ABSTRACT

High-K Isomers and
Mechanisms of K-Violation

Benjamin F. Crowell
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In studying the complex many-body problem of the structure of atomic nuclei, an important simplifying role is played by the identification of conserved quantum numbers. An example, in ellipsoidal nuclei, is the approximately conserved K quantum number, defined as the projection of the total angular momentum along the axis of symmetry. For a given total angular momentum, the states with the lowest energy are most commonly those with K=0, but in a small number of nuclei, high-K modes of excitation have been observed at relatively low energies.

Because of the approximate conservation of K, the decay of these high-K states is subject to selection rules, often causing them to be long-lived isomers. The high-K isomers are able to decay to states of lower K only because of very small admixtures in their wave-functions of the lower values of K. Until the last decade, it had been believed that the physical mechanisms of K-mixing, in the Coriolis force, were fairly well understood, and that the size of the mixing always decreased exponentially as a function of ΔK, as would be expected from perturbation theory. Recent experiments, however, using more sensitive experimental techniques, have uncovered a few instances in which highly K-violating gamma-ray transitions exist, with ΔK as large as 25 ħ.

This presents a conundrum from the point of view of the long-standing phenomenological models, since simple extrapolation from lower values of
\( \Delta K \) suggests that the partial half-lives of these decays should be many orders of magnitude longer than observed. It has proved difficult, however, to make generalizations about the mechanisms which might be responsible, both because of the relative rarity of these highly K-violating decays and because the theoretical models of these decays are still evolving. This thesis presents the results of experiments aimed at detecting such decays, and theoretical calculations to interpret the results. Techniques for detecting the rare decays of high-spin isomers are also developed.

In one experiment, two new high-spin isomers were observed in the nucleus \(^{176}\text{W}\). One, with \( K^\pi=14^+ \), shows an extremely unusual pattern of decay, with several direct branches to \( K=0 \) states. In the few cases where such highly K-violating transitions were previously observed, they were never more than a small fraction of the total decay, with the rest proceeding through states of intermediate \( K \). This isomer, in contrast, shows no detectable decay to any of the available states with \( K>0 \) that were mapped out in this study. In a second experiment, a strict upper limit is placed on a \( K=19 \rightarrow K=0 \) branch in \(^{176}\text{Hf}\).

The newly observed decays in \(^{176}\text{W}\) are discussed in terms of detailed theoretical calculations, with a goal of distinguishing Coriolis mixing and tunneling in the shape degrees of freedom as two possible distinct mechanisms for K-violation. The decays of three different \( K=14 \) states in \(^{176}\text{W}, ^{174}\text{Hf}, \) and \(^{178}\text{W}\) are discussed as a group. While previous models of highly K-violating decays using Coriolis mixing give predictions which are not consistent with the new data, a surprisingly good explanation of the systematics of these decays is achieved through models of tunneling.
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Mechanisms of K-Violation

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Chapter 1
Introduction

1.1 The Challenge of Nuclear Structure

Nuclear structure is a complex many-body problem which has been studied nearly since the advent of quantum mechanics. Efforts towards a comprehensive picture of nuclear spectroscopy have proceeded more slowly than those, for instance, in molecular spectroscopy. Comparison of the time-scales of nuclear and molecular motion helps to explain why nuclear structure has proved the more difficult problem for theoretical treatment. Typical time-scales for rotational, vibrational, and single-particle excitations in nuclei (with mass number $A = 180$) and diatomic molecules [Ner 90] are:

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<th>nuclear</th>
<th>diatomic</th>
<th>molecules</th>
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<td>rotation</td>
<td>$10^{-21}$-$10^{-20}$ sec</td>
<td>$10^{-11}$ sec</td>
<td></td>
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<tr>
<td>vibration</td>
<td>$10^{-21}$ sec</td>
<td>$10^{-13}$ sec</td>
<td></td>
</tr>
<tr>
<td>single-particle</td>
<td>$10^{-21}$ sec</td>
<td>$10^{-15}$ sec</td>
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(Here the time-scales have been defined for vibration as $\omega = \frac{E_{\text{phonon}}}{\hbar}$, for rotation according to the cranking expression $\omega = \frac{(E(J) - E(J-2))}{2\hbar}$ (see chapter 2), and for single-particle excitations as $\hbar/(E_X - E_{g.s.})$, where $E_X$ is the energy of the whole system.¶) In molecules, the different types of excitations occur on very different time-scales, and are therefore approximately independent.

¶ This is the quantity of interest for the dynamics, not $\hbar(E_{\text{unbound}} - E_{\text{single-particle}})$, which is much smaller.
In nuclei, the modes of excitation are, in the most general case, intimately linked. Thus, nuclei are excellent subjects for the study of the rich interplay of collective (rotational and vibrational) and single-particle excitations. If one hopes to identify simpler cases for study among nuclei, the best place to look is where the time-scale for rotation is significantly longer than the time-scales for vibration and single-particle motion. In such cases, it may be justified to assume that the coupling of rotation to the other modes is adiabatic.

Our understanding of these complex systems can be helped by identifying approximately conserved quantum numbers, in addition to the total angular momentum and parity, $J^\pi$, which are strictly conserved because of the properties of the strong force. Such approximately conserved quantum numbers include isospin in light nuclei, and the K quantum number in deformed nuclei. The K quantum number is the focus of this thesis, and will be defined in section 1.3 after the introduction of the necessary preliminary concepts.

1.2 The Mean-Field Approach and the Shell Model

The mean-field approximation is the starting point in almost all models for simplifying the nuclear many-body problem. A strong motivation for this approach comes from the observation that nuclei with the neutron number, $N$, or proton number, $Z$, equal to the magic numbers 2, 8, 20, 28, 50, 82 or 126 are extremely stable. This is analogous to the stability of the noble gases, atoms with the number of electrons equal to 2, 10, 18, 36, 54 or 86. The magic numbers point to the importance of shell structure in
nuclei, which in quantum mechanics is a characteristic of particles bound in a confined space with a mean free path which is long compared to the linear dimensions of the space.\textsuperscript{†} In the mean-field approach, one assumes that the confining force on any given nucleon can be approximated by a potential well, understood as being due to the average attraction exerted by the other nucleons.

As noted above, rotational bands are observed in some nuclei. In quantum mechanics no collective rotation is allowed for a body with a spherical shape, because motion in quantum mechanics is represented by the spatial variation of a wave-function, which for a sphere does not change when it rotates. This implies that nuclei can have shapes which are not spherical. Measurements of the electromagnetic properties of these states indeed show large electric quadrupole matrix elements (see Appendix C), consistent with ellipsoidal shapes. This suggests the introduction of a deformed mean field.

### 1.3 The K Quantum Number

Ellipsoidal shapes may be prolate (like an American football), oblate (like a disk), or non-axially symmetric (with all three major axes of different lengths). When significant and stable ellipsoidal deformations are observed in nuclei, it is found that the equilibrium shapes are nearly always axially symmetric,\textsuperscript{*} and in an axially symmetric mean field there should be conservation of the K quantum number (Fig. 1.1), defined as the projection of

\textsuperscript{†} Unlike a normal fluid, in which the molecules maintain their separation because of electromagnetic forces, the nuclear fluid maintains its volume mainly because of the Fermi exclusion principle.

\textsuperscript{*} It appears that well-deformed ground states and low-spin states essentially always have axial symmetry at equilibrium, but some non-axially symmetric shapes do seem to exist at high spins [Rin 80].
Fig. 1.1: Definition of the $K$ quantum number.
the total angular momentum $J$ along the axis of symmetry. The symbol $\Omega$ is conventionally used to represent the contribution to $K$ of the angular momentum of a single particle. Just as rotational bands are observed in molecular spectra which are built on combinations of vibrational and electronic excitations, rotational bands with $J(J+1)$ level-spacing are observed in deformed nuclei, built on band-heads which have different values of $K$.

The purpose of this thesis is to investigate the mechanisms of $K$-violation in the decay of states with very large $K$ values. Bands with values of $K$ up to a few $\hbar$ are observed in all deformed nuclei. Far less common are states with very large values of $K$. Just as a perfect sphere cannot rotate collectively in quantum mechanics, there can be no collective rotation of an axially symmetric shape about its axis of symmetry. Any angular momentum along the axis of symmetry is therefore not smooth collective rotation; high values of $K$ are formed by aligning the spins of several valence particles along the symmetry axis. The shell structure determines which high-$\Omega$ neutron and proton orbitals are available near the Fermi levels, and depending on this, high-$K$ states may compete with collective rotation and be relatively favored in some nuclei. There is only one small group of nuclei accessible to heavy-ion reactions for which this happens at prolate\footnote{Oblate shapes are usually not stable in nuclei, except perhaps in certain ranges of angular momentum, and it is usually not possible to observe collective rotational bands built on oblate high-$K$ states, since the moment of inertia for collective oblate rotation is low.} deformations: the region with $A \sim 180$. Here, states have been identified, with values of $K$ as high as 25, which occur at energies similar to the energies of states with the same total spin but $K=0$.

Historically, nuclei with stable, axially symmetric ground-state deformations were considered a refuge from the complexity of the most
general problem of nuclear structure, and the guardian of this refuge was the
approximately conserved K quantum number. In a mean field which is only
weakly deformed, the single-particle levels, although in principle pure states
of $\Omega$ (and $j$), are nearly degenerate and easily mixed by shape fluctuations or
residual interactions. K is then not a good quantum number. At larger
deformations, however, the splitting of single-particle states with different
values of $\Omega$ is $\geq 1$ MeV, and the calculated single-particle states have nearly
pure values of $\Omega$ at low temperatures and rotational frequencies. In such
cases, K is good quantum number.

1.4 K-Isomers

Detailed studies of large numbers of deformed nuclei show over and
over a pattern of decay which is intimately connected with K-conservation, a
pattern which is most dramatic for those few nuclei in which high-K states
are observed. This pattern will be referred to here as the normal pattern of
decay of states in deformed nuclei:

1) States in a rotational band decay through in-band
transitions to lower states in the same band, keeping K constant.

2) When the band-head is reached, the decay is to
members of another band with a (usually lower) K value as
close as possible to the original value.

3) When the only possible decay is through a transition
which changes K drastically in one step, the state is isomeric,
and is known as a K-isomer.
The existence of K-isomers is a clear indication of the approximate conservation of the K quantum number. An example of a K-isomer is shown in Fig. 1.2 [Bro 88], and an example of the normal pattern of decay is shown in Fig. 1.3 [Kho 73b], [Kho 73c], [Kho 75], [Kho 76]. The selection rule which determines the existence of an isomer involves the degree of K-forbiddenness, defined as

\[ v = |\Delta K| - \lambda \]  \hspace{1cm} (1.1)

where \( \lambda \) is the multipolarity of the gamma-ray transition. The degree of K-violation \( v \) measures the part of the change in K which cannot be accounted for simply by the angular momentum carried off by the photon. Transitions with \( v > 0 \) are forbidden to first order and are hindered.

The study of K-isomers has led to the accumulation of an overwhelming body of evidence [Löb 68] which shows that in well-deformed states in the range of spin studied to date using detailed spectroscopy, the components of the wave-function with K-values differing by more than a few \( h \) from the dominant value are always very small, never constituting an admixture of more than about one part in \( 10^2 \text{–} 10^4 \). Quantitatively, a measure of the hindrance of a gamma-transition due to K-violation is the hindrance factor

\[ F = \frac{t_{\text{part}}}{t_{\text{W}}} \]  \hspace{1cm} (1.2)

where \( t_{\text{part}} \) is the experimentally determined partial half-life of the decay, and \( t_{\text{W}} \) is the Weisskopf estimate (see appendix C) for the half-life of a single-particle transition. The typical observation [Löb 68] is that the hindrance factor increases by a factor of about 100 for every degree of K-forbiddenness,

\[ F \sim 100^v \]  \hspace{1cm} (1.3)
Fig. 1.2: An example of a K-isomer.
Fig. 1.3: The normal pattern of decay for K-isomers.
This exponential dependence leads to the definition of the *reduced hindrance factor*

\[ f = F^{1/v} \]  \hspace{1cm} (1.4)

which typically gives \( f \approx 100 \).

The obvious question is why K-violating transitions exist at all. One possible mechanism is through Coriolis mixing of states with different values of \( K \)(Fig. 1.4). When a state with a certain value of \( K \) undergoes collective rotation, forming angular momentum \( J > K \), the nucleons experience a Coriolis force. As will be discussed more thoroughly in chapter 2, the Coriolis term in the Hamiltonian is proportional to the projection \( J_x \) of angular momentum along the axis of collective rotation, and therefore does not commute with \( K = J_z \). The symmetry-breaking \( J_x \) operator only has matrix elements connecting states which differ by \( \Delta K = \pm 1 \), and therefore admixtures with \( \Delta K = \pm 2, \pm 3, \ldots \) can only occur through successively higher orders of perturbation theory. This is entirely consistent with the exponential dependence of \( F \) on \( v \), which is part of the normal pattern of K-violation.

The above discussion suggests, then, that very highly K-violating transitions should essentially never be observed. Surprisingly, some transitions have been observed with very large values of \( \Delta K \). An example is given in Fig. 1.5, which shows several transitions in \(^{174}\text{Hf} \) with \( \Delta K = 14 \) [Wal 90]. Other decays with \( \Delta K \) as high as 25 have been observed in \(^{182}\text{Os} \) [Cho 88]. The partial half-lives of the \(^{174}\text{Hf} \) (1.1% branch) and \(^{182}\text{Os} \) decays are 400 μsec and 6μsec, respectively, while extrapolation using the \( f = 100 \) estimate would produce values of \( 2 \times 10^7 \) years and \( 6 \times 10^{26} \) years. As shown in Fig. 1.6, there are enough examples of drastic deviations from \( f = 100 \) to

\[ \dagger \] The symbol \( f_v \) is often used for this quantity.
Fig. 1.4: Schematic representation of the Coriolis mixing mechanism for K-violation. The two states have the same shape and spin, but different orientations of the shape with respect to the spin vector.

\[ K = J_z \]

\[ H_{\text{Coriolis}} = -\omega J_x \]
Fig. 1.5: Example of a highly K-violating decay: $^{174}\text{Hf}$. For simplicity, many of the transitions to states with $K>0$ have been omitted; the branches to the $K=6$ band shown here are the strongest. Transitions from the excited bands to the yrast states are also not shown. Transitions to $K=0$ states are omitted where the intensity was not significant at the 2σ level, or where the structure of the $K=0$ state was not demonstrated conclusively.
Fig. 1.6: Observed hindrance factors for K-violating transitions. Each horizontal line represents a set of transitions with the same multipolarity and degree of K-forbiddenness, from the survey in ref. [Löb 68] of the A~150-200 region. The circles, which are not necessarily meant to be a representative sample, are from more recent experiments. The group with v=12 and 13 are from $^{176}$W ( chapters 5 and 6 of this work) and $^{174}$Hf [Wal 90a], and the point with v=24 is from $^{182}$Os [Cho 88a]. (As suggested in ref. [Löb 68], the data-sets for each multipolarity have been shifted horizontally to take into account the systematic differences in their typical unhindered strengths.)
demonstrate that some qualitatively different mechanism for K-violation is being encountered than had been observed previously.

Motivated by these data, let us re-examine the K quantum number. Its definition implicitly assumes that the nuclear shape is well-defined and static, so that the component of the angular momentum along the symmetry axis can be unambiguously defined. But microscopic bodies do not necessarily have well-defined shapes, and nuclei will therefore exhibit quantum-mechanical fluctuations in their shapes. These fluctuations are typically small (see, e.g., ref. [Bon 90a]), but if the fluctuations have even a small probability of changing the shape of the nucleus drastically, then there may be small admixtures of very different K values. K-isomers can then be considered as a type of shape isomer (Fig. 1.7): a state of a nucleus in which no decay is possible without a large change in the shape, which requires quantum-mechanical tunneling through a potential barrier. The most well-known examples of shape isomers are the fission isomers [Pol 62], [Bjø 80], states in actinide nuclei with extremely large prolate deformations, which may decay, as their name implies, through fission, but which may also decay by barrier penetration to the normally deformed states [Rus 75], [Sch 89]. The sense in which K-isomers can be considered as shape isomers will be explained in more detail in chapter 2.

The goal of this thesis is to find experimental observations and theoretical methods that allow a comparison of the predictive power, for describing the decay of K-isomers, of simple models of tunneling and Coriolis mixing, which have been the two concepts most often used for this purpose. In chapter 2, the theoretical tools to be used are outlined. The experimental techniques are discussed in chapter 3. A large part of the activities leading to this thesis was the development of the Yale BGO Array,
Fig. 1.7: A possible tunneling mechanism for the decay of K-isomers. The precise meaning of the shape coordinate (x-axis) will be explained in ch. 2.
which is described in chapter 4. Chapters 5 and 6 present the results of the \( ^{176}\text{W} \) and \( ^{176}\text{Hf} \) experiments, and in chapter 7 the novel results for \( ^{176}\text{W} \) are compared with the results of detailed calculations. The results are summarized in chapter 8, and possible paths for future investigations are outlined.
Chapter 2
Theory

2.1 The Mean Field

K-isomers are non-collective states in which several valence particles have their angular momenta aligned in the same direction, along the symmetry axis. It is therefore necessary to know which valence states are available to a nucleus, that is, to construct a nuclear shell model. In its simplest form, the shell model consists simply of solving the eigenvalue problem for a spherically symmetric mean field $V(r)$. Two common phenomenological forms for the mean field are the modified harmonic oscillator and the Woods-Saxon model. Descriptions of these are available in many textbooks.

In practice, it is found that not all properties of nuclei can be described using only a one-body potential, and a phenomenological two-body term is therefore introduced in the Hamiltonian. This extra term is known as the residual interaction. A more complete discussion of residual interactions is given in section 2.3.

2.2 Collective Excitations and Geometric Models

In theory, a complete description of nuclear structure can be obtained from the many-body states constructed from properly anti-symmetrized combinations of the single-particle eigenfunctions. In reality, two problems are encountered which, except for nuclei with very few valence particles, limit the usefulness of such an approach:
(1) The size of the space of wave-functions grows quickly with the number of valence particles, and can become astronomically large for mid-shell nuclei.

(2) Both the ground states and the excited states of nuclei far from closed shells often contain highly coherent superpositions of single-particle states. Such coherent effects are observed, for example, in large quadrupole moments of nuclear states and collective enhancements of electromagnetic transitions between states. The calculated wave-functions can therefore be quite complicated and difficult to interpret when expressed in the basis of eigenstates of the spherical shell-model potential.

One would like to find a method which eliminates the need for considering all of the valence particles at once and explicitly constructing their coherent wave-functions; even in a high-K state of a mid-shell nucleus, only a small subset of the valence particles contributes to the angular momentum.†

To find a more useful model for nuclei with many valence particles, it is necessary to identify the most energetically favorable types of collective motion. A convenient starting point is the liquid drop model, which suggests that surface oscillations are the important collective modes at low energy, so that all collective motion can be described entirely in terms of the shape and orientation of the nuclear surface. A common parametrization for the shape of the nucleus is given by

\[ R(\theta, \varphi) = R_0 \left[ 1 + \sum \alpha_{\lambda \mu} Y_{\lambda \mu}(\theta, \varphi) \right] \] (2.1)

where volume conservation is maintained either by adjusting \( R_0 \) as a function of the \( \alpha_{\lambda \mu} \) or by taking \( \alpha_{00} \neq 0 \). Only a few of the \( \alpha_{\lambda \mu} \) are important in most

† This is because the nucleon-nucleon interactions are mainly attractive, which means that the lowest-energy states have as many pairs of particles coupled to spin zero as possible.
cases. The \( \lambda = 1 \) terms correspond only to a motion of the center of mass. The static shapes of nuclei are almost always reflection-symmetric, so that static deformations with other odd values of \( \lambda \) are usually negligible. The surface-tension term in the liquid-drop energy is relatively stiff against deformations with large \( \lambda \), so that \( \lambda = 2 \) (quadrupole) deformations are the most important. For deformations restricted to \( \lambda = 2 \), there are five degrees of freedom, which map onto the coefficients \( \alpha_{2-2}, \ldots \alpha_{22} \). Two degrees of freedom correspond to the intrinsic shape of the nucleus, and three correspond to the Euler angles describing the orientation of the nucleus. The two shape degrees of freedom are usually parametrized by the variables \( \beta \) and \( \gamma \), defined as

\[
\begin{align*}
\alpha_{20} &= \beta \cos \gamma \\
\alpha_{22} &= \frac{1}{\sqrt{2}} \beta \sin \gamma
\end{align*}
\]

(2.2)

(for shapes with their semi-major axes aligned with the x, y, and z axes). The variable \( \beta \) gives the amount of quadrupole deformation. Varying the angle \( \gamma \) allows the inclusion of non-axially symmetric shapes as well. The parameters \( \beta \) and \( \gamma \) can be visualized as polar coordinates in the \( \beta \)-\( \gamma \) plane (Fig. 2.1). The variable \( \gamma \) is equal to 0° or -120° for axially symmetric prolate shapes, and 60° or -60° for axially symmetric oblate shapes, and takes on intermediate values for shapes which are not axially symmetric. The redundancies in \( \gamma \) allow a given shape to be described in more than one orientation, with the coordinate frame defined so that the angular momentum is fixed along the x axis. (Only the interval from -120° to 60° is needed for the present purposes.) Details of the deformations with \( \lambda = 4, 6, \ldots \) are difficult to extract from the data in a model-independent way [Naz 81]; the \( \lambda = 4 \) deformation is discussed further in appendix C. Deformed versions of both the Woods-Saxon and modified harmonic oscillator models are often used. The latter is usually referred to as the Nilsson model.
Fig. 2.1: The beta-gamma plane.
An extremely important consequence of the construction of the $\beta$-$\gamma$ plane is that if the conserved total angular momentum is defined to be along the $x$ axis, then collective and non-collective states with the same shape are represented by different values of $\gamma$ but the same values of $\beta$. In a prolate nucleus, collective rotational states have $\gamma=0$, and a $K$-isomer has $\gamma=-120^\circ$. As was mentioned in chapter 1, this provides a natural way of thinking of $K$-isomers as shape isomers, and raises, as a possible mechanism for the decay of $K$-isomers, tunneling through a barrier in the $\gamma$ degree of freedom. The result would be a mechanism for the direct decay of a $K$-isomer to a $K=0$ collective rotational state.

The method used in the geometric models to simplify the description of collective states is to use a single-particle potential with a deformation corresponding to the deformation of the nucleus. A common notation for the single-particle states in the deformed potential is $\Omega^n[Nn_z\Lambda]$ (or $[Nn_z\Lambda\Omega^n]$) where $\Omega$ is the contribution of the particle to $K$, $N$ is the number of oscillator quanta in the stretched harmonic-oscillator basis, $n_z$ is the number of quanta in the $z$ direction (along the symmetry axis), and $\Lambda$ is the eigenvalue of the $z$-component of the orbital angular momentum. These are often termed the Nilsson quantum numbers, or the asymptotic quantum numbers, since in the limit of large deformation they give the eigenstates of the Nilsson Hamiltonian. Another set of labels is $n\ell j$, where $n=(1/2)(N-\ell+2)$ is the number of nodes in the radial wave-function for $0 \leq r < \infty$ and $\ell$ is written using the letters $s,p,d,f,g,...$. These are referred to as the spherical quantum numbers because they correspond to the eigenstates of the spherical single-particle potential. A quantum number which is always conserved by the model Hamiltonians used in this thesis is the signature quantum number $\alpha$, defined by the equation
\[ R = \exp(i\pi \alpha) \]  \hspace{1cm} (2.3)

where \( R \) is the eigenvalue of the state under rotation by 180 degrees about the cranking axis. Nucleons can contribute \( \alpha = +1/2 \) or \(-1/2\) to the total signature, and nuclei with even \( A \) have \( \alpha = 0 \) for even spins and \( 1 \) for odd spins.

Collective quadrupole excitations may be described as rotational or vibrational. A pure rotational state may be thought of as one in which the shape of the nucleus does not vary, but the orientation does, while a pure vibrational state may be thought of as one in which the shape varies about some equilibrium value while the orientation remains constant. The two modes may be coupled.

Two issues must be faced when using a deformed potential to describe these excitations:

(1) Because the potential is not spherically symmetric, the single-particle states are not states of good angular momentum.

(2) One wishes to avoid the introduction of a potential with time-dependent shape and orientation to describe collective excitations.

In the case of vibrational collectivity or the most general case of mixed vibrational and rotational quadrupole collectivity, there is no simple solution to these problems. In the special case of pure rotational excitations, however, one may use the cranking approximation, in which a transformation is made to the rotating frame in which the nucleus has a fixed spatial orientation. The sole effect of this transformation on the Hamiltonian is the addition of the cranking term

\[ H_{\text{crank}} = -\omega \cdot J \]  \hspace{1cm} (2.4)
which includes both the centrifugal and Coriolis forces. The Hamiltonian in
the rotating frame is known as the Routhian, and is not equal to the total
energy of the system.

The cranking approximation not only eliminates the need for a time-
dependent potential for the description of collective rotational excitations,
but also ameliorates the problem of non-conservation of angular momentum.
The cranking frequency $\omega$ in eq. 2.4 is, in purely mathematical terms, a
Largrange multiplier, which can be adjusted to give a trial wave-function
with any desired value of the expectation value $\langle J_x \rangle$. The trial wave-
function is still not a state of pure angular momentum, however, and for
nuclei in the rare-earth/actinide region, the r.m.s. spread in spin values
contained in a cranking wave-function is typically about $10h$ [Rin 80]. With a
spread this large, it is actually a remarkable fact that so many experimental
observables can be calculated so accurately; this is because the effects of
fluctuations tend to cancel out. The existence of the fluctuations does
however make it impossible to use the cranking wave-functions directly to
calculate electromagnetic transition matrix elements between states. (Semi-
macroscopic approximations which can be used are discussed in Appendix C.)

For axial deformations and $\omega$ perpendicular to the symmetry axis,
$\omega=|\omega|$ can be directly determined from the measured energies of the E2 intra-
band transitions with $\Delta J=2$ according to the relation

$$\omega = \frac{E(J) - E(J-2)}{J_x(J) - J_x(J-2)}$$  \hspace{1cm} (2.5)

where the initial state has angular momentum $J$ and the final state $J-2$, and
for the special case of $K=0$, the denominator is equal to 2. (Equation 2.5 is
plausible when compared with classical electromagnetism, in which an
electric quadrupole rotating with a frequency $\omega_{rot}$ creates E2 radiation with a
frequency $\omega=2\omega_{rot}$.) In classical mechanics, the normal modes of rotation are
those with \( \omega \) along a principal axis of the body, and indeed \( \omega \) is usually taken along the x axis in cranking calculations in nuclear physics. (Since in quantum mechanics there can be no collective rotation of a body about an axis of axial symmetry, \( \omega \) is to be interpreted in such cases purely as a Lagrange multiplier.) For collective rotation and \( K=0 \),

\[
J_x = J
\]  

(2.6)

For non-zero values of \( K \), the approximation [Ben 86a]

\[
J_x = \sqrt{J(J+1) - K^2}
\]  

(2.7)

is often used, where \( J^2 \rightarrow J(J+1) \) is a first-order quantum correction.

### 2.3 The Pairing Interaction

Of the residual interactions in nuclei, the most important is the pairing interaction, which is a short-range, attractive, isoscalar interaction. The pairing force leads to the fact that most nuclei are superfluid. This has two consequences which are important for K-isomer physics:

1. As in any BCS superfluid, the creation of an excited state by breaking a pair requires an expenditure of energy equal to \( 2\Delta \). K-isomers are states in which valence nucleons have been unpaired and coupled together to produce angular momentum, so the competition between high-K states and low-K states depends on the pair gap.

2. The inertial parameters of superfluids are reduced compared to their normal values. Consequently, the moment of inertia for collective rotation is greatly reduced compared to its rigid-body value. Equally important for our purposes is the fact that a gamma-tunneling mechanism for the decay of K-isomers
would depend strongly on the relevant inertial parameter, which behaves like the mass in a collective Schrödinger equation. Superfluidity leads to a reduction in the inertial parameter, which aids in the tunneling process. (The corresponding effect for the \( \beta \) degree of freedom is well-established for tunneling decays of fission isomers.)

The most common residual interaction used in cranking calculations is therefore the nuclear version of the BCS pairing approximation, which reproduces both of the the above effects. Good presentations of this method are available in [Law 80] and [Ben 84]. The single-particle states are replaced by quasiparticle states which are mixtures of particle and hole excitations. They are eigenstates of [Ben 79]

\[
H = H_{\text{def}} - \omega j_x - \Delta(P+P^+) - \lambda N
\]

(2.8)

where the operators \( P \) and \( P^+ \) destroy and create, respectively, a pair of particles in time-reversed orbits, and the term \( -\lambda N \) is used to constrain the system to have the correct expectation value \( \langle N \rangle \) of the particle-number operator. The quasiparticle wave-functions are not eigenstates of particle number, but for small values of the quasiparticle energy they are approximately equal mixtures of particle and hole states, with \( \langle N_{qp} \rangle = 0 \), so that in excited quasiparticle states the ground-state value of \( \langle N \rangle \) is approximately maintained.†

This type of Hamiltonian can be used to describe either \( K \)-isomers (at \( \gamma = -120^\circ \)) or \( K=0 \) states (at \( \gamma = 0^\circ \)). Historically, a Hamiltonian of the form (2.8) was first introduced in order to discuss the phenomenon of "backbending": in the \( K=0 \) states of a deformed nucleus, above a certain

† The typical r.m.s fluctuation in the particle number for nuclei in the rare-earth/actinide region is about 2 [Rin 80].
rotational frequency a new band becomes lower in energy than the ground-state, or g-band. This new band, called the s (for Stockholm) band, contains a broken pair of neutrons or protons with their angular momenta aligned with the axis of collective rotation. The Hamiltonian (2.8) allows one to calculate quantitatively the competition between the Coriolis force, which tends to align the pair, and the pairing force, which tends to keep them coupled with each other to spin zero. S-band states are observed in the nuclei discussed in this thesis, and, as will be discussed in chapter 7, the fact that there is a change in nuclear structure at the g-band/s-band crossing may have important implications for the mechanisms of decay of K-isomers.

All the ingredients have now been introduced for the standard form of the cranking method for nuclei. Some further technical refinements to the model are necessary for the purposes of chapter 7, and these are discussed in appendix B. With these enhancements, cranking models can calculate a total Routhian surface (TRS), which gives the Routhian as a function of $\beta$ and $\gamma$ for a fixed rotational frequency, or a potential energy surface (PES), which gives the total energy as a function of $\beta$ and $\gamma$ for a fixed value of $\langle J_x \rangle$. The latter is preferable because it is the angular momentum, and not the rotational frequency, which is a constant of the motion. An example of a potential energy surface is shown in Fig. 2.2.
Fig. 2.2: A typical potential energy surface. This plot was calculated for $J^\pi=14^+$ in $^{176}$W. The $x$- and $y$-axes are $\beta \cos(\gamma+30^\circ)$ and $\beta \sin(\gamma+30^\circ)$. 
Chapter 3
Experimental Techniques

3.1 Introduction

This chapter will introduce the techniques used in the experiments in this thesis, including discussions of heavy-ion reactions, and detection systems and analysis for gamma-ray spectroscopy. Useful review articles on these subjects can be found in refs. [Cer 74], [Bro 84] and [Kno 89], and only brief discussions will be given here.

To orient ourselves for further discussion of experimental techniques, it is useful to focus on the plane of $E$ vs $J$ (Fig. 3.1), where $E$ is the excitation energy of a state, and $J$ is its angular momentum (also loosely called spin). The set of states which have the lowest $E$ for a given $J$ is approximately a parabola, and is called the yrast line. The yrast states have all their energy tied up in creating angular momentum as efficiently as possible. No states exist below the yrast line. Near the yrast line, the density of states with a given $J$ is small, and the density of states increases with energy above the yrast line, which corresponds to temperature. Since we are interested in high-spin isomers that compete energetically with collective rotational states, our focus is on the yrast or near-yrast states at high angular momentum.

3.2 Heavy-Ion Reactions

Heavy-ion fusion-evaporation reactions are currently our main source of information about high-spin states of nuclei. These are reactions in which two nuclei collide, fuse, and evaporate particles to form a final nucleus...
Fig. 3.1: The E-J plane for $^{176}$W. The solid curve is a parabolic approximation to the yrast line, and the data-points are the known yrast states of $^{176}$W. The entry point is the point in the E-J plane at which $^{176}$W is populated, and after which all decay is by gamma emission. The elliptical region shown is the most probable region for the entry point to fall within.
called the evaporation residue. Fusion-evaporation reactions can populate high spins because of the large mass of the nuclei in the heavy-ion beam, their high velocities, and the large impact parameters involved. The reaction used in chapter 5 will be used in this section as an illustrative example in several places. This reaction is a beam of $^{30}$Si incident on a target of $^{150}$Nd, leading partly to the population of states in $^{176}$W. Some of the quantitative data given here for this reaction comes from calculations using the computer code PACE [Gav 80], which incorporates a statistical model of fusion-evaporation reactions. In such a reaction, the beam nucleus must first overcome the barrier imposed by the Coulomb and centrifugal forces to approach the stationary target nucleus within the range of the strong force. (Here, the Coulomb barrier is 122 MeV in the laboratory frame.) The reactions discussed in this thesis are at non-relativistic energies (< 1 MeV/nucleon, compared to the mass of a nucleon, which is 940 MeV), and the motion up to the point of touching is essentially classical. The partial cross-section for fusion can be approximated by the geometric relation [Bro 84]

$$\sigma_{\text{fus}} = \pi R^2 (l_{\text{max}} + 1)^2$$  \hspace{1cm} (3.1)

where $l_{\text{max}}$ is the highest angular momentum for which the energy is above the barrier.† For this reaction, $l_{\text{max}} = 33 \, \hbar$. Discussions of heavy-ion accelerators may be found in various review articles [Cer 74],[Cer 84].

Partial waves involving non-peripheral collisions lead mostly to the formation of a compound nucleus (although there may be competition from

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† In reactions which do not surmount the barrier, nuclear excitations may still occur through the long-range Coulomb force, in a process called Coulomb excitation. The partial cross-section for Coulomb excitation is typically on the order of 1000 mb, while the partial cross-section for fusion is about 450 mb for the reaction described here. Nuclear inelastic and transfer reactions, which involve grazing collisions, usually have partial cross-sections on the order of 10 mb for the heavy beams and targets considered here.
fission). In the standard picture of the compound nucleus, all degrees of freedom are equilibrated. The only memory which is retained of the incoming channel is contained in the conserved quantities $E_{\text{tot}}, J$, and the parity, $\pi$. All other memory of the incoming channel is lost. The compound nucleus is formed in a hot region of high level density in the $E-J$ plane, and the excitation energy (here, $E_x=62$ MeV) is above the threshold for particle emission. In nuclei in the mass-180 region which are only slightly neutron-deficient, the evaporated particles are almost always (~99%) neutrons. The evaporated neutrons have a mean kinetic energy of about 2 MeV, and with the inclusion of the differences in binding energy, the evaporation of each neutron results in a cooling of approximately 10 MeV. At this beam energy, the neutron multiplicity is strongly peaked at the values 4 and 5, populating the nuclei $^{176}$W and $^{175}$W. Following the evaporation of the final neutron, the residual nucleus then cools towards the yrast line primarily by emission of high-energy statistical $E1$ gamma transitions without significant loss of angular momentum. In the final stage of the decay, the cool nucleus proceeds along the yrast line by gamma-ray emission. It is in this stage of the decay that almost all of the angular momentum is lost. Since the flow of gamma-ray transitions in this final stage is concentrated in a small number of yrast states, these transitions stand out in the spectra compared to the background due to Compton scattering and unresolved weaker transitions. In deformed nuclei the yrast decay occurs almost entirely through rotational cascades, which dominate the gamma-ray spectra. Typical time-scales for the steps of this process are [Gar 86], [Cer 74].
formation of compound nucleus $\sim 10^{-22}$ sec
particle emission $\sim 10^{-19}$ sec
statistical gamma-ray cooling $\sim 10^{-15}$ sec
yrast transitions $\gtrsim 10^{-12}$ sec

Except in cases where an isomer holds up the decay, the time-scale for gamma-ray emission is far shorter than the time resolution of solid-state and inorganic scintillator detectors, which is typically $10^{-9}$ to $10^{-8}$ sec. The sequence of statistical transitions and fast yrast transitions, where a large number of gamma-rays (here $M_{\gamma} \sim 15$) is emitted almost simultaneously, is therefore known as the prompt flash. The prompt flash is both a distinguishing characteristic of fusion-evaporation reactions compared to Coulomb excitation (where relatively few gamma-rays are emitted), and a natural time marker for the beginning of an event. The prompt flash ends either at the ground-state, or in an isomer. If a high-spin isomer is populated, there is also a delayed flash of gamma-rays when the isomer decays.

The principles of gamma-ray detection are discussed in the next section.

### 3.3 Detectors

The high-spin yrast states of nuclei decay entirely by electromagnetic processes, which usually proceed by gamma-ray emission, although part of the transition strength may involve internal conversion or internal pair creation. Gamma-ray detectors are therefore the primary tools for spectroscopy of high-spin states.
In heavy-ion reactions, as discussed in the previous section, the evaporation residue is formed high above the yrast line, and later passes through cold (yrast and near-yrast) states. The goal here is to learn about K-isomers, and so the primary focus is on discrete gamma-ray spectroscopy of the cold states. K-isomers are usually populated very weakly, and trigger detectors are essential for selecting and enhancing the discrete transitions above and below the isomers. The trigger detectors are discussed later in this chapter, and in chapter 4.

Gamma-rays interact in matter, such as the active volume of a detector, through the photoelectric effect, Compton scattering, and pair production. For discrete spectroscopy, useful spectroscopic information is only gained when, through one or more interactions, the gamma-ray deposits all of its energy in the detector; often, due to Compton scattering, only part of the energy is deposited. The probability of interaction is proportional to density and rises quickly with $Z$, and the detection medium should therefore be of the highest density and $Z$ possible. In addition, the sensitivity of the system depends on the energy resolution of the detector.

The standard tool for modern discrete gamma-ray spectroscopy is the high-purity germanium (HPGe) detector [Kno 89]. This is a solid-state detection device made of a semiconducting Ge crystal. The detector is operated at high voltage in a fully depleted reverse-biased mode, and at liquid nitrogen temperature to avoid catastrophic sparking and improve energy resolution by reducing thermal noise. The volume, and therefore the efficiency, of the detector is limited by the depletion depth, which varies as $N^{-1/2}$, where $N$ is the net density of donor impurities minus acceptor

\[\uparrow\text{The dependence is of the form } \sigma \sim Z^n, \text{ with } n=1 \text{ for Compton-scattering, } n=4-5 \text{ for the photoelectric effect, and } n=2 \text{ for pair-production.}\]
impurities. For maximum depletion depth, the density of impurities must be as low as possible, and the Ge crystals used in HPGe detectors are the purest mass-produced substance in the world, with a density of impurities of about $10^{-12}$. No doping is used, and the detector is therefore often called an intrinsic Ge detector, although the net excess of impurities makes it an n-type semiconductor. Crystals may be fabricated in either a planar, disk-shaped geometry, or, for larger detectors, in a cylindrical, coaxial geometry with electrodes on the inner and outer surfaces.

Scintillation detectors form a second class of gamma-ray detection devices [Kno 89]. Such devices have worse energy resolution than Ge detectors, but can be manufactured from inorganic crystals which have greater density and higher Z than Ge crystals, are less expensive, do not require cooling, and have better timing characteristics. The type of scintillator used here is bismuth germanate (BGO). It is used in the present experiments for three purposes:

1. For Compton-suppression of Ge detectors.
2. In a gamma-ray calorimeter and multiplicity filter to act as a selective trigger for high-spin states.
3. As a timing device (using the same hardware as for calorimetry and multiplicity-filtering) to select events in which high-spin isomers are populated, by recording both the prompt and delayed flashes of gamma-rays.

It has already been mentioned that Compton-scattering is an obstacle to discrete gamma-ray spectroscopy, and HPGe detectors are therefore used with Compton-suppression shields made of a scintillating material
surrounding the Ge crystal. When a gamma-ray Compton-scatters out of the Ge, and ends up in the Compton-suppression shield, the signal from the shield is used to veto that particular signal from the Ge.

The use of BGO scintillators as triggers for high-spin states and isomers will be discussed in chapters 4 and 5-7.

3.4 Methods of Spectroscopy

A discussion of high-spin gamma-ray spectroscopy is given in ref. [Cer 74]. In the present experiments, data were collected in coincidence mode, requiring at least two Ge detectors and a minimum number of BGO detectors to fire within a certain time period. The details of the master-gating requirements for each experiment are explained in the relevant chapters. For the reactions and number of Ge detectors considered here, fusion reactions triggering a Ge-Ge coincidence, known as \( \gamma \gamma \), or doubles events, are generally only \(~1\%\) as numerous as those, known as singles events, that trigger only one Ge detector. Modern high-spin experiments, however, are based almost entirely on multi-detector coincidence data. This is because gamma-singles spectra from heavy-ion reactions are extremely complicated, and cannot be used to unambiguously construct a complex decay-scheme.

Instead, coincidences between gamma-ray energies are analyzed. Quantitatively, the coincident intensities can be compared and used to calculate the strengths with which the relevant states are populated, and the branching ratios for the various transitions depopulating a given state. Further constraints on the level scheme can be obtained by the requirement

\[ \text{The scintillator is usually BGO, but part of the Yale shields is NaI. The Ge crystal is completely surrounded by scintillator, except at the front, where gamma-rays enter the Ge crystal, and at the back, where the electrical and thermal contacts are made with the Ge.} \]
that different sequences of transitions linking the same two states should have the same sums of energies.

The method of analysis generally used is one of sorting the event-by-event data into a two-dimensional histogram, or $\gamma$-$\gamma$ matrix, with each axis representing a gamma-ray energy. One-dimensional spectra are then made by summing one or more rows of the matrix, and are studied interactively on a computer. With larger and larger Ge arrays being built, it is becoming possible to conduct studies of triple and higher-order Ge coincidences, with a corresponding increase in sensitivity.

Methods for using Ge and BGO arrays together will be discussed in chapter 4. Some of the more technical aspects of discrete spectroscopy with Ge detectors are discussed in appendix D.
Chapter 4
The Yale BGO Array

4.1 Introduction

The development of the Yale BGO array was an important part of this project. The most common use of BGO arrays is to determine the approximate point in the E-J plane where the gamma-decay originates. This allows the enhancement of weak high-spin transitions, and of the fusion-evaporation channel of interest, since channels which involve the evaporation of different numbers of particles tend to emit different amounts of energy in the form of gamma-rays. The total gamma-ray energy deposited in the array, called the sum energy, gives an approximation to $E$. The number of detectors which fire, called the fold, is a measure of the gamma-ray multiplicity $M$, and $M$ is correlated with $J$, since a single gamma-ray usually carries off 1 or 2 $\hbar$ of angular momentum. The standard notations for the measured quantities are $H$ for the sum energy and $K$ for the fold. The notation $K$ for the fold is highly unfortunate for this thesis, which is concerned with the $K$ quantum number. To avoid confusion, the symbols $H$ and $K$ for the BGO quantities will be avoided (except in appendix A).

The specific detection of high-spin isomers using BGO arrays can be accomplished using two techniques: (1) the timing of the BGO detectors can be used to determine whether there was a delayed flash, and (2) in experiments, such as the Yale experiments described in chapter 7, where beam pulsing is not available, the BGO detectors can be used to determine the timing of the prompt flash, and gamma-rays detected later by the Ge
detectors can be classified as transitions depopulating an isomer. Experiments involving the spectroscopy of high-spin isomers are usually done with pulsed beams. The techniques involving BGO timing were developed as part of this thesis, to improve and optimize the spectroscopy of high-spin isomers without a pulsed beam. In the Argonne experiment described in chapter 5, however, it was also found that BGO timing added a powerful tool for selecting high-spin isomers even when pulsed beams were available.

The experiments performed at Yale used six HP Ge detectors, of which five were Compton-suppressed, plus a 38-element BGO array. The BGO array is shown in Fig. 4.1, and the configuration of the Ge array for use with it is shown in Fig. 4.2. The Argonne experiment used the Argonne/Notre Dame array, which includes 12 Compton-suppressed coaxial HPGe detectors and a 50-element BGO array.

The Yale BGO array consists of 38 hexagonal bismuth germanate detectors in a close-packed geometry. Its large solid angle, approximately 80% of $4\pi$, and high degree of segmentation make it a sensitive tool for measuring the gamma-ray multiplicity. The high density and high average $Z$ of BGO provide an absorption coefficient 2 to 4 times higher than that of NaI for typical $\gamma$-ray energies of interest. For $\gamma$-rays falling within its solid angle, the probability of detection in Yale array is measured to be 90% for $\gamma$ rays with $E_\gamma \leq 1.8$ MeV, so that it can be used as a highly efficient $\gamma$-ray calorimeter. The intrinsic timing resolution of the detectors is 3 ns FWHM, which allows accurate timing of the gamma flash and detection of isomeric decays with very short half-lives. All of this information is available on an event-by-event basis. A decided advantage of BGO multiplicity arrays, compared to arrays composed of NaI detectors, is the
Fig. 4.1: The Yale BGO array. The gaps between the BGO detectors in the horizontal plane are for the Ge detectors to view the target.
Fig. 4.2: Geometry of the Ge and BGO detectors for K-isomer experiments at Yale. Five Compton-suppressed coaxial Ge detectors are used, plus one large-volume unsuppressed Ge.
relatively low scattering probability for neutrons, which eliminates the need for the pulse-shape discrimination electronics used in NaI arrays [Jää 83] for neutron rejection.

In practice, the mapping from fold and sum energy onto E and J is model-dependent. The competition between $\Delta J=1$ and $\Delta J=2$ transitions depends strongly on nuclear structure, except at high temperatures, where $\Delta J=1$ transitions are expected to predominate. The response of the array depends on the $\gamma$-ray energies involved [Jää 83], and there is no unique method for unfolding the data, but certain practical prescriptions have been developed.

In many cases, one is not interested in the exact distribution of E and J that populate a certain state. Instead, one wishes to impose certain cuts on the fold and sum energy with the aim of selecting high-spin events, or events belonging to a certain reaction channel, or both. Empirically, it is found in $\gamma$-ray coincidence spectroscopy that placing cuts on these quantities accomplishes the desired results.

The reverse problem, that of calibrating the response function, \((J,E)\rightarrow(\text{fold, sum energy})\), of the BGO array, has a unique solution for any given gamma-decay. This calibration is especially important in chapter 6, where an experimental upper limit is placed on the strength of a particular K-violating transition in $^{176}$Hf; the upper limit is derived in a way which depends on knowing the response of the array. It is also interesting to learn how well the array performs, since its design is different from that of the other BGO arrays in use. The methods and results are given in appendix A.

The Yale BGO array is the largest array of its type in a university laboratory. The hexagonal geometry of the individual detectors is similar to
that used in the Argonne BGO array, but the Yale array differs from the Argonne array and many others in the method used for allowing γ rays to escape into the solid angle covered by the Ge detectors. Many other arrays employ a certain number of BGO detectors with holes drilled through them to allow the Ge detectors to view the target. This, however, has the disadvantage of greatly interfering with light collection. The Yale array uses a configuration in which some BGO detectors are simply recessed slightly to make peepholes for the Ge detectors and the beam-pipe (Fig. 4.2). The improved light collection results in the best energy resolution of any array of its kind which is currently available (13% FWHM at 661 keV).

4.2 Electronics

A schematic diagram of the electronics used in in-beam experiments to record the BGO information on an event-by-event basis are shown in Fig. 4.3.

The amplifier is used both for its gain, which makes the amplitudes of the pulses great enough to trigger the constant fraction discriminators (CFDs), and in order to prevent ringing and to isolate the energy and timing signal trains. The amplifiers used have a fixed gain, which necessitates the use of a resistive divider on the energy side to divide the amplitudes by ten for input to the energy FERAs. (FERA stands for Fast Encoding and Readout ADC.) The CFDs are CAMAC modules, with thresholds and walk adjustments† programmable in software.

† A CFD works by making a copy of the input signal, delaying it, inverting it, adding it to the original signal, and picking off the timing from the zero-crossing point. The walk adjustment determines the delay which is used.
Fig. 4.3: Electronics for the Yale BGO array.
Two majority logic units (MALUs) are daisy-chained together, and provide two independent outputs which signal when K rises above a selected value. These provide both a trigger for the FERAs and a part of the master-gate. The FERAs digitize the energy and timing information; the BGO electronics are entirely free-running, and are triggered whenever the instantaneous BGO fold rises above the hardware threshold set in the MALUs. When a gamma-flash in the BGO array is not followed by a complete master-gate including a Ge-Ge coincidence, a hardware "fast clear" signal is automatically generated to clear the FERAs and ready them for further processing within a few microseconds. In such cases, the Ge ADCs and the event-handler remain inactive. When a master-gate does occur, the Ge ADCs are triggered, the FERAs are read out through the CAMAC highway, and the Ge information is read out and recorded.

A more detailed example of an electronics setup for a K-isomer experiment at Yale is shown in 4.4.
Fig. 4.4: Electronics for a K-isomer experiment at Yale.
5.1 Introduction

The most important goal of the Yale K-isomer program is to measure systematics of K-violating decays (that is, their variation with N and Z) as a test of different models of the mechanisms responsible for K-violation. If gamma-tunneling is an important mechanism in these decays, then the measured hindrance factors should vary strongly as a function of the height of the potential barrier in the gamma degree of freedom. In the A~180 region, the height of the barrier is predicted to vary rapidly with changing proton number. Detailed calculations will be presented in chapter 7, but qualitatively, the situation is as follows:

- Os (Z=76)  low barrier
- W (Z=74)   intermediate
- Hf (Z=72)  high barrier

Very highly K-violating decays had been observed previously in the Os and Hf isotopes [Cho 88], [Wal 90]. The abnormally low hindrance factors are comparatively higher for the Hf nuclei than for the Os nuclei. If the gamma-tunneling interpretation is correct, then the W isotopes should interpolate smoothly between these extremes. If Coriolis mixing is the dominant mechanism, then the strengths of the decays should be a function of the quasiparticle configuration, the deformation, and the rotational frequency, and should not depend on the shape of the potential energy surface.
The objective of this experiment was to detect high-spin isomers in $^{176}$W. This nucleus is an isotone of $^{174}$Hf, where recent work had shown that highly K-violating branches existed for the $K=14$ isomer in $^{174}$Hf (Fig. 1.5), although these branches were a minute fraction of the decay of the isomer. The nucleus $^{176}$W is therefore a good subject for a quantitative study of the systematics of highly K-violating decays. An intensive study was undertaken not just of the isomeric decays, but of the prompt spectroscopy of this nucleus as well. The broad spectroscopic information contributes vitally to the understanding of the K-isomer physics, for example by identifying many of the 2-qp structures which are the building-blocks for the many-qp K-isomers, and by identifying as completely as possible the set of states available for the decay of the K-isomers. The results of this chapter will be discussed from a theoretical perspective in chapter 7.

### 5.2 Experiment

The experiment was carried out at the ATLAS facility at Argonne National Laboratory. The reaction used was $^{150}$Nd($^{30}$Si, 4n) at a beam energy of 133 MeV. The target consisted of 1.1 mg/cm$^2$ of isotope-separated $^{150}$Nd on a 53 mg/cm$^2$ Pb backing, with a thin layer of Au on the front of the target to prevent oxidation. The intrinsic beam pulsing of the system gave pulses of $\leq 1$ ns duration separated by 82 ns. The gamma-rays were detected by the Argonne/Notre Dame array, which consist of 12 Compton-suppressed Ge detectors and 50 BGO elements. The main trigger, known as the master gate, for the experiment was two
Ge detectors (within 120 ns of each other) in coincidence with at least four BGO detectors.

In order to detect the weakly populated K-isomers, the data from the BGO detectors were analyzed in more detail and using different techniques than is ordinarily done in high-spin gamma-ray spectroscopy. In particular, all of the BGO energy and timing parameters were written to tape individually, on an event-by-event basis. This is discussed in more detail later in this section and in section 5.3. The electronics setup has been described in ref. [Ye 91]. All times were measured relative to the RF signal corresponding to the timing of the beam pulse. A total of $45 \times 10^6$ Ge-Ge coincidence events was recorded.

The energy and timing data from the BGO detectors were used to create several different data-sets of gated Ge-Ge coincidence data. In addition to the fold and sum energy, a delayed fold parameter was created, defined as the BGO fold summed only over those detectors that fired between beam bursts. Gating on the delayed fold provided an extremely sensitive and efficient trigger on decays of high-spin isomers. The gates used to create coincidence matrices were:

<table>
<thead>
<tr>
<th>Ge timing</th>
<th>total</th>
<th>delayed</th>
<th>$\gamma\gamma$ coincidences</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>BGO</td>
<td>BGO</td>
<td>selectively</td>
</tr>
<tr>
<td></td>
<td>fold</td>
<td>fold</td>
<td>enhanced</td>
</tr>
<tr>
<td>prompt-prompt</td>
<td>$\geq 10$</td>
<td></td>
<td>high-spin, non-isomeric</td>
</tr>
<tr>
<td>prompt-prompt</td>
<td>$\geq 4$</td>
<td>$\geq 3$</td>
<td>above isomers</td>
</tr>
<tr>
<td>prompt-delayed</td>
<td>$\geq 4$</td>
<td>$\geq 2$</td>
<td>across isomers</td>
</tr>
<tr>
<td>delayed-delayed</td>
<td>$\geq 4$</td>
<td>$\geq 3$</td>
<td>below isomers</td>
</tr>
</tbody>
</table>
The time parameters were treated as described in appendix D to handle correctly the intrinsic changes in timing of the Ge and BGO detectors with variations in γ-ray energy.

5.3 Level-Scheme

5.3.1 Introduction

The complete decay-scheme is shown in Fig. 5.1. It is divided up into four drawings, with each one roughly corresponding to a range of K values. K values have been assigned to the bands based on their band-head spins. The energies and intensities of the γ-rays observed in 176W are given in Table 5.1 at the end of this chapter. Sections 5.3.2 through 5.3.5 discuss, respectively, the high-K states, the states fed in the decay of the K-isomers, and the K=0 and intermediate-K states. The details of the spin assignments and the construction of the level-scheme are discussed in section 5.3.5.

5.3.2 High-K States

Two high-spin isomers were observed, of which the most strongly populated is a Kπ=14+ state (Fig. 5.1(a)), whose complete patterns of feeding and decay were measured in this experiment. The spin and parity assignment for this state are discussed in detail in section 5.3.5. A hint of the existence of an isomeric state had previously been observed [Dra 78]. An indication of the quality of the present data can be gained from the spectra in Fig. 5.2. The half-life of the isomer is t1/2 = 70 ns, and its decay has been mapped out in detail, including a strongly K-violating
Fig. 5.1(a): Feeding and decay of high-K states in $^{176}$W. Only the members of the low-K bands which receive significant delayed feeding are shown.
Fig. 5.1(b): States in $^{176}$W with $K=0$. 

$^{176}$W 
states with $K=0$
Fig. 5.1(c): Band in $^{176}$W fed with high delayed fold.
Fig. 5.1(d): Bands in 176W built on 2qp states.

Intermediate-K states
Fig. 5.2: Gamma-ray spectra from the delayed-delayed data. Top: Total projection; middle: gate on the 596 keV transition from the g-band; bottom: gate on the 408 keV transition from the non-yrast s-band. All spectra were gated on delayed fold≥1. Transitions involved directly in the decay of the 70-ns isomer are labelled in keV. Transitions in the g-band populated by the decay of the isomer are labelled with the symbol ▼.
branch directly to the 14+ member of the ground-state band, and other
direct branches to states with K=0, as discussed below.

The gate on delayed fold proved to be an extremely powerful tool
for investigating the decay of the isomer. Although the isomer was only
populated with an intensity of 2%, high-spin isomeric decays constituted
15% of the events with delayed fold ≥ 2, and 45% of those with delayed
fold ≥ 3. The power of this method of gating is shown by a comparison
(Fig. 5.3) of total projections of the gamma-ray data, both with and
without a cut on delayed fold. In the ungated projection, the isomeric
decays are not visible against the background, but in the gated
projection, the isomeric decays are quite prominent. In addition to the
70-ns isomer, a second isomer, with a half-life of approximately 10 ns,
was observed (Fig. 5.1(a)).

Decay-schemes (Fig. 5.1(a)) were also constructed for the rotational
bands built on the two isomeric band-heads. The technique used was to
gate on the gamma-rays below the 70-ns isomer, requiring delayed
timing, and look for coincident transitions with prompt timing (Fig. 5.4).
These bands were then investigated further using prompt-prompt data
gated on a highed delayed BGO fold (Fig. 5.4). The half-lives of the two
isomers can be seen in the Ge time spectra in Fig. 5.5.

5.3.3 States Fed by K-Isomers

A novel feature of this experiment is that many weakly populated,
non-yrast structures have been observed, because they are fed
preferentially by the decay of high-spin isomers. These structures,
probably would not have been picked out with only prompt spectroscopy.
A large number of non-yrast states of the g-band and s-band were
Fig. 5.3: Effect of gating on delayed fold. The transitions from the decay of the 70-ns isomer are not visible in the ungated spectrum, but are quite prominent in the gated spectrum.
Fig. 5.4: Gamma-ray spectra for the bands above the isomers. Transitions in
the band built on the 70-ns isomer are labelled by ▼, transitions from the
band built on the 10-ns isomer by ♦, and transitions from the cascade
connecting the two isomers by *. Top: total prompt projection of the prompt-
delayed matrix, gated on delayed fold≥2; middle: prompt spectrum, gated
on the delayed 714 keV transition and delayed fold≥2; bottom: prompt
spectrum, gated on the prompt 223 keV transition and delayed fold≥3.
Fig. 5.5: TDC spectra for transitions depopulating the 10-ns and 70-ns isomers in $^{176}$W. The gating transitions were the 714 keV transition for the 70-ns isomer, and the sum of gates on the 316 and 210 keV transitions for the 10-ns isomer.
observed through their feeding by the decay of the 14+ isomer, as well as
the first excited K=0 band (02+ band) (Fig. 5.1(a)). These non-yrast
structures were first observed because of their delayed feeding, and this
delayed feeding then served as a bootstrap to extend them in prompt
spectroscopy (Fig. 5.1(b)). Table 5.2 gives the intensities of the
transitions depopulating the 70 ns isomer.

The observation of the non-yrast members of the g-band and s-
band is noteworthy, not just because these states are so rarely observed,
but because of their relevance for understanding the highly K-violating
decays. In particular, detailed spectroscopy of these bands in the region
where they cross allows the extraction of a matrix element for the
interaction between them, and as discussed in chapter 7, this has
important implications for testing the role of Coriolis mixing in the decay
of high-K states to the g-band and s-band.

A band was also observed built on a high-K state tentatively
labelled K=(13). The latter band is extremely prominent in spectra gated
on very high delayed fold (Fig. 5.6), and is apparently populated partly
by the decay of another isomer with very high spin (Fig. 5.1(c)).

The decay pattern of the K=(13) band-head could not be mapped
out because of its fragmentation into many weak branches, nor could the
decay scheme of the very high-spin isomer feeding it be constructed.
Arguments are presented in chapter 7 for the tentative K=(13)
assignment for the 4-qp band. In contrast to the 14+ isomer, the decay of
the K=(13) state seems to have a half-life too short to measure with
electronic timing, and seems to decay entirely to the bands with
intermediate K values, including the 6(+) band and possibly the 4(−)
band.
Fig. 5.6: Gamma-ray spectra for the band fed with high delayed fold. Transitions in the band fed with high delayed fold are labelled by ▼. Top: total projection, gated on delayed fold ≥6; middle: sum of gated on the in-band transitions; bottom: gate on the 202 keV. The middle and bottom spectra are not gated on Ge or BGO timing. The 202 keV transition carries some of the flux from the decay of the band-head, and can be seen in the middle spectrum.
5.3.4 *K=0 and Intermediate-K States*

The s-band (Fig. 5.1(b)) and the 2-quasiparticle bands with $K^\pi = 4(^-)$ and $6(^+)$ (Fig. 5.1(d)) have been extended to higher spins. The new $K=0$ and intermediate-$K$ band structures observed are the $0_2^+$ band (Fig. 5.1(b)), and three 2-qp bands with $K^\pi = (5), 7, 8(^-)$ (Fig. 5.1(d)). Examples of some relevant gamma-ray spectra for $K=0$ states are shown in Fig. 5.7.

5.3.5 *Spin Assignments and Further Notes on the Decay-Scheme*

The spins were deduced from the measured by the method of Directional Correlations from Oriented states (DCOs), discussed in appendix D. Fig. 5.8 shows a some of the measured DCO ratios, which for the most part are clearly divided between dipoles, with $R_{DCO}=0.5$, and quadrupoles, with $R_{DCO}=0.95$. The DCO ratios which are important for the spin assignments are given in Table 5.1. Rotational cascades of M1 and E2 transitions are observed throughout the level-scheme, and the observation of these regular rotational patterns also served as a consistency check on the level-scheme.

The spins of the various $K=0$ and intermediate-$K$ rotational bands are all unambiguously determined, except for the $K=(5)$ band, where the DCO ratio of the 1240 keV transition suggests a dipole angular distribution, but is not entirely inconsistent with a quadrupole character; this spin assignment is therefore tentative.

The angular distributions of the transitions depopulating the 70-ns and 10-ns isomer were not measurable, due to their very low intensity, or deorientation of the nucleus during the lifetime of the isomers, or both. The interlocking pattern of branches in the decay of the 70-ns
Fig. 5.7: Gamma-ray spectra for the K=0 states. Top: The sum of several gates on yrast transitions, showing the high-spin s-band; bottom: A similar sum, but for the high-spin g-band. Both spectra are gated on BGO fold ≥10.
Fig. 5.8: DCO ratios from the $^{176}$W experiment.
isomer restricts its spin to $J \leq 14$, assuming that no $\lambda = 3$ transitions are involved.† No branches are observed to the $12^+$ states of the $g$ and $s$ bands, which suggests $J > 13$, since for $J \leq 13$ one would expect that the branches to the $12^+$ states would be stronger, based on their multipolarities and gamma-ray energies. The $M1/E2$ branching ratios are also consistent with those expected for the lowest $K=14$ deformation-aligned configuration, as discussed in chapter 7. The 70-ns isomer is therefore assigned spin 14.

The parities of the $0^+_{2+}$, $6^+(-)$, $8^+(-)$, and $14^+$ bands are assigned based on the observation of fast quadrupole transitions linking states in these bands to bands with significantly different $K$-values; if these were $M2$ transitions, this would imply unprecedentedly low hindrance factors for $K$-violating $M2$ transitions (e.g., $F \leq 1$ for the 397 keV transition depopulating the $8^+(-)$ band, compared to $10^6 - 10^{10}$ for a typical $M2$ transition with $v=6$ [Löb 68]). The negative parity tentatively assigned to the $K=4$ band is based on systematics. This parity assignment is also natural in view of the pattern of decay of this band to the $g$-band, where no quadrupole transitions are observed.

The ordering of the following cascades in the decay of the 70 ns isomer (Fig. 5.1(b)) could not be determined: 885-656 keV, 445-1096 keV, and 519-1068-512 keV. For the 397-441 keV sequence near the $8^+(-)$ band-head, the ordering of transitions is based on rotational patterns of level-spacings. The ordering of the 316-211-267-355 keV cascade depopulating the 10 ns isomer is based on the intensities of the

† If the 917 keV transition was an $E3$, its strength would be 17 W.u., while $E3$ transitions with $v$ as low as 1-2 typically have strengths of $10^{-2} - 10^{-5}$W.u. [Löb 68].
transitions; the time spectra gated on these transitions are also consistent with this ordering.

A consistency check on the level-scheme was carried out by comparing sums of fitted gamma-ray energies for different decay cascades connecting the same states. These sums should be equal. This was done for 73 cases, and the inconsistencies follow a Gaussian distribution, as expected. This emphasizes that qualitative errors in the level-scheme are unlikely to have been made in areas where many transitions interlock, and also that the quantitative estimates of the uncertainties for the gamma-ray energies are reasonable.

The next chapter presents the results of the experiment on $^{176}$Hf, and the experimental results of this chapter will be compared with detailed theoretical calculations in chapter 7.
TABLE 5.1: Energies, intensities, and spin assignments for gamma-rays in $^{176}$W.

<table>
<thead>
<tr>
<th>$E_\gamma$ (keV)±error</th>
<th>$I_\gamma$ (a)±error</th>
<th>$K_\gamma J_\pi \rightarrow K_\alpha J_\pi$</th>
<th>DCO ratio (p)</th>
</tr>
</thead>
<tbody>
<tr>
<td>107.82± .02</td>
<td>25 (b)</td>
<td>$g,2^+$</td>
<td></td>
</tr>
<tr>
<td>174.2 ± .3</td>
<td>.6 ± .1</td>
<td>$4,6^{(-)}$</td>
<td></td>
</tr>
<tr>
<td>186.3 .2</td>
<td>.7 .2</td>
<td>(13,14)</td>
<td>(13,13)</td>
</tr>
<tr>
<td>202.1 .6</td>
<td>~.1</td>
<td>(j)</td>
<td></td>
</tr>
<tr>
<td>210.51 .03</td>
<td>.07(d)</td>
<td>(g)</td>
<td></td>
</tr>
<tr>
<td>218.0 .4</td>
<td>.5 .2</td>
<td>(13,15)</td>
<td>(13,14)</td>
</tr>
<tr>
<td>219.0 .4</td>
<td>.6 .2</td>
<td>(k)</td>
<td></td>
</tr>
<tr>
<td>222.80 .06</td>
<td>1.1 .2</td>
<td>$14,15^+$</td>
<td>$14,14^+$</td>
</tr>
<tr>
<td>230.72 .05</td>
<td>1.0 .2</td>
<td>$s,14^+$</td>
<td>$g,14^+$</td>
</tr>
<tr>
<td>238.2 .1</td>
<td>.9 .2 (d)</td>
<td>$14,16^+$</td>
<td>$14,15^+$</td>
</tr>
<tr>
<td>239.72 .03</td>
<td>82 .6</td>
<td>$g,4^+$</td>
<td>$g,2^+$</td>
</tr>
<tr>
<td>247.7 .2</td>
<td>.6 .2</td>
<td>(13,16)</td>
<td>(13,15)</td>
</tr>
<tr>
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<td>$4,8^{(-)}$</td>
<td>$4,7^{(-)}$</td>
</tr>
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<td>256.2 .1</td>
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<td>$14,16^+$</td>
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<tr>
<td>266.62 .07</td>
<td>.44 .08 (d)</td>
<td>(g)</td>
<td></td>
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<td>.8 .1</td>
<td>$6,8^{(+)}$</td>
<td>$6,6^{(+)}$</td>
</tr>
<tr>
<td>272.3 .1</td>
<td>.6 .1</td>
<td>$4,7^{(-)}$</td>
<td>$4,5^{(-)}$</td>
</tr>
<tr>
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<td>3.0 .2</td>
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<td>$4,4^{(-)}$</td>
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<td>(13,17)</td>
<td>(13,16)</td>
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<td>$14,18^+$</td>
<td>$14,17^+$</td>
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</tr>
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<td>297.8 .1</td>
<td>.20 .04</td>
<td>(h)</td>
<td></td>
</tr>
<tr>
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<td>.4 .1</td>
<td>$4,10^{(-)}$</td>
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</tr>
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<td>(h)</td>
<td></td>
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<td>(g)</td>
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<tr>
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<td>(c)</td>
<td>$14,21^+$</td>
<td>$14,20^+$</td>
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<td>(13,19)</td>
<td>(13,18)</td>
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<td>.27 .05 (d)</td>
<td>(h)</td>
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<td>$4,9^{(-)}$</td>
<td>$4,7^{(-)}$</td>
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<td>$6,8^{(+)}$</td>
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<td>(c)</td>
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<tr>
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<td>.5</td>
<td>g,22+</td>
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Note: The values in parentheses indicate the uncertainties or errors in the measurements.
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<th>Energy (keV)</th>
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<th>Orbital</th>
<th>State</th>
<th>Notes</th>
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<td>g,8+</td>
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<td>808</td>
<td>.3</td>
<td>s,26+</td>
<td>s,24+</td>
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<td>826.6 ± .1</td>
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<td>g,12+</td>
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</tr>
<tr>
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<td>.25</td>
<td>107 .(e) (i)</td>
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<td>.08</td>
<td>03(e,m)</td>
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<td>1541 ± &lt;.06</td>
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<td>g,12+</td>
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(a) Intensities are normalized such that the total intensity (gamma plus internal conversion) of the 108 keV transition is 100%. All intensities are based on the coincident intensities derived from events with Ge fold ≥2 and BGO fold ≥10.

(b) Based on theoretical internal conversion coefficient.

(c) Could not measure.

(d) Intensity taken from total projection of coincidence data gated on prompt-prompt Ge timing and a delayed BGO fold of at least 3. Intensity was normalized according to the intensity of the 223 keV transition.

(e) Intensity taken from total projection of coincidence data gated on delayed-delayed Ge timing and a delayed BGO fold of at least 2. Intensity was normalized according to the intensity with which the 70-ns isomer was populated.

(f) Energy from sums and differences of other gamma-ray energies, could not be determined directly from data.

(g) Cascade connecting the 10-ns isomer to the 70-ns, 14+ state.

(h) Rotational band built on the 10-ns isomer.

(i) Cascade depopulating 70-ns, 14+ state.

(j) Cascade connecting K=(13) band to 6+ band.

(k) Decay of K=(13) band.
(m) Intensity listed is intensity of population through 70-ns isomer; intensity due to prompt population could not be measured.
(n) Observation of this transition is uncertain; only an approximate upper bound is stated for the intensity.
(p) DCO ratios are shown only where they are relevant for establishing spins of band-heads.
TABLE 5.2: Intensities of transitions in the decay of the 70-ns isomer, as a percentage of the total flux through the isomer (100% = 2.4% in Table 5.1).

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<th>$I_\gamma$</th>
<th>$K,J_\pi \rightarrow K,J_\pi$</th>
<th>$E_\gamma$</th>
<th>$I_\gamma$</th>
<th>$K,J_\pi \rightarrow K,J_\pi$</th>
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<td>8 ± 4</td>
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<td>100 17</td>
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<td>5 2</td>
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<td>3 2</td>
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<td>632</td>
<td>3 2</td>
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<td>100 17</td>
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<td>8 4</td>
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<td>558</td>
<td>48 4</td>
<td>$g,12^+$ $g,10^+$</td>
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(a) Upper limit on intensity is based on measured coincident intensities of transitions in the 4$^(-)$ band with transitions above the 70-ns isomer. Intensity not determined directly because of 351 keV g-band transition.

(b) Upper limit on intensity is based on measured coincident intensities of transitions in the 6$^(+)$ band with transitions above the 70-ns isomer. Intensity not determined directly because of 508 keV g-band transition.
Chapter 6
Search for a K-Violating Decay in $^{176}$Hf

6.1 Introduction

As part of the program at Yale to study systematic properties of K-isomers, an experiment was undertaken to search for K-violating decays in $^{176}$Hf. In this nucleus, a series of K-isomers with K=6, 8, 14, 15, 16, 19, 20, and 22 had been observed previously by Khoo et al. [Kho 73b], [Kho 73c], [Kho 75], [Kho 76]. The strongest branches in the decay of each isomer were found to follow the classic pattern of minimum K-violation (see Fig. 1.3). In the neighboring nucleus $^{174}$Hf, the dominant patterns of decay are similar [Wal 83], but recent experiments with greater sensitivity and selectivity have shown abnormal, highly K-violating decays with small branching ratios existing side-by-side with the normal decays [Wal 90]. It is therefore of interest to see whether there are indeed corresponding K-violating transitions in $^{176}$Hf.

This experiment was dependent on the use of more sensitive detection systems for gamma-ray spectroscopy than those previously used. Six hyperpure Ge (HPGe) detectors were used, five with Compton-suppression and one unsuppressed planar Ge detector. As discussed previously in this thesis, BGO arrays with high segmentation provide opportunities for spectroscopy of high-spin isomers beyond those possible with Ge arrays by themselves, and the Yale BGO array was employed in this experiment. Unlike the facility at Argonne, the Yale tandem accelerator does not currently provide pulsed beams. The use of BGO
timing techniques were therefore not a luxury in this case, but a necessity to resolve the weak highly K-violating transitions. The basic method was to use BGO timing to determine when the prompt flash arrived. Given this information, it is possible to distinguish Ge-Ge coincidences where both are delayed from those where both are prompt. In addition, the highly K-violating decays being searched for would have a much higher delayed gamma-ray multiplicity than the normally K-violating cascades, and gates on the BGO fold were therefore used to eliminate a significant part of the background.

6.2 Experiment

States in $^{176}$Hf were populated in the reaction $^{176}$Yb($\alpha$,4n), with a 2.8 mg/cm$^2$ isotope-separated target and a beam energy of 50 MeV. The master gate was a coincidence of at least two Ge detectors and at least two BGO detectors, with the BGO detectors firing within 20 ns of each other and all four detectors firing within a time period of 2 $\mu$s. A total of $22 \times 10^6$ coincidence events was collected. In addition to the Ge energies and times, the energies and times of all the BGO detectors that fired were written to tape on an event-by-event, detector-by-detector basis for the maximum possible flexibility for off-line sorting.

The level of Ge-Ge statistics collected was similar to that achieved in previous experiments using the same reaction with non-Compton-suppressed detectors, and the new information to be gained was therefore only that which resulted from the BGO array (and the improved peak-to-Compton ratios). If a new isomeric decay, with a high degree of K-violation, was to be observed in the present work, using BGO timing
techniques, it would have to satisfy the criterion $M_\gamma(\text{new branch}) \gg M_\gamma(\text{strong branch})$, where $M_\gamma$ is the gamma-ray multiplicity, and the strong branch is the normal decay mode, with cascades of low multiplicity. In addition, it is clear from the previous studies that highly $K$-violating branches, if they exist in this nucleus, are very hindered, and therefore will not be competitive with the strong branches if they are of high multipolarities. Finally, the techniques used here are limited by the timing resolution of the detectors and the rate of random coincidences in the DC beam, resulting in the requirement that the half-life of the isomer be between $\sim 10$ nsec and $\sim 1 \mu\text{sec}$. The only possible candidate satisfying these criteria is a decay (Fig. 6.1) of the $19^+$, 34-nsec isomer directly to the $18^+$ member of the yrast $K=0$ band:

$$K_J^\pi = 19,19^+ \rightarrow 0,18^+, \ E_\gamma=366 \text{ keV}, \ M_\gamma(\text{delayed})=9$$

competing with the previously observed branches

$$19,19^+ \rightarrow 16,18^+, \ E_\gamma=529 \text{ keV}, \ M_\gamma(\text{delayed})=4$$

$$19,19^+ \rightarrow 16,17^+, \ E_\gamma=836 \text{ keV}, \ M_\gamma(\text{delayed})=4$$

Two techniques involving the BGO information were used to select isomeric, highly $K$-violating decays. First, the timing of the Ge detectors was measured relative to the timing of the coincidence between the first two BGO detectors to fire. This allowed the prompt background to be greatly reduced by selecting isomeric events, in which both Ge detectors fired after the prompt coincidence peak in the time spectrum. In experiments without such BGO timing information, the only time which can be measured is the time interval between the firing of the two Ge detectors, which does not allow a distinction to be made between prompt-prompt and delayed-delayed Ge-Ge coincidences. Conceptually, the BGO detectors are being used to determine the timing of the prompt
Fig. 6.1: Decay of the 19+ isomer in $^{176}$Hf.
flash of gamma-rays immediately following the reaction. In practice, it was found that the previously measured decays of the 19\(^+\) isomer were very strongly enhanced by the application of this gate (Fig. 6.2). This shows that the experiment is a very sensitive one for picking out a possible similar decay to the K=0 band. The decay of a known \((19/2)^+\) isomer [Dra 80] in the evaporation residue \(^{175}\)Hf was also observed.

The second technique used was simply to set a gate on the total BGO fold. This allows a distinction to be made between the low-multiplicity normal decay and a possible high-multiplicity decay directly to the K=0 states.

### 6.3 Analysis

No statistically significant 366 keV photopeak was observed in any of the spectra gated on the K=0 transitions, or in the sum of the gated spectra (Fig. 6.3). A quantitative upper limit on the existence of a highly K-violating branch can, however, be set from the present data. The best possible upper limit will be achieved by making cuts on the BGO fold to improve the signal-to-noise ratio. This has the disadvantage that the final result is dependent on a correct calculation of the response of the BGO array, but the greatest possible systematic error that results is not large (on the order of 10-20\% in the branching ratio). The final goal is a lower limit on \(f\), the hindrance factor per degree of K-forbiddenness. Since the degree of K-forbiddenness, \(v\), is 18, \(f\) is proportional to \(R^{-1/18}\), where \(R\) is the branching ratio, and \(f\) is therefore not very sensitive to small errors in \(R\).
Fig. 6.2: Gamma-ray spectra for $^{176}$Hf. Comparison of the ungated spectrum (top) and the delayed-delayed total projection (middle) shows that the 529 keV transition from the decay of the 19+ isomer is strongly enhanced. The delayed-delayed spectrum gated on the 214 keV transition (bottom) shows the very good selectivity available from this experiment.
Fig 6.3: Sum of delayed-delayed gates on yrast transitions in $^{176}$Hf, with a cut on fold$\geq$8. Top: the entire spectrum; bottom: the region around the searched-for 366 keV transition. All the transitions are previously observed. Yrast transitions, present due to leak-through from the prompt-prompt data, are marked with ▼, transitions from the normal decay of the $^{19+}$ isomer with ♦, and the 736 keV transition from the decay of the $^{6+}$ isomer with *. 
The branching ratio to be extracted is

\[ R = \frac{I_\gamma(19^+ \rightarrow K=0)}{I_\gamma(19^+ \rightarrow K=16)} \]

where

- \( I_\gamma \) = weighted average of \( I_{\gamma 1, 2} \)
- \( \gamma 1 \) = K-violating transition depopulating the isomer
- \( \gamma 2 \) = transition below \( \gamma 1 \) and in coincidence with it
- \( I_{\gamma 1, 2} = N_{\gamma 1, 2} (\epsilon_{\gamma 1, 2} \epsilon_{\text{BGO}}) \)
- \( N_{\gamma 1, 2} \) = counts observed in coincidence between \( \gamma 1 \) and \( \gamma 2 \)
- \( \epsilon_{\gamma 1, 2} \) = efficiency of Ge array for detecting a coincidence
  between the two transitions
- \( \epsilon_{\text{BGO}} \) = probability of satisfying the gate on BGO fold

The efficiencies of the two types of Ge detectors were measured as a function of gamma-ray energy using standard radioactive sources. The quantity \( R \) does not depend on the absolute normalizations of these efficiency curves, but does depend on the relative normalization of the curves for the planar Ge and the Compton-suppressed Ge. This relative normalization was determined by taking radioactive source data for all six detectors over the same period of time.

In order to distinguish between the normal, low-multiplicity decay and a highly K-violating, high-multiplicity decay, different gates were set on the BGO fold. For the low-multiplicity decay, no software gate was imposed, so that only the master-gate requirement of BGO fold\( \geq 2 \) was involved. For the high-multiplicity decay, a software gate was applied to select events with BGO fold\( \geq 8 \). Two different methods were used to estimate the quantities \( \epsilon_{\text{BGO}} \) which therefore enter into the extracted branching ratio.
First, the quantities $\varepsilon_{\text{BGO}}$ were derived from the calibration measurements described in appendix A. All the branching ratios below the isomer, except for the one being extracted here, are known, and therefore the probability distribution $P(M_\gamma)$ of the gamma-ray multiplicity from the cascade below the isomer can be calculated. This was done using a simple Monte Carlo calculation. The effects of internal conversion were included. The results are:

<table>
<thead>
<tr>
<th>branch</th>
<th>$M_\gamma=2$</th>
<th>$M_\gamma=3$</th>
<th>$M_\gamma=4$</th>
<th>$M_\gamma=5$</th>
<th>$M_\gamma=6$</th>
<th>$M_\gamma=7$</th>
<th>$M_\gamma=8$</th>
<th>$M_\gamma=9$</th>
<th>$M_\gamma=10$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rightarrow K=0$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>.04</td>
<td>.22</td>
<td>.63</td>
<td>.10</td>
<td></td>
</tr>
<tr>
<td>$\rightarrow K=16$</td>
<td>.03</td>
<td>.15</td>
<td>.46</td>
<td>.36</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
</tbody>
</table>

(The expectation values are $\langle M_\gamma \rangle = 4.2$ for the normal branch and 8.7 for the branch to the $K=0$ yrast band.) In addition to these delayed gammas, it was assumed that an average of three prompt gammas were emitted before populating the isomer. (The isomer, with spin 19, is near the top of the range of spins populated in this reaction.) In events which satisfy the master-gate, however, two of these are required to be detected by the Ge detectors. The average total gamma-ray multiplicity available to the BGO array is therefore approximately 5 for the normal branch, and 10 for the branch to the yrast band. In the relevant range of gamma-ray energies, the calibrations described in appendix A show that the average BGO fold is equal to approximately 0.8 times the gamma-ray multiplicity, and the average BGO fold is therefore approximately 4 for the normal branch, and 8 for the highly $K$-violating decay. The estimated probabilities that the BGO fold will satisfy the cuts are roughly
\[ \varepsilon_{BGO} \sim 80-100\%, \text{ normal branch, fold} \geq 2 \]
\[ \varepsilon_{BGO} \sim 50\%, \text{ highly K-violating branch, fold} \geq 8 \]

As a check on these a priori calculations, direct measurements of \( \varepsilon_{BGO} \) were obtained from the experimental data. This was done by gating on a gamma transition in the Ge spectra to select a given cascade, and then examining the BGO fold distributions. The data were gated on the 529 keV normal K-violating branch, and on the 703 keV \( 18^+ \rightarrow 16^+ \) transition in the yrast band. In the latter case, it was assumed that the gamma-ray multiplicity was similar for populating the \( 18^+ \) yrast state through prompt feeding or through isomeric feeding. A Gaussian was fit to each of the two fold distributions, and the probabilities of satisfying the cuts were found to be

\[ \varepsilon_{BGO} = 80 \pm 10\%, \text{ normal branch, fold} \geq 2 \]
\[ \varepsilon_{BGO} = 35 \pm 5\%, \text{ highly K-violating branch, fold} \geq 8 \]

The agreement between the values of \( \varepsilon_{BGO} \) extracted from the two methods is fairly good, and a conservative upper limit on the relative efficiencies entering into the calculation of the branching ratio is therefore

\[ \frac{(\varepsilon_{BGO}, \text{fold} \geq 2)}{(\varepsilon_{BGO}, \text{fold} \geq 8)} \leq 3. \]

The coincident intensities used in the weighted averages were as follows: for the normal branch, the coincidences of the 529 keV \( 19,19^+ \rightarrow 16,18^+ \) transition with the 186 keV \( 16,16^+ \rightarrow 15,15^+ \) and \( 15,15^+ \rightarrow 14,14^- \) transitions; and for the highly K-violating branch, all the possible coincidences of the 366 keV \( 19,19^+ \rightarrow 0,18^+ \) transition with the yrast transitions leading to the ground state.

The final result for the branching ratio is

\[ R < 0.02 \text{ (2-sigma limit)} \]
which leads to a lower limit on the hindrance factor per degree of K-
forbiddenness of

\[ f > 2.3 \]

A systematic error of as much as 20% in the estimate of the ratio \( (\epsilon_{\text{BGO}}, \text{fold} \geq 2)/(\epsilon_{\text{BGO}}, \text{fold} \geq 8) \) would only result in a change of 0.02 in the value of \( f \). Hindrance factors per degree of K-forbiddenness \( f = 2 \) have been observed in the decays of the K=25 and K=14 isomers in the nearby nuclei \(^{182}\text{Os}\) and \(^{176}\text{W}\) (ref. [Cho 88] and chapter 5). The present upper limit for this decay in \(^{176}\text{Hf}\) therefore sets a constraint on theoretical models, which must account for the differences in hindrance factors between nuclei.
Chapter 7
Discussion and Calculations for 176W

7.1 Introduction

The most dramatic result of the experiments described in this thesis is the observation of highly K-violating decays in $^{176}$W directly from the K=14 isomer to the K=0 states in the g-band, s-band, and $0^+_2$ band. Qualitatively, this is the only known example of the decay of a state with such a high K value in which K=0 states receive essentially all of the flux, while there is no decay at all to the many available states with intermediate values of K. Quantitatively, the hindrance factors for the K=14 $\rightarrow$ K=0 decays are two orders of magnitude lower than those of the K=14 $\rightarrow$ K=0 decays in the isotone $^{174}$Hf. The ability to compare the hindrance factors for the same configuration in two different nuclei makes this potentially a very helpful data-set for exploring the mechanisms of highly K-violating decays, since it is possible to examine the systematics of the K-violating transitions while keeping many of the variables constant. The main purpose of this chapter is to test models of Coriolis mixing and gamma-tunneling against these observations.

Although the focus of this chapter is on understanding the highly K-violating decays, it will also be important to understand the structures of the K=0 and intermediate-K states, since these are the states which are available for the decay of the high-K isomers. Recent attempts at explaining highly K-violating decays in terms of Coriolis mixing, in particular, depend on the structure of the g-band and s-band states, and
the mixing between them in the band-crossing region, since these are the K=0 states available to receive the decay.

In section 7.2, the K=0 states will be discussed, and the intermediate-K states will be taken up in section 7.3. Section 7.4 discusses the rotational bands built on the high-K states and the information that they give on the structures of their band-heads. The available information on the abnormal decays of the high-K isomers is summarized in section 7.5. Finally, tests of the Coriolis mixing and gamma-tunneling mechanisms are developed in section 7.6.

7.2 Rotational Bands With K=0

The yrast K=0 states have been extended through prompt spectroscopy, adding transitions up to the 26+ member of the s-band. More noteworthy is the observation of a variety of non-yrast K=0 structures. This was made possible by the unusual decay pattern of the 14+ isomer, in which 57% of the decay is directly to K=0 states. The decay of the isomer provided a clean trigger for observing these states, most of which are non-yrast and would ordinarily have been difficult to pick out in heavy-ion data. With the decay of the isomer as a starting point, these non-yrast structures were then extended in prompt spectroscopy to cover a wider range of spins. Fig. 7.1 shows the relative excitation energies of these states as a function of J(J+1), which clearly display three rotational bands, with two band-crossings.

The non-yrast parts of the g-band and s-band are easily identified from this plot, and these assignments are confirmed by the pattern of perturbations of the energy levels near the crossing, and by the
Fig. 7.1: Excitation energy as a function of $J(J+1)$ for the g-band, s-band, $0^+_2$ band, and $14^+$ isomer in $^{176}$W. A rotational band with perfect $J(J+1)$ level-spacing would appear on this plot as a straight line. A reference value of $(9 \text{ keV}) \times J(J+1)$ has been subtracted from the excitation energies. This gives a scale on the y-axis which is fine enough to show deviations from $J(J+1)$ level-spacing, which are due to interactions between bands and effects such as centrifugal stretching (see text). The three arrows show the direct decays of the $K=14$ isomer to $K=0$ states, and their intensities as a fraction of the total strength with which the band is populated.
branching ratios between the two bands. The energy perturbations and branching ratios can be compared with theory to derive an interaction matrix element acting between the two bands. It will be assumed that the matrix element is constant with respect to spin, and that only two bands are mixing. Since the energy separation \( E-E' \) is always greater than \( V \), a definite upper limit of \( |V_{gs}| \leq 33 \text{ keV} \) can be stated.

The best source of a lower limit on the matrix element comes from the branching ratios, rather than the energy levels. Such branching ratios have seldom [Kho 73a] been measurable in previous experiments. The ratios used were \( B(E2,\text{out})/B(E2,\text{in}) \) for the strengths of the \( \Delta J=2 \) transitions between bands, where \( B(E2,\text{in}) \) refers to the in-band branch \( (g \rightarrow g \text{ or } s \rightarrow s) \), and \( B(E2,\text{out}) \) refers to the out-of-band branch \( (g \rightarrow s \text{ or } s \rightarrow g) \). Theoretical values were calculated for the ratios as described in appendix C. A narrow range around \( |V_{gs}| =30-33 \text{ keV} \) clearly gives the best fits. The observed branching ratios, and the calculated values for \( |V_{gs}| =32 \text{ keV} \), are as follows:

<table>
<thead>
<tr>
<th>initial state</th>
<th>( \exp. )</th>
<th>( \text{th.} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>14+,s</td>
<td>.008(1)</td>
<td>.004</td>
</tr>
<tr>
<td>16+,s</td>
<td>.24(6)</td>
<td>.56</td>
</tr>
<tr>
<td>18+,g</td>
<td>1.0(3)</td>
<td>.30</td>
</tr>
<tr>
<td>18+,s</td>
<td>1.1(5)</td>
<td>.30</td>
</tr>
</tbody>
</table>
The ratios are largest in the vicinity of the crossing (between $J=16$ and 18), where the mixing is the greatest. Although there is not agreement to within statistical errors, the experimental values span two orders of magnitude, and the calculated values have approximately the right sizes. Values of $V$ outside the range of 30-33 keV give results which are completely out of line with the data. A value of $|V_{gs}| \approx 30$ keV will be adopted for use later in this chapter, where it will be used to investigate possible mechanisms for the K-violating decays. Various second-order effects may be responsible for the discrepancies between the data and the best fit, but the quantities calculated later are not sensitive to $\sim 20\%$ variations in $V$, and the most important piece of data is the upper limit on $V$, which does not depend on the fit to the branching ratios.

The g-band displays a large but gradual increase in its moment of inertia at high spins, similar to that observed in, e.g., $^{179}$W [Wal 91], where the g-band was also observed at high rotational frequencies. This trend is clearly visible in fig. 7.1, where it causes the downward curvature of the g-band. When this type of gradual increase is observed, it has been shown in refs. [Ben 86b], [Ben 89c], [Ben 91] that, based only on the observed energy levels, it is not possible to separate the contributions of centrifugal stretching, Coriolis anti-pairing, and gradual quasiparticle alignment. Calculations using the cranked Nilsson-Strutinski code of ref. [Ben 90] have been carried out as part of the present work, and indicate that the deformation of the g-band rises gradually from $\epsilon_g=.24$ at low spin to $\epsilon_g=.29$ at spin 18, above which

\footnote{For instance variation of V with spin [Kho 73a], differences in deformation between the two bands (discussed in appendix C and later in this section), or interactions with the unobserved states of the $0^+_2$ band having $J>12$}
which it remains approximately constant through spin>30. This naturally will lead to an increase in the moment of inertia, with some contribution as well from Coriolis anti-pairing. No alignment process is predicted, other than the g-band/s-band crossing. (Alignment processes in this region have been discussed previously [Ber 76], [Ben 91]). No strong centrifugal stretching effect is predicted for the s-band, which is expected to maintain $\varepsilon_s=0.22-0.23$ for the range of spins observed in this experiment. This is in agreement with the data, since the s-band does not show any deviation from $J(J+1)$ level-spacing (fig. 7.1) which would be expected if there was significant stretching. Near the g-band/s-band crossing, a rather large difference in deformation ($\varepsilon_s=0.22$, $\varepsilon_g=0.29$) is predicted. This is consistent with the small mixing matrix element extracted for the crossing, since states with different deformations are not expected to mix much. Such deformation changes may be part of the reason for the complicated systematics of the alignment curves of the W isotopes [Ben 91], [Ber 76]. A direct experimental test of this predicted difference in deformations at the crossing is available in principle through comparisons of branching ratios, but the statistical errors in the observed gamma-ray intensities were too great to allow a sensitive comparison with theory.

The third band populated by the decay of the isomer is assigned $K=0$, and identified as the band structure built on the first excited $0^+$ state, based on the following observations:

(1) In the area where this band crosses the s-band, the energy perturbations and branching ratios give an interaction matrix element $30 \leq |V_{0^+,s}| \leq 110$ keV, based on the observed perturbations of the $12^+$ levels. (Insufficient data were available
for a more detailed and accurate fit such as the one performed above for \(|V_{g,s}|\). An interaction this large would be quite anomalous for bands with different \(K\) values.

(2) No signature partner is observed.

(3) The electromagnetic transitions to the \(g\)-band are fairly strong \((B(M1)\sim 0.02\ \text{W.u.},\ \text{assuming}\ \delta=0;\ \text{see}\ \text{appendix C})\) compared to those normally seen [Löb 68] for \(K\)-violating transitions.

(4) Although the depopulation of the band at low spins was too rapid to allow the observation in these data of levels with \(J<6\), two states in \(^{176}\text{W}\) have been observed (at 929 and 1117 keV) in the radioactive decay of the \(3^+\) state in \(^{176}\text{Re}\), which form a clear continuation of the \(J(J+1)\) pattern of excitation energies. The levels in \(^{176}\text{W}\) from \(2^+\) to \(10^+\) form a band nearly identical to the corresponding part of the \(0^2^+\) band in \(^{178}\text{W}\), with gamma-ray energies differing by less than 8 keV, and the band in \(^{178}\text{W}\) has been observed down to the \(K=0\) bandhead. The log \(f_t\) values are very similar (log \(f_t\)=6.7 and 6.9, respectively) for population of the corresponding \(2^+\) states in \(^{176}\text{W}\) and \(^{178}\text{W}\) from the \(^{176}\text{Re}\) and \(^{178}\text{Re}\) decays.

The observation of the crossing between the \(s\)-band and the \(0^2^+\) band is noteworthy, since it has seldom been observed [Kho 73a]. The band built on the first excited \(0^+\) state is often labelled the \(\beta\)-vibrational or "\(\beta\)-band." The extrapolated band-head energy of 0.9 MeV is considerably lower than that expected for a pure 2-quasiparticle state, but attempts to associate such states with simple collective modes like beta-vibrations or pairing vibrations have failed to reproduce the
available data on electromagnetic and particle-transfer matrix elements [Mik 67], [Bes 66], [Bir 75], [Bur 88]. The label "β-band" is therefore avoided in this paper. The measurement in this work of the interaction $V_{02^+,s}$ may test future models of these low-lying excitations.

### 7.3 States With Intermediate K Values

A wide variety of intermediate-K states is observed in this experiment, and most of these can be assigned reasonable 2-qp configurations. Whether similar assignments are appropriate for the $4(-), 6(-), 8(-), ..., \text{and } 5(-), 7(-), 9(-), ..., \text{sequences requires some extra discussion. Based on previous data, it had been suggested [Dra 78] that these sequences might be two of the four members of the octupole-vibrational multiplet with } K^\pi=0^-, 1^-, 2^-, 3^- . \text{This interpretation was motivated by the fact that the excitation energy of the band-head, 1.3 MeV, is slightly lower than the minimum expected for a 2-qp state, since the ground-state pair-gaps are about 0.7 MeV. Two-quasiparticle states would be expected to have an energy of at least twice the pair gap. Such an interpretation, however, would require the existence of previously unobserved band-heads with } K<4 . \text{It had been suggested [Dra 78] that these might not have been observable in previous experiments because of the rapid depopulation of the band at spins higher than the band-head spin. The current experiment is more sensitive than the previous one (where only three Ge detectors were used, without a multiplicity filter), and has failed to reveal any in-band branches leading to } 2^- \text{ and } 3^- \text{bandheads, nor have the other two bands appeared.}
This seems to rule out an interpretation in terms of octupole vibration. It is therefore worth considering the interpretation of these states as simple 2-qp excitations. The discrepancy between the energy of the $4^{-}$ state and the 2-qp energy is rather small. For the lowest proton and neutron configurations with $K^\pi=4^{-}$, the 2-qp energies, calculated using the Woods-Saxon code of ref. [Cwi 87] and the parameters of ref. [Dud 82], are approximately

\[
\begin{align*}
\pi 7/2^+[404] & \otimes \pi 1/2^-[541] & 2.1 \text{ MeV} \\
\nu 7/2^+[633] & \otimes \nu 1/2^-[521] & 1.7 \text{ MeV}
\end{align*}
\]

where the labels $\pi$ and $\nu$ distinguish protons and neutrons, and the ground-state values of the proton and neutron pair-gaps, $\Delta_p=1.0$ MeV, $\Delta_n=0.7$ MeV, have been used. Inaccuracies in the estimated pair gaps and from neglecting neutron-proton interactions may contribute discrepancies of up to a few hundred keV, and the low excitation energy of the band-head is therefore not entirely inconsistent with 2-qp structure. Both the possible 2-qp configurations involve an orbital with $\Omega=\pm1/2$, which would be coupled directly by the $\Delta K=1$ Coriolis matrix elements. This would cause a large admixture of $K=3$, which would make the strengths (2-50 $\mu$W.u.) of the $K$-violating transitions to the $g$-band comparable to the typical values given in ref. [Löb 68] for $E1$ transitions with $\Delta K=3$.

Returning our attention to the 2-qp states as a group, a first step in classifying the intermediate-$K$ band-heads as $\pi^2$ or $\nu^2$ states is to examine the systematics of the band-head energies in nearby nuclei (Fig. 7.2). The $K=4^{-}$ and $7^{-}$ states display approximately constant excitation energies in the W isotopes with $A=172-184$, suggesting that they are $\pi^2$ states. The energies of the $6^+$ states in the Hf isotopes show evidence of
Fig. 7.2: Systematics of intermediate-K 2-qp states in W and Hf isotopes. Top: 4⁻ and 7⁻ band-heads; bottom: 6⁺ band-heads.
minima near $N=104$; this is in agreement with calculations of the quasi-neutron states (see below), which show that the configuration $\nu \frac{5}{2}^{-}[512] \otimes \nu \frac{7}{2}^{-}[514]$ should be favored in this vicinity, rather than $\pi^2$ configurations. (Less extensive data are available for the $6^+$ states in the W isotopes.) Both $\nu^2$ and $\pi^2 8^-$ states have been observed in the Hf-W region, but for $^{176}$W the $\pi^2$ state is expected to be much lower than the $\nu^2$ state. (No attempt will be made to discuss the $K=5$ band in detail, since its parity is unknown, and there is therefore no way to reliably assign it a configuration.) Given these inferences regarding the $\pi^2$ and $\nu^2$ characters of the states, the configurations are unambiguously determined by the available combinations of $\Omega$ values, and the predicted excitation energies can be compared with experiment:

<table>
<thead>
<tr>
<th>$K^\pi$</th>
<th>configuration</th>
<th>$E_x$ (exp)</th>
<th>$E_x$ (theory)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$4^(-)$</td>
<td>$\pi \frac{1}{2}^-[541] \otimes \pi \frac{7}{2}^+[404]$</td>
<td>1.301 MeV</td>
<td>2.1 MeV</td>
</tr>
<tr>
<td>$7^(-)$</td>
<td>$\pi \frac{9}{2}^-[514] \otimes \pi \frac{5}{2}^+[402]$</td>
<td>1.857</td>
<td>2.0</td>
</tr>
<tr>
<td>$8^(-)$</td>
<td>$\pi \frac{9}{2}^-[514] \otimes \pi \frac{7}{2}^+[404]$</td>
<td>1.972</td>
<td>2.0</td>
</tr>
<tr>
<td>$6^(+)$</td>
<td>$\nu \frac{5}{2}^-[512] \otimes \nu \frac{7}{2}^-[514]$</td>
<td>1.656</td>
<td>1.6</td>
</tr>
</tbody>
</table>

The agreement of the calculated and measured excitation energies is satisfactory, except perhaps, as noted above, in the case of the $4^(-)$ state.

In order to describe the high-spin states that make up the rotational bands built on these configurations, it is necessary to associate the states at $\omega=0$, which are eigenstates of $\Omega$, with the quasiparticle trajectories at $\omega>0$, which are, strictly speaking, only
eigenstates of parity and signature. The neutron and proton quasiparticle trajectories are plotted in Fig. 7.3. Orbitals of very different $\Omega$ do not mix appreciably at moderate rotational frequencies, and may therefore be unambiguously extrapolated to $\omega=0$, but states of the same signature and parity which differ only by 1 unit in $\Omega$ will mix even at low rotational frequencies. Changes in structure will therefore occur close to the band-head for states involving these orbitals. The states which can be expected to change character at low frequencies are the orbital

$$\pi 1/2^-[541]$$

which splits into its two signature partners immediately, and the pairs

$$\pi 7/2^+[404] \text{ and } \pi 5/2^+[402]$$

and

$$\nu 5/2^-[512] \text{ and } \nu 7/2^-[514]$$

both of which will mix at low frequencies. For these orbitals, it is therefore equally plausible to continue each $\omega=0$ state either adiabatically, or diabatically to the state with which it mixes. (The construction of diabatic and adiabatic Routhians is discussed in appendix B and in ref. [Ben 86a].) Given this degree of ambiguity in associating band-head configurations with their high-spin continuations, the approach taken here is one of seeking to reproduce the observed 2-quasiparticle Routhians. The orbitals for $\omega>0$ will be labelled using the notation $(\pi,\alpha)_n$, where $n$ is either an integer labelling the states of a given $(\pi,\alpha)$ by excitation energy or, where that notation is ambiguous or misleading, by the letters "H" for high-$\Omega$ orbitals and "L" for low-$\Omega$ orbitals. The signature quantum number is omitted in cases where the two signatures are degenerate (fig 7.3).
Fig. 7.3: Calculated neutron and proton Routhians (see chapter 2 for definitions) for $^{176}W$, calculated with $\beta=.26$, $\beta_4=-.04$, $\gamma=0$. The 2-quasiparticle Routhians shown in fig. 7.4 were constructed by adding these Routhians in pairs (see text).
The K=7 band is used as the reference configuration, and the following orbitals are assigned:

<table>
<thead>
<tr>
<th>band-head</th>
<th>orbitals</th>
</tr>
</thead>
<tbody>
<tr>
<td>4−, α=0</td>
<td>π(−,−1/2)L ⊗ π(+,+1/2)2</td>
</tr>
<tr>
<td>4−, α=1</td>
<td>π(−,−1/2)L ⊗ π(+,−1/2)1</td>
</tr>
<tr>
<td>7−</td>
<td>π(−)H ⊗ π(+)_1</td>
</tr>
<tr>
<td>8−</td>
<td>π(−)H ⊗ π(+)_2</td>
</tr>
<tr>
<td>6+</td>
<td>ν(−)H1 ⊗ ν(−)H2</td>
</tr>
</tbody>
</table>

Fig. 7.4 compares the observed Routhians with the Woods-Saxon Routhians. The agreement is fairly good. There are some differences, including the excitation energy of the 6+ intrinsic state, which is predicted too low, and the signature splitting of the 4− band, which is too large. The over-all description of these states is about as good as can be expected, considering the inaccuracies inherent in a cranking treatment of states with 0<K<J.†

7.4 High-K States

The decay of the K=14+ band-head is the most important experimental result in this thesis, and it is important to analyze both the

† An additional check on the quasipartice assignments comes from the B(M1)/B(E2) values, which are available for the 4(−) band, where both signature partners have been observed. The semi-classical model of Donau and Frauendorf (appendix C) predicts B(M1)/B(E2) values of 0.02-0.06 μN^2/e^2b^2 which are similar to the observed values of 0.02-0.04 μN^2/e^2b^2. (Both the π^2 and ν^2 configurations have |gK−gR|=0.2.)
Fig. 7.4: Comparison of observed and calculated 2-quasiparticle Routhians in $^{176}$W. Each band is labelled with the quantum numbers $K\pi$ or $K\pi,\alpha$. The slopes of the observed Routhians are well reproduced, as are the relative energies of the $6^+$, $7^-$, and $8^-$ Routhians. The two $4^-$ Routhians are calculated too high in energy, but show the correct energies relative to one another, the correct slopes, and the correct alignment behavior.
isomeric band-head and the rotational band built on it for as many clues as possible to its underlying structure. A state with a value of $K$ this large cannot be constructed from the angular momenta of only two quasiparticles. It must therefore be a 4-qp states, since 6-qp states will not occur this low in energy. It has been observed up to rotational frequencies in the neighborhood of the $i_{13/2}$ neutron rotational alignment in the yrast cascade, and neither signature partner shows any sign of rotational alignment. This indicates that the $i_{13/2}$ alignment is blocked by the intrinsic configuration of these bands. The 4-qp state which is predicted by Woods-Saxon calculations to be most nearly yrast is

$$K=14^+, \pi \frac{7}{2}+[404] \otimes \pi \frac{9}{2}^-[514] \otimes \nu \frac{7}{2}+[633] \otimes \nu \frac{5}{2}^-[512]$$

involving a combination of four high-$\Omega$ orbitals (fig. 7.3). The calculated excitation energy is 3.5 MeV, in good agreement with the observed energy of 3.746 MeV. This configuration has a high-$\Omega$ neutron $i_{13/2}$ orbital occupied, consistent with the observed lack of rotational alignment. The observed $B(M1)/B(E2)$ branching ratios agree very well with those calculated for this configuration (Fig. 7.5).

Less can be said about the band containing the 186, 218,... keV transitions (fig. 5.1(c)) and the band built on the 10-ns isomer (fig. 5.1(a)), since the spectroscopic information for these states is so incomplete. Woods-Saxon calculations suggest that the former band may have the configuration

$$K^\pi=13^+, \pi \frac{5}{2}^+[402] \otimes \pi \frac{9}{2}^-[514] \otimes \nu \frac{7}{2}+[633] \otimes \nu \frac{5}{2}^-[512].$$

The $B(M1)/B(E2)$ data for this band are not of high quality, but seem to be consistent with calculations based on this configuration. (The data are not consistent with switching the configurations of the 13+ and 14+ bands, since the calculated $B(M1)/B(E2)$ values are different by an order
Fig. 7.5: Comparison of observed and calculated $B(M1)/B(E2)$ values in the band built on the 70-nsec isomer in $^{176}$W. The calculated values were obtained, as described in appendix C, for the quasiparticle configuration described in the text.
Concerning the band built on the 10-ns isomer, calculations predict a variety of 6-qp states near the yrast line in this range of energy, with both $\pi^4\nu^2$ and $\pi^2\nu^4$ configurations.

### 7.5 Summary of Data on the Decay of the High-K States

The most striking observation in this experiment was that the decay of the 14+ isomer deviates completely from the normal pattern of decay of a K-isomer. The decay is shown in detail in Fig. 5.1c, and the gamma-ray intensities for the decay of the isomer are given in Table 5.2. The majority (57%) of the decay proceeds to K=0 states, and no branches of any detectable strength were found to the states with intermediate K values, even though a very complete set of these states was observed in this experiment, and the apparatus was very sensitive to isomeric decays. This is all the more remarkable because K=14 isomers have also been observed in the neighboring even-even isotope $^{174}$Hf [Sle 90], [Wal 90] and even-even isotope $^{178}$W [Kra 88], [Kra 89], where the highly K-violating decays are only a small fraction (2%) of the total decay (Fig. 7.6).

Detailed data are available on the half-lives and branching ratios for the decays of the K=14 states in $^{176}$W, $^{174}$Hf, and $^{178}$W, and it is therefore possible to compare these data systematically. In this section, a brief summary of the data is given. In the next section, the data will be compared to calculations. In both cases, a spin assignment of K=14 has plausibly been made. Positive parity was assigned to the $^{174}$Hf isomer [Wal 90], and the suggested configuration is identical to that assigned here to the $^{176}$W isomer. For the $^{178}$W isomer, ref. [Kra 89]
Fig. 7.6: A comparison of the decay-schemes of the 14+ isomers in $^{174}$Hf and $^{176}$W. In $^{176}$W, all the observable decay branches are to states with $K=0$, while in $^{174}$Hf, almost all of the decays are to states with higher values of $K$. (The decays to the states in $^{174}$Hf with $K=8$, 11, 12 are weaker than the decays to the $K=6$ band because the gamma-ray energies are much lower.)
argues for a negative-parity assignment, but uses an extended chain of inferences, some of which are model-dependent. Both possible parity assignments will be discussed here. The suggested configurations for the three isomers are:

<p>| | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{176}$W 14$^+$</td>
<td>$\pi 7/2^+[404] \otimes \pi 9/2^-[514] \otimes \nu 7/2^+[633] \otimes \nu 5/2^-[512]$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^{174}$Hf 14$^+$</td>
<td>same as $^{176}$W</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^{178}$W 14$^-$</td>
<td>$\pi 7/2^+[404] \otimes \pi 9/2^-[514] \otimes \nu 5/2^-[512] \otimes \nu 7/2^-[514]$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^{178}$W 14$^+$</td>
<td>$\pi 5/2^+[402] \otimes \pi 9/2^-[514] \otimes \nu 7/2^+[633] \otimes \nu 7/2^-[514]$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The following is a summary of the available data on the $\Delta K=14$ transitions and the K=0 states available as their final states:

<p>| | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>obs. transition from isomer</td>
<td>$\rightarrow g$</td>
<td>$\rightarrow s$</td>
<td>$\rightarrow 0_{2^+}$</td>
<td>$</td>
<td>V_{g,s}</td>
</tr>
<tr>
<td>$^{176}$W</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>$^{178}$W</td>
<td>yes</td>
<td>no</td>
<td>no</td>
<td>no</td>
<td>no</td>
</tr>
<tr>
<td>$^{174}$Hf</td>
<td>yes</td>
<td>*</td>
<td>yes</td>
<td>yes</td>
<td>no</td>
</tr>
</tbody>
</table>

The interaction matrix elements will be important for later discussion of the K-violating decays. The asterisk for the 14$^+ \rightarrow s$ decay in $^{174}$Hf indicates that further evidence is needed for the identification of the state in question as a non-yrast member of the s-band.
The reduced hindrance factors (\(f=F^{1/\nu}\)) for the \(\Delta K=14\) transitions are:

<table>
<thead>
<tr>
<th>final state</th>
<th>176W</th>
<th>174Hf</th>
<th>178W</th>
</tr>
</thead>
<tbody>
<tr>
<td>14+</td>
<td>3.8</td>
<td>5.6</td>
<td>-</td>
</tr>
<tr>
<td>12+</td>
<td>&gt;3.4</td>
<td>5.5</td>
<td>3.2</td>
</tr>
<tr>
<td>14+</td>
<td>3.2</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>12+</td>
<td>&gt;3.0</td>
<td>(3.6)</td>
<td>-</td>
</tr>
<tr>
<td>02+, 14+</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>02+, 12+</td>
<td>2.4</td>
<td>4.5</td>
<td>-</td>
</tr>
</tbody>
</table>

(Transitions have been included in the table only if they are significant at the 2-sigma level.) Although the differences among the reduced hindrance factors for the different nuclei might appear small, these differences represent very large differences in the absolute hindrance factors. For the decays to the 14+ states of the g-bands of \(^{176}\text{W}\) and \(^{174}\text{Hf}\), the absolute hindrance factors are \(4\times10^7\) and \(6\times10^9\), respectively, a difference of two orders of magnitude.

7.6 Calculations of the Decay of the High-K States

7.6.1 Introduction

Since the first observations, in the late 1980s, of gamma-ray transitions with anomalously large values of \(\Delta K\), it has become necessary to reexamine old assumptions about K-violation. One would like to understand the underlying mechanism for the exotic decays. But because the decays arise from small-probability fluctuations in the K quantum number, it is not possible to gain direct insight into these
mechanisms through measurement of static matrix elements, such as the energies, quadrupole moments, and magnetic moments. At present, the mechanisms of K-violation can only be addressed by comparing the K-violating transition matrix elements with theoretical calculations. Since all presently tractable methods for the theoretical treatment of mid-shell nuclei involve truncations and approximations, it should be kept in mind that the mapping from the models to the underlying physics is not necessarily one-to-one.

In this section, the focus will be on understanding why differences of several orders of magnitude exist between the strengths of the $\Delta K=14$ transitions in $^{176}$W and $^{174}$Hf. It was suggested in chapter 1 that two mechanisms which might be responsible for mixing states of different K values are gamma-tunneling [Cho 88], [Ben 89d] and Coriolis mixing. Coriolis mixing involves the mixing together of wave-functions with the same deformation but different orientations of the nuclear shape with respect to the angular momentum vector. Gamma-tunneling involves large-amplitude collective motion in which the shape of the nucleus changes.

The distinction between the two modes of K-mixing is an empirically testable one; Coriolis mixing should manifest itself in systematic variations in K-mixing as a function of configuration and rotational frequency, while gamma-tunneling models predict that K-violating decays should show strong variations in strength as a function of the height and shape of the barrier in the potential energy surface near $\gamma=-60^\circ$. As discussed in chapter 1, the recent observations of highly K-violating transitions in some nuclei would seem to require for their explanation either a drastic change in Coriolis mixing or the introduction
of a gamma-tunneling mechanism. Both concepts will be discussed in light of the present data, and at the end of this section an attempt will be made to place the results in perspective and lay out a conceptual framework for how both models could be combined into a more comprehensive picture.

7.6.2 Calculations of Coriolis mixing

An explanation previously suggested for the $^{174}$Hf results [Wal 90] is that a change in Coriolis mixing occurs because of the change in the structure of the yrast states from $g$-band to $s$-band configurations. The mean $K$-value of the $s$-band states is zero, but because the two quasi-neutrons are in a coupling scheme aligned perpendicular to the symmetry axis, admixtures of various $K$ values are to be expected in the $s$-band wave-function, ranging from 0 to approximately $\pm K_{\text{max}}$, where

$$K_{\text{max}}^2 = j_{\text{max}}^2 - i^2 \quad (7.2)$$

Here $j_{\text{max}}$ is the maximum spin to which the two neutrons can be coupled subject to the Fermi exclusion principle, and $i$ is their rotational alignment. The idea was that the previously observed highly $K$-violating decays can be explained as decays due to Coriolis mixing, with essentially normal values of the hindrance per degree of $K$-forbiddenness, $f \sim 100$, but with a reduced degree of $K$-violation,

$$v \rightarrow v' = v - K_{\text{max}} \quad (7.3)$$

Although decays are also observed to the $g$-band states, such decays are attributed in this model to mixing of the $g$-band and $s$-band configurations in the band-crossing region. This gives rise to the following predictions: (1) there should be a strong correlation between the hindrance factor and the amount of $s$-band admixture in the final state; and (2) for self-consistency, the observed hindrance factors should
not require for their explanation an admixture of \( K = K_{\text{max}} \) in the s-band wave-function with an amplitude of more than 100%. Fig. 7.7 shows a test of the predictions of this model for the two \( 14^+ \) isomers. Although the hypothesis was a reasonable one to account for the \(^{174}\text{Hf}\) data, the addition of the \(^{176}\text{W}\) data to the plot reveals that there is essentially no correlation between the s-band admixture and the hindrance factors, and furthermore the hindrance factors in \(^{176}\text{W}\) are orders of magnitude too low to be explained with even the most drastic assumption of a 100% admixture of \( K = K_{\text{max}} \). This is obviously a highly schematic treatment of Coriolis mixing; the objective was not to calculate the effect in detail but simply to search for the expected trends in the data and evaluate whether the orders of magnitude were in the right range. An entire family of particle-rotor models has been developed for nuclear physics which in theory can calculate Coriolis mixing, but these models are unable to address the issues raised by the decay of a K-isomer because (1) technical difficulties have prevented them from being used for states with more than one quasiparticle; and (2) it is always found that an ad hoc reduction in the Coriolis matrix elements must be introduced, whose physical origin is obscure, and the predictive power of such models for large \( \Delta K \) is therefore doubtful.

7.6.3 Calculations of Gamma-Tunneling

The possible explanation of these decays in terms of gamma-tunneling is now discussed. A schematic picture of the mechanism is given in Fig. 7.8. Dynamical motion in the shape degrees of freedom of nuclei is often very difficult to treat theoretically; as discussed in section 1.1, small-amplitude shape vibrations are usually very strongly coupled to the single-particle degrees of freedom. It is for this reason that
Fig. 7.7: Comparison of the Coriolis mixing calculations with experiment. The x-axis shows the minimum calculated hindrance factor, under the most extreme assumption that 100% of the s-band wave-function has $K=K_{\text{max}}$. 
Fig. 7.8: Schematic picture of the gamma-tunneling model. The thick lines show the potential energy as a function of \( \gamma \), and the thin lines show the wave-function.
tunneling processes can be extremely important to study, because they represent a possible simplification of the dynamics. When a nucleus is tunneling through a classically forbidden region along a path parametrized by a deformation $\gamma$, one expects that exponential attenuation will occur on a scale $\delta \gamma \propto (V-E_0)^{-1/2}$, where $E_0$ is the energy eigenvalue of the state, and the potential, $V(\gamma)$, is defined as the minimum adiabatic value of the energy, $E_{\text{adiabatic}}$, at any value of $\gamma$. Even for the rare cases mentioned above where the decay of a K-isomer occurs with $f$ much less than 100, the total hindrance factor $F$ is on the order of $10^4 - 10^6$ or more, so if a tunneling mechanism is responsible for these decays, the attenuation is very severe. Any motion through non-optimal single-particle configurations with $E>V$ should be extremely strongly attenuated, with the attenuation length reduced to $\delta \gamma \propto (E-E_0)^{-1/2}$. Tunneling processes, then, may be the only type of collective nuclear motion other than rotation for which an adiabatic approximation is justified. For an adiabatic quantum-mechanical tunneling process, the only quantities needed to calculate the tunneling probability are the potential, $V$, and the inertial parameter, $D$, which corresponds to the mass in the Schrödinger equation. Essentially, $D$ measures how much mass has to move around when there is a change in $\gamma$. This will be discussed in more detail below.

In the present work, the potential energy is calculated as a function of $\gamma$ for the $K=14$ isomers in $^{174}\text{Hf}$, $^{176}\text{W}$ and $^{178}\text{W}$ with the cranked Nilsson-Strutinski method, using the computer code described in ref. [Ben 90] (see also chapter 2 and appendix B). In these calculations, the deformation parameters $\varepsilon$ and $\varepsilon_4$, as well as the static pair gaps $\Delta_p$ and $\Delta_n$ were all varied self-consistently, i.e. the gamma-
tunneling path is assumed to be the path of steepest ascent to the saddle-point and steepest descent from the saddle-point. The cranking frequency was varied to provide a constant value of \( \langle J_x \rangle \), and particle-number projection was employed. A step-size of 10° was used for \( \gamma \). Considerable variation was found in \( \epsilon_4 \) as a function of \( \gamma \), with values ranging from 0.00 to 0.05, and it was found that the potential energy surfaces changed significantly if self-consistent variation of \( \epsilon_4 \) was not carried out. The parameters \( \epsilon \), \( \Delta_n \) and \( \Delta_p \) showed variations as functions of \( \gamma \) which remained within about 10% of their average values. The calculated potential energy curves are shown in Fig. 7.9. The following is a summary of some of the calculated parameters:

<table>
<thead>
<tr>
<th></th>
<th>( \langle J_x \rangle )</th>
<th>( \pi )</th>
<th>( V_B )</th>
<th>( \epsilon(\gamma=-120^\circ) )</th>
<th>( \epsilon(\gamma=\gamma_B) )</th>
<th>( \epsilon(\gamma=0^\circ) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>176W</td>
<td>14</td>
<td>+</td>
<td>2.5</td>
<td>.25</td>
<td>.23</td>
<td>.25</td>
</tr>
<tr>
<td>178W</td>
<td>14</td>
<td>-</td>
<td>3.1</td>
<td>.26</td>
<td>.22</td>
<td>.24</td>
</tr>
<tr>
<td>178W</td>
<td>14</td>
<td>+</td>
<td>3.0</td>
<td>.24</td>
<td>.23</td>
<td>.26</td>
</tr>
<tr>
<td>174Hf</td>
<td>14</td>
<td>+</td>
<td>3.4</td>
<td>.26</td>
<td>.25</td>
<td>.26</td>
</tr>
</tbody>
</table>

where \( V_B \) is defined as \( V(\gamma=\gamma_B)-V(\gamma=-120^\circ) \), and \( \gamma_B \) refers to the saddle-point.

In an adiabatic model, the information on the dynamics is contained in the inertial parameter. These dynamics are not presently understood thoroughly enough to allow a calculation of this quantity from first principles. One fact which seems to come out in almost all the microscopic approaches [Ben 89d], [Bar 90] is that superfluidity in nuclei has an important effect on the inertial parameters, with a
Fig. 7.9: Potential energy curves for $^{176}\text{W}$, $^{174}\text{Hf}$ and $^{178}\text{W}$ as a function of gamma.
functional form $D-\Delta^2$. (Two different prescriptions for combining the contributions of the proton and neutron pairing have been proposed [Ben 89d], [Bar 90].)

Another approach is to take a phenomenological functional form for the inertial parameter and to fit adjustable parameters to the available information on properties of nuclei. The results of such an approach [Möl 81] fall within the range of values calculated from the microscopic methods.

It is worth noting, however, that the starting point for such a fit in the approach of ref. [Möl 81] is in the observed transition rates for tunneling processes, the process being the decay of the fission isomers. From this point of view, experiments involving tunneling processes constitute experimental measurements of the inertial parameters, and the inertial parameters may be extracted from the data.

Such a program might be a reasonable one to pursue for K-isomers in the long run, but it first must be established whether or not a gamma-tunneling approach can provide a reasonable description of the data. This is the more modest goal of this chapter; as will be explained in more detail, the results will be calculated in a form which factors out most of the effect of the inertial parameter, and the final results will be inspected to see whether they can approximately reproduce the data for a reasonable choice of the inertial parameter.

The WKB approximation for the tunneling probability, $T$, is

$$T = \exp\left[-\frac{2\hbar^{-1}}{\sqrt{2D(V-E)}}dy\right], \quad (7.4)$$

The tunneling probability $T$ gives the squared amplitude of the part of the wave-function localized in the potential well around $\gamma=0$, which is interpreted as the admixture of the $K=0$, $J=14$ component in the wave-
function of the J=14 state which has predominantly K=14. The absolute normalization of the transition rate (see appendix C) is given by \( B_0 \), which is the strength of the transition from the K=0, J=14 component to the final state. This quantity is estimated as \( B_0 = 0.03-0.3 \) W.u. for M1 transitions and \( B_0 = 10-100 \) W.u. for E2 transitions. These estimates are chosen to cover the typical range of strengths for unhindered transitions. The hindrance factor for the K-violating transition is then

\[
F_{\text{theor}} = T^{-1} B_0^{-1} \tag{7.5}
\]

which can be compared with the measured quantity

\[
F_{\text{exp}} = t/t_w \tag{7.6}
\]

An estimate of the zero-point energy was obtained by calculating the energy for Gaussian trial wave-functions \( \Psi(\gamma) \), with the Gaussian width used as a variational parameter. Since the isomers are \( \sim 1 \) MeV higher in energy than the yrast states with the same spin, it is assumed that the dominant contribution to the decay comes from tunneling from the minimum at \( \gamma = -120^\circ \) to the one at \( \gamma = 0 \), rather than from tunneling in the opposite direction.

As explained above, the philosophy adopted here is one of extracting the inertial parameter \( D \) and comparing it with various a priori estimates. It will be assumed that \( D \) is constant with respect to \( \gamma \). There are two ways in which the inertial parameter enters into the calculated hindrance factors. The more important one is the explicit dependence on \( D \) in eq. (7.4), but the zero-point energy \( E \) also depends on \( D \) (\( E \sim D^{-1/2} \) for a harmonic well). Neglecting the latter for the moment, simple algebra yields

\[
\ln F = -D^{1/2} \times A - \ln B_0 \tag{7.7}
\]

where the quantity
\[ A = -2\hbar^{-1} \int \sqrt{2(V - E)} \, d\gamma \]
is independent of $D$. In other words, if one constructs a log-log plot of $F(\text{exp})$ vs $F(\text{theor})$, then the slope is proportional to $D^{1/2}$, while changing $B_0$ shifts the plot horizontally. This is essentially the method used here to extract values of $D$ from the data, although the dependence of $E$ on $D$ has also been taken into account.

To describe consistently the variations in the tunneling probabilities from nucleus to nucleus, it is necessary to recognize that there will be a certain systematic variation in the inertial parameter from nucleus to nucleus, depending on the deformations and the pair gaps. All microscopic models seem to predict a variation of the inertial parameter proportional to $\Delta^{-2}$, and variation with the deformation is expected [Mol 81] to give a proportionality to $\varepsilon^2$. The following correction has therefore been made:

\[ \ln F(\text{theor}) \rightarrow \ln F(\text{theor}) \times f(\varepsilon,\Delta) \]  

(7.8)

where

\[ f(\varepsilon,\Delta) = \left( \frac{\varepsilon}{\varepsilon^{(0)}} \right) \times \left( \frac{R_A}{R_A^{(0)}} \right)^{1/2} \]  

(7.9)

The value of $\varepsilon^{(0)}$ is fixed at .25. Since the inertial parameter depends on both $\Delta_p$ and $\Delta_n$, a reasonable functional form for $R_A$ must be chosen. Motivated by the method used in ref. [Ben 89d] to calculate the inertial parameter, the form chosen here is

\[ R_A = \frac{\Sigma(G_i \Delta_i^{-2})}{\Sigma G_i} \]  

(7.10)

where the index $i$ refers to neutrons and protons, and the pairing strengths are $G_n = (18 \text{ MeV})/A$ and $G_p = (21 \text{ MeV})/A$. (It should be noted, however, that other authors [Bar 90] have proposed somewhat different functional forms for this dependence on the pair gaps.) Since the calculations do not show very large variations in the pair gaps as a
function of $\gamma$, the pair gaps have been taken simply from the odd-even mass differences [Jen 84]. The quadrupole deformations $\varepsilon$ are taken from the calculated values at the saddle point. The reference value of $R_\Delta$, written $R_\Delta^{(0)}$, is fixed to be 1.21 MeV$^{-2}$, the value for $^{174}$Hf. A summary of the dynamical quantities for the $14^+$ states is as follows:

<table>
<thead>
<tr>
<th>nucleus</th>
<th>$E_{zpm}$ (MeV)</th>
<th>$\varepsilon/\varepsilon^{(0)}$</th>
<th>$(R_\Delta/R_\Delta^{(0)})^{1/2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{174}$Hf</td>
<td>0.53</td>
<td>0.96</td>
<td>1.00</td>
</tr>
<tr>
<td>$^{176}$W</td>
<td>0.43</td>
<td>0.90</td>
<td>0.94</td>
</tr>
<tr>
<td>$^{178}$W</td>
<td>0.44</td>
<td>0.90</td>
<td>0.95</td>
</tr>
</tbody>
</table>

Here $E_{zpm}$ refers to the energy due to zero-point motion in the well around $\gamma=-120^\circ$.

Before comparing theory and experiment for the $14^+$ isomers, a test of the model was performed with a series of $6^+$ isomers in the same region. This is because data are available for a large sample of nuclei having $6^+$ isomers decaying through transitions of the same multipolarity, while data for the $14^+$ decays are available only for a few nuclei. The decays of the $6^+$ isomers to the $K=0$, $4^+$ states of the ground-state bands are therefore an excellent test for establishing whether the model has predictive power. The results of the calculations are shown in Fig. 7.10, where it clear that there is a correlation between the calculated and observed hindrance factors. The correlation between $F^{(\text{exp})}$ and $F^{(\text{theor})}$ is not significantly affected by, e.g., the specific choice of the functional form of $R_\Delta$, although the slope of the line is subject to some change, introducing some ambiguity in extracting an exact value of
Fig. 7.10: Test of the gamma-tunneling explanation for $K=6$ states in the rare-earth region. The dashed line represents an inertial parameter $D=30\text{ MeV}^{-1}\hbar^2\text{rad}^{-2}$. The horizontal error bars show only the uncertainty due to the choice of the unhindered strength $B_0$ (see text).
D. Subject to this uncertainty, the inertial parameter required to fit the data is $D \sim 30 \text{ MeV}^{-1}\hbar^2\text{rad}^{-2}$, which is of the right order of magnitude compared to the macroscopic estimate [Möl 81] of $\sim 13 \text{ MeV}^{-1}\hbar^2\text{rad}^{-2}$.

Returning now to the $K=14$ isomers, the results for $^{176}\text{W}$, $^{178}\text{W}$ and $^{174}\text{Hf}$ are shown in Fig. 7.11. As with the $6^+$ isomers, the tunneling calculations (assuming positive parity for the $^{178}\text{W}$ isomer) predict the right trend in the systematics of the hindrance factors. If negative rather than positive parity is assumed for the $^{178}\text{W}$ isomer, then the hindrance factor for the decay of this state to the g-band is extraordinarily low, and not reproduced by the calculations. It would therefore be interesting to have a reliable determination of the parity of this state. The extracted inertial parameter is $D \sim 60 \text{ MeV}^{-1}\hbar^2\text{rad}^{-2}$. Remembering that the absolute hindrance factors span many orders of magnitude, it is encouraging that the $6^+$ and $14^+$ data can be reproduced with values of the $D$ which are of the right order of magnitude compared to the estimate of ref. [Möl 81]. The increase with spin is also physically reasonable, due to the decreased pair gap and larger number of adiabatic configuration changes. It should be re-emphasized at this point that the goal here was not to extract an accurate value of $D$, but simply to find whether the systematic variations in the hindrance factors could be understood using a physically reasonable choice of $D$. The fitted value of $D$ is quite sensitive to some of the phenomenological assumptions, such as the form of the function $f(\epsilon,\Delta)$. Nevertheless, the inertial parameter is a quantity of great physical interest, and the results given here are encouraging as an indication of the future possibilities for extracting inertial parameters from this type of data, with appropriate refinements to the model.
Fig. 7.11: Test of gamma-tunneling for the 14+ isomers. The dashed line represents an inertial parameter \( D = 60 \text{ MeV}^{-1}\hbar^2\text{rad}^{-2} \). The horizontal error bars show only the uncertainty due to the choice of the unhindered strength \( B_0 \) (see text). The data-points with arrows pointing upwards represent lower limits on the hindrance factors.
Although the gamma-tunneling model seems to give a much better description of the data than the Coriolis mixing calculations, it is worth asking how different the underlying mechanisms described by these two models are, and how both mechanisms could be placed within a broader context. It is also important to understand how the geometric variables such as γ, which are defined within a specific model, can be related to quantum-mechanical observables. As discussed in refs. [Fri 87] and [Fra 91], the quadrupole operator Q_{2\mu} has five degrees of freedom, which can be mapped onto β, γ, and three Euler angles. Of the three Euler angles, one gives only a rotation about the angular momentum vector J, and therefore is only a phase which is unimportant for the dynamics. The other two Euler angles can be given as polar angles θ and φ which define the orientation of the angular momentum vector with respect to the principal axes. Then

\[
\begin{align*}
K &= J \cos \theta \\
J_x &= J \sin \theta \cos \phi \\
J_y &= J \sin \theta \sin \phi
\end{align*}
\]  
(7.11)

Note that everything has been defined in terms of Q and J, which are quantum-mechanical observables. Of the four degrees of freedom β, γ, θ and φ, gamma-tunneling proceeds through γ, and Coriolis mixing through θ. States with different γ values are kinematically coupled by zero-point motion, but states with different θ values have different values of the approximately conserved K quantum number, and are only mixed by the Coriolis force. The angle φ is ignorable for states with axial symmetry (γ=0 and -120°), and calculations for γ≠0,-120° suggest that it is either always zero [Ros 92] or that the potential energy surface is very soft with respect to φ, even in cases where the equilibrium lies at
$0<\varphi<90^\circ$ [Fri 87]. The $\varphi$ degree of freedom has therefore been neglected in this work.

It is now possible to discuss more precisely the approximations and limitations involved in the current calculations, both those which are inherent in a collective picture and those which are simply a result of the limitations of the current state of the art in calculations. First, cranking models as currently practiced involve only cranking along a single principal axis. This allows states with $K=0$ and $K=J$ to be described in a natural way, but allows for less precision in describing states with $0<K<J$. In cranking calculations of the properties of rotational bands, it is usually assumed that the $K$ value of the intrinsic configuration remains constant as a function of increasing rotational frequency, and this question is avoided. In tunneling calculations, however, the motion carries the nucleus through a series of different configurations, and no such simplifying assumption is possible. When calculating the decay of a state with $K=J$ to a state with $K=0$, it is at least a self-consistent assumption to use only principal-axis cranking, since both the initial and final states have their angular momenta aligned with a principal axis. This assumption corresponds to taking the entire space of trial wavefunctions for the variational parameters $\{\gamma, \vartheta ; \varepsilon, \varepsilon_4, \Delta_n, \Delta_p, \ldots\}$ and restricting the calculations to the sub-space $\{\gamma ; \varepsilon, \varepsilon_4, \Delta_n, \Delta_p, \ldots\}$. Historically, a similar approach was taken for calculating the decay of fission isomers, one of gradually increasing the dimensionality of the space. An obvious next step in the area of $K$-isomers is to expand the calculations to include the coordinate $\vartheta$, and it would then be possible to calculate estimates of the decays of a $K$-isomer to the states with
intermediate values of $K$.\footnote{The measured upper limits (table 5.2) on the intensities of such decays for the 70-ns isomer in $^{176}W$ give $f_{\text{int}}=11.0, 8.9, 34.4, \text{and } 43.2$ for $K_{\text{final}}=4, 6, 7, 8$, respectively. It should be noted, however, that these are all E1 transitions, except for the one with $K_{\text{final}}=6$, and values of $f$ for E1 transitions can be misleading, since the relevant unhindered strengths are difficult to estimate, and are usually many orders of magnitude less than 1 W.u. [Löb 68].} Such an approach is currently being pursued [Fra 82], [Fra 91], [Dön 89], [Dön 90a], [Dön 90b], [Dön 90c], but difficulties have been encountered in techniques, interpretation, and computational tractability, and the approximations used to carry out the calculations are currently the subject of controversy. A full treatment of the motion in both $\gamma$ and $\vartheta$ would also make it possible to examine the effects on $K$-mixing of incomplete assaults on the barrier, which may enhance the mixing for small values of $\Delta K$. 
Chapter 8
Conclusions

8.1 Summary

Until the last decade, it appeared that a comfortable systematic understanding of K-violating transitions had been achieved [Löb 68], with universal hindrance factors of about 100 per degree of K-forbiddenness. More recent data [Wal 90], [Cho 88], including the present work, have shown that there are sometimes extreme deviations from the previously established systematics. In particular, the results presented in chapter 5 show that a K=14 isomer exists in $^{176}$W for which these extreme deviations are the only detectable mode of decay, and the more typical modes of decay, following the path of minimum possible K-violation, are not observable at all. This appears even more unusual when compared with the decay of the K=14 isomer, with the same configuration, in the neighboring nucleus $^{174}$Hf. These isomers show similar behavior in their decays, but the ΔK=14 transitions in $^{174}$Hf are hindered by factors that are two orders of magnitude greater. In chapter 7, simple models of Coriolis mixing and gamma-tunneling were employed, to see whether either could account for these observations even to within orders of magnitude. It was found that the $^{176}$W decays had hindrance factors which were simply too low to be explained in a reasonable way by invoking changes in the mechanisms of Coriolis mixing, which had previously been suggested for interpreting the $^{174}$Hf data. Calculations in a gamma-tunneling framework, which were developed and carried out as part of this thesis, however, provide a
natural explanation for the marked differences between the $^{176}$W and $^{174}$Hf decays.

8.2 The Future

Possible directions for future investigations of high-spin isomers and $\text{K}$-violation are summarized in this section.

In the area of theoretical work, the traditional cranked shell model is beginning to show its age. The basic model, with principal-axis cranking, no angular momentum projection, and the independent quasiparticle approximation, has given birth to a several variants, all of which share its limitations. Two areas can be identified in which these limitations are becoming a bottleneck on the interpretation of the experimental data:

(1) A wider assortment of structures is being observed both at higher spins and higher excitation energies, many of which are difficult to interpret with the standard cranked shell model, either because of the presence of non-rotational collectivity, or a need for greater detail in describing the couplings of the different $\Omega$ values, or [Wal 91] possible new physics such as Fermi-alignment.† Some of these limitations have been noted in chapter 7, and the experimental results presented here are at most a very small sample of the complexity of the spectroscopic data now available or soon to be available from arrays such as Gammasphere and Eurogam.

† Fermi-alignment is a newly proposed coupling scheme for nucleons, intermediate between the deformation-aligned and rotation-aligned schemes, in which the spin precesses around an axis which is in a direction intermediate between the deformation axis and the axis of collective rotation.
(2) Whatever role gamma-tunneling may play in some highly K-violating decays, there is a clear necessity for a practical model which can address the issue of dynamical motion in the shape degrees of freedom. The Generator Coordinate Method [Bon 90a], [Bon 90b], [Bon 91] seems to be reaching the stage of maturity where it will soon be able to compete in the high-spin arena, and this promises to be a very fruitful development.

If the gamma-tunneling mechanism for the decay of K-isomers continues to become more firmly established, then a second area for future investigation is the cross-fertilization of these results with studies of the decay of high-spin superdeformed bands [Jan 91], [Shi 92]. Both processes involve tunneling in the $\beta$-$\gamma$ plane at high-spins, but many of the experimental and theoretical difficulties encountered in the case of high-spin superdeformation are not present in the case of K-isomers, for which the decay is not statistical. The development of gamma-tunneling models in the decay of K-isomers could be a useful stepping-stone between the decays of the low-spin fission-isomers and the high-spin superdeformed bands. The K-isomer results can be compared with theory in more detail because of the simpler nature of the final states.

A related question is the possible effect of K-conservation on the decay of high-spin superdeformed states. The corresponding effect in fission isomers has been studied fairly thoroughly [Bjø 80]. There, the gamma branch, through the inner barrier, proceeds through shapes which are not axially symmetric, and K-conservation is therefore probably not important for these decays, but the decay through the outer barrier to fission proceeds through axially symmetric shapes, and the
barrier height is increased by an amount known as the "specialization energy" because of the constraint to pass through configurations with a fixed value of $K$. The corresponding question for the decay of high-spin superdeformed bands is only beginning to be studied [Shi 92], [Bon 90b].

A final area for future activity is the question of the role of the $K$ quantum number in regions of higher spin (or rotational frequency) and excitation energy (or temperature). Coriolis mixing with large $\Delta K$ proceeds through high-order perturbation theory in the operator $\omega_j x$, and such perturbation expansions in many-body fermionic systems experience discontinuities at level-crossings, the first of which, in the case of $\omega_j x$, are the $g$-band/s-band crossings. This is just a restatement of the particular model of Coriolis mixing [Wal 90] in the crossing region discussed in chapter 7. It is logical to wonder whether at sufficiently high rotational frequencies, the Coriolis mixing might become so strong that $K$ would not even be a useful label for states in rotational bands.

The significance of the $K$ quantum number at high temperatures has also been a subject of recent theoretical and experimental investigations. Although some of the theoretical predictions show a transition to chaotic behavior at energies as low as 1-2 MeV above the yrast line [Mat 92a], [Mat 92b], certain experimental data [Rek 90], [Han 91], [Rek 91] have been interpreted as showing evidence that electromagnetic transitions still show signs of $K$-selection at energies as high as 8 MeV above the yrast line. There have even been claims [Sar 90] of experimental evidence for nuclear structure effects in proton evaporation spectra at energies at or near the threshold for particle evaporation, but an attempt to reproduce such a finding in similar, nearby nuclei has not yielded similar results [Blu 91]. It is clear that the
existence of any quantum number, such as K, which may be conserved or partially conserved will tend to postpone until higher temperatures the onset of chaotic behavior, and it may even be possible for some minority of the states to retain well-defined structure in an environment of chaos, if conservation of such a quantum number prevents mixing between the two sets of states. The lack of a consensus implies that there is ample room for future work in this area.
Appendix A
The Response Function of the Yale BGO Array

A.1 Measuring the Response Function

The BGO array is intended to measure the number of gamma-rays emitted and the total energy released in the gamma-decay following a fusion reaction. Calibrating the detector, therefore, would ideally be done by releasing, inside the array, a known number of gamma-rays with known energies. One would then measure the fold, or number of detectors that fire, and the sum-energy deposited in the array.

Since there is no controlled high-multiplicity source of gamma-rays with known energies, a method, using radioactive sources emitting two $\gamma$ rays in coincidence, was used to determine the response of the array. A separate monitor detector, in this case a large-volume unsuppressed Ge detector, is employed in combination with the BGO array. The system is triggered by the observation of one of the two $\gamma$ rays in the Ge detector, which implies the second $\gamma$-ray has been emitted. By measuring the response of the BGO array to this second $\gamma$-ray, the absolute efficiency of the array can be measured. This method can also be extended, as explained below, to allow the determination of the response of the array to a cascade of many $\gamma$ rays. The basic techniques have been outlined previously [Jää 83], but several new techniques were also developed for the present array and electronics, which differ from those used in the previous work. The following table lists the gamma-rays used:
<table>
<thead>
<tr>
<th>SOURCE</th>
<th>TRIGGER</th>
<th>RELEASED INTO BGO</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{207}$Bi</td>
<td>1770 keV</td>
<td>570 keV</td>
</tr>
<tr>
<td>$^{88}$Y</td>
<td>1836 keV</td>
<td>898 keV</td>
</tr>
<tr>
<td>$^{60}$Co</td>
<td>1333 keV</td>
<td>1173 keV</td>
</tr>
<tr>
<td>$^{60}$Co</td>
<td>1173 keV</td>
<td>1333 keV</td>
</tr>
<tr>
<td>$^{88}$Y</td>
<td>898 keV</td>
<td>1836 keV</td>
</tr>
</tbody>
</table>

For convenience of notation, the standard symbols $H$ and $K$ will be used in this appendix for the BGO sum energy and BGO fold, since there is little risk of confusion with the $K$ quantum number.

Fig. A.1 shows $H$ spectra for these $\gamma$-ray energies, with a single gamma-ray. The probability $P(K=x)$ that the BGO fold will take on the value $x$ is shown in Fig. A.2. The data for $K=1$ and $K=0$ provide a measure of the efficiency of the array, and $K=2$ events occur because gamma-rays Compton-scatter from one detector to another.

To simulate the response to a cascade of multiplicity $M$, a series of $M$ events can be concatenated, and the energy deposited in each detector is summed over the events. A difficulty in the simulation of cascades is presented by the non-linearity of $K$. In general,

$$K^{(\text{event }1+\text{event }2)} \neq K^{(\text{event }1)} + K^{(\text{event }2)}$$  \hspace{1cm} (A.1)

This non-linearity is due to two effects:

1. Two photons may deposit energies $E^{(1)}$ and $E^{(2)}$ in the same detector, with both of the individual energies below the threshold for detection, but with the sum $E^{(1)}+E^{(2)}$ rising above the threshold.
Fig. A.1: BGO sum energy spectra from radioactive sources.
Fig. A.2: BGO fold from radioactive sources, when a single gamma-ray is emitted from the center of the array.
(2) It is also possible to have both $E^{(1)}$ and $E^{(2)}$ above threshold, with the resulting contribution to $K$ being 1 rather than 2.

The latter is particularly important for large $M$. Because of these non-linear effects, the contribution of a given detector to $K$ must be defined for the purposes of this measurement according to a threshold with respect to the total histogrammed energy, rather than according to whether the CFD fires during the individual events.

The main quantities of interest are the first two moments of the distributions of $H$ and $K$, $\langle H \rangle$, $\sigma(H)$, $\langle K \rangle$, and $\sigma(K)$. Since $H$ is linear with respect to summing over events, the central limit theorem implies that for sufficiently large $M$ the distribution of $H$ becomes a normal (Gaussian) distribution, and the moments $\langle H \rangle$ and $\sigma(H)$ suffice to characterize the distribution of $H$ completely. Empirically, it is found that the distribution is very nearly Gaussian even for small $M$. The moments of $H$ for a cascade of multiplicity $M$ can be determined from their measured $M=1$ values according to the relations

$$\langle H(M) \rangle = \langle H(M = 1) \rangle \times M \quad (A.2)$$

$$\sigma(H(M)) = \sigma(H(M = 1)) \times \sqrt{M} \quad (A.3)$$

which are exact due to the linearity of $H$. For $M=1$, typical values are

<table>
<thead>
<tr>
<th>$E_\gamma$</th>
<th>$\langle H \rangle$</th>
<th>$\sigma(H)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>898 keV</td>
<td>550(9) keV</td>
<td>400(17) keV</td>
</tr>
<tr>
<td>1836 keV</td>
<td>940(14) keV</td>
<td>830(10) keV</td>
</tr>
</tbody>
</table>

Note that the values of $\langle H \rangle$ and $\sigma(H)$ are calculated for the entire distribution of the random variable $H$, which includes Compton scattering, and a spike at $H=0$ from events in which no BGO detector fired. If one calculated $\langle H \rangle$ and $\sigma(H)$ for the photopeak only, then we would have much
larger values of $\langle H \rangle = E_\gamma$ and much lower values of $\sigma(H)$, but this would not be a consistent way of treating $H$.

For $K$, the natural distribution with which to compare is the binomial distribution $B(N,p)$, where $N=38$ is the number of detectors and $p=\langle K \rangle/N$. That is, one could imagine a model of the response of the array in which the firing of each BGO detector constituted a statistically independent trial with a probability $p$ of success; the BGO fold, $K$, would then have, by definition, the binomial distribution $B(N,p)$. The idea of a similarity of the $K$ distribution with a binomial distribution motivates a consideration of the transformation of $K$ to the variable $K'$, defined as

$$K' = -N \ln \left( 1 - \frac{K}{N} \right)$$

(Equation A.4)

For small $K$, the variable $K'$ has the property

$$K \rightarrow K'$$

(Equation A.5)

and for $K$ having a binomial distribution,

$$K'(\langle K^{\text{event 1 + event 2}} \rangle) = K'(\langle K^{\text{event 1}} \rangle) + K'(\langle K^{\text{event 2}} \rangle)$$

(Equation A.6)

Systematic deviations from the binomial distribution are to be expected due to anti-correlations between detectors. These anti-correlations arise because the conditional probability that detector $i$ will fire, given that detector $j$ has fired, is lower than the unconditional probability, since detector $j$ is 80-100% likely to absorb the entire $\gamma$-ray energy. For example, imagine for a moment that the array covers 100% of $4\pi$ and that a single gamma-ray is emitted into the array, with an energy so low that there is a negligible chance of Compton scattering. We have $\langle K \rangle = 1$ and $\sigma(K)=0$, since exactly one detector is guaranteed to fire, but for $\langle K \rangle = 1$, the binomial approximation gives $\sigma(K)=0.987$. This is because the binomial
approximation assumes that it is still possible for any BGO to fire, even given the fact that another one has fired.

Despite these anticorrelation effects, equation (A.6) for $\langle K \rangle$ is found to be extremely accurate when compared to the results of simulated cascades. A comparison is given in Fig. A.3, showing that equation (A.6) is nearly exact to within statistical errors. The binomial approximation to $\sigma(K)$ is unfortunately not as good. It is found in practice (Fig. A.3) that for $E_\gamma$ greater than about 0.9 MeV, $\sigma(K)$ is approximately 10-20% less than the binomial value, which is $\sqrt{Np(1-p)}$. The reduction becomes as much as 40% for $E_\gamma \sim 0.6$ MeV, for which Compton-scattering between detectors is less likely.

One can therefore conclude that analytic calculations give accurate results for $\langle H \rangle$, $\sigma(H)$, and $\langle K \rangle$, while the simulation of cascades of $\gamma$ rays is necessary for calculating $\sigma(K)$, and for providing a pictorial representation of the distributions of $H$ and $K$.

An important advantage of the analytic method lies in the simplicity of (1) the subtraction of the Compton background in the Ge spectrum under the photopeak on which a trigger is made; and (2) the subtraction of random coincidences. In the analytic method, these subtractions are relatively simple to carry out for the $M=1$ moments, and no further treatment is necessary for higher multiplicities. In the method of simulated cascades, two difficulties arise.

First, the subtraction of Compton background in the Ge trigger becomes complex (see section A.2), since for large $M$ it may be subject to extremely large statistical fluctuations unless high statistics and very good peak-to-background ratios are available.
Fig. A.3: Top: additivity of the BGO fold, compared with the analytic approximation ($E_{\gamma}=0.57$ MeV). The error bars on the y-axis are all less than 0.3 units of $\langle K \rangle$. Bottom: Statistical spread in the BGO fold, compared with the analytic approximation ($E_{\gamma}=0.57$ and 0.90 MeV). The error bars for $y$ are all less than 0.03 units of $\sigma(K)$. 
The second difficulty is related to the removal of the effects of random coincidences. As explained above, it is not possible, due to the non-linearity of $K$, to exploit the information from the timing of the BGO pulses relative to the firing of the Ge trigger detector when simulating the distribution of $K$. Furthermore, the gain of the detection system depends strongly on the degree to which the analog pulse overlaps the charge-integration window, so that the effects of random coincidences on $H$ cannot be taken into account by any background-subtraction procedure involving the subtraction of events falling in a time-random window from those falling in a prompt coincidence window.

The combined effect of these two difficulties is to allow the use only of sources with strengths in a relatively restricted range ($\sim 0.1$ to $\sim 1 \mu$Ci) for the method of simulated cascades using charge-sensing ADC's. The methods used for background-subtraction and removal of random-coincidence effects are explained in sections A.2-3.

In Fig. A.4 are shown distributions of $K$, calculated using the method of simulated cascades, for $\gamma$-ray energies of 1173 and 1836 keV and multiplicities 10, 20, and 40. In Fig. A.5, the $K$ resolution $\sigma(K)/K$, for $M=15$, of the 38-element Yale BGO array is compared with that of the 71-element array at Chalk River. A reduction in resolution for 38 detectors as opposed to 72 detectors is observed. The reduction in resolution will mainly be important at very large multiplicities ($M \geq 30$) which would tend to saturate the smaller Yale array, making the sensitivity $dK/dM$ small.
Fig. A.4: BGO fold from the method of simulated cascades.
Fig. A.5: BGO fold resolution compared to that of the Chalk River BGO array.
A.2 Background-Subtraction

For the M=1 calibrations, the background-subtraction procedure is relatively simple. BGO data are obtained by gating on the relevant photopeak in the Ge spectrum. These data contain contributions both from the photopeak, which we label P, and from the Compton background underneath the photopeak, which we label B. To remove the effects of Compton background, one can set a gate on the continuum next to the photopeak and subtract, giving \( P = (P + B) - B \). Now imagine the situation for \( M=2 \), where two events are concatenated. There are two Ge gates to set, each of which has both terms, \( P + B \). The data gated on both Ge photopeaks therefore provide four possible combinations, \( PP + PB + BP + BB \), which may be written symbolically as \( (P + B)^2 \). To remove the effects of background, one must construct \( [(P + B) - B]^2 = (P + B)^2 - 2B(P + B) + B^2 = P^2 \). Note that the middle term carries twice the weight of the other two terms, which magnifies its statistical fluctuations by a factor of 2.

To generalize this discussion, a series of \( M \) events, when concatenated, will contain a certain number of events, which we label \( P \), due to the complete absorption of the triggering \( \gamma \)-ray in the Ge detector and a certain number of events, labelled \( B \), due to triggering on the continuous part of the spectrum. Two gates can be created, one surrounding the photopeak in the Ge spectrum, and another nearby gate of equal width which includes only Compton background. Events in the first gate are labelled \( P + B \), and events in the second gate are labelled \( B \). The background-subtraction to be carried out is then

\[
[(P + B) - B]^M = \sum_{j=0}^{M} \binom{M}{j} (-1)^j B^j (P + B)^{M-j} \quad (A.7)
\]
where B and P+B stand for ensembles of events, and products of these symbols are to be understood as ensembles of concatenated events in which the given number of events in the P+B and B gates are present. If the peak-to-background ratio (P+B)/B is much greater than M, then the sum converges very quickly, but if the peak-to-background ratio is not favorable, then the extremely large values of the combinatorial factor for large M can cause the sum to consist of many positive and negative terms, many of which are astronomically large. Even for lower values of M, high peak-to-background levels and high statistics are needed for the method to work, requiring long runs and the use of strong sources.

A.3 Random Coincidences

Due to the non-linearity of the definition of K, timing information from the CFD’s cannot be used to remove the effects of random coincidences in the method of simulated cascades. The resolving time for random coincidences is therefore approximately equal to the width, w, of the charge-integration gate. Most of the radioactive sources commonly used for calibration in γ-ray spectroscopy have strengths of about 10 μCi, causing an average of ~1 random coincidence per event for w=800 ns. Sources with strengths of ≤1 μCi must therefore be found for simulating of cascades.

The difficulties involving random coincidences can be completely avoided when determining the moments of H using the analytic method. The effects of random coincidences depend on both the number of random coincidences and on the gain of the ADC as a function of how well the BGO pulse overlaps with the charge-integration gate. Both of these effects have been calculated in closed form as part of this work, and the following
analytic results apply for the contributions of random coincidences to the moments of $H$:

$$
\langle H_{\text{random}} \rangle = R \tau_\mu \left( \langle H^{(1)}_{\text{prompt}} \rangle + \langle H^{(2)}_{\text{prompt}} \rangle \right) \quad (A.8)
$$

$$
\sigma^2(H_{\text{random}}) = R \tau_\sigma \left[ \sigma^2(H^{(1)}_{\text{prompt}}) + \sigma^2(H^{(2)}_{\text{prompt}}) \right]
+ \left( \langle H^{(1)}_{\text{prompt}} \rangle + \langle H^{(2)}_{\text{prompt}} \rangle \right)^2 \quad (A.9)
$$

where the superscripts (1) and (2) refer to the two $\gamma$-rays, and $R$ is the absolute rate of radioactive decays in the source. The constants $\tau_\sigma$ and $\tau_\mu$ have the dimensions of time, and give characteristic time-scales over which the system is sensitive to random coincidences. If the output pulse of the BGO detector was a delta-function, instead of an exponential, then we would have $\tau_\sigma = \tau_\mu = w = 800$ ns. For exponential pulses, they become slightly different from $w$:

$$
\tau_\mu = \frac{w}{1 - \exp[-a(w - h)]} = 970 \text{ nsec}
$$

$$
\tau_\sigma = \frac{w - a^{-1}(1 - e^{-aw})}{1 - \exp[-a(w - h)]} = 710 \text{ nsec} \quad (A.10)
$$

where $a$ is the exponential decay constant of the output pulses from the BGO detectors and $h$ is the time between the start of the gate and the leading edge of the pulse. These results are exact except for angular-correlation effects between the two $\gamma$ rays emitted in a random coincidence, which should be very small due to the nearly complete coverage by the BGO array of the solid angle surrounding the source. The linear equations (A.8-9) can be solved for the moments of $H_{\text{prompt}}$ in terms of the measured moments of $H_{\text{prompt}} + H_{\text{random}}$. Measurements with a variety of sources of varying strengths verifies the consistency of these equations to within statistical errors for sources of various strengths, and the results are also consistent for sources with strengths so low that the effects of random coincidences are negligible.
For the moments of $K$, data are collected for sources of two different strengths but composed of the same isotope. The quantities $K'(\langle K \rangle)$ and $\sigma^2(K)$ are then extrapolated linearly with respect to $R$ to find the values appropriate for $R \to 0$, thereby removing the effects of random coincidences.
Appendix B
Notes on the Calculations

B.1 Extensions to the Models

The mean-field approach as described in chapter 2 has several parameters which must be adjusted to fit a nucleus or a region of nuclei. These include the deformation parameters and the pair gaps. If one wishes instead to predict these parameters, one must construct trial wave-functions $|\Psi\rangle$ and minimize the total energy of the nucleus $<\Psi|H|\Psi>$ with respect to the parameters. Two difficulties are encountered when doing this in the cranking formalism. First, the trial wave-functions are constructed at a given rotational frequency, but the minimization should actually be done for a given angular momentum, since it is the angular momentum which is a constant of the motion and a quantum-mechanical observable. Second, the total energy cannot be calculated correctly by adding up the single-particle energies, since the replacement of the two-body interaction $V_{ij}$ with the one-body mean-field $V_i$ results in double-counting when summing the single-particle energies, $<T_i>+<V_i>$, over the index $i$. The errors in the predicted energies are also amplified when summing over all of them, as opposed to summing over a few valence particles [Rin 80]. For the calculations performed here, the first difficulty is avoided by interpolating in $\omega$ to construct states with a given average angular momentum $<J>$ [Ben 90], and the second difficulty is avoided by using Strutinski smearing [Str 67]. Good explanations of Strutinski smearing are available in the literature [Rin 80],[Nil 69], and only a short and qualitative description will be given here. Imagine adding up the energies of a set of filled orbitals in a nucleus, starting with all the orbitals
empty and then putting particles into orbitals one by one, adding each one's energy to the sum. The energy is a function of the particle number, $E=E(N)$, and contains a smoothly varying part, which could be approximated, for example, by a polynomial. It also contains a part which fluctuates quickly with $N$, depending on which valence orbitals are near the Fermi level. The idea of Strutinski smearing is to take the smooth part of $E(N)$ from the liquid drop model, and then to calculate a shell correction $\delta E$, so that $E=E_{\text{liquid drop}}+\delta E$. The shell correction is calculated by first summing up the calculated single-particle energies to get $E_0$, and then randomly perturbing all the energy levels so as to obliterate (smear) the shell structure, and calculating an average energy $E_{\text{no shell}}$ summed over the possible perturbed energy spectra, finally giving $\delta E=E_0-E_{\text{no shell}}$.

Two further improvements were used in these calculations:

- the automatic elimination [Ben 89a] of virtual interactions between quasiparticle states, defined as interactions which are artifacts of the cranking approximation
- the use of Strutinski renormalization in the calculation of $\langle J \rangle$ as well as $\langle H \rangle$ [Ben 75], which compensates for an unintended effect of the Nilsson $V_{\ell\ell}$ term on the average moment of inertia at high spins [Ben 89b]

The total energy and angular momentum at a given rotational frequency are calculated as [Lan 91]

$$E_{\text{tot}} = E_{\text{RLD}} + \delta E_{\text{shell}} + E_{\text{pair}}$$
$$J_{\text{tot}} = J_{\text{RLD}} + \delta J_{\text{shell}}$$  \hspace{1cm} (B.1)

where $E_{\text{RLD}}$ and $J_{\text{RLD}}$ are the energy and angular momentum for a rotating liquid drop with the rigid-body moment of inertia, and the Strutinski-smeared quantity $\delta X$ is defined, for a one-body operator $X$, as
the difference \(<X>-\langle X\rangle_{\text{Strut}}\) between its ordinary expectation value and its Strutinski-smeared expectation value.

**B.2 The Nilsson and Woods-Saxon Models**

The calculations of the potential energy curves in chapter 7 were carried out using a cranked Nilsson code [Ben 90], which incorporates the features described above. (A simpler version of the code has been described in ref. [Lan 91].) The only place in this thesis in which a different type of calculation was used was for the discussion of the 2-quasiparticle bands in chapter 7. There, the more elaborate features described in the previous section were not needed, since the calculations were being carried out at a fixed shape, and a Woods-Saxon code [Cwi 87] was chosen, because the Woods-Saxon results are generally slightly more accurate [Ben 89b], and a universally applicable set of input parameters is available [Dud 82] which avoids some of the worries associated with choosing the input parameters for a Nilsson calculation [Cro 92].

A higher-order expansion than the one given in chapter 2 for relating the Nilsson and Woods-Saxon deformation parameters is

\[
\varepsilon = 0.944\beta - 0.122\beta^2 + 0.154\beta\beta_4 - 0.199\beta_4^2
\]
\[
\varepsilon_4 = -0.852\beta_4 + 0.141\beta_4^2 + 0.122\beta\beta_4 + 0.295\beta^2
\]

(B.2)

Ref. [Ben 89b] gives a more complete investigation of the correspondence between the Nilsson model and the Woods-Saxon model, and ref. [Bjø 80] gives a summary some other sets of deformation parameters commonly encountered in the literature.
Appendix C  
Electromagnetic Properties

The electromagnetic properties of states in nuclei are often far more sensitive indicators of the structure than the energy levels. Gamma-ray transition rates in nuclei are usually expressed in terms of either the reduced matrix elements, $B(\lambda)$, or Weisskopf units. $B(\lambda)$ values are proportional to the square of the electromagnetic matrix element of the transition connecting the initial and final states, and include a factor which removes the variation of the strength with gamma-ray energy. Values in Weisskopf units (W.u.) are proportional to those expressed as $B(\lambda)$, but the systematic variations with respect to the size of the nucleus (i.e., the spatial extent of the single-particle wave-functions) are also removed. The result is a typical strength for a single-particle (non-collective) transition. The partial half-lives for one-W.u. transitions are

$$t_w(E\lambda) = (1.90 \times 10^{-22} \text{sec}) \kappa_\lambda A^{-23/3} \left( \frac{164 \text{ MeV}}{E_{\gamma}} \right)^{2\lambda+1} \quad (C.1)$$

$$t_w(M\lambda) = (3.255)A^{2/3}t_w(E\lambda) \quad (C.2)$$

where

$$k_\lambda = \left( \frac{\lambda}{\lambda + 1} \right) \left( \frac{3 + \lambda}{3} \right)^2 (2\lambda + 1)!!^2 \quad (C.3)$$

Experimentally, the most easily extracted electromagnetic quantities for a rotational band with two signature partners linked by M1 transitions are the ratios $B(M1)/B(E2)$ for the M1 and E2 transitions depopulating the same state, which can provide useful information about the structure of the states. The ratio is given by
Here I and E are the intensities and energies of the gamma-ray transitions. Donau and Frauendorf [Dön 83] (see also [Hil 89]) have derived a simple theoretical expression for these ratios in a semi-microscopic approximation,

\[
\frac{B(M1)}{B(E2)} = \frac{(0.693 \text{ MeV} \mu_N^2 / e^2 b^2) I_v(M1)}{I_v(E2)} \times \frac{E_g^2(E2)}{E_g^2(M1)}
\]

(C.4)

where

\[
B(M1)/B(E2) = \frac{(16/5) Q_t^{-2} \times (1-K^2/(J-1/2)^2)^{-2} \times (K/J)^2}{(1\pm\Delta e'/\hbar \omega)^2 \times \{(g_{\text{coup}}-g_R)[(J^2-K^2)^{1/2}-i_{\text{coup}}] \}}
\]

(C.5)

\[
J^2 \rightarrow (J-1/2)^2
\]

(C.6)

is a first-order quantum correction. The factor involving the signature splitting $\Delta e'$ is to account for the oscillation of the magnetic dipole moment between the states $K$ and $-K$. The resulting $B(M1)$ values are increased (decreased) for the transitions with $E_\gamma = \omega - \Delta e'$ ($E_\gamma = \omega + \Delta e'$). The subscript "coup" refers to strongly-coupled quasiparticles with their angular momenta precessing around the deformation axis, and the subscript "dec" refers to quasiparticles whose angular momenta have been decoupled from the deformation axis by the Coriolis force. The rotational alignments of the two types of particles are denoted by $i_{\text{coup}}$ and $i_{\text{dec}}$. The collective rotational $g$-factor $g_R$ is often approximated as $Z/A$, but can be derived more accurately from cranking calculations [Rin 80]; the latter method will be used in this work.

The transition quadrupole moment $Q_t$ is
\[ Q_t = (0.01 \text{ b}/\text{fm}^2) (9/5\pi)^{1/2} Z e R_0^2 \beta \times f(\beta, \gamma, \beta_4) \times (V/V_0)^{-5/3} \]

where the factor for normalizing the volume is given by

\[ V/V_0 = 1 + 0.238 \beta^2 \]

and

\[ f(\beta, \gamma, \beta_4) = \cos \gamma - 0.577 \sin \gamma - 0.360 \beta + 0.720 \beta \cos^2 \gamma + 0.294 \beta \cos \gamma \sin \gamma \\
+ \beta_4 [0.965 \cos \gamma - 1.396 \sin \gamma (5/6 \cos^2 \gamma + 1/6) \\
+ (0.403 \sin \gamma - 0.164 \cos \gamma) \sin 2\gamma - 0.543 \sin^3 \gamma] \]

(C.7)

(derived as part of this work; see also [Rin 80] for the volume factor). Terms of order $\beta^3$ and $\beta_4^2$ have been neglected in the expansion of $f$, and to this order, the effect of the volume normalization factor is negligible. As noted in ref. [Naz 81], the correct method for combining $\beta_4$ and $\gamma$ deformations is not trivial. The calculations in this thesis use the definition given in ref. [Lan 91], which corresponds to parametrization [A] in ref. [Naz 81].

Another case is that of two bands which are not signature partners, and which interact through some matrix element. A situation of this type is encountered in section 7.2, where the goal is to extract the interaction matrix element between the g-band and s-band, for later use in models of Coriolis mixing. The experimental information available is the branching ratio of the in-band and out-of-band E2 transitions from an initial state of angular momentum $J$ to final states with angular momenta $J-2$ in both bands. If the g- and s-components of a state with angular momentum $J$ are denoted by

\[ g_J = \langle g | J \rangle \]
\[ s_J = \langle s | J \rangle \]

(C.8)
then the following equations hold:

$$g_j s_j^2 = 1$$

$$g_j s_j = \frac{V}{(E - E')^2}$$

where \( V \) is the interaction matrix element between the two bands, and the primed and unprimed symbols distinguish the two states which are mixing. If the deformations of the two bands are different, then the change which is introduced is

$$\left( \frac{s_j s_j' - \eta g_j g_j'}{s_j s_j' - g_j g_j'} \right)^2 \rightarrow \left( \frac{s_j s_j' - \eta g_j g_j'}{s_j s_j' - \eta g_j g_j'} \right)^2$$

where \( \eta = \beta_g / \beta_s \), and \( \beta_g \) and \( \beta_s \) are the equilibrium deformations of the \( g \)-band and \( s \)-band configurations.

For a discussion of the different definitions of the electromagnetic matrix elements used in the literature, see ref. [Kra 73].
Appendix D

Technical Issues in Discrete Spectroscopy

D.1 Coincident Intensities

Some of the essential quantities measured in discrete gamma-ray spectroscopy are the coincident intensities between pairs of gamma-rays in a cascade. This appendix describes some of the standard methods used for analyzing this type of data. The raw coincident intensities, which are the number of counts in the coincidence photopeaks in the $\gamma$-$\gamma$ matrix, are extracted by fitting the gated spectra with Gaussian peaks and a smooth background, using the program GELIFIT [Rad 92]. The fitted areas must be corrected for the efficiency of the Ge detectors. Gamma-singles data are collected for calibration purposes with radioactive sources, and the fitted centroids and areas are used to derive a linear energy calibration, and an efficiency curve of the form

$$\ln \text{eff} = [P(x)^G + Q(x)^G]^{-1/G}$$

(D.1)

where $x = \ln(E_\gamma/E_0)$ and P and Q are quadratic polynomials which determine the efficiency at low and high energies. The widths of the photopeaks are calibrated using the functional form

$$w_E^2 = R(E_\gamma)$$

(D.2)

where $w_E$ is the FWHM and R is a quadratic polynomial. All coincident intensities were derived from gaussian fits with the widths fixed to the correct values from this calibration.

In order to extract coincident intensities between photopeaks, it is also important to subtract the contribution from the background under the
photopeaks. The background is composed partly of Compton-scattered gamma-rays and partly of a large number of weak photopeaks which are below the limits of the resolution of the detectors. A slice taken from a $\gamma$-$\gamma$ matrix, which gates on a given photopeak, will also contain the spectrum in coincidence with the background underneath the photopeak. A standard simple and effective method for background-subtraction is as follows: It is found in practice that the spectrum in coincidence with background is essentially the same as the total projection of the gamma-gamma matrix, but scaled down by some factor. The projection is therefore scaled down by the appropriate factor, and subtracted channel-by-channel from the raw slice. This standard procedure is a simple and effective method for background-subtraction, and can actually be carried out once for the entire matrix.

**D.2 Corrections to Ge Timing**

Constant-fraction discriminators (CFDs) were used for both Ge and BGO timing. If all pulses have the same shape except for an over-all linear rescaling, then CFDs have the property that the timing does not depend on the amplitude of the pulse. In practice, it is found that low-energy pulses have slower rise-times. Regardless of how carefully the threshold and walk adjustments on the CFD are made, variations as a function of energy are observed both in the mean timing and the timing resolution [Cro 92]. This effect is demonstrated in Fig. D.1. Spectroscopy of isomers requires an accurate determination of the timing of the detected $\gamma$-rays, in order to distinguish the $\gamma$-rays emitted in the (usually rare) decay of an isomer from the profuse background of $\gamma$-rays emitted in a single prompt flash. The
Fig. D.1: Contour plot of $E_{Ge}$ vs $t_{Ge}$, showing the effect of pulse height on the timing of Ge detectors. The data histogrammed are from the $^{176}$Hf experiment in chapter 6. The changes in the centroid and width of the timing distribution are clearly visible for $E_{Ge}<100$ keV. The arrows show the position of the ±1.7σ sliding TDC gate for energies of 100 and 300 keV.
timing parameters for both BGO and Ge detectors were therefore corrected in software according to the rule

\[ t \rightarrow t + aE^b \gamma \]  \hspace{1cm} (D.3)

where \( a \) and \( b \) are fitted parameters. When setting a gate on the prompt-timing peak in the Ge TDC spectra, it is necessary as well to vary the width of the gate (and the width of the gate for subtracting random coincidences) to accept a fixed fraction of the peak, regardless of energy. (This effect is not subsumed in the efficiency calibration, for which there is no timing reference and therefore no time parameter.) The observed widths of the TDC peaks were fitted to the functional form

\[ w_t = cE^d \gamma \]  \hspace{1cm} (D.4)

and the widths of the gates were set accordingly (usually to cover the range of \( t_0 \pm 1.7 \) sigma) as a function of energy (see Fig. D.1).

**D.3 DCO Ratios**

Spin assignments in nuclear spectroscopy can be made by the measurement of angular distributions of gamma-rays or angular correlations between gamma-rays. These non-isotropic effects arise because the evaporation residues formed in heavy-ion reactions have their spins strongly aligned in the plane perpendicular to the beam. When the spectra are complex and information is required on very weak transitions, it is not possible to extract the necessary information from gamma-singles, and the angular correlations must be measured from gamma-gamma coincidence data. This is known as the DCO method, for Directional Correlations from Oriented states, discussed in refs. [Kra 73], [Krä 89], [Ye 91] and [Bar 87]. Although in principle these correlations may be extremely complicated [Bar 87], [Kra 73], simplifying approximations can
be made when the multipolarities $\lambda$ of the transitions involved are small compared to the nuclear spins $J$. First, even when there are intervening transitions between the observed ones, the resulting deorientation effect is negligible [Kra 73]. Second, the observation of one $\gamma$-ray perturbs the distribution of magnetic substates, causing angular correlations with the other $\gamma$-ray, that is, a joint angular distribution which is different from the uncorrelated product of the two angular distributions. For $\lambda \ll J$, this effect is small [Bar 87]. It is therefore possible to construct ratios of sums of coincident intensities which are mainly sensitive to the multipolarity of only one of the two gamma-rays. These are known as DCO ratios [Bar 87], [Kra 73].

Such a ratio has been defined in ref. [Ye 91], for the Argonne/Notre Dame Ge array used in the $^{176}$W experiment described in chapter 5, as

$$R_{DCO}(\gamma_1) = I_{\gamma_1\gamma_2}(fb,fb)/I_{\gamma_1\gamma_2}(s,fb)$$

(D.5)

The labels $fb$ and $s$ refer to the detection of the $\gamma$-ray in the forward/backward or side rings of the array, at $\theta=34.5^\circ/145.5^\circ$, or $90^\circ$, respectively. The two coincident intensities are summed over all possible combinations of detectors. Measuring $R_{DCO}$ allows a distinction to be made between dipole and quadrupole transitions (see fig. 5.8).
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