Search for Shape Isomers in $^{68}$Ni

by

Bronislaw Konstanty Dichter

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ABSTRACT

Search for Shape Isomers in $^{56}$Ni

Bronislaw Konstanty Dichter

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The $^{40}$Ca($^{16}$O, $^{28}$Si*)$^{28}$Si* reaction excitation function has been studied in search of resonant structure correlated with resonant structure observed in the $^{28}$Si($^{28}$Si, $^{28}$Si*)$^{28}$Si* reaction excitation function. The existence of such correlations would indicate that the resonances originate from a nuclear structure effect in the compound nucleus $^{56}$Ni. Deformed shell model calculations predict the existence of a potential energy minimum at extremely large prolate deformations (3:1 axis ratio) for $^{56}$Ni at spins between 30 and 50. In an analogy with shape isomers in very heavy nuclei, the states built upon this minimum would be hindered from mixing with the much more numerous other compound nuclear states, and their lifetimes would be greatly enhanced. The decay of these long-lived states in $^{56}$Ni should give rise to resonant structures in the excitation functions of nuclear systems that can form the $^{56}$Ni compound nucleus.

The $^{40}$Ca($^{16}$O, $^{28}$Si*)$^{28}$Si* reaction excitation function has been measured between 74.925 and 77.25 MeV in 75 keV steps for angles between 30° and 60° in the laboratory. The Q-value distributions and angular distributions at each energy have also been measured. A statistical analysis of the $^{40}$Ca($^{16}$O, $^{28}$Si*)$^{28}$Si* and $^{28}$Si($^{28}$Si, $^{28}$Si*)$^{28}$Si* reaction
excitation functions, once the average behavior of the excitation functions is factored out, shows that they are correlated at an 80% confidence level. Analysis of the narrow structures reveals that, if they represent the decay of resonances, the reduced width for the decay of the resonance to the elastic $^{28}\text{Si} + ^{28}\text{Si}$ channel is approximately an order of magnitude larger than the reduced width for the decay into the elastic $^{16}\text{O} + ^{40}\text{Ca}$ channel, in qualitative agreement with the calculations. The analysis of the Q-value spectra and angular distributions indicates that the nonresonant yield from this reaction comes from a process with a long-lived intermediate state with a lifetime of the order of $10^{-19}$ sec or longer. The data are equally consistent with a fusion-fission process or a deep-inelastic process with orbiting.
Search for Shape Isomers in $^{56}\text{Ni}$

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CHAPTER 1
INTRODUCTION

1.1 MOTIVATION

The discovery of resonances in $^{12}\text{C} + ^{12}\text{C}$ scattering [Br60] twenty-five years ago opened up a new area of research in nuclear physics. In the time since the initial discovery, a great deal of work, both experimental and theoretical, has been devoted to understanding the processes underlying heavy-ion resonances. Extensive accounts of these efforts can be found in review articles [Er77, Br78, Br82].

For many years, it was thought that resonances were confined to only three systems, $^{12}\text{C} + ^{12}\text{C}$, $^{12}\text{C} + ^{16}\text{O}$ and $^{16}\text{O} + ^{16}\text{O}$. This belief was contradicted by the discovery of resonance-like behavior in the $^{16}\text{O} + ^{28}\text{Si}$ system [Br77]. Since that time resonances, or at least evidence for the formation of a long-lived intermediate state during a scattering process, have been observed in a number of systems including $^{24}\text{Mg} + ^{24}\text{Mg}$ [Zu83], $^{28}\text{Si} + ^{28}\text{Si}$ [Be81] and $^{12}\text{C} + ^{60}\text{Ni}$ [Sh84]. The formation of a long-lived intermediate nuclear complex may be a relatively common and, therefore, important feature of nuclear interactions. It is clearly important to understand the mechanisms that underlie this phenomenon.

In the entrance channel models (see section 5.1), the resonances arise when two colliding ions become trapped in a pocket of a potential and then rotate together as a dinuclear molecule. These models, introduced by several groups [Im68, Ab75, Sc70] around 1970, were successful in reproducing the observed structure in the cross sections of the $^{12}\text{C} + ^{12}\text{C}$ and $^{12}\text{C} + ^{16}\text{O}$ systems. Recently, Thiel, Greiner and
Scheid [Th84] have extended these calculations to the $^{28}\text{Si} + ^{28}\text{Si}$ system.

The present work was motivated by the discovery of resonances in the elastic and inelastic scattering of $^{28}\text{Si} + ^{28}\text{Si}$ [Be81a] at about twice the Coulomb barrier energy, and by theoretical calculations that suggested that a different mechanism may be responsible for the resonances in this system ($^{56}\text{Ni}$ compound nucleus). Deformed shell model calculations [Pl81, Be84a] indicate that at very high spins ($J \sim 40$), $^{56}\text{Ni}$ has a local minimum in the potential energy surface corresponding to a highly elongated, prolate shape with an axis ratio of 3:1. Betts [Be84b] suggested that in analogy with the shape isomers in very heavy nuclei [R180, p. 67], quasibound states in that minimum may be long-lived and that their decay can give rise to the observed resonant structure.

The work presented here is a qualitative test of the following prediction made by the deformed shell model calculations. If the resonances seen in $^{28}\text{Si} + ^{28}\text{Si}$ scattering are of compound nuclear nature, then these resonances should also be seen in other reactions that lead to the same compound nucleus, excitation energy and spin. The reaction studied in the present work is the $^{40}\text{Ca} (^{16}\text{O}, ^{28}\text{Si}^*) ^{28}\text{Si}^*$ reaction. The $^{16}\text{O}$ beam energies used in the present experiments result in the same range of excitation energies in $^{56}\text{Ni}$ as studied by Betts et al. [Be81b]. In addition, the grazing angular momenta of the two systems in this excitation energy range of $^{56}\text{Ni}$ are the same within $2h$, thus minimizing the angular momentum mismatch as a reason why the resonances should be seen in one but not the other reaction (the $^{28}\text{Si} + ^{28}\text{Si}$ data indicate resonance spins near to the grazing angular
momentum). The goal of the experiments was to search for structure in the \( ^{40}\text{Ca}(^{16}\text{O},^{28}\text{Si}^*)^{28}\text{Si}^* \) reaction excitation function, and if structure was found, to check if there was a correlation with structure in the \( ^{28}\text{Si}(^{28}\text{Si},^{28}\text{Si}^*)^{28}\text{Si}^* \) reaction, as implied by the shell model calculations.

The excitation functions of \( ^{28}\text{Si} + ^{28}\text{Si} \) elastic and inelastic scattering \([\text{Be81b}]\) are shown in Figure 1.1. Back angle angular distributions taken across the broad \((\Gamma = 2-3 \text{ keV})\) structures in the total yield excitation function are well fit by a single, squared Legendre polynomial; the degree of the polynomial is equal to the value of the grazing angular momentum (see Figure 1.2). At energies that correspond to the minima between the broad structures, the angular distributions cannot be fitted without using a sum of a large number of Legendre polynomials. The broad structures are broken up into a number of much narrower \((\Gamma = 130 \text{ keV})\) structures that are extremely well correlated \([\text{Sa84}]\) among all the measured elastic and inelastic exit channels. The picture that is implied by the data is that the broad structures are potential scattering resonances (shape resonances), with the resonant \( l \) being very close to the grazing angular momentum; the shape resonances are in turn coupled to the much narrower resonances. The shape resonances are well understood; the mechanism underlying the narrower structures is not. The purpose of this work is to investigate the nature of these narrow resonances using the \( ^{40}\text{Ca}(^{16}\text{O},^{28}\text{Si}^*)^{28}\text{Si}^* \) reaction.
Figure 1.1

The excitation functions of elastic and inelastic \(^{28}\text{Si} + ^{28}\text{Si}\) scattering [Be81]. The excitation function labeled "TOTAL" includes events with \(Q > -22\) MeV. The detection efficiency was 100% for events with \(Q = 0\) MeV and dropped almost linearly to 0% for events with \(Q = -22\) MeV.
Figure 1.2

a) The elastic differential cross section of $^{28}\text{Si} + ^{28}\text{Si}$ scattering at a beam energy of 120 MeV [Be85]. The data show a smooth falloff with increasing angle beyond the grazing bump, near $\theta = 20^0$, interrupted at $\theta = 60^0$ by a transition to an oscillatory behavior.

b) Back angle angular distributions at beam energies that correspond to the peaks of the broad structures observed in the $^{28}\text{Si} + ^{28}\text{Si}$ scattering excitation function [Be81a]. The solid and dashed curves are $[P_L(\cos \theta)]^2$ shapes for the indicated $L$ and $L-2$, respectively. Typical error bars are shown on points near $90^\circ$. 
$^{28}\text{Si} + ^{28}\text{Si}$

$E_{\text{lab}} = 120$ MeV

$x \times 100$
1.2 ORGANIZATION

Chapter 2 contains the description of a preliminary measurement of the $^{40}$Ca($^{16}$O,$^{28}$Si*)$^{28}$Si* reaction done at Brookhaven National Laboratory and a discussion of these early results. In Chapter 3, the construction and principles of operation of the gas ionization counter used in the final experiment are described. Chapter 4 deals with the final experiment done at the A.W. Wright Nuclear Structure Laboratory comparing these data with the $^{28}$Si + $^{28}$Si data. Sections 1 and 2 of Chapter 5 are devoted to a discussion of the entrance channel models and the deformed shell model calculations. In the remainder of Chapter 5, simple models of the $^{40}$Ca($^{16}$O,$^{28}$Si*)$^{28}$Si* reaction are used to extract information about the lifetime of the intermediate state formed during the collision, about the nuclear shapes at the point of scission and about the decay widths. Finally, Chapter 6 contains the concluding remarks.
CHAPTER 2

PRELIMINARY $^{40}\text{Ca}(^{16}\text{O},^{28}\text{Si}^*)^{28}\text{Si}^*$ REACTION MEASUREMENT

The $^{40}\text{Ca}(^{16}\text{O},^{28}\text{Si}^*)^{28}\text{Si}^*$ reaction excitation function was measured initially at the Brookhaven National Laboratory (BNL) Tandem Van de Graaff accelerator using a technique whereby both fragments from the reaction were detected in coincidence. The advantage of a coincidence technique, at least until the dynamics of a reaction are well understood, is the relatively clean identification of the two-body $^{28}\text{Si} + ^{28}\text{Si}$ final state. In a singles measurement, the Si yield may result from other than two-body processes. For example, the Si may be a product of the $^{40}\text{Ca}(^{16}\text{O},^{32}\text{S}^*)^{24}\text{Mg}^*$ reaction followed by an $\alpha$-decay of the $^{32}\text{S}$. A disadvantage of the coincidence experiment for this reaction is that, in practice, it is very difficult to cover the full range of recoil angles, and consequently, the detection efficiency can depend on the reaction Q-value and the corresponding angle between the two fragments. Another possible disadvantage of the two detector coincidence techniques is the difficulty in rejecting a) random coincidences, and b) true coincidences, where two particles from a multibody final state are detected (for example, a three-body final state with two heavy ions and an $\alpha$-particle, with one of the heavy ions and the $\alpha$-particle being detected by the detectors). A full discussion of this problem is deferred to section 2.3.1. In this Chapter, the coincidence measurement will be described; this description will be followed in Chapter 4 by a discussion of a complementary measurement of
the $^{40}\text{Ca}(^{16}\text{O},^{28}\text{Si}^*)^{28}\text{Si}^*$ reaction excitation function where only one of the two fragments was detected and identified.
2.1 EXPERIMENTAL TECHNIQUE

2.1.1 Two PSD Coincidence Technique

In this experiment, a coincidence was set up between two Ortec position sensitive surface barrier detectors (PSDs). Using the detector signals, the masses of the two detected particles can be calculated, and then the Q-value, assuming a two-body process, can also be determined. The principles of the two PSD techniques as well as their application to this experiment will be discussed in this section.

It is simple to show that in a two-body process, \( A + B \to C + D \), the masses of the two particles in the final state are given by

\[
M_C = \frac{E_A}{E_C} \frac{M_A}{E_A} \frac{\sin^2 \theta_D}{\sin^2 (\theta_C + \theta_D)}
\]

(2.1)

\[
M_D = \frac{E_A}{E_D} \frac{M_A}{E_A} \frac{\sin \theta_C}{\sin^2 (\theta_C + \theta_D)}
\]

(2.2)

where \( E_C \) and \( \theta_C \) are the laboratory energy and angle of motion of particle \( C \); \( E_D \) and \( \theta_D \) are the same quantities for particle \( D \), and \( E_A \) and \( M_A \) are the energy and mass of the beam particle. The type of beam particle and the beam energy are selected by the experimenter, and the remaining quantities in eqs. (2.1) and (2.2) are measured by the PSDs.
The calculated masses are plotted in a $M_C - M_D$ plot. In case of $^{16}\text{O} + 40\text{Ca}$ scattering, there are strong peaks in this spectrum in regions that correspond to channels where both nuclei are $4N$ nuclei ($\alpha$-particle nuclei), $^{16}\text{O} + 40\text{Ca}$, $^{20}\text{Ne} + 36\text{Ar}$, $^{24}\text{Mg} + 32\text{S}$ and $^{28}\text{Si} + 28\text{Si}$. Once an event is found to have $M_C$ and $M_D$ values that fall within a region identified as one of these peaks, the exact masses of the nuclei in the channel corresponding to that peak are assumed for the two detected particles and the $Q$-value

$$Q = E_C + E_D - E_A$$ (2.3)

can be calculated for that event by combining eqs. (2.1), (2.2) and (2.3).

$$Q = -E_A \left[ 1 - \frac{M_A}{M_C^E M_D^E} \frac{M_C^E \sin^2 \theta_D + M_D^E \sin^2 \theta_C}{\sin^2 (\theta_C + \theta_D)} \right]$$ (2.4)

where $M_C^E$ and $M_D^E$ are the assumed exact masses of the particles C and D. Eq. (2.4), rather than eq. (2.3), was used to obtain the $Q$-value because the angular resolution of the PSDs used in the experiment was better than the energy resolution.
2.1.2 Calibration of Position and Energy Signals

The position calibration was performed in the following way. Masks with vertical slits were mounted on the front faces of the detectors. The angles of the centers of the slits with respect to the beam axis were determined by optical surveying. A position spectrum was then generated by measuring the scattering of a 75 MeV $^{16}$O beam from a thin Au target (a discussion of the data acquisition system used is found in section 2.2). The locations of the slits in the position spectrum were related to the measured slit angles, thus giving the absolute angular calibration. The energy signals from the PSDs were not used in calculating the Q-value (see eq. (2.4)) and eqs. (2.1) and (2.2) only require that the two PSD energy signals be calibrated relative to each other. This was accomplished by sending identical pulser signals, of known amplitude, to both PSDs and noting the locations in the respective energy spectra of the resulting peaks. A relative calibration was then determined by comparing the peak positions in the energy spectra of the two detectors.

2.1.3 Experimental Setup

In this experiment, the excitation function for the $^{40}$Ca$(^{16}$O,$^{28}$Si*)$^{28}$Si* reaction was measured for beam energies between 75 and 78.25 MeV in 100 keV steps (72 keV in the center of mass frame). The beam current, as measured in the Faraday cup after the target, was approximately 360 nA throughout the duration of the experiment. The
average beam charge state, after passing through the target, was $7.6\times 10^9$, as calculated using an expression derived by Shima et al. [Sh82]. The target was a 10 $\mu g/cm^2$ carbon foil with a $^{40}Ca$ layer deposited on the side of the target facing the beam. Result of target analysis using elastic $^{16}O + ^{40}Ca$ scattering at 15° showed the $^{40}Ca$ layer to be 22.8 $\mu g/cm^2$ thick. The energy loss of 76 MeV $^{16}O$ in this Ca layer is 80 keV.

The primary detectors in the experiment were two Ortec PSDs. The active area of these detectors is 47 mm wide and 8 mm high, and the depth is 100 $\mu m$. PSD$_L$ was mounted with its center at 60° with respect to the beam and 10 cm away from the target; a mask, limiting the vertical aperture to 4 mm, was installed on the front face. The angular range covered by PSD$_L$ was $48° < \theta_{lab} < 72°$. The other detector, PSD$_R$, was also at 60°, but only 8 cm from the target, and its vertical aperture was limited to 7 mm by a mask. In this geometry, the solid angle was defined by PSD$_L$. The slightly larger solid angle of PSD$_R$ ensured a 100% coincidence efficiency for detecting $^{28}Si + ^{28}Si$ final state events with $Q > -8$ MeV.

Two standard Ortec surface barrier detectors were mounted at 15° on either side of the beam axis, to be used as monitor detectors. Each was 32.5 cm from the target and had a 0.12 cm diameter aperture. The beam collimation was checked by placing a paper target in the beam for several seconds and then measuring the burn mark corresponding to the beam spot. The beam spot was found to be roughly square in shape, 1.5 mm on a side. The experimental setup is shown in Figure 2.1.
Figure 2.1

A schematic diagram of the experimental setup.
FARADAY CUP

MON-L

MON-R

PSD-L

PSD-R

TARGET

$23 \mu g/cm^2 \ ^{40}Ca$
on carbon backing

$^{16}O$ BEAM
The arrangement of the monitor detectors allows for a normalization insensitive to beam angle shifts. At 15° in the lab and at the beam energies of interest, the $^{16}O + ^{40}Ca$ elastic scattering cross section is approximately 1.2 times the Rutherford cross section and has the characteristic $1/\sin^4(\theta_{cm}/2)$ behavior. The quantity used to normalize the yields in the PSDs, $MON$, is given by

$$MON = (MON_R \cdot MON_L)^{1/2}$$

(2.5)

where $MON_l$ is the $^{16}O + ^{40}Ca$ elastic yield in the right or left monitor. If the beam shifts in angle, the yield in one detector will increase and the yield in the other will decrease, but the product of the two yields will be unaffected to first order. For example, a beam shift of 0.5° will change MON by only 0.2% from the unshifted value.
2.2 DATA ACQUISITION

The signals from the detectors were first processed by an array of standard NIM electronic modules (see Figure 2.2), the processed signals were then sent to the inputs of scalers and analog to digital converters (ADCs). The outputs of these devices were in turn set to the BNL data acquisition system, consisting of coupled Xerox Sigma 6 and Sigma 7 computers. A full discussion of this stage of data processing for a similar data acquisitions system at WNSL is found in section 4.4. The monitor ADC signals were stored directly into 2048 channel spectra in the computer memory. These spectra were transferred to magnetic tape at the end of each run. For events when signals from PSD\(_R\) and PSD\(_L\) occurred within ± 400 nsec of each other, the outputs from the 2048 channel ADCs handling the PSD signals were transferred to the magnetic tape, event by event, for later replay.

The dead time was measured by sending the output of the beam current integrator (BCI) to a computer gated scaler and a scaler independent of the computer. The differences in the total number of counts in the two scalers can be related to the dead time of the computer driven data acquisition system. For all the runs, the computed dead times varied between 0.4% and 0.8%. Other uncertainties in the experiment were considerably larger than the dead time corrections, and so the data was not corrected for dead time effects.
Figure 2.2

A schematic diagram of the electronics.
2.3 DATA ANALYSIS

The data analysis was done entirely off-line by replaying the accumulated data tapes. The logic of the replay program is shown in Figure 2.3. The analysis was done in two steps. First, the two-body final state events were selected from all PSD\(_R\) - PSD\(_L\) coincidence events, and second, these events were processed identifying the masses of the nuclei and calculating the Q-value for the event.

2.3.1 Selection of Two-Body Final State Events

The rate of random coincidences can be shown to be negligible by considering the total rate of particles striking each PSD. This rate is approximately 40 Hz, while the experimentally defined coincidence time interval is 800 nsec. Therefore, the probability of a random coincidence is vanishingly small, about 1 in \(10^5\).

The rejection of events with more than two particles in the final state is a more difficult problem. These final states can be divided into two classes: A) three heavy ions; and B) two heavy ions and a light particle in the final state. At the bombarding energies of this experiment, less than 5 MeV/u, it is assumed that events with more than three bodies in the final state are rare enough so that they can be neglected. In the following discussion, it will be argued that class A events are relatively rare, and class B events can be experimentally excluded.
Figure 2.3

Flow chart of the logic of the data analysis program.
The results of a recent study [Gr84] of a nearby system,
$^{12}\text{C} + ^{40}\text{Ca}$ at $^{12}\text{C}$ beam energies of 121 and 186 MeV, can be used to argue that class A events are not important. In that coincidence experiment, two detectors measured the nuclear charge of outgoing fragments. It was found that at 186 MeV, the two detected fragments had on the average six units of charge fewer than the compound nucleus; at 121 MeV, the fragments had only four units of missing charge. Therefore, it is reasonable to assume that in the $^{16}\text{O} + ^{40}\text{Ca}$ reaction at a beam energy of 75 MeV, any two outgoing fragments will have a missing charge of less than four units. This assumption implies that class A events do not occur in $^{16}\text{O} + ^{40}\text{Ca}$ scattering at the energies studied in this work.

Class B events, with one heavy ion and a light particle (e.g., an $\alpha$-particle) detected by the PSDs and the second heavy ion undetected, can be identified on a plot where the total mass of the two fragments, $M_{\text{tot}}$, is plotted against the total kinetic energy of the fragments, $E_{\text{tot}}$. A typical $M_{\text{tot}} - E_{\text{tot}}$ plot, together with a gate used to exclude these class B events, is shown in Figure 2.4. The PSDs are 100 µm deep, so that the most energy that an $\alpha$-particle can deposit in the detector is 12.5 MeV (protons deposit even less energy), or typically 10 MeV less than is deposited by heavy ions from $^{16}\text{O} + ^{40}\text{Ca}$ scattering. The angular range of the PSDs is small enough so that the angular factors in eqs. (2.1) and (2.2) are similar for these class B events and two-body events, and therefore, the small size of the maximum energy loss for an $\alpha$-particle makes the calculated mass large. As a result, in the $M_{\text{tot}} - E_{\text{tot}}$ plot, these class B events tend to lie along a broad line parallel to the $M_{\text{tot}}$ axis and extending from $M_{\text{tot}} = 56$ u to larger values of $M_{\text{tot}}$. The gate in the $E_{\text{tot}} - M_{\text{tot}}$ spectrum was constructed to
A typical $M_{\text{tot}} - E_{\text{tot}}$ plot. The solid lines define a gate used to reject some three-body final state events (see text). The dashed line indicates a region of greatest density of events. The $E_{\text{tot}}$ scale is in arbitrary units.
exclude as much of this line as possible. Two-body events with
Q < -22 MeV are experimentally indistinguishable from the heavy-ion - α
events and fall outside this gate; they are not included in the
subsequent analysis.

Class B events with both heavy ions detected by the PSDs and the
light particle undetected cannot be distinguished on the M_{tot} - E_{tot}
plot from two-body events. However, in a M_R - M_L plot, where M_R and M_L
are masses calculated for particles in PSD_R and PSD_L respectively, the
calculated masses for these class B events may be shifted away from
regions on the plot which correspond to two-body final states of the
^{40}\text{Ca}(^{16}\text{O},^{28}\text{Si}^*)^{28}\text{Si}^* reaction, and thus be experimentally
distinguishable.

2.3.2 Processing of Selected Events

All events that fall inside the gate in the M_{tot} - E_{tot} spectrum
are plotted in a M_R - M_L spectrum such as the one shown in Figure 2.5.
It is evident that there is an enhancement of yield around the line
corresponding to M_R + M_L = 56 u. The events lying on or near that line
are broken up into several clusters corresponding to four exit channel
mass splittings, ^{16}\text{O} + ^{40}\text{Ca}, ^{20}\text{Ne} + ^{36}\text{Ar}, ^{24}\text{Mg} + ^{32}\text{S} and
^{28}\text{Si} + ^{28}\text{Si}. There are seven such groups instead of four, since the
mass asymmetric breakup events can be detected in two different ways,
lighter particle or heavier particle in the PSD_L detector. Seven gates
were constructed on the M_R - M_L spectrum defining the regions of the
different mass splittings.
Figure 2.5

A spectrum showing mass of a particle detected in $PSD_L$ vs. the mass of a particle detected in $PSD_R$. For clarity, only cells with more than five counts have been plotted.
If the $M_R$ and $M_L$ values calculated for an event were within one of these gates, the exact masses of the nuclei in that exit channel were assumed, and the energy deposited in each detector was calculated by solving eqs. (2.1) and (2.2) for $E$ in terms of the masses and angles. A Q-value for each such event was then calculated using eq. (2.4), and, finally, a $Q_\text{lab}$ spectrum was created for each mass splitting. These spectra made it, in principle, possible to examine the Q-value distributions and angular distributions for the various two-body final states of the $^{16}O + ^{40}Ca$ reaction. In practice, because of poor coincidence efficiency (see section 2.6) in all but the mass symmetric exit channel, only the $^{28}Si + ^{28}Si$ final state Q-value and angular distributions and the $^{16}O + ^{40}Ca$ elastic scattering angular distribution were useful.
2.4 ELASTIC AND INELASTIC $^{16}O + ^{40}Ca$ SCATTERING

In this section, the Q-value distribution of the $^{40}Ca(^{16}O, ^{28}Si^*)^{28}Si^*$ reaction and the angular distribution of elastic $^{16}O + ^{40}Ca$ scattering will be examined in order to demonstrate the accuracy of the angular and Q-value calibrations. The Q-value spectrum of the $^{40}Ca(^{16}O, ^{28}Si^*)^{28}Si^*$ reaction from a run at a beam energy of 75.6 MeV is shown in Figure 2.6. The elastic peak is evident and is divided between bins with $Q = 0$ and 1 MeV. The width of the peak is in agreement with the expected Q-value resolution. The error in $Q$ due to experimental uncertainties is

$$\Delta Q = \left( \frac{30}{38} \Delta \theta_R^2 + \frac{30}{38} \Delta \theta_L^2 \right)^{1/2}$$

The position resolution of the P series PSD used in this experiment is 0.5 mm, which corresponds to an angular resolution of 0.3° for this experimental setup. The value of $\frac{30}{38}$, obtained by differentiating eq. (2.4), is about 80 MeV/rad for elastic $^{16}O + ^{40}Ca$ scattering, so that $\Delta Q = 0.6$ MeV.

The angular distribution of the events in the elastic peak is shown in Figure 2.7. For comparison, an unnormalized angular distribution calculated using the optical model parameters (parameter set 2a from the work of Vigdor et al. [Vi79]) is also shown in the Figure. Despite the poor counting statistics, the data and the calculated cross section agree very well.
Figure 2.6

The Q-value spectrum for the $^{40}\text{Ca}(^{16}\text{O},^{16}\text{O}^*)^{40}\text{Ca}$ reaction.
Figure 2.7

The measured elastic $^{16}O + ^{40}Ca$ differential cross section is plotted together with the cross section calculated using optical model parameter set 2a of Vigdor et al. [Vi79]. The solid circles represent events where the $^{16}O$ was detected in PSD$_L$, the open circles represent events where the $^{16}O$ was detected in PSD$_R$, and the solid line is the optical model calculation.
The elastic $^{16}\text{O} + ^{40}\text{Ca}$ scattering yields in the PSDs were normalized in the following way. The experimental center-of-mass differential cross section is given by

$$\frac{d\sigma}{d\Omega_{\text{el}}} (\theta_{\text{cm}}) = 2.67 \frac{q_b}{q_{\text{tot}}} \cdot \frac{A_t}{t} \cdot \frac{Y(\theta_{\text{lab}})}{\Delta \Omega(\theta_{\text{lab}})} \cdot \text{JAC}(E_{\text{beam}}, \theta_{\text{lab}}, \theta_{\text{cm}})$$

(2.7)

where $q_b$ is the mean charge state of the beam particles after passing through the target, $q_{\text{tot}}$ is the total collected charge in the Faraday cup in $\mu$C, $A_t$ is the target atomic number in u, $t$ is the target thickness in $\mu$m/cm$^2$, $\Delta \Omega$ is the solid angle, $Y$ is the measured yield and JAC is the Jacobian that connects the laboratory and center of mass frames. The factor $q_b/q_{\text{tot}}$ can be expressed by using an equation analogous to eq. (2.7) in terms of the elastic $^{16}\text{O} + ^{40}\text{Ca}$ yield in the 15° monitors, the known elastic cross sections and the monitor solid angles. Substituting this expression for $q_b/q_{\text{tot}}$ into eq. (2.7) yields

$$\frac{d\sigma}{d\Omega_{\text{el}}} (\theta_{\text{cm}}) = \frac{d\sigma}{d\Omega_{\text{M}}} (\theta_{\text{lab}}) \cdot \frac{\Delta \Omega_{\text{M}}}{\Delta \Omega(\theta_{\text{lab}})} \cdot \frac{Y(\theta_{\text{lab}})}{\text{MON}} \cdot \text{JAC}(E_{\text{lab}}, \theta_{\text{cm}}, \theta_{\text{cm}})$$

(2.8)

where $d\sigma/d\Omega_{\text{M}}$ is the laboratory 15° elastic $^{16}\text{O} + ^{40}\text{Ca}$ cross section (calculated using parameter set 2a from Vigdor et al. [V179]), $\Delta \Omega_{\text{M}}$ is the solid angle of the monitor, and MON is defined by eq. (2.5)).
2.5 MASS DISTRIBUTIONS

The error in the calculated mass values, \( \Delta M \), can be computed from an equation analogous to eq. (2.6) using eqs. (2.1) and (2.2) to obtain the partial derivatives with respect to \( \theta_L, \theta_R, E_L \), and \( E_R \). The largest contribution to \( \Delta M \) comes from the error in the energy measurement in the PSD which is estimated to be 3% for heavy ions. This spread includes the effects of the nonlinearities in the energy measurement as a function of position in the detector. A typical uncertainty in the extracted value of \( M \) for events of interest is approximately \( \pm 0.8 \) u or 2 u FWHM.

Figure 2.8 shows a typical \( M_{\text{tot}} = M_R + M_L \) spectrum. It is strongly peaked about \( M_{\text{tot}} = 56 \) u and has a FWHM of 6 u. The spread of the \( M_{\text{tot}} \) peak is consistent with the experimental uncertainties in determining \( M_R \) and \( M_L \) and the error propagated when they are summed together. The distribution of masses along the \( M_R + M_L = 56 \) u line in the \( M_R - M_L \) spectrum is shown in Figure 2.9. This plot was obtained by constructing gates centered on the even masses and with a range of about 2 u. The main feature of the data, the enhancement of the yields of 4N nuclei is evident, showing that two-body exit channels in the scattering of \(^{16}\text{O} + ^{40}\text{Ca} \) at large angles are dominated by final states with two "\( \alpha \)-like" nuclei.
Figure 2.8

A spectrum of $M_{\text{tot}} = M_R + M_L$ values for events not rejected by the gate in the $M_{\text{tot}} - E_{\text{tot}}$ spectrum (see text).
Figure 2.9

The distribution of masses along the $M_{\text{tot}} = 56$ u line detected in PSDR.
2.6 RESULTS IN THE $^{28}$Si + $^{28}$Si EXIT CHANNEL

The experimental arrangement was designed to maximize the coincidence efficiency for $Q = 0$ events in the $^{28}$Si + $^{28}$Si exit channel. The coincidence efficiencies for the mass asymmetric breakup channels were consequently very low, and no reliable angular and Q-value distributions could be extracted. The detection efficiency for $^{28}$Si + $^{28}$Si final state events over the entire angular range of PSD$_L$ was 100% only for events with $Q > -8$ MeV and then dropped linearly with decreasing Q-value down to 55%, if a $1/\sin(\theta_{\text{cm}})$ angular distribution is assumed for events with $Q = -20$ MeV. Therefore, only the $^{40}$Ca($^{16}$O,$^{28}$Si$^*$)$^{28}$Si$^*$ reaction data were analyzed in detail.

2.6.1 Angular Distributions and Q-Value Distributions

The angular distribution for $85^\circ < \theta_{\text{cm}} < 97^\circ$, from a run at a beam energy of 75.3 MeV, is shown in Figure 2.10. The detection efficiency drops off rapidly for $\theta_{\text{cm}} < 85^\circ$ and $\theta_{\text{cm}} > 97^\circ$, so that only a very limited angular range can be examined. Within this range, the angular distribution is consistent with a $1/\sin(\theta_{\text{cm}})$ behavior, characteristic of an object whose lifetime greatly exceeds its rotational period. The small range of the angular distribution data makes a more quantitative analysis impossible.

The Q-value distribution from the same run is shown in Figure 2.11. No efficiency corrections have been applied, but the general features of the data can still be examined. The Q-value distribution is
The angular distribution from the $^{40}\text{Ca} \left(^{16}\text{O}, ^{28}\text{Si}^*\right)^{28}\text{Si}^*$ reaction for events with $-20 < Q < 0$ MeV at a beam energy of 75.3 MeV. The data points have been only plotted for the angles within a region of 100% coincidence efficiency. The dashed line is the average value of the cross section in this angular region.
Figure 2.11

Q-value distribution from the $^{40}\text{Ca}(^{16}\text{O},^{28}\text{Si}^*)^{28}\text{Si}^*$ reaction at a beam energy of 75.3 MeV. The solid line shows the coincidence efficiency as a function of Q-value, assuming that the differential cross section is proportional to $1/\sin(\theta_{\text{cm}})$. 
peaked around $Q = -15$ MeV and is roughly symmetrical about the centroid with a FWHM of about 8-10 MeV. The large negative $Q$-value of the peak of the distribution indicates that the $^{40}\text{Ca}^{(16}_0,28\text{Si}^*)28\text{Si}^*$ reaction is a strongly damped process. Just as in the case of the angular distributions of a more quantitative analysis requires more extensive data.

The shape of the $Q$-value spectrum can also be used to rule out the possibility of a substantial part of the yield in the $^{28}\text{Si} + 28\text{Si}$ exit channel being missed because one of the $^{28}\text{Si}$'s $\alpha$-decayed to $^{24}\text{Mg}$. The effective threshold for $\alpha$-decay of $^{28}\text{Si}$, $E_{th}$ is given by

$$E_{th} = BE(28\text{Si}) - BE(24\text{Mg}) - BE(\alpha) + E_c$$

(2.9)

where $BE$ is the nuclear binding energy and $E_c$ is the Coulomb energy of spherical, touching $^{24}\text{Mg}$ and $^4\text{He}$ nuclei. Note that the inclusion of an angular momentum barrier would make $E_{th}$ even larger. Evaluation of eq. (2.9) gives $T = 16.1$ MeV. The energy available for excitations, $E^*$, of the $^{28}\text{Si}$ nuclei is given by

$$E^* = Q + BE(16_0) + BE(40\text{Ca}) - 2BE(28\text{Si})$$

(2.10)

or $E^* = Q + 3.4$ MeV. If the excitation energy is shared in proportion to the fragment masses or by assuming equal nuclear temperatures, then
the excitation energies of the two $^{28}\text{Si}$ will be equal. In that case, the Q-value required to produce 16.1 MeV of excitation energy in the $^{28}\text{Si}$ is -28 MeV. The symmetrical shape of the Q-value spectrum centered on $Q = -15$ MeV indicates that there are very few events with Q-values more negative than -28 MeV. The energy sharing may not be strictly equal; for events with $Q > -20$ MeV, though, the excitation energy would have to be shared in ratio of 2:1 before one of the $^{28}\text{Si}$ had enough excitation energy to α-decay; this is unlikely, compared to the expected 1:1 sharing.

2.6.2 Normalization of Data

The normalization of the $^{28}\text{Si}$ yields from runs at different energies, used to construct the $^{40}\text{Ca}(^{16}\text{O},^{28}\text{Si}^*)^{28}\text{Si}^*$ reaction excitation function was done in two steps. The first step was to obtain the relative energy-to-energy normalization, and the second was to fix the absolute cross section scale by calculating the total cross section for a single run. The absolute cross section scale was obtained for a run at a beam energy of 76 MeV. The experimental center of mass differential cross section, $d\sigma/d\Omega$, was calculated using an equation analogous to eq. (2.8) and summed over all possible Q-value and $\theta_{\text{lab}}$ combinations in the Q-θ$_{\text{lab}}$ spectrum. The differential cross section was assumed to be proportional to $1/\sin(\theta_{\text{cm}})$ at all angles

$$\frac{d\sigma}{d\Omega} = \frac{A}{\sin \theta_{\text{cm}}} \quad (2.11)$$
so that the value of A was determined by fitting in the region of 100% coincidence efficiency for events with \( Q > -20 \) MeV (the restricted angular range \( 85^\circ < \theta_{\text{cm}} < 97^\circ \)). The total cross section \( \sigma_{\text{tot}} \) was then obtained by integration eq. (2.9)

\[
\sigma_{\text{tot}} = 2\pi A. \tag{2.12}
\]

The uncertainty of this procedure is estimated to be about 25%. Most of the error comes from uncertainties in the solid angle values and in the \( A/\sin(\theta_{\text{cm}}) \) function fit to a small set of data points, each of which has a large error associated with it. If the cross section is more strongly forward peaked than \( 1/\sin(\theta_{\text{cm}}) \), this method of calculating \( \sigma_{\text{tot}} \) will underestimate its true value. The total cross section at a beam energy \( E_b \) was calculated relative to the total cross section of the run at \( E_b = E_0 = 76 \) MeV using

\[
\sigma_{\text{tot}}(E_b) = \sigma_{\text{tot}}(E_0) \frac{Y(E_b)/\text{MON}(E_b)}{Y(E_0)/\text{MON}(E_0)} \tag{2.13}
\]

where \( Y \) is the yield for events with \(-20 < Q < 0 \) MeV and \( \text{MON} \) is defined by eq. (2.5).
2.6.3 Excitation Function

The measured $^{40}\text{Ca}(^{16}\text{O},^{28}\text{Si}^*)^{28}\text{Si}^*$ reaction excitation function for events with $-20 < Q < 0$ MeV is shown together with the $^{28}\text{Si}(^{28}\text{Si},^{28}\text{Si}^*)^{28}\text{Si}^*$ reaction excitation function for events with $Q > -22$ MeV in Figure 2.12. There are no structures in the excitation function at a level above 5%. The experimental errors of the $^{16}\text{O} + ^{40}\text{Ca}$ data are about 3.5%, and structures at a level below 5% cannot be ruled out. In fact, there is some evidence suggesting the existence of structures at energies $E_{\text{cm}} = 68.2, 68.5, 69.3$ and 70.2 MeV. In the $^{28}\text{Si} + ^{28}\text{Si}$ data, there are prominent structures near these energies. Therefore, a more detailed study of the $^{40}\text{Ca}(^{16}\text{O},^{28}\text{Si}^*)^{28}\text{Si}^*$ reaction is needed to resolve the question of whether or not there are structures at these energies which would provide evidence for the existence of shape isomers.

The improved measurement of the $^{40}\text{Ca}(^{16}\text{O},^{28}\text{Si}^*)^{28}\text{Si}^*$ reaction excitation function must have better counting statistics and, if practical, a smaller energy step size. Improvements in the measurement of angular and Q-value distributions are also needed for a better understanding of the mechanism of the $^{40}\text{Ca}(^{16}\text{O},^{28}\text{Si}^*)^{28}\text{Si}^*$ reaction. The angular distributions measured in the present experiment have an extremely limited angular range, and an extension of this range is necessary to learn the true shape of the angular distribution. If the distribution has a shape proportional to $1/\sin(\theta_{\text{cm}})$, this would indicate the existence of a long-lived, high-spin intermediate state (for a more detailed discussion, see section 5.3). A more forward peaked component of the distribution, if one were seen, would indicate
The excitation functions of the $^{40}\text{Ca}(^{16}\text{O},^{28}\text{Si}^*)^{28}\text{Si}^*$ and $^{28}\text{Si}(^{28}\text{Si},^{28}\text{Si}^*)^{28}\text{Si}^*$ reactions plotted as a function of the excitation energy of the compound nucleus $^{56}\text{Ni}$.
that a rapid direct reaction contributes to the cross section as well.

The Q-value spectrum can also be used to extract information about the dynamics of the reaction. If the mean Q-value does not change with the center of mass frame angle, this indicates that the intermediate state of the reaction has a large lifetime compared with the time that it takes to damp out the energy of relative motion. In addition, the mean Q-value may be related to the shapes of the \(^{28}\text{Si}\) nuclei at the moment of scission (see section 5.5).

In summary, although the coincidence experiment was inconclusive with respect to the question of structure in the \(^{40}\text{Ca}(^{16}\text{O},^{28}\text{Si}^*)^{28}\text{Si}^*\) reaction excitation function and was an unsatisfactory way of obtaining angular and Q-value distributions, it yielded useful information by defining how a better experiment might be done. On the basis of the results of the coincidence experiment described in this Chapter, it is clear from the concentration of yield in the \(4N\) (\(\alpha\)-like) nuclei and from the Q-value spectra that a singles \(\text{Si}\) yield would be dominated by the yield from the \(^{40}\text{Ca}(^{16}\text{O},^{28}\text{Si}^*)^{28}\text{Si}^*\) reaction for Q-values greater than \(-20\) MeV. Therefore, a detector was designed and built (see Chapter 3) in order to carry out an improved measurement with better angle and Q-value information. This measurement (described in Chapter 4) was a singles experiment, but in which the detected ions were identified by their nuclear charge.
CHAPTER 3

GAS IONIZATION DETECTOR

3.1 DETECTOR DESCRIPTION

In order to carry out an improved measurement of the excitation function of the \( ^{40}\text{Ca}(^{16}\text{O},^{28}\text{Si}^*)^{28}\text{Si}^* \) reaction, we designed and built a gas ionization counter capable of measuring the energy, the rate of energy loss and the incident angle of motion of the detected ions. This detector was based on earlier designs of gas counters for use in focal planes of magnetic spectrometers [Sh75, Er76] and was intended as an all-purpose, heavy-ion detector to be used in the 30" Ortec scattering chamber. The construction, the principles of operation and the performance of the counter will be described below. A schematic diagram of the detector is shown in Figure 3.1.

Incident ions enter through a thin polypropylene window (≈ 80 \( \mu \text{g/cm}^2 \)). Along their paths, the particles ionize gas molecules in the region between the cathode and the first grid producing electron-ion pairs. The electrons are swept toward the anode by the applied electric field with typical drift velocities on the order of 1 cm/\( \mu \text{sec} \), while the much heavier positive ions move toward the cathode with drift velocities of the order of 1 cm/msec.

The energy of the incident ion is measured using the double grid technique of Fulbright [Sh75] where the cathode and the first grid, \( G_1 \), are capacitively coupled and form a single element with respect to AC signals. This assembly can be regarded as a Faraday cage, and as long
Figure 3.1

a) A schematic side view of the detector.

b) Top view of the detector.

c) Front view of the detector.
as the electrons are between the cathode and G1, no signal is generated. A signal begins to develop once the electrons drift past G1; after they have drifted past the second grid, G2, the charge induced on the cage is proportional to the number of positive ions left in the cage. The positive ion drift velocities are smaller than the electron velocities by a factor of 1,000 so that during the time it takes to remove the electrons, the positive ions do not move appreciably. As long as there are no large recombination effects, the induced signal is proportional to the number of electron-ion pairs created by the incident particle which corresponds to the energy that it deposited in the gas. The positive ions drift to the cathode within milliseconds and are collected with no deleterious effect on the fast signal.

Until the electrons pass G2, the anode is electrically shielded from them by that grid. Once they pass G2, however, the electrons induce a positive charge on the segmented anode. A 1.0 cm wide anode segment near the front of the counter is used to provide the energy loss, ΔE, information. The E_res segment of the anode is used only to maintain uniform electric fields in the counter. The electrons swept near the position wire located at the front of the counter are used to obtain the incident particle's angle of motion. The wire is held at a high positive potential with respect to its housing, so that near the wire surface the electric field is high enough to accelerate these electrons to energies large enough to ionize gas molecules, thus producing more electrons. In this way, an avalanche is formed at a spot on the wire directly above the ionization track. A charge division technique is then used to determine the position of this spot which, together with the location of the beam spot on the target, determines
the angle of motion of the incident particle. A thin anode segment between
the wire housing and the ΔE strip, the guard strip, is used to
shield the ΔE strip from stray electric field associated with the wire assembly.

A continuous gas flow system is used with this counter. The gas flows in at the back and is pumped out at the front, in order to increase the rate of exchange of the gas in all parts of the counter. Three gases were tested in the counter: isobutane, methane and CF₄. Isobutane was found to give the best energy resolution.
3.2 CONSTRUCTION AND ELECTRICAL CONNECTIONS

The detector is enclosed in a 1/8" stainless steel box. The entrance window is 9.23 cm long and 1 cm high, although during normal use the entrance aperture width is reduced to approximately 6 cm by a mask mounted on the front plate. The active region of the counter between the cathode and G1 is 20 cm deep and 3.15 cm high. The distances between G1 and G2 and between G2 and the anode are both 0.5 cm.

The grids were wound by hand with a 1 mm spacing over aluminum frames, using 0.004" Nichrome wire. After winding, the grids were transferred to grooved, trapezoidal shaped frames that fit in the counter. The transfer was accomplished by gluing the wires to the new frame with epoxy and cutting off all excess wire. Conducting paint was applied to ensure electrical contact between the frame and the wires.

Two Teflon pieces (approximately 1" x 1" x 8.5") are the interior framework of the counter. They are screwed into the back plate and extend along the entire length of the detector. The cathode, an aluminum plate, is mounted on the bottom of the Teflon support pieces; G1 is attached to the top. Teflon spacers separate G1 from G2 and G2 from the anode. The anode is made of circuit board. The ΔE strip, the guard strip and the $E_{res}$ segment are defined by etching on the lower side; the upper side was etched out to provide the layout for the counter's electrical circuitry.

The position wire housing is an aluminum cylinder with a 1.2 cm outside and 0.8 cm inside diameter. A section about 4mm wide has been removed along the entire length of the tube to allow the electrons to
drift inside. A high resistivity (2,000 Ohms/ft), 0.0006" diameter wire is stretched along the central axis of the tube. At both ends the wire is threaded through a hole in a copper rod and soldered in place. The rods are held in place in the wire housing and electrically isolated from it by nylon spacers. There is an electrical connection at each end of the wire so that signals from each end can be read out.

The interior surface of the side walls of the detector, except next to the entrance window, is lined with circuit board, etched into five horizontal 3 mm wide strips. These strips carry voltages progressing from just above the cathode voltage on the lowest strip to just below the Gi voltage on the highest. The purpose of this arrangement is to minimize the electric field edge effects near the walls of the counter, and its effect is to improve the resolution of the energy and position signals.

The entrance window is normally a stretched polypropylene foil, although 0.1 mil mylar foils are used when high gas pressures are required. The polypropylene is obtained commercially as 1 mil foil; it is then stretched by clamping a piece of it in a circular frame which is then pulled over the open end of a 6" diameter metal cylinder. The edge of the cylinder was gently and smoothly rounded and was also covered by a nylon stocking to help prevent the foil from tearing during the stretching. Near the end of the process the foil, initially transparent, becomes highly colored, changing progressively from green to blue to red as it becomes thinner. The foil does not stretch uniformly, so that different colors appear in different parts of the foil. In practice it was found that only the foils which are predominantly blue with some red areas are sufficiently thin and uniform to be used for the
windows.

The thickness of one such foil was measured using an α-gauge [Ba65]. Four sections of the foil were measured, and the results ranged from 70 to 85 µg/cm², a variation of 20%. A polypropylene foil of this thickness is not strong enough to withstand a gas pressure of even 30 torr, and therefore, it must be supported by wires on the vacuum side of the entrance aperture. The 0.004" diameter wire is used and the spacing between these wires is normally 2 mm, thus giving 96% transmission. Even with these support wires, the limiting pressure is about 80 torr; when higher pressures are needed, 0.1 mil mylar foils must be used. The 0.1 mil mylar foils were tested to 250 torr in the counter without showing any ill effects.

The circuit diagram of the detector is shown in Figure 3.2. All capacitors labeled C₁ are necessary to short to ground large transient signals, presumably caused by sparking inside the counter. This prevents the buildup of charge on the various electrodes and eliminates the resulting cross talk that was a serious problem in an early version of the counter. In addition, these capacitors and the 122 MΩ resistance between the ΔE strip and the power supply effectively isolate the ΔE signal from large AC signals that might originate, for example, on the position wire housing and then reach the ΔE strip through the anode power supply.

The energy resolution of the detector is sensitive to the value of the coupling capacitance of the cathode and G1. The resolution improves with increasing capacitance because, as the cathode-G1 coupling capacitance increases, the relative importance of the stray capacitances
Figure 3.2

Circuit diagram for the detector. The capacitances $C_1$ and 0.1 μf, $C_2$ is 0.25 μf; the resistance $R$ is 22 MΩ, $R_2$ is 100 MΩ and $R_3$ is 200 MΩ.
in the system (particularly the capacitance between the cathode and the enclosing box) decreases. The capacitance used in the final counter configuration, 0.25 µf, is the largest that could be incorporated into the detector. Undoubtedly, the energy resolution can be further improved if high capacitance, higher voltage condensers which are smaller in size can be found to replace the present ones. The 200 MΩ resistor keeps the G1 DC level at ground while forcing any AC signal across the capacitors coupling the cathode and G1 and then to the input of the preamplifier.
3.3 GRIDS

In a parallel plate ionization chamber, the signal size on the anode depends on the position and orientation of the ionization rack in the chamber. It is easy to show that the effective charge, $q_{\text{eff}}$, transported across the chamber as an electron, moves to the anode is

$$q_{\text{eff}} = ex/D \tag{3.1}$$

where $D$ is the distance between the electrodes, $x$ the distance from the point on the ionization track where the electron-ion pair is created to the anode, and $e$ is the charge of the electron. A similar result applies to the positive ion generated signal, hence the total charge transported by an ion pair is $e$. This unwanted $x$ dependence of the fast electron generated signal may be eliminated by the use of Frisch grid [Fr44].

The grid is located between the cathode and anode, and it approximates the effect of a conducting plane. It shields the anode from electrical activity on the cathode side of the grid. With the grid in place, the $x$ term in eq. (3.1) is equal to the anode-grid distance for all electrons regardless of their point of origin in the volume between the grid and cathode. This effectively eliminates any postion dependence of the anode signal.

The two important parameters of a grid are the shielding efficiency, $\sigma$, and the transmission, $\lambda$. Buneman et al. [Bu49] derived formulas to calculate these quantities for the parallel plate geometry. The electrical shielding efficiency, $\sigma$, is given by
\( \sigma = 1 - \frac{d}{1\pi r} \ln \left( \frac{d}{2\pi r} \right) \)  \hspace{1cm} (3.2)

where \( d \) is the grid wire spacing, \( r \) is the wire radius and \( p \) the grid-anode distance. In this counter, \( \sigma = 0.96 \), that is the grids are 96% effective in shielding the anode from having signals induced on it by the motion of electrons below \( G2 \) and in shielding the \( G1 \)-cathode assembly from effects of electron motion above \( G2 \).

The transmission of the grid is the percentage of the electric field lines that bypass the grid. This is roughly equal to the percentage of the electrons that make it through the grid, as the electrons diffuse along the field lines [Bu49]. Provided that

\[
\frac{1 - \rho}{1 + \rho} < \frac{E_p}{E_Q} < \frac{1 + \rho}{1 - \rho} \hspace{1cm} (3.3)
\]

where \( E_Q \) is the electric field between the cathode and the grid, and \( E_p \) is the electric field between the grid and the anode and \( \rho = 2\pi r/d \), the transmission, \( \lambda \), is given by [Bu49]

\[
\lambda = \frac{E_p - E_Q}{\pi E_Q} \left\{ (C-1)^{1/2} - \cos^{-1}(1/C) \right\} \hspace{1cm} (3.4)
\]

where

\[
C = \frac{E_p + E_Q}{E_p - E_Q} \cdot \rho \hspace{1cm} (3.5)
\]

Condition 3.3 was satisfied in all the work done with the counter. During the measurement of the \(^{40}\text{Ca}(^{16}\text{O},^{28}\text{Si}^*)^{28}\text{Si}^* \) reaction excitation
function, the following voltages were applied to the electrodes—
cathode: -800V; Gl: 0V; G2: +200V; and anode: +500V. The
transmission of G1, with G2 acting as the anode, is 99.3% with this
configuration. The transmission of G2, with G1 acting as the cathode is
98.7%, so that the total transmission is 98.1%. 
3.4 ENERGY AND ΔE SIGNALS

The energy resolution of the counter was determined by using only a small angular range of the counter to measure the energy spectrum produced by elastic scattering of a 30 MeV $^{28}\text{Si}$ beam from a thin Au target. In this case, the kinematic spread in the scattered particle energy was about 200 keV. The spread caused by the electronic noise of the counter and the associated electronics (measured by injecting a pulser signal between the detector and preamplifier) was 860 keV (FWHM). The total spread caused by these two effects, obtained by adding them in quadrature, is 880 keV (FWHM), while the actually observed resolution was 1.6 MeV (FWHM). The largest contributions to the remaining spread in the energy measurement are probably from inadequate grid shielding and form the loss of induced signals because of the incompleteness of the Faraday cage formed by the cathode and G1. This energy resolution, of the order of 3-5% (FWHM), was entirely adequate for the $^{16}\text{O} + ^{40}\text{Ca}$ experiment. The resolution can be further improved if necessary by increasing the coupling capacitance between the cathode and G1.

The ΔE signal is collected by the 1.0 cm wide strip which is one of the anode segments. This signal must be corrected for the incident particle's angle of motion, $\theta$, since the length of the ionization track under the E strip depends on that angle, $\ell = 1.0 / \cos(\theta - \theta_c)$, where $\theta_c$ is the angle that the line connecting the beam spot on the target with the center of the counter entrance aperture makes with the beam axis. The nuclear charge resolution of the ΔE signal is sufficient to resolve Si from Al and S so that it is at least 7%.
3.5 POSITION SIGNAL

The electric field inside the cylindrical geometry of the position wire housing is proportional to $1/r$, where $r$ is the distance from the central axis. The position wire is located along that axis, and since its diameter is very small, 0.0006", the field near the wire is very large. As the electrons drift up from the ionization track into the housing opening, they are accelerated toward the wire. Near the wire, the electric field is large enough to give the electrons enough energy to ionize gas molecules; typical energies necessary are approximately 20 eV. The electrons released form these molecules in turn ionize other molecules, and an avalanche develops. Large amplification of the number of electrons reaching the wire (up to a factor of $10^8$) can be obtained this way [Sa77].

Once the charge from the avalanche is deposited on a spot on the wire above the particle's track, the resistance to either side of the wire determines how much of the charge will flow to that end of the wire. The resistivity of the wire is uniform so that the amount of charge moving toward each end is proportional to the distance between the avalanche spot and that end of the wire. By forming the quotient of the signal from one side to the sum of the signals from both sides, the position of the avalanche spot and, consequently, the particle's track can be determined.

The position wire resolution was measured by mounting a mask, with 0.22 cm slits separated by 0.18 cm distances across the front plate of the counter and scattering a 30 MeV beam of $^{28}\text{Si}$ off a thin Au target. The yield in two of the slits and a fit which assumes that the
uncertainty in the resolution is described by a Gaussian function with a standard deviation of 0.04 cm (FWHM = 0.1 cm) is shown in Figure 3.3. This linear resolution corresponds to an angular resolution of 0.5°, since the detector is designed so that the position wire is about 12.3 cm from the target.

Two of the main contributions to the spread in position resolution are expected to be a) the multiple scattering in the window and gas, and b) the electronic noise. The multiple scattering angle (in radians) can be calculated [C169] and is given by

\[ \Delta \theta_{\text{FWHM}} = 1.26 \times 10^{-7} \frac{Z_1(Z_1 + 1)Z_2^2 t}{E^2 M} \ln \left( \frac{212t(Z_1 + 1)}{MZ_1(Z_1^{2/3} + Z_2^{2/3})} \right) \]

where \( Z_1 \) is the atomic number of the target material, \( Z_2 \) is the average charge state of the projectile in the target, \( t \) is the target thickness in \( \mu g/cm^2 \), \( M \) is the target mass in amu, and \( E \) is the projectile energy in MeV. For compounds, \( Z \) is given by

\[ Z = \frac{1}{N} \sum_{i=1}^{N} Z_i^2 \]

where \( Z_i \) is the atomic number of the constituent atoms of the molecule. \( Z_2 \) is calculated using an expression derived by Shima et al. [Sh82]. For a 30 MeV \( ^{28} \text{Si} \) ion going through an 80 \( \mu g/cm^2 \) polypropylene window plus 1 cm of gas (30 torr of isobutane) before it gets to the
Figure 3.3

The solid line is a part of a position spectrum generated with the mask mounted on the entrance aperture. The dashed line is the fit, described in the text, that assumes a 0.1 cm FWHM resolution for the position wire.
position wire, the calculated $\Delta e^2_{\text{FWHM}}$ corresponds, in this geometry, to a spread in the position measurement at the wire with a FWHM of 0.04 cm. The electronic noise contribution was measured using a pulser and corresponds to a $\delta_{\text{FWHM}}$ of 0.05 cm. An additional source of error is the finite size of the beam spot on the target. This effect, estimated using the size of the beam spot on the target (see section 4.3) corresponds to a $\delta_{\text{FWHM}}$ of 0.05 cm. The total width from these sources, added in quadrature, is 0.08 cm or 0.4°.
4.1 MOTIVATION

The results from the coincidence experiment done at BNL suggest that there is intermediate structure in the $^{40}\text{Ca}(^{16}\text{O},^{28}\text{Si}^*)^{28}\text{Si}^*$ excitation function. The experimental limitations made it impossible to rule out, to a high degree of probability, that the observed structure was caused by statistical fluctuations. As indicated in section 2.6, this initial experiment was limited by both counting statistics and by a restrictive Q-value range. An experiment designed to improve on these problem areas was done at the A.W. Wright Nuclear Structure Laboratory (WNSL) at Yale University.
4.2 BEAM AND TARGET

The $^{16}$O beam was provided by the WNSL MP Tandem Van de Graaff accelerator.  O$^-$ ions were extracted from a Middleton type ion source [Mi74] and injected into the Tandem. The source injection energy was about 180 keV. At the Tandem terminal, a 5 $\mu$/cm$^2$ carbon foil was used to strip the electrons from the O$^-$ ion. During the experiment, either a 6$^+$ or a 7$^+$ charge state of oxygen was used, depending on the desired beam energy. After acceleration, the momentum of the beam was analyzed by the 90° bending magnet. The typical beam current during the running of the experiment was calculated using an empirical expression derived by Shima et al. [Sh82] which assumes an equilibrium charge state distribution is reached by the beam particle as it traverses the target. The charge state of the beam estimated from results of elastic scattering into MON1 and MON2 (see section 4.2) was in good agreement with the calculated value.

The energy increment of the excitation function was only 75 keV, so that a degaussing technique [Ov69] was used on the analyzing magnet to ensure that the energy at which each data point was taken was exactly reproducible. The steps of the degaussing procedure are as follows: the magnet is taken to full field for three minutes, then it is taken through zero to a slightly reversed field ($I = -3$ amps) for three minutes, and finally it is brought up to the desired field.

After the completion of the measurement of the $^{40}$Ca($^{16}$O,$^{28}$Si$^*$) excitation function, the 90° bending magnet was calibrated to obtain an absolute beam energy calibration. Elastic $^{12}$C + p scattering has a $3/2^-$ resonance at 14.231 MeV [Le69] that can be used for this purpose. The
NMR frequency at the magnet setting, at which the beam at the resonance energy passes through the analyzing slits, is recorded and can be used to calculate the magnet calibration factor, \( K \), which is given by [Ov69]

\[
K = \frac{ME}{q^2f^2} \left[ 1 + \frac{E}{2Mc^2} \right] \tag{4.1}
\]

where \( M \) is the mass of the beam ion in amu, \( q \) is its charge state, \( E \) is the resonance energy in keV, \( f \) the NMR frequency in MHz and \( c \) is the velocity of light. The value of \( K \), found using \(^{12}\text{C} + p\) scattering was \( 44.201 \pm 0.013 \text{ keV u/MHz}^2 \). Once \( K \) is determined, eq. (4.1) can be solved for \( E \) for arbitrary \( q, f \) and \( M \). At each bombarding energy used during the experiment, the NMR frequency was recorded and later used to calculate the true beam energy.

The target used in the measurement of the \(^{40}\text{Ca}^{(16}_0,^{28}\text{Si}^*)^{28}\text{Si}^*\) reaction was made by the decomposition of \( \text{CaCO}_3 \) and the subsequent reduction of \( \text{CaO} \) in a tantalum boat [St85]. The \( \text{CaCO}_3 \) was enriched in \(^{40}\text{Ca} \) to the level of 99.9%. The tantalum boat used in the evaporation was a tube with flattened ends and a 7 mm\(^2 \) circular hole on the top.

In order to clean and deoxidize the boat surface before the Ca evaporation was done, the boat was heated to a temperature of 2,150° C, measured with a Leeds and Northrup optical pyrometer, at a pressure below 10\(^{-5} \) torr and held there for approximately three minutes. After the boat cooled off to an ambient temperature, it was removed from the evaporator and 20 mg of \( \text{CaCO}_3 \) was placed into it. The boat and a 10 \( \mu \)g/cm\(^2 \) carbon foil were then mounted in the evaporator. The foil was mounted on the Ortec chamber vacuum target transport device about 9 cm.
above the boat. The transport device was used to move the target from
the evaporator to the Ortec chamber while under vacuum. Pure Ca
oxidizes extremely readily, so that this step is crucial in avoiding an
enormous oxygen build-up on the target.

The carbon foils were initially retracted and shielded from the
tantalum boat. The boat temperature was then gradually raised over a
period of 10 minutes to 1100° C. The pressure in the evaporator
chamber, which was about $2 \times 10^{-6}$ torr at the start of the heating,
would rise as $\text{CO}_2$ was released following the dissociation of $\text{CaCO}_3$ in the
$\text{CaCO}_3 \rightarrow \text{CaO} + \text{CO}_2$ reaction. The increase in temperature was controlled
by the requirement that the pressure in the evaporator be kept below
$2 \times 10^{-5}$ torr during the decomposition of $\text{CaCO}_3$. After all the $\text{CaCO}_3$
decomposed, the pressure fell back to its starting value. The carbon
foil was then moved into place directly over the hole in the boat
surface at a distance of 9 cm, and the temperature was raised to
1,600° C and held there for four minutes. At this temperature, the $\text{CaO}$
reacts with the tantalum via the $5\text{CaO} + 2 \text{Ta} \rightarrow \text{Ta}_2\text{O}_3 + 5\text{Ca}$, oxidizing
the boat surface and evaporating the calcium.

The target used during the production runs was analyzed using a 30
MeV $^{28}\text{Si}$ beam before the start of the experiment. This analysis showed
the target to have the following content: $^{40}\text{Ca} - 28 \mu\text{g/cm}^2$, carbon -
12 $\mu\text{g/cm}^2$, oxygen - 2 $\mu\text{g/cm}^2$ and tantalum - 1.5 $\mu\text{g/cm}^2$. The
identification of tantalum requires a comment. The energy resolution of
the surface barrier detector used in the target analysis was not
sufficient to uniquely identify an element as heavy as tantalum. The
kinematics deduced for the "heavy element" were consistent with
tantalum, and since the evaporation boat was made of that material, it was assumed that the heavy element was in fact tantalum.
4.3 EXPERIMENTAL SETUP

The experiment was performed in the 30" diameter Ortec chamber on the L15 beam line at WNSL. The primary detector was the gas ionization counter described in the previous chapter. This counter measured the total energy, $E$, the rate of energy loss, $\Delta E$, and the entrance position (angle) of the incident ions, $\theta$. The energy can only be measured properly for ions that stopped in the gas. A continuous flow gas system kept the counter filled with 99.5% pure isobutane ($C_4H_{10}$) at a pressure of 30 torr. This pressure was sufficient to stop Si ions at all energies accessible during the experiment. The most energetic ions with $Z < 12$, however, were not stopped in the gas.

The center of the entrance window of the detector was 11.3 cm from the target and at a 45° angle with respect to the beam. A mask mounted on the window limited the aperture to 1 cm in height and 5.7 cm in width. This corresponded to a 31° bite in angle in the laboratory frame, from $30^\circ \leq \theta_{\text{lab}} \leq 60^\circ$.

For best performance, the counter gas must be as free of water vapor and other impurities as possible. Therefore, before flowing through the counter, the isobutane was passed through two heat exchange coils. The first coil was immersed in an ice-water bath to cool the gas and condense out impurities. The second coil, immersed in a water bath, returned the gas to ambient temperature. Isobutane has a high boiling point of -11.7 °C, so that it was not possible to cool the gas to a temperature much lower than 0 °C.

In addition to the gas counter, there were three silicon surface barrier detectors in the Ortec chamber. Two 300 µm deep detectors
served as small angle monitors. MON1 was at 15°, 29 cm from the target with a 1/16" diameter collimator; MON2 was at 25°, 20 cm from the target with a 3/32" diameter collimator. A large area (600 mm²), 60 μm deep detector, MON3, was located 10 cm from the target at 112° and had a 1.062" diameter collimator. This detector monitored the back angle elastic scattering off the tantalum on the target. The entire experimental setup is diagramed in Figure 4.1.

Before the beginning of the \( ^{40}\text{Ca}(^{16}\text{O},^{28}\text{Si})^{28}\text{Si}^* \) reaction excitation function measurement, the gas counter and the monitor detectors were calibrated by scattering a \(^{28}\text{Si}\) beam at energies of 30, 40, 50 and 60 MeV off a thin Au target. The energy and angular calibration of the gas ionization counter was done by placing a mask with slits in front of the entrance window and then following the procedure similar to the one described in section 2.1.2. The \(^{28}\text{Si}\) beam was chosen for this use so that the ions used in calibrating the gas counter would be of the same type and energies as the ions of interest during the experiment.

The beam collimation consisted of two tantalum colimators. The upstream collimator was 51.3 cm from the target and had a 3/32" diameter aperture; the downstream one was 6.5 cm from that target and had a 1/16" diameter, electropolished aperture. The electropolishing process removes most of the imperfections that are found on edges of a machine made hole. A 3/16" diameter antiscatter slit was placed about 1.5 cm from the target. The maximum size of the beam spot allowed by this collimation system was approximately 2 mm in diameter. The actual beam spot on the target was found to be roughly that size.
Figure 4.1

Detector geometry used to measure the $^{40}\text{Ca}(^{16}\text{O},^{28}\text{Si}^*)^{28}\text{Si}^*$ reaction at WNSL.
MON I

MON 2

IONIZATION COUNTER

TARGET

28 $\mu g/cm^2$ $^{40}$Ca
on carbon backing

$^{16}$O BEAM
In order to minimize the carbon build-up, a nitrogen cooled cold trap was mounted around the target. The active area of the trap was a copper cylinder with a vertical hole to allow the target ladder to move and a horizontal cut to allow the beam to strike the target and the scattered particles to reach the detectors. This type of cold trap is very effective in removing residual gases, for example hydrocarbons, that have boiling points above that of nitrogen. The cracking of hydrocarbons by the beam could produce free carbon atoms which can then be swept by the beam onto the target to form a carbon layer on the upstream side of the target. This layer degrades the beam energy, and since the energy step size in this experiment was only 75 keV, it was important to keep the carbon build-up to a minimum. The gas used in the counter was a hydrocarbon, so that great care was taken to ensure that there was no leak from the counter. At the pressure of $4 \times 10^{-7}$ torr, which was held in the Ortec chamber during the experiment, no evidence of a leak was seen except when the detector window failed, as described below.

The subsequent data analysis was greatly complicated by the repeated failure of the polypropylene entrance windows of the gas ionization counter. After about a day of data taking, a window would develop a leak which would grow worse rapidly and necessitate mounting a new window. Inspection of the windows indicated that the failure was probably a consequence of radiation damage. During the experiment, five different windows were used for the counter. Each window foil had a slightly different thickness, and this resulted in differences in calibration and in slightly different conditions inside the counter. The data were therefore divided into five groups of runs corresponding
to the different windows. Each group had slightly different calibrations and were analyzed separately. There were many overlap points between the five sets of runs, and they were used to provide proper relative normalizations.

The cause of the window problems was probably the bombardment by electrons which are copiously produced during beam-target collisions. When the fifth window was mounted, four strong, rare earth magnets were installed in the two dipole configurations, designed to bend electron trajectories away from the window. After the experiment was over that window was examined, and no signs of radiation damage were found.
4.4 DATA ACQUISITION

Data acquisition was a multistep process. The signals generated by the detectors were first processed by an ensemble of NIM electronic modules (the electronics diagram is shown in Figure 4.2). The resulting signals were sent to the inputs of the analog to digital converters, (ADCs) and scalers. ADC and scaler outputs, in turn, were sent to the IBM 4341 computer by the Front End, an interfacing device. Once received by the computer, the data were handled and analyzed by the data acquisition program. The program caused the incoming data to be written on magnetic tape for later off-line replay and also stored the data on a disk so that it could be examined on-line.

The NIM electronic system generates two types of signals: data signals and event triggers. The data signals are pulses with voltage levels proportional to the charge of the pulses generated by the detectors; they are the inputs to the ADCs. Typical preparation of a data signal consists of three stages: amplification, timing adjustment and pulse shaping. An example of these stages is the monitor detector circuit shown in Figure 4.2. The preamplifier and the amplifier amplify the original detector signal; the delay amplifier adjusts the relative timing between the data signal and the event trigger (described below) and possibly other data signals (for a full description of timing requirements see [Nu67]). Finally, the linear gate stretcher shapes the data signal so that it is compatible with the requirements of the ADC.

The event triggers are pulses with a fixed voltage level, independent of the detector generated voltage levels, provided that they are above a threshold set by the experimenter. Event trigger signals
Figure 4.2

Schematic diagram of the electronics.
GAS IONIZATION COUNTER CIRCUIT

TYPICAL MONITOR DETECTOR CIRCUIT
are sent to scalers, where they are counted, and also directly to specific inputs of the Front End. An event trigger received at the Front End causes an instruction to be issued to the computer to read the ADCs and scalers associated with that event. For example, if the gas counter event trigger is received at the Front End, the contents of the ADCs to which the $E$, $\Delta E$ and the position wire signals, $P_{up}$ and $P_{dn}$, are sent, will be read into the computer.

The three silicon surface barrier detectors, MON1, MON2 and MON3 had a single 1024 channel ADC recording the energy deposited in the detector associated with their events. The gas ionization counter had four 4096 channel ADCs and one 1024 channel ADC associated with its event. $E$, $\Delta E$, $P_{up}$ and $P_{dn}$ data signals were sent to the 4096 channel ADCs while the pulser flag signal (see section 5.4.1) was sent to the 1024 channel one. A coincidence of the $E$ and $\Delta E$ signals defined the event trigger.

A scaler event had 11 scalers associated with it. The scalers were: the beam current integrator, dead time scalers for the MON1, MON2, MON3 and gas counter events, scalers counting the number of event triggers for the different events and a 10 Hz timer. The event trigger for this event was sent to the Front End every time that the charge collected on the Faraday cup increased by 0.16 $\mu$C, roughly once a second.

The performance of the gas counter depends on the density of the gas in the detector, and therefore on the temperature and pressure. Those two parameters, measured every 10 seconds, were the data signals of the last event. The temperature was measured by a thermocouple...
located inside the counter and the pressure by a digital pressure transducer located in the gas line just before the entrance to the counter. The signals were sent to a 1024 channel ADC. No significant changes in either parameter were detected during the experiment.
4.5 DATA ANALYSIS

4.5.1 Pulser Events and Gain Shifts

The pulser generated events provided an important, continuous calibration of all the devices, NIM electronics and ADCs associated with the gas counter event. About once a second, pulser signals of known amplitude were injected into the circuit between the detector and the preamplifier (see Figure 4.2). Thus, the pulser signals were processed by the system as if they were signals from the detector. The pulser flag, a nonzero signal sent to the 1024 channel pulser ADC (one of the five ADCs associated with the gas counter event), distinguished these events from true gas counter events.

Gain shifts in the electronic NIM modules and ADCs can be corrected for by using the pulser signals. With no beam on target, the pulser peak was only three to four channels wide in the 4096 channel E, \( \Delta E \), \( P_{\text{up}} \) and \( P_{\text{dn}} \) spectra. When there was beam on target, the electrical activity in the counter caused a high level of electronic noise. As a result, the FWHM of the pulser peaks increased to about 30 channels, and it was no longer possible to distinguish between pulses that were far away from the centroid of the pulser peak because of a gain shift or because of the level of electronic noise in the counter at the time of the event.

Therefore, the following method was used to extract the gain shift information. Every run was divided into ten parts with each part corresponding to 10,000 beam current integrator counts (100 \( \mu \text{C} \) of collected charge). The centroids of the pulser peaks in the \( E \), \( \Delta E \), \( P_{\text{up}} \)
and $P_{dn}$ spectra were calculated for each of the ten parts. The ratio of these centroids to the standard values (the mean values of the centroids for all runs with the same entrance window) was the correction applied to the signals in that part of the run.

The only measureable gain shift that was found was in the Canberra 2020 amplifier used for amplifying the $E$ signal. The gain drifted by up to 3% from run to run and by about as much during some runs. The selection algorithm for Si ions, as will be seen below, was quite sensitive to the errors in the $E$ signal. Thus, without the corrections, a reliable analysis would not have been possible.

4.5.2 Selection and Handling of Si Events

The goal of the analysis was to identify Si ions and, with the assumption that the Si ion was produced in the $^{40}\text{Ca}(^{16}\text{O},^{28}\text{Si}^*)^{28}\text{Si}^*$ reaction, calculate the $Q$-value, $Q$, and the center of mass frame angle, $\theta_{cm}$, for each Si event. The need to extract $Q$ and $\theta_{cm}$ will be made clear in sections 4.6 and 4.8. Although preliminary analysis was done on-line, a complete analysis of the data was done off-line by replaying the accumulated data tapes. The logic of the analysis program is shown schematically in Figure 4.3.

The first step in the analysis was to correct the $E$ signal for the gain shift, as was discussed in section 4.5.1. Next, the angle independent $\Delta E$ signal was calculated. An ion, entering the counter at an angle $\theta$, travels a distance $D(\theta)$ under the $\Delta E$ strip of the anode
Figure 4.3

Flowchart of the off-line data analysis program.
START

GET NEXT EVENT

IS IT A HLC EVENT?

IS IT A PULSER EVENT?

CORRECT FOR GAIN SHIFT

CALCULATE $\Delta E_c$

IS IT A EVENT?

CALCULATE $\theta_{cm}$ and Q

UPDATE $Q$ vs $\theta_{cm}$ ANALYZER

END OF RUN?

END
\[ D(\theta) = D_0 / \cos(\theta - \theta_c) \] (4.2)

where \( \theta_c \) is the angle that the line connecting the beam spot on the target with the center of the detector's aperture makes with respect to the beam axis. If the rate of energy loss of an ion moving under the \( \Delta E \) strip is constant, which is a good approximation since \( D_0 \) is small (=1 cm), then the observed \( \Delta E \) signal is directly proportional to the path length \( D(\theta) \). The quantity actually used in the subsequent analysis is the angle independent \( \Delta E \) signal, \( \Delta E_0 \), given by

\[ \Delta E_0 = \Delta E(\theta) \cos(\theta - \theta_c) \] (4.3)

where \( \Delta E(\theta) \) is the observed \( E \) signal.

An \( E \) vs. \( \Delta E \) plot was generated in the next phase of the analysis (see Figure 4.4). The Bethe-Bloch formula for stopping powers can be approximated by \( E\Delta E \sim Z^2 \), where \( Z \) is the nuclear charge of the ion. Within this approximation, ions with different nuclear charges will fall onto sections of different hyperbolic contours on an \( E \) vs. \( \Delta E \) plot. A free-form gate was drawn around the group identified as Si on an \( E-\Delta E \) plot from one of the \(^{16}O + ^{40}Ca \) runs. The Si location was determined by scattering a \(^{28}Si \) beam off a thin Au target at several beam energies, before the start of the \(^{16}O + ^{40}Ca \) runs. With the gas pressure optimized for Si detection, the higher energy particles with \( Z < 12 \) were not stopped in the active region of the counter, and therefore, less than their total energy was deposited in the gas. This results in the contours for these ions "bending over" at higher energies in the \( E-\Delta E \) plot.
Once the E and ΔE values for an ion are found to fall within the gate in the E-ΔE spectrum, it is assumed to come from the 
\(^{40}\text{Ca}(^{16}\text{O},^{28}\text{Si}^*)^{28}\text{Si}^*\) reaction (this assumption will be examined further in section 4.6). A shift in the E gain will move events into or out of the E-ΔE gate and can have a large effect on the measured \(^{28}\text{Si}\) yield. For example, a 1% shift in E gain can have a nearly 2% effect on the Si yield. Once the mass, energy and angle of motion of one particle of a two-body final state are known, the kinematics of the reaction are completely determined, and \(\theta_{\text{cm}}\) and Q of the reaction may be calculated. These two quantities are given by the following relationships

\[
\theta_{\text{cm}} = \tan^{-1} \left( \frac{\sin \theta_3}{\cos \theta_3 - (V_{\text{cm}}/V_3)} \right) \quad (4.4)
\]

\[
Q = E_3 \left[ 1 + \frac{\sin^2 \theta_3}{\sin^2 \theta_4} \right] - E_{\text{beam}} \quad (4.5)
\]

\[
\theta_4 = \tan^{-1} \left( \frac{\sin \theta_3}{\frac{M_{\text{Si}} E_{\text{beam}}}{M_{\text{Ox}} E_3} - \cos \theta_3} \right) \quad (4.6)
\]

where \(M_{\text{Si}}\) is the mass of silicon, \(M_{\text{Ox}}\) the mass of oxygen, the subscript 3 refers to the detected Si ion and the subscript 4 to the undetected one.
The gas ionization detector was sensitive to Si ions with energies above 10 MeV, however, the E-ΔE gate (see Figure 4.4) cuts out allsilicons with less than about 20 MeV of kinetic energy. Si ions withless than 20 MeV lose most of their energy in the ΔE region and so areindistinguishable from other ions which also lose most of their kineticenergy in that region. This experimental cut-off corresponds to eventswith Q = -40 MeV at the most forward angles and Q = -20 MeV at the mostbackward ones. Therefore, the detector has close to full efficiency atall laboratory angles for detecting $^{28}$Si from events with Q > -20 MeV,but the efficiency for detecting $^{28}$Si from events with -40 < Q < -20 MeVvaries from nearly 100% at $\theta_{\text{lab}} = 30^\circ$ to 0% at $\theta_{\text{lab}} = 60^\circ$. A typical Q-value vs. $\theta_{\text{cm}}$ plot is shown in Figure 4.5. As can be seenfrom this Figure, there is full detection efficiency for events fallingin the rectangle bounded by the lines Q = 0 MeV, Q = -20 MeV $\theta_{\text{cm}} = 55^\circ$and $\theta_{\text{cm}} = 90^\circ$. This feature was used in obtaining the absolute crosssection scale for the $^{40}$Ca($^{16}$O, $^{28}$Si*)$^{28}$Si* reaction (see section 4.8).
Figure 4.4

E-ΔE plot from a run at $E_{\text{beam}} = 75.075$ MeV.

Events with $E$ and $\Delta E$ values corresponding to the region inside the solid line gate were identified as Si.
Figure 4.5

A Q-value vs. $\theta_{\text{cm}}$ plot from a run at $E_{\text{beam}} = 75.075$ MeV.
4.6 SINGLES EXPERIMENT JUSTIFICATION

In the data analysis, two assumptions were made about the detected Si ions. It was assumed that $^{28}\text{Si}$ ions were observed, and it was also assumed that these ions came from a two-body exit channel of the $^{16}\text{O} + ^{40}\text{Ca}$ reaction. Both assumptions were based on the results of the coincidence experiment described in chapter 2, as well as some theoretical considerations. In this section the evidence for both assumptions will be reviewed.

The two-body final state assumption can be tested by comparing the singles Si yield from the present experiment with the coincident Si yield from the BNL study in the angular range common to both experiments, $45^\circ < \theta_{\text{lab}} < 60^\circ$. A large three-body contribution to $^{16}\text{O} + ^{40}\text{Ca}$ scattering with a Si in the final channel would be evidenced by an enhancement of the singles yield over the coincidence yield. Figure 4.6 shows the Q-value spectra for $-20 < Q < 0$ MeV from both experiments, normalized to the elastic $^{16}\text{O} + ^{40}\text{Ca}$ scattering at 15°. The Yale data have been corrected for the slight loss of efficiency in detecting Si ions from events with $Q < -15$ MeV at back angles, while the BNL data have been corrected for coincidence efficiency. Within experimental errors the two spectra are indistinguishable.

Figure 2.9 shows that yields of $\alpha$-like nuclei, $^{16}\text{O}$, $^{20}\text{Ne}$, $^{24}\text{Mg}$, $^{28}\text{Si}$, $^{32}\text{S}$, $^{36}\text{Ar}$ and $^{40}\text{Ca}$, are significantly enhanced over all other nuclei. This suggests that the $Z = 14$ ions from this reaction are $^{28}\text{Si}$ nuclei rather than other isotopes of silicon. A recent calculation, within a framework of a dynamic theory of deep inelastic scattering by Saroha et al. [Sa85a], qualitatively reproduces the observed mass yields.
Figure 4.6

A comparison of the singles and coincidence Si yields with $-20 < Q < 0$ MeV. The solid line is the singles Si yield (WNSL) and the dashed line is the coincidence Si yield (BNL). The beam energy is 75.075 MeV.
(i.e. the preferential production of α-like nuclei). Figure 4.7 shows the calculated mass distribution. It is evident that the mass yields are strongly peaked at the "α-particle" nuclei, in agreement with the data.

4.6.1 The $^{40}\text{Ca}(^{16}\text{O},^{24}\text{Mg}^*)^{32}\text{S}^*$ Reaction

The simplest, most likely contaminant process involves the $^{40}\text{Ca}(^{16}\text{O},^{24}\text{Mg}^*)^{32}\text{S}^*$ reaction followed by α-decay of $^{32}\text{S}$ to $^{28}\text{Si}$ which is subsequently detected by the gas ionization counter. The results of the coincidence experiment rule out a large contribution from this process only for events with $Q > -20$ MeV. A Monte Carlo simulation of the $^{40}\text{Ca}(^{16}\text{O},^{24}\text{Mg}^*)^{32}\text{S}^*$ reaction was therefore carried out to gain insight into the effects for events with even more negative Q-values. A simple model of the $^{40}\text{Ca}(^{16}\text{O},^{24}\text{Mg}^*)^{32}\text{S}^*$ reaction was used to generate the angular distributions and excitation energies of $^{32}\text{S}$ ions. If the excitation energy was above the α-emission threshold, the α-decay of $^{32}\text{S}$ was simulated, and the laboratory frame angle, $\theta_{\text{lab}}$, and energy, $E$, of the daughter $^{28}\text{Si}$ was calculated. For all cases where $\theta_{\text{lab}}$ was such that the $^{28}\text{Si}$ ion would be detected by the gas counter, the apparent Q-value was calculated using the values of $\theta_{\text{lab}}$, $E$, and eqs. (4.5) and (4.6), under the (false) assumption that the ion came from the $^{40}\text{Ca}(^{16}\text{O},^{28}\text{Si}^*)^{28}\text{Si}^*$ reaction. In this way, a Q-value spectrum of contaminant events was obtained.

The $^{40}\text{Ca}(^{16}\text{O},^{24}\text{Mg}^*)^{32}\text{S}^*$ reaction was assumed to have the distribution of Q-values similar to the $^{40}\text{Ca}(^{16}\text{O},^{28}\text{Si}^*)^{28}\text{Si}^*$ reaction (see section 4.7). For ease of computation, the Q-value distribution
Figure 4.7

A comparison of the observed mass yields from $^{16}O + ^{40}Ca$ scattering and the mass yields calculated by Saroha et al. [Sa85]. The upper curve shows the experimental mass distribution; the lower two are calculated distributions corresponding to excitation energies of $^{56}Ni$ of 22.9 MeV ($\Theta = 2$) and 17.3 MeV ($\Theta = 1.75$). $\Theta$ is the thermodynamic nuclear temperature.
was chosen to be a Gaussian, centered at $Q = -17$ MeV and with a standard deviation of 5.5 MeV. It should be noted that calculations with different values of centroid and standard deviation were also carried out, and the final results were found to be rather insensitive to these values. The available excitation energy, $E^*_\text{tot} = \Delta\text{BE} + Q$ ($\Delta\text{BE}$ is the difference in binding energies between the entrance and exit channels), was shared between the $^{24}\text{Mg}$ and the $^{32}\text{S}$ in one of two ways, either in proportion to their masses or by assuming equal nuclear temperatures. The first method of energy sharing gives $E^*(^{32}\text{S}) = 32/56 E^*_\text{tot}$. Equal temperatures energy sharing mode requires that

$$\frac{E^*_1 - \Delta_1}{a_1} = \frac{E^*_2 - \Delta_2}{a_2} \quad (4.7)$$

where particles 1 and 2 are the $^{24}\text{Mg}$ and $^{32}\text{S}$, $a$ and $\Delta$ are level density and pairing energy parameters taken from a tabulation by Gilbert and Cameron [G165], and $E^*_1 + E^*_2 = E^*_\text{tot}$. In either case, the distribution of the excitation energies of the $^{32}\text{S}$ is sharp, that is for each $Q$-value there is a single value of the $^{32}\text{S}$ excitation energy.

The next step in the calculation is to "smear out" the distribution. The new value of the $^{32}\text{S}$ excitation energy, $E^*_n(^{32}\text{S})$, is given by

$$E^*_n(^{32}\text{S}) = E^*_\text{sh}(^{32}\text{S}) + R_n \quad . \quad (4.8)$$

where $E^*_\text{sh}(^{32}\text{S})$ is the "sharp" value and $R_n$ is a normally distributed
random number; the mean of the distribution is 0 and the width is an adjustable parameter $\sigma$. $E_n^{*\text{Mg}}$ is then determined by the conservation of energy, $E_n^{*\text{tot}} = E_n^{*\text{Mg}} + E_n^{*\text{S}}$ and the constraints $E_n^{*\text{S}} > 0$ and $E_n^{*\text{Mg}} > 0$.

The $\alpha$-emission threshold of $^{32}\text{S}$, $E_{\text{th}}$ is approximated by

$$E_{\text{th}} = BE^{^{32}\text{S}} - BE^{^{28}\text{Si}} - BE(\alpha) + E_c$$  

(4.9)

where $BE$ is the binding energy of a nucleus and $E_C$ is the Coulomb energy of spherical, touching $^{28}\text{Si}$ and $^4\text{He}$ nuclei. Evaluation of eq. (4.9) gives $E_{\text{th}} = 12.7$ MeV. Equation (4.9) gives the lower limit of the threshold because it only holds for the $s$-wave decay. In general, the centrifugal barrier will make the real threshold larger than that given by eq. (4.9). If the $E_n^{*\text{S}}$ is above the emission threshold, a decay is assumed to take place, and the differential cross section is approximated by a constant $d\sigma/d\theta$ in the rest frame of the $^{32}\text{S}$. The $^{28}\text{Si}$ energy and angle of motion are then found from two-body kinematics. The quantity of interest is the Q-value spectrum of these contaminant events, more precisely, the allowed Q-value range for these events.

The resulting Q-value spectrum for these contaminant events is very insensitive to the mode of excitation energy sharing. If the value of the "smearing" parameter, $\sigma$, is fixed at 0 (the $E_n^{*\text{S}}$ distribution is given by the sharp distribution), the contaminant events always have an apparent Q-value $Q < -25$ MeV. As $\sigma$ is increased, the upper limit of the distribution shifts to less negative Q-values. Even for $\sigma$ as large as 2.5 MeV, however, there are no events with $Q > -20$ MeV. Figure 4.8 shows several contaminant event Q-value distributions as a function of
Figure 4.8

Apparent Q-value spectra for $^{28}\text{Si}$ ions from the $^{40}\text{Ca}(^{16}\text{O},^{24}\text{Mg}^*)(^{32}\text{S}^*)(\alpha)^{28}\text{Si}^*$ reaction for different values of the smearing parameter $\sigma$. The area under each curve corresponds to 10,000 counts.
the smearing parameter \( \sigma \). The results of this calculation are in agreement with the experimental finding that the singles Si yield for \( Q > -20 \text{ MeV} \) is free from contaminants; however, they indicate that this may not be true for \( Q < -20 \text{ MeV} \).

The arguments and calculations in this section justify the use of the assumptions made in the data analysis but only for events with \( Q > -20 \text{ MeV} \). For these events, the evidence indicates that the measured singles Si yield is very close to the true \( { }^{28}\text{Si} \) yield from the \( { }^{40}\text{Ca}(^{16}\text{O},{ }^{28}\text{Si}^*){ }^{28}\text{Si}^* \) reaction.
4.7 Q-VALUE AND ANGULAR DISTRIBUTIONS

In the process of measuring the $^{40}$Ca($^{16}$O,$^{28}$Si*)$^{28}$Si* reaction excitation function, the Q-value and angular distributions of the $^{28}$Si nuclei were measured at each bombarding energy. In this section, the properties of the Q and $\theta$ distributions will be examined in detail, and the behavior of the Q-value distribution as a function of the bombarding energy will also be discussed. In the next section, the absolute and relative normalizations of the data will be discussed.

The uncertainties in the derived Q-values can be related to the experimental uncertainties in the ion energy, E, and the anode of motion, $\theta$. These quantities are uncorrelated, and therefore, the uncertainty in Q is given by

$$\Delta Q = \left\{ \left( \frac{\Delta Q}{\Delta E} \right)^2 \cdot \Delta E^2 + \left( \frac{\Delta Q}{\Delta \theta} \right)^2 \cdot \Delta \theta^2 \right\}^{1/2} \quad (4.10)$$

Note that $\Delta E$ here is the error in E, not the rate of energy loss of the ion in the counter. The partial derivatives are easily evaluated using eqs. (4.4) and (4.5). In the region of interest, 20 < E < 60 MeV and 30° < $\theta_{lab}$ < 60°, $\Delta Q/\Delta E$ ranges from 0.8 to 1.4, and $\Delta Q/\Delta \theta$ from 0.6 to 1.2 MeV/rad. Typical values for E and $\theta$ are 0.4 MeV and 0.6° (see sections 3.4 and 3.5). Using these values in eq. (4.10) yields $\Delta Q \sim 0.5$ MeV. A similar calculation gives $\theta_{cm} \sim 0.7^\circ$. In addition, there can be a systematic uncertainty in the derived Q-values resulting from differences in the thickness of the counter entrance foils. This
effect was corrected for in the following analysis by comparing spectra obtained at the same beam energy but with different windows (see Appendix A). Figure 4.9 shows the experimentally obtained Q-value spectrum for 30 MeV $^{28}\text{Si} + \text{Au}$ elastic scattering data. The observed resolution of $Q = 0.5$ Mev is consistent with the above analysis of experimental uncertainties.

A typical Q-value spectrum for the $^{40}\text{Ca}(^{16}\text{O},^{28}\text{Si}^*)^{28}\text{Si}^*$ reaction, taken from a run at 76.125 MeV, is shown in Figure 4.10. In the range $-20 < Q < 0$ MeV, the spectrum is roughly Gaussian in shape with a standard deviation of about 4 MeV; superimposed on the Gaussian shape is a large negative Q-value tail. The discussion in section 4.6 indicates that secondary $\alpha$-evaporation events from the $^{40}\text{Ca}(^{16}\text{O},^{26}\text{Mg}^*)^{32}\text{Si}^*$ reaction may contribute to this low Q-value yield. Although the shapes of the Q-value spectra do not change with bombarding energy in the energy range studied, there is a general shift of the spectrum towards more negative Q-values with increasing beam energy. This trend is well illustrated by Figure 4.11a, where $\langle Q \rangle$, the experimentally defined average Q-value, is plotted against the beam energy. This shift comes about for two reasons. As the energy of a collision increases, more energy becomes available for excitations of the nuclei in the exit channel, and therefore, large and negative Q-values become more probable in the reaction. In addition, for a fixed Q-value, the $^{28}\text{Si}$'s have more kinetic energy and events with very low Q-values (experimentally undetectable at lower energies) become detectable. The average Q-value is independent of the center of mass angle, as is evident from Figure 4.12, indicating that the lifetime of the intermediate state is longer than the time needed to completely damp out the energy of relative motion [Co77].
Figure 4.9

Q-value spectrum for elastic $^{28}\text{Si} + \text{Au}$ scattering at 30 MeV.
Figure 4.10

The Q-value spectrum for the $^{40}\text{Ca}(^{16}\text{O},^{28}\text{Si}^*)^{28}\text{Si}^*$ reaction at $E_{\text{beam}} = 75.075$ MeV. The dashed curve shows a Gaussian curve with a standard deviation of 4 MeV.
Figure 4.11

a) \( \langle Q \rangle \) is plotted as a function of beam energy. Note that each set of runs with a different window is plotted with its own symbol.

b) \( \langle Q \rangle_t \) is plotted as a function of beam energy. \( \langle Q \rangle_t \) is defined in Appendix B.
Figure 4.12

The average Q-value as a function of the center of mass angle. Beam energy is 75.075 MeV.
A typical angular distribution, for events with $-20 < Q < 0$ MeV at a beam energy of 76.2 MeV, is shown in Figure 4.13. For $55^\circ < \theta_{\text{cm}} < 90^\circ$, the distribution is consistent with a $d\sigma/d\Omega \sim 1/\sin(\theta_{\text{cm}})$ behavior, or equivalently, $d\sigma/d\theta_{\text{cm}} = \text{constant}$. Outside these limits, the detection efficiency drops quickly to 0. An angular distribution described by a constant $d\sigma/d\theta_{\text{cm}}$ is characteristic of a decay of a rotating body whose lifetime is large compared to its rotational period, indicating that the $^{28}\text{Si}$ is a decay product of a long-lived intermediate state. A more quantitative analysis of these aspects of the angular distributions will be deferred to section 5.4.
The center of mass angular distribution for the $^{40}\text{Ca}(^{16}\text{O},^{28}\text{Si}^*)^{28}\text{Si}^*$ reaction; $-20 < Q < 0 \text{ MeV}$. The dot-dashed line shows a fit to the data with a function proportional to $1/\sin \theta_{\text{cm}}$. Note the rapid decrease of efficiency for $\theta_{\text{cm}} < 54^\circ$ and $\theta_{\text{cm}} > 92^\circ$. 

Figure 4.13
4.8 NORMALIZATION OF DATA

The normalization of the data was a three-step process. First, yields from runs with the same gas counter entrance window were normalized relative to each other using $^{160}\text{O} + ^{181}\text{Ta}$ elastic scattering yields in the 112° monitor (MON3). The second step consisted of normalizing the five sets of runs (one set for each entrance window) relative to each other by using Si yields from runs taken at the same beam energy but with different windows. Finally, the absolute scale of the cross section was determined by calculating the total cross section for a single run (run 18 at 75.3 MeV) based on the elastic $^{160}\text{O} + ^{40}\text{Ca}$ scattering into the 15° monitor (MON1), the known cross section for this process [Vi79] and the known solid angles of the MON1 and the gas counter.

The elastic $^{160}\text{O} + ^{40}\text{Ca}$ cross section changes very rapidly with angle near 15° and for MON1 a 0.1° shift in beam angle, the maximum allowed by the collimation, changes the yield by 3%. Although small compared to the uncertainty in the absolute normalization (see below), this is an unacceptable error in the relative normalization, since the structures being searched for are expected to be roughly that size. The elastic $^{160}\text{O} + ^{181}\text{Ta}$ scattering yield in MON3 is very insensitive to beam angle shifts and, therefore, is suitable to be used for the relative normalization. The $^{160} + ^{18}\text{Ta}$ elastic differential cross section was not used for the absolute normalization because the manner of calculating the cross section made it uncertain by approximately 20%.
4.8.1 RELATIVE NORMALIZATION

The relative, energy to energy, normalization of the Si yield, $N_{rel}$, is given by

$$N_{rel} = \frac{\sigma}{Y_3 \cdot DT_3}$$

(4.11)

where $\sigma$ is the calculated average cross section for elastic $^{16}O + ^{181}Ta$ scattering into MON3 and $Y_3$ and $DT_3$ are the yield and dead time correction for MON3 for a given run. The uncertainty in this procedure largely reflects the uncertainty in $\sigma$. Unlike for $^{16}O + ^{40}Ca$ scattering which has been extensively studied in the energy and angular range of interest, the optical model parameters for $^{16}O + ^{181}Ta$ elastic scattering are derived from an experiment done at somewhat higher energies.

Videbaek et al. [V177] determined the optical model parameters for $^{16}O + ^{181}Ta$ elastic scattering at beam energies of 83, 90 and 96 MeV by fitting measured angular distributions. The depth and diffusivity of the real potential were kept fixed at $V_0 = 100$ MeV and $a = 0.45$ fm respectively, the diffusivity of the imaginary potential was also fixed at 0.45 fm, while the radius parameter, $r_0$, and the depth of the imaginary potential, $V_I$, were allowed to vary. The resulting fitted parameters are shown by the solid circles in Figure 4.14; in addition, possibilities of extrapolating these values into the energy region of interest are also shown. Several parameter sets, corresponding to different combinations of the extrapolated parameter values and
The optical model parameters determined by Videbaek et al. [Vi77] for $^{16}O + ^{18}Ta$ elastic scattering as a function of beam energy are shown as solid dots. The solid lines are linear extrapolations of these parameters to lower energies, and the dashed line is a quadratic extrapolation.
Videbaek's original 83 MeV parameter set, were used to calculate \( \sigma \), to test the sensitivity of \( \sigma \) to changes in \( r_0 \) and \( V_I \) values. The parameter sets are tabulated in Table 4.1.

The resulting cross sections are shown in Figure 4.15. While the scale of the cross sections varies by up to 20% with different parameter sets the cross section curves have essentially the same shape so that the relative normalization is not very sensitive to the choice of optical model parameters. Parameter set PI was chosen to be the one used in calculating \( \sigma \) used in eq. (4.11).

The excitation function of \( \sigma_{\text{tot}} \) is shown in Figure 4.16. Runs for each of the five windows are plotted with different symbols. It is clear that a slight adjustment is needed to make the data from all the windows consistent with each other. A number of repeat points, runs at the same beam energies but with different windows, were taken to ensure that this adjustment could be made in a quantitative way. The adjustment was made by requiring that the average of the yields in the overlap runs between different windows be equal to each other. As a result, window 1 yields needed to be increased by 5%, those in runs with window 4 decreased by 5% and all others were left unchanged. The result of this normalization are shown in Figure 4.17.

4.8.2 Absolute Normalization

The absolute normalization of run 18 proceeded as follows. The center-of-mass differential cross section for the Q-value range \(-20 < Q < 0 \) MeV was a function of \( \theta_{\text{cm}} \) was calculated using
Table 4.1

Optical model parameters for $^{16}\text{O} + ^{181}\text{Ta}$ elastic scattering.

<table>
<thead>
<tr>
<th>Parameter Set</th>
<th>$V_I$</th>
<th>$r_o$</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td>125.9</td>
<td>1.226</td>
</tr>
<tr>
<td>P2</td>
<td>126.0</td>
<td>1.210</td>
</tr>
<tr>
<td>P3</td>
<td>170.0</td>
<td>1.226</td>
</tr>
<tr>
<td>P4</td>
<td>170.0</td>
<td>1.210</td>
</tr>
<tr>
<td>P5</td>
<td>150.0</td>
<td>1.226</td>
</tr>
<tr>
<td>P6</td>
<td>160.0</td>
<td>1.220</td>
</tr>
</tbody>
</table>
Figure 4.15

The $^{160} + ^{181}Ta$ elastic cross sections at 112° calculated with the parameter sets tabulated in Table 4.1. Parameter set P4, which represents the linear extrapolation of $r_0$ and a quadratic extrapolation of $V_I$, leads to a cross section virtually identical to that generated using P1. Parameter P6 results in a cross section very close to that obtained using P5.
Figure 4.16

Excitation function of the \( ^{40}\text{Ca}(^{16}\text{O},^{28}\text{Si}^*)^{28}\text{Si}^* \) reaction. Note that each set of runs with a different window is plotted with its own symbol. No adjustment using overlap runs is made (see text). The energy scale is the excitation energy of the compound nucleus, \( ^{56}\text{Ni} \).
Excitation function of the $^{40}\text{Ca}(^{16}\text{O},^{28}\text{Si}^*)^{28}\text{Si}^*$ reaction. Note that each set of runs with a different window is plotted with its own symbol. An adjustment using overlap runs has been made (see text).
\[ \frac{d\sigma}{d\Omega} (\theta_{cm}) = 2.67 \frac{q_b}{q_{tot}} \frac{A_t}{t} DT \sum_i \frac{Y_i \text{JAC}_i}{\Delta\Omega(\theta_{lab})} \text{ (mb/sr)} \]  

(4.12)

where \( q_b \) is the mean beam charge at the Faraday cup, \( q_{tot} \) is the total collected charge in the Faraday cup in \( \mu \)C, \( A_t \) is the atomic mass of the target in amu, \( DT \) is the gas ionization counter dead time correction, \( \Delta\Omega(\theta_{lab}) \) is the solid angle of the detector in mSr, \( t \) is the target thickness in \( \mu \)g/cm\(^2\) and \( Y_i \) and \( \text{JAC}_i \) are the yield and Jacobian for a given \( Q \)-value and \( \theta_{lab} \). The sum over \( i \) was carried over all allowed values of \( Q \), and the corresponding values of \( \theta_{lab} \) (\( Q \) is in 0.5 MeV bins and \( \theta_{lab} \) is in 1° bins). The solid angle \( \Delta\Omega(\theta_{lab}) \), for a 1° bite in angle, is given by

\[ \Delta\Omega(\theta_{lab}) = 1.53 \cos(\theta_c - \theta_{lab}) \text{ mSr} \]  

(4.13)

where \( \theta_c \) is the angle of the center of the detector (45°) and the cosine term corrects for the fact that while the center of the detector is 11.3 cm from the target this distance increases with increasing \( |\theta_c - \theta_{lab}| \) and consequently, \( \Delta\phi \), the differential azimuthal angle, decreases as well. Figure 4.18 shows the relevant geometry for the derivation of the correction term. The quantity \( q_b/q_{tot} \) may be obtained by using a modified form of eq. (4.12) and considering the elastic \(^{16}\text{O} + ^{40}\text{Ca} \) scattering in MON1,
Figure 4.18

The side and top views of the gas counter entrance aperture and the relevant geometry for calculating the solid angle.
\[ \Delta \Omega (\theta) = 1.53 \cos (\theta - \theta_c) \text{ msr/deg} \]
The differential cross section is obtained by using the optical model code PTOLEMY [Ma78] with the parameter set 2a from the work of Vigdor et al. [Vi79]. Combining eq. (4.12) and (4.14) gives the required differential cross section for the $^{40}\text{Ca}(^{16}\text{O},^{28}\text{Si}^*)_2^{28}\text{Si}^*$ reaction in terms of the yields and solid angles of MON1 and the gas counter and the elastic $^{16}\text{O} + ^{40}\text{Ca}$ cross section at 15°. The estimated uncertainties are: 10% for $\Delta(\bf{\omega})_{lab}$, 6% for $\Delta(\bf{\omega})_{lab}$ and 5% for $\Delta(\Sigma Y_1 JAC_1)$, giving a total uncertainty of the absolute calibration of approximately 15%. In the measured angular range, the differential cross section of the $^{40}\text{Ca}(^{16}\text{O},^{28}\text{Si}^*)_2^{28}\text{Si}^*$ reaction is consistent with a $1/\sin(\theta_{\text{cm}})$ behavior (see Figure 4.13). This behavior was assumed to continue to 0° and 180° in determining the total cross section.

The excitation function obtained by averaging all the runs taken at the same beam energies is shown in Figure 4.19. The cross section increases roughly linearly with increasing beam energy with some structures superimposed on the smoothly increasing component of the cross section. There are prominent structures near excitation energies of the compound nucleus, $^{56}\text{Ni}$, $E_{\text{cm}} = 68.1, 68.9$ and 69.2 MeV, and there is a hint of a smaller one near $E_{\text{cm}} = 67.45$ MeV. A statistical analysis and a detailed comparison of the $^{40}\text{Ca}(^{16}\text{O},^{28}\text{Si}^*)_2^{28}\text{Si}^*$ and $^{28}\text{Si}(^{28}\text{Si},^{28}\text{Si}^*)_2^{28}\text{Si}^*$ reaction excitation functions will be carried out in the following section.

\[
\frac{q_b}{q_{\text{tot}}} = t \frac{\Delta \Omega_1 \frac{d\sigma}{d\Omega_{\text{lab}}}^{(EL)}}{2.67 A_1 Y_1^{(EL)} DT_1} \quad (4.14)
\]
Final excitation function of the $^{40}\text{Ca}(^{16}\text{O},^{28}\text{Si}^*)^{28}\text{Si}^*$ reaction. All relative and absolute normalization corrections have been applied. Data points at energies where more than one run was taken were obtained by averaging the yields from those runs.
4.9 DISCUSSION OF RESULTS

4.9.1 Comparison of Experiments

Before the comparison of the $^{40}\text{Ca}(^{16}\text{O},^{28}\text{Si}^*)^{28}\text{Si}^*$ and the $^{28}\text{Si}^{(^{28}\text{Si},^{28}\text{Si}^*)^{28}\text{Si}^*}$ reaction excitation functions can be made, some of the differences between the two measurements must be considered. Following the considerations of detector efficiency and the effects of contamination from the secondary $\alpha$-decay of $^{32}\text{S}$, discussed in earlier sections, the $Q$-value range of the excitation function was limited to $Q > -20$ MeV. This range is similar to that in the study of the $^{28}\text{Si}^{(^{28}\text{Si},^{28}\text{Si}^*)^{28}\text{Si}^*}$ reaction by Betts et al. [Be81b], the range $-22 < Q < 0$ MeV. It must be noted that in the $^{28}\text{Si} + ^{28}\text{Si}$ study, the detection efficiency was 100% at $Q = 0$ MeV and dropped almost linearly to 0% at $Q = -22$ MeV, while in the present work the detection efficiency is close to 100% for all events with $Q > -20$ MeV. Since the experiments were done at different laboratories, one at BNL and the other at WNSL, there may have been differences in the beam energy calibrations. The WNSL magnet was calibrated (see section 4.2) just after the completion of the $^{16}\text{O} + ^{40}\text{Ca}$ experiment, so that the energies of its beams are well-known. However, in retrospect it was not possible to determine an accurate calibration of the BNL analyzing magnet at the time of the $^{28}\text{Si} + ^{28}\text{Si}$ experiment, and it is possible that the beam energies from the BNL Tandem were off from their nominal values. The drift in the calibration of the Yale Tandem analyzing magnet was about a part in $10^4$, a similar shift of the BNL magnet corresponds to an energy shift of
about 100 keV in the nominal beam energies of the $^{28}$Si + $^{28}$Si experiment. Therefore, a relative energy shift between the two excitation functions of the order of 100 keV is possible.

4.9.2 Comparison of Excitation Functions and Statistical Analysis

The excitation functions of the $^{40}$Ca($^{16}$O,$^{28}$Si*)$^{28}$Si* and $^{28}$Si($^{28}$Si,$^{28}$Si*)$^{28}$Si* reactions are plotted together for comparison in Figure 4.20. The data suggest that the structures in the $^{28}$Si($^{28}$Si,$^{28}$Si*)$^{28}$Si* reaction excitation function at $E_{\text{CN}} = 67.4$, 68.2, 68.9 and 69.3 have counterparts in the $^{40}$Ca($^{16}$O,$^{28}$Si*)$^{28}$Si* reaction excitation function, while the structures at $E_{\text{CN}} = 67.7$ and 68.5 do not. A more quantitative statement can only be made after performing a detailed statistical analysis of the data.

The results of the statistical analysis, which follows a method outlined by Saini and Betts [Sa84], presented in this section are a quantitative measure of the degree of correlation between the intermediate ($\Gamma \sim 200$ keV) width structures in the $^{40}$Ca($^{16}$O,$^{28}$Si*)$^{28}$Si* and $^{28}$Si($^{28}$Si,$^{28}$Si*)$^{28}$Si* reaction excitation functions. The two excitation functions have very different large-scale structure and, therefore, new excitation functions, with the average behavior subtracted out, must be constructed from the data. The new functions, $D_1(E)$ and $D_2(E)$, allow a direct comparison of the intermediate width structures in the two data sets. In the following discussion, the superscript 1 will refer to the $^{40}$Ca($^{16}$O,$^{28}$Si*)$^{28}$Si* reaction, and the subscript 2 will refer to the $^{28}$Si($^{28}$Si,$^{28}$Si*)$^{28}$Si* reaction data.
Figure 4.20

Comparison of the $^{40}\text{Ca}(^{16}\text{O},^{28}\text{Si}^*)^{28}\text{Si}^*$ and $^{28}\text{Si}(^{28}\text{Si},^{28}\text{Si}^*)^{28}\text{Si}^*$ reaction cross sections.

The $^{28}\text{Si} + ^{28}\text{Si}$ data are from [Be81b].
The D(E) functions, or deviation functions, are in turn used to construct two new functions: the summed deviation function \( S(E) \) and the cross correlation function, \( C(E) \)

\[
S_{12}(E) = \frac{1}{2} \left[ D_1(E) + D_2(E) \right] \quad (4.15)
\]

\[
C_{12}(E) = D_1(E) \cdot D_2(E). \quad (4.16)
\]

In addition to \( S_{12}(E) \) and \( C_{12}(E) \), the functions \( S(E) \) and \( C(E) \) and their frequency distributions can be calculated for completely uncorrelated deviation functions. A comparison of the results using functions generated from the data and the results expected for uncorrelated data sets yields a quantitative measure of the correlations.

The deviation functions are constructed in the following way. First, the average cross section \( <\sigma(E)> \) is calculated by averaging \( \sigma(E) \) over an interval of 430 keV. Figures 4.21a and 4.21b show the two data sets and their \( <\sigma(E)> \) curves. The averaging interval corresponds to four data points to the right and left of the central point for the WNSL data; therefore, \( <\sigma(E)> \) is undefined for the first and last four points in the data set. The \( ^{28}\text{Si} + ^{28}\text{Si} \) data were not measured at the same energies as the \( ^{16}\text{O} + ^{40}\text{Ca} \) data and had to be interpolated to the Yale energies. Next, new excitation functions \( Y_1(E) \) were then generated according to

\[
Y_1(E) = \frac{\sigma_1(E)}{<\sigma_1(E)>} \quad (4.17)
\]
a) The $^{40}\text{Ca}(^{16}\text{O},^{28}\text{Si}^*)^{28}\text{Si}^*$ reaction excitation function and $\langle\sigma(E)\rangle$.

b) The $^{28}\text{Si}(^{28}\text{Si},^{28}\text{Si}^*)^{28}\text{Si}^*$ reaction excitation function and $\langle\sigma(E)\rangle$.

c) The frequency distribution of the deviation function, $P(D)$, for the $^{16}\text{O} + ^{40}\text{Ca}$ data. The dashed line is the theoretically expected frequency distribution with mean 0 and variance 1.

d) The frequency distribution of the deviation function, $P(D)$, for the $^{28}\text{Si} + ^{28}\text{Si}$ data. The dashed line is the theoretically expected frequency distribution with mean 0 and variance 1.
where $i = 1,2$. Finally, a deviation function $D(E)$ can be constructed from $Y(E)$,

$$D_i(E) = \frac{Y_i - \langle \langle Y_i \rangle \rangle}{\sqrt{\langle \langle Y_i^2 \rangle \rangle - \langle \langle Y_i \rangle \rangle^2}}$$

(4.18)

where the double bracket $\langle \langle \rangle \rangle$ denotes averaging over the entire energy range of the data set. By construction, the experimental and theoretical frequency distributions of $D_i(E)$ are expected to be approximately normally distributed with the mean and variance of 0 and 1 respectively. This property of the deviation functions will be used to calculate the expected values of $S(E)$ and $C(E)$ for uncorrelated data. The frequency distributions for $D_1(E)$ and $D_2(E)$ are shown in Figures 4.21c and 4.21d.

If the deviation functions are statistically independent (uncorrelated), then it can be shown [Me73] that the $S(E)$ frequency distribution, $P(S)$, is normally distributed with $\mu = 0$, and variance $\sigma^2 = 0.5$. $P(S)$ for the two data sets has a mean $\mu = 0.0$ and a variance $\sigma^2_{\text{exp}} = 0.67$ (the $^{28}\text{Si} + ^{28}\text{Si}$ data have been shifted by -25 keV to maximize $\sigma^2_{\text{exp}}$). The value of $\sigma^2_{\text{exp}}$ indicates that there is a correlation between the two data sets at an 80% confidence level [Ab72]. $S_{12}(E)$ and $C_{12}(E)$ are graphed together in Figures 4.22a and 4.22b. In order to clarify the statistical significance of the size of correlations implied by $S_{12}(E)$ and $C_{12}(E)$, confidence levels are plotted.
Figure 4.22

a) The summed deviation function $S(E)$ is shown, together with confidence limits.

b) The cross correlation function $C(E)$ is shown, together with confidence limits.
along with $S_{12}(E)$ and $C_{12}(E)$. $P(S)$ used in calculating confidence limits was the theoretically calculated distribution with $MU = 0$ and $\sigma^2 = 0.5$, $P(C)$ was calculated using a Monte Carlo method with randomly generated spectra.

The peaks at $E_{cn} = 67.4$, 68.2 and 68.9 are at the 95% confidence limit of $S(E)$, which implies a correlation between the two data sets at these energies. The two other peaks, at $E_{cn} = 68.5$ and 69.2, further than one standard deviation away from the mean value of $S(E)$ are at an 80% confidence level. In the cross correlation function, the $E_{cn} = 68.2$ and 68.9 peaks show up at better than 90% confidence level, the $E_{cn} = 67.4$ and 69.2 MeV at a 70% level (about one standard deviation), and at $E_{cn} = 68.5$ MeV there is no evidence of correlation.

The statistical analysis strongly suggests that there are correlations between the excitation functions of the $^{40}\text{Ca}(^{16}\text{O},^{28}\text{Si}^*)^{28}\text{Si}^*$ and $^{28}\text{Si}(^{28}\text{Si},^{28}\text{Si}^*)^{28}\text{Si}^*$ reactions, but a conclusive statement is not possible because of two major difficulties. One is the narrowness of the structures compared to the energy step size of the excitation function; the other is the limited number of data points in the energy region covered by both experiments (once the eight points at the edges are eliminated, only 37 points remain to be used in the analysis). The first difficulty is not easily resolved, since it is not practical to measure this excitation function over an energy range of several MeV in much smaller steps than were used in the present experiment. The second problem could be solved by measuring the $^{40}\text{Ca}(^{16}\text{O},^{28}\text{Si}^*)^{28}\text{Si}^*$ reaction excitation function in an energy range several MeV broader than the range covered in the present work.
CHAPTER 5

5.1 ENTRANCE CHANNEL MODELS

Entrance channel models were introduced by Imanishi [Im68,Im69], Scheid, Greiner and collaborators [Sc70, Fi72] and Abe [Ab75] as part of an effort to understand the intermediate width structure (Γ = 200 keV) seen in the $^{12}$C + $^{12}$C elastic scattering [Br60]. All three of these models rely on a similar mechanism: a coupling of the broad (Γ ~ 2-3 MeV) potential scattering resonances in the elastic channel to much narrower resonances in the inelastic channels. The double resonance model of Scheid et al. [Sc70] will be described in section 5.1.1 as an example of the entrance channel models, and results obtained within the framework of that model in the $^{28}$Si + $^{28}$Si system will be discussed in section 5.1.2.

5.1.1 Double Resonance Model

The double resonance model can be understood by considering the resonances in a potential well (see Figure 5.1a). The resonances can be classified into two groups: quasibound and virtual resonances. The quasibound resonances ($l = 0, 4, 6$ levels in Figure 5.1a) are localized inside the attractive well, since their energies are below the barrier top. If no other channels are open, the width of the quasibound states is small because in order to go back to the elastic channel the nuclei must tunnel through a potential barrier. The small width of these
Figure 5.1

a) Potentials for several values of $\ell$ between 0 and 12 in the $^{12}\text{C} + ^{12}\text{C}$ system and the resonances in those potentials [Fi72]. The resonances drawn with solid lines ($\ell = 0, 4, 6$) indicate quasibound resonances, while those drawn with dashed lines ($\ell = 8, 10$) indicate virtual resonances. The resonance for $\ell = 12$ (not shown) is also a virtual resonance.

b) Diagram illustrating the capture from a virtual resonance to a quasibound resonance by exciting one of the nuclei to an excited state with an excitation energy $E^*$. This Figure is taken from the work of Fink et al. [Fi72].
resonances gives rise to the intermediate width structure in the cross section. The virtual resonances (l > 8 levels in figure 5.1a) lie above the barrier in energy and so have large widths back into the entrance channel. The virtual resonances give rise to the broad structures in elastic scattering.

The intermediate width structures result from the following process. Consider nuclei moving in the top potential in figure 5.1b with a kinetic energy $E_1$, corresponding to a virtual resonance in the elastic channel, and an angular momentum $L^*$. If one of the nuclei has an excited state characterized by an excitation energy $E^*$ and angular momentum $L^*$ and if the potential for an $L$, such that $|L_1 - L^*| < L < |L_1 + L^*|$, has a quasibound resonance at an energy $E = E_1 - E^*$, then the resonance excitation of that nucleus will trap the nuclei in the quasibound state. The initial energy in the entrance channel must correspond to a virtual resonance energy because only the resonating partial wave has a large amplitude to excite the quasibound states [Fi72].

A crucial condition for the existence of the intermediate width structures is that the spreading width (the partial width for the decay into a compound nuclear state) of the quasibound resonance be small [Fi72]. This in turn requires that the absorption out of the elastic channel be small or equivalently that the imaginary part of the potential be small. A large spreading width will smear out the resonance structures and contribute only to the background.
5.1.2 Results in the $^{56}\text{Ni}$ Compound Nucleus System

A microscopic calculation of the $^{28}\text{Si} + ^{28}\text{Si}$ and $^{16}\text{O} + ^{40}\text{Ca}$ scattering within the framework of the generator coordinate method [La82] indicates that in the $^{28}\text{Si} + ^{28}\text{Si}$ system, quasibound resonances exist for each partial wave, up to $l = 36$, while none exist in the $^{16}\text{O} + ^{40}\text{Ca}$ system in the range $l = 30-40$. The existence of quasibound resonances is a necessary condition for the double resonance mechanism. Thiel et al. [Th84] have carried out a double-resonance calculation that reproduced qualitatively some of the resonant features in the $^{28}\text{Si} + ^{28}\text{Si}$ system. The real potential used in the calculation [Ko82] explicitly includes the coupling of the ground state configuration of $^{28}\text{Si} + ^{28}\text{Si}$ to the single excitation of the first excited $2^+$ state in $^{28}\text{Si}$. The effects of all the other channels are taken into account with an imaginary potential. The experimental [Be79] and the calculated $90^\circ$ excitation function of elastic and inelastic (first $2^+$ state) scattering of $^{28}\text{Si} + ^{28}\text{Si}$ are shown in Figure 5.2. The calculations for this low resolution data (energy step size of 1 MeV) agree qualitatively with the data; the widths and sizes of the structures in the calculated excitation function are comparable to those in the measured excitation function.

The double resonance calculation, however, fails to reproduce the measured elastic angular distribution (see Figure 1.1a). The distribution calculated by Thiel [Th85] gives a good fit to the oscillatory structure at back angles, but it also has large oscillations at forward angles ($\theta < 60^\circ$) which are manifestly not present in the data. This difficulty of the model in the $^{28}\text{Si} + ^{28}\text{Si}$ system may be
Figure 5.2

The experimental and theoretical $\theta_{cm} = 90^\circ$ excitation functions for $40 < E_{cm} < 75$ MeV in 1 MeV steps. The data (dashed curve) is from [Be79] and the theoretical prediction (solid curve) is from [Th84].
$^{28}\text{Si} + ^{28}\text{Si}$

$\theta_{\text{CM}} = 90^\circ$

Elastic Scattering

Inelastic Scattering $2^+$

$\frac{d\sigma}{d\Omega}$ (mb/sr)

$E_{\text{CM}}$ (MeV)
significant since the oscillations are due to the fact that the imaginary potential is small, and (as was discussed in section 5.1.1) the model does not allow a large imaginary potential that could damp out the unwanted oscillations.

Two final points must be made concerning the double resonance model and the $^{40}\text{Ca}(^{16}\text{O},^{28}\text{Si}^*)^{28}\text{Si}^*$ reaction. As there are no quasibound resonances in the $^{16}\text{O} + ^{40}\text{Ca}$ system in the angular momentum range of interest, the model predicts that no intermediate width structures should be seen in elastic and inelastic $^{16}\text{O} + ^{40}\text{Ca}$ scattering. However, since this model as presently formulated has no mechanism for a large mass transfer, it cannot be used to describe the $^{40}\text{Ca}(^{16}\text{O},^{28}\text{Si}^*)^{28}\text{Si}^*$ reaction.
5.2 DEFORMED SHELL MODEL

The resonances in the $^{28}\text{Si} + ^{28}\text{Si}$ system may reflect the existence of long-lived states in the $^{56}\text{Ni}$ compound nucleus rather than the special features of the entrance channel. This possibility can be explored by studying the structure of high spin states in $^{56}\text{Ni}$ within the framework of the deformed shell model. The theory underlying such calculations and recent results [Be84a] obtained for $^{56}\text{Ni}$ will be described in this section.

5.2.1. Outline of Calculation

The goal of the calculation is to obtain the total energy of the nucleus as a function of spin and of the various parameters describing the shape of the nucleus. The minima in the multidimensional energy surface correspond to relatively stable configurations, since they are more tightly bound than configurations with slightly different values of the parameters. If $\alpha_i$ are the various shape parameters, the total energy, $E_{\text{tot}}$, is given by

$$E_{\text{tot}}(\alpha_i, I) = E_{\text{LD}}(\alpha_i, I=0) + \Delta E_{\text{sh}}(\alpha_i, I)$$  \hspace{1cm} (5.1)

where $I$ is the spin, $\Delta E_{\text{sh}}$ are the shell model energy corrections calculated using the Strutinsky prescription (see section 5.2.2) and $E_{\text{LD}}$
is the liquid drop model (LDM) energy [Co74]. The residual two-body interactions and the pairing interaction are neglected in the calculation by Bengtsson et al. which will be discussed in section 5.2.4.

5.2.2 Strutinsky Method and Cranking Model

A procedure combining the best features of the LDM and the shell models was proposed by Strutinsky [St67, St68]. The LDM reproduces very well those properties of the nucleus that depend smoothly on the number of particles but fails to reproduce the rapid fluctuations in some nuclear properties that occur for particle numbers near closed shells. Shell model calculations, on the other hand, give a good account of the fluctuations but fail to reproduce the experimental values of the quantities that vary smoothly with particle number. In the Strutinsky prescription, the value of a physical quantity (in this case, the binding energy) is the sum of the LDM value and the shell model corrections. The corrections are generated by dividing the shell model energy into an oscillating part, $\Delta E_{sh}$, and a smoothly varying part, $\hat{E}_{sh}$,

$$E_{sh} = \sum_i e_i = \hat{E}_{sh} + \Delta E_{sh}$$  \hspace{1cm} (5.2)

where $e_i$ are the single particle energies.
The decomposition of the shell model energy is done in the following way. The shell model level density, \( g(e) \) is given by

\[
g(e) = \sum_i \delta(e-e_i)
\]  

(5.3)

where \( \delta \) is the Dirac delta function. If the number of particles in the nucleus is \( A \), then

\[
A = \int_{-\infty}^\lambda g(e) \, de
\]  

(5.4)

where \( \lambda \) is the Fermi energy; in the shell model, \( \lambda \) can be arbitrarily chosen to be between the last filled and the first unfilled level. Finally, the shell model energy is

\[
E_{sh} = \int_{-\infty}^\lambda e \, g(e) \, de
\]  

(5.5)

The shell model energy levels are grouped into bunches with the average spacing between groups corresponding to the average spacing between major shells, \( 41A^{-1/3} \) MeV [Mo47 p. 469]. Therefore, \( g(e) \) has oscillations with roughly this period.
A smooth part of the level density, a continuous function \( \tilde{g}(e) \), is now introduced. It has the same average behavior as \( g(e) \) but does not contain the oscillations. The corresponding Fermi level, \( \tilde{A} \) is determined by the requirement that

\[
\tilde{A} = \int_{-\infty}^{\tilde{\lambda}} \tilde{g}(e) \, de \quad (5.6)
\]

and so the smooth part of the shell model energy is given by

\[
\tilde{E}_{sh} = \int_{-\infty}^{\tilde{\lambda}} e \, \tilde{g}(e) \, de \quad (5.7)
\]

Substituting eqs. (5.6) and (5.7) into eq. (5.2) and solving for \( \Delta E_{sh} \) gives the required shell corrections.

The calculation of \( \tilde{g}(e) \) may be computationally complicated, but it is simple in principle. The smoothed level density is obtained using a folding procedure

\[
\tilde{g}(e) = \frac{1}{\tau} \int_{-\infty}^{\infty} g(e') \, f\left( \frac{e' - e}{\tau} \right) \, de' \quad (5.8)
\]

where \( f \) is an averaging function which smears out \( g(e) \) and \( \tau \) is an adjustable parameter approximately equal to \( 41A^{-1/3} \) MeV. The averaging
procedure should leave $\hat{g}(e)$ unchanged, if it is averaged again using
the same procedure, so that

$$\hat{g}(e) = \int_{-\infty}^{\infty} \hat{g}(e') f\left(\frac{e' - e}{\Delta}ight) \, de'$$

(5.9)

In general, eqs. (5.8) and (5.9) cannot be simultaneously satisfied, but
for the actual level densities and a smooth function $\hat{g}(e)$ the function
$f$ is given by [Ri84, p. 87]

$$f(x) = P(x) \cdot W(x)$$

(5.10)

where $P(x)$ is the generalized Laguerre polynomial [Ab72, Chap. 22] and
$W(x)$ is

$$W(x) = (\pi)^{-1/2} \exp(-x^2)$$

(5.11)

Although this procedure was developed for an infinite potential well, it
can be generalized for more realistic finite potentials [St75].

In the shell model, angular momentum can be generated by unpaired
nucleons in valence orbitals, by paired nucleons with aligned angular
momenta or by rotating the potential and, consequently, the nucleus
itself about an axis. The cranking model provides a way of calculating
the single particle energies needed in eq. (5.2) for a nucleus which is
rotating with frequency \( \omega \) in the laboratory frame. It is straight
forward to show that, if the rotation is about one of the principal axes
of the nucleus, then the single particle energies in the laboratory
frame, \( e_{\omega i} \) are given by [Ri84, p. 128]

\[
e_{\omega i} = e_{oi} + \hbar \omega \langle j_{xi} \rangle
\]

(5.12)

where \( j_{xi} \) is the projection of the total angular momentum of particle \( i \)
on the rotation axis and \( e_{oi} \) are the single particle energies in the
frame rotating with the nucleus. The total energy in the laboratory
frame, \( E_\omega \) is obtained summing eq. (5.12) over all the particles

\[
E_\omega = \sum_i e_{oi} + \hbar \omega \langle J_x \rangle
\]

(5.13)

where \( J_x = \sum_i j_{xi} \). For large values of the total angular momentum, \( J \),
the quantum mechanical wobbling of the \( J \) vector may be neglected and
\( \langle J_x \rangle = J \).
5.2.3 Nilsson Potential

The single particle energies, \( e_{01} \) may be calculated using a variety of shell model potentials. Bengtsson et al. [Be84] used the Nilsson potential [Ni55, Gu67] and the deformed Woods-Saxon potentials in calculating the properties of \(^{56}\text{Ni}\). Both will be briefly described below.

The Nilsson potential, as used by Bengtsson et al., has three parameters describing the shape of the nucleus, \( \varepsilon_2, \varepsilon_4, \) and \( \gamma \). The parameter \( \gamma \), which has a range of \( 0^\circ \leq \gamma < 360^\circ \), is the triaxiality degree of freedom [Ri84, p.7]. For example, for values which are a multiple of \( 60^\circ \), the nucleus will be axially symmetric, whereas for all other values the nucleus will have all three principal axes with different lengths. The elongation of the nucleus is described by \( \varepsilon_2 \) [Ni55]. If \( \gamma = 0 \), then shapes with \( \varepsilon_2 = 0 \) are spherical, with \( \varepsilon_2 > 0 \) prolate and \( \varepsilon_2 < 0 \) oblate. The parameter \( \varepsilon_4 \) defines the size of the hexadecupole deformation. The full potential for \(^{56}\text{Ni}\) is then

\[
V_{\text{Ni}} = \frac{1}{2} \hbar \omega_0 \rho \left\{ 1 - \left[ \frac{2}{3} \varepsilon_2 (4\pi/5)^{1/2} \right] \left[ \cos(\gamma) Y_{20} + \sin(\gamma) (Y_{20} + Y_{2-2})/\sqrt{2} \right] + 2\varepsilon_4 P_4(\cos\theta) \right\} - 2\hbar \omega_0 L^* S , \tag{5.14}
\]

where \( \rho \) is a dimensionless radial coordinate, \( \rho = r (\hbar / m \omega_0)^{1/2} \), \( Y_{lm} \) are the spherical harmonics, and \( P_L \) is a Legendre polynomial. The general Nilsson potential includes a term proportional to \( L^2 - \langle L \rangle^2 \); this term is 0 in the calculation by Bengtsson et al. The parameters of the spin-orbit term were taken to be \( \kappa = 0.08 \) and \( \omega_0 = 41A^{-1/3} \); \( \omega_0 \) was set to \( 41A^{-1/3} \).
5.4.2 Deformed Woods-Saxon Potential

The parametrization of the nuclear shape using $\epsilon_2$, $\epsilon_4$, and $\gamma$ cannot take into account the shapes characterized by necking and reflection asymmetry degrees of freedom. A deformed Woods-Saxon potential can be written to explicitly include these degrees of freedom as well as those included in the Nilsson potential. The effect of the necking coordinate [Fa79], $N$, can be seen in Figures 5.3a and 5.3b. When $N = 1$, there is no neck, and as $N$ decreases, the cross-sectional area of the neck decreases until the two parts are joined at a single point at $N = 0$. The mass asymmetry parameter [Fa81], $M_r = M_1/(M_1 + M_2)$, is equal to the ratio of mass on one side of the neck to the total mass of the nucleus. The shapes in Figures 5.3a and 5.3b have $M_r = 0.5$, while the shape in Figure 5.3c is mass asymmetric and has $M_r \neq 0.5$.

The deformed Woods-Saxon potential has the form

$$ V_{ws}(\hat{r}) = \frac{V_0}{1 + \exp(S/a)} , \quad (5.15) $$

where $a$ is the diffusivity and $S$ is in general a complicated function of the spatial coordinates describing the shape of the nucleus as parametrized by $\epsilon_2$, $\gamma$, $r$, and $M_r$. For the simple case of a spherically symmetric potential, with $N=1$ and $M_r = 0.5$, $S = R - r$, where $R$ is the nuclear radius and $r$ is the radial coordinate, and eq. (5.15) reduces to the usual form of the Woods-Saxon potential. For other cases, an expression for $S$ given by Gotz et al. [Go71] must be used.
Figure 5.3

a) The nuclear shape at the 3:1 superdeformation (see section 5.2.5). This shape has $\epsilon = 1.06$, $\gamma = 0^\circ$, and the necking parameter, $N$, is 0.7.

b) The nuclear shape that minimizes the LDM energy at high spins $30 < J < 50$. This shape has $\epsilon = 106$, $\gamma^0 = 0$, and the necking parameter, $N$, is 0.25.

c) A schematic picture representing an axially symmetric ($\gamma = 0^0$) and reflection asymmetric ($M_\pi \neq 0.5$) shape with a neck, $N < 1$. 
The Hamiltonian used in the Woods-Saxon calculation is

\[ H_{ws} = T + V_{ws} + V_c + V_{so} \quad (5.16) \]

where \( T \) is the kinetic energy, \( V_{ws} \) is the potential given by eq. (5.15), \( V_c \) is the Coulomb potential, and \( V_{so} \) is the spin-orbit term. \( V_{so} \) is

\[ V_{so} = -\hbar \sigma \cdot [\mathbf{V}_{ws} \times \mathbf{r}] \quad (5.17) \]

where \( \sigma \) represents the Pauli matrices and \( \mathbf{V}_{ws}^{so} \) is a potential of the form given by eq. (5.15).

5.2.5 Discussion of Results

The results of an extensive investigation of \(^{56}\text{Ni}\) using the cranked Nilsson and Woods-Saxon potentials by Bengtsson et al. [Be84a] will be summarized in this section. The authors studied the energy surfaces of \(^{56}\text{Ni}\) at spins between 30 and 50 as a function of \( \varepsilon_2, \varepsilon_4 \), and \( \gamma \) using the Nilsson potential. In addition, the binding energy was also investigated as a function of the necking and mass asymmetry coordinates using the deformed Woods-Saxon potential.
Energy surfaces in the \((\varepsilon_2, \gamma)\) plane at spins 28, 32, 36, 40, 44 and 48, were calculated using the Nilsson potential, minimizing the energy at each grid point with respect to \(\varepsilon_4\). The results of the calculation are shown in Figure 5.4. At spin 28, there are two minima in the surface; the deeper one is at a small deformation \(\varepsilon_2 = 0.2\), and the other one at a very large prolate deformation \((\varepsilon_2 = 1.06, \varepsilon_4 = 0.16, \text{ and } \gamma = 0)\), roughly corresponding to a 3:1 axis ratio. In addition, indications of a third minimum are seen at a triaxial deformation \((\varepsilon_2 = 0.6, \gamma = 40^\circ)\). With increasing spin, the region of the surface corresponding to the triaxial shape becomes a well-defined local minimum and both highly deformed configurations come down in energy relative to the \(\varepsilon_2 = 0.2\) minimum. The superdeformed 3:1 configuration becomes yrast (lowest energy at a given spin) for spins above 40.

The 3:1 configuration corresponds in shape to two prolate \(^{28}\text{Si}\) nuclei lined up with their axes of symmetry along the same line. The single particle structure of this configuration is \((0)^2(1)^6(2)^6(3)^6(4)^4(5)^2(6)^2\) for the protons, and an identical one for the neutrons (in this notation, \((m)^n\), \(m\) is the major shell number and \(n\) is the number of particles in that shell).

This minimum in the energy surface reflects the facts that the LDM energy changes slowly with \(\varepsilon_2\) for prolate shapes with high angular momenta and that the shell energy has a very deep minimum for \(\varepsilon_2 = 1.06\). The shell energy minimum is very stable against changes in the rotational frequency. Therefore, once the LDM energy, at large rotational frequencies, begins to favor large deformations, the shell energy corrections will ensure the stability of the \(\varepsilon_2 = 1.06\) shape.
Potential energy surfaces in the $(\epsilon_2, \gamma)$ plane as calculated in the cranked Nilsson model for $^{56}\text{Ni}$ at spin values 28, 32, 36, 40, 44 and 48. At each grid point in the $(\epsilon_2, \gamma)$ plane, the energy is minimized with respect to $\epsilon_4$ and only positive parity configurations are considered. The energy separation between solid lines is 4 MeV and between a solid line and a dashed line in 2 MeV. The numbers beside the lines mark the energy relative to the lowest energy in each plot. The lowest energy value is written below each plot, and the corresponding deformation is marked by solid circle. This Figure is from the work of Bengtsson et al. [Be84a].
Real neck formation is not allowed in the Nilsson model; therefore, in order to investigate neck formation in the superdeformed configuration, the $N$ dependence of the total energy was studied using the deformed Woods-Saxon potential. It was found that the energy minimum corresponding to the superdeformed shape has $N = 0.7$, indicating only very slight neck formation. The $\epsilon_2 = 1.06$, $\gamma = 0^\circ$ and $N = 0.7$ shape as well as the shape that minimized the LDM energy are shown in Figures 5.3a and 5.3b, respectively.

The LDM energy for the superdeformed shape has a maximum at $M_\tau = 0.5$, implying a tendency for mass asymmetric breakup. The Woods-Saxon calculation shows, however, that the total energy, as given by eq. (5.1) has a minimum at $M_\tau = 0.5$. This indicates that the breakup of the superdeformed configuration into the symmetric $^{28}\text{Si} + ^{28}\text{Si}$ exit channel will be enhanced.

Figure 5.5, taken from the work of Bengtsson et al. shows the calculated energies of the superdeformed configuration and the $\epsilon_2 = 0.2$ configuration as a function of spin and of the energies and spins of the broad structures in the $^{28}\text{Si}(^{28}\text{Si},^{28}\text{Si^*})^{28}\text{Si^*}$ excitation function [Be81a, Be81b]. The moment of inertia $\Theta$ is given by

$$
\Theta = \frac{J(J+1)\hbar^2}{2 \varv_{\text{rot}}} \quad , \quad (5.18)
$$

where $\varv_{\text{rot}}$ is the rotational energy of the nucleus. The total energy of a nucleus, $E_{\text{tot}}$, is just

-133-
Figure 5.5

Excitation energy of $^{56}\text{Ni}$ is plotted as a function of $J(J + 1)$ for the data of Betts et al. [Be81a, Be81b], the slightly deformed shape ($\epsilon_2 \approx 0.2$) and the 3:1 superdeformed shape ($\epsilon_2 \approx 1.06$). For clarity, the abscissa is marked by the $J$ values rather than the $J(J + 1)$ values.
\[ E_{\text{tot}} = E_{\text{rot}} + E^* \quad , \quad (5.19) \]

where \( E^* \) includes all nonrotational energy. Therefore, substituting eq. (5.18) in eq. (5.19) and differentiating with respect to \( J(J+1) \) yields the result

\[ \frac{dE_{\text{tot}}}{d(J(J+1))} = \frac{n^2}{2 \Theta} \quad (5.20) \]

The slope of the \( E \) vs. \( J(J+1) \) curve of the 3:1 configuration and the slope defined by the \( ^{28}\text{Si} + ^{28}\text{Si} \) data are similar, indicating that their moments of inertia are similar as well.

Betts [Be84b] has suggested that the superdeformed configuration and the states that are built upon it could provide an explanation for the resonances seen in the \( ^{28}\text{Si} + ^{28}\text{Si} \) system. These states may be prevented by reasons of structure from mixing with the very numerous other compound nuclear states (see section 5.6), and so their lifetimes could be greatly lengthened compared to those states. If the density of the superdeformed states is not too large (3–5 MeV\(^{-1}\)), their decay can result in the structure observed in the \( ^{28}\text{Si}(^{28}\text{Si},^{28}\text{Si}^*)^{28}\text{Si}^* \) reaction excitation function. The superdeformed states should decay preferentially to \( ^{28}\text{Si} + ^{28}\text{Si} \) (with both \( ^{28}\text{Si} \) nuclei having a prolate shape), and therefore, this model would predict that the reduced width for the decay into the \( ^{16}\text{O} + ^{40}\text{Ca} \) channel, in particular, should be much
smaller than the reduced width for the $^{28}\text{Si} + ^{28}\text{Si}$ channel (see section 5.7.3).
5.3 ANGULAR MOMENTUM CONSIDERATIONS

Whether the structures observed in the $^{40}\text{Ca}(^{16}_0, ^{28}\text{Si}^*)^{28}\text{Si}^*$ reaction excitation function are resonant or not, there still remains a substantial background yield to be accounted for. There are two general classes of processes that can lead to a reaction with such a large mass transfer: a compound nuclear process and deep inelastic scattering. In the first case, a fully equilibrated compound nucleus is formed, and all particles in the exit channel come from the subsequent decay of the compound nucleus. This process results in a $1/\sin(\theta_{\text{cm}})$ behavior of the differential cross section for compound nuclei with large spins. In the deep inelastic process, no compound nucleus is formed; in the center of mass, the total kinetic energy of the outgoing fragments in the exit channel is close to the Coulomb energy, indicating that most of the available kinetic energy has been used to excite the internal degrees of freedom in the exit channel nuclei. The differential cross sections of deep inelastic scattering can have a $1/\sin(\theta_{\text{cm}})$ shape, if an orbiting process is involved, or they can be more forward peaked than that, if a fast, direct reaction takes place. In sections 5.3, 5.4 and 5.5, the background yield will be analyzed in terms of both types of processes.

A consideration of the nature of the compound nuclear and deep inelastic processes leads to approximate limits on the range of angular momenta that contributes to the $^{40}\text{Ca}(^{16}_0, ^{28}\text{Si}^*)^{28}\text{Si}^*$ reaction. The upper limit, $L_{\text{max}}$, is derived from elastic scattering data and is the same for both types of reactions, while the lower limit, $L_{\text{min}}$, must be determined separately for each type.
L\text{max} is obtained using the "quarter point recipe" of Blair [Bl54]. Blair showed that in a simple, strong absorption model, there is an angular momentum \( l_{1/4} \), such that all partial waves with \( l \leq l_{1/4} \) are absorbed out of the elastic channel, and all those with \( l > l_{1/4} \) only contribute to elastic scattering. Therefore \( l_{1/4} \) is the largest angular momentum in any reaction. Classically, \( l_{1/4} \) is related to the "quarter point" angle, \( \theta_{1/4} \), at which the ratio of the elastic cross section to the Rutherford cross section is 0.25, by the relation

\[
l_{1/4} = n \cot(\theta_{1/4}/2),
\]

where \( n \) is the Sommerfield parameter. The \( l_{1/4} \) values for the energy range of interest were obtained by calculating the elastic cross section to the Rutherford cross section ratio using the optical model code PTOLEMY [Ma78] with parameter set 2a taken from the work of Vigdor et al. [Vi79]. At the beam energy of 75 MeV \( l_{1/4} = 37.4 \) and the \( l_{1/4} \) values increase smoothly to 38 at 77.25 MeV. At a given beam energy, \( L_{\text{max}} \) is identified with \( l_{1/4} \) at that energy.

5.3.1. \( L_{\text{min}} \) FOR THE COMPOUND NUCLEUS

The symmetric fission of the \(^{56}\text{Ni}\) compound nucleus leads to the \(^{28}\text{Si} + ^{28}\text{Si}\) exit channel, thus a consideration of the fission barrier for \(^{56}\text{Ni}\) can be used to obtain \( L_{\text{min}} \) for the fusion-fission reaction. The height of the fission barrier, \( B_f \), for \(^{56}\text{Ni}\) [Mu82, Sa85b] is shown in Figure 5.6. Nuclei with \( B_f > 9 \) MeV are expected to decay predominantly
Figure 5.6

The fission barrier for $^{56}$Ni as a function of the angular momentum of the compound nucleus.
by light particle evaporation, while for those with $B_f < 9 \text{ MeV}$, fission is expected to be the dominant mode of decay [Ba80, p. 334], so that an appreciable fission yield can be expected only for $l > 32$. Thus, in the compound nucleus fusion-fission picture, $L_{\text{min}} = 32$.

It must be noted here that the assignment of the fusion-fission yield to the angular momentum range $32 < L < L_{\text{max}}$ ($L_{\text{max}}$ varies with energy as discussed above) is apparently in disagreement with the results of the sharp cut-off model of fusion discussed by Vlgdor et al. [Vi79]. The fusion cross section is given by

$$\sigma_{\text{fus}} = (\pi/k^2) \sum_{l=0}^{\infty} (2l+1) T_L P_L$$

where $T_L$ is the potential barrier transmission coefficient, and $P_L$ is the probability for fusion once the barrier is overcome. In the sharp cut-off model it is assumed that there exists a critical angular momentum, $l_{\text{fus}}$, such that for $l < l_{\text{fus}}$, $T_L P_L = 1$ while for $l > l_{\text{fus}}$, $T_L P_L = 0$. With this substitution, eq. (5.22) is modified to

$$\sigma_{\text{fus}} = (\pi/k^2) l_{\text{fus}} (l_{\text{fus}} + 1) .$$

Vlgdor et al. measured the $^{160} + ^{40}\text{Ca}$ fusion cross section and using eq. (5.23) determined the limiting angular momentum; in the energy range of interest, $l_{\text{fus}} = 32 \pm 0.7$. This would seem to exclude all but the
lowest partial wave in the range \( l_{\text{min}} < l < l_{\text{max}} \) from taking part in the \( {^{40}}\text{Ca}(^{16}_{\text{O}},^{28}_{\text{Si}})^{28}_{\text{Si}}* \) reaction.

In fact, the sharp cutoff model does not exclude any of the partial waves in the range \( l_{\text{min}} < l < l_{\text{max}} \). The reason for this is that the sharp cut-off model, as applied to \( ^{16}_{\text{O}} + ^{40}_{\text{Ca}} \) fusion, is not sensitive to small admixtures to the fusion cross section from partial waves with angular momenta greater than \( l_{\text{fus}} \). The fusion cross section at a beam energy of 74.4 MeV is measured to be \( 1,172 \pm 53 \) mb [V179], while even if the entire \( ^{28}_{\text{Si}} + ^{28}_{\text{Si}} \) yield from the \( {^{40}}\text{Ca}(^{16}_{\text{O}},^{28}_{\text{Si}})^{28}_{\text{Si}}* \) reaction came from a fusion-fission process, it would only account for less than 20 mb (see [Sa85c]). If the physically unrealistic sharp cut-off requirement on \( T_{p}^{P_{2}} \) term is relaxed and the term is parametrized by a smooth function (equal to 1 for \( l < 32 \) and rapidly decreasing for \( l > 32 \)) then an admixture of \( l \) values between \( l_{\text{fus}} \) and \( l_{\text{max}} \) to the fusion cross section can easily be justified.

5.3.3 \( l_{\text{min}} \) for the Deep Inelastic Reaction

The sharp cut-off model of fusion can be used to estimate the angular momentum range allowed for deep inelastic scattering in the \( {^{40}}\text{Ca}(^{16}_{\text{O}},^{28}_{\text{Si}})^{28}_{\text{Si}}* \) reaction. This model indicates that virtually all the flux with \( l < l_{\text{fus}} \) is absorbed into the fusion channel and allows only small admixtures of partial waves with \( l > l_{\text{fus}} \) to contribute to fusion. Therefore, within this model, the angular momenta available for reactions other than fusion and elastic scattering must lie between \( l_{\text{min}} = l_{\text{fus}} \) and \( l_{\text{max}} = l_{1/4} \).
5.4 LIFETIME OF THE INTERMEDIATE STATE

The lifetime of resonances in the cross section can be estimated from their widths, but it is also possible to deduce the lifetime of the process that gives rise to the smooth background yield in the $^{40}\text{Ca}(^{16}O,^{28}\text{Si}^*)^{28}\text{Si}^*$ reaction from the shape of its angular distributions. Barrette et al. [Ba75] showed that the angular distribution from a decay of a spinning body is given by

$$\frac{d\sigma}{d\theta} = \text{const.} \left[ \exp(-\theta/\omega\tau) + \exp((2\pi - \theta)/\omega\tau) \right]$$

where $\tau$ is the mean lifetime and $\omega$ is the angular frequency. In the fusion-fission process the spinning body is the compound nucleus itself; in a deep inelastic process, the spinning body is the rotating dinuclear complex. The experimentally obtained angular distributions from the $^{40}\text{Ca}(^{16}O,^{28}\text{Si}^*)^{28}\text{Si}^*$ reaction will be used in this section to obtain limits on the lifetime of the intermediate state formed for either process.

5.4.1 Angular frequency of the Compound Nucleus

The angular frequency of the rotating $^{56}\text{Ni}$ compound nucleus, $\omega_{\text{cm}}$, can be calculated if simplifying assumptions about the shape and angular momentum of the nucleus are made. The compound nucleus spin will be
assumed to be $J = 35$, a value in the middle of the range defined by $L_{\min}$ and $L_{\max}$. The shape of the nucleus will be assumed to be the LDM saddle point shape, with the moment of inertia $[Co74] \Theta_{cm} = 40 \Theta_o$ where $\Theta_o$ is the moment of inertia of a rigid sphere. The value of $\Theta_o$, assuming a radius parameter $R_o = 1.16 \text{ fm}$, is $441 \text{ u fm}^2$, so that

$\Theta_{cm} = 1760 \text{ u fm}^2$. Finally, since $J = \Theta \omega$, the angular frequency of the compound nucleus is $1.3 \times 10^{21} \text{ sec}^{-1}$.

5.4.2. Angular Frequency of the Rotating Dinuclear Complex

The angular frequency of the rotating dinuclear complex, $\omega_{di}$, will be calculated using the scattering model of Tsang [Ts74] together with the additional assumption of a strong friction force, implying no rolling or sliding motion of the two nuclei relative to each other while they are in contact ("sticking limit"). This added assumption is motivated by the fact that in deep inelastic scattering, the relative kinetic energy of the interacting nuclei is almost completely damped out. In Tsang's model, the reacting nuclei move in a common potential and are subject to a friction force which conserves total angular momentum, while coupling the angular momentum of relative motion to the spins of the individual nuclei. The equations of motion are then solved for the angular momenta transferred from the initial angular momentum of relative motion, $L_0$, to the reacting nuclei, $L_1$ and $L_2$, and the angular momentum of rigid rotation of the two nuclei about the center of mass of the dinuclear complex, $L_r$. In the sticking limit, $L_1$, $L_2$ and $L_r$ are given by
where \( \mu \) is the reduced mass and \( R \) is the separation of the centers of the two nuclei.

For the purpose of this calculation, the \( ^{40}\text{Ca}(^{16}\text{O},^{28}\text{Si}^*)^{28}\text{Si}^* \) reaction may be modeled as the collision of \( ^{16}\text{O} + ^{40}\text{Ca} \) nuclei followed by a fast exchange of nucleons which forms the two \( ^{28}\text{Si} \) nuclei, which then rotate together before separating. The distance between the two rotating nuclei in similar systems has been found by several groups [Sh84, R185] to be roughly parametrized by \( R_{12} = R_1 + R_2 + 2 \text{ fm} \). If the initial angular momentum is chosen to be \( L_0 = 35 \) (for the same reason as in the compound nuclear case) then for a dinuclear system composed of two spherical \( ^{28}\text{Si} \) nuclei, \( \omega_{d1} = 1.4 \times 10^{21} \text{ sec}^{-1} \).
5.4.3 Lifetimes

Figure 5.7 shows an angular distribution from the $^{40}_{\text{Ca}}(^{16}_{\text{O}},^{28}_{\text{Si}}{^*})^{28}_{\text{Si}}{^*}$ reaction and curves calculated using eq. (5.24) with $\omega_{d1}$ for different values of the mean lifetime $\tau$ (within the accuracy of this calculation, $\omega_{cm}$ and $\omega_{d1}$ are equal). Evidently, the shape of the angular distribution implies that $\tau_{cm} = \tau_{d1} \gg 10^{-19}$ sec. The data shown in Figure 5.6 cannot exclude the possibility that another much faster process characterized by a lifetime $\tau_f$ is also contributing to the $^{40}_{\text{Ca}}(^{16}_{\text{O}},^{28}_{\text{Si}}{^*})^{28}_{\text{Si}}{^*}$ reaction. Such a process would lead to a forward peaked $d\sigma/d\theta$ component (see eq. (5.24)). However, using eq. (5.24) and the fact that this component is too small to be seen at $\theta_{cm} = 50^\circ$, it is possible to set an upper limit on $\tau_f$. This limit is $\tau_f < 10^{-22}$ sec.

The calculated lifetimes fit consistently with the two pictures of the $^{40}_{\text{Ca}}(^{16}_{\text{O}},^{28}_{\text{Si}}{^*})^{28}_{\text{Si}}{^*}$ reaction. The value of $\tau_{cm}$ is of the order of magnitude expected for compound nuclei in this mass range and excitation energy range [Ba80, p. 214]. The fact that $\tau_{d1} \omega_{d1} \gg 1$ indicates that the dinuclear compound is very long-lived in the sense that it completes a large number of revolutions before scissioning. The time needed for nucleon exchange cannot be much smaller than $10^{-22}$ sec [Ba80, p. 215], so that the limit on $\tau_f$ is not stringent enough to rule out a fast, direct type process.
Figure 5.7

The angular distribution from the $^{40}\text{Ca}(^{16}\text{O},^{28}\text{Si}^*)^{28}\text{Si}^*$ reaction at a beam energy of 75.3 MeV. The dashed lines are calculations using eq. (5.24) for different volume of the mean lifetime; the angular frequency is $\omega_{\text{di}} = 1.4 \times 10^{21} \text{ sec}^{-1}$.
5.5 NUCLEAR SHAPES AT THE SCISSION POINT

5.5.1 Outline of Calculation

The simple model of the $^{40}\text{Ca}(^{16}_0,^{28}\text{Si}^*)^{28}\text{Si}^*$ reaction described in the previous section, together with energy balance considerations, can be used to extract information about the shape of the $^{28}\text{Si}$ nuclei at the moment of scission. The results obtained from this analysis should approximately hold for the fusion-fission picture as well, since just prior to symmetric fission, the highly deformed compound nucleus is similar in shape to two rigidly rotating $^{28}\text{Si}$ nuclei. The calculation is semiclassical in nature, so that it can only be expected to hold for mean values of measured quantities.

Energy conservation requires that the total energy before the collision be equal to the total energy at the scission point, which depends on the deformation of the two $^{28}\text{Si}$'s and their separation. This requirement can be expressed as

$$E_{\text{cm}} = \Delta BE + E_{\text{pot}} + E_{\text{rot}} + V_1 + V_2$$  \hspace{1cm} (5.27)$$

where $\Delta BE$ is the difference in binding energies of the entrance ($^{16}_0 + ^{40}\text{Ca}$) and exit ($^{28}\text{Si} + ^{28}\text{Si}$) channels, $E_{\text{pot}}$ is the potential energy of the two nucleus system, $E_{\text{rot}}$ is the rotational energy of orbiting motion, $V_1$ and $V_2$ are the excitation energies of the two $^{28}\text{Si}$'s and $E_{\text{cm}}$ is the center of mass bombarding energy. It is assumed that at
the scission point, the frictional force damps out all radial motion. The experimentally measured Q-value is just $-(\Delta E + U_1 + U_2)$, so that eq. (5.27) can be rewritten as

$$E_{\text{pot}} + E_{\text{rot}} = E_{\text{cm}} + Q \quad (5.28)$$

Because the right hand side of eq. (5.28) consists of quantities that are experimentally measured, this model can be used to determine the quantity $E_{\text{pot}} + E_{\text{rot}}$ which in turn can be used to extract information about the nuclear deformation as described below.

5.5.2 Nuclear Shape Parametrization and Moments of Inertia

The shape of the $^{28}\text{Si}$ can be parametrized in terms of an ellipsoid of revolution. The lengths of the half-axes expressed in terms of the Hill-Wheeler elongation parameter, $\beta$, [Ri80, p. 8], are given by

$$a = R_0 \exp \{ \beta (5/4\pi)^{1/2} \} \quad (5.29)$$

$$b = R_0 \exp \{ -0.5 \beta (5/4\pi)^{1/2} \} \quad (5.30)$$
where $2a$ is the length of the symmetry axis of the ellipsoid and $2b$ is the length of the other two axes. Positive values of $\beta$ imply prolate deformations and negative values imply oblate deformations. The axis ratio can be calculated using eqs. (5.29) and (5.30) and is given by

$$a/b = \frac{(4\pi/5)^{1/2} + \beta}{(4\pi/5)^{1/2} - 0.5\beta}$$  \hspace{1cm} (5.31)$$

for small values of $\beta$.

The moment of inertia of an ellipsoid of revolution with half-axes of length $a$, $b$ and $b$ about one of the two identical axes (revolution about an axis of symmetry is quantum mechanically forbidden) is given by

$$\Theta = 0.2MR^2 \left\{ (a/b)^{4/3} + (b/a)^{2/3} \right\}$$  \hspace{1cm} (5.32)$$

where $R$ is the radius of a sphere with a volume equal to that of the ellipsoid. For nuclei, $R$ is the sharp sphere radius, $R = r_0A^{1/3}$ with $r_0 = 1.16$ fm [Kr79]. Davis and Nix [Da767] calculated the correction to the moment of inertia, $\Delta \Theta_d$, caused by the diffuseness of the nuclear surface; it is given by

$$\Delta \Theta_d = 2Md^2$$  \hspace{1cm} (5.33)$$
where $M$ is the mass of the nucleus and $d$ is the surface diffusivity ($d = 1$ fm for nuclei [Bl77]). Combining eqs. (5.32) and (5.33) gives the result for $\Theta$ in $u\cdot fm^2$

$$\Theta = 0.2AR^2 \left\{ \left(\frac{a}{b}\right)^{4/3} + \left(\frac{b}{a}\right)^{2/3} \right\} + 2A, \quad (5.34)$$

where $A$ is the atomic number of the nucleus. If the diffusivity corrections are neglected and $a = b$, then eq. (5.34) reduces to the well-known result for the moment of inertia of a sphere, $\Theta = 0.4MR^2$.

5.5.3 Potential and Rotational Energies

The rotational energy term, $E_{rot}$, can be expressed as

$$E_{rot} = \frac{L_r(L_r + 1) \hbar^2}{2\mu(R_{12} + s)^2} \quad (5.35)$$

where $R_{12} = r_0(A^{1/3} + A^{1/3})$ with $r_0 = 1.16$ fm, $s$ is the distance between the surfaces of the nuclei and $L_r$ is the angular momentum of orbiting motion. $L_r$ is given by eq. (5.26) where the moments of inertia are now functions of $\beta$ and are calculated using eqs. (5.31) and (5.34).
The potential energy term, $E_{\text{pot}}$, is a sum of the nuclear potential $V_n$ and the Coulomb potential energy $V_c$. A double-folding potential of Krappe et al. [Kr79] which depends explicitly on the deformation parameter $\beta$ and on $s$ is used to calculate $V_n(\beta, s)$. In the range of $\beta$ and $s$ values of interest ($\beta = 0 - 0.3$ and $s = 1 - 3$ fm) $V_n$ is small, typically about $-2$ MeV. The Coulomb energy of two ellipsoids is approximately given by [Ha78]

$$V_c = \frac{Z_1 Z_2 e^2}{r_{12} + s} \left( 1 + \frac{6}{5} \cdot \frac{(a/b) - 1}{(a/b) + 2} \cdot \frac{R_1^2 + R_2^2}{(r_{12} + s)^2} \right)$$  \hspace{1cm} (5.36)$$

where eq. (5.31) connects $a/b$ with $\beta$. This term is much larger in absolute value than $V_n$, and in the $\beta$ and $s$, range of interest is about 30 MeV.

5.5.4 Results of Calculation

The calculations using eq. (5.28) (Figure 5.8 shows the relevant geometry) were performed at a beam energy of 75.675 MeV, where the mean $Q$-value was best determined, $Q = -16.5 \pm 0.7$ MeV (see Appendix B). The results, sets of related values of $s$ and $\beta$ satisfying eq. (5.28) for angular momenta between 32 and 38, are plotted in Figure 5.9. Figure 5.10 shows the effect of the uncertainty in $Q$ on the calculation for $I = 35$. There is additional evidence that $s = 2$ fm. Ritzka et al. [Ri85] studied a similar heavy ion system, $^{16}_0 + ^{48}_\text{Ti}$, and found that a
Figure 5.8

A cross section of two ellipsoidal nuclei with their axes of symmetry along the line joining the centers of mass of the nuclei. The axis ratio for each ellipsoid is 1.5:1.
A contour plot of $\beta$ and $s$ values that satisfy eq. (5.28) for different values of the initial angular momentum (see text).
Figure 5.10
A plot showing the effect of uncertainty in Q on the s and β values that satisfy eq. (5.28). The solid line is the \( \lambda = 35 \) contour for \( Q = -16.5 \) MeV, the dashed lines are contours for \( \lambda = 35 \) and \( Q = -16.5 \pm 0.7 \) MeV.
value of $s$ close to 2 fm was needed to fit the observed spins of the outgoing fragments as well as the angular distribution of the fragments [Tr84]. In addition, the rotating liquid drop model [Co74] predicts a saddle-point for $^{56}$Ni in the spin and excitation energy range of interest that corresponds to $s \sim 2$ fm and $\beta \sim 0.2$ (the difference between saddle-point and scission point configurations is expected to be small in a system as light as $^{56}$Ni [Da77]).

If in this model the value of $s = 2$ is chosen, then for $L_0 = 35$ the $^{28}$Si nuclei will have prolate shapes with $\beta = 0.17$. At $s = 2$ fm, the uncertainty in $\beta$ due to the spread in $\langle Q \rangle$ is roughly $\pm 0.08$, so that $a/b = 1.17 \pm 0.08$. A configuration of two $^{28}$Si nuclei with these values of $\beta$ and $s$ joined along their symmetry axis corresponds to a highly deformed $^{56}$Ni with a 3:1 axis ratio.
The decay of compound nuclear states characterized by a width \( \Gamma \) and spin \( J \) can give rise to fluctuations in the excitation function. In this section, the statistical theory of the decay of the compound nucleus will be applied to calculate these fluctuations. The important quantities to be calculated are the coherence width, \( \Gamma_c(U,J) \) and the level density, \( \rho(U,J) \), of the compound nuclear states at a given excitation energy, \( U \), and spin, \( J \). It will be shown that \( \Gamma_c \) and \( \rho \), for the region of interest in \( ^{56}\text{Ni} \), are inconsistent with the type of structures observed in the \( ^{40}\text{Ca}(^{16}\text{O},^{28}\text{Si}^*)^{28}\text{Si}^* \) reaction, thus ruling out statistical compound nucleus fluctuations as the cause of these structures.

The coherence width of a compound nucleus as a function of excitation energy and spin is given by [Er60]

\[
\Gamma_c(U,J) = \left( \frac{2 \pi \rho_{\text{cn}}(U,J)}{2 \pi} \right)^{-1} \varepsilon_{\text{max}} \sum_{\nu} \int_{0}^{\varepsilon_{\text{max}}} \varepsilon \, d\varepsilon \, T_{\nu}(\varepsilon) \varepsilon \sum_{s=|J-l|}^{J+l} \rho_{\nu}(E_{\nu},j) \tag{5.37}
\]

where \( \rho_{\text{cn}}(U,J) \) is the level density of the compound nucleus at an excitation energy \( U \) and spin \( J \), \( \nu \) is an index which labels the different emitted particles, \( i_{\nu} \) is the spin of the emitted particle, \( T_{\nu}(\varepsilon) \) is the transmission coefficient, \( j \) is the spin of the level populated in the corresponding residual nucleus and \( \rho_{\nu}(E_{\nu},j) \) the corresponding level density and \( \varepsilon_{\nu} \) is the kinetic energy of particle \( \nu \) in the compound nucleus rest frame, and \( E_{\nu} \) the excitation energy of the residual
nucleus. These quantities are related by

\[ \epsilon_v + E_v + B_v + E_r = U \]  (5.38)

where \( B_v \) is the binding energy of particle \( v \), and \( E_r \) the recoil energy of the residual nucleus. Barrette et al. [Ba79] found that the contribution to \( \Gamma_c \) from particles other than neutrons, protons and \( \alpha \)-particles is negligible, and consequently all complex fragments were neglected in the calculation.

The key element in the calculation is the evaluation of the expressions for the transmission coefficients and the level densities. The transmission coefficients are parametrized by

\[ T_1(\epsilon_v) = \frac{C_v}{1 + \exp((B_{1v} - \epsilon_v)/\Delta_{1v})}. \]  (5.39)

where the parameters \( C_v, B_{1v}, \) and \( \Delta_{1v} \) are obtained by fitting eq. (5.39) to the optical model transmission coefficients [Go81]. A back-shifted Fermi gas level density [Di73] of the form

\[ \rho(U,J) = \frac{1}{24\sqrt{2}} \frac{2J + 1}{\sigma a^{1/4}} \frac{\exp\{2(U-a)^{1/2} - J(J+1)/2\sigma^2\}}{(U - \Lambda + t)^{5/4}} \]  (5.40)
was adopted, where the thermodynamic temperature, \( t \), is defined by [La54]

\[
U - \Delta = at^2 - t
\]  
(5.41)

The spin cutoff parameter, \( \sigma \), is given (for large excitation energies) by [Di73]

\[
\sigma^2 = 0.015tA^{5/3}
\]  
(5.42)

A parametrization by Dilg et al. [Di73] was used to obtain the values of the level density parameter, \( a \), and the backshift parameter, \( \Delta \).

A note of caution must be added about the use of the parameters, \( a \), \( \Delta \), and \( \sigma \) with eq. (5.40). The parameters \( a \) and \( \Delta \) have been extracted by fitting data for large numbers of nuclei at excitation energies from 10 to 30 MeV, while \( \sigma \) has been derived from purely theoretical considerations [Di73]. The use of these parameters for this calculation (\( U = 68 \) MeV) is an extrapolation beyond the range where the parameters have been tested. Nevertheless, since eq. (5.37) involves only the ratios of level densities of similar nuclei at similar excitation energies, the results obtained from it should be less sensitive to the errors in the parameters than the level densities themselves.

Calculations using eq. (5.37) were carried out for \( ^{56}\text{Ni} \) for excitation energies covered in the present study of the \( ^{40}\text{Ca}(^{16}\text{O},^{28}\text{Si}^*)^{28}\text{Si}^* \) reaction excitation function (\( U = 69 \) MeV) and for spins between 32 and 37. The calculations show that \( \Gamma_c \) is of the order
of 400 - 500 keV in this region. The average level spacing, \( D(U,J) \), is just the inverse of \( \rho(U,J) \) so that it is readily computed, although owing to the uncertainty in the parameter values, as noted above, it is no better than an order of magnitude estimate. In this region \( D(U,J) \) is of the order of eV, which shows that the compound nuclear states are strongly overlapping. With \( \Gamma_c \gg D \), no isolated resonances are expected in the cross section. It is evident from considering Figure 4.19 that the measured excitation function is not consistent with the statistical fluctuation picture. The excitation function is generally smooth with only a few isolated structures whose widths are at least a factor of 2 smaller than \( \Gamma_c \).
5.7 ANALYSIS OF STRUCTURES IN THE EXCITATION FUNCTION

Important insight into the structure of a resonance can be gained by measuring its partial decay widths, that is the fractions of the total width that go into the various decay channels. The statistical analysis of section 4.9.2 suggests a correlation between structures in the \( {}^{28}\text{Si}(^{28}\text{Si}, ^{28}\text{Si}^*)^{28}\text{Si}^* \) and \( {}^{40}\text{Ca}(^{16}\text{O}, ^{28}\text{Si}^*)^{28}\text{Si}^* \) reaction excitation functions at several energies. If the tentative evidence of correlations is accepted, the structures near \( E_{\text{cm}} = 67.4, 68.2 \) and \( 69.2 \) MeV in the \( {}^{40}\text{Ca}(^{16}\text{O}, ^{28}\text{Si}^*)^{28}\text{Si}^* \) reaction are evidence for the existence of resonances in the \( ^{56}\text{Ni} \) compound nucleus, as suggested by the deformed shell model calculations in section 5.2. Extraction of the elastic \( ^{16}\text{O} + ^{40}\text{Ca} \) partial width from the data and a comparison to the \( ^{28}\text{Si} + ^{28}\text{Si} \) partial width is a first step in exploring the spectroscopy of these resonances.

5.7.1 Resonant Part of the Cross Section

A strong assumption about the relative phases of the resonant and nonresonant amplitudes in the \( {}^{40}\text{Ca}(^{16}\text{O}, ^{28}\text{Si}^*)^{28}\text{Si}^* \) reaction must be made if the elastic \( ^{16}\text{O} + ^{40}\text{Ca} \) width can be extracted from the excitation function. This assumption allows the cross section, \( \sigma_{\text{tot}} \), to be divided into a resonant, \( \sigma_{\text{res}} \), and a nonresonant, background part, \( \sigma_{\text{bg}} \). The three terms are related by

\[
\sigma_{\text{tot}} = \sigma_{\text{res}} + \sigma_{\text{bg}} \quad (5.43)
\]
An estimate of $\sigma_{\text{res}}$ can then be made from inspection of the excitation function and can be used in the calculation of the widths.

Eq. (5.43) is motivated in the following way. The term resulting from interference between $\sigma_{\text{res}}$ and $\sigma_{\text{bg}}$, in a reaction with a large number of possible final states, is a sum of a large number of dot products between the resonant and nonresonant amplitudes. If the phase angles of the resonant and nonresonant amplitudes in the various final states accessible to the system are uncorrelated, then this sum will, on the average, be 0.

5.7.2 Resonant Cross Section and Widths

The observed widths can be related to $\sigma_{\text{res}}$ at the resonance energy by [B152]

$$\sigma_{\text{res}} = 4\left(\frac{\pi}{k^2}\right) (2L_{\text{res}} + 1) \frac{\Gamma_{\text{in}} \Gamma_{\text{out}}}{\Gamma_{\text{tot}}^2}$$

(5.44)

where $\Gamma_{\text{in}}$ and $\Gamma_{\text{out}}$ are the partial widths in the entrance and exit channels, respectively, and $\Gamma_{\text{tot}}$ is the total resonance width. However, in addition to information about the dynamics of the reaction, $\Gamma_{\text{in}}$ and $\Gamma_{\text{out}}$ also contain Coulomb and centrifugal barrier penetrability factors, $P_f$. A quantity which contains only the dynamical information about the reaction is the reduced width, $\gamma^2$. The reduced width, for a given channel, is proportional to the absolute square of the matrix element $\langle \bar{\psi} | \psi_{\text{res}} \rangle$, where $\psi_{\text{res}}$ is the wave function of the resonance.
and \( \Psi \) is the wave function in that channel. More simply, \( \gamma^2 \) is the overlap between the resonance and channel wave functions. The reduced width is given by

\[
\gamma^2 = \frac{\Gamma}{2P_x^2}
\]

(5.45)

the penetrability, \( P_x \), can be written in terms of the regular and irregular Coulomb wave functions

\[
P_x = \frac{kR}{\sqrt{P_x^2 + G_x^2}}
\]

(5.46)

The radius \( R \) roughly corresponds to the distance at which the nuclear potential becomes negligible. In this calculation, \( R \) was taken to be \( 1.6 \left( A_1^{1/3} + A_2^{1/3} \right) \) fm.

5.7.3 Calculation of the \( ^{28}\text{Si} + ^{28}\text{Si} \) Elastic Width

The calculation of the width requires the values of the \( L_{\text{res}} \), \( \Gamma_{\text{tot}} \), and \( \sigma_{\text{res}} \). \( L_{\text{res}} \) of the resonance at \( E_{\text{cm}} = 68.2 \) will be taken as 36. The angular momentum of the broad structure in the \( ^{28}\text{Si}(^{28}\text{Si},^{28}\text{Si}^*)^{28}\text{Si}^* \) reaction excitation function at that energy was identified by Betts et al. [Be81a] as being in the range between 36 and 38. If the lower value is taken, then \( L_{\text{res}} \) falls within the allowed range for the \( ^{40}\text{Ca}(^{16}\text{O},^{28}\text{Si}^*)^{28}\text{Si}^* \) reaction (see section 5.3). \( \Gamma_{\text{tot}} \) estimated from Figure 4.19, is roughly 200 keV.
The value of $\sigma_{\text{res}}$, also estimated from Figure 4.19, is 0.06 mb. Eq. 5.44 can now be evaluated and

$$\Gamma_{\text{rel}}   \Gamma_{\text{tot}} = 7.7 \text{ keV}^2$$

(5.47)

where $\Gamma_{\text{in}}$ is the elastic $^{16}O + ^{40}Ca$ partial width and $\Gamma_{\text{out}}$ is the partial width for the decay into any elastic or inelastic $^{28}Si + ^{28}Si$ channel.

The total silicon partial width, $\Gamma_{\text{tot}}^{^{28}Si + ^{28}Si}$, needed to evaluate $\Gamma_{\text{rel}}^{^{16}O + ^{40}Ca}$ has not been measured, but an estimate of it can be made from the results of a study of a similar reaction. The elastic and inelastic scattering of $^{24}Mg + ^{24}Mg$ shows the same characteristics as the $^{28}Si + ^{28}Si$ system. The excitation function has broad, 2-3 MeV wide structures, with angular momenta close to the grazing angular momenta, which are in turn broken up into isolated structures with widths of about 200 keV. All the narrower structures are extremely well correlated in the various elastic and inelastic channels [Zu83]. The distribution of the partial widths into these channels has been studied [Be84b], with the result that the total $^{24}Mg + ^{24}Mg$ width was found to be approximately 30% of $\Gamma_{\text{tot}}$.

If $\Gamma_{\text{tot}}^{^{28}Si + ^{28}Si}$ is estimated by $\Gamma_{\text{tot}}^{^{24}Mg + ^{24}Mg}/\Gamma_{\text{tot}}$, then $\Gamma_{\text{tot}}^{^{28}Si + ^{28}Si} = 60$ keV, and eq. (5.47) gives $\Gamma_{\text{rel}}^{^{16}O + ^{40}Ca} = 130$ ev. The value of $\Gamma_{\text{rel}}^{^{28}Si + ^{28}Si}$ has been estimated by Saini and Betts [Sa84] to be 1-2 keV, so that the ratio of the elastic reduced widths $\gamma_{^{28}Si + ^{28}Si}/\gamma_{^{16}O + ^{40}Ca}$ is approximately 7. This indicates that the resonance wave function is more closely related to that of two touching $^{28}Si$ nuclei than the touching $^{16}O + ^{40}Ca$ nuclei.
This result is in qualitative agreement with the deformed shell model calculations of section 5.2. The ratio of the elastic reduced widths in each channel to the Wigner limit [Te52] in that channel is less than 0.1% for both \(^{28}\text{Si} + ^{28}\text{Si}\) and \(^{16}\text{O} + ^{40}\text{Ca}\), indicating that the resonance is not a simple molecular configuration. The various quantities of interest are tabulated in Table 5.1.
Table 5.1

Parameters for Structure near $E_{cn} = 68.2$ MeV

<table>
<thead>
<tr>
<th>System</th>
<th>$\Gamma$ (keV)</th>
<th>$\gamma^2$ (eV)</th>
<th>$\gamma^2/T_W$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{16}_O + ^{40}_Ca$</td>
<td>0.13</td>
<td>5</td>
<td>$7 \times 10^{-5}$</td>
</tr>
<tr>
<td>$^{28}_Si + ^{28}_Si$</td>
<td>1-2</td>
<td>23-45</td>
<td>$7 \times 10^{-4}$</td>
</tr>
</tbody>
</table>

$T_W$ is the Wigner limit.
CHAPTER 6

CONCLUSIONS

In this study, the excitation function of the $^{40}\text{Ca}(^{16}\text{O},^{28}\text{Si}^*)^{28}\text{Si}^*$ reaction has been measured in 75 keV steps from 73.95 to 77.25 MeV. At each energy, the angular distributions and Q-value distributions were measured. The $1/\sin(\theta_{\text{cm}})$ shape of the angular distribution and the fact that the Q-value is independent of center of mass angle indicate that the reaction proceeds through a long-lived, equilibrated intermediate state. The excitation function has a smooth component which increases linearly with bombarding energy and a fluctuating component which produces isolated structures ($\Gamma = 200$ keV), with a magnitude of approximately 3% of the size of the smooth component.

The smooth background yield from the $^{40}\text{Ca}(^{16}\text{O},^{28}\text{Si}^*)^{28}\text{Si}^*$ reaction has been analyzed in terms of two models: the fusion-fission model and the deep inelastic model. The data from this work are equally consistent with either model. However, results from a recent study of inclusive cross sections of the $^{16}\text{O} + ^{40}\text{Ca}$ reaction leading to fragments with $20 < A_{\text{fragment}} < 28$ and of the $^{16}\text{O} + ^{40}\text{Ca}$ reaction leading to fragments with $20 < A_{\text{fragment}} < 30$, at beam energies between 67.3 and 87.3 MeV [Sa85c], suggest that the fusion-fission process represents at least a substantial part of the cross sections for the two reactions.

Sanders et al. [Sa85c] calculated the expected fission yields for $^{56}\text{Ni}(^{16}\text{O} + ^{40}\text{Ca})$ and $^{60}\text{Ni}(^{16}\text{O} + ^{40}\text{Ca})$ using fission barriers generated with the Krappe-Nix-Sierk potential [Kr79]; these barriers include the
effects of surface diffuseness and the finite range of the nuclear force. These effects result in substantially lower fission barriers and, consequently, a larger fission yield than estimates based on the standard liquid drop model. The energy dependence of the calculated fission yields tracks closely with the measured energy dependence of the fission-like yield. This indicates that the fusion-fission process is an important part of the cross section.

The analysis of the structures in the excitation function suggests that they may be correlated with structures in the $^{28}\text{Si}(^{28}\text{Si},^{28}\text{Si}^*)^{28}\text{Si}^*$ reaction excitation function. Under the assumption that the structures are correlated, the size of the fluctuations, compared to those in the $^{28}\text{Si} + ^{28}\text{Si}$ system, was related to the ratio of reduced widths of the resonance to the $^{16}\text{O} + ^{40}\text{Ca}$ and $^{28}\text{Si} + ^{28}\text{Si}$ elastic channels. The reduced width in the symmetric $^{28}\text{Si} + ^{28}\text{Si}$ elastic channel was found to be approximately an order of magnitude larger than the reduced width in the $^{16}\text{O} + ^{40}\text{Ca}$ elastic channel. These results are in qualitative agreement with the predictions of the deformed shell model calculations for $^{56}\text{Ni}$.

Deformed shell model calculations are qualitatively confirmed by results of studies of other Ni isotopes as well. The calculations predict a strong superdeformed minimum at high spins for $^{56}\text{Ni}$ and the gradual disappearance of this minimum, with increasing neutron number, in $^{58}\text{Ni}$ and $^{60}\text{Ni}$ [Be84a]. The existence of resonant structures in the $^{28}\text{Si} + ^{28}\text{Si}$ system is well documented [Sa84]; while the evidence for the existence of corresponding resonant structures in the $^{16}\text{O} + ^{40}\text{Ca}$ system is not conclusive, it does suggest that they exist. The data for the other Ni isotopes are shown in Figure 6.1. The $^{28}\text{Si} + ^{28}\text{Si}$ ($^{56}\text{Ni}$)
Figure 6.1

A plot showing the excitation function from the $^{28}_{}$Si + $^{28}_{}$Si , $^{28}_{}$Si + $^{30}_{}$Si and $^{30}_{}$Si + $^{30}_{}$Si reactions. The experimental uncertainties for the $^{28}_{}$Si + $^{30}_{}$Si and $^{30}_{}$Si + $^{30}_{}$Si systems are the same as the ones plotted for the $^{28}_{}$Si + $^{28}_{}$Si system.
\[ 28\text{Si} + 28\text{Si} \]
\[ 28\text{Si} + 30\text{Si} \]
\[ 30\text{Si} + 30\text{Si} \]
elastic and inelastic excitation function is characterized by a large number of both broad ($\Gamma = 2\text{-}3\text{ MeV}$) and narrow ($\Gamma = 200\text{ keV}$) structures. The $^{28}\text{Si} + ^{30}\text{Si}$ excitation function is much smoother, although a hint of broader structures remains, and, finally, the $^{30}\text{Si} + ^{30}\text{Si}$ excitation function is completely smooth within the experimental resolution.

Another system where the deformed shell model calculations have been qualitatively borne out is the $^{48}\text{Cr}$ system. The calculated energy surfaces of $^{48}\text{Cr}$ at high spin are very similar to $^{56}\text{Ni}$ [Be84a]. The $^{24}\text{Mg}(^{24}\text{Mg},^{24}\text{Mg}^*)^{24}\text{Mg}^*$ reaction cross section (as was discussed in section 5.7.3) is very similar in features to the $^{28}\text{Si}(^{28}\text{Si},^{28}\text{Si}^*)^{28}\text{Si}^*$ reaction cross section. The broad structures in the excitation function, characterized by angular momenta close to the grazing angular momenta, are broken up into much narrower resonant structures. In addition, structures correlated with those in the $^{24}\text{Mg}(^{24}\text{Mg},^{24}\text{Mg}^*)^{24}\text{Mg}^*$ reaction have been found in $1\alpha$ and $2\alpha$ transfer channels, $^{24}\text{Mg}(^{24}\text{Mg},^{20}\text{Ne}^*)^{28}\text{Si}^*$ and $^{24}\text{Mg}(^{24}\text{Mg},^{16}\text{O}^*)^{32}\text{S}^*$ reaction cross sections [Sa85d].

The evidence for the existence of shape isomers can only be qualitative at present. No calculations of the spectrum of levels built upon the superdeformed minimum or their partial widths are currently available. Once such calculations are performed, the large body of data available for the $^{56}\text{Ni}$, $^{58}\text{Ni}$, $^{60}\text{Ni}$ and $^{48}\text{Cr}$ systems can be used for quantitative comparison with the deformed shell model predictions.
APPENDIX A

As can be seen from Figure 4.11a, the values of $\langle Q \rangle$ for runs with window 2 are about 0.8 MeV more positive than those with the other windows. This effect is most likely caused by a slight difference in thickness of the polypropylene entrance window. It is possible to estimate the difference needed to account for the observed effect by making some simplifying assumptions. Using eq. (4.5) for runs at the same beam energy (windows 2 and 5) yields

$$dQ = (E_{W2} - E_{W5}) \cdot (1 + \frac{\sin^2 \theta_3}{\sin^2 \theta_4})$$  \hspace{1cm} (A.1)

where $E_{W1}$ is the energy measured by the detector with window 1 in place. The factor containing $\theta_3$ and $\theta_4$ does not change greatly for events with the same $\theta_3$ (laboratory frame angle of motion of the detected Si ion) but with different Q-values. The average value of this factor when folded with the measured angular distribution is about 1.6. Substituting this value into eq. (A.1) gives $E_{W2} - E_{W5} = 0.5$ MeV. This means that on the average, 0.5 MeV more energy must be lost by the Si ion in window 5 than in window 2.

The energy loss, $\delta E$, is related to the thickness of the window, $t$, by

$$\delta E = (\frac{dE}{dt})t$$  \hspace{1cm} (A.2)
where \( \frac{dE}{dt} \) is the stopping power in MeV/\( \mu g/cm^2 \) and \( t \) is in units of \( \mu g/cm^2 \). The stopping power for polypropylene (C\(_3\)H\(_6\)) is given by Bragg's rule

\[
\left( \frac{dE}{dt} \right)_{C_3H_6} = \frac{36}{42} \left( \frac{dE}{dt} \right)_C + \frac{6}{42} \left( \frac{dE}{dt} \right)_H
\]  

(A.3)

The stopping powers were obtained from the tabulation of Northcliffe and Schilling [No70]. The \(^{28}\)Si stopping power averaged over the observed energy spectrum is approximately 0.025 MeV/\( \mu g/cm^2 \). The typical polypropylene window thickness is 80 \( \mu g/cm^2 \) and, therefore, an energy loss of 0.5 MeV corresponds to 20 \( \mu g/cm^2 \). A difference in thickness of the order of 25% can account for the shift in \( <Q> \) values between the windows. This type of difference is well within the limits of accuracy inherent in foil stretching process (see section 3.2).
The true mean Q-values of the $^{40}\text{Ca}(^{16}\text{O},^{28}\text{Si}^*)^{28}\text{Si}^*$ reaction, $Q_t$, as a function of the bombarding energy cannot be simply extracted from the measured Q-value spectra. The simplest approximation to $\langle Q \rangle_t$ is $\langle Q \rangle$, the experimentally determined average Q-value, but the presence of contaminant events in the Q-value spectrum for $Q < -20$ MeV, makes $\langle Q \rangle$ more negative than $\langle Q \rangle_t$. The most probable value of the Q-value distribution is another approximation to $\langle Q \rangle_t$, but the difficulty with his prescription is that with the limited counting statistics and a bin size of 0.5 MeV, the most probable value may be up to 1.5 MeV away from $\langle Q \rangle_t$. An unsuccessful attempt was made to get $\langle Q \rangle_t$ using three different gaussian fits to the Q-value spectrum. The spectra were fit with a single Gaussian, a single skewed Gaussian and a double Gaussian with one peak near $Q = -17$ MeV and the other in the low Q-value tail region. Unlike $\langle Q \rangle$ values which vary smoothly with beam energy, the centroids from the various fits varied erratically by up to 1.5 MeV between runs separated by as little as 75 keV. The failure of this method must be attributed to the relatively low counting statistics in each Q-value bin and the unsuitability of Gaussians to fit the shape of the spectrum. Although the double Gaussian fits resulted in good fits to the data, with chi-squared per degree of freedom of about one, the relatively flat shape of the low Q-value tail made the position of the low Q Gaussian centroid highly variable, and this in turn made the position of the centroid of the main peak unstable as well.

The counting statistics were improved by adding together the spectra from three runs taken at the beam energy of 75.675 MeV (each
with a different window but with nearly the same \( \langle Q \rangle \). Using this composite spectrum, three methods were used to approximate \( \langle Q \rangle_t \): 1) a double Gaussian fit was performed; 2) an average Q-value was calculated (not over the entire Q-value range but only over a part of it which is roughly symmetrical, \( Q = -17 \pm 4 \) MeV; and 3) a value of \( \langle Q \rangle_t \) was chosen by visual inspection. The values given by these three methods were -16.65, -16.25 and -16.5 MeV, respectively; the average was -16.5 MeV. This value was adopted as \( \langle Q \rangle_t \) at the beam energy of 75.675 MeV. The uncertainty in \( \langle Q \rangle_t \) is estimated to be \( \pm 0.7 \) MeV.

Once \( \langle Q \rangle_t \) at one beam energy is known, a parameter \( K(E_{\text{beam}}) \) can be defined such that

\[
K(E) = \frac{\langle Q(E) \rangle_t}{\langle Q(E) \rangle}
\]  

Using the values of \( \langle Q \rangle_t \) and \( \langle Q \rangle \) at 75.675 MeV gives \( K(75.675 \text{ MeV}) = K_0 = 0.82 \). The parameter \( K(E) \) is simply the value by which \( \langle Q \rangle \) must be multiplied to give \( \langle Q \rangle_t \). The difference between the highest and lower beam energies during the experiment was 4%, so that the shapes of the Q-value spectra should change very little. Indeed, a study of the Q-value spectra showed that at the level of experimental uncertainty there is no difference in shape between the various spectra. To a good approximation, then, \( K(E) \) = \( K_0 = 0.82 \). The plot of \( \langle Q \rangle_t \) as a function of beam energy is shown in Figure 4.11b. The values from window 2 have been multiplied by the correction factor 1.04 (see Appendix A) and values at energies where more than one run was done were calculated by averaging the values from these runs.
BIBLIOGRAPHY


Ba80  R. Bass, Nuclear Reactions with Heavy Ions (Springer-Verlag, Heidelberg, 1980).


Be84a T. Bengtsson, M. Faber, M. Ploszajczak, I. Ragnarsson and S. Aberg, Lund MPh - 84/01 preprint.


Be85  R.R. Betts, private communication.


<table>
<thead>
<tr>
<th>Reference</th>
<th>Authors</th>
<th>Journal/Book Title</th>
<th>Volume/Issue</th>
<th>Page Numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Br78</td>
<td>D.A. Bromley</td>
<td>in <em>Nuclear Molecular Phenomena</em>, edited by N. Cindro</td>
<td>(North Holland, Amsterdam, 1978)</td>
<td>p. 3.</td>
</tr>
<tr>
<td>Er63</td>
<td>T. Ericson</td>
<td>Ann. of Phys.</td>
<td>23 (1963)</td>
<td>390.</td>
</tr>
<tr>
<td>Fr44</td>
<td>O.R. Frisch</td>
<td>BR - 49,</td>
<td></td>
<td>1944.</td>
</tr>
<tr>
<td>Reference</td>
<td>Author(s)</td>
<td>Journal/Book Title</td>
<td>Year</td>
<td>Pages</td>
</tr>
<tr>
<td>-----------</td>
<td>-----------</td>
<td>--------------------</td>
<td>------</td>
<td>-------</td>
</tr>
<tr>
<td>Ha78</td>
<td>R.W. Hasse</td>
<td>Pramana</td>
<td>1978</td>
<td>441</td>
</tr>
<tr>
<td>Im68</td>
<td>B. Imanishi</td>
<td>Phys. Lett.</td>
<td>1968</td>
<td>256</td>
</tr>
<tr>
<td>Im69</td>
<td>B. Imanishi</td>
<td>Nucl. Phys.</td>
<td>1969</td>
<td>33</td>
</tr>
<tr>
<td>Ma78</td>
<td>M.H. Macfarlane and S.C. Pieper</td>
<td>ANL-76-11</td>
<td>1978</td>
<td></td>
</tr>
<tr>
<td>Me73</td>
<td>W. Mendehall and R.L. Schaefer</td>
<td>Mathematical Statistics with Applications</td>
<td>1973</td>
<td></td>
</tr>
<tr>
<td>Mo57</td>
<td>S.A. Moszkowski</td>
<td>Handbuch der Physik, Vol. 39</td>
<td>1957</td>
<td>411</td>
</tr>
<tr>
<td>No70</td>
<td>L.C. Northcliffe and R.F. Schilling</td>
<td>Nuclear Data Tables 7</td>
<td>1970</td>
<td>233</td>
</tr>
<tr>
<td>Nu67</td>
<td>The Nuclear Data Acquisition Group</td>
<td>IBM, TN 21.575-21</td>
<td>1967</td>
<td></td>
</tr>
<tr>
<td>Ov69</td>
<td>J.C. Overley, P.D. Parker and D.A. Bromley</td>
<td>NIM 68</td>
<td>1969</td>
<td>61</td>
</tr>
<tr>
<td>P181</td>
<td>M. Ploszajczak</td>
<td>private communication</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ri74</td>
<td>A. Richter</td>
<td>in Nuclear Spectroscopy and Reactions</td>
<td>1974</td>
<td>343</td>
</tr>
<tr>
<td>R180</td>
<td>P. Ring and P. Schuck</td>
<td>The Nuclear Many Body Problem</td>
<td>1980</td>
<td></td>
</tr>
<tr>
<td>Sa77</td>
<td>F. Sauli</td>
<td>CERN 77-09</td>
<td>1977</td>
<td></td>
</tr>
</tbody>
</table>

Sa85b  S.J. Sanders, private communication.


Sa85d  S. Saini et al., to be published.


St83  J.D. Stinson, private communication.

Te52  T. Teichman and E.P. Wigner, Phys. Rev. 87 (1952) 123.


Th85  A. Thiel, private communication.


