}$ reaction probably implies the presence of single-particle configurations mixed into these states.

The transitions to the levels at 4.295 Mev and 4.464 Mev have angular distributions which are similar and look very much like either an $l=0$ or an $l=1$ transfer. Both the proton groups to these levels stand out strongly in the spectrum and have peak widths which are slightly broader than observed for the other peaks. This could indicate that there is a closely spaced doublet in each case of that the states have natural widths. The level at 4.295 Mev can be identified tentatively as the same level excited at 4.289 Mev by the neutron total absorption experiments and assigned a spin of $3/2^-$. The total width of the state was found to be about 5.0 kev in the neutron experiments and thus would not be expected to contribute extra width in this study where the instrumental width was about 12 kev. This level could be populated via a fragment of $4p_{3/2}$ single-particle strength. The second state at 4.464 Mev could correspond to the level observed in the neutron studies ($E_x=4.463$ Mev) which was assigned a spin of $5/2^+$, however, the angular distributions observed for this level in the (d,p) reaction could not be fit very well by an $l=2$ transfer assignment. Thus the spin and parity of the 4.464 Mev state seen in the (d,p) study
remains in doubt. In the neutron work a large resonance possessing a width of 58 kev was observed at 0.504 Mev ($E_x = 4.441$ kev) and reported as an $t=0$ capture and spin $1/2^+$ (FK65). It might have been expected that a state with such a large width would be seen strongly in the (d,p) reaction. However, unless the energy measurements disagree by about 25 kev (which would put it in agreement with the level seen at 4.464 Mev in the (d,p) study) there is no evidence that this level was seen in this experiment. This level is of some interest since it has been interpreted as an example of a doorway state (FK65).

2. Prédictons of Model Calculations

Under the assumption that these unbound states are populated by a one-step reaction process, the transitions to these states must be proceeding either through ground state correlations or through admixtures of single-particle configurations in the more complex final states. Based on the RPA calculations (See Table VI-3) which predict that no 2p-1h states at this excitation should be excited via correlations, it is concluded that the transitions to these states are populated via single-particle admixtures. Only Hamamoto (HA70) has made predictions of the admixtures of single-particle configurations in the 2p-1h states above 4.0 Mev excitation.

In these calculations she has computed the single-particle admixtures in the members of the $(\text{Pb}^{208}(3^-) \times j_{15/2}) (\text{Pb}^{208}(3^-) \times d_{5/2})$, and $(\text{Pb}^{208}(4^+) \times g_{9/2})$ particle-vibration multiplets. In Table VI-12 are listed the mixing probabilities of the
Table VI-12

Predicted Fragmentation of Single-Particle Strength

<table>
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<tr>
<th>$^J\pi$</th>
<th>Excitation Energies</th>
<th>Predicted Configurations</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/2$^+$</td>
<td>4.59 MeV</td>
<td>$\sqrt{.131}</td>
</tr>
<tr>
<td>3/2$^+$</td>
<td>4.28 MeV</td>
<td>$\sqrt{.041}</td>
</tr>
<tr>
<td>5/2$^+$</td>
<td>4.22 MeV</td>
<td>$\sqrt{.027}</td>
</tr>
<tr>
<td>7/2$^+$</td>
<td>4.33 MeV</td>
<td>$\sqrt{.021}</td>
</tr>
<tr>
<td>9/2$^+$</td>
<td>4.32 MeV</td>
<td>$\sqrt{.009}</td>
</tr>
<tr>
<td>11/2$^+$</td>
<td>4.44 MeV</td>
<td>$\sqrt{.091}</td>
</tr>
<tr>
<td></td>
<td>4.08 MeV</td>
<td>$\sqrt{.006}</td>
</tr>
</tbody>
</table>
positive parity single-particle configurations predicted to be found in these particle-vibration states. About 13% of the 4s_{1/2} single-particle strength is predicted to be located in the 1/2^+ member of the (Pb^{208}(4^+) \times g_{9/2}) multiplet at 4.59 Mev. This could correspond to the level seen at 4.464 Mev in the (d,p) study which as a spectroscopic factor of S = .1093 if a 1/2^+ spin assignment is assumed. The mixing probabilities of 0.041 and 0.027 are predicted for the d_{3/2} and d_{5/2} admixtures in the particle vibrations states at 4.28 Mev and 4.22 Mev excitation respectively. These predictions could correspond to two of the levels in the excitation region 3.9 Mev to 4.15 Mev which have possible t=2 assignments. No t=4 or t=6 transfers were observed in the states found above 3.9 Mev excitations which can be identified with the particle-vibrations predicted to have admixtures of g_{7/2}, g_{9/2} or i_{11/2} single-particle configurations. In particular, no evidence was found for the existence of the approximately 10% fragment of the g_{9/2} single-particle strength predicted in the (Pb^{208}(3^-) \times j_{15/2})_{9/2^+} particle-vibration state. This state is of interest since the predicted wave function for the 9/2^+ ground state which contained a mixture of the (Pb^{208}(3^-) \times j_{15/2})_{9/2^+} configuration was used in the calculation of the decay of the 15/2^- state at 1.424 Mev to the ground state which resulted in a prediction which was in good agreement with the experimental results.

In Table VI-13 are shown the predicted mixing
Table VI-13

Predicted Single-Particle Admixtures in the \([\text{Pb}^{208}(3^{-}) \times d_{5/2}]\) Multiplet

<table>
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<th>(J^\pi)</th>
<th>Excitation Energies</th>
<th>Predicted Configurations</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/2(^-)</td>
<td>4.13</td>
<td>(\sqrt{0.039}</td>
</tr>
<tr>
<td>3/2(^-)</td>
<td>4.13</td>
<td>(\sqrt{0.017}</td>
</tr>
<tr>
<td>5/2(^-)</td>
<td>4.12</td>
<td>(\sqrt{0.007}</td>
</tr>
<tr>
<td>7/2(^-)</td>
<td>4.19</td>
<td>(\sqrt{0.020}</td>
</tr>
<tr>
<td>9/2(^-)</td>
<td>4.12</td>
<td>(\sqrt{0.002}</td>
</tr>
<tr>
<td>11/2(^-)</td>
<td>4.13</td>
<td>(\sqrt{0.076}</td>
</tr>
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</table>
probabilities of the single particle components mixed into the negative parity particle-vibration states of the $(\text{Pb}^{208}(3^-) \times d_{5/2})$ multiplet located at about 4.1 Mev excitation. These states might be expected to be identified with the group of states in the excitation region 3.9 to 4.15 Mev, however, since all the states observed in this excitation have $\ell=2$ or 3 assignments only the $5/2^-$ and $7/2^-$ particle-vibration states can be considered for comparison, and in these two cases there is no way of deciding whether any one of the 9 observed levels corresponds to the predicted levels.

3. Conclusions

The interpretation of the transitions to the unbound levels is not clear. The observation of 9 transitions with very similar angular distribution in the group of states between 3.90 and 4.14 Mev must be considered unusual in view of the large density of states in this region of different spins and parities. It is not clear whether this is a result of the reaction process or the nuclear structure, but in either case it is an extremely interesting phenomenon. The calculations of Hamamoto predict that particle-vibration states at this excitation should be populated via single-particle admixtures of states in both the shell $N=182-184$ and the shell $N>184$ with all $\ell$-values between 0 and 7. In view of the observation of nine similar angular distributions, it appears that Hamamoto's predictions concerning these states are not found experimentally.
The two transitions to the states at 4.295 Mev and 4.464 Mev were identified as possible \( t = 0 \) or \( t = 1 \) transitions, given tentative spin assignments of \( 3/2^- \) and \( 1/2^+ \) respectively, and tentatively interpreted as being populated via admixtures of \( 4p_{3/2} \) and \( 4s_{1/2} \) single-particle admixtures. In these cases Hamamoto's predictions of admixtures of the \( 4p_{3/2} \) and \( 4s_{1/2} \) single-particle configurations in the particle-vibration states are in reasonable agreement with the spectroscopic factors obtained.

Thus while it appears probable that the unbound states excited in the \((d,p)\) reaction are populated through single-particle admixtures, the interpretation of the transitions to these states remains a mystery. As has been discussed in some detail the question of whether these states contain fragments of the positive-parity single-particle strength is of great interest and further investigation of the structure of these states should be made.
VII. SUMMARY

In the present study of the Pb$^{208}(d,p)$Pb$^{209}$ reaction we observed, in addition to the strong transitions to the single-particle states, over 35 weak transitions leading to states of 2p-1h character in Pb$^{209}$ between excitation energies of 2.0 to 4.5 Mev. Most of these transitions were found to have typical stripping patterns and hence were assumed to occur by a direct neutron transfer mechanism. They were assigned $l$-values and spectroscopic factors by the usual DWBA analysis of the angular distributions. The results were then compared with the calculations which predict that 2p-1h states may be excited in two ways, namely, by direct transfers proceeding through correlations in the Pb$^{208}$ ground state or through single-particle admixtures in the final states. Based on these comparisons it was concluded that it is possible to interpret the results under the assumption that the weak transitions are predominantly of a one-step character without resorting to a two-step explanation of the reaction process.

The excitation of most of the 2p-1h states below 4.0 Mev in Pb$^{209}$ provides evidence for the presence of 2p-2h admixtures in the Pb$^{208}$ ground state. Our quantitative measurements which indicate admixtures of the order of about 1% were limited to only a small number of the possible 2p-2h configurations predicted to be in the Pb$^{208}$
ground state. The RPA calculations predict that of the many 2p-1h states in Pb\(^{209}\), only a relative few should be populated with sufficient strength to be observed and these few appear in our spectra and agree with our measurements. However, when all of the RPA contributions are summed including those more complex admixtures such as 4p-4h, 6p-6h, etc, a substantial probability results for finding particles above the Fermi level in the Pb\(^{208}\) ground state. Thus the qualitative agreement of the RPA with the present results, as far as a comparison is possible, makes plausible a description for the Pb\(^{208}\) ground state which contains a substantial probability for particles to exist outside the "closed shells".

Concerning the level structure of Pb\(^{209}\), evidence was presented that the \(1j_{15/2}\) single-particle state, 60% of whose strength appears at 1.42 Mev, is fragmented into at least three additional states in the excitation region 3.0 to 4.0 Mev. Although this is the only single-particle state in the shell \(N=126-184\) to be fragmented below 4.0 Mev, we have seen evidence for single-particle strength from the \(N>184\) shell in the region below 4.0 Mev. For example, strength of the unbound \(2h_{11/2}\) single-particle state is found in several bound 2p-1h states. Finally, our results indicate that the 2p-1h states in Pb\(^{209}\) are qualitatively well described in terms of a particle or hole built on the core states of Pb\(^{208}\) or Pb\(^{210}\), respectively, although the details of the structure are
not yet reproduced by the particle-core coupling.

While we have drawn several conclusions concerning the structure of $^{208}\text{Pb}$ and $^{209}\text{Pb}$ which we believe are qualitatively correct, it must be remembered that the analysis of the weak transitions studied contain certain ambiguities that affect the quantitative aspects of our conclusions. Nevertheless, this should not detract from a broader conclusion to be drawn from this study which is that the weak transitions in the $(d,p)$ reaction can provide information about nuclear structure in addition to what is obtained from the more commonly studied strong transitions.

To fully exploit the experimental information gained in this study in order to obtain detailed quantitative nuclear structure information, it is now necessary to analyze this data by a method in which the intimate relationship between the reaction process and the nuclear structure has been taken into account. Thus in order to perform the $(d,p)$ reaction calculation correctly, we must first obtain the proper form factor from a calculation of the structure of the $^{208}\text{Pb}$ ground state. Furthermore, reaction calculations of two-step processes should also be performed using this nuclear structure information. Only at this point will it be possible to draw more quantitative conclusions concerning the character and magnitude of the $^{208}\text{Pb}$ ground state correlations, the extent of mixing of the single-particle states in the continuum with the bound $2p-1h$ states and the importance of two-step processes in these transitions.
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